REYER GERLAGH

THE EFFICIENT AND SUSTAINABLE USE OF ENVIRONMENTAL RESOURCE SYSTEMS

The efficient and sustainable use of environmental resource systems

Reyer Gerlagh

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When I started studying econometrics and mathematics at the Vrije Universiteit, I found myself not unexpectedly in the field of mathematical economics. I was offered a student-assistent job for Prof. Gerard van der Laan. His gentle supervision provided valuable guidance through my student years.

After finalizing my Master's thesis for econometrics under supervision of Michiel Keyzer in 1992, it took another year to finalize the thesis for mathematics. In 1993, I came to work at the Institute for Environmental Studies (IVM), where Harmen Verbruggen offered me the opportunity to carry out the theoretical research that has led to this Doctoral dissertation. I am still grateful for this, since the primary focus of the institute is directed more towards applied analysis.

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1. INTRODUCTION

1.1. INTRODUCTION

At the end of the 20^{th} century, the world is confronted with several global environmental problems, such as deforestation, stratospheric ozone depletion, and climate change, that are generally attributed to human intervention. They constitute unprecedented challenges to the stability and resilience of ecological systems on earth. At stake is nothing less than the earth's capacity to support human life with the supply of food, fresh air, and favorable environmental conditions.

Yet current environmental policies seem inadequate to ensure an efficient and sustainable use of the environmental systems. In some instances, 'strong sustainability' policies are implemented that physically constrain the use of the environmental resources in order to guarantee the conservation of a high environmental quality. Such policies could, however, impose unnecessary limitations on the present use of the environment, and cause inefficiency. In other cases, environmental resources have been privatized to stimulate their efficient use, or are managed publicly in conformity with market conditions. Still, this cannot prevent the irreversible deterioration of the resources, thereby putting the welfare of future generations at risk, and cause unsustainability.

One reason for the inadequacy of policy interventions may be the present lack of scientific understanding of the environmental processes at play [Holling 1986]. The climate change issue is a case in point. The debate in this field has focused on reductions of greenhouse gas emission [UNFCCC 1998], but the size and speed of reduction that is required remains a matter of great controversy. While loss of biodiversity is expected to constitute the major damaging consequence, present knowledge, even of the importance of the various processes, is limited [IPCC 1996a, Section 6.2.13] [IPCC 1996b, Section 1.3.2]. This results in policy recommendations that are based on highly subjective perspectives. Those who consider experiments with the resilience of the earth unacceptable, see the high stakes as sufficient reason to demand strong sustainability checks on environmental resource use. Their opponents find the worries about environmental resource systems overstated, and play down the issue in several ways. Some argue that massive use of environmental resources is only temporary, and that technological innovation by itself, and without strict policies, will move the economy towards environmental-extensive production. Others go further and deny the influence of antropogenic environmental resource use on global phenomena such as climate change, finding support in recent findings that exogenous fluctuations such as irregular solar radiation partly explain the phenomena.

Not only are there great scientific uncertainties in terms of the physical and biological processes involved, economics offer no appropriate recipe for environmental policies either, and the debate on the proper definition of sustainability continues. Some consider an allocation sustainable if and only if environmental resources do not deteriorate, and call for a strong sustainability policy, while others find an allocation sustainable if and only if social welfare does not decrease over time. The latter definition is hard to operationalize, since it requires the specification of social welfare aggregation over different consumers within a period, but it has the advantage that the policies it leads to need not conflict with efficiency. Nonetheless, the economic literature provides only few examples of policies that are able to combine both efficiency and sustainability [Mourmouras 1993].

The lack of an adequate theoretical basis for environmental policy leads to the use of so called 'integrated assessment models' (IAMs) that do not assure both efficiency and sustainability [Peck and Teisberg 1992], [Nordhaus 1994], [Manne *et al.* 1995], [Manne and Richels 1995], [Nordhaus and Yang 1996] [Tol 1996b]. The models follow the standard policy rules and either specify strict conservationist measures that constrain greenhouse gas (GHG) emissions, or calculate efficient emission reduction paths irrespective of unsustainability.

The present study aims at contributing to the theoretical basis of environmental policy by formulating economic rules and possible institutions that bring forward an optimal use of the environmental resources, while taking care of both efficiency and sustainability. It adopts the definition of sustainable development by the World Commission on Environment and Development, of a development "that meets the needs of the present, without compromising the ability of future generations to meet their own needs" [WCED 1987]. We notice that this does not require that all environmental resources remain intact, or that welfare increases over time.

This study also chooses a certain perspective on the nature of the environmental processes at play. It presumes that environmental concerns are to be taken seriously, and that public intervention is required. It views the agreements that have been reached for the protection of the ozone layer in Montreal in 1987, for the preservation of biodiversity in Rio de Janeiro in 1992, and for the control of climate change in Kyoto in 1998 as first steps that will have to be expanded in the future.

As a formal analytical framework, we use the concept of competitive equilibria [Arrow and Debreu 1954] that lies at the heart of the welfare economics literature. More specifically, we seek to include the environmental resources in the formal format of the competitive economy. This raises some difficulties, because it is not immediately clear whether environmental systems produce economic goods in the usual sense, with homogeneity properties and an unambiguous unit of measurement. For environmental services such as fresh air, the unit of measurement is not always obvious. Hence, it is not surprising that, though the IAMs referred to above claim to represent competitive economies, they lack markets for environmental resources, and property rights over these resources.

In addition, the analytical framework should allow for infinite time horizons, to account for the time horizons associated with the major environmental problems. Climate change, for example, has time delays between cause and effects that cover several centuries. Because of this, the environmental problems link various generations with one another that do not live at the same time. Therefore, our analysis will use a framework with infinite time horizon that links all present and future consumers through a sequence of overlapping generations.

However, to introduce the issue within such a framework, we need to characterize the issue and the context in more precise terms. To begin with, we have to take account of the various perspectives on environmental systems. There is disagreement on some of the system's properties, such as its resilience, but other properties are commonly accepted, such as the systems capacity to supply resource amenities over an indefinite time period. We also need to pay more attention to the normative context of the economic analysis, since the subject has a strong potential for friction between the efficiency and sustainability objective. Finally, the intertemporal and intergenerational aspects need some discussion, as they link the dynamics of environmental resources to the intergenerational distribution of welfare. This will enable us to motivate our approach, and to present the plan of this study.

1.2. DESCRIBING ENVIRONMENTAL RESOURCE SYSTEMS

1.2.1. Current environmental concerns

We will now seek to characterize the present environmental concerns, and to identify the characteristics of the environmental resource systems that are most relevant for our purposes. Most current environmental concerns date back to the 1960s and the early 1970s, when several studies were published that marked a turning point in the nature and scale of environmental problems. Before the 1960s, environmental problems were mainly perceived in relation to food production and population growth [Malthus 1798], resource shortage and industrial production [Jevons 1906], and local pollution by industry [Carson and Darling 1962].

The 1960s and 1970s elevated these concerns to a global platform. Ehrlich and Ehrlich [1970] argue that pollution caused by economic activities decreases potential food production, and thereby decreases the earth capacity to support human life. They explicitly discuss several environmental resource systems that are essential for human life, including the biogeochemical cycles underlying climate change, and conclude that the disturbance of the systems has to be contained without further delay. Forrester [1971] develops a model on world dynamics that includes the interactions between the environment, the economy, and demography, and predicts global disasters, unless stringent environmental policies were followed to reduce the impact of economic growth on environmental resource systems. The publication of 'Limits to growth', known as the (second) report to the Club of Rome [Meadows 1972] and subsequent studies continue this work.

The basic reason for present concerns is the awareness that a geometric increase of economic activities is incompatible with the limited carrying capacity of complex environmental resource systems. The prediction of a break down of the environmental systems, based on the extrapolation of past trends, is analogous to the arguments by Malthus, but the insights that emerged in the 1970s add three significant notions to the discussion. First, it is nowadays being stressed that the environmental problems have expanded to a global scale. Physical bounds of the earth limit the further expansion of activities of mankind. Secondly, life supporting systems are included in the analysis, fresh air to breathe, clean water to drink, which is a departure from the classic focus on the exhaustibility of resources. Life supporting systems are renewable in the sense that they have the capacity to regenerate themselves. Yet there exist certain thresholds, beyond which degeneration becomes irreversible. Thirdly, account is taken of natural delays in ecological processes. Any delay in the cause-effect chain between the release of a pollutant and its appearance in a harmful form, implies an equally long time before the policy to control that pollutant becomes effective in reducing its harmful effects. In other words, any pollution policy that is based on controls that become effective only when some harm has already been detected, cannot avoid a further deterioration of the situation before an improvement can be achieved. The destruction of the ozone layer is a good example. Though the emission of relevant pollutants has been reduced substantially since the Montreal protocol in 1987, it is expected that the ozone layer will not recover from its degeneration before the year 2010. Therefore, to prevent deterioration, ecological systems necessarily require actions that are based on forecasts, rather than on the measurement of actual environmental problems.

The shift in the perception of environmental problems has evolved gradually. Before the publication of the report to the Club of Rome, Barnett and Morse [1963] rejected, on the basis of historic data, the hypothesis that exhaustible resources such as metals had become scarcer. After the oil crises and the boom in commodity markets of 1972-1974, during which it was claimed that the depletion of resources could have catastrophic implications, the trends in natural resource and commodity prices from before the oil crisis continued, as if nothing had happened. The findings of Barnett and Morse were confirmed by later studies [Howe 1979], that find no substantial increase of scarcity for most minerals. As a result, fear of resource depletion slowly faded away.

This does not imply that exhaustible resource use is without problems. Quarries produce 'lunar landscapes', and remainders of ore pollute the local environment. Yet typically, environmental concerns are not about the exhaustible resource itself but its side-effects. Energy use and its associated environmental concerns are a good example. In the 1970s, these concerns focused on the exhaustion of fossil fuels, but at present, they are directed to the potential enhanced greenhouse effects of combustion [Cline 1992]. It is believed that the antropogenic emissions of CO_2 in particular and other so called greenhouse gases will probably increase the future surface temperature of the earth. This will also affect the weather conditions, and could lead to global climate changes.

Climate change is actually an environmental problem in *optima forma*. It has all the three characteristics mentioned that make its management so difficult. First, the enhanced greenhouse effect is a global problem. Secondly, the cause of the problem is described in terms of pollution of the biogeochemical system[1] that carries biological life, rather than in terms of depletion of an exhaustible resource. Thirdly, there is an immense time delay between the emissions of greenhouse gases and the appearance of various effects in the biogeochemical system. For this reason, climate change seems to offer an ideal case to illustrate the effects of various environmental policies, and to put to the test the environmental policy that we develop as an alternative for the current policies which either rely on strong sustainability oriented measures, or on efficiency oriented measures. As an introduction to illustrations in coming chapters, let us briefly characterize the current status and trends for the relevant climate change variables.

1.2.2. Climate change: current status and trends[2]

During the past two centuries, human activities have significantly altered the composition of the global atmosphere. The atmospheric content of natural constituents has changed and many new, man-made components have been added. It is becoming clear that the

[1] For convenience, we will use the term biogeochemical system to refer to the complete environmental system that determines regional and global climate. The biogeochemical system includes the atmosphere, the hydrosphere, the geosphere and the biosphere. It also includes cycles between living organisms and the various spheres, including biological, chemical and geological processes [Ehrlich and Ehrlich 1970] [Manahan 1994].

[2] An extensive and accessible article about climate change is [Chadwick and Sartore 1998].

consequences of these changes are serious. Of most importance are the enhanced greenhouse effect, stratospheric ozone depletion, acidification and smog. Of these four environmental problems, the enhanced greenhouse effect seems to be the most difficult to handle. Since the industrial revolution, atmospheric CO_2 concentrations[3] have risen by 25 per cent, from 280 parts per million by volume (ppmv) to more than 355 ppmv. Meanwhile, the concentrations of other greenhouse gases (GHGs) such as nitrous oxide (N_2O), methane (CH_4), ozone (O_3) and chlorofluorocarbon (CFC), as well as the concentrations of aerosols, have increased even more.

It is estimated that the present natural atmospheric insulation keeps the global average temperature of the earth's surface about 33 °C above the non-insulated level [IPCC 1990, p. xiv]. The current trend of GHG emissions leads to a doubling of the CO_2 concentrations in the atmosphere, relative to pre-industrial levels, in about forty years. Part of past emissions, the airborne fraction, remains in the atmosphere, and accumulates with present emissions. It is expected that the main consequences of current and past GHG emissions will occur within a period ranging from several decades to some hundred years. Global average temperatures are expected to rise by 1 to 4.5 degrees Celsius in the next century. However, even 'best estimate' projections have a large range of uncertainty. Large parts of ocean-atmosphere-biosphere interactions are not well understood, because of the many complex relations between the atmosphere, the oceans and terrestrial biota, and additional feedback systems [IPCC 1995].

It has been argued that antropogenic GHG emissions do not cause any enhanced greenhouse effect, and that recent global temperature rises can be attributed to irregular solar radiation [Lassen and Friis-Christensen 1995]. However, statistical analysis seems to reject the hypothesis of natural variability, and points to antropogenic GHG emissions as the most probable source for the current rise in temperature [Tol 1996b].

GHG emissions are also special because of their irreversibility. The natural cycles that constitute the atmospheric system have a capacity for regeneration. It seems that most of the emissions are being absorbed in one way or another and become harmless. For example, carbon is stored in trees and linked to the carbon content of the atmosphere through growth and decay of forests. The carbon in trees does not add to the greenhouse effect, and consequently, the biogeochemical system might be able to absorb large parts of the GHG emissions without effects for the atmosphere. Moreover, the biogeochemical system is at least partly a renewable system. On the whole, a complex system of interrelated cycles determines the system's capacity to cope with past, present and future emissions, and the limits on absorption are not well known. Yet detailed simulation

[3] CO_2 contributes more than half of the radiative force of all antropogenic greenhouse gases [IPCC 1995].

models suggest that the absorption capacity is limited [Maier-Reimer and Hasselman 1987], and that part of GHG emissions leads to irreversible and cumulative changes.

Besides the limits on absorption, a further reason why GHG emissions might lead to irreversible changes is that the climate system could possibly respond to cumulative emissions through sudden changes rather than gradual adjustments. A possible so called 'runaway' greenhouse effect cannot be ruled out. When temperatures start rising slowly, additional GHGs could be released by vegetation, oceans and soils. Then, through positive feedback relations, the greenhouse effect could become a self-accelerating phenomenon. However, it is also possible that the earth responds by absorbing more GHGs through shifts in ocean and terrestrial plant growth. In the latter case, the biogeochemical system is stable and human disturbances have limited effects only. Several additional feed backs between the biosphere and the climate system can be identified, for which it is not clear so far whether they enhance or slow down the changes.

The overall consequences of the enhanced greenhouse effect are not limited to rising temperatures. The changed biogeochemical system might also cause a change in global climate patterns. This human-induced climate change includes changes in the temperature and circulation of ocean water, water levels, ice coverage, moisture, precipitation, and changes in the occurrence of extreme weather events.

All this might have important negative consequences for the structure and functioning of ecosystems, and for human activities. We mention two examples. First, if the sea level rises by one meter, a consequence of thermal expansion and the melting of land ice, which is estimated to occur by the year 2100, 120 million people will be at risk of ocean flooding, compared to an estimated 50 million people at present. Many small island countries lose a significant part of their land area, or disappear completely. Secondly, a temperature increase of 3^0 Celsius is expected to increase the malaria incidence by 50-80 million people, and many more effects have been listed [IPCC 1996b, p.9-12].

It is difficult to quantify the projected impacts. Nonetheless, several economic 'integrated assessment models' have been developed that attempt to calculate optimal emission reductions, balancing present costs of GHG emission reductions and future damages if these reductions do not take place.

To conclude, climate change is a global phenomenon. The effects of GHG emissions will occur everywhere, and they do not discriminate on the location of emissions. Moreover, the biogeochemical system underlying climate change can be considered renewable up to some (unknown) threshold. Continued emissions will probably lead to irreversible changes, though there is much controversy on this issue. Finally, climate change is characterized by significant time delays. These three characteristics call for cautious management. However, it is not clear what type of caution, or good stewardship, is indicated in this case, because the course to be pursued depends fully on the perspective adopted as regards environmental stability.

1.2.3. Four perspectives on environmental stability

Large systems, such as the biogeochemical system, are often of such complexity that it is impossible to determine their stability features in an analytical manner. Moreover, it is not clear whether the representation of a real world system through an analytical (mathematical) model adequately represents the dynamic features of the real world system. This also applies to the issue of climate change, for which the appropriateness of model simulations can only be established in a distant future. There is no present evidence for a stable climate system, for an unstable system, or for a chaotic system, and if one of these characterizations applies for a certain model, this largely reveals the perspective of the model builder.

Given the uncertainties that cover the global climate system, the question becomes which policy strategy to choose if different models give different results. Natural scientists are unable to resolve this issue, and at present they seem to pursue 'a battle of perspectives' [Janssen 1996]. Let us characterize the main differences in perspectives by applying a typology developed by Schwarz and Thompson [1990]. Accordingly to this typology, different people have different perspectives on the stability and resilience of environmental systems, and consequently, on the required management to protect future generations. Interestingly, this difference is not restricted to scientific issues, but has manifest social and psychological dimensions.

There are two extreme perspectives, referred to as 'nature ephemeral' and 'nature benign'. The first perspective, symbolized in Figure 1.1, considers the dominant dynamics as unstable. In the figure, it can be seen that only a small push causes the ball to tumble off the hill. Under this perspective, minimal perturbation becomes the overriding moral imperative, and small becomes beautiful. Trials can go ahead only if it is certain that there will be no errors. From this perspective, the present scale of antropogenic GHG emissions is a global experiment with climate change which in the worst case might altogether deprive the global environment of its capacity of supporting human life, and has to be checked.

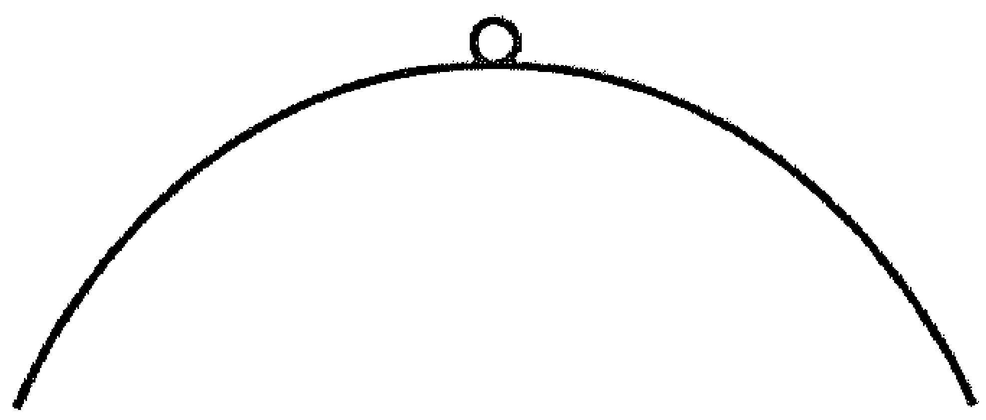

FIGURE 1.1. *'Nature ephemeral'* → *'Small is beautiful'*

Natural scientists who have studied species that became extinct, and ecosystems that were destroyed, might cling to this perspective. Destroying a rain forest can be a matter of years, restoring it will take centuries, if possible at all. The imperative for management is then to conserve as much environmental resources as possible.

The opposite perspective, symbolized in Figure 1.2, considers the dominant dynamics as stable. As can be seen in the figure, the ball will always return after a kick. There is no need for urgent action. A 'laissez faire' policy is naturally associated to this view. Researchers who adopt this perspective dispute the urgency of environmental problems. With regard to climate change, they argue that the most important GHGs, CO_2 and NH_4 are natural compounds of the atmosphere rather than pollutants, and that the biogeochemical cycles which drive the global substance flows will naturally absorb the gases without many risks. Another argument says that current emissions will decrease autonomously as a result of technological innovation [Chakravorty *et al.* [1997], and yet another that the worries for future generations are grossly overstated because future welfare from the consumption of man-made goods will exceed present welfare by a magnitude, providing more than adequate compensation for possible environmental losses [Schelling 1992].

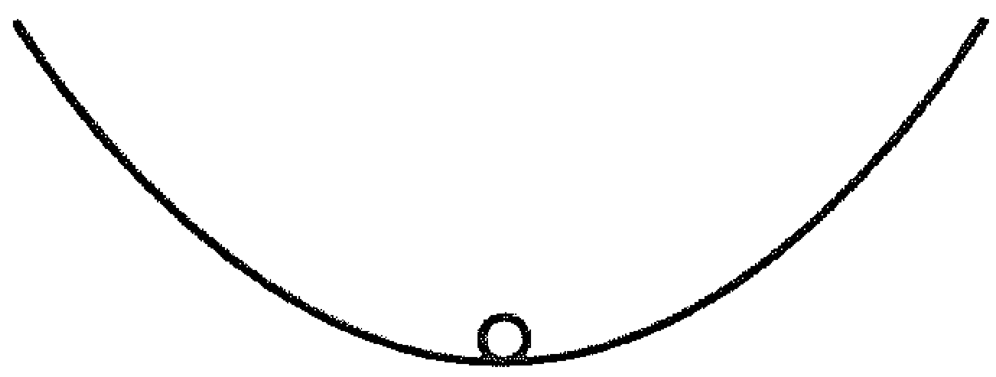

FIGURE 1.2. *'Nature benign'* → *'Laissez faire'*

Economists who were educated with the wisdom of the 'invisible hand' of the free market often choose this perspective, believing that markets are capable of guiding the distribution of scarce resources towards their optimal use, once they have been set up and property rights over resources have been distributed. Because of the assumed stability of the environmental system, a delay in actions does not cause insuperable problems, and it is thus not urgent to hasten the internalization of environmental resources in the market system.

In between the two extreme perspectives, Schwarz and Thompson [1990] distinguish two other. The third perspective considers the dominant dynamics as locally stable. The system can cope with small perturbations, but large impulses drive the ball over the edge, and thereafter the system will not return to the 'natural' state (Figure 1.3). In this perspective, the climate change policy requires scientific research to study the precise effects of GHG emissions, and to define 'safe corridors', development paths that produce no threat to stability of the climatic system.

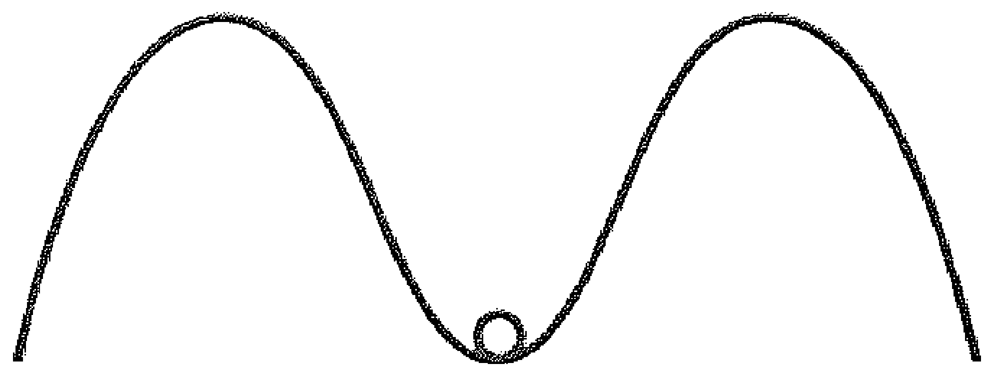

FIGURE 1.3. *'Nature tolerant/perverse'* → *'Hierarchical control'*

A hierarchical social ordering is naturally linked to this perspective. Strong social controls are required as well as a precise knowledge of the border between equilibrium and disequilibrium.

The fourth perspective is 'nature capricious', and relates to a chaotic system behavior. A chaotic system could be deterministic, but still unpredictable. The ball might roll to the right, but after another small kick, suddenly roll to the left, and thereafter to the right again (Figure 1.4). In this perspective, life is experienced as a lottery. All one can do is try to cope, as best as one can, with the prevailing situation. Small peasants, who do not use irrigation and greenhouses to create an artificial environment, might find the weather an unpredictable cause of plenty and shortages. At macro level, this perspective considers the climate itself as unpredictable, and climate change policy will consist of adapting to unforeseen changes with the means available. Any attempt to anticipate is almost pointless.

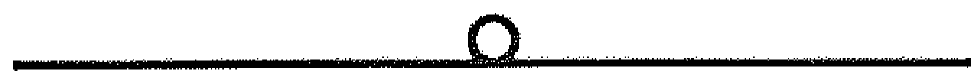

FIGURE 1.4. *'Nature capricious'* → *'Fatalism'*

To summarize, we described four different perspectives on the system stability, which are primarily associated to psychological attitudes. The present understanding of complex environmental resource systems, such as the biogeochemical system, is insufficient to discriminate between the perspectives. Too many different global climate variables are linked: GHG emissions, atmospheric concentrations, and absorption by biotic and abiotic media.

Implicitly, in most parts of this study, we adopt the third perspective of local stability within certain thresholds. In fact, it is hardly meaningful to study sustainability issues through economic analysis under other premises. The first perspective, of a truly globally unstable system, requires such cautious actions, that implementation of sustainability policies reduces to strong constraining measures, and it leaves no room for institutions that match social and individual objectives. In our view, this perspective underrates the capacity of ecological systems to adapt and recover from damages. The second perspective, of a globally stable system does not require sustainability policies, since in the long run the environment always returns to its natural level. This perspective, however, neglects the irreversibilities of some real world processes, such as species extinction. The fourth perspective of an unpredictable environment, gives no handles for policies other than short term adaptation. This perspective is valid for many phenomena at the micro-level, but at the macro-level, system dynamics typically persist within a relatively wide interval. Indeed, the environmental resources taken for illustrations throughout this study all have the characteristic that, if properly managed, they can produce an indefinite flow of valuable services, but if improperly managed, they can become over-exploited, and decline irreversibly.

1.3. MANAGING THE ENVIRONMENTAL RESOURCES

1.3.1. The early debates

Clearly, these perspectives on environmental stability have far reaching implications for the economic policy to be pursued. Malthus [1798] is an early exponent of the nature-ephemeral viewpoint. Subsequent nineteenth century writers often opted for a more 'benign' view. Malthus used a relatively simple argument to predict that food scarcity would become an ever lasting problem. He based his theory on two presumptions. First, he emphasized the necessity of a secure food supply for the existence of man, and supposed that food output could increase at most in a linear sequence, due to the law of diminishing returns. Secondly, he assumed that, though the 'passion between the sexes' is necessary for the survival of mankind, its intensity causes a geometric increase of population, and eventually population will exceed food supply.

Whereas Malthus postulated an approximately linear growth rate for food supply, Ricardo [1817] treated the issue of food scarcity as an economic question of optimal land use. He recognized that land is a heterogeneous resource, with differential fertility on the individual parcels of land. Assuming that the better lands would be used first, he found a continuous decline in land productivity, with increasing food demand resulting in increasing resource scarcity. Since the induced scarcity effects for the better lands is a gradual process, food production shows decreasing marginal returns to inputs of other production factors such as labor.

Godwin [1820] became the main opponent of these ideas at the time. He rejected the implicit doom laid on mankind by the hypothesis of autonomous population growth. In his view, this was based on unscientific reasoning, and in conflict with the empirical evidence that showed great diversity of growth rates of population in the world and fast changes in these rates. Godwin found no inherent arguments against a modest demographic growth.

He tried to develop a more scientific theory on the checks of population growth, based on historical evidence, rather than "referring everything to occult causes." In his opinion, the main checks on population growth were wars, pestilence and famine, which were not caused by population pressure *per se*, but were caused by bad government and underdevelopment. Accordingly, social development might keep society away from misery, and combined with good governance that would prevent wars, pestilence and famine, a modest optimal growth in population could result.

Mill [1848] further continued Ricardo's economic analysis of production, but more in line with Godwin's positive perspective, he added the notion that technological growth postpones food scarcity. Mill believed that the Malthusian "extensive limits of the earth" were not a controlling force, because Ricardian scarcity comes into play first, raising counter-forces to Malthusian scarcity. Mill argued that the principle of 'progress' compensates for diminishing returns. The increased scarcity of land induces an extension of knowledge that increases production. If additional (food) demand faces progressively harder terms, the law of increasing marginal production costs is suspended, or temporarily controlled, by whatever adds to the general power of mankind over nature. Furthermore, he observed that the power of population to increase geometrically was seldom, if ever, used to the utmost.

1.3.2. Social welfare as the aim of production

Mill's analysis already views economic activities as the means for achieving welfare. This approach is in contrast with the classic economic analysis which attributed the value of commodities to the use of scarce (physical) production factors such as labor, and addressed the environmental concerns in this perspective. In the beginning of the nineteenth century, economic analysis directed its attention to the utility consumers derive from their consumption [Kregel 1985], and this led to the so called 'neoclassical' principles. Bentham put forward the utilarian principle that the origin of value lies in the capacity of commodities to generate individual happiness or utility. At the end of the nineteenth century, the concept of utility as the end of economic production had become the corner stones of neoclassical economics [Marshall 1890].

The neoclassical principles are important for several reasons. First, if production is not an autonomous process, but only a means to an end, there is no reason to expect an extrapolation of past and current trends, because the system of production has no driving force of its own.

Secondly, the neoclassical principles offer a natural approach to analyze environmental resources and other production factors within a unified framework. If the value of economic goods stems from the consumers, who value their consumption goods, non-consumable goods are valued indirectly according to their contribution to the production process. In case a part of the consumption value cannot be attached to certain production factors, it is regarded as rent or profit. Hence, there is an accounting identity that keeps the value of total consumption equal to the value of all production factors plus rents and profits. Environmental resources, which are important production factors, can be included in the analysis in the same way as other production factors. The value of environmental resources depends on their contribution to the welfare of consumers.

Thirdly, the neoclassical principles give a normative content to economic analysis, as it becomes the objective of economic policy to increase utility of the consumers, with all the distributional complexities this entails.

The early neoclassical economists relied on mathematical analysis to study the optimal use of (natural) resources. Their analysis formally confirmed the classic idea of Adam Smith [1776] that optimization of social welfare is compatible with individual selfish behavior. In an economy with free markets, the benefit of the individual does not challenge the benefit of the society. Pigou [1920] proposed a correction of this principle for those cases in which production processes produce side-effects for others that are not taken into account by the agent producing the side-effects. Such unintentional linkages are known as externalities, and they often appear in relation to the non-exclusive use of environmental resources. Rivers are used both for drinking water, and for waste disposal, and if the effect of waste disposal on water quality is not taken into account, this produces an externality. Externalities distort the efficient functioning of the economy, because they obscure the optimal use and valuation of production factors. Pigou recognized the problem as a conflict between social and private interests. His solution calls for governments to implement economic instruments, e.g. taxes, that reconcile individual and social benefits, so that it becomes the advantage of the individual to act in the benefit of whole society. This is known as the internalization of externalities in the economy. In the example of competing river use, internalization can be reached by levying a Pigovian pollution tax for waste disposal, assuming that the drinking water being used is of such small quantities that this has no external effects. In the Pigovian tradition, many other so-called market failures have been identified subsequently, including open access to common resources, and market dominance by some economic agents, that require specific measures to reconcile individual and social benefits.

1.3.3. Efficiency and decentralization through markets

The classic and early neoclassic economists considered the maximization of social welfare and its distribution as the main objective of economic production. Social welfare was understood as a 'utilarian', i.e. unweighted, aggregate of individual welfare, and hence, equity was a positive part of the theory. Redistributive taxes, which transfer income from the rich to the poor, were positively accepted, because, it was believed, a marginal unit of income is of more value for the poor than it is for the rich [Edgeworth 1897]. In the beginning of the twentieth century, this perspective was challenged by the school of new welfare economics that disputed the comparability of utilities among consumers. Exemplary is the claim by Kaldor [1939] who states that distributional issues are not a subject for economic analysis, which should only be concerned with the optimization of production. The principles of new welfare economics are based on three value judgments, for which the basis was laid by Pareto [1927]:

(i) Each individual is the best judge of his own welfare.

(ii) The welfare of society depends on the individual welfare of its citizens.

(iii) If the welfare of one individual increases, without the decrease of welfare of others, the welfare of society increases.

The most pronounced common characteristic of these judgments is that they build welfare economics on an individualistic approach. The welfare of society is reduced to the sum of its parts, where individual preferences are assumed to be autonomous (i.e. independent), and incomparable. No paternalism is allowed; no individual may conclude for any one else what is in his or her best interest. To distinguish individual from social welfare, the former is often referred to as utility.

These value judgments make it possible to attribute normative content to the optimality conditions studied in neoclassical analysis. However, the three Paretian value judgments are of little practical use without an additional set of assumptions on individual behavior and preferences, because they relate utility and social welfare, but do not link utility to economic allocations. The following three assumptions make it possible to interpret different states of the economy in terms of different utility levels.

(i) Each individual behaves rationally

(ii) Only the end states determine the welfare of the individuals

(iii) There is no malevolence between individuals

The rationality assumption is a refinement of the first value judgment. Together, both state that each individual chooses the action bundle that yields the highest personal welfare. Thus, if a consumer can choose between two consumption bundles A and B, and he chooses A, this reveals that A yields higher utility than B.

The second assumption, on the 'end state', asserts the independence of welfare on the action by which an end state is reached. Because of this, the efficiency of the allocation mechanism is of primary concern, not the sovereignty of individuals. A dictatorial system is not ruled out, but by the first value judgment, a maximal social welfare is much easier in a system where individual freedom prevails.

The third assumption excludes perversities, whereby some individuals prefer other individuals to be worse off.

In general, an allocation based on rational and free choices potentially provides Pareto optimal solutions, that is solutions which do not allow for further increase of social welfare in the sense of value judgment (iii). The competitive equilibrium we are concerned with in this study is such an allocation. In this equilibrium, all goods carry a price, such that every consumer determines his supply and demand of commodities so as to maximize utility given prices and his income, and every producer maximizes profits at given prices and subject to his production possibility set, while aggregate supply matches with aggregate demand. The competitive equilibrium has gained widespread acceptance from the moment it has formally been described and formal proofs were given for its existence [Arrow and Debreu 1954]. Apart from the assumptions on the production possibility sets and preferences, a most significant property of the competitive equilibrium is that prices guide the economy to its equilibrium, where supply matches demand. Both consumers and producers take prices for given, but there is no mechanism that leads prices to their equilibrium.

The competitive equilibrium has become a key concept of economics, because of its role in the two fundamental theorems of welfare economics. The First Welfare Theorem states that (under suitable assumptions on the endowments and preferences of consumers and the production possibilities of producers) any competitive equilibrium yields a Pareto optimal use of resources, and the Second Welfare Theorem (under slightly more restrictive assumptions) that the inverse relation is also valid; every Pareto optimal allocation can be realized as a competitive equilibrium with income transfers [Debreu 1959, Theorem 6.3 and 6.4].

The first welfare theorem is of greatest importance; it claims that in order to ensure optimal resource use, it is sufficient to establish markets for all commodities, including environmental resources, and to ensure that all agents act as price takers. There should be no monopoly or other market dominance, but if this is successfully accomplished, then there is no need for any central control of resource use. The private interests will drive the economy towards an efficient equilibrium. This result marks another important step in normative economics. If welfare maximization in the sense of Pareto efficiency is the end of economic policy, then property rights and competitive markets are the means.

Within this line of thought, an alternative for Pigovian taxes consists of establishing property rights and markets for the environmental resources concerned. In this case, users pay the owners for their use, instead of paying taxes to the government. To continue with the example above, if the river used for waste disposal and drinking water is held as private property, the owner will maximize its value and will ensure that all users pay a fair price. Yet for the welfare theorems to apply, markets should function well. Dominance and monopolies are not allowed. Despite the limited circumstances under which all assumptions are satisfied, the implications are appealing, and they surely have supplied important argument in favor of so called free market solutions to environmental problems.

1.3.4. Equity and sustainability, the need for policy intervention

The Paretian principles do not allow the ranking of (Pareto) optimal allocations, and hence, they provide no basis for distributive policies. Moreover, they take the optimality of individual decisions for granted. This has led Koopmans [1957] to argue: "If beyond a certain stage above the bare subsistence level welfare is comparative rather than absolute, should not criteria of fairness in the distribution of opportunities rank above criteria of efficient allocation."

Notwithstanding the importance of the efficiency aspect, the economic system should also meet other concerns. It should be sustainable, for both present and future generations. This calls for additional value judgments.

The classic utilarian welfare function aggregates the unweighted welfare of all individuals, and maximizes this social welfare function. A well-known alternative welfare function maximizes the minimal utility level, known as the Rawlsian [1972] maximin criterion. However, the use of such a social welfare function does not ensure that all equity requirements are met. In dynamic economies with infinite horizons, it is not improbable that the optimization of an exogenous welfare function leads to an inequitable result [Pezzey 1992]. Hence, it is useful to combine a social welfare function with additional equity constraints.

Yet, in normative economic analysis, the concept of equity causes some problems. The first welfare theorem provides a clear handle for decentralizing Pareto optima through property rights and competitive markets, and in that context, the main task of the government is to provide an economic 'structure', that is to ensure that property rights are being respected, monopolies avoided, and so on. However, equity might also require the government to redistribute income, and in practice, the taxes that are used for this purpose often distort the competitive markets, causing inefficiencies. The government has to compromise between equity and efficiency, and as few general principles are available, it will have to rely on central control and the use of computable models.[4]

At this juncture, it is important to mention that the approach we propose to pursue in this study will view the sustainability problem as an equity problem, that is, it will treat the sustainability problem as in terms of intergenerational distribution of welfare. Consequently, the question becomes whether there is a trade off between efficiency and sustainability, and the task is to identify the minimal degree of central planning that is necessary to allow decentralization of a sustainable allocation.

This treatment of sustainability is not common practice. On the one hand, natural scientists often interpret sustainability as the (physically) sustainable use of environmental resources, that is a development pattern that does not exhaust environmental resources [Boulding 1966].

On the other hand, economists who are more familiar with the welfare approach and specify sustainability accordingly, tend to use a broad variety of criteria [Pearce *et al.* 1989]. In Chapter 4, we discuss two of these. In this study, we choose to characterize sustainability in accordance with the WCED [1987] report as an allocation path in which both present and future generations can meet their own needs, and consider an allocation path sustainable if utility levels of both present and future generations do no decrease below a predefined critical level.

1.3.5. Intergenerational distribution of welfare: the role of capital transfers

The (re)distribution of income and welfare over generations raises a special issue. Within one period, it is possible to imagine a transfer of either physical commodities or income among consumers. However, to transfer commodities or income towards the future requires durable commodities, in general terms referred to as capital goods, whose value is referred to as the wealth that is passed over to the next generation. To pass over wealth to a future generation beyond the immediate successor requires a chain of recurring transfers. Furthermore, no wealth can be transferred backwards in time. Clearly, intertemporal redistribution has limitations of its own, and requires special treatment.

A standard framework in which the welfare distribution and capital transfers are often analyzed is the Ramsey [1928], or dynastic model. The dynamic model developed by Ramsey is based on the concept of a central (dynastic) planner who maximizes aggregated weighted welfare in present and future periods, and who, for this purpose, determines the optimal capital stock transferred to the future.

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[4] See Mirrlees [1971] and Stiglitz [1987] for a formal analysis of the trade off between efficiency and equity, and see Gunning and Keyzer [1995] and Ginsburgh and Keyzer [1997] for the use of applied general equilibrium models in applied analysis.

Early analysis of this model focused on the transfer of physical man-made capital (as opposed to non-physical man-made capital such as knowledge), jointly used with labor to produce consumer goods and new capital goods. Once the equipment has been built, its use is limited by its design, and therefore, it requires a cautious investment decision based on some forecasts. Nonetheless, since physical man-made capital typically decays within a finite time horizon, the Ramsey model was mostly used for studying savings and investments.

Later on, when it became clear that the production in industrialized economies depends on substantial inputs of exhaustible resources such as minerals, these environmental resources were also recognized as valuable assets. The Ramsey model turned out to offer a practical framework to analyze the rules for optimal exploitation [Hotelling 1931]. Since minerals are then treated as being similar to man-made capital, it is clear that they can also be conceived of as capital, broadly defined as stocks that may contribute to future welfare. The Ramsey model is still used in economic theory to characterize optimal environmental resource use, and we will also rely on it. However, its application to optimal environmental resource use has important limitations.

Unlike in man-made capital that depreciates relatively rapidly, the optimal investments in environmental resources have to take the very long term into consideration. This is so because deterioration of the environment can be viewed as negative investments that possibly do not depreciate. For the short term, one might consider the concept of a central planner as a practical abstraction that represents the intertemporal preferences of the consumers. However, the planner in the Ramsey model is like a dynastic dictator who decides about the welfare levels of both present and future generations. For the long term, this can lead to unsustainable solutions [Pezzey 1992] in which, for example, entire fish populations are harvested and become extinct [Clark 1973].

The alternative framework used to study capital transfers and welfare issues is the overlapping generations (OLG) model. In this model, consumers save a part of their income when young and buy capital goods, which they sell when old to pay for their pension. The next generation has to buy the capital stock when young from the previous generation that is old. It does not receive the capital for free.

In the dynastic framework, if sustainability is at stake in the sense that the future welfare becomes too low, the obvious solution consists of changing the intergenerational preferences of the central planner. In the OLG framework, no such option is available, and this raises the question as to which government intervention might ensure sustainability in this framework, and how this could interfere with efficiency.

To summarize, if we want to analyze the efficient and sustainable (intergenerationally equitable) use of environmental resources, we need a framework that allows for capital transfers and is capable of handling very long time horizons. Both the dynastic and the OLG framework satisfy these requirements, but they assume different institutional settings.

1.4. THESIS AND OVERVIEW

1.4.1. Physical characteristics of the environmental resource systems

As discussed in Section 1.2.1, present environmental concerns tend to focus on environmental resource systems that have a global component, are essential for life-support, and have substantial time delays between causes and environmental effects. In the coming chapters, we describe economies with environmental resources that possess some of these properties. However, to facilitate the formal analysis, we will have to introduce several abstractions and cannot claim that the analysis fully captures the complexity of physical environmental resource systems with feed back relations of various spatial and time scales. Therefore, we will often use the term 'environmental resources', to stress that we pay limited attention to the biogeochemical system itself. The resources to be considered have the following characteristics:

(i) They are essential for the production of welfare.

(ii) They are 'renewable' in the sense that they have the capacity to regenerate themselves indefinitely without maintenance costs and to produce an indefinite flow of services.

(iii) They are 'exhaustible' in the sense that over-exploitation irreversibly reduces their future productive capacity.

(iv) There are substantial time delays in the resource dynamics.

In the formal analysis, we abstract from the spatial indication 'global', but use the term 'essential' (and also 'non-negligible'). This emphasizes that the resources under consideration pose no purely local problems. The second, third and fourth characteristic are self explanatory, though the terms 'renewable' and 'exhaustible' may need some clarification. Fish populations are (up to some threshold level) truly renewable, insofar that the habitat ecosystem remains intact. If the habitat is damaged, this can lead to irreversible changes in the ecosystem. One might think of the habitat as a relatively static resource stock that becomes exhausted if overexploited. The fish populations can be considered as the amenities from the resource stock. Therefore, we will often refer to a resource with the second and third characteristic as an 'exhaustible resource with amenity value'.

If we contrast these characteristics of environmental resources with man-made capital, we find that they share the first characteristic (i.e. both are valuable assets) but that the other three characteristics are distinctive. Man-made capital has no potential for regenerating itself, it only depreciates. Moreover, because it is man-made, it is replaceable and (abstracting from cultural monuments) there is no irreversible loss, and one might imagine that man-made capital has the potential of growing continuously [Solow 1956]. Finally, though there is a gestation lag between investments and the use of capital, this lag cannot stand the comparison with the delays in environmental resource processes that can span decades or centuries.

Given these four characteristics, and given the distinction between environmental and man-made capital, we are now in a position to formulate our perception of environmental problems. The Malthusian concerns focus on the impossibility of checking population growth in another way than by vice and misery, while it is impossible to increase environmental production (food) in the same way as one can increase the production of man-made goods. Degeneration of the environment does not enter Malthus' analysis in which the environment is treated as an 'expendable' resource. Consequently, there is not much scope for environmental economics, and birth control is the sole remedy.

The debates on resource depletion of the 1960s and 1970s also express concerns of a more limited scope. Because extraction of minerals is physically limited, ultimately, the economy has to be based on recycling of materials or renewable substitutes. The second and fourth characteristics do not apply to these resources. The efficient use of exhaustible resources has been thoroughly analyzed; and the literature provides several variations on Hotelling's rule. In this context, sustainability calls for a relatively fast industrial transformation towards a recycling economy, but this is largely a matter of appropriate timing.

Finally, the more recent environmental issues, such as biodiversity loss and climate change, make it necessary to address the problem of 'exhaustion' (global degeneration) of complex environmental resource systems. There are parallels with mineral exhaustion, as ultimately, the exhaustion of environmental resource systems has to be brought to a halt, unless we accept an ever-degrading environment. However, because of the amenity value, the analysis should also attempt to answer the question which exhaustion level can be considered sustainable and how to ensure that the economy remains within a safe zone. These environmental problems require that due attention be paid not only to all first three characteristics, but also to the fourth, that adds some complexity, since it reduces the incentive to conserve the environment, as damages occur in a distant future. The two main questions in this study, reflected in the title, can now be stated as follows:

(i) *How to represent environmental resources with the four characteristics within a dynamic, competitive economy, and* (ii) *how to specify environmental policies that guarantee the efficient and sustainable use of these resources, and do not require day-to-day intervention.*

1.4.2. Approach

This study analyzes these two questions within the theoretical framework of welfare economics. As mentioned in Section 1.3.3, the literature on environmental protection has gradually evolved from strictly conservationist measures that disregard efficiency criteria, to Pigovian taxes that restore efficient pricing, and, more recently, to the attribution of property rights over environmental resources. This attribution is based on the idea that efficient use of environmental resources is best served by enforcing their competitive use. Yet because we want to analyze sustainability as well as efficiency, we must pay due attention to dynamic aspects of competitive economies, and the incorporation of environmental resources.

To accommodate the formal analysis in next chapters, we will use strict, though common theoretic assumptions, and take agents to be perfectly rational. Consumers maximize utility and producers maximize profits, we abstract from any social and cultural processes that could drive preferences and private and public choice. It is assumed that individual preferences can be represented by utility functions which satisfy standard assumptions, that individual production possibilities can be represented by transformation functions which also satisfy standard assumptions, that markets clear, and that all agents have complete information on future prices (perfect foresight).

Thus, we describe an 'ideal' abstract world, and in fact, we disregard many practical and fundamental questions that might inflict upon the efficient and sustainable resource use. To mention some, we abstract from distorted markets, endogenous technology, endogenous preferences, uncertainty, specific physical and spatial aspects of environmental processes, and not in the least, from endogenous demographic change. Remarkably, despite these idealized properties, non-sustainability can persist.

Besides the general theorems that will be stated and proven, we also develop a stylized model, ALICE[5], to which the theory can be applied. We begin with a one-good OLG exchange economy, and subsequently add an environmental resource, time delays, and

[5] Applied Long-term Integrated Competitive Equilibrium model.

man-made renewable capital. The stylized model allows us to analyze in more detail the effects of different policies on consumption, production, and welfare dynamics.

Throughout the study, we use climate change for illustrative purposes, and apply ALICE to give numerical support. Climate change is a notable example of an environmental problem that exhibits all four characteristics. An efficient and effective management of potential climate change requires the recognition of all services provided by the biogeochemical system underlying the global climate systems, the valuation of these services, awareness of their indefinite provision unless irreversible changes occur due to antropogenic GHG emissions, and finally the time delays that make it very difficult to estimate the ultimate consequences of present-day policies. We do not incorporate the uncertainties in the analysis, but choose a specification that conforms with the prevailing practice in the literature on economic integrated assessment. It will be assumed that GHG emissions lead to some irreversible changes, and that the climatic conditions associated to the 'natural' or 'pre-industrial' atmospheric compound were optimal, that is, that any change leads to a less favorable environment. These two assumptions are in line with the literature [IPCC 1996a], but we reiterate that they are still subject of heated scientific debate.

1.4.3. Overview

This study is organized as follows. In Chapter 2, both types of dynamic economies (dynastic and OLG) are formally specified, and existence of equilibrium is proven. We pay particular attention to the consequences of including exhaustible resources with amenity values (characteristics two and three). It is shown that the equilibrium paths exhibit the specific features of path-dependence (to our knowledge, a property that was not noticed in the economic literature before). This property implies that present policies have non-diminishing effects on future welfare, and points once more to the urgency of policy interventions.

Chapter 3 focuses on efficiency aspects and on the capacity of environmental resources to produce an indefinite stream of valuable services, the first and second characteristic. These resources will be referred to as 'non-negligible', because they produce a strictly positive share of total welfare. Their presence is of particular importance for OLG economies, as these can in principle become dynamically inefficient, i.e., have competitive equilibria that are not Pareto optimal. It is proven that non-negligible environmental resources ensure (dynamic) efficiency of the OLG economy.

The chapter also introduces ALICE in its simplest version. This applied model has a single environmental resource that possesses the first three specific characteristics: (i) the resource has non-negligible amenity value and is therefore valuable, (ii) it is exhaustible, but, (iii) if no extraction takes place, the resource produces an indefinite stream of valuable services (the amenity value). We provide an example of strictly conservationist policies that create inefficiencies, and we show that efficiency is restored if property rights over the resource are given to the present generation, a policy known as grandfathering. However, we also show that, compared to the strictly conservationist policy, grandfathering improves welfare of the present generation while reducing it for future generations. Indeed, an unsustainable equilibrium path cannot be ruled out. Next, parameters are chosen such that the numerical outcomes of the stylized model become comparable with those of existing integrated assessment models that include climate change. The numerical results suggest that the biogeochemical cycles represent an immense value, and confirm the earlier findings that future generations do not benefit from the efficiency gain achieved by grandfathering.

Chapter 4 focuses on sustainability. The literature has shown that competitive equilibria are capable of producing unsustainable allocations in which future welfare drops to zero. To prevent this from happening, we develop additional policy measures that ensure sustainability.

Our main result is to show that a trust fund can be set up to perform this task while obeying relatively simple and recursive decentralized rules of conduct. The trust fund needs a once-and-for-all computation of a feasible sustainable allocation path, but once it has been set up, its actions are entirely specified in terms of current prices. Thus, it represents an amendment of the competitive economy, as it requires public intervention in the initial period, but it requires no central planning for any period in the future. This makes the trust fund a suitable instrument for restoring sustainability while maintaining efficiency in a standard competitive OLG economy.

The illustration of Chapter 3 is pursued in Chapter 4, now focusing on the sustainability aspects associated to the use of an exhaustible resource with amenity value. We elaborate on the result of Chapter 3, and show that if environmental resources exhibit time delays (characteristic four), an unsustainable equilibrium in which welfare levels decrease to zero becomes more probable. Application of the theoretical findings to the numerical model with climate change shows that the trust fund can significantly raise welfare and reduce optimal GHG emission levels.

In Chapter 5, we broaden our scope and review some of the abstractions adopted in the formal analysis. We start with a further elaboration of the numerical illustration for climate change that includes explicit physical variables such as atmospheric CO_2 concentrations, and global average temperatures. Thereafter, we discuss in more general terms how physical environmental processes fit with the regular economic production structure. We connect the amenity value of environmental resources to the distinction between the quality and quantity of environmental resources.

Finally, we return to Malthus' concerns on environmental resource scarcity: the confrontation of increasing environmental resource demand by increasing populations with a supply that has only limited growth capacity. In an OLG economy with endogenous fertility, where the parents decide about fertility with the best interests of their unborn children in mind, the sustainability problem does not arise in per capita terms, but human population can become extinct, unless resources are preserved.

Chapters 2-4 are relatively technical. Readers mainly interested in the general line of the argument can limit themselves to the abstracts of these chapters and possibly their concluding sections, and proceed to chapter 5 and 6. A brief technical documentation for the ALICE model is contained in annexes to chapters 3 and 4. A full documentation is given in Gerlagh [1998b].

2. AN INFINITE HORIZON OLG/DYNASTIC ECONOMY

An infinite time horizon economy is developed with overlapping generations (OLG) and constant returns to scale in production, to be used throughout the study. The existence of both dynastic and OLG competitive equilibria is proven and examined. It is shown that the presence of exhaustible resources with amenity values produces a continuum of steady states and path dependent equilibrium paths. Consequently, effects of present-day policies do not diminish over time. Determinacy of the infinite horizon equilibrium and the approximation of infinite horizon OLG equilibria by truncated economies are briefly discussed. An infinite horizon mixed OLG/dynastic economy is constructed for which equivalence is established with a finite horizon mixed OLG/dynastic economy. The chapter concludes with a brief discussion of model extensions including an illustration that highlights the conceptual difference between dynastic and OLG models.

2.1. INTRODUCTION

In this chapter, a theoretical model of a competitive economy is developed that will be used throughout the study and will be extended for specific purposes. Next chapters will use this model to study the effects of different environmental resource assumptions and policies on efficiency and sustainability with stylized examples and numerical applications to the climate change problem.

To analyze environmental resources and associated policies, the model has to include several elements. First, it should be capable of handling the very long term. For example, the enhanced greenhouse effect is an environmental problem where effects of current pollution probably appear up to hundreds of years later. The models that are used to make simulations in order to find optimal policies to manage the enhanced greenhouse effect often cover hundreds of years, e.g., [Peck and Teisberg 1992, Nordhaus 1994, Manne *et al.* 1995, and Nordhaus and Yang 1996]. In this respect, the ideal model for fundamental analysis is one which horizon can be extended to infinity. The ideal applied model is one in which a truncation after some period can be justified.

Secondly, the model should be able of describing competitive equilibria in which efficient use of the environmental resources is ensured by distribution of property rights. The growing interest in property rights presumably emanates both from the perception that markets can only function if private agents themselves have an incentive to secure payments for environmental resource use, and from the understanding that the sums at stake might be substantial. In an intergenerational setting, this requires a so called overlapping generations (OLG) approach. So far, concerning the subject of optimal use of environmental resources, OLG types of models have mainly been applied for theoretic analysis. Howarth [1991] and Howarth and Norgaard [1992] develop a simple OLG model with finite horizon to discuss some basic notions of resource depletion in an intergenerational context. Based on the model analysis, they argue that intergenerational distribution and efficiency are to be considered separately. Mourmouras [1993] uses a one-commodity one-resource OLG model with infinite horizon to show that a specific (tax) policy that keeps resources 'in reserve' rather than distributing their full value to the present generations is required to guarantee sustainability. In this context, sustainability requires that consumption or utility does not decrease over time, and it is thus a form of intergenerational equity. Without the tax policy, consumption and utility can decrease without bound. John and Pecchenino [1992, 1994] use an OLG model to study economic growth and environmental dynamics.

This ambition brings us to the third required element of the model; it should be capable of incorporating typical environmental characteristics. These features are to be incorporated in such a way that property rights can be attributed. The lack of an adequate representation of environmental flows and stocks is a major drawback in current applied climate change models. All climate models referred to above are incapable of distributing environmental resource endowments. Consequently, environmental consumption, e.g., pollution, does not enter the budget constraint. In next chapters, we will present an alternative integrated assessment model that has these capacities.

This chapter will specify a theoretical model with infinite horizon. Section 2.2 introduces the generations and producers, and defines the competitive equilibrium for the economy. The rules for notation are given in Annex 2A. Section 2.3 describes the dynastic and OLG equilibria in more detail, and gives the existence proofs (Theorem 2.1 and Theorem 2.2). For the infinite horizon OLG equilibrium, the existence proof is based on the construction of a sequence of truncated equilibria that converge to the infinite horizon equilibrium, following Balasko *et al.* [1980]. We contribute to the theory by providing a simplified fixed point mapping from the welfare weights unit simplex onto itself (equations 2.32-2.34), which is the basis for the existence proof for the truncated equilibria. However, the main parts of these sections follow standard theory, and we rely to a large extent on Ginsburgh and Keyzer [1997, Ch.8], and many references given there.

Section 2.4 analyzes some features of the equilibrium, the steady states and the convergence of equilibrium paths, if there are exhaustible resources with amenity value. This issue is of relevance for environmental economics because many environmental resources cannot fully regenerate after deterioration which implies that they are in some way exhaustible. However, the literature on dynamic equilibria and steady states has not paid much attention to these resources, and in Section 2.4.1 we argue that this is one of the reasons for the feeling of dissent among so called 'ecological economists' with what they call 'neoclassical economics'. We give the general equations that define the steady states in Section 2.4.2. Next, in Section 2.4.3, a stylized economy is described with a single exhaustible resource that has amenity value, and it is shown that this property produces a continuum of steady states. The example is generalized, and we show that in economies with exhaustible resources that have amenity values, steady states form a continuum (Theorem 2.3).

Section 2.5 shifts the attention to an issue of relevance for applied models, namely the approximation of infinite horizon equilibria by truncated economies. Section 2.5.1 informally discusses the relation between stability of steady states, determinacy of dynamic equilibria, and the approximation of infinite horizon equilibria by equilibria of truncated economies. Section 2.5.2 provides a mixed OLG/dynastic economy in which the infinite horizon economy coincides with the truncated economy (Theorem 2.4).

Finally, Section 2.6 briefly discusses the flexibility of the model, paying attention to some minor model extensions: multiple period overlapping generations in Section 2.6.1, and multiple dynasties in Section 2.6.2. An illustration of a multiple dynasties economy is given that also highlights the difference between dynastic and OLG models. It is shown that the slavery implied by the dynastic model leads to extreme inequitable consumption patterns. In Section 2.6.3, we point to a possible weakening of the utility and endowments assumptions which maintain irreducibility of the OLG economy. The chapter concludes with Section 2.7 that gives a summary of the results.

2.2. Basic model formulation

2.2.1. Model set up

To begin with, this section specifies the elements of the economy used throughout the study. Let there be infinitely many discrete periods, $t = 1,\ldots,\infty$, and infinitely many generations, living two successive periods each. In every period, a new generation is born, so that in every period, there is a young and an old generation alive. Generations are of different sizes, denoted by n_t and are identified by the first period in which they live. Let there be H goods, C stocks, and J firms. Let $x_{t,t}, x_{t,t+1} \in \mathbf{R}_+^H$ be the consumption bundle of generation t, when young and old respectively, and $\omega_y, \omega_o \in \mathbf{R}_+^H$ the *per capita* initial endowments vectors for the young and the old generation. The first generation is denoted by t=0, since it has lived one period before t=1, and hence $x_{0,0} \in \mathbf{R}_+^H$ is exogenous to the model (notice that the generation born in period t=1 is the *second* generation). Let $y_{j,t} \in \mathbf{R}^H$ be the production vector of firm $j=1,\ldots,J$, in period t, $k_{j,t} \in \mathbf{R}_+^C$ the stocks used for production in period t, and $k_{j,t}^p \in \mathbf{R}_+^C$ the planned stocks. Planned stocks are decided upon one period in advance, so that they are endogenous for periods $t=2,\ldots,\infty$. In the first period, planned stocks are exogenous, and owned by the first generation. Let $p_t \geq 0$ be the price vector in period t for the H commodities, and $\psi_{j,t}\geq 0$ the price vector at the beginning of period t for the C stocks of firm j.

The following sub-sections describe the agents, generations (consumers) and producers, in more detail. Thereafter, the competitive equilibrium is defined.

2.2.2. Consumers

It is assumed that the dynamics of generations are determined by an autonomous demographic process:

$$n_{t+1} = \phi(n_t). \tag{2.1}$$

The size of a generation is a real number. Hence, it is implicitly assumed that each member of a generation is infinitesimally small. The size is bounded from above and below, $n_t \in [\underline{n}, \overline{n}] \subseteq \mathbf{R}_{++}$, where the double '+' sign denotes the strictly positive orthant. The regeneration function satisfies the following assumption:

ASSUMPTION 2.1. *The demographic regeneration function* $\phi:[\underline{n}, \overline{n}] \to [\underline{n}, \overline{n}]$ *has a fixed point,* $\phi(n^*) = n^*$, *to which the sequence* n_t *converges, thus*

N1: $n_1, n_2, \ldots \to n^*$.

Generations maximize utility derived from consumption in the first and second period of their life. Each generation is assumed to be homogeneous, hence within a generation, each person has the same consumption bundle and the same utility as a function of consumption per capita. The utility of the generation as a whole is taken to be the sum of all these utilities, that is the size of the generation multiplied by the utility derived from average consumption: $n_t\mathrm{u}(x_{t,t}/n_t, x_{t,t+1}/n_t)$. Where convenient, the extended utility function will be written as $\mathrm{U}(x_{t,t}, x_{t,t+1}; n_t)$. We notice that there is no need to impose homogeneous distribution of consumption over all members of a generation. Instead, one can assume aggregate welfare maximization within a generation, or maximization of minimal utility within the generation. However, it is essential to assume that all members of the generation have the same utility function.

The consumption vector that maximizes utility has to satisfy the budget constraint: expenditures on consumption cannot exceed revenues from endowments plus net transfers, Φ_t, received from other generations or a public agency. The first generation has a somewhat different budget constraint, as its endogenous consumption is restricted to a single period, and as its endowments include initial capital stocks:

$$p_1 x_{0,1} \leq p_1 n_0 \omega_o + \textstyle\sum_j \psi_{j,1} \mathrm{k}_{j,1}^p + \Phi_0 \tag{2.2}$$

is the budget constraint for the first generation, where $n_0\omega_o$ is the total amount of flow endowments of the first generation, and $k^p_{j,1}$ is the holding of initial stocks of firm *j* by the first generation. Note that flow endowments are multiplied by the size of a generation, as they are defined *per capita*. One might think of flow endowments independent of population, e.g., absorption capacity for pollution. Thus, the abstract model specification might slightly differ from an applied model specification. However, such a specification is in some sense arbitrarily, as environmental services might also be thought of as products of natural processes instead of endowments, captured in the variable *y*. For convenience, we will not include such considerations in the analysis, but restrict ourselves to the per capita assumption. The other generations, t=1,...,∞, are constrained by

$$p_t x_{1,t} + p_{t+1} x_{1,t+1} \leq p_t n_t \omega_y + p_{t+1} n_t \omega_o + \Phi_t \ . \tag{2.3}$$

In the basic model, it is implicitly assumed that the size of a generation remains constant between the two periods. No member of the generation dies before his time.

The other assumptions are common in the literature, see [Ginsburgh and Keyzer 1997, Ch. 2]. The utility function satisfies the following assumptions.

ASSUMPTION 2.2 *The utility function,* $u: \mathbf{R}^H_+ \times \mathbf{R}^H_+ \to \mathbf{R}$ *is*

U1: *continuous,*

U2: *concave, and*

U3: *strictly increasing in a commodity k in both periods.*

In addition, the utility function satisfies

U4: $u(0,0) > 0$.

Continuity of the utility functions on the interior of the domain follows from concavity. Assumption *U1* extends the property to the borders of the domain. The continuity of the utility function implies that consumption can be obtained in continuous quantities. In this respect, the assumption is similar to the assumed continuous size of generations: it assumes that because the indivisible units are (relative) small, the quantity measure can be treated as a continuum.

Concavity is also a common assumption. We refer to [Ginsburgh and Keyzer 1997, Ch.8] for a discussion. It is common in infinite horizon economies to assume continuous differentiability and strict concavity of the utility functions [Balasko *et al.* 1980, Kehoe, Levine and Romer 1987]. We will use a modified existence proof that enables us to relax the assumptions.

The third assumption states a specific case of non-satiation. There is one strictly desired commodity, for which utility always increases as the consumption increases. This assumption derives its relevance in combination with the next assumption of a strictly positive endowment of the desired good. Herewith, every generation is linked to the next and previous generations, as those are endowed with a commodity that they desire and *vice versa*. Together, these assumptions ensure the irreducibility of the economy[6], as defined by McKenzie [1959]. In a competitive equilibrium, this ensures that every consumer has strictly positive income and consumption.

The fourth assumption, u(0,0)>0, is used to anchor utility. A useful consequence of this assumption is that utility exceeds expenditures, as will be shown for the dynastic welfare program in the next section. The fourth assumption ensures that the extended utility function $U(x_{t,t},x_{t,t+1};n_t)$ is well-defined, concave, homogeneous of degree 1, strictly increasing in the population size, and that $U(0,0;n_t)>0$ for strictly positive population size (Lemma 2.34 in Annex 2B).

The following assumptions on endowments guarantee a minimal income level from endowments for all generations if the desired commodity has a strictly positive price in all periods.

ASSUMPTION 2.3. *The endowments are*

$\Omega 1$: *non-negative:* $\omega_y,\omega_o\geq 0$, *and*

$\Omega 2$: *strictly positive for the desired commodity k:* $\omega_y^k,\omega_o^k>0$.

The utility maximization is a 'convex' program: the objective function is concave (as it should be convex for a minimization program) and the set of feasible allocations is convex. As noted above, the price of the desired commodity has to be strictly positive in equilibrium, as consumption would become infinite otherwise. Then, if all prices are finite for any period, the budget constrained consumption set has a strict interior. Now, the optimal consumption vector can be completely characterized by the Kuhn-Tucker conditions. Given prices p, the optimal consumption, $\hat{x}_{t,t},\hat{x}_{t,t+1}$, maximizes utility subject to the budget constraint, if, and only if, (a) $\hat{x}_{t,t},\hat{x}_{t,t+1}$ maximizes $U(x_{t,t},x_{t,t+1};n_t)-\lambda(p_tx_{t,t}+p_{t+1}x_{t,t+1})$, for some positive dual variable λ, and (b) the budget constraint is binding. This characterization is used later to prove the second welfare theorem. The first condition is satisfied for any solution to a welfare maximizing program. The second condition can be satisfied by adjusting transfers between generations afterwards so that each generation exactly uses disposable income.

[6] Which means that the economy does not consist of two (or more) independent sub-economies.

2.2.3. Producers

Producers maximize profits every period, i.e. they maximize the value of net production flows plus the value of planned stocks minus the value of used stocks, $p_t y_{j,t} + \psi_{j,t+1} k^P_{j,t+1} - \psi_{j,t} k_{j,t}$. Following [Ginsburgh and Keyzer 1997, Ch. 7], the set of feasible production bundles $(y_{j,t}, k_{j,t}, k^P_{j,t+1})$ is described by transformation functions: $F_j: \mathbf{R}^H \times \mathbf{R}^C_+ \times \mathbf{R}^C_+ \to \mathbf{R}$.

$$F_j(y_{j,t}, -k_{j,t}, k^P_{j,t+1}) \leq 0 \tag{2.4}$$

for t=1,...,∞; j=1,...,J. The transformation functions are firm specific but not time dependent. The time independence presumes that technology is included in the stock variable, endogenous to the model. The transformation functions satisfy the following assumptions.

ASSUMPTION 2.4. *For every j=1,...,J, the transformation function* $F_j: \mathbf{R}^H \times \mathbf{R}^C_+ \times \mathbf{R}^C_+ \to \mathbf{R}$ *is*

F1: *continuous,*

F2: *convex, and*

F3: *homogeneous of degree one.*

For given bounded initial stock, $k^P_{j,1}$, *there is a time independent upper bound,* $\bar{k}_j$, *for produced capital:*

F4: $\exists \bar{k}_j \geq k^P_{j,1}: k_j \leq \bar{k}_j \wedge y_j \geq -\bar{n}(\omega_y + \omega_o) \wedge F_j(y_j, -k_j, k^P_j) \leq 0 \Rightarrow k^P_j \leq \bar{k}_j$.

For given bounded stocks, production is bounded from above:

F5: $\exists \bar{y}_j: k_j \leq \bar{k}_j \wedge F_j(y_j, -k_j, k^P_j) \leq 0 \Rightarrow y_j \leq \bar{y}_j$.

Finally, aggregate production can maintain strictly positive consumption:

F6: $\exists k < k^P_1, y: \forall j: F_j(y_j, -k_j, k_j) \leq 0 \wedge \sum_j y_j + \underline{n}(\omega_y + \omega_o) >> 0$.

The convexity of the transformation function ensures that the set of feasible production vectors is convex. Homogeneity of degree one reflects a constant returns to scale technology: if $(y_t, -k_t, k^P_t)$ is a feasible production bundle, than $(ay_t, -ak_t, ak^P_t)$ for $a \geq 0$ is a feasible production bundle as well. Particularly, there is the possibility of inaction: $F_j(0,0,0) \leq 0$. Assumption *F4* asserts the existence of a uniform upper bound for the capital stock (i.e., independent of the period), $\bar{k}_j$, such that (for any period), (i) if the actual capital stock (at the beginning of the period) is below this level, and (ii) if the production inputs are bounded by the constant *per capita* endowments and population upper bound (*N1*), then the planned capital stock (for the next period) does not exceed the uniform upper bound. By forward induction, (*F4*) and (*N1*) imply a uniform bound for the planned stocks, $\bar{k}_j$, and (*F5*) extends the uniform bound to the production vector. These assumptions allow for exponential economic growth up to the upper bound, and are used to prove compactness of the equilibrium space (Lemma 2.6), which is used to prove boundedness for the dynastic value function (Lemma 2.10), and existence of the OLG equilibrium (Theorem 2.2). Assumption *F4* can be relaxed to allow for sustained growth, but to maintain boundedness of the dynastic value function, this would require an upper bound for the utility function [Ginsburgh and Keyzer 1997, Ch. 8, Assumption CD1], and to maintain compactness of the OLG equilibrium space, this would require a redefinition of the product space in the infinite time horizon economy comparable with the period-t prices construct in Definition 2.19. We notice that Assumption *F4* applies to the commodity balance, and not only to the technology space. Together with Assumption *F5*, it ensures that the common assumptions on F(.) are satisfied: being non-decreasing, and the intersection of F(.) with the positive orthant being equal to {0}. Together, they guarantee that the transformation function is non-satiated in y and k and hence the intersection of feasible production bundles, y, with the positive orthant is the zero vector. Thus, it is not possible to produce something from nothing. Finally, Assumption *F6* is formulated for the aggregate production set. It guarantees a strict interior for the welfare program defined in the next section. If convenient, we will refer to aggregate production only, not distinguishing individual producers: $y_t \equiv \sum_j y_{j,t}$, and $k_t \equiv \sum_j k_{j,t}$.

A specific consequence of homogeneity is that profits are zero in competitive equilibrium. If profits were positive, production could be extended in fixed proportions and profits could increase without bounds. On the other hand, the possibility of inaction ensures profit maximization yielding non-negative profits. Finally, it must be stressed that the imposition of the upper bound does not imply a choice for an 'environmentalist' perspective that constrains economic activities: the bound can be chosen arbitrarily large.

2.2.4. Competitive equilibrium

In a competitive equilibrium, generations maximizes utility subject to their budget constraints, e.g., $p_t x_{t,t} + p_{t+1} x_{t,t+1} \leq p_t n_t \omega_y + p_{t+1} n_t \omega_o + \Phi_t$, for generations t=1,...,∞. Producers maximize profits, $p_t y_{j,t} + \psi_{j,t+1} k^p_{j,t+1} - \psi_{j,t} k_{j,t}$, subject to the transformation constraints. Prices ensure that demand matches supply for all commodities and stocks. For each finite time period, the norm of the price vector has to be finite. However, the sequence of prices does not need to be bounded. The commodity balance holds: consumption cannot exceed production plus endowments. The commodity balance is binding for all commodities with strictly positive price. For commodities with zero prices, supply can strictly exceed demand. The complementarity is represented by '$\perp$' in the following equation[7],

$$x_{t-1,t} + x_{t,t} \leq \sum_{j=1,..,J} y_{j,t} + n_{t-1}\omega_o + n_t\omega_y \perp p_t \geq 0\,, \tag{2.5}$$

for t=1,...,∞.

By analogy to the commodity balance, used stocks cannot exceed planned levels,

$$k_{j,t} \leq k_{j,t}^p \perp \psi_{j,t} \geq 0\,, \tag{2.6}$$

for j=1,...,J, and t=1,...,∞. The conditions for a competitive equilibrium are summarized in the following definition.

DEFINITION 2.5. *A competitive equilibrium is a bundle* $(\hat{x},\hat{y},\hat{k},\hat{k}^p,\hat{p},\hat{\psi},\hat{\Phi})$[8], *such that*

- *E1:* *prices are non-zero and bounded for any period:* $\forall$ t=1,...,∞: $0<|p_t|,|\psi_{j,t}|<\infty$,
- *E2:* *consumption maximizes utility subject to the budget constraints for given transfers* $\hat{\Phi}$, *and*
- *E3:* *in every period, every firm maximizes profits subject to the transformation constraints.*

Moreover,

- *E4:* *the commodity balances* (2.5) *hold and are binding for non-zero commodity prices, and*
- *E5:* *stock balances* (2.6) *hold and are binding for non-zero stock prices.*

Note that the definition does not specify the source of transfers. In a dynastic economy, transfers will arise because of the preferences of the dynastic planner, whereas in the OLG equilibrium, transfers are supposed to be given by a transfer function.

With this definition, it is already possible to establish some characteristics of the equilibrium. We begin with the following lemma:

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[7] The complementarity used in '$a \geq 0 \perp b \geq 0$' implies that both the inequalities hold, and that a'b=0 (the vector inproduct). A convenient interpretation is that the equality of the left constraint is binding conditionally upon the right inequality.

[8] Variables evaluated at equilibrium are denoted by a hat when convenient.

LEMMA 2.6. *Under assumptions N1, U1-U4, F1-F6, capital stocks, production vectors, consumption vectors and utilities are elements of compact spaces* **K**, **Y**, **X**, and **U**: $k_{j,t}, k^{P}_{j,t} \in \mathbf{K} \subseteq \mathbf{R}^{C}_{+}$, $y_{j,t} \in \mathbf{Y} \subseteq \mathbf{R}^{H}$, $x_{t,t}, x_{t-1,t} \in \mathbf{X} \subseteq \mathbf{R}^{H}_{+}$, *and* $n_t u_t \in \mathbf{U} \subseteq \mathbf{R}_{+}$. *The infinite horizon commodity/stock/utility product space is compact.*

Proof. The bounds on production of planned stocks (*F4*), together with the stock balance (*E5*) ensures that capital stocks are uniformly bounded, so that $k_{j,t}, k^{P}_{j,t} \in \mathbf{K} \subseteq \mathbf{R}^{C}_{+}$, where **K** is a compact subspace. Given $k_{j,t} \in \mathbf{K}$, assumption *F5* sets an upper bound on production, whereas the commodity balance (*E4*) sets a lower bound. Thus, $y_{j,t} \in \mathbf{Y} \subseteq \mathbf{R}^{H}$, where **Y** is compact. The commodity balance and the upper bound on the population size ensures that $x_{t,t}, x_{t-1,t} \in \mathbf{X} \subseteq \mathbf{R}^{H}_{+}$, where **X** is compact. The continuity of utility (*U1*) together with the bounds on the generation's size ensure that utility lies within a compact set: $n_t u_t \in \mathbf{U} \subseteq \mathbf{R}_{+}$, where u_t is the *per capita* utility. Taking together, the commodity/stock/utility space for an equilibrium can be restricted to a countable infinite product of compact spaces, which is itself compact in the product topology (Tychonoff theorem, see [Bourbaki 1965, Part I, Section 9.5, Theorem 3] or [Ward 1972, Theorem 86]). ■

Notice that the price sequences can increase without bound, so that the price space is not compact in the product topology. Compactness for a modified price space will be established later as a part of the existence proof for an OLG equilibrium.

The next property of an equilibrium to be considered is the relation between the value of consumption and production factors. From the commodity balance (2.5), it follows that in every period the value of endowments plus production equals the value of consumption,

$$\hat{p}_t \hat{x}_{t-1,t} + \hat{p}_t \hat{x}_{t,t} = \sum_{j=1,\ldots,J} \hat{p}_t \hat{y}_{j,t} + \hat{p}_t n_{t-1} \omega_o + \hat{p}_t n_t \omega_y \,. \tag{2.7}$$

The stock balances (2.6), together with the zero profit property of the producers ensure that the value of used stocks equals the value of production plus next period stocks,

$$\hat{\psi}_{j,t} \hat{k}_{j,t} = \hat{p}_t \hat{y}_{j,t} + \hat{\psi}_{j,t+1} \hat{k}_{j,t+1} \,, \tag{2.8}$$

for $j=1,\ldots,J$. After summation of the two equations, we have:

$$\hat{p}_t (\hat{x}_{t-1,t} + \hat{x}_{t,t}) + \sum_{j=1,\ldots,J} \hat{\psi}_{j,t+1} \hat{k}_{j,t+1} = \hat{p}_t (n_{t-1} \omega_o + n_t \omega_y) + \sum_{j=1,\ldots,J} \hat{\psi}_{j,t} \hat{k}_{j,t} \,. \tag{2.9}$$

The equation states that in every period, the value of production factors, endowments plus used capital stocks, is equal to the value of consumption plus next period capital stocks. In order to derive the accounts for consumption plus net investments, we subtract the value of present stocks at future prices, $\psi_{t+1}k_t$, from both the left and right side, and derive

$$\hat{p}_t(\hat{x}_{t-1,t} + \hat{x}_{t,t}) + \sum_{j=1,\dots,J} \hat{\psi}_{j,t+1}\Delta\hat{k}_{j,t} = \hat{p}_t(n_{t-1}\omega_o + n_t\omega_y) - \sum_{j=1,\dots,J} \Delta\hat{\psi}_{j,t}\hat{k}_{j,t}\,. \tag{2.10}$$

The value of consumption plus net investments equals the value of endowments plus rents from stocks. Note that in many equilibria prices decrease, so that the rent of capital is positive: $\Delta\psi_t = \psi_{t+1} - \psi_t \leq 0$. Furthermore, note that net investments in stocks are valued at prices at the end of the period, whereas rents are taken over stocks at the beginning of the period. This is arbitrary, as can be seen by subtracting $\Delta\psi_t\Delta k_t$ from both sides,

$$\hat{p}_t(\hat{x}_{t-1,t} + \hat{x}_{t,t}) + \sum_{j=1,\dots,J} \psi_t\Delta\hat{k}_{j,t} = \hat{p}_t(n_{t-1}\omega_o + n_t\omega_y) - \sum_{j=1,\dots,J} \Delta\psi_t\hat{k}_{j,t+1}\,. \tag{2.11}$$

The accounting equations do not distinguish between man-made stocks and natural stocks. Extraction of a valuable exhaustible resource is represented by a negative value of $\psi_{j,t}\Delta\hat{k}_{j,t}$. Hence, it is considered a negative investment [Hartwick's 1990] [Mäler's 1991].

If prices converge to zero sufficiently fast to keep the sums bounded, (2.9) can be summed over the infinite time horizon, and

$$\sum_{t=1,\dots,\infty} (\hat{p}_t\hat{x}_{t,t-1} + \hat{p}_t\hat{x}_{t,t}) = \sum_{j=1,\dots,J} \hat{\psi}_{j,1}\,\mathrm{k}^p_{j,1} + \sum_{t=1,\dots,\infty} (\hat{p}_t n_{t-1}\omega_o + \hat{p}_t n_t\omega_y)\,. \tag{2.12}$$

This equation states that the value of consumption, aggregated over time, equals the value of endowments plus the value of initial stocks. Moreover, at the left hand side is the value of all consumers expenditures, where on the right side all income except from transfers is aggregated. From the equality, it can be concluded that transfers sum to zero,

$$\sum_{t=0,\dots,\infty} \hat{\Phi}_t = 0\,. \tag{2.13}$$

This completes the definition of the economy and its equilibrium, except for the welfare optimization that characterizes a dynastic framework, or the transfer specification that characterizes an OLG framework. These are specified below.

2.3. DYNASTIC OPTIMA AND OLG EQUILIBRIA

2.3.1. Dynastic welfare optimum

For the purpose of this study, we discuss two types of competitive equilibria, commonly referred to as the 'dynastic' and the 'OLG' approach. We introduced both concepts in Section 1.3.5, and confront them in several places throughout the study. Our general position will be that the OLG framework is superior. In Section 2.6.2, an illustration highlights the slavery that is implied by the dynastic framework; endowments of future generations can be sold by their ancestors so that they inherit a debt. In Section 4.3.1, we argue that the dynastic framework does not give a clue as to which institutions and policy instruments can be used to ensure sustainability. In Section 4.4.3, we conclude from a numerical analysis that the assumption of a constant discount rate which is standard in most dynastic models, conflicts with the aim of combining efficiency and sustainability. On the other hand, the OLG approach offers a natural context for studying institutions and policy rules as we will see in Section 4.3.3. In Chapter 3, we will also show that the limitation of the OLG model, that it may lead to inefficient allocations, can be avoided particularly easily in the present context. Nonetheless, the dynastic framework has firm ground in the literature, and we will give it due attention.

We first study the dynastic economy, in which there is a planner whose objective is to maximize a social welfare function, given the physical constraints imposed by the commodity balances, stock balances and transformation constraints. As the physical constraints are fixed (without endogenous parameters), the dynastic planner is fully characterized by the welfare function that aggregates utilities of different generations into one welfare measure.

A commonly used welfare function aggregates utilities by weights that are proportionate to the size of the generation, and decline geometrically in time. Such a welfare function is referred to as having constant pure time preference. The form can be found in all text books on growth models. It can be traced back as far as Ramsey [1928]. Welfare is written:

$$w_0 = \sum_{t=0,\dots,\infty} \beta^t \, \mathrm{U}(x_{t,t}, x_{t,t+1}; n_t), \tag{2.14}$$

where β is the constant utility discount factor. Another commonly used form is the so called maximin, or Rawlsian welfare function. It is particularly popular among advocates of intergenerational equity. A discussion of its merits in relation to Hartwick's rule for optimal capital planning can be found in [Solow 1974; 1986]. In this form, welfare is taken to be equal to the lowest per capita utility level of all generations:

$$w_0 = \min_{t=0,..,\infty} \mathrm{u}(x_{t,t}/n_t, x_{t,t+1}/n_t). \tag{2.15}$$

Both welfare functions can be represented in recurrent form, as $w_t = n_t u_t + \beta w_{t+1}$, and $w_t = \min\{u_t, w_{t+1}\}$, respectively, where u_t is the utility per capita. Koopmans [1960] shows that there is a broad class of welfare functions that can be written in such a recursive way. We use the general recursive welfare function:

$$w_t = \mathrm{H}(\mathrm{U}(x_{t,t}, x_{t,t+1}; n_t), w_{t+1}), \tag{2.16}$$

where H is called aggregator function. The aggregator function studied by Koopmans [1960] and Koopmans *et al.* [1964] is continuous, nondecreasing, and concave. The authors show that if welfare is to be finite, the aggregator function needs to exhibit a 'time perspective': welfare needs to discount future utility in some way to arrive at a finite aggregate. This time perspective or discounting will appear as the contracting property in the aggregator function to be discussed below.

If welfare is allowed to become infinite, an alternative ordering is required to compare different allocation sequences that both yield infinite welfare. Such orderings can be based on principles such as 'overtaking optimality' [Carlson and Haurie, 1987]. As a short digression, let us consider an example where the aggregator function sums all future utilities. Then, one allocation is overtaken by another if from period T on, the summed utilities for t=1,...,T of the latter allocation exceeds the summed utilities of the former:

$$\forall t \geq T\colon \sum_{i=0,...,t} \mathrm{U}(\tilde{x}_{i,i}, \tilde{x}_{i,i+1}; n_i) \leq \sum_{i=0,...,t} \mathrm{U}(\hat{x}_{i,i}, \hat{x}_{i,i+1}; n_i) \tag{2.17}$$

where i denotes the generations, the hat denotes the overtaking optimal allocation, and the tilde denotes the former allocation. However, orderings on sequences with unbounded values are typically incomplete[9]. The assumptions for the aggregator functions specified below will ensure that aggregate welfare is finite.

The following assumptions on the aggregator function characterize a welfare structure that is equivalent to the welfare structure implied by the standard assumptions in the literature. In [Lucas and Stokey 1984] and [Keyzer 1991], a slightly different aggregator function is used, incorporating the utility function as an element of the aggregator function. In [Stokey and Lucas 1989, part I and II], an extensive analysis is given of the recursive value function with constant time preference.

Returning to the aggregator function, we require that it satisfies the following assumptions:

[9] An ordering is complete if all pairs of allocations can be ordered. If an ordering is incomplete, there exists a pair for which the ordering cannot determine which allocation is better.

ASSUMPTION 2.7. *The aggregator function* H: $\mathbf{R}_+ \times \mathbf{R}_+ \to \mathbf{R}_+$ *is*

H1: *continuous,*

H2: *concave, and*

H3: *strictly increasing in both arguments.*

H(.) *satisfies*

H4: H(0,0)=0,

and H(.) *is strictly contracting in the second argument. More precisely, there is a* $\beta \in (0,1)$ *such that*

H5: $|\mathrm{H}(u, \tilde{w}) - \mathrm{H}(u,w)| \le \beta |\tilde{w} - w|$

The contractor $\beta<1$ enables us to set an upper bound on aggregated welfare, given an upper bound on utility. Let $\bar{u}$ be the upper bound for extended utility. Then, we have that for any feasible extended utility u:

$$\mathrm{H}(u,w) \le \mathrm{H}(\bar{u},w) \le \mathrm{H}(\bar{u},0) + \beta w, \tag{2.18}$$

where the first inequality follows from *H3* and the second inequality follows from *H5*. If we take an upper bound $\bar{w}$ for w_{t+1} that satisfies $\bar{w} > \mathrm{H}(\bar{u},0)/(1-\beta)$, it follows that:

$$u_t \le \bar{u} \ \wedge w_{t+1} \le \bar{w} \Rightarrow \mathrm{H}(u_t, w_{t+1}) < \bar{w}. \tag{2.19}$$

which implies that $\bar{w}$ is an upper bound for w_t as well. Hence, we can consistently assume that $\bar{w}$ is an upper bound for welfare in all periods.

Through the aggregator, the welfare of the whole sequence of utilities is incorporated in the variable w, evaluated at t=0. The dynastic welfare program can then be written as the maximization of w_0.

PROGRAM 2.8. *Infinite time horizon dynastic welfare program*

$$\mathrm{v}(k_1^P, x_{0,0}; n_0) = max\ w_0$$

$0 \le w_t \le \bar{w}$ for t=0,..,∞

$x_{t-1,t}, x_{t,t} \ge 0, k_{j,t}, k_{j,t+1}^P \ge 0$, and $y_{j,t}$ for $j=1,\ldots,r$, $t=1,\ldots,\infty$

subject to

$w_t \leq \mathrm{H}(\mathrm{U}(x_{t,t}, x_{t,t+1}; n_t), w_{t+1})$ (μ_t) $t=0,\ldots,\infty$

$x_{t-1,t} + x_{t,t} \leq n_{t-1}\omega_o + n_t\omega_y + \sum_j y_{j,t}$ (p_t) $t=1,\ldots,\infty$

$k_{j,t} \leq k^p_{j,t}$ $(\psi_{j,t})$ $j=1,\ldots,J;$ $t=1,\ldots,\infty$

$\mathrm{F}_{j,t}(y_{j,t}, -k_{j,t}, k^p_{j,t+1}) \leq 0$ $(\lambda_{j,t})$ $j=1,\ldots,J;$ $t=1,\ldots,\infty$

for $n_{t+1}=\phi(n_t)$, where variables in brackets are Lagrange multipliers. The population size is regarded as a parameter in the value function, rather than as a variable. We will briefly consider economies with endogenous populations in Chapter 5.

It is not evident that the value function of the above welfare program is well defined. It has to be shown that (i) the welfare program has at least one feasible solution, that (ii) the welfare program has an upper bound, and that (iii) the supremum is actually reached as a maximum. Additionally, we will have to show that (iv) the dual variables are well defined and bounded for any period.

(i) The existence of a feasible solution follows immediately from the maintainable positive production (*F6*). (ii) The upper bound is imposed explicitly in the program. Notice that the upper bound on w_t is necessary but not binding.[10] Hence, it is not necessary to consider the dual variable of this constraint. (iii) Attainment of the supremum as a maximum requires a more thorough analysis; it is stated as Lemma 2.10 below. Finally, property (iv) is given as a part of the proof of Theorem 2.1.

It has been shown by Lucas and Stokey [1984, Theorem 2] that the set of feasible allocations of the dynastic welfare program is closed. From this it follows that the sequence for which welfare converges to the supremum has a sub-sequence that converges within the feasibility space. Thus, for any particular value of initial stocks, consumption and endowments, the welfare program returns a unique value.

Before we state formal lemma's and theorems for the dynastic economy, it is convenient for further reference to specify standard assumptions:

ASSUMPTION 2.9. *The standard assumptions for a dynastic economy are*

D1: *N1, U1-U4, Ω1, Ω2, F1-F6, and H1-H5*

---

[10] Let us take the constant time preference aggregator function as an example. To check that the upper bound is necessary, it is sufficient to notice that one can reach any w_0, adjusting w_t, by forward induction. This is possible if there is no upper bound. On the other hand, to show that the constraint is not binding for any feasible solution, an indirect demonstration is needed. Assume that the upper bound on w is binding for period t. It follows from equation (2.19) that $\overline{\mathrm{w}} = \mathrm{H}(\mathrm{u}_t, w_{t+1}) \Rightarrow w_{t+1} > \overline{\mathrm{w}}$, which is not feasible. This special use of a necessary but non-binding constraint is possible because of the infinite time horizon.

We can now state as a theorem:

LEMMA 2.10. *If Assumptions D1 are satisfied, then the value function* v(.), *defined by the dynastic welfare Program* 2.8 *is well defined.*

Proof. Note that the variable space can be restricted to a compact space, $\mathbf{K}^{\mathbf{N}}$, $\mathbf{Y}^{\mathbf{N}}$, and $\mathbf{X}^{\mathbf{N}}$, where **N** is the set of discrete time periods, since it is a countable infinite product of compact spaces **K**, **Y**, and **X**. Consider the set of infinite allocations that satisfy the constraints of the welfare program for the first T periods. Let this set be denoted by $\mathbf{W}_T$. As all constraints are defined by continuous functions (*U1*, *F1*, *H1*), it follows that $\mathbf{W}_T$ is closed. Clearly, $\mathbf{W}_1 \supseteq \mathbf{W}_2 \supseteq \ldots$ From this, it follows that the set of feasible allocations, $\mathbf{W}_\infty = \bigcap_{T \geq 1} \mathbf{W}_T$ is closed. As the variable space is compact in the product space (Lemma 2.6), so is $\mathbf{W}_\infty$. Thus, every sequence for which w_0 converges to the supremum has a converging sub-sequence with a limit point in $\mathbf{W}_\infty$ [Ward 1972, Theorem 83]. The value of w_0 of this limit point attains the supremum as a maximum, and this value is returned by the value function. ■

The next step is to show that the value function has other desired properties: continuity, concavity and non-decreasingness. To prove such, a recursive welfare program is defined that represents the infinite horizon welfare program by dynamic optimization. The arguments follow Lucas and Stokey [1984] in their proof of Theorem 1. The aggregator function H defines an operator P_H on the value function: $v \rightarrow P_H v$ as follows:

PROGRAM 2.11. *Recursive welfare program, defining operator* P_H.

$$P_H v(k_t^p, x_{t,t}; n_t) = max\ H(U(x_{t,t}, x_{t,t+1}; n_t), v(k_{t+1}, x_{t+1,t+1}; \phi(n_t)))$$

$$x_{t,t+1}, x_{t+1,t+1} \geq 0, k_{j,t}, k_{j,t+1}^p \geq 0, \text{ and } y_{j,t} \text{ for } j=1,\ldots,J$$

subject to

$$x_{t,t+1} + x_{t+1,t+1} \leq n_t \omega_o + \phi(n_t)\omega_y + \textstyle\sum_j y_{j,t} \qquad (p_t)$$

$$k_{j,\tau} \leq k_{j,\tau}^p \qquad (\psi_{j,\tau}) \qquad j=1,\ldots,J; \quad \tau=t,t+1$$

$$F_j(y_{j,t}, -k_{j,t}, k_{j,t+1}^p) \leq 0 \qquad (\lambda_{j,t}) \qquad j=1,\ldots,J$$

Again, the dual variables are included in the program, though it has still to be proven that they are well defined. Operator P_H, defined by the recursive welfare program has the following properties.

LEMMA 2.12. *If Assumptions D1 are satisfied, then the operator* P_H, *defined by Program* 2.11 *is a contraction, and has a unique fixed point value function* $\hat{v}(.)$, *which is continuous, concave and non-satiated in its arguments.*

Proof. We only give a brief description of the main lines of the arguments. The contracting property of P_H follows immediately from the contraction property of the aggregator function H(.) (*H5*). Moreover, note that the operator preserves continuity, concavity and non-decreasingness. A thorough discussion of this point can be found in [Stokey and Lucas 1989, Ch.4]. The property follows from the continuity, concavity and non-decreasingness of the elements of the operator: the utility function, the transformation functions and the aggregator function (*U1, U2, U3, F1, F2, F3, F4, H1, H2, H3*).

As noted above, welfare is bounded. The operator maps any element of the Banach space of continuous bounded value functions on the same Banach space. The contraction of the operator ensures that there is a unique fixed point.

As the operator is contracting on the broader class of all bounded (possibly non-continuous) value functions, any such function converges uniformly to the fixed point as the operator P_H is iterated infinitely many times. As P_H preserves continuity, concavity and non-decreasingness properties, it is sufficient to start the iteration with a continuous, non-decreasing, concave, value function to prove that the fixed point function $\hat{v}$ is continuous, concave and non-decreasing in the variables. ■

The next step is to show that the fixed point $\hat{v}(.)$ of the operator equals the valuation function resulting from the infinite horizon welfare program. The proof follows closely the one given in [Stokey and Lucas 1989, Ch.4] for recursive welfare with constant time preference, and therefore, will be very brief.

LEMMA 2.13. *Under Assumptions D1, the fixed point* $\hat{v}$ *of the operator* P_H, *defined by Program* 2.11, *is the value function of the infinite horizon welfare program. Moreover, the optimal allocations of Program* 2.8 *and Program* 2.11 *coincide.*

Proof. Clearly, the value function defined by Program 2.8 is a fixed point of the operator P_H. As the fixed point is unique, it follows that they coincide. Thus, the infinite program is represented by the iterated recursive program in the following way: The *T*-period welfare program associated to $P_H^T\hat{v}$, yields the same *T*-period optimal allocation as the first *T* periods of the infinite horizon program. By extending *T* to ∞, the operator yields an infinite sequence of allocations that coincides with the optimal allocation of the infinite horizon program. ■

Finally, it is shown that the dynastic welfare Program 2.8 yields a competitive equilibrium with transfers. This is the second welfare theorem applied to our model.

THEOREM 2.1. *If Assumptions D1 are satisfied, then the infinite horizon welfare Program* 2.8. *has an optimal allocation, which is a competitive equilibrium with transfers (Definition* 2.5).

Proof. The existence of an optimal allocation is proven by the preceding two lemmas. The next step is to show that the dual variables are well defined and satisfy the regular Kuhn-Tucker conditions. From this, the five conditions (*E1*,...,*E5*) of an equilibrium can be proven to be satisfied.

As was noted in the previous proof, a finite horizon welfare program, defined by the iterated recursive welfare program, can be used to generate optimal allocations for the first *T* periods. This finite horizon program has a strict interior (*F6*). Hence, dual variables exist that satisfy the Kuhn-Tucker characterization. As the Kuhn-Tucker conditions give a complete characterization of the optimum in any period, it follows that the optimal (possibly set valued) variables and dual variables do not change for extensions of the horizon after that period. Thus, for any finite period, the dual variables evaluated at the optimum are well defined, particularly bounded and Kuhn-Tucker conditions can be used (Property (iv) of Program 2.8).

Let the optimal allocation and dual variables be denoted by a hat. It follows from the recursive welfare program that for every period, prices are bounded (*E1*). Secondly, $\hat{w}_t$ maximizes $\hat{\mu}_{t-1}\,\mathrm{H}(.,w_t)-\hat{\mu}_t w_t$. The aggregator function is strictly increasing in the second argument, so that either both $\hat{\mu}_{t-1}$ and $\hat{\mu}_t$ are zeros, or both are positive. As w_0 is in the objective function, it follows that $\hat{\mu}_0 = 1$, so that all $\hat{\mu}_t$ are strictly positive. Consumption, $\hat{x}_{t,t},\hat{x}_{t,t+1}$, maximizes $\hat{\mu}_t\,\mathrm{H}(\mathrm{U}(x_{t,t},x_{t,t+1};n_t),\hat{w}_{t+1}) - \hat{p}_t x_{t,t} - \hat{p}_{t+1}x_{t,t+1}$. Hence, if $\hat{p}_t x_{t,t} + \hat{p}_t x_{t,t} \le \hat{p}_t\hat{x}_{t,t} + \hat{p}_t\hat{x}_{t,t}$, then $\hat{\mu}_t\,\mathrm{H}(\mathrm{U}(x_{t,t},x_{t,t+1};n_t),\hat{w}_{t+1}) \le \hat{\mu}_t\,\mathrm{H}(\mathrm{U}(\hat{x}_{t,t},\hat{x}_{t,t+1};n_t),\hat{w}_{t+1})$. As the aggregator function is strictly increasing in the first argument, it can be concluded that $\mathrm{U}(x_{t,t},x_{t,t+1};n_t) \le \mathrm{U}(\hat{x}_{t,t},\hat{x}_{t,t+1};n_t)$. Thus consumption maximizes utility subject to a budget constraint (*E2*). As a consequence, prices of the strictly desired commodity *k* are strictly positive so that the price vector is non-zero. Thirdly, $(\hat{y}_t,-\hat{k}_t,\hat{k}^p_t)$ maximizes $\hat{p}_t y_{j,t} - \hat{\psi}_{j,t}k_{j,t} + \hat{\psi}_{j,t+1}k^p_{j,t+1} - \hat{\lambda}\,\mathrm{F}(y_{j,t},-k_{j,t},k^p_{j,t+1})$. As the price is non-zero, the dual variable λ is non-zero. This ensures that the transformation function F(.) is zero in the optimum. From this it follows that if $\mathrm{F}(y_{j,t},-k_{j,t},k^p_{j,t+1}) \le \mathrm{F}(\hat{y}_{j,t},-\hat{k}_{j,t},\hat{k}^p_{j,t+1})$ =0, then $\hat{p}_t y_{j,t} - \hat{\psi}_{j,t}k_{j,t} + \hat{\psi}_{j,t+1}k^p_{j,t+1} \le 0$. In other words, profits are maximized (*E3*). The remaining two conditions on the commodity and stock balances (*E4*, *E5*) follow directly from the recursive welfare program. ■

To summarize, the dynastic welfare program has an optimal solution. This welfare optimum can be decentralized as a competitive equilibrium using transfers. Note that the Kuhn-Tucker conditions imply that because of (*U4*)

$$\hat{w}_0 \geq \hat{p}_1 \hat{x}_{0,1} + \hat{\mu}_1 \hat{w}_1 \geq \hat{p}_1 \hat{x}_{0,1} + \hat{p}_1 \hat{x}_{1,1} + \hat{p}_2 \hat{x}_{1,2} + + \hat{\eta}_2 \hat{w}_2 \geq \ldots \; , \tag{2.20}$$

from which follows that

$$\hat{w}_0 \geq \sum_{t=1,\ldots,\infty} \hat{p}_t (\hat{x}_{t-1,t} + \hat{x}_{t,t}) \; . \tag{2.21}$$

Hence, the infinite sum of consumption values can be summed, (2.13) holds and transfers sum to zero. In particular, as the value of endowments can be summed, it follows that their value converges to zero, and that the income-expenditure budget (2.12) holds.

2.3.2. OLG equilibrium

In an OLG equilibrium, transfers do not follow from a dynastic welfare program, but are determined by 'transfer functions'. Till now, net transfers entered the budget constraint, without any reference to the intergenerational payments underlying these transfers. These payments become visible if net income transfers for each separate generation are converted into transfers between subsequent generations.

To describe the relation between net transfers to generations and a transfer sequence between subsequent generations, we start with the first generation. Let the subsequent transfer in period t from the old generation to the (next) young generation be denoted by Γ_t. The first generation (born before the first period, therefore denoted by t=0) receives a net transfer Φ_0. Because in every period, net transfers have to be equal to zero, the transfer to the old have to be paid by the second generation that is young in the first period, thus $\Phi_0 = -\Gamma_1$, or equivalently, $\Gamma_1 = -\Phi_0$. For the entire life-cycle, the second generation (born in the first period) receives a net transfer Φ_1. Because this generation has already received a transfer Γ_1 in the first period, it will pay the surplus as a transfer Γ_2 in the second period to the young generation in that period, $\Gamma_2 = \Gamma_1 - \Phi_1$. Using induction for all future periods, we have $\Gamma_{t+1} = \Gamma_t - \Phi_t = -\sum_{\tau=0,\ldots,t} \Phi_\tau$. Given transfers summing to zero (2.13), this results in $\Gamma_T = \sum_{t=T,\ldots,\infty} \Phi_t$, in every period, the old generation pays an amount to the young generation that is equal to the cumulative net transfers of all future generations. For the equilibrium existence proof given hereafter in this section, we describe an economy in which all generations living after a period T have a common budget, and are represented by one central planner. Then, we might interpret Γ_T as the income transfer of this planner, and use

Φ^T for convenience, where the superscript distinguishes the planner from the individual generation.

The choice between using transfers from and to a public authority, and transfers from old to young, depends on the purpose of the transfer system. This is illustrated below in two examples. The first example uses income transfers to redistribute initial endowments.

EXAMPLE 2.14. *Intergenerational transfers by means of intra-temporal endowments redistribution*

Consider an economy with constant population, n=1, and constant endowments ω_y and ω_o. Assume that it is preferable, for some reason, to have a redistribution of income that can be associated to an alternative endowments distribution $\tilde{\omega}_y, \tilde{\omega}_o$, where $\tilde{\omega}_y + \tilde{\omega}_o = \omega_y + \omega_o$. The transfer system that implements the redistribution is given by $\Gamma_t = p_t(\tilde{\omega}_y - \omega_y) = p_t(\omega_o - \tilde{\omega}_o)$. ■

Next, in anticipation of the discussion in Chapter 4, we use transfers to generate income from rights for a physical share in the output of a production process. To maintain flexibility, that is to prevent inefficient production rigidities, these are formulated as 'pseudo endowments'.

EXAMPLE 2.15. *Intergenerational transfers by means of 'pseudo endowments'*

Consider an economy where a part of the initial stock of firm j, $0 \le \overline{k}_j \le k_j^0$, has the potential to generate a sustained positive flow of output: $\overline{y}_j \ge 0$, $F_j(\overline{y}_j, -\overline{k}_j, \overline{k}_j) \le 0$. It is possible to take the value of the stock from the first generation, and to redistribute the value as pseudo endowments $\overline{y}_j$ to future generations. The transfer scheme that implements this idea is given by $\Phi_0 = -\psi_{j,1}\overline{k}_j$, and $\Phi_t = (\psi_{j,t} - \psi_{j,t+1})\overline{k}_j$ for t=1,...,∞. From the Kuhn-Tucker conditions for profit maximization, it follows that $\Phi_t \ge p_t \overline{y}_{j,t} \ge 0$: every generation, except the first, receives a net transfer when young equal to or greater than the pseudo endowment $\overline{y}_j$. Note that the same transfer system can be represented by $\Gamma_t = \psi_{j,t}\overline{k}_j$. However, the income effect is more clearly expressed in the former representation. ■

To generalize these examples, we define transfer functions, $\Lambda:(p,\psi) \to \Gamma$. The transfer function returns a vector of transfers for all periods. Its domain is the set of all possible price vectors: $\Lambda: \mathbf{R}_+^{H\times\infty} \times \mathbf{R}_+^{C\times\infty} \to \mathbf{R}^\infty$. Transfers will play an important role establishing efficiency in Chapter 3 and sustainability in Chapter 4. Transfer functions have to satisfy the following conditions.

ASSUMPTION 2.16. *The transfer function* $\Lambda:(p,\psi)\rightarrow\Gamma$

$\Lambda 1$: *is a linear function of prices,*

$\Lambda 2$: *depends on present and past prices* $(p_1,\ldots,p_t,\ \psi_1,\ldots,\psi_{t+1})$ *only,*

$\Lambda 3a$: *for generation* $t=1,\ldots,\infty$, *it leaves a strictly positive real income measured in units of the desired commodity, i.e., there is an* $\varepsilon>0$:

$p_t n_t \omega_y + p_{t+1} n_t \omega_o + \Lambda_t(p,\psi) \geq (p_t^k + p_{t+1}^k)\varepsilon + \Lambda_{t+1}(p,\psi)$ *and*

$\Lambda 3b$: *it leaves a strictly positive real income in the first period measured in units of the desired commodity, i.e., there is an* $\varepsilon>0$: $p_t n_t \omega_y + \Lambda_t(p,\psi) \geq p_t^k\varepsilon$

where p^k *is the price of the desired commodity k. Moreover, the transfers leave the first generation with a strictly positive share of his income from capital,*

$\Lambda 4$: *there is an* $\varepsilon>0$: $p_1 n_0 \omega_o + \psi_1 k_1^P - \Lambda_1(p,\psi) \geq p_1^k\varepsilon + \varepsilon\psi_1 k_1^P$.

There are several notes to be made regarding the transfer functions. First, the restriction to linear transfer functions has the advantage that these preserve convexity (when applied to a set) which simplifies the equilibrium existence proof given further in the chapter. Existence can also be proven for the more general class of continuous transfer functions, but the proof would become more complex. The restriction to linear transfers does not mean that transfers are overly simplified, as we will show in Chapters 3 and 4 where two non-trivial linear transfer functions are constructed that establish efficiency and sustainability respectively. Dependence of these transfer functions on past and present prices only implies that they do not require perfect foresight of the authority implementing the transfers. Moreover, the existence proof should be kept simple. The third condition ensures that after applying the transfers, all generations have strictly positive income and can make non-negative savings from their first period to their second period of life. The fourth condition tightens condition ($\Lambda 3b$) when applied for the first generation.

As for the dynastic economy, we take the assumptions for an OLG economy together before we define the OLG equilibrium:

ASSUMPTION 2.17. *The standard assumptions for an OLG economy are*

O1: *N1, U1-U4,* $\Omega 1$, $\Omega 2$, *F1-F6, and* $\Lambda 1$-$\Lambda 4$

Given the conditions for transfer functions, an OLG equilibrium can be defined.

DEFINITION 2.18. *An OLG competitive equilibrium is a bundle* $(\hat{x},\hat{y},\hat{k},\hat{k}^P,\hat{p},\hat{\psi},\hat{\Phi})$ *that satisfies the conditions E1-E5 for a competitive equilibrium as well as the additional requirement that*

E6: transfers are given by a transfer function, $\Phi_t - \Phi_{t+1} = \Lambda_t(p,\psi)$, *as in Assumption* 2.16.

As the OLG equilibrium is characterized by the existence of an explicit transfer system, the question now arises how transfers affect the equilibrium. One way by which transfers can affect the competitive equilibrium is through the balance of savings and capital. Savings enter the model as an explicit variable once the budget constraint is split into two budget decisions for each period in which the generation is alive. When the generation is young, expenditures on consumption plus savings are equal to revenues from endowments plus transfers received by the old generation.

$$p_t x_{1,t} + S_{t+1} = p_t n_t \omega_y + \Gamma_t \tag{2.22}$$

Note that the subscript for savings refers to the period, not to the generation. S_{t+1} denotes the savings at the end of period t, i.e. the beginning of period $t+1$, by generation t. This practice has the advantage that subscripts correspond in the capital-savings balances written below. When old, expenditures on consumption plus payments to the 'new' young generation match revenues from endowments plus cashed savings.

$$p_{t+1} x_{1,t+1} + \Gamma_{t+1} = p_{t+1} n_t \omega_o + S_{t+1}, \tag{2.23}$$

for $t=1,\ldots,\infty$. Note that in competitive equilibrium, the budget constraint is binding so that we can consider equalities, rather than constraints.

Let us consider the first period. Summing the budget equations for both generations alive in that period, the commodity equation (2.7) for the first period, and the stocks equation (2.8) for the first period, we have:

$$S_{t+1} = \sum_{j=1,\ldots,J} \psi_{j,t+1} k_{j,t+1} \tag{2.24}$$

for $t=1$. Summation of the same equations for next periods shows that equation (2.24) is also valid for all other periods. In words, the value of the capital transferred to the next period is equal to private savings. This might suggest that financial transfers to future generations have no effect on capital transfers to future generations. This, however, is not the case. If we use the other representation of transfers, assuming the existence of a public authority that implements all transfers to and from generations in their first period, transfers to future generations are described as positive public savings, denoted by A_t:

$$A_t = \sum_{\tau=t,\ldots,\infty} \Phi_\tau = -\sum_{\tau=0,\ldots,t-1} \Phi_\tau = \Gamma_t . \tag{2.25}$$

The two-period budget equation becomes:

$$p_t x_{t,t} + S_{t+1} = p_t n_t \omega_y + \Phi_t , \tag{2.26}$$

$$p_{t+1} x_{t,t+1} = p_{t+1} n_t \omega_o + S_{t+1} . \tag{2.27}$$

Now, the resulting capital-savings equation is:

$$S_{t+1} + A_{t+1} = \sum_{j=1,\ldots,J} \psi_{j,t+1} k_{j,t+1} . \tag{2.28}$$

From a partial analysis point of view, transfers to future generations result in positive public savings (2.25) that increase the capital stock (2.28). This result does not conflict with (2.24). The point is that in the absence of a public authority, transfers to future generations are held as private savings, and thus, they do not enter the capital balance explicitly.

Finally, existence of an OLG competitive equilibrium has to be extablished. The proof uses the concept of a T-period competitive equilibrium that shrinks to the infinite horizon equilibrium for $T \to \infty$, similar to the argument used to show that an allocation actually attains the welfare optimum in Lemma 2.10. The proof follows [Balasko and Shell 1980] [Balasko *et al.* 1980], and [Ginsburgh and Keyzer 1997, Sections 8.2 and 8.3]. A T-period competitive equilibrium is a competitive equilibrium where all conditions are only imposed for the first T periods. For this T-period equilibrium, the existence proof is based on a modified Negishi welfare program [Negishi 1960] [Gerlagh 1998b]. As T is delayed to ∞, the T-period competitive equilibrium satisfies all conditions *E1-E6* of the (infinite) competitive equilibrium. To prove that there exists a T-period competitive equilibrium for $T \to \infty$, it is necessary that both the variable and the dual space of prices be compact. For this, we will use the concept of period-t prices.

The infinite sequence of prices, p_t, and $\psi_{j,t}$, $t=1,\ldots,\infty$, is divided in two-period price vectors normalized for period t, $(\hat{p}_t^t, \hat{p}_{t+1}^t, \hat{\psi}_{j,t}^t, \hat{\psi}_{j,t+1}^t)$, such that the period-$t$ price is equal to the infinite price vector at period t and t+1 up to a constant: $\exists\lambda:(\hat{p}_t^t, \hat{p}_{t+1}^t, \hat{\psi}_{j,t}^t, \hat{\psi}_{j,t+1}^t) = \lambda_t(\hat{p}_t, \hat{p}_{t+1}, \hat{\psi}_{j,t}, \hat{\psi}_{j,t+1})$. The infinite price vector can be represented by an infinite sequence of (bounded) period-t prices. Demanding that every period-t price lies within the compact unit simplex, we ensure that the infinite sequence is in a compact product space (Tychonoff's theorem). The period-t prices contain all necessary information at period t for both generations and producers.

To prove existence of a T-period competitive equilibrium, we use a truncated economy, in which generations $T,T+1,\ldots,\infty$ are replaced by a dynastic planner which maximizes $\bar{\beta}\,\mathrm{U}(x_{T,T}, x_{T,T+1}, n_T) + \bar{\psi}k_{T+1}$ for some fixed scalar $\bar{\beta}$ and vector $\bar{\psi}$. To construct a compact set of possible value functions, we demand $(\bar{\beta},\bar{\psi}) \in S^{J\times C}$ and $x_{T,T+1}\in$X. The objective vector $\bar{\psi}$ of the dynastic planner is restricted to non-trivial stocks, i.e., stocks for which there is a feasible production plan which produces a strictly positive level of that stock by that firm in period T+1. Let this sub-set be denoted by an apostrophe, $(J\times C)'\subseteq J\times C$, thus $(\bar{\beta},\bar{\psi}) \in S^{(J\times C)'}$. In this truncated economy, prices are also presented as period-t prices.

Now, the T-period competitive equilibrium is defined by.

DEFINITION 2.19. *An OLG T-period competitive equilibrium is a bundle* $(\hat{x},\hat{y},\hat{k},\hat{k}^P,\hat{p},\hat{\psi},\hat{\Phi})$, *where the prices refer to sequences of period-t prices, such that*

- *E1a':* *for all periods, period-t prices are non-zero* $0 < \|\hat{p}_t^t\|, \|\hat{p}_{t+1}^t\|, \|\hat{\psi}'^t_{j,t}\|, \|\hat{\psi}'^t_{j,t}\|$, *on the unit simplex,* $\|(\hat{p}_t^t, \hat{p}_{t+1}^t, \hat{\psi}'^t_{j,t}, \hat{\psi}'^t_{j,t+1})\|_1 = 1$, *and*
- *E1b':* *period-t prices are consistent over time:* $\forall t{:}\exists\lambda_t{:}(\hat{p}_{t+1}^{t+1}, \hat{\psi}^{t+1}_{j,t+1}) = \lambda_t(\hat{p}_{t+1}^t, \hat{\psi}'^t_{j,t+1})$, *and*
- *E2a':* *consumption maximizes utility subject to the budget constraints for period-t prices for periods t=1,…,T–1,*
- *E2b':* *the dynastic planner maximizes* $\bar{\beta}\,\mathrm{U}(x_{T,T}, x_{T,T+1}, n_T) + \bar{\psi}k_{T+1}$ *subject to his budget constraint, and*
- *E3':* *production maximizes profits subject to the transformation constraints for period-t prices for t=1,…,T.*

Moreover,

- *E4':* *the commodity balances hold and are binding for non-zero prices for periods t=1,…,T,*
- *E5':* *stock balances hold and are binding for non-zero stock prices for periods t=1,…,T+1, and*
- *E6':* *for generations 0,…,T–1, transfers are given by the transfer function. The dynasty receives a transfer given by* $\Phi^T = -\sum_{t=0,\ldots,T-1}\Phi_t$,

where $\Phi^T = \Gamma_T$ is the transfer to the dynastic planner. The superscript T is used to mark the difference from the transfer to generation T as it would be generated by the transfer function. Assumption (*A3b*) ensures that the dynasty has strictly positive income.

Let the set of equilibria for this truncated economy (Definition 2.19) be denoted by $\mathbf{E}_T$. From the definition, it follows immediately that $\mathbf{E}_{T+1} \subseteq \mathbf{E}_T$. To prove non-emptiness of $\mathbf{E}_T$, we will define a fixed point mapping on the welfare weights of a truncated welfare program. The welfare program aggregates natural logarithms of the extended utilities, which are homogeneous of degree one and this will produce a fixed point mapping Q(.) specified below by (2.32)-(2.34) that simplifies the existing fixed point mappings [Gerlagh 1998b].

The Negishi welfare program that supports the T-period equilibrium is given by:

PROGRAM 2.20. *Negishi welfare program for truncated economy*

$$max\ w = \sum_{t=0,\ldots,T-1} \alpha_t \ln(\upsilon_t) + \alpha_T \ln(\overline{\beta}\upsilon_T + \overline{\psi}k_{T+1} + \eta_T)$$

$\eta_t \geq \varepsilon n_t$ and $\upsilon_t \geq U(0,0,n_t)$ for $t=0,\ldots,T$, $x_{t-1,t}$, $x_{t,t} \geq 0$, and $y_{j,t}$ for $j=1,\ldots,J$, $t=1,\ldots,T$, $k_{j,t} \geq 0$ for $t=1,\ldots,T+1$, $k^P_{j,t} \geq 0$ for $t=2,\ldots,T+1$

subject to

$\eta_t \leq n_t$	(q_t)	$t=0,\ldots,T$
$\upsilon_0 \leq U((\eta_0/n_0)x_{0,0}, x_{0,1}, \eta_0)$	(ζ_0)	
$\upsilon_t \leq U(x_{t,t}, x_{t,t+1}, \eta_t)$	(ζ_t)	$t=1,\ldots,T-1$
$\upsilon_T \leq U(x_{T,T}, (\eta_T/n_T)x_{T,T+1}, \eta_T)$	(ζ_T)	
$x_{t-1,t} + x_{t,t} \leq n_{t-1}\omega_o + n_t\omega_y + \sum_j y_{j,t}$	(p_t)	$t=1,\ldots,T$
$k_{j,t} \leq k^P_{j,t}$	$(\psi_{j,t})$	$j=1,\ldots,J;\ \ t=1,\ldots,T+1$
$F_{j,t}(y_{j,t}, -k_{j,t}, k^P_{j,t+1}) \leq 0$	$(\lambda_{j,t})$	$j=1,\ldots,J;\ \ t=1,\ldots,T$

for $\alpha \in S^T$, and $0<\varepsilon<1$. We make two remarks. First, the lower bounds for η and υ are necessary to keep the objective function well defined and continuous. However, as the objective function is (via aggregate utilities) increasing in η and υ (Lemma 2.34), it follows that the dual variables associated with the lower bound must be zero. Therefore, we do not include these bounds in the program, and only use them to define the range for η and υ. Secondly, the asymmetric extended utility functions for generation 0 and T are needed to ensure homogeneity in the variables ($x_{0,0}$ and $x_{T,T+1}$ are not variable in the program). Because of constant returns to scales of the extended utility functions, the optimum of the welfare program satisfies

$$p_1 x_{0,1} + q_0 \eta_0 = \alpha_0 \tag{2.29}$$
$$p_t x_{t,t} + p_{t+1} x_{t,t+1} + q_t \eta_t = \alpha_t \ (1 \le t \le T-1) \tag{2.30}$$
$$p_T x_{T,T} + \psi_{T+1} k_{T+1} + q_T \eta_T = \alpha_T \tag{2.31}$$

Now, define the mapping $Q(.;\bar{\beta},\bar{\psi},x_{T,T+1}) : S^T \to S^T : \alpha \to \tilde{\alpha}$ by

$$\tilde{\alpha}_0 = p_1(\alpha) n_0 \omega_o + q_0(\alpha) n_0 + \psi_1(\alpha) k_1^p + \Phi_0(\alpha) \tag{2.32}$$
$$\tilde{\alpha}_t = p_t(\alpha) n_t \omega_y + p_{t+1}(\alpha) n_t \omega_o + q_t(\alpha) n_t + \Phi_t(\alpha) \ (1 \le t \le T-1) \tag{2.33}$$
$$\tilde{\alpha}_T = p_T(\alpha) n_T \omega_y + q_T(\alpha) n_T + \Phi^T(\alpha) \tag{2.34}$$

where $\bar{\beta},\bar{\psi},x_{T,T+1}$ are parameters of the fixed point mapping Q(.), we explicitly wrote that the dual variables p and q and the transfers Φ (which are based on the linear transfer function Λ(.) using dual prices) are correspondences of the welfare weights. Assumption (Λ2) ensures that the transfers do not depend on future prices not generated by the program. Transfers are linear in prices (*Λ1*), and hence, the fixed point mapping Q(.) is linear in the dual variables, which are upper-semi-continuous and convex valued (maximum theorem [Ginsburgh and Keyzer 1997, Theorem A..3.1], using *U1, U2, U4, F1, F2, F4, F5, F6*). It follows that the mapping $Q: \alpha \to \tilde{\alpha}$ is upper-semi-continuous and convex valued. Moreover, because of constant returns to scale of production (*F3*), we have

$$\sum_{t=0,\dots,T} \tilde{\alpha}_t = \sum_{t=0,\dots,T} \alpha_t = 1, \tag{2.35}$$

and because every generation has strictly positive real income after transfers (*Λ3a*), the image of Q(.) is on the unit simplex. We can now state and prove:

LEMMA 2.21. *If Assumptions O1 are satisfied, then for any* $(\bar{\beta},\bar{\psi},x_{T,T+1}) \in S^{(J\times C)'} \times \mathbf{X}$, *the mapping* $Q(.;\bar{\beta},\bar{\psi},x_{T,T+1})$ *has a fixed point,* $Q(\hat{\alpha};\bar{\beta},\bar{\psi},x_{T,T+1}) = \hat{\alpha}$, *which has strictly positive welfare weights.*

Proof. The existence of a fixed point follows from Kakutani's fixed point theorem. Before we prove that welfare weights are strictly positive, we note that, in the fixed point, budget constraints are satisfied as the 'population constraint' $\eta_t \le n_t$ satisfies the complementarity condition, $q_t \eta_t = q_t n_t$ $(0 \le t \le T)$.

Because we consider the simplex, there must be one strictly positive welfare weight. If any welfare weight α_t for $t=0,\dots,T-1$ is positive, it follows that the strictly desired commodity has strictly positive prices in the periods t and $t+1$, and hence, that the preceding generation $t-1$ and subsequent generation $t+1$ have positive income as well, which implies that their welfare weights must be strictly positive. In this way, we construct a 'chain' of positive welfare weights, from which it follows that all welfare weights must be strictly positive. There is one exception to this rule: if all welfare weights $\alpha_0,\ldots,\alpha_{T-1}$ are zero, α_T is unity, and $\overline{\beta} = 0$. Now, the strictly desired commodity has a zero price in all periods, and we cannot produce the chain. We will prove that this occurrence is impossible, and that there must be another strictly positive α_t. If $\overline{\beta} = 0$, then $\overline{\psi}$ must be non-zero for non-trivial stocks, so that $\psi_{T+1}k_{T+1}>0$, and either $\psi_T k_T>0$ or $p_T y_T<0$. The second possibility results in $p_T x_{T-1,T}>0$ as $\overline{\beta} = 0$, and hence implies that $\alpha_{T-1}>0$ in which case the chain is restored. If $\psi_T k_T>0$, it follows that $\psi_{T-1}k_{T-1}>0$ or $p_{T-1}y_{T-1}<0$. By inductive reasoning, we conclude that either some α_t is positive, or $\psi_1 k_1>0$. But now, (*A4*) states that the first generation has strictly positive income so that its welfare weight is strictly positive, and again, the chain is restored. ■

The next lemma provides the necessary conditions to show that the sequence of T-period equilibria converges to the infinite horizon OLG competitive equilibrium.

LEMMA 2.22. *If Assumptions O1 are satisfied, than the set of equilibria for the truncated economy (Definition* 2.19), E_T, *is non-empty and closed.*

Proof. The set of equilibria E_T consists of the optimal solutions, including dual variables, of Program 2.20 for a given set of $(\hat{\alpha}, \overline{\beta}, \overline{\psi}, x_{T,T+1}) \in S^T \times S^{(J\times C)'} \times \mathbf{X}$, where $\hat{\alpha}$ is a fixed point of Q(.): $E_T \sim \{(\alpha, \overline{\beta}, \overline{\psi}, x_{T,T+1}) \in S^T \times S^{(J\times C)'} \times \mathbf{X} \mid Q(\alpha; \overline{\beta}, \overline{\psi}, x_{T,T+1}) = \alpha\}$. Non-emptiness follows from the previous lemma. As the fixed point mapping is continuous, the set is closed. By the maximum theorem, the optimal allocations are a continuous function of the parameters $(\hat{\alpha}, \overline{\beta}, \overline{\psi}, x_{T,T+1})$, hence the set of equilibrium allocations is also closed.

■

We can now state and prove:

THEOREM 2.2. *If Assumptions O1 are satisfied, then there exists an infinite horizon OLG equilibrium (Definition* 2.18).

Proof. We follow Balasko *et al.* [1980]. The sequence of equilibria is decreasing $\mathrm{E}_1 \supseteq \mathrm{E}_2 \supseteq \ldots$ and the infinite intersection satisfies all conditions of an infinite horizon OLG equilibrium for all periods, so that it is sufficient to prove that the infinite intersection $\mathrm{E}_\infty = \cap_{t=1}^{\infty} \mathrm{E}_T$ is non-empty. As $\mathrm{E}_1 \supseteq \mathrm{E}_2 \supseteq \ldots$ is a decreasing sequence of non-empty closed sets within the compact commodity/price product space (Lemma 2.6), non-emptiness of the intersection follows [Ward 1972, Theorem 24]. The infinite price sequences for commodities and stocks can easily be recovered from the period-t prices. Moreover, prices are bounded for any finite period. It is clear that these prices, together with the allocations satisfy all conditions (*E1,...,E6*) for the OLG infinite horizon competitive equilibrium. Hence, a competitive equilibrium exists. ■

This completes the description of the economy. Existence of the infinite horizon competitive equilibrium for both the dynastic and the OLG economy has been established, for which the analysis closely followed the literature. The use of logarithmic extended utility functions enabled us to specify a linear feed back mapping Q(.), and therefore, we were able of relaxing the assumptions concerning continuous differentiable and strictly concave utility functions.

The next section discusses some properties of the competitive equilibrium in the presence of environmental resources that possess the typical characteristics mentioned in Section 1.4.1.

2.4. EXHAUSTIBLE RESOURCES, STEADY STATES AND EQUILIBRIUM PATHS

2.4.1. Introduction

In this section, we analyze the infinite horizon allocations for economies in which there are exhaustible resources with amenity value. The theory of long term behavior has been concerned with the equilibrium path followed by the variables, to find out whether it converges, diverges, cycles, or behaves chaotically. Traditionally, the emphasis has been on the convergence of a competitive equilibrium to a steady state, a condition in which consumption, production and stocks are constant over time. If convergence takes place, the steady state can be identified with the long term equilibrium. There is a huge body of literature on steady states, but it has not been recognized that there is an interesting case if there are exhaustible resources with amenity or use values (that is, resources that exhibit the characteristics 2 and 3 mentioned in Section 1.4.1). We show that, in that case, steady states form a continuum, and the equilibrium paths exhibit path dependence.

Exhaustible resources with amenity or use values have been studied many years ago in terms of their option value [Krutilla 1967], and in this context, they are referred to as 'irreplaceable', which implies that they cannot be regenerated, and which makes clear that they have limited scope for substitution [Henry 1974]. Typical analysis of option values is partial and emphasizes uncertainty, and the value of reducing this uncertainty, because the uncertainty requires a cautious management which possibly blocks the most profitable management. Here, we study another consequence of these resources: their effects on the steady states and the equilibrium paths in a competitive equilibrium with perfect foresight.

The literature on steady states has not given much attention to exhaustible resources with amenity values. This might be explained by the history of environmental resource economics. In the 1970s, growth theory was mainly concerned with the question how future growth would be influenced by limited availability of environmental resources and the inputs they provide [Solow 1997]. Models were developed that paid extensive attention to the substitution between the extracted resource and man-made capital or technology, cf. [Stiglitz 1974a].

During the 1980s, awareness grew that the vulnerability of overexploited ecosystems might be a much greater threat to future human welfare. Such ecosystems can formally be modeled through exhaustible resources with amenity value [Krautkraemer 1985]. However, these refinements had little significance. This might be explained as follows. If the extraction of the resource is an essential production factor as in the classic models, the main effect of the amenity value is that it possibly restricts the exhaustion to a limited part of the initial resource stock, for example because of a threshold resource level below which damages become irreversible and unacceptable. The extraction reduction over time and the need for substitution remains the same as in the classic model, and therefore, the basic properties of the growth model remain unaltered: the substitution of renewable resources for extracted exhaustible resources determines whether economic growth can sustain, and this is not affected by an amenity value. Thus, exhaustible resources with amenity value have little impact.

However, sustained physical growth over an infinite horizon is a narrow perspective: On the one hand, if it is believed that sustained growth is possible, there is little reason to give up today's consumption for the sake of future's welfare. If, on the other hand, substitution of man-made capital for the decreasing exhaustible resources is insufficient, a doom scenario of decreasing production is unavoidable, and the concern for future generations is ambiguous because this doom can only be postponed. Thus, the growth perspective is useful if one is concerned with the existence of a sustainable equilibrium path, but it is of no use if one is not concerned with the existence issue but with the selection of an (un)sustainable equilibrium path.

Daly [1997] continually attacks the growth perspective, arguing that production is a physical activity, bounded to mass balances and the laws of thermodynamics. Therefore, continued growth requires the continued increase of material throughput, which he believes is incongruous with the bounds of planet earth. In Daly's perspective, the economy should become stationary, without growth or decline. Though his arguments are not always satisfactory (see replies by Solow [1997] and Stiglitz [1997]), they reflect a general feeling of dissent among ecological economists with what they regard neoclassical economics which, in their view, does not pay sufficient attention to natural regularities [Opschoor 1997].

The ecological economists do not consider the substitution between exhaustible resources and renewable alternatives as the main concern for future welfare. They point out that if welfare in the long term decreases because of economic activities in the short term, it is primarily because of the impossibility to recover or substitute for degraded 'renewable' resources such as ecosystems [Clark 1997]. They argue that environmental damages are not automatically compensated for in the future by economic growth. Indeed, the analysis in the next sections shows that if we forget about sustained economic growth and assume that the economy converges to a steady state, the present use of exhaustible resources changes the equilibrium allocation in a way that does not diminish over time. Postponing environmental policies has indefinite consequences, and intervention should not be delayed.

2.4.2. Steady states, general specification

For our steady state analysis, it will be practical to use the primal utility and transformation functions as in [Keyzer 1991] and [Lucas and Stokey 1984], though the steady state is often formulated using the excess demand format in which demand and production are invariant functions of prices (e.g. [Kehoe 1990], [Ginsburgh and Keyzer 1997, Sections 8.1.5, 8.2.5, 8.3.5]). Let us define a steady state with the help of the dynastic welfare Program 2.8 of Section 2.3.1 that uses the primal utility and transformation functions, and for convenience, assume a constant population level, $n_t=n$. Recall that if we start the welfare program in period t, it takes the available capital stock, k_t^p, and the consumption of the currently old when they were young, $x_{t-1,t-1}$, as inputs. Assume that we solve the program in period t, and find that the optimal planned stock in the next period t+1 is equal to the initial stock available, $k_{t+1}^p = k_t^p$, and that the consumption of the young is equal to the consumption of the currently old when they were young, $x_{t,t}=x_{t-1,t-1}$. We then have a steady state, since solving the program again at t+1 with the new values will reproduce the same outcome for period t+2, and so forth.

As all variables are constant, we can specify steady state variables as $x_y=x_{t,t}$, $x_o=x_{t,t+1}$, $y=y_t$, $k=k_t$, $k^p=k_t^p$ and $w=w_t$, where the subscripts 'y' and 'o' denote the young and old generations, respectively. Furthermore, it follows from the welfare program that the welfare weights μ_t decrease at a constant factor β, $\mu_{t+1}=\beta\mu_t$, which satisfies

$$\beta = \mathrm{H}_2(\mathrm{U}(x_y, x_o; n), w). \tag{2.36}$$

Solution of the welfare program for the consumption of the young generation in period t and t+1 gives

$$p_t \geq \mu_t \, \mathrm{H}_1(.) \, \mathrm{U}_1(x_y, x_o; n) \;\perp\; x_y \geq 0, \tag{2.37}$$

and

$$p_{t+1} \geq \mu_{t+1} \, \mathrm{H}_1(.) \, \mathrm{U}_1(x_y, x_o; n) \;\perp\; x_y \geq 0, \tag{2.38}$$

from which we conclude that $p_{t+1}=\beta p_t$, if we abstract from commodities with zero consumption. Similarly, $\psi_{t+1}=\beta\psi_t$, and $\lambda_{t+1}=\beta\lambda_t$. We can thus specify the steady state prices p and ψ, and dual variables μ and λ up to normalization by $p_t=\beta^t p$, $\psi_t=\beta^t\psi$, $\mu_t=\beta^t\mu$, and $\lambda_t=\beta^t\lambda$. The constraints of the welfare program are thus rewritten for the steady state:

$$w = \mathrm{H}(\mathrm{U}(x_y, x_o; n), w) \tag{2.39}$$

$$x_y + x_o \le n\omega_o + n\omega_y + \textstyle\sum_j y_j \ \perp p \ge 0 \tag{2.40}$$

$$k_j \le k_j^p \perp \psi_j \ge 0 \qquad j=1,\ldots,J \tag{2.41}$$

$$\mathrm{F}_j(y_j, -k_j, k_j^p) \le 0 \ \perp \ \lambda_j \ge 0 \qquad j=1,\ldots,J \tag{2.42}$$

The first-order optimality conditions for consumption are:

$$p \ge \mu\,\mathrm{H}_1(.)\,\mathrm{U}_1(x_y, x_o; n) \ \perp \ x_y \ge 0, \tag{2.43}$$

$$\beta p \ge \mu\,\mathrm{H}_1(.)\,\mathrm{U}_2(x_y, x_o; n) \ \perp \ x_o \ge 0, \tag{2.44}$$

and for production they are:

$$p = \lambda\nabla_1 \mathrm{F}_j(y_j, -k_j, k_j^p), \qquad j=1,\ldots,J \tag{2.45}$$

$$\psi_j \ge \lambda\nabla_2 \mathrm{F}_j(y_j, -k_j, k_j^p) \ \perp \ k_j \ge 0, \qquad j=1,\ldots,J \tag{2.46}$$

$$\beta\psi_j \le \lambda\nabla_3 \mathrm{F}_j(y_j, -k_j, k_j^p) \ \perp \ k_j^p \ge 0. \quad j=1,\ldots,J. \tag{2.47}$$

The number of equations is equal to the number of variables, w, p, ψ_j, λ_j, β, x_y, x_o, y_j, k_j, k_j^p, where we choose $\mu=1$ to normalize prices. If the system of equations (2.36), (2.39)-(2.45) has a regular Jacobian, it follows that steady state values are distinct, provided they exist.

The steady state conditions for the OLG economy are similar, but we must exclude the welfare equation (2.39), and replace the equations (2.36), (2.43) and (2.44) associated with the aggregator function by the budget equation and two marginal utility conditions:

$$px_y + \beta p x_o = p\omega_y + \beta p\omega_o, \tag{2.48}$$

$$p \ge \mu\,\mathrm{U}_1(x_y, x_o; n) \ \perp \ x_y \ge 0, \tag{2.49}$$

$$\beta p \ge \mu\,\mathrm{U}_2(x_y, x_o; n) \ \perp \ x_o \ge 0. \tag{2.50}$$

Thus, the OLG economy has one steady state equation and one variable less than the dynastic economy (the welfare measure w). Again, if the Jacobian of the system of equations is regular, steady state values are distinct, provided they exist. We formally define the steady states:

DEFINITION 2.23. *A steady state allocation is a bundle* $(\hat{x}, \hat{y}, \hat{k}, \hat{k}^p, \hat{p}, \hat{\psi})$ *that satisfies the system* (2.36), (2.39)-(2.45) *for a dynastic economy or* (2.40)-(2.42), (2.45), (2.48)-(2.50), *for an OLG economy.*

So far, the analysis does not deviate from the literature. Usually, the Jacobian is assumed to be regular almost everywhere, which implies that singularity of the Jacobian is considered exceptional (Sard's theorem, see e.g. [Ginsburgh and Keyzer 1997, Appendix A, Theorem 7.5]). Recent literature indicates that structural singularity could arise in steady states of monetary economies with financial assets but without fiat money [Gottardi 1996]. However, the example below makes clear that even in a real economy without money, the assumption of a structurally regular Jacobian is not valid if there are exhaustible resources with amenity or use values. This has important consequences.

2.4.3. A continuum of steady states, illustration, generalization, and discussion

We set up a simple OLG economy in which a single exhaustible resource with amenity value produces a continuum of steady states, and discuss the consequences for the competitive equilibrium path. A more elaborated example for a continuous time dynastic economy is given in [Gerlagh and Keyzer 1998a].

EXAMPLE 2.24. *An exhaustible resource with use value generating a continuum of steady states.*

Let there be a one-good economy, whose resource has regeneration factor of 2. The extracted resource can be used to produce the consumption good: $x_y + x_o = 1 + y$ and $y_t + k_{t+1}^p \le 2k_t$. To prevent unbounded accumulation of capital (Assumption *F4*), we may additionally assume that there is some capacity constraint in production, which however, does not become binding in the example below. We therefore omit the explicit specification of this constraint. Generations maximize a Cobb-Douglas utility function in which the second period has expenditure share 1/3. The young generation has one (unit of) endowment, the old generation none. The steady state equations are:

$$x_y + x_o = 1 + y \tag{2.51}$$
$$k = k^p \tag{2.52}$$
$$y + k^p \le 2k \perp \lambda \ge 0 \tag{2.53}$$
$$x_y + \beta x_o = 1 \tag{2.54}$$
$$\beta x_o = 1/2\, x_y \tag{2.55}$$
$$1 = \lambda \tag{2.56}$$
$$\psi = 2\lambda \tag{2.57}$$
$$\beta\psi = \lambda \tag{2.58}$$

where prices are normalized by p=1. The solution is given by x_y=2/3, x_o=2/3, y=1/3, k=k^p=1/3, β=1/2, ψ=2, λ=1. In the steady state, savings, 1–x_y=1/3, balance with capital, βψk=1/3. So far, this steady state is not unusual. Now, let us assume that the resource is exhaustible. Extraction of the resource for consumption is still possible, but it is not possible to recover the resource afterwards by abandoning extraction. This property is formally represented by an additional constraint, $k_{t+1}^p \leq k_t$, which implies that any extraction of the resource is irreversible. The steady state system changes into

$$x_y + x_o = 1 + y \tag{2.59}$$
$$k = k^p \tag{2.60}$$
$$y + k^p \leq 2k \perp \lambda_1 \geq 0 \tag{2.61}$$
$$k^p \leq k \perp \lambda_2 \geq 0 \tag{2.62}$$
$$x_y + \beta x_o = 1 \tag{2.63}$$
$$\beta x_o = 1/2\, x_y \tag{2.64}$$
$$1 = \lambda_1 \tag{2.65}$$
$$\psi = 2\lambda_1 + \lambda_2 \tag{2.66}$$
$$\beta\psi = \lambda_1 + \lambda_2 \tag{2.67}$$

There is one new equation (2.62), and one new variable, λ_2. The previous solution is still valid, for λ_2=0. However, the Jacobian of the system of equations has become structurally singular. The second and fourth equation partly coincide, and as a result, the steady states form a continuum. To generate the continuum of steady states, take any 0<k≤1/3, and we then have the solution x_y=2/3, x_o=1/3+k, y=k^p=k, β=1/(1+3k), ψ=1+1/3k, λ_1=1, λ_2=1/3k–1. Again, life-cycle savings, 1–x_y=1/3, balance with capital, βψk=1/3. ■

Though no special assumption was made, the steady state of this economy has a structurally singular Jacobian and a continuum of steady states. Interestingly, the steady state to which the economy converges is not indeterminate; it is selected as a part of the competitive equilibrium path.

For the continuum of steady states to be meaningful, the resource should be exhaustible, and it should also produce an irreplaceable good (the amenity value). It is not necessary to require that welfare should drop to zero in absence of the resource, as in [Dasgupta and Heal 1974]. However, if there exists a perfect substitute, all equilibria in the continuum will yield the same welfare. To clarify this point, consider the example above, and assume that there is an alternative man-made resource (which is not exhaustible) that provides the same services as the exhaustible environmental resource. If the environmental resource level drops below 1/3, an increase of the man made resource is fully capable of compensating this loss. The steady states form a continuum, and all yield the same welfare.

Next, let us develop the point in terms of the general model. Assume that firm j uses an exhaustible resource c. Usually, the limited regeneration capacity will be treated as part of the transformation function F_j, but for the analysis here, we need to treat this constraint separately. We append the constraint for the exhaustible resource to the steady state equation (2.42),

$$\bar{1}^c k_j^p \leq \bar{1}^c k_j \ \perp \ \lambda_{2,j}^c \geq 0, \tag{2.68}$$

where $\bar{1}^c$ denotes the vector with its c-th element equal to unity and other elements equal to zero, $\lambda_{2,j}^c \geq 0$ is a scalar. Equations (2.46) and (2.47) are changed into:

$$\psi_j \geq \lambda \nabla_2 F_j(y_j, -k_j, k_j^p) + \lambda_{2,j}^c \bar{1}^c \ \perp \ k_j \geq 0 \tag{2.69}$$

$$\beta \psi_j \leq \lambda \nabla_2 F_j(y_j, -k_j, k_j^p) + \lambda_{2,j}^c \bar{1}^c \ \perp \ k_j^p \geq 0 \tag{2.70}$$

We have added one equation (2.68) and one variable, $\lambda_{2,j}^c \geq 0$. The original solution to the steady state system remains valid, for $\lambda_{2,j}^c = 0$. The new equation (2.68) partly coincides with the original equation (2.42), and therefore, the Jacobian of the new steady state system is structurally singular, and solutions might exist for other given positive values of $\lambda_{2,j}^c$. We state and prove:

THEOREM 2.3. *If Assumptions D1 or O1 are satisfied for a dynastic or OLG economy respectively, and if the steady state equations* (2.36), (2.39)-(2.45), (2.69), (2.70) *(or* (2.40)-(2.42), (2.45), (2.48)-(2.50), (2.69), (2.70) *for the OLG economy) have a regular Jacobian, then the existence of an essential exhaustible resource satisfying equation* (2.68), *causes the steady states to form a continuum.*

Proof. Take the solution of the steady state equations for, say, a dynastic economy, (2.36), (2.39)-(2.45), (2.69), (2.70). Because of regularity of the Jacobian, the steady state equations can also be solved for small variations in the equations by continuous adjustment of variables (inverse function theorem). Hence, there exists an $\varepsilon>0$, such that for any $0 \leq \lambda_{2,j}^c \leq \varepsilon$, which results in small variations in equations (2.69) and (2.70), a solution exists that is based on a continuous adjustment of variables. The set of these solutions forms a continuum. ■

It follows from the proof that under the stated assumptions, the dimension of the continuum is equal to the number of exhaustible resources with amenity values. Sard's theorem implies that the probability is zero that a solution for the equations without the exhaustible resource constraint (2.68) and dual variable $\lambda_{2,j}^c \geq 0$ has a singular Jacobian, see e.g. [Ginsburgh and Keyzer 1997, Appendix A, Theorem 7.5], which implies that almost all steady states of economies with exhaustible resources that have use or amenity value form continuums. If we identify the steady states with the value of $\lambda^c_{2,j}$, we find one end of the continuum that is given by $\lambda^c_{2,j} = 0$. The other end of the continuum can be open in case $\lambda^c_{2,j}$ can be increased without bound (Example 2.24). It is closed if, for a certain value of $\lambda^c_{2,j}$, an additional constraint becomes binding, for example, if some consumption level becomes zero and cannot further decrease. Figure 2.2 gives an illustration. We conclude that exhaustible resources with amenity values substantially affect the properties of the competitive equilibrium.

We are now in a position to relate the perspective of ecological economists (Section 2.4.1) to this continuum of steady states. Let us assume that the equilibrium path converges to a steady state. Usually, the equilibrium converges to one of the distinct and stable steady states of an economy, each of which is associated with a domain of initial conditions, called basin of attraction[11]. Within every basin, small changes have no long term consequences, and the allocation converges to the same steady state. Figure 2.1 shows the stock dynamics of an imaginary two-resource economy; basin boundaries are represented by dashed lines; steady states by a dot. There are three basins with a stable steady state. Here stability means that differences in the allocations within a basin diminish over time. There is one unstable steady state where the basins meet. Consequently, the long term allocation only depends on the (environmental) policy in the long run. If some environmental policy, for example a more efficient use of environmental resources, is implemented somewhere in a distant future, postponing this policy will have no long term effect, as long as this delay does not lead the economy outside the original basin. Environmental policy might attempt to shift the equilibrium to another basin with a more attractive steady state, but within a basin, the long term equilibrium is invariant to present-day policies.

[11] For convenience, we abstract from unstable steady states and stable cycles.

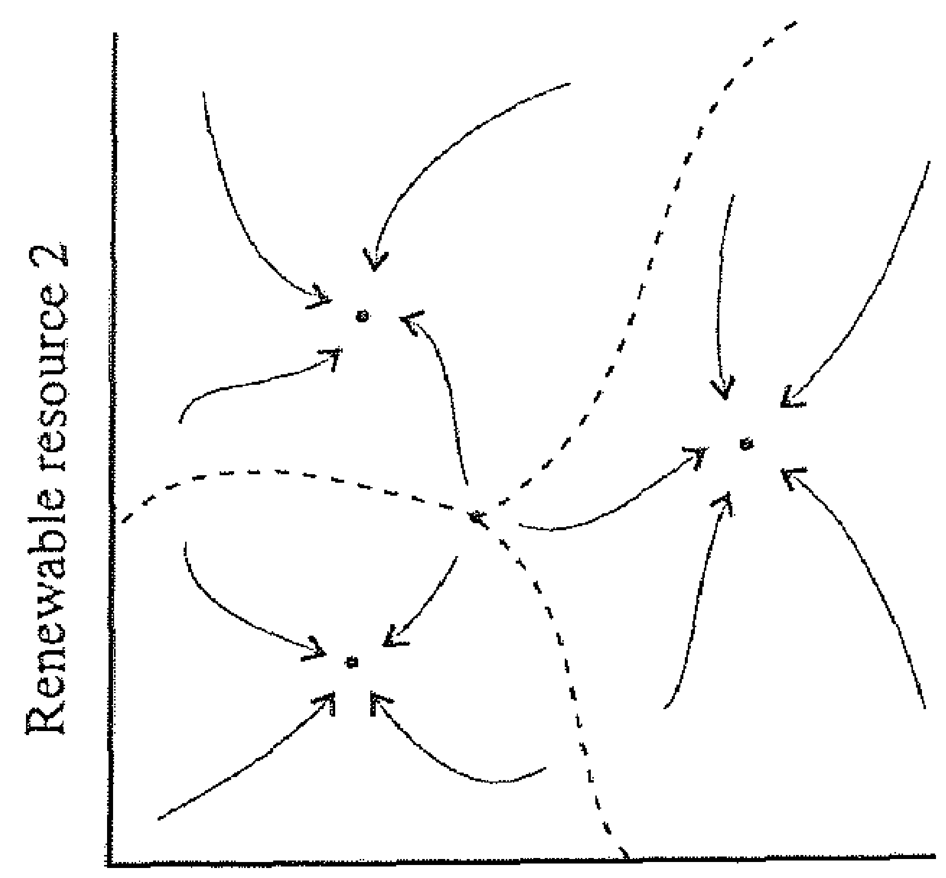

FIGURE 2.1. *Convergence paths for distinct steady states*

When the steady states form a continuum, the converse is true, as the steady state allocations lie 'arbitrary close' to one another, and even within one basin any small difference in the initial conditions might lead to the selection of another steady state allocation. One might say that the equilibrium path converges to the 'nearest' steady state. In an economy with only one state variable (the resource stock), as in Krautkraemer [1985] and in Example 2.24 above, the stock either adjusts downwards in the direction of the unique steady state with maximum resource stock, or remains constant if the initial level happens to lie below that maximum steady state stock, in which case the equilibrium instantaneously moves to that steady state. However, the equilibrium path becomes more complex if the state space dimensions are larger [Gerlagh and Keyzer 1998a].

Let us illustrate what happens under exhaustibility. Consider Figure 2.2, which represents a similar economy as Figure 2.1, but, the second resource is exhaustible and has amenity value. We abstract from the possibility of multiple basins, and assume that the steady states form a line in the two-dimensional state space (i.e. the Jacobians only have a rank deficiency of one). Figure 2.2 presents the continuum of steady states by a line to which all equilibrium paths within the basin converge. Along such a path, the exhaustible resource stock can only decline (all arrows are directed downwards). The upper-right end of the continuum represents the steady state in which the exhaustibility of the resource poses no constraint to the economy ($\lambda^c_{2,j} = 0$, see proof of Theorem 2.3 and discussion thereafter). The lower end of the continuum is determined by the constraint that the exhaustible resource level cannot be negative. A small shift in the present state affects the equilibrium path for the indefinite future; the effects do not vanish over time. Hence, a temporary policy that succeeds in bringing the economy on another equilibrium path, has everlasting consequences, and though sustainability is defined in terms of the long term allocation, it is tightly linked to the present-day policy.

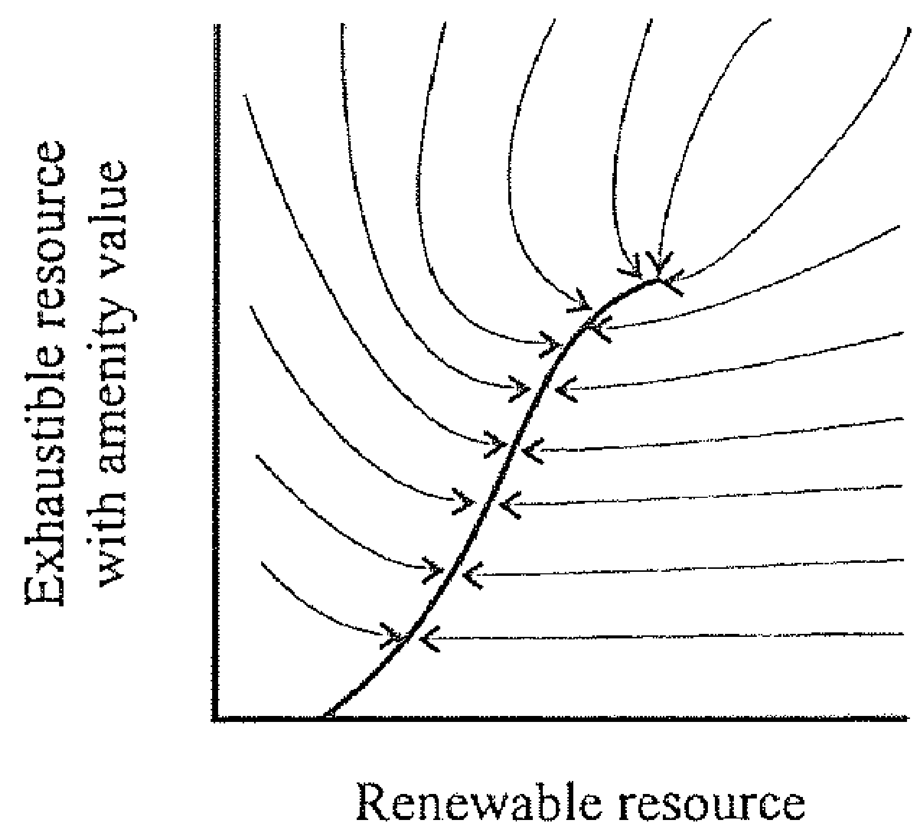

FIGURE 2.2. *Convergence paths to one continuum steady state segment*

To summarize, if some exhaustible resource is 'irreplaceable', present-day environmental policy will affect the long term equilibrium. Consequently, one cannot postpone sustainability policies without risking a lasting and substantial decrease of welfare for future generations.

2.5. USING TRUNCATED ECONOMIES FOR APPLICATIONS

2.5.1. Determinacy and finite approximation of OLG economies

Apart from the properties of the steady state itself, the main concern in applied modeling is the dynamic behavior towards the steady state, or more precisely, the relation between convergence to the steady state, (in)determinacy of the infinite horizon equilibrium, and approximation of the infinite horizon equilibrium by a truncated economy. The so called determinacy of the equilibrium path plays a central role in the analysis of these issues. In general, an equilibrium is said to be indeterminate if there exists a manifold of equilibria, instead of distinct equilibria. We will use the term 'manifold' as having the same mathematical meaning as 'continuum', to distinguish the indeterminacy that emerges from the existence of the 'manifold' of dynamic equilibria from the selection that emerges from the 'continuum' of steady states. It is possible that the steady state is a continuum while the equilibrium is not a manifold, in which case the equilibrium 'automatically' selects one steady state allocation from the continuum and converges to it, provided the equilibrium path converges. On the other hand, if there is a manifold of competitive equilibria, there is no selection mechanism within the economy for one particular equilibrium. The selection has to be made exogenously to the economy, and there is no justification for the selected equilibrium to be more realistic than other equilibria.

The OLG literature often analyzes indeterminacy around steady states, using first-order approximations of demand and supply functions, e.g., [Kehoe and Levine 1990] and [Kehoe 1991]. It implicitly assumes that the properties of the first-order approximations apply to the equilibrium path towards the steady state. We will follow the same approach, and since our main purpose is to describe an unresolved problem rather than to present a clear-cut solution, the presentation will be informal.

The first-order analysis of excess demand shows that OLG equilibria posses a certain turnpike property. Local stability around a steady state can be characterized in terms of eigenvectors and eigenvalues. A steady state equilibrium is determinate if half of the eigenvectors of price deviations converge to zero (the stable eigenvectors) while the other half diverges (the unstable eigenvectors). The stable eigenvalues ensure that the equilibrium will converge exponentially to the steady state from the first period onwards. The unstable eigenvalues ensure that a truncated equilibrium will converge exponentially to the steady state backwards from the truncated period. If the turnpike property holds, the allocation in the first period is not sensitive to small changes in the last period, while the allocation at the last period is not sensitive to small changes in the first period. In general terms, deviations from the steady state decrease exponentially during the first periods, and increase exponentially towards the end. In the OLG, indeterminacy arises when the number of stable eigenvectors is too large, and the number of unstable eigenvectors too small; too many first period allocations converge to the steady state, and this makes the equilibrium allocation in the initial period indeterminate.

Indeterminacy of the infinite horizon OLG equilibrium also affects the truncated economy. Though the truncated equilibrium is determinate (because it is of finite dimension), the equilibria exhibit a typical response under variations in the end period. In the terminal period, the dimension of the deviation from the steady state exceeds the number of unstable eigenvectors, so that these are not sufficient to let the equilibrium converge to the steady state backwards from the terminal period on. As a result, the equilibrium of a truncated T-economy becomes sensitive to the last period specification [Kehoe and Levine 1990].

Here, we conjecture that if in a truncated equilibrium all variables remain within a distance ε from a given steady state during the central periods ('turnpike' behavior, see Figure 2.3), then (i) the infinite horizon model is determinate, (ii) the equilibrium of the infinite horizon model converges to the steady state, (iii) the truncated model is insensitive to the truncation conditions, and (iv) in the first periods, the truncated model remains within the range ε from the infinite horizon equilibrium.

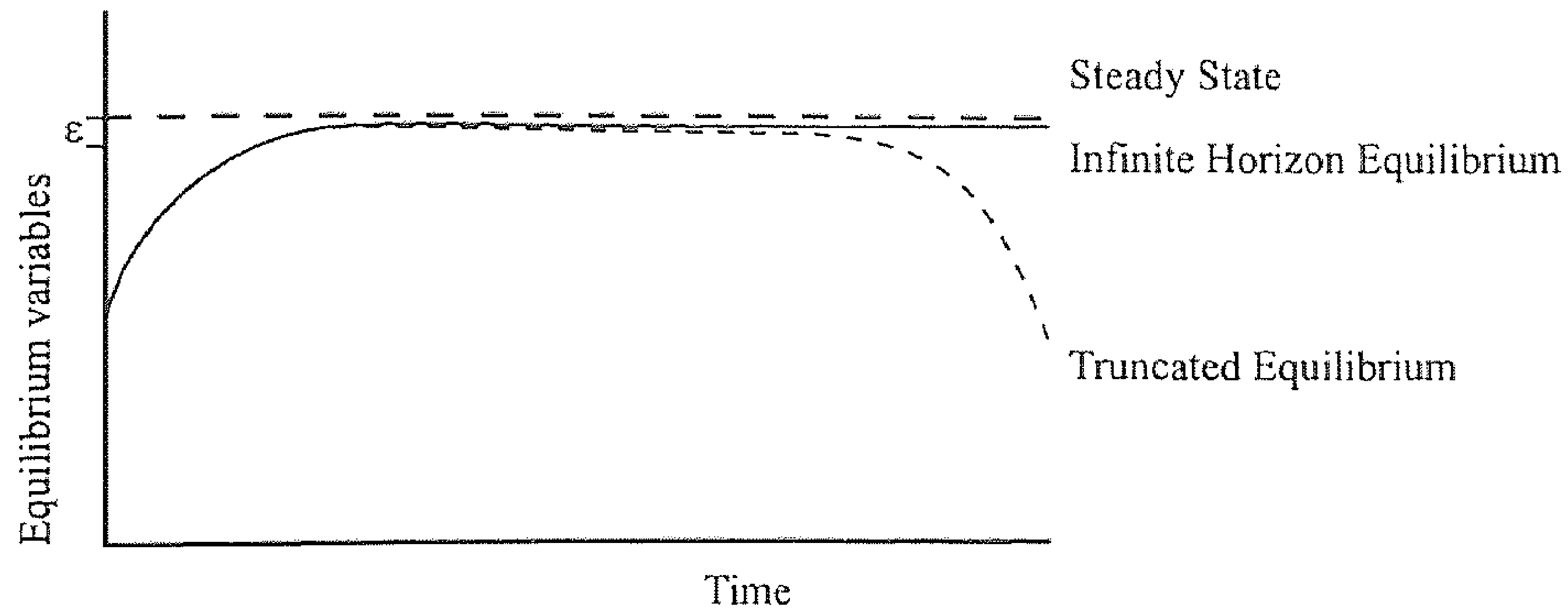

FIGURE 2.3. *Turnpike behavior and finite approximation.*

Our argument runs as follows. (i) The 'turnpike' behavior shows that deviations from the steady state propagate exponentially both onwards and backwards, suggesting that half of the eigenvalues is stable, and the other half is unstable. The infinite horizon model is determinate. (iii) As a result, the truncated model is insensitive to the truncation conditions. (ii) At any point in time, the deviation from the steady state can be attributed to both the initial period conditions and the truncation period T conditions. The effects of the latter diminish for any fixed t if the truncation period is postponed indefinitely, $T \rightarrow \infty$, so that the equilibrium allocation converges. Thus, for any t, we have an equilibrium allocation if $T \rightarrow \infty$, and the set of all these allocations is the infinite horizon equilibrium. In this equilibrium, the 'central period' in which the equilibrium approaches the steady state is extended to the infinite horizon. (iv) If the truncated economy has approached the steady state in period t within the range ε, then both the disturbance attributed to the first period and the truncation period T are within this range, and because of the latter, the difference with the infinite horizon model is also within this range. As the effects of the truncation decrease backwards, calculated allocations for preceding periods also comply with the infinite horizon equilibrium within the range ε.

The properties (i)-(iv) will be used in next chapters to approximate infinite horizon equilibrium paths in numerical applications of OLG models. Usually, we will first compute a steady state and use the value shares for consumption and capital stocks in the steady state as inputs for the valuation function in the truncated economy, e.g. for the parameters $\overline{\beta}$ and $\overline{\psi}$ of Program 2.20. If the truncated economy exhibits the turnpike behavior discussed above, then we will assume that, in the first periods, the truncated equilibrium is a good approximation of the infinite horizon equilibrium. For a similar approach in the literature, see e.g. [Auerbach and Kotlikoff [1987]. However, we notice that the issue of truncation is still not resolved in the literature [Kehoe 1991]. The reader is referred to the literature for a further discussion of determinacy: an extensive analysis of a two-sector economy is given by [Galor 1992]. Efficiency and determinacy are analyzed within the general context of abstract OLG economies in [Geanakoplos and Polemarchakis 1991]. A discussion of efficiency and determinacy within the context of applied general equilibrium models can be found in [Ginsburgh and Keyzer 1997, Sections 8.2 and 8.3].

2.5.2. A mixed OLG/dynastic economy

In this section, a mixed OLG/dynastic model is presented. It is shown that the same economy can be described both as an infinite horizon economy, and as a finite horizon economy, and that both are equivalent. This can be considered a theoretic advantage compared to an arbitrary alternative truncation of the pure OLG model. In practical terms, the mixed OLG/dynastic economy will be useful if the pure OLG economy has no steady state, is inefficient, or indeterminate.

Having defined the dynastic as well as the OLG models, we find that both models have their drawbacks. The major argument against the dynastic approach is that it supposes a kind of slavery. The dynastic planner redistributes the value of endowments which implies that every endowment, e.g., including labor, is considered as the property of the dynasty [Keyzer and Merbis 1995]. The advantage is that the associated equilibrium is defined in a straightforward manner, and is both efficient and determinate. On the other hand, the OLG economy does not need to impose slavery. Transfers can be specified explicitly, and their properties can be analyzed. Unfortunately, this OLG economy has the potential of generating inefficient and even indeterminate equilibria.

The economy in this sub-section combines the OLG and dynastic approaches. The objective is to define an economy for which an equilibrium exists, for which all equilibria are efficient and determinate, and for which as many as possible generations have their own budget constraint. The economy that satisfies this demand is one in which the first T generations have their own budget constraints, whereas the remaining generations are represented by a dynastic planner as in [Keyzer 1991]. Note that the dynastic planner has to own a 'non-negligible' share in endowments. This is the price to pay for ensuring efficiency and determinacy. While the solution is convenient for our purpose, from a mathematical point of view, the mixed economy is a special case of a dynastic model. However, this does not exclude economic analysis based on the OLG approach for the first T generations. Because of this, the economy will still be referred to as a mixed OLG/dynastic economy. In analogy to the pure dynastic and OLG economies, we take the required assumptions together.

ASSUMPTION 2.25. *The standard assumptions for a mixed OLG/dynastic economy are*

M1: $N1$, $U1$-$U4$, $\Omega 1$, $\Omega 2$, $F1$-$F6$, $H1$-$H5$, *and* $\Lambda 1$-$\Lambda 4$

Let us define the mixed economy more precisely. For generations 0 to $T-1$, the economy is an OLG economy where consumers maximize utility given an exogenous transfer function, and where producers maximize profits given their transformation constraint. From generation T onward, generations are represented by a dynastic planner. The dynastic planner is an agent in the economy, like the first T generations. The dynastic planner maximizes welfare, given the aggregator function, as a function of consumption from generation T onward. However, as we have seen before (Theorem 2.1), the economy from period T onward can be decentralized as a competitive equilibrium, using ('dynastic') transfers. The resulting economy is defined by:

DEFINITION 2.26. *An infinite horizon mixed OLG/dynastic competitive equilibrium is a bundle* $(\hat{x}, \hat{y}, \hat{k}, \hat{k}^P, \hat{p}, \hat{\psi}, \hat{\Phi})$, *where prices refer to sequences of period-t prices, such that conditions E1,...,E5 hold of Definition* 2.5, *and condition E6 of Definition* 2.18 *is replaced by:*

E6a'': For generations 0,...,T–1, transfers are given by a transfer function.

E6b'': For generations T,...,∞, transfers are the outcome of dynastic welfare optimization.

We prove the existence of a mixed dynastic/OLG competitive equilibrium using the Negishi welfare program for the economy. The welfare program maximizes a weighted sum of utilities of all consumers who have their own budget constraint. In case of the mixed OLG/dynastic economy, weights are assigned to the first generations $0,\ldots,T-1$, and to the dynastic planner, which maximizes w_T.

PROGRAM 2.27. *Negishi welfare program for infinite horizon OLG/dynastic economy*

$$max \sum_{t=0,\ldots,T-1} \alpha_t \, U(x_{t,t}, x_{t,t+1}; n_t) + \alpha_T w_T$$

$$0 \le w_t \le \overline{w} \text{ for } t=T,..,\infty$$

$$x_{t-1,t}, x_{t,t} \ge 0, k_{j,t}, k^P_{j,t+1} \ge 0, \text{ and } y_{j,t} \text{ for } j=1,\ldots,r,\ t=1,\ldots,\infty$$

subject to

$$w_t \le H(U(x_{t,t}, x_{t,t+1}; n_t), w_{t+1}) \qquad (\mu_t) \qquad t=T,\ldots,\infty$$

$$x_{t-1,t} + x_{t,t} \le n_{t-1}\omega_o + n_t\omega_y + \sum_j y_{j,t} \qquad (p_t) \qquad t=1,\ldots,\infty$$

$$k_{j,t} \le k^P_{j,t} \qquad (\psi_{j,t}) \qquad j=1,\ldots,J;\ t=1,\ldots,\infty$$

$$F_{j,t}(y_{j,t}, -k_{j,t}, k^P_{j,t+1}) \le 0 \qquad (\lambda_{j,t}) \qquad j=1,\ldots,J;\ t=1,\ldots,\infty$$

Welfare weights have to be adjusted such that all budget constraints precisely hold for generations $0,\ldots,T-1$, where transfers are determined by a transfer function. All other generations are represented by the dynastic planner, having one common budget constraint.

$$\sum_{t=T,\ldots,\infty} p_t x_{t,t} + p_{t+1} x_{t,t+1} \leq \sum_{t=T,\ldots,\infty} p_t n_t \omega_y + p_{t+1} n_t \omega_o + \sum_{t=T,\ldots,\infty} \Phi_t \tag{2.71}$$

To find the welfare weights representing the competitive equilibrium, we represent the program as a finite horizon program. Recall the dynastic program from period T on yields a continuous concave non-decreasing value function (Lemma 2.10, 2.12, and 2.13). This value function $v(k_{T+1}, x_{T,T}; n_T)$ is substituted for w_T and the feasibility constraints for period $T+1,\ldots,\infty$. We define a competitive equilibrium for the finite horizon economy:

DEFINITION 2.28. *A finite horizon mixed OLG/dynastic competitive equilibrium is a finite bundle* $(\hat{x}, \hat{y}, \hat{k}, \hat{k}^p, \hat{p}, \hat{\psi}, \hat{\Phi})$, *where the prices refer to sequences of period-t prices, such that conditions E1',...,E6' of the T-period competitive equilibrium (Definition* 2.19) *hold and*

E7: consumption of the dynasty $(x_{T,T}, k_{T+1})$ *maximizes the dynastic value function subject to a dynastic budget constraint.*

Before discussing the dynastic budget constraint, we specify the finite horizon welfare program:

PROGRAM 2.29. *Negishi welfare program for finite horizon OLG/dynastic economy*

$$\max \sum_{t=0,\ldots,T-1} \alpha_t \, U(x_{t,t}, x_{t,t+1}; n_t) + \alpha_T \, v(k_{T+1}, x_{T,T}; n_T)$$

$x_{t-1,t}, x_{t,t} \geq 0$, and $y_{j,t}$ for $j=1,\ldots,J$, $t=1,\ldots,T$
$k_{j,t} \geq 0$ for $t=1,\ldots,T+1$, $k^p_{j,t} \geq 0$ for $t=2,\ldots,T+1$

subject to

$x_{t-1,t} + x_{t,t} \leq n_{t-1}\omega_o + n_t \omega_y + \sum_j y_{j,t}$	(p_t)		$t=1,\ldots,T$
$k_{j,t} \leq k^p_{j,t}$	$(\psi_{j,t})$	$j=1,\ldots,J;$	$t=1,\ldots,T+1$
$F_{j,t}(y_{j,t}, -k_{j,t}, k^p_{j,t+1}) \leq 0$	$(\lambda_{j,t})$	$j=1,\ldots,J;$	$t=1,\ldots,T$

The equivalence between the infinite and finite OLG/dynastic welfare programs can be used to prove the existence of an equilibrium for the OLG/dynastic economy. In the finite economy, the objective of the dynastic planner is to maximize $v(k_{T+1},x_{T,T};n_T)$ subject to the budget constraint

$$p_T x_{T,T} + \sum_j \psi_{j,T+1} k_{j,T+1} \leq p_T n_T \omega_y + \Phi^T \tag{2.72}$$

where Φ^T is the transfer to the dynastic planner. The superscript is used to mark the difference with the transfer to generation T as for the truncated pure OLG model. The dynastic planner has strictly positive income given transfers that are based on a transfer function.

The result of this analysis is that the finite mixed OLG/dynastic economy with transfers satisfies common assumptions. Hence, this economy has an equilibrium with non-zero consumption for all consumers. Using the finite horizon Negishi program, there exist welfare weights so that the allocation of the finite welfare program coincides with the competitive equilibrium. The same welfare weights can be used for the infinite horizon OLG/dynastic welfare program, where it follows immediately that the optimum does satisfy the conditions of a mixed infinite OLG/dynastic economy. We can now state and prove:

THEOREM 2.4. *If assumptions M1 are satisfied, then there exists a competitive equilibrium for both the infinite horizon and the finite horizon mixed OLG/dynastic economy (Definition* 2.26 *and* 2.28 *respectively), which are efficient and determinate and coincide for the first T periods.*

Proof. The existence of the finite horizon equilibrium (Definition 2.28) follows from the existence of welfare weights for which budget constraints are satisfied. The proof is the same as in the fixed point analysis of Lemma 2.35. The conditions for the infinite horizon OLG T-competitive equilibrium (Definition 2.26) are implied by the conditions for the finite mixed OLG/dynastic equilibrium. Particularly, they coincide for the first T periods. For efficiency, note that the finite economy is both efficient and determinate. The dynastic planner ensures efficiency and determinacy for the economy after period T, so that the infinite horizon economy is efficient and determinate for the whole infinite period. ■

2.6. MODEL EXTENSIONS AND AN ILLUSTRATION

2.6.1. Multiple period OLG

Section 2.5 discussed the approximation of infinite horizon equilibria through truncated economies, and presented a mixed dynastic/OLG economy in which the infinite horizon and truncated horizon equilibria coincide. This section will briefly review possibilities for extensions of the model to check its flexibility in applications.

The main properties of the OLG model are not described through explicit assumptions, but are implicitly present in the set up of the model. We will briefly discuss two implicit assumptions, the first regarding the number of periods that every generation lives, the second regarding the heterogeneity of dynasties, or in terms of the OLG economy, generations that live in the same period. The relaxation of the second implicit assumption will also be used to highlight some conceptual differences between dynastic and OLG economies.

The first implicit assumption is the two-period life of generations, which is rather arbitrary. To apply such a model, long periods are required to match a life time of about sixty or eighty years in two periods. Even though childhood can be considered as part of the parents' economic life, e.g. [Auerbach and Kotlikoff 1987] and [Auerbach *et al.* 1989], the periods still consist of two or three decades. In case a proper description of dynamics of the state variables requires shorter periods, the number of periods within a generation's life-cycle has to be adjusted.

OLG models in which consumers live more than two periods can be converted into a two-period OLG model. An algorithm for such conversion is provided by Balasko *et al.* [1980] and is briefly discussed by Kehoe [1991]. However, the economy constructed in this way cannot be compared to the simple two-period model sketched above, because in the converted two-period model, consumers will have different utility functions.

Thus, this conversion does not enable us to apply the theorems of this chapter to a multi-period OLG economy. Fortunately, it is not necessary, as the theorems of this chapter apply rather trivially to the more general multi-period case once assumptions are extended in the obvious way. The main difference is the more complex notation required. For example, let us consider an OLG model where agents live 3 periods. The value function, defined by the dynastic welfare program has to be changed from $\mathrm{v}(k_1^p, \mathrm{x}_{0,0}; n_0)$ into $\mathrm{v}(k_{-1,0}^p, k_{0,0}^p, x_{-1,-1}, x_{-1,0}, x_{0,0}; n_{-1}, n_0)$, where the first subscript for initial stocks denotes generations. As such extensions make the notation tedious without adding substance to the analysis, we leave this subject aside.

2.6.2. Multiple dynasties, an illustration for OLG versus dynastic modeling

Another implicit assumption that seems restrictive refers to the homogeneity of generations. We can relax it if we extend the model to include different dynasties. One can think of several homogenous social groups with relatively few marriages between the groups. For applied global models, regions can serve as dynasties, e.g. [Manne 1995] and [Nordhaus and Yang 1996]. Each dynasty might have its own population dynamics and utility functions, as long as all individuals satisfy the same assumptions. The OLG economy can be extended without any difficulties to a multiple dynasties OLG economy.

However, the situation is different in the dynastic equilibrium, and also in the mixed OLG/dynastic equilibrium, where several difficulties arise. The welfare program for a multiple dynasty model becomes:

PROGRAM 2.30. *Infinite time horizon multiple dynasty welfare program*

$$\mathrm{v}(k_0^p, x_{0,0}; \alpha, n_0) = max \sum_i \alpha_i w_{i,0}$$

$$0 \le w_{i,t} \le \overline{\mathrm{w}} \text{ for } t=0,..,\infty$$

$$x_{i,t-1,t}, x_{i,t,t} \ge 0, k_{\mathrm{j,t}}, k_{j,t+1}^p \ge 0, \text{ and } y_{\mathrm{j,t}} \text{ for } j=1,\ldots,r,\ t=1,\ldots,\infty$$

subject to

$$w_{i,t} \le \mathrm{H}(\mathrm{U}(x_{i,t,t}, x_{i,t,t+1}; n_{i,t}), w_{i,t+1}) \qquad (\eta_{i,t}) \qquad i=1,\ldots,I, \quad t=0,\ldots,\infty$$

$$\sum_i x_{i,t-1,t} + x_{i,t,t} \le \sum_i n_{i,t-1}\omega_o + n_{i,t}\omega_y + \sum_j y_{\mathrm{j,t}} \qquad (p_t) \qquad t=1,\ldots,\infty$$

$$k_{\mathrm{j,t}} \le k_{j,t}^p \qquad (\psi_{j,t}) \qquad j=1,\ldots,J; \quad t=1,\ldots,\infty$$

$$\mathrm{F}_{j,t}(y_{j,t}, -k_{j,t}, k_{j,t+1}^p) \le 0 \qquad (\lambda_{\mathrm{j,t}}) \qquad j=1,\ldots,J; \quad t=1,\ldots,\infty$$

where dynasties are denoted by $i=1,\ldots,I$, and α_i is the welfare weight of the dynasty in the welfare program. Notice that we omitted differences between dynasties with respect to endowments and production technologies that are available. Such specifications can be included easily.

The dynastic competitive equilibrium is given by welfare weights for which each dynasty satisfies its budget constraint. Lucas and Stokey [1984] comprehensively analyze such an economy, though they do not prove existence of the competitive equilibrium. The specific case of multiple dynasties with constant time preference is analyzed by Kehoe, Levine and Romer [1990], who include an existence proof for the competitive equilibrium. Ginsburgh and Keyzer [1997, Proposition 8.2] prove existence for the general dynastic economy with recursive welfare functions. Both proofs assume continuously differentiable and strictly concave utility functions. Considering the use of the simplified fixed point mapping Q(.) in the proof of Theorem 2.2, we conjecture that it should be possible to prove existence while relaxing differentiability and strict concavity [Gerlagh 1998b].

The multiple-dynasty equilibria have important characteristics distinguishing them from single-dynasty equilibria. Most remarkably, in multi-dynasty equilibria, the slavery that is possible by the dynastic perspective has far-reaching implications for the trade patterns over time, which can become very strange. As this behavior highlights the conceptual difference between dynastic and OLG models, we develop this point through an example.

We refer to the economy considered in [Nordhaus and Yang 1996], who, however, do not describe the peculiarities with respect to the trade patterns. Consider a model of the global economy, where regions are taken to be represented by different dynasties. Assume that the dynastic planners have constant time preferences. It is commonly accepted that a necessary implicit assumption for this model to be relevant is that all dynasties should have the same time preference; otherwise, only the most patient dynasty will survive eventually. Furthermore, assume that there is one consumer good freely traded on a world market, so that for that good prices are equal around the world. This implies equal price changes over time. If all dynasties have the same intertemporal elasticity for consumption, it follows that consumption growth is the same for all regions, which implies that the regional share of global consumption does not change over time. However, if two regions have different production growth rates and equal consumption growth rates, then the fast growing region imports much of its consumption at the beginning of the simulation period, redeeming the resulting international debt by exports in the end. This leads to very large trade flows.

A stylized version of the Nordhaus and Yang [1996] model may serve as an illustration. Consider a pure exchange economy with two regions, labeled 'North' and 'South'. For convenience, we abstract from production technologies and consider the potential output as endowments. The endowment of the North is initially 20, increasing by 2.5 per cent per year, and the endowment of the South is initially 1, increasing by 4 per cent per year. It takes 2 centuries before both regions have converged to the same level of endowments. The growth factor is denoted by $\delta_N^{\omega}=1.025$ and $\delta_S^{\omega}=1.04$, where the superscript is used to distinguish growth of endowments from growth of consumption. Eventually, after these 2 centuries, the endowments of both regions are assumed to converge.

In both regions, consumption is determined by a dynastic planner which maximizes a weighted sum of period-wise logarithmic utility where welfare weights decrease exponentially by a factor σ per year. The aggregator function supporting this welfare function is given by $H(u,w)=\ln(u)+\sigma w$. To simplify the following analysis, we assume that utility is specified similarly, as

$$u(x_{i,t,t},x_{i,t,t+1})=\ln(x_{i,t,t})+\sigma^c\ln(x_{i,t,t+1}), \tag{2.73}$$

where the superscript c denotes the consumer's time preference, as opposed to the planner's time preference. The resulting welfare function is given by

$$w_{i,0} = \ln(x_{i,0,0}) + \sum_{t=1,\dots,\infty} (\sigma)^t ((\sigma^c/\sigma)\ln(x_{i,t-1,t}) + \ln(x_{i,t,t})), \tag{2.74}$$

from which it follows that in every period consumption is distributed between young and old according to a fixed share so that the analysis can be reduced to consumption aggregated over the old and young generation, $x_{i,t}$, with welfare given by

$$w_{i,1} = \sum_{t=1,\dots,\infty} (\sigma)^t \ln(x_{i,t}). \tag{2.75}$$

We calibrate the value for σ such that prices decrease by 5 per cent during the first periods, denoted by β_t=0.95 for t=1. If there were no trade between both regions, the consumption growth rate would be equal to the growth rate in endowments so that the logarithmic welfare function would give

$$\sigma_i = \beta\delta_i^{\omega}, \tag{2.76}$$

for $i=N,S$, so that σ_N=0.974 and σ_S=0.988. However, recall that the time preference factor σ has to be the same for both regions. To calculate the common time preference, we first approximate the wealth in the first period, $W_{i,1}$, of both regions, ignoring the growth rates after 2 centuries and possible changes in the price ratios β_t,

$$W_{i,1} \approx \frac{\omega_{i,1}}{1-\beta_1\delta_i^{\omega}}, \tag{2.77}$$

for $i=N,S$, where $\omega_{i,1}$ is the initial endowment. The calculated values are $W_{N,1}$=762, and $W_{S,1}$=83. Total consumption in the first period is given by

$$x_{N,1}+x_{S,1}=(1-\sigma)(W_{N,1}+W_{S,1}), \tag{2.78}$$

which has to be equal to the endowments which gives σ=0.975. We readily calculate consumption in the initial period at $x_{N,1}$=18.9 and $x_{S,1}$=2.1, respectively. In other words, in the first year, the South consumes more than twice its endowments, importing this consumption from the North. Growth of consumption in both regions is the same, given by

$$\delta_t^x=\sigma/\beta_t, \tag{2.79}$$

which gives δ_t^x=1.026 for t=1. Endowments increase faster than consumption in the South and they match each other after 55 years. From that period onward, the South starts repaying its debt to the North. One century later consumption of the South consists of only 25 per cent of its endowments. Though the income levels of the North and South region converge after 2 centuries, their share in consumption remains fixed at the same rate (1:10) as in the first period.

This example reveals that, in dynastic models, well-behaved preferences can be associated with unrealistic consumption and trade patterns, because the dynastic planner does not connect present consumption with present endowments and future consumption with future endowments. The consumption patterns might become more acceptable under an alternative aggregator function H(.). However, it should be noted that any aggregator function for which relative consumption growth is directly related to price changes, as in (2.79), causes the same problem. This is because the consumption share of different dynasties does not change over time. More specifically, the characteristic is independent of the intertemporal elasticity of substitution.

2.6.3. Irreducibility, a weakening of the strictly desired commodity assumption

We conclude with a brief discussion of the irreducibility of the OLG economy and the presence of a strictly desired commodity. The combined assumption of an always desired good (*U3*), together with the strictly positive endowments for this good (Ω*2*) is meant to ensure positive income for every generation at equilibrium (see the proof of Lemma 2.21). However, in applied models, it can be useful to accept strictly positive endowments for labor only, without labor entering the utility function (no preference for leisure). Such a model does not satisfy the assumptions.

McKenzie [1959] introduced alternative assumptions that suffice to prove existence. An economy is 'irreducible' if any partition of consumers in two sets (two disjunct sets that cover the whole set) enables the first group of consumers to Pareto-improve the welfare of the other group by a reallocation of endowments, and *vice versa*. For the OLG model, an assumption can be stated that makes the economy irreducible.

ASSUMPTION 2.31 *The OLG economy satisfies for any* $T=0,\ldots,\infty$

R1: *The first generations* $0,\ldots,T$ *are capable of producing a commodity in period* $T+1$ *that is desired by generation* $T+1$, *and*

R2: *generation* $T+1$ *is endowed with a commodity that is desired by the first generations* $0,\ldots,T$, *possibly using the production technology.*

Let us check that Assumptions *R1-R2* guarantee the satisfaction of McKenzie's conditions by indirect demonstration. Suppose that the OLG economy were reducible, that is, there is a sub-set of generations, $\mathbf{A}=\{t_1, t_2, \ldots\}$, either finite or infinite, that can be disconnected from the other generations, $\mathbf{B}$, the complement of $\mathbf{A}$; in formal terms, $\mathbf{A}\cup\mathbf{B}=\{0,1,\ldots,\infty\}$, and $\mathbf{A}\cap\mathbf{B}=\emptyset$. Without loss of generality, we can assume that for some T the first generations $0,\ldots,T$ are in $\mathbf{A}$. Then, because of *R1*, the generations in $\mathbf{A}$ can increase the welfare of the generations in $\mathbf{B}$, and because of *R2*, the opposite holds as well. Hence, the sets are connected, and the economy is irreducible.

If assumption *R1* and *R2* hold, assumptions *U3* and *Ω2* can be relaxed. First, it is clear that the assumption above requires non-satiated utility functions. There does not need to be one specific desired commodity (*U3*), but there is always some commodity that is desired. Secondly, the assumption requires that generation t has to offer something in period t. Thus, endowments of the young cannot be zero, but they do not have to be positive for the desired commodity (*Ω2*).

The second set of assumptions that can be relaxed concerns the time independence of the physical aspects of the economy, e.g., the assumption that there is a strictly positive stationary production (*F6*). Though from a physical point of view, it seems unlikely that the laws of nature will change, in applied models, it might be preferable to use an exogenous technology path, as such an explicit assumption might be easier to justify than the specification of any dynamic relationship. Such adjustments clearly do not alter any of the theoretical results. The stylized illustrations in coming chapters will allow for time dependence, and will not satisfy all assumptions.

2.7. CONCLUDING REMARKS

Recall from the introduction that we wanted to develop a model with an infinite horizon, with generation specific property rights, with the capacity to incorporate and analyze 'typical' environmental production functions, and finally with a flexible set of assumptions that can be adjusted if applications so require. Let us review our progress so far.

Theorem 2.1 and 2.2 ensure existence of infinite horizon dynastic and OLG equilibria if Assumptions *D1* and *O1* are satisfied, respectively. Theorem 2.3 makes clear that the second and third environmental characteristics mentioned in Section 1.4.1 cause present policies to have everlasting consequences because the steady states form a continuum, and the equilibrium becomes path dependent. Section 2.5.1 provides the arguments to determine whether an approximation of the infinite horizon equilibrium by a truncated equilibrium satisfies. Theorem 2.4 gives a specific mixed OLG/dynastic economy for which there is a perfect match between an infinite horizon equilibrium and a truncated equilibrium. The final Section 2.6 gave a brief general digression on some basic model extensions that can be useful for applied models.

The OLG model makes it possible to use generation-specific property rights, and explicit intergenerational transfer functions. This property will be useful for the analysis in coming chapters. The model is general and uses vector notations, so that results of formal analysis apply to more specific models. The assumptions are based on prevalence in the literature, with a slight modification for the transformation technology to account for environmental issues, a bounded production space in particular.

The main theoretical contribution of this chapter is stated in Theorem 2.3: exhaustible resources with amenity values generate a continuum of steady states. Provided the equilibrium converges to the steady state, the equilibrium path selects one element of the continuum. Therefore, present-day policies have everlasting effects that do not diminish over time.

In Chapter 3 and 4, the model will be used for the analysis of (dynamic) efficiency and sustainability. Every next chapter will also provide some illustrations related to the issue of climate change.

ANNEXES TO CHAPTER 2

2A. NOTATION

Denote the n-dimensional real space by $\mathbf{R}^n$, the non-negative orthant by $\mathbf{R}^n_+$, and the strictly positive orthant by $\mathbf{R}^n_{++}$. Let $x=(x_1,\dots,x_h,\dots,x_n)'$ and $z=(z_1,\dots,z_h,\dots,z_n)'$ be two vectors in $\mathbf{R}^n$, and let $y_j\in\mathbf{R}^n$, $j=1,\dots,J$ and $k_t\in\mathbf{R}^n$, $t=1,\dots,\infty$ be sets of vectors in $\mathbf{R}^n$. The following notations are equivalent:

Enumeration	all $h \Leftrightarrow h=1,\dots,n$
	all $j \Leftrightarrow j=1,\dots,J$
	all $t \Leftrightarrow t=1,\dots,\infty$
Unit vector	$\bar{1} \equiv (1,1,\dots,1)$
Unit h-vector	$\bar{1}^h \equiv (0,\dots,0,1,0,\dots,0)$, h-th element is equal to unity
Selection of h-th element	$\bar{1}^h x \equiv x_h$
Summation	$\bar{1}x \equiv \Sigma_h x_h \equiv \sum_h x_h \equiv \sum_{h=1,\dots,n} x_h \equiv \sum_{h=1}^{n} x_h$
	$\Sigma_j y_j \equiv \sum_j y_j \equiv \sum_{j=1,\dots,J} y_j \equiv \sum_{j=1}^{J} y_j$, and similar for t
Complementarity slackness	$(x \ge 0 \perp z \ge 0) \Leftrightarrow (x \ge 0,\ z \ge 0$, and $xz = 0)$
Inner product	$xz \equiv x'z \equiv \sum_{h=1}^{n} x_h z_h$
Maximal element	$\max_h x_h \equiv \max\ \{x_h \mid h=1,\dots,n\}$
Vector norm	$\lVert x\rVert_p \equiv (\Sigma_h \lvert x_h\rvert^p)^{1/p}$ for given $1\le p<\infty$
	$\lVert x\rVert_\infty \equiv \max_h \lvert x_h\rvert$
	$\lVert x\rVert \equiv \lVert x\rVert_p$ for some p to be specified
Inequality	$x \ge z \Leftrightarrow x_h \ge z_h$ for all h, and similar for $\le$
	$x > z \Leftrightarrow x_h > z_h$ for all h, and similar for $<$
Derivatives	$\mathrm{F}'(x) \equiv \partial \mathrm{F}(x)/\partial x$
	$\nabla \mathrm{F}(x,y) \equiv \partial \mathrm{F}(x,y)/\partial (x,y)$
	$\mathrm{F}_1(x,y) \equiv \mathrm{F}_x(x,y) \equiv \nabla_1 \mathrm{F}(x,y) \equiv \nabla_x \mathrm{F}(x,y) \equiv \partial \mathrm{F}(x,y)/\partial x$
Mutation	$\Delta k_t \equiv k_{t+1}-k_t$

The notation for a mathematical program will be introduced by means of an example. Define the functions $\mathrm{u}:\mathbf{R}^n_+ \to \mathbf{R}$ and $\mathrm{F}_j:\mathbf{R}^n \to \mathbf{R}$, $j=1,\dots,J$ and consider the mathematical program:

max $\mathrm{u}(x)$

$x \geq 0, y_j$, all j

subject to

$x \leq \Sigma_j y_j$ (p)

$\mathrm{F}_j(y_j) \leq 0$ (λ_j) $(j=1,\ldots,J)$

where the choice variables x and y_j are placed under the maximand. Here, x is constrained to be non-negative, while y_j is unconstrained. We can either use 'all j' or 'j=1,...,J', to denote the set of choice variables for y_j and the set for which the second constraint applies. The brackets (p) denote the vector Lagrange multiplier associated with the constraint.

2B. EXTENDING CONCAVE (CONVEX) FUNCTIONS

The lemmas in this section are based on Theorem 1.5 and its proof in [Ginsburgh and Keyzer 1997, Appendix A]. Lemma 2.34 is an extension of Theorem 1.5 in the sense that it includes a criterion of when the extended concave function contains the origin in its domain.

LEMMA 2.32. *Let* $\mathrm{f}(x,y)$*:* $\mathbf{R}^a\times\mathbf{R}_+^b\rightarrow\mathbf{R}$*: be continuous and homogeneous of degree* 1 *on its whole domain and concave on the extended unit simplex* $\mathbf{R}^a\times\mathbf{S}^b$*, defined by* $\bar{1}y = 1$*, then* f(.) *is concave on its whole domain* $\mathbf{R}^a\times\mathbf{R}_+^b$.

Proof. Take two variable pairs (x_a, y_a) and (x_b, y_b), f(.) satisfies

$$\begin{aligned}
&\mathrm{f}(\gamma x_a + (1-\gamma)x_b, \gamma y_a + (1-\gamma)y_b)\\
&= (\gamma\bar{1}y_a + (1-\gamma)\bar{1}y_b)\,\mathrm{u}(\frac{\gamma x_a + (1-\gamma)x_b}{\gamma\bar{1}y_a + (1-\gamma)\bar{1}y_b}, \frac{\gamma y_a + (1-\gamma)y_b}{\gamma\bar{1}y_a + (1-\gamma)\bar{1}y_b})\\
&= (\gamma\bar{1}y_a + (1-\gamma)\bar{1}y_b)\,\mathrm{u}(\eta v_a + (1-\eta)v_b, \eta w_a + (1-\eta)w_b)
\end{aligned}$$

where $\eta = \gamma\bar{1}y_a / (\gamma\bar{1}y_a + (1-\gamma)\bar{1}y_b)$ and $(v_a, w_a),(v_b, w_b)\in\mathbf{R}^a\times\mathbf{S}^b$ is defined by $v_a = x_a/\bar{1}y_a$, $w_a = y_a/\bar{1}y_a$, and the analogous for b. Because of concavity of f(.) on $\mathbf{R}^a\times\mathbf{S}^b$, we have

$$\begin{aligned}
&\geq (\gamma\bar{1}y_a + (1-\gamma)\bar{1}y_b)(\eta\,\mathrm{f}(v_a, w_a) + (1-\eta)\,\mathrm{f}(v_b, w_b))\\
&= \gamma\bar{1}y_a\,\mathrm{f}(v_a, w_a) + (1-\gamma)\bar{1}y_b\,\mathrm{f}(v_b, w_b)\\
&= \gamma\,\mathrm{f}(x_a, y_a) + (1-\gamma)\,\mathrm{f}(x_b, y_b)
\end{aligned}$$

which is the desired result. ■

LEMMA 2.33. *Let* $f(x,y)$:$\mathbf{R}^a \times \mathbf{R}_+^b \rightarrow \mathbf{R}$ *be continuous and concave on the extended unit simplex* $\mathbf{R}^a \times \mathbf{S}^b$, *defined by* $\bar{1}y = 1$, *then* $\tilde{f}:\mathbf{R}^a \times \mathbf{R}_+^b \setminus \{0\} \rightarrow \mathbf{R}$ *defined by*

$$\tilde{f}:(x,y) = \bar{1}y\, f(\frac{x}{\bar{1}y}, \frac{y}{\bar{1}y})$$

is continuous, concave and homogeneous on its whole domain $\mathbf{R}^a \times \mathbf{R}_+^b \setminus \{0\}$. *The origin can be included in the domain for y if the domain for* (x,y) *is restricted to a cone,* $|x| \leq A\, \bar{1}y$ *for some positive* A. *In that case,* $\tilde{f}(0,0)=0$.

Proof. Homogeneity of $\tilde{f}(.)$ follows from construction. Furthermore, $\tilde{f}(.)$ is concave (convex) on the extended simplex, defined by $\bar{1}y = 1$, so that concavity on its whole domain follows from the previous lemma. The domain has to exclude y=0 because $\lim_{n \downarrow 0} n\, f(x/n, y)$ can depend on y. However, restricting the domain to the cone ensures $(x/\bar{1}y, y/\bar{1}y)$ to be within a compact space, so that f is bounded, and $\bar{1}y \cdot f(.)$ converges to zero for $\bar{1}y \downarrow 0$. ■

LEMMA 2.34. *Let* $f(x)$:$\mathbf{R}^a \rightarrow \mathbf{R}$*: be continuous and concave, let* $f(0) \geq 0$, *and let* $f(.)$ *be bounded from below by* $f(x) \geq -A|x|$ *for some positive finite* A, *then* $\tilde{f}:\mathbf{R}^a \times \mathbf{R}_+ \rightarrow \mathbf{R}$ *defined by*

$$\tilde{f}(x,y) = y\, f(\frac{x}{y}) \text{ for } y \neq 0, \text{ and}$$

$$\tilde{f}(x,0) = \lim_{y \downarrow 0} y\, f(\frac{x}{y})$$

is well defined, continuous, concave, homogeneous of degree one and non-decreasing in y on its whole domain $\mathbf{R}^a \times \mathbf{R}$. *If* $f(0)>0$, *the extended function* $\tilde{f}(.)$ *is strictly increasing in y.*

Proof. First, we prove $\tilde{f}(.)$ to be well defined for y=0. Note that $\tilde{f}(.)$ is increasing in y, as the first derivative with respect to y satisfies $\tilde{f}_y(x,y) = f(x/y) - (x/y)f'(x/y) \geq f(0) \geq 0$. Thus, any strictly decreasing sequence $y \downarrow 0$ causes $\tilde{f}(x,y)$ to be strictly decreasing as well, and as $\tilde{f}(.)$ is bounded from below by –A, the sequence converges. Moreover, it is easily shown that concavity of $\tilde{f}(.)$ extents to its boundary y=0, so that $\tilde{f}(x,0)$ is continuous in x. Homogeneity on the interior follows from construction. We are only left to prove that $\tilde{f}(.)$ is continuous on its whole domain (i.e., between the boundary and the interior).

Consider a sequence $(x,y)_i \rightarrow (\hat{x},0)$, for $i=1,\ldots,\infty$. As the sequence converges, all $(x,y)_i$ are elements of the same compact set $[\underline{x},\bar{x}]\times[0,\bar{y}]$. Now, define $a(x,y)=\tilde{f}(x,y)-\tilde{f}(x,0)$, and $b(y)=\max\{a(x,y)|x\in[\underline{x},\bar{x}]\}$. Because of continuity and compactness, we have $\forall\delta:\exists\varepsilon:\forall x\in[\underline{x},\bar{x}]: y<\varepsilon \Rightarrow a(x,y)<\delta$ which implicitly states that b(.) is continuous. This completes the proof for continuity as $|\tilde{f}(x_i,y_i)-\tilde{f}(\hat{x},0)| \le b(y_i) + |\tilde{f}(x_i,0)-\tilde{f}(\hat{x},0)|$ which are both continuous functions converging to zero. ■

In case that f(.) is not bounded from below by $f(x)\ge -A|x|$, then $\tilde{f}(.)$ is defined for strictly positive y only. Because in our model, we explicitly assume the population size to be strictly positive, $n>0$, the extended utility function is well defined. However, in case that the population size becomes endogenous, it may be necessary to extent Assumptions *U1-U4* with a *U5*: $u(x_y,x_o) > -A\|(x_y,x_o)\|_2$.

3. Dynamic efficiency in OLG economies

This chapter analyzes dynamic inefficiencies that can occur in the OLG model developed in Chapter 2 and formulates additional assumptions under which dynamic efficiency is restored. These assumptions apply to consumers (time preference, endowments, empathy between generations) and producers (existence of a 'non-negligible environmental resource'). If these assumptions are not satisfied, efficiency can be restored by a public authority that issues a freely tradable claim which gives the owner a constant share of flow endowments in every period. The claim is given to the first generation which thereby obtains a non-negligible share of total income.

The issue is illustrated by means of a numerical simulation with an applied model, ALICE. The analysis shows that an expected increase in life-expectancy (aging) in the 21st century is capable of shifting a dynamic efficient equilibrium towards a dynamically inefficient course of the equilibrium. Next, we show that if the claim is issued, this leads to transfers which restore efficiency, and we examine the transition towards the efficient equilibrium. Finally, we introduce climate change in the model and show how the biogeochemical system underlying this process can be viewed as a non-negligible environmental resource that ensures dynamic efficiency of the competitive equilibrium.

3.1. Introduction

This chapter deals with dynamic (in)efficiency of the competitive OLG equilibria. The adjective 'dynamic' is used to distinguish the inefficiencies under study from those in static and finite horizon models which are caused by market imperfections. Dynamic inefficiency refers to equilibria that seem efficient when considered period-wise, but that turn out to be inefficient in an infinite horizon context. The possibility of dynamic inefficiency implies that (period-wise) competitive behavior with a complete set of markets is not sufficient for Pareto optimality, and thus, the first Welfare Theorem (which takes the number of commodities to be finite[12]) does not apply.

The possibility of dynamic inefficiency of OLG equilibria was already noticed by Samuelson [1958] who found that an OLG equilibrium without money is capable of generating competitive equilibria that are not Pareto optimal, whereas the use of fiat money removed the inefficiency. After this result, many other examples of inefficient OLG equilibria have been constructed [Balasko *et al.* 1980]. Once the basic problem is understood, it is trivial to generate more examples. A typical case runs as follows.

Example 3.1. *Dynamic inefficiency*

Let there be one commodity, and one stock. All generations have population of unit size, and utility functions given by $u(x_y, x_o) = \sqrt{x_y x_o}$, where x_y is consumption when

[12] A commodity is not only characterized by its physical properties, but also by its place and time of production and consumption. The same physical entity produced in another period is therefore another commodity [Debreu 1959]. There are thus infinitely many commodities in an infinite horizon OLG model.

young, and x_o consumption when old. The endowments are given by ω_y=4 and ω_o=0. The transformation function is given by $F(y_t, -k_t, k_{t+1}^p) = y_t - k_t + 2k_{t+1}^p$. Finally, the initial stock is equal to unity. Consider the following OLG equilibrium with zero transfers. Prices for commodities are equal to prices for stocks, increasing by a factor two in every period: $p_t=\psi_t=2^t$. Consumption is given by $(x_y,x_o)=(2,1)$. Production is given by $(y_t, -k_t, k_{t+1}^p) = (-1,-1,1)$. Consumers and producers satisfy the assumptions *O1*, and behave optimal in the equilibrium. Producers maximize profits period-wise, converting one unit of the flow commodity plus one unit of the stock into one unit of the stock for the next period. As prices double, this transformation maximizes profits for each period. However, when taken over all periods, production is nothing but annihilation of goods that are stored as capital without use. A simple division of endowments over both generations, so that $(x_y,x_o)=(2,2)$ would yield higher utility for every generation. ■

The example shows a clear welfare loss from dynamic inefficiency. This raises the question whether the inefficiency and induced welfare loss is an artifact of an unrealistic model, or that this represents a more fundamental characteristic. To clarify this issue, we look for assumptions and institutions that restore dynamic efficiency. If these conditions seem realistic, we do not expect dynamic inefficiency to be a relevant issue for realistic economies. However, if the conditions seem artificial, the issue is a matter of serious concern.

The chapter is organized as follows. Section 3.2 gives the general analysis, starting with two lemmas in Section 3.2.2 that state sufficient conditions for dynamic efficiency: boundedness of the value of all aggregate endowments, or equivalently, the presence of one consumer with a non-negligible income share. The analysis proceeds in Section 3.2.3 with the search for assumptions concerning utility functions and endowments that establish dynamic efficiency (Theorem 3.1).

Section 3.2.4 looks at the production side of the economy. In the literature, a frequently used assumption requires that goods can be stored as stocks, and be used to reproduce themselves. For a formal elaboration, see [Ginsburgh and Keyzer 1997, Proposition 8.8]. The condition implies that production can grow without bound and without a necessary input of a non-produced good. We consider this assumption to be rather strong, and formulate an alternative condition requiring that some environmental resource should be 'non-negligible', that is posses the capacity to produce a good in which utility is non-satiated for the indefinite future. It is shown that such a non-negligible resource restores dynamic efficiency if given to the first generation (Theorem 3.2) [Geanakoplos and Polemarchakis 1991]. We argue that this condition is likely to be fulfilled for an economy in which the environmental resources are incorporated. In certain circumstances, land satisfies the assumptions [Rhee 1991], but, more in general, environmental resources that have amenity values (the first two characteristics mentioned in Section 1.4.1) also satisfy. In the absence of a non-negligible environmental resource, or if such a resource is unpriced, dynamic inefficiency can easily occur.

Section 3.2.5 introduces a public agency that restores efficiency without requiring special assumptions on preferences and technology (Theorem 3.3). This is accomplished by means of a fictitious non-negligible resource that realizes dynamic efficiency in the same way as a physical non-negligible environmental resource. In fact, the agency generates intergenerational transfers by issuing a real claim on the perpetual flow of future (human) endowments which than accrues to the first generations. This completes the general analysis.

The chapter continues with an application of the non-negligible claim to a stylized economy in Section 3.3.1. We depart from an efficient equilibrium, and show that a gradual increase of life-expectancy, for convenience referred to as 'aging', is capable of shifting the efficient equilibrium to a dynamically inefficient one. Initially, generations live two periods of 20 years (childhood is presumed to be a part of the life-cycle of the parents, which implies that in total, people are assumed to live 60 years), which increases to three periods. In the specified model, the increased life-expectancy induces increased savings and a decreased interest rate which, in turn, causes the total value of endowments to become unbounded and the competitive equilibrium to become inefficient.

Though the representation of 'aging' is highly stylized, it reflects an observed trend. Because of improved hygiene, health care, and changing ways of life, life expectancy is expected to increase further in the course of the next centuries. The World Bank estimates a global average life expectancy of 66 years for generations born between 1990 and 2000, increasing to 85 years for generations born around 2100 [WB 1994].

Section 3.3.2 describes the issuing of the non-negligible claim in this economy, and the transition towards the efficient steady state. It is shown that this efficiency policy can improve welfare of all generations.

In Section 3.4, the steady state model is developed into a stylized simulation model with an exhaustible resource that has amenity value in order to illustrate the non-negligible resource. The model distinguishes periods of 20 years duration, the first of which corresponds to the interval 2000-2020. The transition from generations that live two periods to generations that live three periods is supposed to take place during the 21st century. Section 3.4.1 specifies the simulation model and two environmental policies: a 'strong sustainability' policy based on strictly conservationist measures, and a 'grandfathering' policy where the value of the resources accrues to the first generation. It is shown that the exhaustible resource with amenity value is non-negligible, and therefore Theorem 3.2 applies: if the resource is grandfathered, the simulated economy is dynamically efficient. To restore efficiency, the 'strong sustainability' policy needs to be accompanied by a non-negligible claim given to the first generation.

Section 3.4.2 analyzes the steady states of the economy for the two policies. Theorem 2.3 of Chapter 2 applies: the exhaustible resource generates a continuum of steady states (Theorem 3.3). It is noticeable that, in case of the grandfathering policy, the welfare level of future generations is decreased compared to the strong sustainability policy notwithstanding the fact that grandfathering restores efficiency (Theorem 3.4). Section 3.4.3 applies the simulation model to climate change, and presents some numerical findings. To test the sensitivity of the results with respect to assumptions that do not directly relate to the issue of climate change, we add a third scenario that excludes aging. Section 3.5 concludes.

3.2. Formal analysis

3.2.1. Introduction

The general analysis in this chapter uses the formal model developed in Chapter 2. For convenience, we summarize its basic set up. There are infinitely many discrete periods, $t = 1,\ldots,\infty$, and infinitely many generations, living two successive periods each. In every period, a new generation is born, so that in each period, there is a young and an old generation alive. Generations are of different sizes, denoted by n_t and are identified by the first period in which they live. There are H goods, C stocks, and J firms. The consumption bundle of generation t, when young and old respectively, is denoted by $x_{t,t}, x_{t,t+1} \in \mathbf{R}^H_+$ and $\omega_y, \omega_o \in \mathbf{R}^H_+$ denotes the *per capita* initial endowments vectors for the young and the old generation. If convenient, $\omega_{i,t}$ is used to denote aggregate endowments of generation i in period t. The production vector of firm $j = 1,\ldots,J$, in period t, is denoted by $y_{j,t} \in \mathbf{R}^H$, and $k_{j,t} \in \mathbf{R}^C_+$ denotes the stocks used for production in period t, and $k^P_{j,t} \in \mathbf{R}^C_+$ the planned stocks. In the first period, planned stocks are exogenous, and assumed to be endowments of the first generation. Let $p_t \geq 0$ be the price vector in period t for the H commodities, and $\psi_{j,t} \geq 0$ the price vector at the beginning of period t for the C stocks of firm j.

Generations maximize utility derived from consumption in the first and second period of life, denoted by $U(x_{t,t}, x_{t,t+1}; n_t)$, subject to the budget constraints. Producers maximize profits in each period, $p_t y_{j,t} + \psi_{j,t+1} k^P_{j,t+1} - \psi_{j,t} k_{j,t}$, subject to the production constraint $F_j(y_{j,t}, -k_{j,t}, k^P_{j,t+1}) \leq 0$. In equilibrium, profits are zero because of constant returns to scale. In the first period, the old generation has a claim on the capital stock $k^P_{j,1}$. There are no other initial claims. As a result, in every period, life-cycle savings balance with the value of the capital stock.

3.2.2. Dynamic efficiency and the value of endowments

Throughout the chapter, dynamic efficiency is established by proving finiteness of the value of all aggregate endowments, or equivalently, by proving that one particular consumer owns a non-negligible share of total endowments [Geanakoplos and Polemarchakis 1991, p.1902]. We state this property and the equivalence as Lemma 3.2 and Lemma 3.3. The first lemma states that the boundedness of the total value of endowments is a sufficient condition for dynamic efficiency. A slightly stronger proposition for an exchange economy can be found in [Ginsburgh and Keyzer 1997, Proposition 8.6]. The second lemma states the equivalence with the presence of one generation with a non-negligible income.

LEMMA 3.2. *A competitive OLG equilibrium in which* $\sum_{t=1,\ldots,\infty} \hat{p}_t(\omega_{t-1,t}+\omega_{t,t})+\hat{\psi}_1 k_1^p<\infty$, *that is in which the value of endowments is bounded, is (dynamically) efficient.*

Proof. In this proof, for convenience, we mark the competitive equilibrium with bounded value of endowments with a hat. In the equilibrium, for every generation, the value of consumption is equal to income, and boundedness of income implies boundedness of the value of consumption:

$$\sum_{t=1,\ldots,\infty} \hat{p}_t(\hat{x}_{t,t}+\hat{x}_{t-1,t})<\infty. \tag{3.1}$$

Hence, the elements $\hat{p}_t(\hat{x}_{t,t}+\hat{x}_{t-1,t})$ must converge to zero, for $t\to\infty$, and

$$\sum_{t=T,\ldots,\infty} \hat{p}_t(\hat{x}_{t,t}+\hat{x}_{t-1,t})\downarrow 0 \tag{3.2}$$

for $T\to\infty$.

We will now show that there is no Pareto improvement of the competitive equilibrium possible, by proving that this would violate the convergence above.

Let us assume that there is an alternative Pareto superior allocation, denoted by a tilde, that strictly increases utility of generation τ, without decreasing utility of any other generation. From utility maximization for generations t=0,…,T, where T>τ, it follows that

$$\hat{p}_1\hat{x}_{0,1}\le\hat{p}_1\tilde{x}_{0,1}, \tag{3.3}$$

for the first generation,

$$\hat{p}_t\hat{x}_{t,t} + \hat{p}_{t+1}\hat{x}_{t,t+1} \le \hat{p}_t\tilde{x}_{t,t} + \hat{p}_{t+1}\tilde{x}_{t,t+1}, \tag{3.4}$$

for generations $1,\ldots,\tau-1,\tau+1,\ldots,T$, and

$$\hat{p}_\tau\hat{x}_{\tau,\tau} + \hat{p}_{\tau+1}\hat{x}_{\tau,\tau+1} + \varepsilon \le \hat{p}_\tau\tilde{x}_{\tau,\tau} + \hat{p}_{\tau+1}\tilde{x}_{\tau,\tau+1}, \tag{3.5}$$

for generation τ for some $\varepsilon>0$. Profit maximization of the firms gives

$$\sum_{t=1,\ldots,\infty}\hat{p}_t\tilde{y}_t \le \sum_{t=1,\ldots,\infty}\hat{p}_t\hat{y}_t, \tag{3.6}$$

and the commodity balance gives

$$\sum_{t=1,\ldots,\infty}(\hat{p}_t\tilde{x}_{t-1,t} + \hat{p}_t\tilde{x}_{t,t}) \le \sum_{t=1,\ldots,\infty}(\hat{p}_t\omega_{t-1,t} + \hat{p}_t\omega_{t,t} + \hat{p}_t\tilde{y}_t), \tag{3.7}$$

while the value of supply and demand is equal in the original allocation

$$\sum_{t=1,\ldots,\infty}9\hat{p}_t\hat{x}_{t-1,t} + \hat{p}_t\hat{x}_{t,t}) = \sum_{t=1,\ldots,\infty}(\hat{p}_t\omega_{t-1,t} + \hat{p}_t\omega_{t,t} + \hat{p}_t\hat{y}_t). \tag{3.8}$$

We can now sum the equations (3.3), (3.4) (3.5), (3.6), and subtract (3.7) and have

$$\sum_{t=T+1,\ldots,\infty}(\hat{p}_T\tilde{x}_{T,T+1} + \hat{p}_{T+1}\tilde{x}_{T+1,T+1}) + \varepsilon \le \sum_{t=T+1,\ldots,\infty}(\hat{p}_T\hat{x}_{T,T+1} + \hat{p}_{T+1}\hat{x}_{T+1,T+1}). \tag{3.9}$$

The equation states that if the value of consumption for generation τ increases by ε, then the value of consumption of the remaining generations must decrease by at least the same amount. However, the value of consumption cannot decrease for any generation $1,\ldots,T$ (3.4), and this makes it necessary to decrease the value of consumption after generation T (3.9), however, this becomes unfeasible for $T\to\infty$, as the value of future consumption (the right hand side) decreases to zero (while the left hand side is bounded from below by ε). ■

LEMMA 3.3. *If there is at least one consumer with a non-negligible income,* $\exists\varepsilon>0$: $\hat{p}_1\omega_{0,1} + \hat{\psi}_1 k_1^P + \hat{\Phi}_0 \ge \varepsilon\sum_{t=1,\ldots,\infty}(\hat{p}_t\omega_{t,t} + \hat{p}_{t+1}\omega_{t,t+1})$, *or* $\hat{p}_i\omega_{i,i} + \hat{p}_{i+1}\omega_{i,i+1} + \hat{\Phi}_i \ge \varepsilon\sum_{t=1,\ldots,\infty}(\hat{p}_t\omega_{t,t} + \hat{p}_{t+1}\omega_{t,t+1}) + \varepsilon\hat{\psi}_1 k_1^P$ *for some $i>0$, then the total value of endowments will be bounded,* $\sum_{t=1,\ldots,\infty}\hat{p}_t(\omega_{t-1,t} + \omega_{t,t}) + \hat{\psi}_1 k_1^P < \infty$, *and vice versa.*

Proof. Let there be one generation with a non-negligible income. This income must be bounded as prices are bounded in every period, and hence, the total value of endowments is bounded from above by the income of this generation divided by its share ε. To prove the lemma the other way around, assume that the total value of endowments is finite. As prices are non-zero, there is a generation with positive income. Dividing this income with the finite value of total endowments results in a strictly positive share $\varepsilon>0$. ■

The lemmas do also apply to economies with externalities in utility and non-rival consumption such as in Section 3.2.3. To see this, first, we notice that an economy in which there is non-rival consumption of a public good is equivalent to a competitive economy with auxiliary goods, one for every consumer, and an auxiliary sector that produces these 'non-rival' goods as joined output, as long as utility functions are non-decreasing in all goods [Milleron 1972]. Since the lemmas apply for this equivalent competitive economy, they apply for the original economy as well. Secondly, we notice that a utility externality as in Assumption 3.6 produces an economy that is equivalent to one in which a 'utility good' is non-rivally produced by a 'utility sector', and used both as a consumption good and as an input for the utility sector in the next period. Again, since the lemmas apply to the equivalent economy, they also apply to the original economy. The lemmas have thus a broad validity.

3.2.3. Time preference and empathy

If an equilibrium is dynamically inefficient, it is possible through transfers to obtain an allocation where both the net receivers and the net payers of transfers gain. If making a gift does not harm the benefactor, one might expect that such gifts could possibly be generated by rational consumer behavior, in which case there would be no need for government intervention. However, in the dynamically inefficient equilibrium, the possibility of increasing one's own welfare by renouncing part of one's income is not necessarily recognized by the benefactor: a transfer to another generation still seems to reduce the own income, and hence welfare. In this section, we give examples of an economy in which transfers restore efficiency, but where they follow from the altruism of the benefactor.

The analysis is specific in the sense that the efficiency relies on strong assumptions with respect to utility functions and endowments. The next sub-section gives an alternative assumption regarding the production technology that is less specific and also suffices to attain dynamic efficiency.

Consider an economy with constant population, with each generation of size $n=1$, where consumers live two periods, and where the endowments when old are a significant share of total endowments:

ASSUMPTION 3.4. *For every commodity, endowments are not too strongly skewed in favor of the first period:* $\omega_y \leq \delta\omega_o$, *for some* $\delta>0$.

In addition, we assume that the utility function has a Cobb-Douglas structure with a strictly positive time preference that exceeds the rate of decline in endowments:

ASSUMPTION 3.5. *The utility function is a CD function with constant expenditure share* $0<\sigma<1/\delta$ *for consumption when old*: $\mathrm{u}(x_y, x_o) = \tilde{\mathrm{u}}(x_y)^{\frac{1}{1+\sigma}}\, \tilde{\mathrm{u}}(x_o)^{\frac{\sigma}{1+\sigma}}$, *where* $\tilde{\mathrm{u}}(.)$ *is linearly homogeneous.*

As follows from utility maximization, expenditures have fixed proportions:

$$p_{t+1}x_{t,t+1} = \sigma \cdot p_t x_{t,t} \ . \tag{3.10}$$

Moreover, assume that life cycle savings are non-negative (the young cannot borrow, paying back when old), then consumption when young is less than endowments,

$$p_t x_{t,t} \le p_t \omega_y \ , \tag{3.11}$$

while consumption when old exceeds endowments.

$$p_{t+1}\omega_o \le p_{t+1}x_{t,t+1} \, . \tag{3.12}$$

After summation of the last three equations, we have

$$p_{t+1}\omega_o \le \sigma \cdot p_t \omega_y , \tag{3.13}$$

which implies that the distribution of endowments, Assumption 3.4, guarantees that the value of endowments decreases over time, at an exponential rate not less than $\sigma\delta$:

$$p_{t+1}(\omega_o + \omega_y) \le \sigma\delta \cdot p_t(\omega_o + \omega_y) \, . \tag{3.14}$$

Hence, if the rate of time preference exceeds the rate of decline in endowments, the competitive equilibrium will be efficient.

We extend the example by introducing another kind of time preference: empathy. Empathy induces transfers between generations on a purely voluntary basis, unlike the efficiency policy described later. Empathy can be directed forward, to future generations, or backward, to past generations. As the solution to avoid dynamic inefficiencies requires that we give present generations a non-negligible share in future endowments, it is backward empathy, transferring income from future to present generations, that will prove helpful.

We assume that utility also partly depends on the utility of the previous generation, using a nested Cobb-Douglas structure:

ASSUMPTION 3.6. *The utility function is a nested CD function with constant expenditure share* $0<\gamma<1$ *transferred to the previous generation*: $\upsilon_t = \tilde{\tilde{u}}(x_y, x_o, \upsilon_{t-1}) = \upsilon_{t-1}^{\gamma}\, u(x_y, x_o)^{1-\gamma}$, *where* υ_t *is the utility of generation t.*

The empathy induces generation t to transfer a share γ of its expendable income to the previous generation, for example if the young pay for a part of the pensions of the old (the previous generation), a so called 'pay as you go' system.

Expendable income of generation t itself includes a transfer from generation t+1, and by induction, from generation t+s. Disregarding transfers that are expressed in prices other than for periods t and t+1, the budget equation for the first period for generation t yields:

$$p_t x_{t,t} \le p_t \omega_y - \gamma p_t \omega_y - \gamma p_{t+1} \omega_o - \gamma^2 p_{t+1} \omega_y \; . \tag{3.15}$$

For the second period, we have:

$$p_{t+1} \omega_o + \gamma p_{t+1} \omega_y \le p_{t+1} x_{t,t+1} . \tag{3.16}$$

Together with equation (3.10), this leads to:

$$(1+\gamma\sigma) p_{t+1} \omega_o + \gamma (1+\gamma\sigma) p_{t+1} \omega_y \le \sigma (1-\gamma) p_t \omega_y \; . \tag{3.17}$$

This result is stronger than equation (3.13), since it implies that

$$p_{t+1} \omega_o \le \frac{1-\gamma}{1+\gamma\sigma} \sigma p_t \omega_y , \tag{3.18}$$

which can be used to show that the value of endowments decreases geometrically over time. This can be summarized as a theorem.

THEOREM 3.1. *If consumers satisfy Assumptions* 3.4-3.6, *transfers are voluntary and non-negative and* $\sigma\delta < (1+\gamma\delta)(1+\sigma\gamma)/(1-\gamma)$, *in addition to Assumptions O1, the infinite horizon OLG equilibrium (Definition* 2.16) *is dynamically efficient.*

Proof. Substitution of $(1-\gamma)/(1+\gamma\delta)$ units of ω_y for $\delta(1-\gamma)/(1+\gamma\delta)$ units of ω_o (Assumption 3.6) gives:

$$p_t (\omega_y + \omega_o) \le \frac{\gamma(1+\delta)}{1+\gamma\delta} p_t \omega_y + \frac{(1+\delta)}{1+\gamma\delta} p_t \omega_o , \tag{3.19}$$

and by (3.17) we have:

$$\frac{\gamma(1+\delta)}{1+\gamma\delta}p_t\omega_y+\frac{(1+\delta)}{1+\gamma\delta}p_t\omega_o\le\frac{(1+\delta)(1-\gamma)}{(1+\gamma\delta)(1+\gamma\sigma)}\sigma p_t\omega_y. \qquad (3.20)$$

Again, if we substitute $(1-\gamma)/(1+\gamma\delta)(1+\gamma\sigma)$ units of ω_y for $\delta(1-\gamma)/(1+\gamma\delta)(1+\gamma\sigma)$ units of ω_o, we have:

$$\frac{(1+\delta)(1-\gamma)}{(1+\gamma\delta)(1+\gamma\sigma)}\sigma p_t\omega_y\le\frac{(1-\gamma)}{(1+\gamma\delta)(1+\gamma\sigma)}\delta\sigma p_t(\omega_y+\omega_o). \qquad (3.21)$$

Summation of the equations gives the desired result:

$$p_{t+1}(\omega_y+\omega_o)\le\frac{(1-\gamma)}{(1+\gamma\delta)(1+\gamma\sigma)}\delta\sigma p_t(\omega_y+\omega_o), \qquad (3.22)$$

which implies that if $\sigma\delta<(1+\gamma\delta)(1+\sigma\gamma)/(1-\gamma)$, then $p_{t+1}(\omega_o+\omega_y)<p_t(\omega_o+\omega_y)$; the value of endowments decreases geometrically and the aggregate value is bounded, and Lemma 3.2 can be invoked. ■

The theorem combines time preferences for consumption with empathy between generations. If empathy between generations is part of the restored efficiency ($\gamma>0$), the equilibrium is characterized by intergenerational transfers. The young voluntarily transfer a part of their income to the old ($\Gamma_t<0$, in the notation of Chapter 2), though these transfers are not imposed by a public authority.

3.2.4. A non-negligible environmental resource

As an alternative for consumers' time preferences, efficiency can also be established if the production structure satisfies additional assumptions with regard to the production technology. In the literature, input-output growth models usually assume that all commodities can be reproduced every period. Under this assumption, giving up one unit of consumption for all goods in period t yields an increased consumption in period t+1 of strictly more than one unit (of all goods). Hence, prices must fall in time by at least the regeneration rate. As prices decrease exponentially, the aggregate value of endowments is finite, and dynamic efficiency is restored. [Ginsburgh and Keyzer 1997, Section 8.3.2]

However, there are two objections against this common assumption. First, it is rather strong. This is reflected in the resulting exponentially decrease in prices, which is not required for dynamic efficiency. Secondly, the assumption would contradict the previous Assumption *F4* in this study: continuation of the regeneration would increase the capital stock without bound. As we are living in a finite world, this seems unrealistic.

As an alternative, let us assume that there is a resource which produces non-negative services, $\bar{y}$, that always increase utility. Moreover, assume that such an output can be produced by a resource stock $\bar{k}$ that does not require maintenance costs. These properties define an environmental resource that has the first two characteristics mentioned in Section 1.4.1. We may think of a natural renewable resource, such as fish and timber which can be produced indefinitely. For the precise formulation, we divide the output $\bar{y}$ in a part given to the old and the young generation, denoted by $\bar{y}_o$ and $\bar{y}_y$ respectively. The resource has to satisfy: $F_j(\bar{y}, -\bar{k}, \bar{k}) \le 0$, where j denotes the technology associated to the resource. We take the view that either one firm (resource) suffices to produce the non-negligible output, or that a combination of firms is required, and omit the subscript j for convenience. Thus, if we state that there is 'an environmental resource', we formally demand that aggregate technology supports the indefinite output stream.

ASSUMPTION 3.7. *There is a non-negligible environmental resource: i.e. a resource with a feasible sustainable production vector $\bar{y}$ for which utility is non-satiated.*

F7: $\exists \bar{k} < k_1^P, \bar{y} \ge 0$: $F(\bar{y}, -\bar{k}, \bar{k}) \le 0$ *and*
$\forall x_y, x_o \in \mathbf{X}: u(x_y + \bar{y}_y, x_o + \bar{y}_o) > u(x_y, x_o)$

In addition, the utility of the first generation (living only one period) is also taken to be increasing in this commodity bundle. The feasible sustainable production vector $\bar{y}$ is a fixed parameter, exogenous to the equilibrium. We will now state and prove that utility increases uniformly in this parameter $\bar{y}$. More precisely, that there is a fixed ε by which utility increases for the given $\bar{y}$.

LEMMA 3.8. *Under standard Assumptions O1 of Chapter 2, if* $\forall x_y, x_o \in \mathbf{X}: u(x_y + \bar{y}_y, x_o + \bar{y}_o) > u(x_y, x_o)$, *that is if utility is non-satiated in $\bar{y}$, utility increases uniformly in $\bar{y}$:*

$$\exists\ \varepsilon > 0: \forall x_y, x_o \in \mathbf{X}:\ u(x_y + \bar{y}_y, x_o + \bar{y}_o) \ge (1+\varepsilon)\, u(x_y, x_o) \tag{3.23}$$

Proof. Notice that we could have written $\varepsilon(\bar{y})$ to stress that ε depends on the sustainable output $\bar{y}$, but as the sustainable output itself is fixed, this is not necessary. Recall from Chapter 2 (Lemma 2.7) that consumption is bounded. Let μ be the minimum of $u(x_y + \bar{y}_y, x_o + \bar{y}_o)$ over the compact consumption set $\mathbf{X}$. As u(.) is bounded from below by zero (*U3*), Assumption *F7* implies that $u(x_y + \bar{y}_y, x_o + \bar{y}_o)$ is strictly positive and continuous (*U2*), and hence, μ is well defined and $\mu > 0$. Now, define the continuous function $e(x_y, x_o) = (u(x_y + \bar{y}_y, x_o + \bar{y}_o) + \mu) / (u(x_y, x_o) + \mu) - 1$, and let ε be its minimum over **X**. It follows that $u(x_y + \bar{y}_y, x_o + \bar{y}_o) + \mu \geq (1+\varepsilon)(u(x_y, x_o) + \mu)$, and from that, (3.23) follows. ■

A resource satisfying Assumption *F7* will be referred to as non-negligible, because it has the capacity to increase utility uniformly, that is, with an amount that does not diminish over time. The theorem below states that the mere existence of a non-negligible environmental resource is sufficient to establish efficiency.

THEOREM 3.2. *If standard Assumptions O1 are satisfied, and if there is a non-negligible environmental resource (Assumption F7), then the infinite horizon OLG equilibrium (Definition 2.19) with no transfers exists and is efficient.*

Proof. Existence is proven in Theorem 2.2 in Chapter 2. For efficiency, we show that the first generation owns a non-negligible share of total income. Lemma 3.2 is then used to establish efficiency. We recall that the value of initial stocks equals the aggregate value of production over time, being at least equal to the value of any alternative production scheme:

$$\psi_1 k_1^p = \sum_{t=1,\dots,\infty} p_t y_t \geq \sum_{t=1,\dots,\infty} p_t \bar{y} \ . \tag{3.24}$$

Let the welfare weights α_t be chosen to satisfy:

$$(p_t, p_{t+1}) \geq \alpha_t \nabla u(x_{t,t}, x_{t,t+1}) \ \perp \ (x_{t,t}, x_{t,t+1}) \geq 0 \ . \tag{3.25}$$

where ∇ is the sub-differential (and the obvious adjustment for t=0). Concavity of the utility function ensures that

$$\alpha_t \, u(x_{t,t}, x_{t,t+1}) \geq p_t x_{t,t} + p_{t+1} x_{t,t+1} \ , \tag{3.26}$$

because u(0,0)≥0, and that

$$p_t \bar{y}_y + p_{t+1} \bar{y}_o \geq \alpha_t \, u(x_{t,t} + \bar{y}_y, x_{t,t+1} + \bar{y}_o) - \alpha_t \, u(x_{t,t}, x_{t,t+1}) \ . \tag{3.27}$$

Because utility increases uniformly (Lemma 3.8), the value of the sustainable production vector is a non-negligible share of utility, and (3.27) becomes

$$p_t \bar{y}_y + p_{t+1} \bar{y}_o \geq \varepsilon \alpha_t \, u(x_{t,t}, x_{t,t+1}) , \tag{3.28}$$

for ε>0. Now, substituting (3.26) results in:

$$p_t \bar{y}_y + p_{t+1} \bar{y}_o \geq \varepsilon (p_t x_{t,t} + p_{t+1} x_{t,t+1}) \tag{3.29}$$

for t=1,...,∞, and the equivalence,

$$p_1 \bar{y}_o \geq \varepsilon p_1 x_{0,1} \,, \tag{3.30}$$

for generation t=0. Summation over all periods, including the commodity balances gives

$$\sum_{t=1,\ldots,\infty} p_t \bar{y} + \varepsilon \sum_{t=1,\ldots,\infty} p_t y_t \geq \varepsilon \sum_{t=1,\ldots,\infty} p_t (\omega_{t,t} + \omega_{t-1,t}) \,, \tag{3.31}$$

which, because of (3.24), leads to the required result:

$$\psi_1 k_1^p \geq \frac{\varepsilon}{1+\varepsilon} \sum_{t=1,\ldots,\infty} p_t (\omega_{t,t} + \omega_{t-1,t}) \,. \tag{3.32}$$

As $\psi_1 k_1^p$, is a non-negligible share of total endowments and as all the stocks are owned by the first generation (no transfers), it follows that this generation has a non-negligible share of total income, and Lemma 3.2 and Lemma 3.3 can be invoked. ■

Assumption *F7* requires that the environmental resources must always be capable of increasing utility, which seems plausible. It is not required that the resource stock be measured in the same units as the output flow, nor that the resource stock is renewable. Exhaustible resources with amenity values also satisfy the assumption as we illustrate in Section 3.4. The global ecosystem might be an example. It produces many life-support elements that undoubtedly increase utility, and it is capable of regeneration if properly managed. At the same time, its regeneration capacity is limited, once it has deteriorated beyond a threshold level. Thus, in a stylized context, it can be represented as an exhaustible resource with amenity value.

3.2.5. A non-negligible income claim

An alternative mechanism for securing dynamic efficiency of an infinite horizon competitive OLG equilibrium is based on policy measures, rather than on characteristics of the consumers and producers. Here, we avoid the use of additional assumptions on preferences (empathy) and technology (non-negligible resource). Instead, a transfer scheme is introduced that restores efficiency. The essential element of the policy is to endow one consumer with a non-negligible share of total future endowments, using transfers if necessary.

To implement the intergenerational transfers, we assume that there is a public authority that can levy taxes, now and in the future. This public authority issues a freely tradable claim and pays the owner a real interest, e.g. the value of a given bundle of goods. More precisely, in every period, the claim yields the owner a small share, γ, of all flow endowments. The payments by the public authority to the owner are balanced with taxes levied during the same period, so that the claim induces an income transfer from all owners of endowments to the owner of the claim. The confidence in the public authority, especially its recognized right to levy taxes safeguards the value of the claim. Let the value of the claim be denoted by χ, which satisfies

$$\chi_t = \gamma \sum_{\tau=t,\dots,\infty} p_\tau(\omega_{\tau,\tau} + \omega_{\tau-1,\tau}) \tag{3.33}$$

In the first period, the claim is given to the first generation so that net transfers are given by

$$\Phi_0 = \chi_1 - \gamma p_1 \omega_{0,1} \tag{3.34}$$

for the first generation, and

$$\Phi_t = -\gamma(p_t \omega_{t,t} + p_{t+1}\omega_{t,t+1}) \tag{3.35}$$

for the other generations.

The claim can be compared with real bonds, such as index-linked bonds based on the consumer price index [Cecco *et al.* 1997]. Whereas the payments of the latter are indexed on the value of a standard bundle of consumer goods, this claim is indexed on the value of endowments. We note, however, that the comparison might be misleading because the context of our analysis is different from one in which index-linked bonds are analyzed usually. Our model has perfect foresight and complete markets (including asset markets), which implies that all financial assets are equivalent [Modigliani and Miller 1958]. Hence, all monetary issues such as inflation, uncertainty, and asset market incompleteness are not subject of analysis. Here, the claim is purely a redistributional device between the first and subsequent generations that restores efficiency:

THEOREM 3.3. *The infinite horizon OLG equilibrium (Definition 2.16 under standard assumptions O1) exists and is (dynamically) efficient if the transfers are supported by a claim with a non-negligible real interest rate, that is given to the first generation as a transfer (equations* (3.33), (3.34), and (3.35)).

Proof. The implied transfers do not only depend on present and past prices. This implies that Assumption A2 is violated and Theorem 2.2 does not apply. However, the economy is equivalent to an artificial economy without transfers for which the theorem applies. In the artificial economy, endowments of all generations are reduced by a factor γ. Assumptions ($\Omega 1$) and ($\Omega 2$) will still be satisfied. Next, a resource is defined with a stock that regenerates itself and generates an output equal to γ times the original endowments. This resource satisfies assumptions (*F1-F6*), and (*F7*) because utility is strictly increasing in the own endowments. Therefore, the constructed economy has an equilibrium, and is efficient (Theorem 3.2). Because of the equivalence, the original economy has also an efficient equilibrium. ■

We notice that the policy can be modified so as to restrict the use of the claim to situations where dynamic inefficiency might arise. For this, we endow the first generation with an option to buy the claim, instead of giving the claim for free. If the option is exercised, the first generation receives the claim at a certain price, say the value of total endowments in the first period. The option value will be positive only if the claim value exceeds the value of endowments, i.e., if prices decrease too slowly. If prices decrease at a sufficient rate, the option will not be exercised. Dynamic efficiency will be ensured, since the claim 'automatically' enters the economy if prices decrease too slowly.

Future commitments of the public authority can be understood as public debt, and in a partial analysis, we could interpret the issuing of the non-negligible claim as a reduction of the capital stock. To see this, we recollect from Chapter 2, that in the standard OLG equilibrium, life-cycle savings balance with the value of capital under the assumption that the first generation owned no claim other than for the initial capital stock. The existence proof for the equilibrium above suggests how this balance will change in the OLG equilibrium with the non-negligible claim. In the artificial economy, life-cycle savings S_t balance with the value of capital $\psi_t k_t$ plus the value of the claim χ_t:

$$S_t = \psi_t k_t + \chi_t \tag{3.36}$$

To be precise, life-cycle savings consist of income minus consumption expenditures in the first period, where income excludes the tax payments made to the public authority associated to the claim.

Therefore, it might be suggested that an ordinary ad-hoc public debt policy offers an easier way to attain dynamic efficiency. However, the advantage of the claim is that it determines the proper level of public debt within the competitive equilibrium. Though the efficiency policy does not satisfy Assumption A2 in the formal sense, it is 'simple' as intended: the public authority does not need to have perfect foresight to implement the required transfers. It only issues the claim and gives it to the first generation (not necessarily for free), balancing its commitments in every period with taxes in the same period.

To conclude, we compare the efficiency effect of the environmental resource with the non-negligible claim. From the proof of Theorem 3.3, it is clear that the non-negligible claim functions as a fictitious non-negligible resource. The question that naturally arises is whether the claim still is of any use if efficiency is already ensured by the existence of environmental resources. On this issue, two comments are in order. First, it might not be desirable to give the environmental resource to the generation that unveils its value, as grandfathering environmental resources might cause unsustainability. This is analyzed in full detail in the next chapter. Secondly, the efficiency established by the non-negligible claim is more robust than if it is based on a non-negligible environmental resource. Once the claim has been issued, it ensures that the value of the consumption of the old is always a non-negligible share of the net present value of all future endowments. This property is robust, and probably also applies to less ideal economies with market distortions and without perfect foresight. In such economies, the claim will ensure that the value of the consumption of the old is always a non-negligible share of the *expected* net present value of all future endowments. Establishing perfect markets and perfect foresight in a given period would immediately ensure perfect dynamic efficiency from that period onwards. On the other hand, the environmental resource argument for efficiency is frail. If the resource is not used optimally, its value might become less than the value of the indefinite stream of potential future output. Moreover, if the environmental resource is exhausted, the dynamic efficiency argument vanishes readily. We conclude that the non-negligible claim offers a more robust way of attaining dynamic efficiency, and we will see in the next chapter that it can be combined with a policy instrument that achieves sustainability.

3.3. THE NON-NEGLIGIBLE CLAIM, AN ILLUSTRATION

3.3.1. Aging population causing dynamic inefficiency

In this and next chapters, we will develop a series of stylized models that illustrate the general analysis. The common model used for the illustrations will be referred to as ALICE, an acronym for Applied Long-term Integrated Competitive Equilibrium model. The steady state model described in this section, ALICE 0, is a one-good exchange model in which the substitution effect of an increase in the interest rate exceeds the income effect, i.e., an increased interest rate implies increased life-cycle savings. This property explains most of the results in the next two sub-sections. In Section 3.4, ALICE is developed into a simulation model which includes a basic description of the biogeochemical cycles determining the global climatic system as an exhaustible environmental resource with amenity value. There is no man-made capital, and in the long run, the production side reduces to an exchange economy for 'man made' goods and a constant production of environmental services. The model will also be extended and used in Chapter 4, where some welfare properties will be proven in relation to a specified sustainability policy. Finally, in Chapter 5, the model will be further developed into

ALICE 2 which includes explicit geophysical variables such as CO_2 concentrations and the global mean temperature.

ALICE 0 uses simulation periods of 20 years duration, and it is used to compare steady states before and after a demographic transition, and to analyze the transition between different steady states. Before the demographic transition, generations are assumed to live two adult periods. These two periods can be thought of as describing the life-cycle from 20 to 60 years. The first twenty years are not incorporated in the own economic life-cycle, but are assumed to fall within the parents' economic life-cycle. Initially, everyone is supposed to reach the age of 60, no individual dies before his time, neither does anyone live beyond.

The utility function is of the CES type with an intertemporal elasticity of consumption $\rho>0$, and time preference factor $\sigma>0$. The endowments grow at a stationary rate by $\delta>1$ per period. In each period, $\omega_{t-1,t}=\omega_{t,t}=\delta$, endowments are evenly distributed over the young and old generation. Whether we assume the growth of endowments to be caused by population growth or by increasing per capita endowments does not matter for our analysis, as the size of population does not affect consumption behavior, due to linear homogeneity. The model has no stocks nor other claims, hence ALICE 0 is a simple one-good two-period exchange economy in which savings are zero. In other words, consumption exactly equals endowments, $(x_{t-1,t},x_{t,t})=(\omega_{t-1,t},\omega_{t,t})$, and $x_{t-1,t}=x_{t,t}=\delta^t$. In the steady state before the demographic transition, the price dynamics between two periods are given by

$$\beta = \sigma\delta^{-1/\rho} \tag{3.37}$$

where $\beta=p_{t+1}/p_t$ is the (constant) price ratio between two periods. The rate of decrease of prices, $\beta^{-1}-1$, is referred to as the (real) interest rate. The parameters, given in the annex, are chosen such that prices decrease by 5 per cent annually. The economy is (assumed to be) efficient: prices drop faster than the economy grows, $\beta\delta<1$. This completes the description of the steady state before the demographic transition

After the demographic transition, we assume for convenience that people reach 80 years of age (As we already indicated in Section 3.1, the World Bank [WB 1994] assumes a life expectancy of 85 years in 2100). Their economic life (excluding childhood) will now span three periods. However, endowments do not extent uniformly over the life-cycle. Endowments of the old are only 40 per cent of the endowments of the young and middle aged. The time preference for consumption is assumed to remain constant over the lifetime. Thus, consumption will increase at the same rate. To satisfy the budget constraint, consumption in the first two periods will be less than endowments, whereas consumption in the third period will exceed the endowments: life-cycle savings are positive because of aging. Yet, because we assumed that there are no initial claims in the competitive equilibrium, which implies that there are no claims in the steady state, aggregate period-wise savings have to be zero in the pure exchange economy. The price ratio between two periods will have to adjust to ensure this.

The nature of the resulting shift in the steady state equilibrium depends on various assumptions, including the specification of the retirement system. One possibility is a so called 'pay as you go' system of social security, which implies that transfers from present to retired generations increase. Auerbach *et al.* [1989] analyze 'aging' within this context. Alternatively, a fully funded system assumes that all generations pay for their own retirement. This will be our assumption.

In case of a fully funded system, the effect of an increased interest rate on life-cycle savings depends on the substitution and income effects of the interest rate. If the interest rate increases, consumption when old becomes relative cheap compared to consumption when young, and life-cycle savings will increase to substitute consumption when young for consumption when old. On the other hand, if the interest rate increases, decreased savings when young suffice for the same consumption levels when old, the so called income effect. It is in general unknown which effect dominates, but as noted by Blanchard and Fisher [1989, Section 3.5 and Problem 4], the substitution effect usually exceeds the income effect if the utility function is separable in time and if endowments are substantial throughout the life-cycle. As this applies to our economy, we may expect that aging causes the steady state interest rate to decrease because of compensation for the tendency towards increased life-cycle savings described above.

For the formal analysis, we follow Kehoe [1991] and define demand functions, d_y, d_m, d_o: $\beta \rightarrow \mathbf{R}^+$, where $d_y(\beta)=\hat{x}_y$, $d_m(\beta)=\hat{x}_m$, $d_o(\beta)=\hat{x}_o$, which maximizes utility, $u(x_y, \delta x_m, \delta^2 x_o)$, subject to the budget constraint $x_y+\beta\delta x_m+\beta^2\delta^2 x_o=\omega+\beta\delta\omega_m+\beta^2\delta^2\omega_o$, where the subscripts '*y*', '*m*', and '*o*' denote the young, middle aged and old. Thus, the demand functions satisfy:

$$d_y(\beta)+\beta\delta d_m(\beta)+\beta^2\delta^2\, d_o(\beta)=\omega+\beta\delta\omega_m+\beta^2\delta^2\omega_o. \tag{3.38}$$

As there is no production, excess demand $z(\beta)$ is given by:

$$z(\beta)=d_y(\beta)+d_m(\beta)+d_o(\beta)-(\omega_y+\omega_m+\omega_o)\,. \tag{3.39}$$

Multiplying equation (3.38) by $\beta^{-2}\delta^{-2}$ and subtracting equation (3.39), results in the savings balance multiplied by a factor $(1-\beta^{-1}\delta^{-1})$:

$$(1-\beta^{-1}\delta^{-1})((1+\beta^{-1}\delta^{-1})(\omega_y-d_y(\beta))+\omega_m-d_m(\beta))=z(\beta), \tag{3.40}$$

which is zero in equilibrium. Hence, either the interest rate is equal to the growth rate, $\beta\delta=1$, or life-cycle savings, S, are equal to zero:

$$S = (1+\beta^{-1}\delta^{-1})(\omega_y - d_y(\beta)) + \omega_m - d_m(\beta)) = 0. \tag{3.41}$$

Life cycle savings of the young are equal to the excess value of endowments, $\omega_y - d_y(\beta)$. The middle aged generation has savings from the current period, $\omega_m - d_m(\beta)$, plus savings from the preceding period, when the generation was young. But the savings of the young one period ago differ by a factor δ^{-1} from current savings of the young, while their value has increased by a factor β^{-1}. Thus, total savings of the middle aged amount to $\beta^{-1}\delta^{-1}(\omega_y - d_y(\beta)) + \omega_m - d_m(\beta)$.

If the interest rate is equal to the growth rate, $\beta\delta=1$, the budget equation (3.38) reduces to the commodity balance and excess demand must be zero as well, $z(\delta^{-1})=0$. This type of steady state is called monetary, because it is supported by fiat money to balance non-zero (private) savings. There is another steady state, $z(\beta)=0$, for which $\beta\delta\neq1$, called the real steady state. It follows from (3.40) that savings are zero in the real steady state. Since we have not introduced claims in the economy, the monetary steady state is unattainable. But, it is of interest since we will show that the efficiency policy described in Section 3.3.2 based on the issuing of the non-negligible claim attains a steady state arbitrary close to the monetary one.

Table 3.1 (first 2 column entries) summarizes the properties of the 'monetary' and 'real' steady states for the parameters specified in Table 3.3 of the annex.

TABLE 3.1. *Characterization of steady states for ALICE 0*

	Real	Monetary	Claim	
Net tax rate			0.01	0.1
Private savings[1]	0.	1.6	2.1	3.8
Interest rate (yr^{-1})	0.005	0.020	0.025	0.046
Welfare	1.048	1.064	1.062	1.020
Consumption by young	1.140	1.012	0.972	0.802
Consumption by middle aged	0.757	0.783	0.788	0.800
Consumption by old	0.503	0.606	0.640	0.798

[1] Ratio to annual income
See the text for explanation of variables, and Annexes 3A.2 and 3A.3 for the computations.

The first row entry corresponds to private life-cycle savings which are non-zero for the monetary steady state. Private savings are given as a ratio to the income level in the same period. The second row entry gives the interest rate. The interest rate for the monetary steady state is equal to the economic growth rate, 2 per cent per year, whereas the interest rate of the real steady state lies below the growth rate. This confirms our earlier statement that the interest rate would decrease because of aging. It implies that the value of the future endowments increases and in our numerical model, the real steady state is inefficient.

The third row entry gives a measure of welfare defined as relative utility derived from consumption compared to (potential) utility derived from endowments, $w \equiv u(x_y, \delta x_m, \delta^2 x_o)/u(\omega_y, \delta\omega_m, \delta^2\omega_o)$. Obviously, the welfare measure is constant in time for both steady states. The golden rule applies to this economy, which states that of all possible steady states satisfying the commodity balance, stationary welfare is maximal for the monetary steady state. The last three row entries give the distribution of total consumption (summing to 2.4) over the young, middle aged and old generations.

The OLG competitive equilibrium without intergenerational transfers appears to be determinate (see Section 2.5.1). We ran the truncated model for generations 1,...,30, and periods 3,...,30 (see Annex 3A.1 for a more detailed description). Figure 1 reveals the turnpike dynamics for the welfare measure (defined for generations 1,...,30) and the interest rate (defined for periods 3,...,29). The figure shows that the equilibrium has approached the real steady state in period 6, and only deviates again after period 27. This characteristic is a sufficient condition for determinacy as it shows that half of the eigenvalues is stable, whereas the other half is unstable. Moreover, it follows that the truncation does not affect the equilibrium during the 'central periods'. Therefore, in the next sub-section, we will put the efficiency policy in effect during these periods.

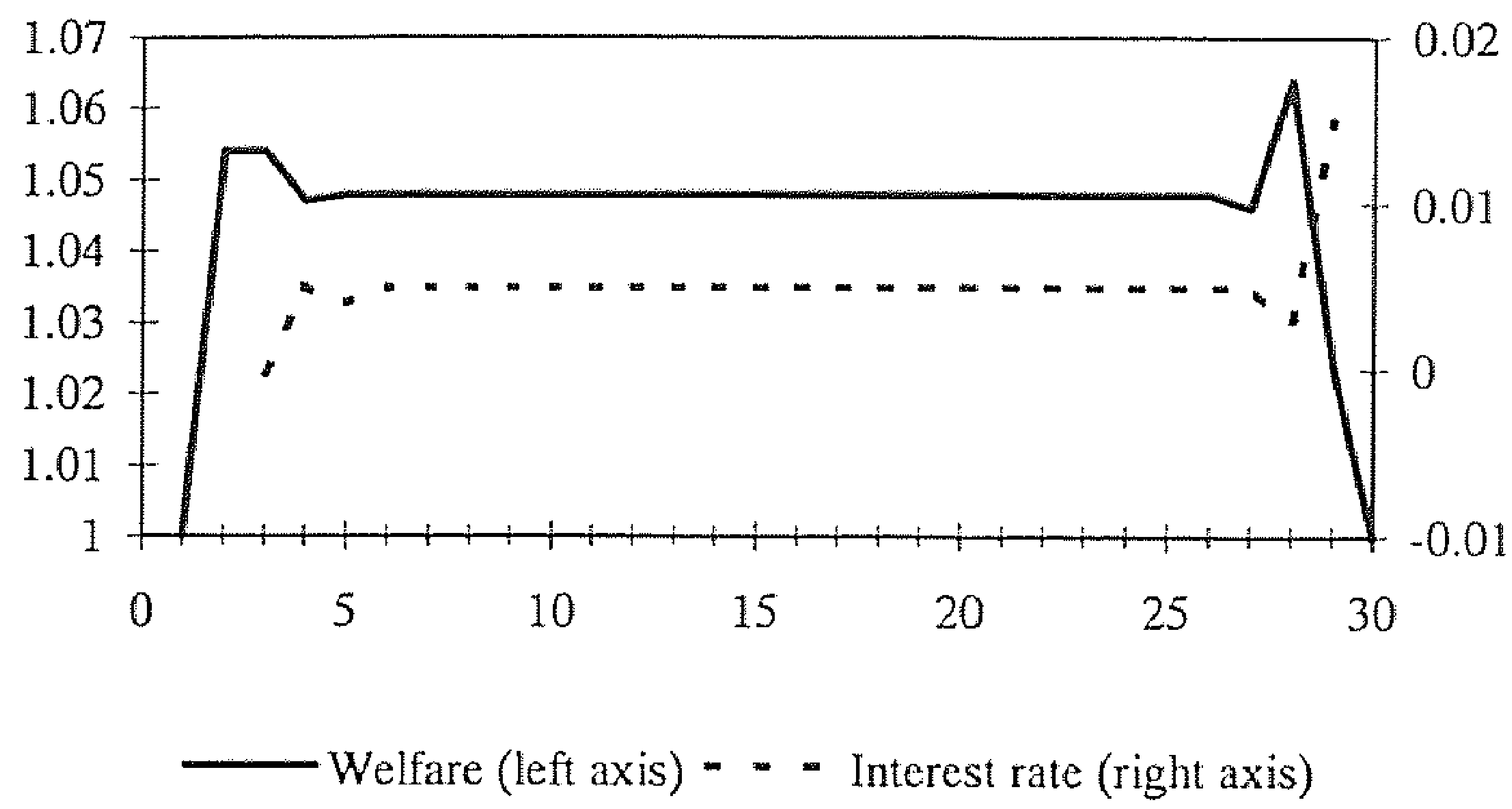

FIGURE 3.1. *Welfare and interest turnpikes for truncated economy without transfers*

3.3.2. Restoring efficiency by issuing a non-negligible claim

Having described the economy without transfers, we can study the effects of the efficiency policy. To restore efficiency, the public authority issues the claim in period $\tau=11$, giving the claim for free to the young generation. In each subsequent period, the public authority pays the owner of the claim the value of a fixed share γ of endowments by levying the required tax on all generations alive in that period. The economy will be seen to converge rapidly to a new steady state after period τ, where the budget equation (3.38) becomes:

$$x_y+\beta\delta x_m+\beta^2\delta^2 x_o=(1\text{-}\gamma)(\omega_y+\beta\delta\omega_m+\beta^2\delta^2\omega_o)\ . \tag{3.42}$$

The demand functions for the steady state, $d_y(\beta,\gamma)$, $d_m(\beta,\gamma)$, $d_o(\beta,\gamma)$, are now continuous functions of the price ratio and the tax rate. Excess demand is adjusted in the same way, so that it is also continuous in the price ratio and the tax rate, $z(\beta,\gamma)$, for $\beta>0$, $0\leq\gamma<1$. Because of continuity of $z(\beta,\gamma)$, for small $\gamma>0$, there are two solutions β for $z(\beta,\gamma)=0$ close to the original solution for $z(\beta,0)=0$ that represented the monetary and real steady states. However, as we will show, the solution for $z(\beta,\gamma)=0$ close to the real steady state is not a proper steady state, and hence, only the 'monetary' steady state exists.

To show that the steady state close to the real steady state is unfeasible, we apply the accounting rules derived in Section 3.2.5. As the claim is given for free and as there is no capital in the model, equation (3.36) becomes:

$$S_t=\chi_t \tag{3.43}$$

which states that private savings are equal to the value of the claim. In the steady state, we have

$$\chi=(\gamma\,/\,(1-\beta\delta))(\omega_y+\omega_m+\omega_o)\,, \tag{3.44}$$

where the claim value is measured in current prices. The value of the claim is only finite for $\beta\delta<1$, otherwise, the value of the claim is unbounded and the steady state does not exist. This rules out a steady state close to the real steady state.

Consequently, imposing a small efficiency tax will not cause a minor shift in equilibrium from the real steady state to another close steady state, but a jump from the real steady state to a steady state close to the previous monetary steady state.

Two scenarios are run, for $\gamma=0.01$ and $\gamma=0.1$, representing a net tax on endowments of future generations of 1 per cent and 10 per cent, respectively. The feasible steady states with taxes close to the monetary steady state are listed in the last two column entries of Table 3.1. The first row entry of the table gives the value of the claim as a share of the value of endowments, which is equal to private savings transferred from the previous to the present period. The results can be checked easily. The net present value of the claim is equal to 1 per cent of production cumulated over an indefinite future where the interest rate exceeds economic growth by 0.5 per cent per year, which gives 0.01/0.005=2 times the value of annual endowments. The same calculation applies for the 10 per cent tax. The claim value is 0.1/0.026=3.8 times the value of annual endowments.

After the economy has moved towards the steady state with the non-negligible claim for γ=0.01, welfare has increased as compared to the inefficient real steady state (Table 3.1, fourth row entry). Future generations are better off paying the tax associated to the claim. The generation receiving the claim for free (generation 11) finds its utility increasing with 20 per cent (Figure 3.2). The 1 per cent tax policy is a win-win policy which is possible because of the inefficiency of the original real steady state. However, as the equilibrium has become efficient, any further increase of the tax rate will decrease utilities of some generations and increase utilities of others. This is confirmed by the results associated with the 10 per cent tax claim, where a fall in welfare of future generations below the real steady state value is set off by a further increase of utility for the generation receiving the claim (Figure 3.2).

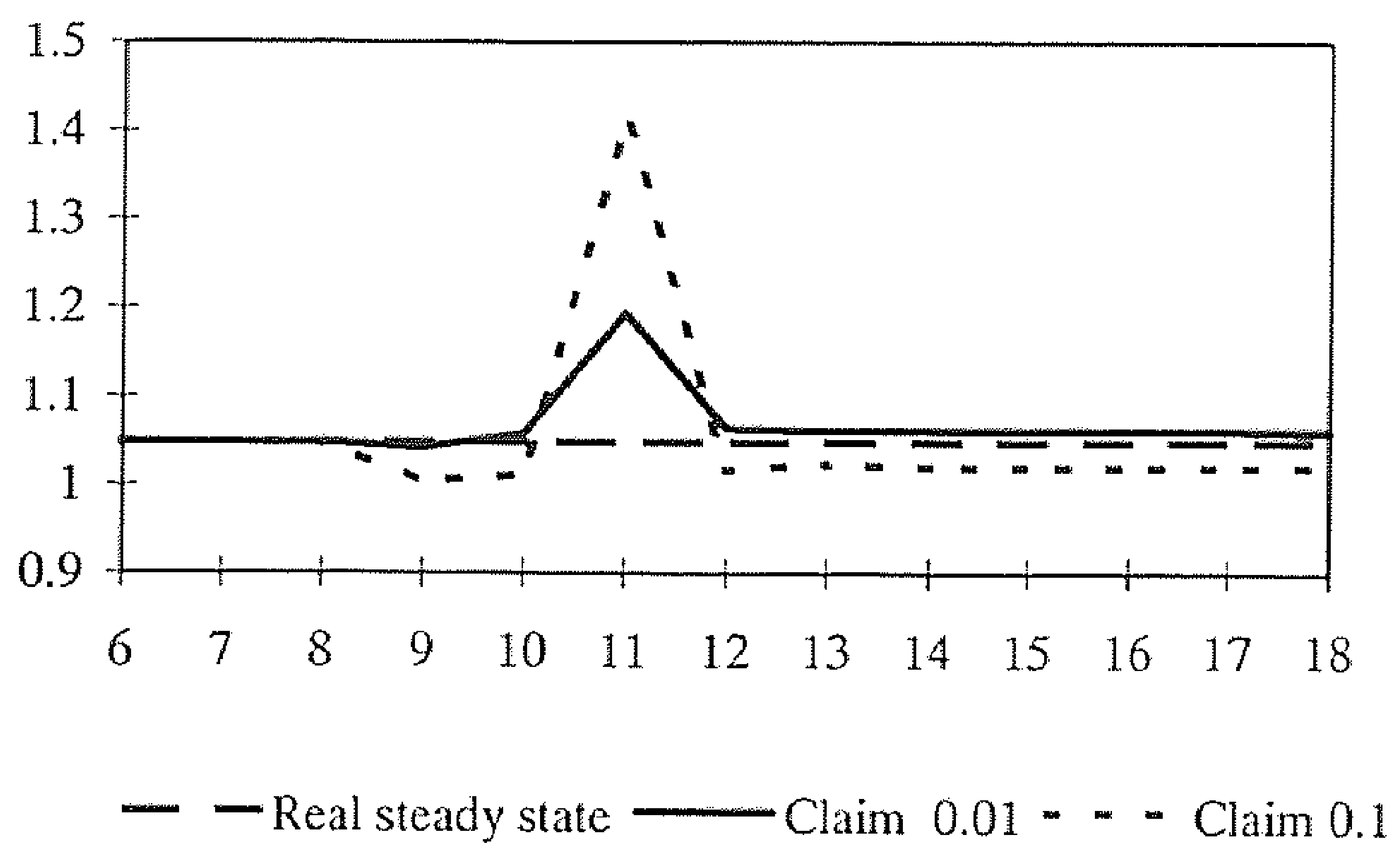

FIGURE 3.2. *Relative welfare during the transition*

The transition process from the real steady state to the 'monetary' steady state (the steady state close to the monetary steady state) is more clearly shown by the interest rate (Figure 3.3). For reference, we have also drawn the interest rate of 2 per cent that corresponds to the monetary steady state.

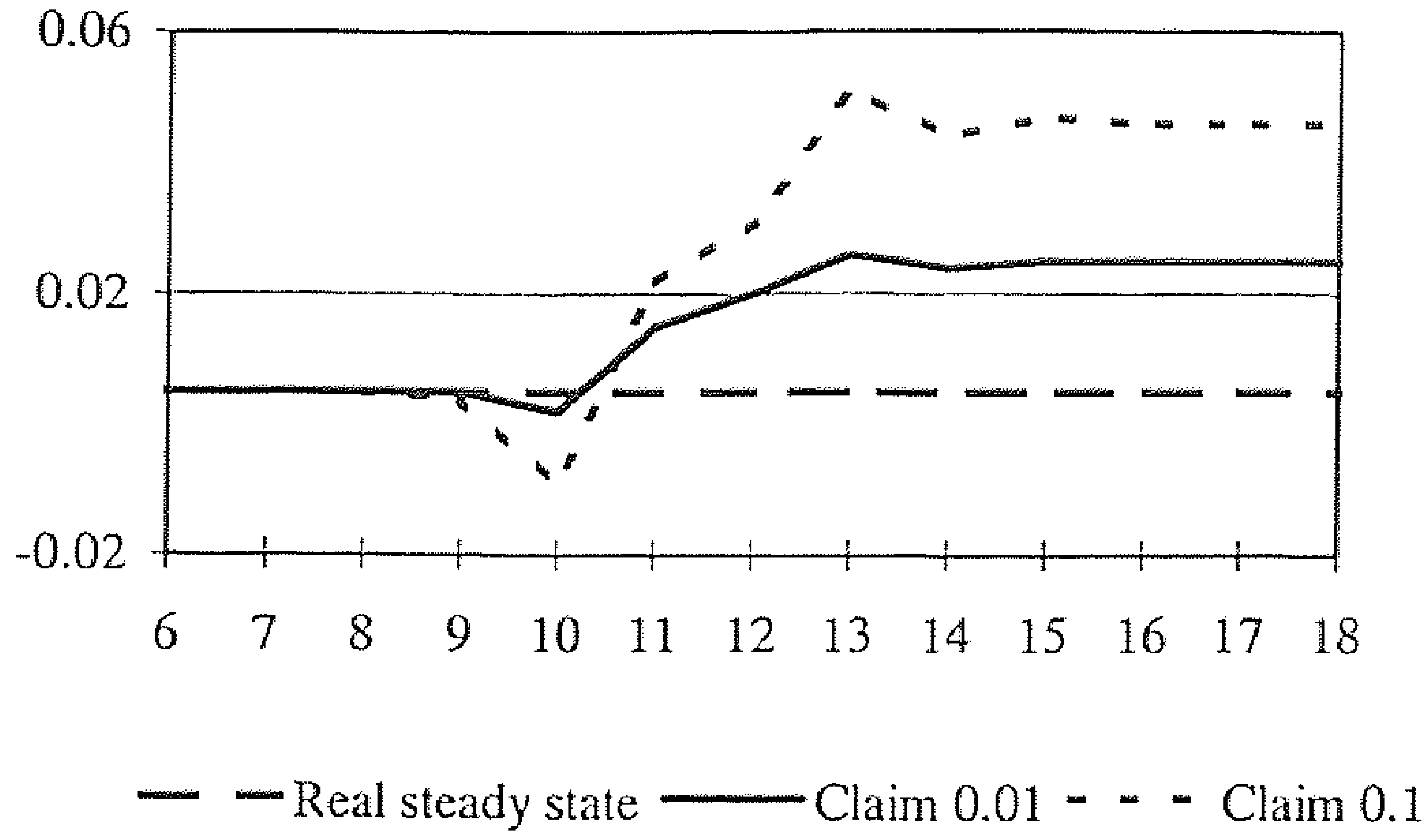

FIGURE 3.3. *Interest rates during the transition*

An interesting question is what happens if the tax rate γ that supports the claim becomes arbitrary small. We see that the associated steady state after the implementation of the policy will lie arbitrarily close to the monetary steady state. The claim has a strict positive value, though it pays only an arbitrarily small amount to its owner; it has become fiat money.

To summarize, we have seen that under specified assumptions, an increased life-expectancy can shift an efficient equilibrium towards an inefficient one. Issuing the non-negligible claim restores efficiency, and shifts the equilibrium from an inefficient to an efficient steady state. The generation that receives the claim benefits most, but if the claim is not too large, other generations benefit as well.

3.4. A NON-NEGLIGIBLE RESOURCE, AN ILLUSTRATION

3.4.1. An exhaustible resource with amenity value, model specification

The economy analyzed in this section is based on the stylized model described in [Gerlagh and Keyzer 1998b]. It extends the steady state model given above to a simulation model, ALICE 1.0, that distinguishes periods of 20 years duration, the first of which corresponds to 2000-2020. The rise in life-expectancy (represented by the transition from generations that live two periods to generations that live three periods) is supposed to take place during the 21^{st} century (the first five periods). Table 3.5 in Annex 3B.1 compares the stylized demographic transition used in the model with data from the literature.

If the economy were the same exchange economy of the preceding section, the model would only describe the demographic transition and the associated shift towards the dynamic inefficient equilibrium. However, the economy extends the OLG model of pure exchange described above in two respects: it contains an exhaustible resource with amenity value and specifies a transfer mechanism that distributes the value of this resource to consumers. This Section 3.4.1 describes the single-resource economy in general terms, and specifies two alternative environmental policies that can be applied to this environmental resource. Section 3.4.2 analyzes the steady states of the economy. The demographic transition plays no other role in the analysis of Sections 3.4.1 and 3.4.2, then that it causes the need for the non-negligible resource or the non-negligible claim to restore efficiency. Section 3.4.3 concludes the illustration with a numerical application to climate change, and briefly discusses the role of the life-expectancy issue.

Let the resource exploitation be performed by a firm denoted by *j*='*m*'. The resource itself is denoted by a superscript '*r*'. Let $k^r_{m,t}$ be the resource stock from which $y^r_{m,t}$ units are subtracted each period:

$$k^{p,r}_{m,t+1} \leq k^r_{m,t} - y^r_{m,t} . \tag{3.45}$$

where $k^{p,r}_{m,t+1}$ is the planned stock at the end of period *t*. The exhaustible resource has amenity value, $y^b_{m,t}$, denoted by a superscript *b*[13]. We follow Krautkraemer [1985] and assume that the amenity level is proportional to the stock level, relative to the initial stock $\overline{\mathrm{k}}^r = k^{p,r}_{m,1}$:

$$y^b_{m,t} \leq k^r_{m,t} / \overline{\mathrm{k}}^r . \tag{3.46}$$

Thus $y^b_{m,t}$ is measured as an index, with maximum output $y^b_{m,t} = 1$. Because of constant returns to scale in (3.45) and (3.46), profit maximizing behavior by the environmental firm can be represented through the zero-profit condition:

$$\psi^r_{m,t} k^r_{m,t} = \sum_{\tau=t,\ldots,\infty} (p^r_\tau y^r_{m,\tau} + p^b_\tau y^b_{m,\tau}), \tag{3.47}$$

where p^r_t and p^b_t are the prices of the extracted resource *r* and the amenity level *b* in period *t* respectively, and $\psi^r_{m,t}$ is the price of the resource stock at the beginning of period *t*. The equation states that the value of the resource, $\psi^r_{m,t} k^r_{m,t}$, is equal to the value of its output, which also can be written recursively,

$$\psi^r_{m,t} k^r_{m,t} = p^r_t y^r_{m,t} + p^b_t y^b_{m,t} + \psi^r_{m,t+1} k^r_{m,t+1} . \tag{3.48}$$

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[13] The letter *b* anticipates Section 3.4.3 where the amenity value is said to represent the level of 'biodiversity'.

A second firm, denoted by j='n', uses the extracted resource and 'labor', denoted by a superscript l, for the production of a single consumer good, denoted by a superscript c:

$$y_{n,t}^{c} = y_{n,t}^{l-} g_t(y_{n,t}^{r-} / y_{n,t}^{l-}), \tag{3.49}$$

where the superscript minus signs denote inputs, $g_t(.)$ is a continuous, differentiable, concave production function, with $g_t(0)=1$, and $0<g_t'(0)<\infty$. The subscript t is maintained to allow for technological innovation. Because $g_t(0)=1$ and the restriction on the slope, the extracted resource is valuable, but not essential for the production. The assumptions differ from the prevalence in neoclassical growth models with exhaustible resources, and this reflects a difference in focus discussed in Section 2.4; we do not analyze the possibility of sustained growth through substitution of exhaustible by renewable resources and technological progress, but concentrate on the consequences of resource extraction in the presence of amenities. The condition that $g_t(0)=1$ also implies that labor is measured in output units. We assume that over time the production functions converge to some function $g_\infty(.)$ that also meets the requirements on $g_t(.)$. As for the resource exploitation, profit maximization can, in view of constant returns to scale, be represented through the zero-profit condition:

$$p_t^c y_{n,t}^{c} = p_t^l y_{n,t}^{l-} + p_t^r y_{n,t}^{r-}. \tag{3.50}$$

This completes the description of producer behavior.

Generations maximize their lifetime utility $U(x_i^c, x_i^{nr,b}; n_i)$ derived from rival consumption of the consumer good $x_i^c = (x_{i,i}^c, x_{i,i+1}^c, x_{i,i+2}^c)$ and non-rival consumption of the resource amenity $x_i^{nr,b} = (x_{i,i}^{nr,b}, x_{i,i+1}^{nr,b}, x_{i,i+2}^{nr,b})$. Their utility function U(.) is assumed to be non-negative, differentiable, concave, and strictly increasing in all arguments. The generations maximize utility subject to the budget constraint:

$$max\ \{\, U(x_i^c, x_i^{nr,b}; n_i) \mid \sum_{t=i,\dots,i+2} p_t^c x_{i,t}^c + \varphi_{i,t}^b x_{i,t}^{nr,b} \le \sum_{t=i,\dots,i+2} p_t^l \omega_{i,t}^l + H_{i,t} \,\}, \tag{3.51}$$

where p_t^c, p_t^l denote the given prices of the (rival) consumer good and labor, respectively, $\varphi_{i,t}^b$ are the given Lindahl prices for non-rival consumption of the resource amenity of generation i in period t, $\omega_{i,t}^l$ denotes the labor endowment and $H_{i,t}$ is the income from the environmental resource received by generation i in period t:

$$\sum_{i=0,\dots,\infty} H_i = \psi_{m,1}^r k_{m,1}^{p,r}. \tag{3.52}$$

where H_i denotes the aggregate income of generation i. The first generations not living three periods have adjusted utility functions and budget constraints in the obvious way. We notice that generation t=0 only has a claim H_0 to the resource, and no other initial claim, and we also want to make clear that we use income claims H_i instead of the intergenerational transfers Φ_i to emphasize that we do not assume *a priori* that the environmental resource is given to the first generation.

We assume that endowments converge to stationary values, and this stylized economy satisfies the standard Assumption *O1*. Utility is strictly increasing in the resource amenity, and therefore the resource satisfies Assumption *F7*, and is thus non-negligible.

Finally, for every period t, there is a commodity balance for the resource stock,

$$k^{r}_{m,t} \le k^{p,r}_{m,t} \;\perp\; \psi^{r}_{m,t} \ge 0\,, \tag{3.53}$$

for the extracted resource,

$$y^{r-}_{n,t} \le y^{r}_{m,t} \;\perp\; p^{r}_{t} \ge 0\,, \tag{3.54}$$

for labor,

$$y^{l-}_{n,t} \le \omega^{l}_{t-2,t} + \omega^{l}_{t,t} + \omega^{l}_{t,t} \;\perp\; p^{l}_{t} \ge 0\,, \tag{3.55}$$

and for the consumer good,

$$x^{c}_{t-2,t} + x^{c}_{t-1,t} + x^{c}_{t,t} \le y^{c}_{n,t} \;\perp\; p^{c}_{t} \ge 0\,. \tag{3.56}$$

Nonrivalness of the demand for the resource amenity is expressed through a commodity balances for all living generations in every period:

$$x^{nr,b}_{i,t} \le y^{b}_{m,t} \;\perp\; \varphi^{b}_{i,t} \ge 0\,, \tag{3.57}$$

which indicates that contemporary consumers should agree about the amenity level. Associated to these balances are Lindahl prices which should add up to the production price:

$$p^{b}_{t} = \varphi^{b}_{t-2,t} + \varphi^{b}_{t-1,t} + \varphi^{b}_{t,t}\,. \tag{3.58}$$

This completes the description of the model, and we can now define an equilibrium:

DEFINITION 3.9. *A competitive equilibrium of model* (3.45)-(3.58) *is an allocation supported by prices* $p^{c}_{t}, p^{l}_{t}, p^{e}_{t}, p^{b}_{t}, \varphi^{b}_{i,t}$, ψ_t, for $p^{c}_{t}, p^{l}_{t}, p^{e}_{t}, p^{b}_{t}$ *normalized on the simplex, in which production maximizes profits subject to the technology constraints* (3.45)-(3.50), *consumption maximizes utility subject to lifetime budget constraints* (3.51), *a transfer mechanism distributes the value of the environmental resource over the generations* (3.52), *markets clear as in* (3.53)-(3.57), *and Lindahl prices satisfy* (3.58).

Next, we can formulate a 'strong sustainability' and 'grandfathering' policy as specific rules for managing the environmental resource, and distributing its value, while maintaining the budget constraint (3.51). Our first, 'strong sustainability' scenario constrains the use of the environmental resource by banning all extraction:

$$y^r_{m,t} = 0. \tag{3.59}$$

Consequently, the level of the resource amenity is maximal and stationary, and $x^{nr,b}_{i,t} = y^b_{m,t} = 1$ for $i \in \{t-2, t-1, t\}$. An important element of the strong sustainability scenario is that it exempts all generations from paying for the non-rival consumption of the resource amenity. This can be represented through an income claim that is exactly equal to the value of non-rival consumption,

$$H_{i,t} = \varphi^b_{i,t}, \tag{3.60}$$

so that the budget equation becomes:

$$\sum_{t=i,i+1,i+2} p^c_t x^c_{i,t} = \sum_{t=i,i+1,i+2} p^l_t \omega^l_{i,t}. \tag{3.61}$$

The strong sustainability policy treats the environmental resource as an exogenous factor, and reduces the economy to a one-good exchange economy, comparable to ALICE 0. Because the environmental resource is not incorporated in the economy, dynamic efficiency is not ensured, and for this purpose, a non-negligible claim is issued in the first period (2000-2020), denoted by $\gamma > 0$, and given to the first generation. For future generations, the budget constraint therefore becomes:

$$\sum_{t=i,i+1,i+2} p^c_t x^c_{i,t} = (1-\gamma) \sum_{t=i,i+1,i+2} p^l_t \omega^l_{i,t}, \tag{3.62}$$

Since in (3.49) labor is measured in output units and the resource amenity is exogenous to the economy, in equilibrium the price of 'labor' will be equal to the price of the consumer good in every period. Because the resource is not incorporated in the economy, life-cycle savings balance with the value of the non-negligible claim, χ_t, (3.36) applies:

$$S_t = \chi_t. \tag{3.63}$$

As a strong sustainability policy can be viewed as the imposition of an additional, linear restriction on the production technology, it does not pose specific problems for existence of equilibrium, Theorem 3.3 applies. But since it operates like a quota, the resulting equilibrium will in general be inefficient.

The second scenario grandfathers the entire environmental resource to the first generation, so that

$$H_0 = \psi^r_{m,1} k^{p,r}_{m,1} \tag{3.64}$$

and

$$H_i = 0, \tag{3.65}$$

for i=1,..., ∞. There is no further intergenerational transfer. The environmental resource becomes a 'normal' capital good which value is equal to life-cycle savings (3.36)

$$S_t = \psi^r_{m,t} k^r_{m,t} \,. \tag{3.66}$$

Unlike the 'strong sustainability' policy, the extracted resource can now be bought by the firms and all generations have to pay for their non-rival use of the resource amenity. The budget constraint for all generations after the first becomes

$$\sum_{t=i,i+1,i+2} p^c_t x^c_{i,t} + \varphi^b_{i,t} x^{nr,b}_{i,t} = \sum_{t=i,i+1,i+2} p^l_t \omega^l_{i,t} \,. \tag{3.67}$$

As the economy satisfies the standard assumptions *O1* and *F7*, Theorem 3.2 applies; the equilibrium exists and is efficient.

3.4.2. Steady state analysis

We now turn to a steady state analysis of the model. We recollect from Section 2.4 that, due to the existence of a non-renewable resource with amenity value, this analysis cannot follow the usual procedure. In an OLG-model with renewable resources, the steady states are in general locally unique and are the solution of a static fixed-point problem (see e.g. [Kehoe 1991]). However, when the resource is non-renewable, steady states form a continuum and every dynamic allocation that converges to a steady state will select one allocation from this continuum, given the initial conditions and intertemporal preferences and other parameters that are specified.

This characteristic has far-reaching implications. If all environmental resources were renewable, they could, starting from arbitrary initial conditions, always return to a 'natural' steady state, provided all stocks remain positive (e.g., to rule out full extinction of species) and there would be no sustainability problem in the long run. In this case initial conditions can only determine the choice among given steady states if there are many, but not the welfare in these steady states. However, if one accepts that some vital resources are clearly non-renewable and have amenity value, the point of convergence will depend upon initial conditions and actions in early periods.

Let us analyze the issue in terms of our ALICE 1.0 model, starting with the case that the biogeochemical system is renewable. Time subscripts will be dropped for all steady state variables and subscripts '0', '1', and '2', will denote 'young', 'middle aged' and 'old' consumers. Prices are normalized with the use of the utility function, and the intertemporal price ratio is denoted by the constant β: $p_{t+1}=\beta p_t$. Parameters $\omega^{l}_{i,t}$ and the production technology $g_t(.)$ are supposed to have reached their stationary values. The assumption of renewability amounts to replacing (3.45) by

$$k^{p,r}_{m,t+1} \le \mathrm{f}(k^r_{m,t}) - y^r_{m,t}\,, \tag{3.68}$$

where f(.) is a concave regeneration function with f(0)=0, f′(0)>1, f′(∞)<1, which ensures that there is some k>0 for which f(k)=k and f′(k)<1. We also assume that this steady state is attainable in the sense that it can be reached from k_1 and that the dynamic model converges to it.

Profit maximization for the environmental resource (treated as one firm) leads to the following four steady state conditions determining the stock and output levels k^r_m, y^b_m, y^{r-}_m, and the stock price ψ, given prices p^r, and p^b,

$$k^r_m \le \mathrm{f}(k^r_m) - y^r_m \quad \perp \quad \beta\psi^r_m \ge 0\,, \tag{3.69}$$

$$y^b_m \le k^r_m / \bar{\mathrm{k}}^r \quad \perp \quad p^b \ge 0\,, \tag{3.70}$$

$$p^r \le \beta\psi^r_m \quad \perp \quad y^r_m \ge 0\,, \tag{3.71}$$

$$\psi^r_m \le \beta\,\mathrm{f}'(k^r_m)\psi^r_m + \mathrm{f}'(k^r_m)p^r / \bar{k}^r \quad \perp \quad k^r_m \ge 0\,, \tag{3.72}$$

where the ⊥-sign refers to complementarity conditions; for every commodity, either the constraint on the left or the constraint on the right side must be binding.

Profit maximization for the firm that produces the consumer good gives three conditions that determine the input and output levels for the consumer good, given the prices p^r, p^l, and p^c,

$$y_n^c \le y_n^{l-} \,\mathrm{g}(y_n^{r-} / y_n^{l-}) \;\perp\; p^c \ge 0\,, \tag{3.73}$$

$$p^c(\mathrm{g}(y_n^{p-} / y_n^{l-}) - (y_n^{p-} / y_n^{l-})\mathrm{g}'(y_n^{p-} / y_n^{l-})) \le p^l \;\perp\; y_n^{l-} \ge 0\,, \tag{3.74}$$

$$p^c \,\mathrm{g}'(y_n^{p-} / y_n^{l-}) \le p^p \;\perp\; y_n^{p-} \ge 0\,. \tag{3.75}$$

Utility maximization gives the price and its dynamics for the consumer good and the relative prices of the resource amenity,

$$\partial\,\mathrm{u}(x^c, x^{nr,b}) / \partial x_i^c \le \beta^i p^c \;\perp\; x_i^c \ge 0\,, \qquad i=0,1,2 \tag{3.76}$$

$$\partial\,\mathrm{u}(x^c, x^{nr,b}) / \partial x_i^{nr,b} \le \beta^i \varphi_i^b \;\perp\; x_i^{nr,b} \ge 0\,, \qquad i=0,1,2 \tag{3.77}$$

where we normalized prices by taking them to be equal to marginal utility. The commodity balances, and summing Lindahl prices to the production price yields

$$\sum_{i=0,1,2} x_i^c \le y_n^c \;\perp\; p^c \ge 0\,, \tag{3.78}$$

$$y_n^{l-} \le \sum_{i=0,1,2} \omega_i^l \;\perp\; p^l \ge 0\,, \tag{3.79}$$

$$y_n^{p-} \le y_m^p \;\perp\; p^p \ge 0\,, \tag{3.80}$$

$$x_i^{nr,b} \le y_m^b \;\perp\; \varphi_i^b \ge 0\,, \qquad i=0,1,2 \tag{3.81}$$

$$\sum_{i=0,1,2} \varphi_i^b \le p^b \;\perp\; y_m^b \ge 0\,. \tag{3.82}$$

Finally, the steady state budget constraint is given by

$$\sum_{i=0,1,2} \beta^i (p^c x_i^c + \varphi_i^b x_i^{nr,b}) = \sum_{i=0,1,2} \beta^i ((1-\gamma) p^l \omega_i^l + H_i)\,, \tag{3.83}$$

with γ=0 and H_i=0 for the grandfathering scenario. We can now derive : p^c, p^l, p^r, p^b, φ_i^b, $x_i^c, x_i^{nr,b}$, $y_n^c, y_n^{l-}, y_n^{p-}, y_m^r, y_m^b$, k_m^r, ψ_m^r, and β from the equations (3.69)-(3.83), given the parameters $\omega_{i,t}^l$ where the real claims H_i are determined by the distributive policy. There may be several solutions, but in general their number is finite, i.e. the steady states are distinct.

Next, we turn to the case of an exhaustible resource. The function f(.) now becomes an identity and the steady state conditions derived from the profit maximization for the biogeochemical system are:

$$k_m^r \le k_m^r - y_m^r \quad \perp \quad \beta\psi_m^r \ge 0\,, \tag{3.84}$$

$$y_m^b \le k_m^r / \bar{k}^r \quad \perp \quad p^b \ge 0\,, \tag{3.85}$$

$$p^p \le \beta\psi_m^r \quad \perp \quad y_m^r \ge 0\,, \tag{3.86}$$

$$\psi_m^r \le \beta\psi_m^r + p^b / \bar{k}^r \quad \perp \quad k_m^r \ge 0\,. \tag{3.87}$$

Hence the resource stock k_m^r has dropped from equation (3.69), which now reduces to $y_m^p = 0$ (3.84), and from (3.72) which becomes identical to (3.86). We apply Theorem 2.3 from Chapter 2 to this economy:

THEOREM 3.4. *The model* (3.73)-(3.87) *in which the environmental resource is exhaustible,* f(*k*)=*k*, *has a steady state that corresponds to segments for* k_m^r.

Finally, we notice that because resource extraction is zero in every steady state (recollect from the properties of g(.) that the extraction of the exhaustible resource with amenity value is not essential for production), the commodity balance for the consumer good (3.73) and (3.78) reduces to:

$$\sum_{i=0,1,2} x_i^c = \sum_{i=0,1,2} \omega_i^l\,, \tag{3.88}$$

This reduced commodity balance will now be used in the comparison of steady state welfare levels among the strong sustainability and grandfathering policies. For this, we also need the following assumption:

ASSUMPTION 3.10. *The utility function is a nested CES function with constant expenditure share* 0<ν<1 *for the resource amenity, and constant intertemporal elasticity of substitution* ρ>0.

The assumption boils down to preferences for rival consumption of the consumer good which are independent of the amenity level, provided these are stationary as is the case in the steady state. We can now state and prove:

THEOREM 3.5. *Under Assumption* 3.10, *and if the non-negligible claim satisfies* γ=ν, *then the strong sustainability policy improves the steady state welfare level compared to the grandfathering policy, while the steady state interest rate does not change.*

Proof. The model (3.73)-(3.87) is based on the full incorporation of the environmental resource, but can easily be adjusted to account for the strong sustainability policy by making two changes. First, the complementarity condition, $y_m^r \geq 0$, in equation (3.86) is removed, and secondly, the resource level is set equal to the initial level, $k_m^r = k_{m,1}^{p,r}$.

Assumption 3.10 ensures that for constant amenity level, the iso-utility curves with respect to the consumer good are (up to a constant) independent of the resource amenity level. Furthermore, the same commodity balance (3.88) applies to both scenarios, and the same budgets associated to the rival consumption of the consumer good (3.83):

$$\sum_{i=0,1,2} \beta^i p^c x_i^c = (1-\nu) \sum_{i=0,1,2} \beta^i p^l \omega_i^l , \tag{3.89}$$

for the grandfathering policy, while the strong sustainability policy requires that we substitute γ for ν. Thus, if γ=ν, the consumption allocation of consumer good over the life-cycle, and the supporting price depreciation β, is the same for both policies.

Finally, we compare the amenity levels. It is obvious that under grandfathering, the amenity level cannot exceed the initial level, which is equal to the strong sustainability level. Because utility is strictly increasing in the amenity level, the strong sustainability policy results in the highest utility in the steady state. ■

The theorem implies that grandfathering improves efficiency as compared to the strong sustainability policy but that it also favors the first generation which receives the entire environmental resource, leaving future generations with a welfare reduction, because they have to pay for a degenerated environment. This is not obvious, because there would have been scope for a Pareto improvement with respect to the inefficient strong sustainability policy. In Chapter 4, we will introduce a trust fund policy that restores efficiency without favoring the first generation, and see that this trust fund is capable of producing a Pareto improvement compared to the strong sustainability policy.

3.4.3. Numerical application to climate change

The stylized model developed above can be used for integrated assessment analysis of the climate change problem in the same way as other so called 'integrated assessment models' (IAMs). Let us first briefly recollect from Section 1.2.2 the issue at hand. Emissions of greenhouse gases are supposed to increase the insulation capacity of the atmosphere and raise the global average temperature which disturbs the biogeochemical system. This is likely to result in a reduction in environmental functions. In an economic model, we can treat emissions as a flow input into the economic process of the good 'fresh air', extracted from an (atmospheric) stock that has amenity value because it performs environmental functions. The IAM literature often refers to this good as 'emission permits' but we refer to an 'emission unit' because a permit is a license to use a good rather than the good itself.

There are good reasons for treating the supply of 'emission units' as exhaustible. Maier-Reimer and Hasselman [1987, Table 1] estimate that between 13 and 17 per cent of emissions remain in the atmosphere for an indefinite time. This would imply, first, that to stabilize the biogeochemical system in the long run within an IAM, net emissions must be banned altogether and secondly, that the selection of the long term steady state of the biogeochemical system should depend on past cumulative emissions, and cannot be determined within a conventional steady state analysis.

It must be reiterated, however, that the environmental functions are threatened in many ways other than greenhouse gas emissions, and that the relation between these emissions and environmental functions is poorly understood. The IPCC [1996, Section 6.2.13] notes: ecological systems and biodiversity are an area where 'losses from climate change could be among the largest, yet where past research has been the most limited'. In fact the subject is still a matter of great controversy. Some studies suggest that measured climate change is not caused by human activity [Lassen and Friis-Christensen 1995], some argue that the problem will be of short duration since there will anyway in the near future be an endogenous shift towards fossil free fuels [Chakravorty *et al.* 1997], while others claim that the potential damages do not warrant a reduction in economic growth [Schelling 1992].

The issue cannot be settled, nor does it have to be, in this section where we only use the problem of climate change as an illustration of basic policy choices that have to be faced in various areas of environmental protection. In this chapter, we compare strictly conservationist measures (*strong sustainability*) with a free market without intergenerational redistribution (*grandfathering*). In Chapter 4, a free market policy with intergenerational redistribution (*trust fund*) will be added to the comparison. We use climate change as an example to calibrate our numerical calculations to IAMs that were developed on this subject.

In the early IAMs, there is a single global planner that optimizes production and consumption, and distribution of property rights is irrelevant [Peck and Teisberg 1992] [Nordhaus 1994]. Subsequent IAMs focus on the issue of optimal international distribution of emission reductions [IPCC 1992, Ch.3] and introduce an explicit regional ownership distribution of production factors. Yet they altogether ignore the distribution of emission units [Manne *et al.* 1995] [Nordhaus and Yang 1996], or give limited attention to the subject [Manne and Richels 1995], from which we conclude that the environmental resource under study is only half-way incorporated in the competitive equilibrium. The issue of intergenerational distribution has gained increasing importance. Howarth and Norgaard [1992] use an OLG model in their general discussion of intertemporal efficiency and intergenerational distribution. Finally, Stephan *et al.* [1997] describe an OLG/IAM for climate change. In every period, optimal emission units are shared between the young and the old generations. In this model the distribution has no significant effect on the optimal level of emission reduction. However, this might be due to the fact that this model ignores the income effects of the environmental functions altogether. To analyze welfare distributions and efficiency of different policies with the same model, we need an IAM with climate change that attributes property rights over the entire biogeochemical system, including the environmental functions.

For this, we run simulations with ALICE 1.0, an applied version of the model specified above. This OLG/IAM has a stylized representation of the biogeochemical system and its relation to the economy. It distinguishes time periods of 20 years duration, starting in the year 2000; r stands for the use of 'emissions units' or 'fresh air' and the stock of unused units, b is an indicator for the environmental functions, referred to as 'biodiversity' for short.

The parametric forms of the production function $g_r(.)$ and the utility function u(.) of ALICE 1.0 are shown in equations (3.119) and (3.120) of Annex 3B.2 For $g_r(.)$ the parameters were calibrated so as to satisfy two conditions: first, maximum production and associated emissions follow the IS92a scenario [IPCC 1992], and secondly, emissions decrease by 1 per cent point for each 4 US$/tC increase in the price of emission units. This production function implies a quadratic cost function of emission reduction, and since it has no lagged variables, it also means that we abstract from any transition costs associated with a switch towards fossil free energy carriers. As it is not our objective to provide new insights into the issue of climate change itself, the remaining parameters are calibrated to provide numerical results that replicate the common IAM-scenarios of the models mentioned above.

The calculations cannot be used for applied policy. It is commonly recognized that calculated optimal emission reductions and supporting prices are sensitive to several assumptions, see [Nordhaus 1994, Table 6.5] for a brief list. One of the crucial variables is the discount rate, a general term that refers to either the pure time preferences in welfare functions or to real interest rates for consumer goods. And if the complexity of the model is increased (either its environmental or its economic sub-system) while the explanatory power for the price dynamics remains limited, the uncertainty range for optimal emission pricing is not likely to improve significantly. To illustrate this particular point, we add a third scenario, similar to the basic grandfathering scenario with the difference that 'aging' of the population is omitted. We have seen in Section 3.3 that aging causes a decrease in the interest rate. Conversely, leaving out aging causes an increase in the interest rate. The third scenario therefore reveals the significance of aging, and in general, the sensitivity of the simulation results with respect to assumptions that affect the interest rate.

The outcomes from the scenario simulations are summarized in Table 3.2 below, in terms of four pairs of variables related to emissions, assets, interest rates and welfare, respectively.

TABLE 3.2. *Scenario results for ALICE 1.0*

Policy	Units	Strong Sustainability	Grandfathering	Grandfathering
Aging		yes	yes	no
Tax based claim (γ)		0.1	n/a	n/a
Price of emission units in first period	US$/tC	400.	13.	9.
Cumulative Emissions	GtC[1]	0.	1000.	2000.
Asset value in first period [2]		2.4	2.8	2.3
Real interest rate (2020) [3,4]		0.083	0.083	0.086
Real interest rate after 2400 [3]		0.023	0.023	0.055
Welfare of first generation		1.00	1.05	0.88
Welfare after 2400		1.00	0.95	n/a

1. CO2 equivalents in Gigaton Carbon
2. Measured relative to annual production, see Annex 3A.3
3. Price ratio between two periods minus unity, on an annual basis, see Annex 3A.3
4. Between first two periods

The first indicator is the price of emission units, and is equivalent to the so called CO_2 tax in other models. The table only shows the emission price for the first period because this is the relevant measure (of consequences) for the present day economy. Recall that the reduction of emissions is assumed to be linear in its price: a 1 per cent point reduction per 4 US$/tC. Thus, the emission price in the strong sustainability scenario (400 US$/tC) reduces emissions by 100 per cent. The emission price of 13 US$/tC for the grandfathering scenario reduces emissions by 3 per cent. Omitting aging from the model decreases the emission price to 9 US$/tC, reducing emissions by 2 per cent.

The second indicator is the cumulative amount of emissions over the whole simulation period and is a measure of cumulative environmental degradation. There are no emissions in the strong sustainability scenario, whereas the grandfathering scenario results in cumulative emissions of 1000 GtC, two-third of the 1500 GtC cumulative emissions reached in 2100 according to the IS92a scenario [IPCC 1993]. However, the model result for this indicator is sensitive with respect to its assumptions related to aging. If aging is left out, cumulative emissions increase to 2000 GtC, which implies that there is no significant emission reduction in the 21st century. Since we do not claim that the carry-over of aging in our model is a true representation of reality, it is not possible to point out the cumulative emission in the first grandfathering scenario as more realistic than the calculated level in the second grandfathering scenario.

The third indicator is the asset value of the non-negligible claim and the exhaustible resource that is given to the first generation in the strong sustainability and grandfathering policy, respectively. These grants to the first generation ensure efficiency, and in this respect, the non-negligible claim in the strong sustainability scenario is comparable with the value of the environmental resource in the grandfathering scenario. The numerical values are sensitive to the model specification, but it seems credible that environmental resources are valuable assets.

The fourth indicator gives the 'real' interest rates, defined as the price ratio for consumer goods between two periods minus unity. Their value is expressed on an annual basis. Both the grandfathering and strong sustainability scenario with the non-negligible claim and with aging population have the same interest rate, confirming Theorem 3.5. Leaving out aging increases the interest rate as expected.

Finally, the welfare measures presented in the last two rows give the welfare relative to the strong sustainability scenario. The assets given to the first generations, both in the strong sustainability and in the grandfathering scenario, substantially increase their welfare. On the other hand, the welfare levels of future generations decrease as they have to pay for the assets. Grandfathering the environmental resource decreases the steady state welfare level further because of the deteriorated environment.

To summarize, grandfathering ensures Pareto efficiency while the strong sustainability policy does not. However, future generations experience reduced welfare, as the efficiency gain improves welfare of the early generations only. Grandfathering can even lead to a persistent fall in welfare. In the next chapter, this aspect is discussed in more detail. Furthermore, the optimal reduction in emissions appears to depend on assumptions that have no direct connection with climate change, such as the assumptions on demographic transition. This illustrates once more that quantitative results can be sensitive to assumptions that are not easily recognized, irrespectively of the complexity of the model, and that great care has to be exercised when using numerical outcomes of such models as a basis for actual environmental policy formulation.

3.5. Conclusions

Dynamic inefficiency can occur relatively easily in an OLG model with perfect foresight. We gave an example where the increase of life-expectancy expected by the World Bank in the next century [WB 1994] shifts an efficient equilibrium towards an inefficient equilibrium. We studied ways to restore dynamic efficiency.

If the economy contains a 'non-negligible' environmental resource, the dynamic inefficiency can be avoided via a claim on this resource. An environmental resource is non-negligible if it can produce a stationary stream of a strictly desired output without maintenance costs (Assumption *F7*, Theorem 3.2), that is if it has the first two characteristics mentioned in Section 1.4.1. An IAM with climate change was used for a numerical illustration. The biogeochemical system underlying climate change was shown to constitute a non-negligible environmental resource in this model.

As an alternative to the non-negligible environmental resource, a fictitious environmental resource can be used, as a freely tradable claim, that gives its owner a constant share of the endowments at every period. The claim is issued by a public authority that guarantees the payments and that balances its commitments by a lump-sum tax in every period. The non-negligible claim is given to the first generation which thereby obtains a non-negligible income, ensuring dynamic efficiency (Theorem 3.3).

Annexes to Chapter 3

3A. Model description for ALICE 0

3A.1. Model set up

ALICE 0 uses simulation periods of 20 years each, denoted by t=1, ...,T, where T=30. Generations live three periods, and are identified with the first period in which they live. The first two generations, t=0,1, and last two generations, $t=T^{-}1,T$, live only one or two periods. There is only one good and no production, hence all variables represent scalars. ALICE 0 is written in excess demand format (with iteration over prices), in contrast to the other versions which use less simple production and utility functions, and for which the Negishi approach (with iteration over welfare weights) is more tractable.

Consumption of generation t in period t,t+1,t+2 is given by demand functions, $d_y(W_t,p_t,p_{t+1},p_{t+2})$, $d_m(W_t,p_t,p_{t+1},p_{t+2})$, $d_o(W_t,p_t,p_{t+1},p_{t+2})$, that maximize utility, a CES function,

$$u(x_{t,t},x_{t,t+1},x_{t,t+2}) = \left(\left(x_{t,t}\right)^{\frac{\rho-1}{\rho}} + \sigma\left(x_{t,t+1}\right)^{\frac{\rho-1}{\rho}} + \sigma^2\left(x_{t,t+2}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \qquad (3.90)$$

subject to the budget constraints

$$p_t x_{t,t} + p_{t+1}x_{t,t+1} + p_{t+2}x_{t,t+2} \leq p_t\omega_{t,t} + p_{t+1}\omega_{t,t+1} + p_{t+2}\omega_{t,t+2}. \qquad (3.91)$$

The right hand side, the wealth of generation t, is denoted by W_t. Here, the capital W for wealth is not to be confused with the small w for welfare. Demand functions for the first two and last two generations are adjusted in the obvious way. Endowments grow by a factor δ per period, with constant shares for the old, middle and young generations.

$$(\omega_{t-2,t},\omega_{t-1,t},\omega_{t-1,t}) = \delta^t(0.4,1,1) \qquad (3.92)$$

The parameters are listed in Table 3.3.

TABLE 3.3. *Parameter values for ALICE 0*

Parameter	symbol	value	annual rate[1]
Intertemporal elasticity of substitution	ρ	0.5	
Economic growth	δ	1.5	0.02
Time preference	σ	0.82	0.01
Interest rate[2]	β	0.38	0.05

1. The annual rate of decrease or increase for variables below or above unity.
2. The interest rate is a resulting variable, not a parameter.

The intertemporal elasticity of consumption is within the wide range of estimates found in different studies (See [Davies 1981, p.573] and [Hurd 1989, p.799] for a discussion of several estimates, typically ranging from ρ=0.25 to ρ=1.5). The consumers' time preference is calculated from equation (3.37), given the economic growth of 2 per cent per year.

The budget constraint can be rewritten as

$$x_{t,t} + \beta_t x_{t,t+1} + \beta_t \beta_{t+1} x_{t,t+2} \leq \tilde{W}_t \tag{3.93}$$

where $\tilde{W}_t = W_t / p_t$ denotes relative wealth, and $\beta_t = p_{t+1}/p_t$ are relative prices over time. This leads to adjusted demand functions $\tilde{d}_y(\tilde{W}_t, \beta_t, \beta_{t+1})$, $\tilde{d}_m(\tilde{W}_t, \beta_t, \beta_{t+1})$, and $\tilde{d}_o(\tilde{W}_t, \beta_t, \beta_{t+1})$. From the budget constraint, it follows that the demand functions satisfy

$$\tilde{d}_y(\tilde{W}_t, \beta_t, \beta_{t+1}) + \beta_t \tilde{d}_m(\tilde{W}_t, \beta_t, \beta_{t+1}) + \beta_t \beta_{t+1} \tilde{d}_o(\tilde{W}_t, \beta_t, \beta_{t+1}) = \tilde{W}_t \tag{3.94}$$

Relative wealth for the first two generations is deflated by prices in the first period, $\tilde{W}_1 = W_1 / p_3$ and $\tilde{W}_2 = W_2 / p_3$, and the demand functions are adjusted in the obvious way. Wealth is then given by

$$\tilde{W}_t = \omega_{t,t+2} \qquad (t=0) \tag{3.95}$$
$$\tilde{W}_t = \omega_{t,t+1} + \beta_{t+1}\omega_{t,t+2} \qquad (t=1) \tag{3.96}$$
$$\tilde{W}_t = \omega_{t,t} + \beta_t \omega_{t,t+1} + \beta_t \beta_{t+1} \omega_{t,t+2} \qquad (t=2,\ldots,T-2) \tag{3.97}$$
$$\tilde{W}_t = \omega_{t,t} + \beta_t \omega_{t,t+1} \qquad (t=T-1) \tag{3.98}$$
$$\tilde{W}_t = \omega_{t,t} \qquad (t=T) \tag{3.99}$$

Competitive equilibrium is defined by zero excess demand,

$$d_o(\tilde{W}_{t-2}, \hat{\beta}_{t-2}, \hat{\beta}_{t-1}) + d_m(\tilde{W}_{t-1}, \hat{\beta}_{t-1}, \hat{\beta}_t) + d_y(\tilde{W}_t, \hat{\beta}_t, \hat{\beta}_{t+1}) = \omega_{t-2,t} + \omega_{t-1,t} + \omega_{t,t} \tag{3.100}$$

where $\hat{\beta}$ denotes the equilibrium price dynamics, with the obvious adjustments for the demand functions for the first two and last two generations.

Now, the competitive equilibrium is altered by the issuing of the claim in period τ. The relative value of the claim, $\tilde{\chi}_t = \chi_t / p_t$, is recursively defined by

$$\tilde{\chi}_t = \beta_t \tilde{\chi}_{t+1} + \gamma(\omega_{t-2,t} + \omega_{t-1,t} + \omega_{t,t}), \tag{3.101}$$

for t=τ, ...T−1, and

$$\tilde{\chi}_T = \gamma(\omega_{T-2,T} + \omega_{T-1,T} + \omega_{T,T}), \tag{3.102}$$

for the last period.

The wealth equations change into

$\tilde{W}_t = \omega_{t,t+2}$	(t=0)	(3.103)
$\tilde{W}_t = \omega_{t,t+1} + \beta_{t+1}\omega_{t,t+2}$	(t=1)	(3.104)
$\tilde{W}_t = \omega_{t,t} + \beta_t\omega_{t,t+1} + \beta_t\beta_{t+1}\omega_{t,t+2}$	(t=2,...,τ−3)	(3.105)
$\tilde{W}_t = \omega_{t,t} + \beta_t\omega_{t,t+1} + (1-\gamma)\beta_t\beta_{t+1}\omega_{t,t+2}$	(t=τ−2)	(3.106)
$\tilde{W}_t = \omega_{t,t} + (1-\gamma)(\beta_t\omega_{t,t+1} + \beta_t\beta_{t+1}\omega_{t,t+2})$	(t=τ−1)	(3.107)
$\tilde{W}_t = (1-\gamma)(\omega_{t,t} + \beta_t\omega_{t,t+1} + \beta_t\beta_{t+1}\omega_{t,t+2}) + \tilde{\chi}_t$	(t=τ)	(3.108)
$\tilde{W}_t = (1-\gamma)(\omega_{t,t} + \beta_t\omega_{t,t+1} + \beta_t\beta_{t+1}\omega_{t,t+2})$	(t=τ+1,...,T−2)	(3.109)
$\tilde{W}_t = (1-\gamma)(\omega_{t,t} + \beta_t\omega_{t,t+1})$	(t=T−1)	(3.110)
$\tilde{W}_t = (1-\gamma)\omega_{t,t}$	(t=T)	(3.111)

which of course sums to the total value of endowments. Again the competitive equilibrium is defined by zero excess demand.

3A.2. Steady states

Both the real and monetary steady state are calculated by solving β and $\tilde{W}$ from the following two equations

$$\mathrm{d}_y(\tilde{W},\beta,\beta) + \mathrm{d}_m(\delta^{-1}\tilde{W},\beta,\beta) + \mathrm{d}_o(\delta^{-2}\tilde{W},\beta,\beta) = 2.4 \tag{3.112}$$

$$\tilde{W} = 1 + \delta\beta + 0.4\delta^2\beta^2 \tag{3.113}$$

The steady state for the equilibrium with the tax claim is calculated by changing the wealth equation into

$$\tilde{W} = (1-\gamma)(1 + \delta\beta + 0.4\delta^2\beta^2) \tag{3.114}$$

and demanding that

$$\beta\delta < 1 \tag{3.115}$$

to exclude steady states with an unbounded value of the tax claim.

3A.3. Accounting

The welfare measure, w, has been defined (below Table 3.1) by $w \equiv u(x_y, \delta x_m, \delta^2 x_o)/u(\omega_y, \delta\omega_m, \delta^2\omega_o)$. Let N be the number of years per period, the annual interest rate is defined by

$$r_t = \beta_t^{1/N} - 1 \tag{3.116}$$

The asset value is equal to the value of private savings, which value at the end of the period is given by equation (3.41) from which we calculate average private savings as a ratio to annual endowments

$$S=(1+\beta^{-1}\delta^{-1})(\omega_y - d_y(\beta)) + \omega_m - d_m(\beta))(0.5+0.5\beta/\delta)(N/\omega) \tag{3.117}$$

where we omit the time subscripts, N=20 is the number of years per period, and ω (without subscript) denotes the total endowments within a period.

3B. Model description for ALICE 1.0[14]

3B.1. Demographic transition

ALICE 1.0 is based on a Negishi welfare program for which welfare weights are calculated such that budget constraints are satisfied [Ginsburgh and Keyzer 1997, Section 3.1]. The simulation periods are 20 years each, denoted by $t \in T = \{1,\ldots,T\}$, starting in 2000. In every period, a new generation is born, living two or three periods, identified with the first period in which it lives, $i \in \{0,\ldots,T\}$. So, in each period, a young, a middle-aged and possibly an old generation coexist. Increasing life-expectancy, because of the transition from two life-periods towards three life-periods, is supposed to take place during the first century. In the first period, there is only a young and middle-aged cohort; there is no old-aged cohort. Twenty per cent of the young in the first period lives another third period, hence, in the third period, there is a small group of old consumers. Of the next generation (the 'young' of the second period), forty per cent lives three periods, while the others live only two periods. Life-expectancy continues to increase until all members of the generation living from 2060 to 2120 complete the full three periods.

Let n_i denote the size of generation i when born, and $n_{i,t}$ denote the size of generation i in period t. We assume that no member of a generation dies before the second period of

[14] A more comprehensive description of the calibration and calculation of ALICE 1 is given in [Gerlagh 1998b]

his live, $n_{t,t+1}=n_{t,t}=n_t$, for all t. The number of people living the full three periods increase linearly: $n_{1,3}=0.2n_{1,2}$, $n_{2,4}=0.4n_{2,3}$, and so forth, until $n_{5,7}=n_{5,6}$. The size of a generation is defined recursively according to a logistic growth curve:

$$n_{t+1} = (a - (a - 1)(n_t/\overline{n}))n_t \tag{3.118}$$

where a is the growth factor for n_t close to zero, and $\overline{n}$ is the maximal size of a cohort. The parameters are calibrated on the World Bank data [WB 1994] and listed in Table 3.4.

TABLE 3.4. *Parameter values for demographic transition in ALICE 1.0*

Parameter	symbol	value	annual rate[1]
Initial growth factor	a	1.85	0.03
Maximum size of a cohort (billion people)	$\overline{n}$	2.970	
Size of first cohort (1960-2020, billion people)	n_0	1.459	

[1] The annual rate of decrease or increase for variables below or above unity.

The resulting population dynamics are shown in Table 3.5 (the population in period t includes the children that enter the model one period later).

TABLE 3.5. *Population and Life-expectancy in ALICE 1.0*

	1960	1980	2000	2020	2040	2060	2080	2100	2200
Population WB[1]	n/a	n/a	6.1	7.7	9.0	9.9	10.6	11.0	n/a
Population ALICE 1.0	n/a	n/a	6.1	7.7	9.0	10.0	10.7	11.1	11.1
Life expectancy at birth WB1	n/a	63.5	67.4	71.2	74.7	77.9	80.3	82.6	n/a
Life expectancy at birth ALICE 1.0	60.0	64.0	68.0	72.0	76.0	80.0	80.0	80.0	80.0

[1] Population in billion people, life-expectancy in years, calculated from [WB 1994]

The consumption behavior of generations from which not all members live three periods is based on some further assumptions. For any member of a generation, until the beginning of the third period, only the probability of living three periods is known. At the beginning of the third period, any member is either alive or not. Each member is supposed to maximize expected life-time utility. There are no non-intended bequests to future generations due to uncertain lifetime [Hurd 1989] because there is an intra-generational life-insurance company to which all members of a generation pay their savings in the second period of life; the insurance company repays the savings to the living members of the generation in the third period. Under this conditions, the generation can be described by one representative consumer that maximizes aggregate utility subject to one budget constraint.

3B.2. Function and parameter specifications

In ALICE 1.0, the production function for the consumer good has the parabolic shape:

$$g_t(y_{n,t}^{r-} / y_{n,t}^{l-}) = 1 + \eta(y_{n,t}^{r-} / y_{n,t}^{l-})(1 - y_{n,t}^{r-} / 2\zeta_t y_{n,t}^{l-}), \tag{3.119}$$

where ζ_t is the exogenous emission intensity for which production is maximal, and η is the (constant) maximal marginal productivity of emission units. Thus, $g_t(.)$ is continuous, differentiable, concave, $g_t(0)=1$, and $g_t'(0)=\eta$. The supply of labor and the parameters η and ζ are chosen so that maximum production and associated emissions follow the IS92a scenario [IPCC 1992], and emissions decrease by one per cent point for every 4 US$/tC increase in the price of emission units.

Consumers maximize utility, $U(x_i^c, x_i^{nr,b}; n_i)$ with both x_i, x_i^{nr} and n_i vectors over time. The utility function is of a nested CES type.

$$U(x_i^c, x_i^{nr,b}; n_i) = \left(\sum_{t=i,\ldots,i+2} \sigma^{t-i} n_{i,t} \left((x_{i,t}^c / n_{i,t})^{1-\nu} (x_{i,t}^{nr,b})^{\nu} \right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{3.120}$$

where $\rho>0$ denotes the intertemporal elasticity, $0<\nu<1$ the expenditure share for 'biodiversity', and σ reflects the consumers pure time preference. Notice that the utility function is homogeneous of degree one in consumption, x_i, x_i^{nr}.

The rate of increase in labor efficiency over the life cycle is denoted by δ_l. Additionally, labor efficiency increases due to technological innovation, denoted by λ_t. The endowments are given in labor units (efficiency equivalents) by

$$(\omega_{t-2,t}, \omega_{t-1,t}, \omega_{t,t}) = \lambda_t (0.4\delta_l^2 n_{t-2,t}, \delta_l n_{t-1,t}, n_{t,t}) \tag{3.121}$$

The parameter λ is such that production runs parallel with the IS92a scenario [IPCC 1993]. The time preference is set equal to zero, which is consistent with a 2.5 per cent increase of consumption per year over the life-cycle, given a 5 per cent real interest rate. See Auerbach *et al.* [1989,p.119] for a further discussion. For generations that do not live three periods, utility functions are truncated in the obvious way. Table 3.6 lists the parameter values.

TABLE 3.6. *Parameter values for consumers in ALICE 1.0*

Parameter	symbol	value	annual rate[1]
Intertemporal elasticity of substitution	ρ	0.67	
Value share of 'biodiversity'	ν	0.10	
Time preference	σ	1.00	0.
Labor efficiency growth	δ	1.50	0.02

1. The annual rate of decrease or increase for variables below or above unity.

3B.3. Computation

The numerical results for the first two periods, presented in Table 3.2, are based on the welfare program:

PROGRAM 3.11 *Negishi welfare program for ALICE 1.0*

$$max \sum_{i=0,\ldots,\infty} \alpha_i \ln(U(x_i^c, x_i^{nr,b}; n_i))$$

$$x_{i,t}^c, x_{i,t}^{nr,b} \geq 0,\ y_{n,t}^c, y_{n,t}^{l-}, y_{n,t}^{r-}, y_{m,t}^r, y_{m,t}^r \geq 0, \text{ and } k_{m,t}^r, k_{m,t+1}^{p,r} \geq 0 \text{ for } t \in \mathbf{T},\ i \in \{t-2, t-1, t\}.$$

subject to

$$x_{t-2,t}^c + x_{t-1,t}^c + x_{t,t}^c \leq y_{n,t}^c \qquad (p_t^c)$$

$$y_{n,t}^{l-} \leq \omega_{t-2,t}^l + \omega_{t-1,t}^l + \omega_{t,t}^l \qquad (p_t^l)$$

$$y_{n,t}^{r-} \leq y_{m,t}^r \qquad (p_t^r)$$

$$x_{i,t}^{nr,b} \leq y_{m,t}^b \qquad (\varphi_{i,t}^b) \qquad i \in \{t-1, t\}$$

$$y_{n,t}^c \leq y_{n,t}^{l-} + \eta y_{n,t}^{r-}(1 - y_{n,t}^{r-} / 2\zeta_t y_{n,t}^{l-}) \qquad (p_t^c)$$

$$k_{m,t}^r \leq k_{m,t}^{p,r} \qquad (\psi_{m,t}^r)$$

$$k_{m,t+1}^{p,r} \leq k_{m,t}^r - y_{m,t}^r \qquad (\lambda_{m,t+1})$$

$$y_{m,t}^b \leq k_{m,t}^r / \overline{k}^r \qquad (p_t^b)$$

for $t \in \mathbf{T} = \{1, \ldots, \infty\}$, where $k_{m,1}^{p,r} = \overline{k}^r$ is the (exogenous) initial amount of unused emission units. The initial stock of emission units is set to 5000 GtC. The IS92a scenario assumes accumulated emissions of 1500 GtC by 2100, so that the stock of unused emission units, and hence the production of biodiversity, is reduced by 30 per cent. As biodiversity accounts for 10 per cent of expenditures, this amounts to a loss of GDP of 3 per cent, being in line with common assumptions in the models mentioned earlier.

The values for the first two periods are calculated using a truncated model, in which the number of periods is limited by the computer accuracy of calculations because of the exponential decrease of welfare weights and prices, and ranges from 10 periods for the grandfathering scenario till 20 periods for the strong sustainability scenario. The long term values were calculated using the steady state model. We do not repeat the equations for the steady state, as these were given in full detail in Section 3.4.2.

The truncated model contains a value function for the capital stocks after the last period, and it uses the outcomes of the steady state calculations to specify the parameters of the value function. In the steady state, the young and middle aged generation save a share θ_y and θ_m of their income. Together, these savings balance the capital stock, and the share of each stock is denoted by $\tilde{\theta}_j^c$, where $\sum_{j,c}\tilde{\theta}_j^c = 1$. Now, we assume that in the last period of the truncated economy, these expenditure and savings shares are equal to their steady state values, which is consistent with the following welfare objective:

$$max \sum_{i=0,\dots,T-2}\alpha_i \ln(\upsilon_i) + \sum_{i=T-1,\dots,T}\alpha_i((1-\theta_i)\ln(\upsilon_i) + \theta_i \sum_{j,c}\tilde{\theta}_j^c \ln(k_{j,T+1}^{p,c}) \tag{3.122}$$

where υ_i is the utility of generation i, which is based on the consumption in two or one periods only for the last two generations, respectively.

The capital shares θ are given by the steady state analysis and exogenous to the truncated economy. The welfare weights α are determined as part of the equilibrium calculations. The various OLG scenarios follow from the specification of different income definitions. Let D_t for t=0,...,T denote the relative budget deficits of generation t, and let $B_t=\alpha_{t+1}/\alpha_t$ for t=0,...,T–1 denote the so called time preference profile. Program 3.11 returns budget deficits as a function of the time preference profile, and this can be written in reduced form as $D=\Psi(B)$. Calculation of the equilibrium now amounts to finding the time preference profile for which $\Psi(\hat{B})=0$. This problem is solved by a quasi-gradient method, that is based on a local linear approximation of $\Psi(.)$, and we also use the continuation approach. More precisely, we attempt to follow the homotopy path B_s for $s\in[0,1]$, that satisfies $\Psi(B_s)=(1-s)D$, where s=0 represent the initial time preference profile, and s=1 represents the equilibrium [Ginsburgh and Keyzer 1997, Appendix A.8]. In case we do not succeed to follow the homotopy path, we use a genetic algorithm (stochastic search) to ensure convergence, and to provide new data to estimate $\Psi'(.)$. See Gerlagh [1998a] for more details.

The truncated welfare program is optimized by use of GAMS programming language [Brooke *et al.* 1992]. The calculated truncated equilibria show clear 'turnpike' behavior, from which we conclude that they are determinate and insensitive to truncation conditions (Section 2.5.1).

3B.4. Accounting

Calculation of the entries of Table 3.2 is straightforward. The emission price (in US$/tC, the so called CO_2 tax in other IAMs) is defined as the ratio between the price for emission units and the consumer good,

$$P_t^r = p_t^r / p_t^c, \tag{3.123}$$

and cumulative emissions (tC) are given by

$$E = k_1^r - k_{T+1}^r. \tag{3.124}$$

The interest rates and relative asset values are defined in analogy to ALICE 0 (Annex 3A.3).

4. SUSTAINABILITY IN DYNASTIC AND OLG ECONOMIES

This chapter analyzes sustainability in dynastic and in OLG economies. For dynastic economies, direct inclusion of sustainability criteria is compared with the indirect approach that uses a lower discount rate. For OLG economies, a sustainability policy is formulated, based on a trust fund that gives future generations a real share of environmental resources. It is shown that the trust fund ensures efficient use of the environmental resources and at the same time operates as a guardian of the interests of unborn generations.

To illustrate the operation of the trust fund, we introduce it into an applied model with a single exhaustible resource that has amenity value, and compare it with two alternative policies. The first alternative describes a 'strong sustainability' policy in which environmental degradation is controlled by strict conservationist measures. The second considers grandfathering, i.e. endows present generations with full ownership over all resources. Though both the trust fund and grandfathering lead to efficient allocations, only the trust fund improves welfare of all generations as compared to the strong sustainability policy. Finally, the analysis is numerically applied to the issue of climate change, and it is shown that the trust fund substantially increases emission reductions, or alternatively increases the stock of man-made capital to compensate for the environmental loss.

4.1. INTRODUCTION

4.1.1. Sustainability in a competitive economy

For a long time, the efficient pricing of environmental scarcities was seen as the cure to most problems of environmental degradation because this problem was mainly perceived as an issue of inefficient use of environmental resources. Along this line of thought, Pigovian taxes figured prominently among the proposed instruments. More recently, however, it has been suggested that efficient pricing cannot be implemented without an adequate specification of private and public property rights over environmental resources, since markets can only function if private agents have an incentive to prevent free use of resources. These ideas are reflected in the first Welfare Theorem, which states that private property rights, in conjunction with a complete set of competitive markets, ensure the efficiency of equilibrium allocations.

In practical terms, the intuition is that under appropriate management, the environment should have the capacity of generating an indefinite stream of environmental services. This capacity can be expressed as a stock which value is equal to the net present value of this stream of services. Indeed, under appropriate pricing, the sustainable management of the natural environment could potentially become a most profitable venture, in view of the vital role of environmental services in the world economy. For example, Costanza *et al.* [1997] estimate the present value of the (potentially indefinite) stream of environmental services at up to 54 trillion US dollars in 1997 at world level, though obviously such calculations are always debatable.

However, general equilibrium theory has shown that establishing a complete set of markets for all environmental resources and attributing property rights over all of these might not be sufficient to prevent serious environmental degradation. A simple dynastic (Ramsey-type) model can be used to show that exhaustion of environmental resources, including renewable ones could persist under competitive markets [Pezzey 1992]. The same problem arises within an overlapping generations (OLG) model [Mourmouras 1993]. The exhaustion of environmental resources can lead to a gradual fall in welfare, and though the allocation might be efficient, the intergenerational distribution becomes extremely uneven. Whereas in the dynastic model the exponential decrease of welfare weights of future generations can be said to express a lack of empathy for future generations [Broome 1992], the standard OLG model does not allow for any empathy of this kind. Within the OLG format, it is the distribution of property rights for resources over generations that determines the competitive equilibrium allocation. In the standard version of this model, both man-made and environmental capital accrue to the present generation that will sell its property to its successor in order to provide for old age. This is known as grandfathering.

This chapter studies sustainability in competitive equilibria for both dynastic and OLG economies. The core of the analysis centers on the set up of a trust fund, an institutional device for an OLG economy. The description of the trust fund is based on capital accounting rules, treating environmental resources on equal footing with man-made capital. However, this so called 'Capital Theory Approach' has been criticized for its reduction of sustainability to a one dimensional issue, i.e. whether total capital increases or decreases. An example of such criticism is given by Stern [1997] who argues that instead of looking for a solution for unsustainability with the help of the capital theory approach, it would be wiser to bring about an 'organization as the representative or guardian of future generations that would act in a similar way to that of independent central banks in their zealous fights against inflation'. This chapter meets this criticism and shows that the capital theory approach is particularly useful to specify the rules for such an organization.

The chapter is organized as follows. In Section 4.1.2, sustainability is defined in formal terms. Section 4.2 briefly studies sustainability in the dynastic framework, comparing a direct inclusion of sustainability constraints in Section 4.2.1 with alternatives that attempt to improve sustainability by introducing either increasing welfare weights of future generations, Section 4.2.2, or the inclusion of physical constraints, Section 4.2.3. Section 4.3 analyzes sustainability in an OLG economy, following the same line of analysis as for the analysis of efficiency in Chapter 3: first, in Section 4.3.1, additional assumptions are formulated that establish sustainability without a transfer policy and secondly, in Sections

4.3.2 and 0, a simple sustainability policy is constructed working as a redistributional device between generations in case that the additional assumptions are not met. This sustainability policy uses a trust fund that transfers income to future generations.

Thereafter, Section 4.4 explores the consequences of the theory applying the trust fund in an economy with a single resource that has amenity value. The model and environmental resource policies are set up in Section 4.4.1. A steady state analysis is given in Section 4.4.2 (Theorem 4.4, Theorem 4.5 and Theorem 4.6), and the model is applied numerically to climate change in Section 4.4.3 (Table 4.2). Section 4.5.1 further develops the model, assuming a substantial delay between cause and effect of environmental resource use. The numerical analysis in Section 4.5.2 shows that this delay increases the need for a trust fund to guard sustainability of the competitive equilibrium (Table 4.3). So far, the stylized model describes an economy without man-made capital. This is of significance because the loss of environmental resources might be compensated by an increase in the man-made capital stock, which is (by definition) a renewable resource. In Section 4.6.1, we add man-made capital, and indeed, the numerical findings in Section 4.6.2 suggest a substantial substitution of the renewable resources for the exhaustible one (Table 4.4). Finally, conclusions are drawn in Section 4.7.

4.1.2. Formal definition of sustainability

We recollect the main notions from Section 1.3.3. In the literature, intergenerational issues of equity are captured under the term sustainability (see [Pearce 1989] for a list). There are many different definitions of sustainability, causing some confusion. Using welfare economic concepts, sustainability is usually identified with a condition for the utility levels. Weak sustainability refers to conditions that are relative easy to meet while strong sustainability requires stricter conditions more difficult to meet.

However, the same terms 'weak' and 'strong sustainability' are also used for another differentiation made by so called 'ecological economists' [Klaassen and Opschoor 1991]. The ecological economists emphasize the vulnerability of global systems and brought to the fore limited substitutability between environmental and man-made resources. Thus, there is a need for strict and stringent environmental regulation, because environmental damage cannot be compensated by an increased stock of man-made goods. The maintenance of environmental resources is referred to as strong sustainability. Within this perspective, weak sustainability refers to an allocation in which man-made resources may substitute for a loss of environmental resources. The typical ecological economist will not find this possibility of substitution too realistic.

Throughout this study, a strong sustainability policy is based on strict and stringent rules that ensures an environment of the highest quality. A weak sustainability policy accepts substitution between environmental and man-made capital, and only demands that every generation can meet its own needs [WCED 1987, Ch.2].

For the formal definition, we assume that there is a common 'basic needs consumption vector' with which weak sustainability can be identified, i.e., an allocation is weakly sustainable if and only if every generation can consume its basic needs. The advantage of this definition is that it requires minimal comparability only, and that it does not discriminate between intra- and intergenerational equity: a weakly equitable allocation is one in which every consumer now and in the future can meet its basic needs.

Let the basic needs consumption vector be denoted by $\overline{\mathrm{x}}$, which is assumed to be the same for both the young and the old, and utility derived from this consumption be referred to as critical utility, denoted by $\overline{\mathrm{u}} = \mathrm{u}(\overline{\mathrm{x}},\overline{\mathrm{x}})$. Recall from Chapter 2 that u(.) denotes the per capita utility function, where for convenience we omit the time subscript, which takes consumption when young and when old as inputs, while U(.) denotes the aggregate utility over all members of a generation. The basic needs define weak sustainability:

DEFINITION 4.1. *An allocation $x_{t,t}$, $x_{t-1,t}$, $t=1,\ldots,\infty$, is weakly sustainable if per capita utilities exceed the critical level,* $\mathrm{u}(x_{t,t}/n_t, x_{t,t+1}/n_t) \geq \mathrm{u}(\overline{\mathrm{x}},\overline{\mathrm{x}})$, $t=0,\ldots,\infty$.

Within this framework, the distinction between strong and weak sustainability can be demonstrated as follows. Strong sustainability requires that every consumer actually satisfies his basic needs:

$$(x_{t,t}/n_t, x_{t,t+1}/n_t) \geq (\overline{\mathrm{x}},\overline{\mathrm{x}}), \tag{4.125}$$

whereas weak sustainability allows for substitution, and only demands that utility derived from consumption exceeds (utility derived from) the basic needs. Similarly, strong sustainability requires that the environmental resource stocks physically remain intact, whereas weak sustainability allows for substitution. Yet, if there are no substitution possibilities, then, in the formal framework, both approaches coincide.

We also note that the above definition seems preferable to the alternative $\mathrm{u}(x_{t,t}/n_t, x_{t,t+1}/n_t) \geq \overline{\mathrm{u}}$, because it emphasizes that the definition of sustainability does not depend on the utility function, nor does it require comparability of utilities between different consumers. It would have made no difference had we specified generations specific utility functions. The evidence for a sustainable equilibrium allocation indeed does not require any information on the utility functions; it is sufficient to know that every generation has the option of satisfying its basic needs:

LEMMA 4.2. *A competitive OLG equilibrium, in which all generations have sufficient income to buy the basic needs consumption vector is weakly sustainable. Formally:*

$$p_t x_{t,t} + p_{t+1} x_{t,t+1} \geq n_t p_t \overline{x} + n_t p_{t+1} \overline{x} \Rightarrow u(x_{t,t}/n_t, x_{t,t+1}/n_t) \geq u(\overline{x}, \overline{x}) \tag{4.1}$$

where consumption and price vectors refer to competitive equilibrium values.

Proof. In a competitive equilibrium, consumers maximize utility, and, therefore, the equilibrium consumption allocation is preferred to any alternative consumption allocation within the budget space, including the basic needs. ■

Throughout this chapter, we take all standard assumptions (*M1*) of Chapter 2 to apply, but we also need to formulate assumptions that ensure feasibility of the weak sustainability condition. For this, we proceed as follows. Let $\overline{y}$ be a stationary production flow vector, supported by a stationary stock $\overline{k}$, $F(\overline{y}, -\overline{k}, \overline{k}) \leq 0$. We can think of an environmental resource that produces a constant output without maintenance costs as described in Chapter 3 (Assumption *F7*), but this is not necessary for the formal analysis to come. The stock $\overline{k}$ might have strictly positive maintenance costs as well, reflected in negative elements for the production flow $\overline{y}$. Furthermore, in every period, let the stationary production flow be distributed between the old and young generations, $\overline{y} = \overline{y}_y + \overline{y}_o$. Consumption of the old and young generation, $x_{t-1,t}$, $x_{t,t}$, is then given by $n_{t-1} x_{t-1,t} = n_{t-1}\omega_o + \overline{y}_o$ and $n_t x_{t,t} = n_t \omega_y + \overline{y}_y$, respectively. Weak sustainability can be guaranteed if these consumption levels exceed the basic needs vector for all population levels between the lower and upper bound, $n_t \in [\underline{n}, \overline{n}]$. This is formalized as follows

ASSUMPTION 4.3. *There exists an interior stationary production allocation,* $(\overline{y}, \overline{k})$, *which is feasible and supports weak sustainability.*

F8: $\exists (\overline{y}, \overline{k}) \in \mathbf{Y} \times \mathbf{K} : \overline{k} < k_1^p \wedge F(\overline{y}, -\overline{k}, \overline{k}) \leq 0,\ \overline{y} = \overline{y}_y + \overline{y}_o,$ *with* $\forall n \in [\underline{n}, \overline{n}]$:
$n\overline{x} < n\omega_y + \overline{y}_y$ *and* $n\overline{x} < n\omega_o + \overline{y}_o$.

The assumption may seem rather crude. It essentially states that there should exist a physically sustainable production level. The absence of substitution between different commodities, and stationarity of the production flow is introduced for the sake of convenience. The analysis in this chapter can readily be extended under weaker assumptions in this respect.

Throughout this chapter, intergenerational equity will be identified with 'weak sustainability' (Definition 4.1). For completeness, we mention two alternative definitions of intergenerational equity that are more commonly found in the literature. The first requires *per capita* utilities not to decrease over time [Pezzey 1992]:

$$u(x_{t,t}/n_t, x_{t,t+1}/n_t) \geq u(x_{t-1,t-1}/n_{t-1}, x_{t-1,t}/n_{t-1}), \tag{4.2}$$

for $t=1,\ldots,\infty$. Non-decreasing utility requires much stronger assumptions regarding comparability between consumers than a critical utility level. Using the classification of Roberts [1980], non-decreasing utility requires 'ordinality and level comparability' of the utility functions[15].

Nonetheless, in the literature, it has gained widespread acceptance. To take account of the problem that, in practice, it is difficult or even impossible to measure utility, one has looked for a measurable equivalent sustainability criterion. Solow [1974] and Hartwick [1977] have shown that under certain conditions, a constant utility level is equivalent with 'zero net investments', where 'net investments' refer to the value of the mutations of the man-made and environmental capital stock. This is known as Hartwick's rule, but it has to be noticed that it is based on several rather stringent assumptions, and will not hold in a general competitive economy. Nonetheless, this rule has led to the popular sustainability criterion that the total capital stock (in the broad sense, including man-made and environmental resources) should not fall. In particular, if environmental resources degenerate, this has to be compensated by an increase in man-made stocks that are at least as valuable.

The second alternative equity criterion even goes one step further, and requires that an allocation is only equitable if welfare of the generation that is worst off is maximized. This is known as the 'maximin' criterion. The condition was brought forward by Solow [1974] as an extension of the principle of the 'veil of ignorance'[16] to the issue of intergenerational distribution, though Rawls did himself not consider the principle useful for intergenerational issues of equity (see [Solow 1974] for reference and comment). The condition also requires ordinality and comparability of ordinal utility levels. The criterion leads to an allocation with stationary utility.

$$u(x_{t,t}/n_t, x_{t,t+1}/n_t) = u(x_{t-1,t-1}/n_{t-1}, x_{t-1,t}/n_{t-1}), \tag{4.3}$$

The non-decreasing utility and stationary utility conditions are rather restrictive, and not always compatible with competitive equilibria. We return to this aspect towards the end of Section 4.2.1.

Moreover, we would argue that the increasing utility criterion is not 'equitable enough' while the maximin criterion in 'too equitable'. By this, we mean that the increasing utility criterion has no natural extension to interpersonal equity within a generation, since the interpersonal distribution of welfare can be extremely uneven. It seems difficult to justify

[15] Some authors do not hesitate to assume maximal comparability between consumers. Harsanyi [1976] argues that all people are physically comparable, which implies comparability of utility as well.

[16] The veil of ignorance refers to a situation in which an imaginary consumer chooses the preferred distribution of welfare over all consumers while he does not know which consumer he himself is [Rawls 1972].

the concern for future generations without any concern for the distribution of welfare among present generations [Schelling 1995]. On the other hand, the maximin criterion brings us to the other extreme if applied to intra-generational issues. Its natural extension would require a flat distribution throughout, which is such a strong demand, that it seems beyond any conceivable equity policy.

To summarize, most of the analysis in this chapter is based on the weak sustainability criterion of Definition 4.1, which only requires the recognition of basic needs, and does not call for a distinction between interpersonal and intergenerational equity, and for which existence proofs can proceed under common assumptions. We will see that the policy that safeguards weak sustainability is straightforward (Section 0), and that it has substantial consequences (Section 4.4, 4.5 and 4.6).

4.2. THE DYNASTIC FRAMEWORK

4.2.1. Incorporating sustainability criteria

Within the dynastic framework, all generations depend on the empathy of the dynastic planner, and a sustainability criterion can be conceived as an additional manifestation of empathy. From this perspective, the inclusion of an appropriate constraint is the obvious way to implement a sustainability criterion. We will do so using the infinite horizon dynastic welfare program of Chapter 2 with the aggregator function.

PROGRAM 4.4. *Infinite horizon sustainable dynastic welfare program*

$$v(k_1^p, x_{0,0}; n_0) = max\ w_0$$

$$x_{t-1,t}, x_{t,t} \geq 0, k_{j,t}, k_{j,t+1}^p \geq 0, \text{ and } y_{j,t} \text{ for } j=1,\ldots,J,\ t=1,\ldots,\infty,\ , \upsilon_t, w_t \geq 0 \text{ for } t=0,\ldots,\infty$$

subject to

$w_t \leq H(\upsilon_t, w_{t+1})$	(η_t)	$t=0,\ldots,\infty$
$\upsilon_t \leq U(x_{t,t}, x_{t,t+1}; n_t)$	(α_t)	$t=0,\ldots,\infty$
$x_{t-1,t} + x_{t,t} \leq n_{t-1}\omega_o + n_t\omega_y + \Sigma_j y_{j,t}$	(p_t)	$t=1,\ldots,\infty$
$k_{j,t} \leq k_{j,t}^p$	$(\psi_{j,t})$	$j=1,\ldots,J,\ \ t=1,\ldots,\infty$
$F_{j,t}(y_{j,t}, -k_{j,t}, k_{j,t+1}^p) \leq 0$	$(\lambda_{j,t})$	$j=1,\ldots,J,\ \ t=1,\ldots,\infty$

for $n_t = \phi^t(n_0)$. We recall the basic properties of this program from Chapter 2 under assumptions *D1*. It is feasible and has a strict interior. All welfare weights, α_t , are strictly positive, and expenditures are bounded from above by weighted utilities, $p_t x_{t,t} + p_{t+1} x_{t,t+1} \leq \alpha_t \upsilon_t$. The dynastic welfare program represents a competitive equilibrium because all welfare weights are non-zero. The equilibrium is efficient because aggregated utility is finite as it is bounded by present welfare, w_0.

We include within the dynastic welfare program the weak sustainability constraint:

$$n_t \bar{u} \leq \upsilon_t , \qquad (\mu_t) \qquad (4.4)$$

for $t=0,\ldots,\infty$ and state and prove that this additional constraint leads to a weakly sustainable competitive equilibrium.

THEOREM 4.1. *Let the standard Assumptions D1 and the additional Assumption F8 be satisfied, and let us include a weak sustainability constraint* (4.4) *in Program* 4.4. *Then, the associated welfare optimum yields an efficient sustainable* (*Definition* 4.1) *competitive equilibrium with transfers.*

Proof. Modify the dynastic program as follows. The objective function is changed into $(1-b)w_0+b\tilde{w}$, constructing a homotopy for $b\in[0,1]$; instead of constraint (4.4), we append the constraint $n_t\tilde{w}\leq\upsilon_t$ with dual Lagrange variable μ_t,

$$n_t\tilde{w}\leq\upsilon_t \qquad (\mu_t) \qquad (4.5)$$

for $t=0,\ldots,\infty$.

For $b=0$, the program reduces to the original program, whereas $b=1$ yields the program associated with the *maximin* condition. For any $0\leq b<1$, the welfare weights α remain strictly positive. (For $b=1$, some welfare weights might become zero.) Concavity of the aggregator function and utility functions ensures that the value of endowments which is equal to the value of consumption is bounded by the objective function which itself is bounded from above. Hence, the program modification amounts to adjusting welfare weights α, and the optimal allocation is an efficient competitive equilibrium (with transfers). For $b=1$, the lower bound utility exceeds the critical level, $\tilde{w}>\underline{u}$ because Assumption *F8* requires a strict inequality for the commodity balance and utility is non-satiated in consumption. As the optimal allocation is upper-semi-continuous in b, it follows that there is a $b<1$ for which the optimal lower bound utility is equal to the critical level: $\tilde{w}=\underline{u}$. This b provides an efficient competitive equilibrium with transfers which is weakly sustainable. The resulting welfare program is equivalent to one in which the original weak sustainability constraint is included. ■

In the literature, the use of critical utility constraints within the welfare program usually serves different purpose. Blackorby and Donaldson [1984] and Blackorby *et al.* [1995] subtract the critical utility from the actual utility levels before aggregation into a social welfare measure, and use this method to analyze optimal population levels. This is not relevant in the present context where population is exogenous, but we return to the issue in Chapter 5, when we discuss the relation between sustainability and fertility.

The satisfaction of the alternative increasing and stationary utility criteria has been analyzed extensively in the literature. The stationary utility condition associated to the maximin principle requires aggregate net investments to be zero, and this has become known as Hartwick's rule [Hartwick 1977], [Dixit *et al.* 1980], [Dasgupta and Mitra 1983] and [Solow 1986]. The inference for environmental economics is that to ensure sustainability, environmental resource degradation, seen as desinvestments in the economic sense, should to be balanced by equal positive investments in man-made capital. Though Hartwick's rule is itself straightforward, it is not always compatible with competitive equilibria. One has to impose stronger assumptions on the economy to ensure the existence of a competitive equilibrium with non-decreasing utility [Dasgupta and Mitra 1983]. Riley [1980] replaces the condition of non-decreasingness by a 'non-exploitation' condition which allows for decreasing utilities when this is inevitable.

While it appears to be difficult to meet the criteria in the theoretical model, in most integrated assessment models (IAMs), there is no problem, because the exogenous technological progress is more than sufficient. Yet the constraint of non-decreasing utility is rarely applied in these models. To guarantee sustainability, modellers of IAMs often proceed by raising welfare weights of future generations, and by specifying additional physical constraints. Both options are briefly discussed below.

4.2.2. Small or zero discount rates

Most climate change IAMs are dynastic models that have exponentially decreasing welfare weights. As noted in Chapter 2, this can be represented by the aggregator function: $H(u,w)=u+\beta w$. The rate of decrease, $\beta^{-1}-1$, is called the pure time preference or discount rate. It has been argued that the use of a positive discount rate reflects myopia and is ethically indefensible [Broome 1992]. Unsustainable optimal allocations are believed to be the consequence of the negligible weights of future generations in present day decision making. A simple calculation can be used to demonstrate these diminishing weights: a discount rate of 5 per cent per year throughout one century eventually causes a decrease in welfare weights by a factor 100. If the discount rate is the cause for unsustainability, the obvious solution is to increase the welfare weights of future generations by decreasing the discount rate.

When taken to the extreme, the ethical argument against myopia results in zero discount rates, and equal weights for all generations. An argument against this practice is that welfare becomes unbounded if summing non-weighted utilities for an indefinite future, and it is difficult to rank two allocations which both yield infinite welfare. In case the two allocations converge exponentially to one another, their difference can be valued and used for the ranking. However, for the general case, more complex definitions of optimality are necessary [Carlson and Haurie 1987]. To apply the standard ranking of allocations by comparing finite welfare measures, some kind of time preference is necessary [Koopmans 1960] [Koopmans *et al.* 1964].

An alternative for zero discounting is the use of 'low' discount rates. Discount rates are 'low' if they are exogenously set below the value which is believed to be the empirical estimate, e.g. the value used for calibration of a model (see the use of 'standard' and 'low' discount rates in [Manne *et al.* 1995]). This raises the question as to which small but positive discount rate should be used for intergenerational welfare analysis. There is no unique threshold discount rate below which the optimal allocation becomes sustainable. Every environmental resource has its own threshold discount rate above which the optimal management becomes unsustainable. A different set of assumptions results in different conditions that describe the knife-edge between sustainable and unsustainable optimality programs, e.g. see Clark [1973] for an analysis of biological populations with varying regeneration rates, Dasgupta and Heal [1974] for an analysis with a constant absolute reproduction level, and Solow [1974] and Stiglitz [1974] for other specifications. Thus, one has to specify the 'sustainable' discount rate depending on the application, but then, it seems more natural to have the discount rate endogenously determined by adding an explicit sustainability constraint as in Section 4.2.1.

Another argument against the use of a 'low' discount rate, is its 'non-specificity'. In a dynastic welfare program, the discount rate influences many variables, not only the distribution of welfare. One such effect is the immediate and substantial increase of the savings rate, which has been emphasized as an unrealistic characteristic of sustainability policies based on low discount rates [Nordhaus 1994] [Manne *et al.* 1995]. We conclude that the manipulation of the discount rate is not an effective parameter to achieve sustainability.

4.2.3. Imposing physical bounds

If sustainability can be linked to the state of the environment, an effective measure for securing sustainability might be the imposition of stringent boundary conditions for physical variables. In general, this would cause distortionary inefficiencies in the economy, and it is therefore undesirable. However, the notion of efficiency does not exclude physical boundaries that mark the boundaries of a physical sub-system, as is common practice for production processes, see [Ginsburgh and Keyzer 1997, Section 2.1.2] for a discussion.

Physical boundaries can also be imposed as a representation of limited knowledge about the environmental processes at play. If, for example, the emission absorption capacity is poorly understood, the best approach may be to define tolerable levels of emission absorption and periodically distribute tradable licenses to pollute. Unfortunately, if it is not possible to explicitly link environmental resource use and the negative externalities, welfare seems to increase if one releases the constraints, and this undermines public support for environmental policies.

The imposition of physical boundaries for a moral reason, such as the protection of future welfare levels, falls under what was referred to as a strong sustainability policy. The application in Section 3.4, and the applications in this chapter in Sections 4.4, 4.5 and 4.6 make use of such a policy scenario as a benchmark for the grandfathering and the trust fund policy described in Section 0.

4.3. THE OLG FRAMEWORK

4.3.1. Intergenerational dependence

The previous section discussed sustainability measures within the dynastic perspective. However, though sustainability can be included relatively simply in the dynastic framework, as we have seen in Section 4.2.1, it gives no indication as to the economic institutions or policy instruments that should be created to implement the policy in practice. The dynastic framework simply presumes that a benevolent dynastic dictator distributes income according to his optimal choice. This section describes an overlapping generations economy in which every generation is free to spend its own income. In line with the analyses of Chapter 3 regarding efficiency, two strategies are followed to establish sustainability. We start by introducing sufficient assumptions for consumers and producers, and subsequently formulate a transfer policy for cases in which such assumptions do not apply.

The concern for sustainability expresses the empathy of present generations for future generations, and hence, empathy is a candidate extension of the standard model of Chapter 2 to look for, if we want to establish sustainability in the competitive equilibrium. Recall that we have already briefly studied empathy in Section 3.2.2, in relation to dynamical efficiency, where we did not succeed in formulating general assumptions concerning empathy that suffice to restore dynamical efficiency.

The literature on OLG models with empathy attempts to explain the accumulated wealth in the form of man-made capital stocks as bequests or, alternatively, as life-cycle savings. Kotlikoff and Summers [1981] estimate that half of accumulated wealth (capital) can be attributed to bequests. This claim is however questioned, for example by Modigliani [1988] who provides an overview of estimates for the share of wealth originating from bequests. Hurd [1989] goes one step further and asks whether empathy is the real cause of bequests, arguing that most bequests are accidental, and occur as a result of uncertainty about the date of death.

Here, we choose an alternative approach that does not include empathy as such within the model, and express the empathy of present generations for future generations indirectly in the willingness to set up an institution that ensures sustainability.

Let us imagine a continuum of economies, with simple autarkic economies at one extreme, and complex economies with many interdependencies at the other. In the autarkic economy without production, sustainability requires that the endowments exceed the basic needs consumption vector, $\omega_y, \omega_o \geq \overline{x}$, or alternatively, $u(\omega_y, \omega_o) \geq \overline{u}$. If consumers exchange, they will only do so if this improves their utility, and in the competitive equilibrium, utility always exceeds utility derived from the basic needs. In this case, no sustainability policy is required. It can easily be seen that in an economy without production, if endowments are differently distributed, it is relatively simple to construct a sustainability policy, as this only requires a redistribution of endowments between the young and old generations within the periods.

Next, we include production in the economy. Let us assume that the young generations do not consume their entire endowments, but store a part of these as wealth, $x_y+y_y=\omega_y$, where the subscript '*y*' denotes the young generation, and y_y denotes the consumption given up when young. The capital stock generated with y_y is denoted by k, and satisfies $F(y_y,-k,0)\leq 0$. When old, the capital stock is used to increase the consumption level by y_o, which satisfies $F(y_o,0,k)\leq 0$. Now, if both x_y and $x_o=\omega_o+y_o$ exceed the basic needs, the allocation is sustainable for the generation concerned. This is generalized in the assumption below.

ASSUMPTION 4.5. *Every generation can meet its own basic needs:*

F9: *There is a feasible production pair* $(\overline{y}_y, \overline{y}_o)$, *supported by a stock* $\overline{k} \geq 0$, *such that endowments plus production are sufficient to meet the basic needs:* $F(\overline{y}_y, -\overline{k}, 0) \leq 0$ $F(\overline{y}_o, 0, \overline{k}) \leq 0$, $\overline{x} + \overline{y}_y \leq \omega_y$, *and* $\overline{x} \leq \omega_o + \overline{y}_o$.

Assumption *F9* ensures that every generation can meet its own basic needs without the help of other generation, and it implies Assumption *F8*. There is no environmental problem and weak sustainability is straightforwardly proven:

THEOREM 4.2. *Let the standard assumptions O1 and the additional Assumption F9 be satisfied, then the competitive OLG equilibrium without transfers is weakly sustainable, i.e. every generation can meet its own basic needs (Definition* 1.1).

Proof. Weak sustainability follows directly from Lemma 4.2: utility maximization and the observation that in competitive equilibrium, because of profit maximization, the costs of the required production pair is non-positive, $p_t \bar{y}_y + p_{t+1} \bar{y}_o \leq 0$, ensures weak sustainability. ■

Though Assumption *F9* may seem extremely restrictive, it deserves a brief discussion since it offers a natural introduction to the creation of a trust fund in Section 0. We recall from Coase [1960, Section 10] that we should not think of production factors as physical commodities, but as legal rights to use physical commodities in a certain way. Whether Assumption *F9* is satisfied does not only depend on physical characteristics of an economy, but on property rights systems as well. The following example may illustrate this point:

EXAMPLE 4.6. *Sustainability and the distribution of property rights*

Consider an agricultural economy that is capable of producing its own life-support, using land and labor as the main production factors. Moreover, assume that labor is uniformly distributed over the life-cycle, and abstract from land degradation so that there is no physical sustainability problem at hand. We specify two different property right systems that define two different economies, of which only one satisfies Assumption *F9*. The first property rights system treats land as private property that is held by the old as life-cycle savings but that does not generate life-cycle income (the value of its revenues is equal to the price at which it is bought). As land is an essential production factor to produce the basic needs while land itself cannot be produced by labor, Assumption *F9* is not satisfied. Though the economy does not suffer from physical unsustainability, it might suffer from economic unsustainability as there is no guarantee that income from labor is sufficient to buy the basic needs. The second property rights system distributes property rights for the flows of services produced by the land. Every citizen has the right to farm a certain amount of land during his life-time. In the economic model, the services provided by the land have become a flow endowment, and there are no economic stocks. This property right system satisfies Assumption *F9* and thereby guarantees weak sustainability. ■

4.3.2. Environmental policies in OLG economies

The economies to be considered next lie at the complex end of the spectrum: generations are not self supporting because stocks are essential in the production process. No generation can produce a critical utility for itself without making use of the stocks left by the previous generations. In the standard version of the OLG framework, generations cannot count on the empathy of a dynastic planner or on the empathy of other generations. Every generation maximizes its own utility subject to its own budget constraint. It is therefore important to specify a policy that safeguards the interests of different generations.

We might distinguish four archetypes of policies, characterized by their attention for efficiency and sustainability (Table 4.1). A *laissez faire* policy ignores the environmental resources altogether, which implies that markets are incomplete, and this will probably lead to an inefficient and unsustainable environmental resource use. A *grandfathering* policy gives the environmental resources to the present generations which maximize profits and thereby ensure their efficient use (assuming that the competitive economy itself has the institutions that make it Pareto efficient). Unfortunately, it can be optimal to exploit the entire environmental resource for present-day use, leaving a steadily decreasing environmental resource level for future generations, for example if the net present value of an indefinite sustainable output stream is lower than the revenues from a short term exhaustion of the resource [Clark 1973]. A *strong sustainability* policy imposes strict conservationist measures, based on strong environmental quality standards that ensure sustainability but violate efficiency as they constrain substitution possibilities. Future generations might be willing to lower the environmental quality standards in exchange for increased man-made capital, or *vice versa*. Finally, a *trust fund* policy issues property rights over environmental resources, thus ensuring their efficient use, but shares these with future generations (via the trust fund) to guarantee sustainability. The rules of conduct for the trust fund can be understood as an economic set of rules that underlie a legal intergenerational system of rights and obligations [Weiss 1989].

TABLE 4.1. *Environmental policy options*

	Inefficient	Efficient
Unsustainable	Laissez faire	Grandfathering
Sustainable	Strong sustainability	Trust fund

We notice that the trust fund policy asks the present generation to renounce its claims to (a part of) the environmental resources and to transfer the claims to a trust fund that uses them to guard the interests of future generations. The creation of the fund does not lead to Pareto improvement. Therefore the trust fund can only come into play through empathy of present generations for their descendants which, though not explicitly represented within the model, is comparable to a policy that guards the interests of the poor. The trust fund gives future generations real income claims which can be understood as income from 'pseudo property rights' or 'pseudo endowments'. In this sense, the trust fund extends the idea of property rights which, in the standard competitive equilibrium, are defined for actual stock and flow endowments only.

4.3.3. A trust fund for attaining sustainability

Sharing property rights over environmental resources between present and future generations is not an easy task, since there is no possibility of direct communication between the parties. Hence the need for an institution that operates according to pre-specified rules, and that could function as follows. First, at a given point in time (the initial year of the analysis), the ownership of all environmental resources which were previously treated as free goods are attributed to a public trust fund. Secondly, a law is passed that all consumers and firms should pay for the environmental resources they use, and a perfect enforcement mechanism is put in place. Thirdly, trade is opened in the ownership titles over these resources: ownership is divided over many small shares and the private sector can buy these shares. Also, the trust fund can buy shares in the private sector. Finally, the trust fund gives a real claim to every present and future consumer. At birth, every person receives a claim for a certain amount of environmental services over the life-cycle which gives the owner income, and which can be used to consume the environmental services themselves, or if preferred, to consume other goods. The environmental resources thus give additional income to the consumers, as is the case in the economy of Example 4.6 with the property rights distribution that satisfies Assumption *F9*.

The next question is how much the trust fund can afford to pay the consumers over an indefinite future. To determine this, the trust fund calculates the maximal level of production of environmental services that can be sustained forever, say, of fish, timber, clean air and water; this output level is referred to as the maximal sustainable output. This is not necessarily the output level that would prevail in a steady state of the environment, since substitution could be allowed for. This maximally sustainable output will be allocated across consumers as a real income claim.

We can envisage the operation of this trust fund as follows. Let us begin imagining a steady state in which the environment produces the maximal sustainable output in every period. There is neither environmental degradation, nor economic growth. The trust fund holds assets equal to the value of the environmental resources which value is equal to the net present value of all future output. In other words, the interest on the asset precisely matches the required payments to the consumers. Every consumer receives exactly the income required to pay for the maximally sustainable output. Thus, the pricing of environmental services does not cost the consumer any income. Next, we must ensure that the trust fund could operate properly under non-steady state conditions as well. For this, its financing should be specified in more detail.

We assume that the trust fund's management is instructed to maintain financial assets up to a level that is sufficient to fulfill the fund's commitments. In a general equilibrium model, it will be sufficient for the trust fund to keep its asset value equal to the current value of the environmental stock needed to generate the sustainable output. The mechanism is that the value of this hypothetical stock cannot be less than the value of the maximal sustainable output in the same period, plus the value of the associated stock in the next period. Otherwise, there would be an activity that makes a strictly positive profit: buying the stock, selling the output (the maximal sustainable output) and the remaining stock at the end of the period. Thus, if the instruction is followed in period 1, the trust fund will be able to pay the value of the maximal sustainable output to the consumers for that period and to follow the instruction for the next period, and so on.

If environmental degradation happens to persist, future generations can no longer consume the quantity of environmental services on which the claim was based. In that case their income from the trust fund will exceed actual expenditures on the environment (that is limited by the actual state of the environment) and will be spent on other commodities. This might be interpreted as a compensation to future generations for the degraded environment. The difference in the distribution of property rights over environmental resources between grandfathering and the trust fund is also known as the difference between 'willingness to pay' and 'willingness to accept' [Krutilla and Fisher 1975, Ch.2]. If future generations find the preservation of environmental resources essential, then, in the general equilibrium model, the mechanism will help to prevent degradation, since it enables future generations to send the appropriate price signal to its predecessors. At first sight, it might seem inconceivable that preferences of future generations affect current allocations while these future generations do not exist, but we have to keep in mind that we assume existence of complete asset markets which implies that future preferences and future scarcities are represented through the prices of these assets.

Now, let us formally represent the trust fund and the associated transfers. In the model of Chapter 2, income claims are described as a transfer system satisfying conditions *A1*-*A4*. The idea of the sustainability policy is to endow all generations with real income claims, represented by the production flows $\bar{y}_o$ and $\bar{y}_y$, in addition to income derived from conventional endowments $(n_t\omega_y, n_t\omega_o)$. The task of providing the additional income claims is accomplished by the trust fund.

The fund's management is told to maintain its assets equal to the current value of the environmental stock needed to generate the sustainable output,

$$A_t = \psi_t \bar{k}, \tag{4.6}$$

where A_t denotes the value of the assets hold by the fund. We can easily show that this rule of conduct is sufficient to enable the fund to pay all its future commitments. The mechanism is as follows. In competitive equilibrium, denoted by a hat, production maximizes profits which is zero, so that $p_t\,\bar{y}+\psi_{t+1}\,\bar{k}-\psi_t\,\bar{k}\leq\; p_t\hat{y}+\psi_{t+1}\hat{k}_{t+1}-\psi_t\hat{k}_t=0$. Thus, the value of the hypothetical stock $\bar{k}$ cannot be less than the value of the maximal sustainable output in the same period, plus the value of the stock in the next period:

$$A_t\;\geq p_t\,\bar{y}+A_{t+1}\,. \tag{4.7}$$

If the instruction is followed in period 1, the fund will be able to pay the value of the maximal sustainable output to the consumers for that period and to follow the instruction for the next period, and so on. The surplus of the fund, or pseudo profit π_t^g, is given by

$$\pi_t^g\;=\psi_t\,\bar{k}-p_t\,\bar{y}-\psi_{t+1}\,\bar{k}\geq 0\,, \tag{4.8}$$

which can be divided between the young and old generation, according to some shares θ_y and θ_o, θ_y+θ_o=1. The income claim generation t receives from the fund is now given by:

$$\Phi_{t,t}\;=p_t\,\bar{y}_y+\theta_y\pi_t^g\,, \tag{4.9}$$

when young, and

$$\Phi_{t,t+1}\;=p_{t+1}\,\bar{y}_o+\theta_o\pi_{t+1}^g\,, \tag{4.10}$$

when old, for t=1,...,∞.Compared to the standard OLG model in which the resource accrues to the first generation, this generation has to pay for the set up of the fund.

$$\Phi_{0,1}\;=p_1\,\bar{y}_o+\theta_o\pi_1^g\;-\psi_1\,\bar{k}\,, \tag{4.11}$$

However, if the resources were previously treated as free goods, the first generation faced a zero price for its purchase and is thus not paying for the set up. Usually, capital stocks are bought by the young generations to provide for income when old. At the beginning of every period, the capital stocks are property of the old generation. Grandfathering previously free environmental resources extends this principle, with only one difference: the grandfathered resources have not been bought by the old generation when young. Giving these resources to the trust fund instead of to the old generation does not deprive the old generation of any resource, but only limits the gift they receive. Consequently, it will be more difficult to find support for the set up of a trust fund after the resources have been grandfathered, because resources will have to be alienated.

Given the transfers described by (4.8)-(4.11), all generations t=1,...,∞ have sufficient income to buy the basic needs. We state and prove:

THEOREM 4.3. *Under Assumptions O1 and Assumption F8, if the transfers are supported by a trust fund as in equations* (4.6), (4.8)-(4.11), *then the infinite horizon OLG equilibrium (Definition* 2.16) *exists and is sustainable.*

Proof. As transfers are linear in present and past prices, every generation has non-zero income and the first generation owns a non-zero part of the initial stock ($\bar{k} < k_1^p$), the trust fund policy satisfies assumptions *A1*-*A4* and the existence proof of Theorem 2.2 applies. Weak sustainability follows from Lemma 4.2. ■

The holdings of the trust fund A_t can be considered public savings, to which the same accounting rules apply as in Chapter 2. The transfers paid by the trust fund ensure that its dynamics (4.6) satisfy:

$$A_{t+1} = A_t - \Phi_{t-1,t} - \Phi_{t,t} . \tag{4.12}$$

and the capital balance (2.28) applies:

$$S_{t+1} + A_{t+1} = \psi_{t+1} k_{t+1} ; \tag{4.13}$$

private life-cycle savings plus the trust fund holdings balance with the value of capital.

Since the trust fund ensures sustainability, we can conclude that it avoids environmental deterioration. Each generation receives an income from the trust fund plus its standard endowments. If this is sufficient to satisfy all subsistence requirements and future generations consider the preservation of environmental resources as essential for their welfare, then in the general equilibrium model, sustainability will be maintained, and in this way, if environmental degradation causes unacceptable welfare losses, the trust fund will prevent it.

4.3.4. Combining efficiency and sustainability

In Chapter 3, we developed an efficiency policy supported by a claim on future flow endowments given to the first generation. In the sub-section above, we developed a sustainability policy supported by claims on environmental resource services given to future generations. To distinguish between these two, we will denote them as the forward and backward claim respectively, and if convenient refer to the latter shortly as the trust fund. In principle, there is no difficulty combining both policies.

If the trust fund and endowment claim are combined, life-cycle savings S plus the trust fund A balance with the value of capital ψk plus the value of the forward claim χ,

$$S_t + A_t = \psi_t k_t + \chi_t . \tag{4.14}$$

It is clear from this equation, that both policies have opposite effects, and they cancel out when $A_t=\chi_t$. Nonetheless, their combination can be useful because they 'automatically' determine the required level of public savings/debt, and thereby they elevate the task of the public agent.

4.4. A SINGLE EXHAUSTIBLE RESOURCE WITH AMENITY VALUE

4.4.1. Model and policy specification

After having described the principles of the fund, we can now illustrate its operation using ALICE 1.0, described in Section 3.4. We recall the basic set up of the model. The resource exploitation is performed by a firm denoted by *j='m'*. The resource itself is denoted by a superscript *r*. Let $k^r_{m,t}$ be the resource stock from which $y^r_{m,t}$ units are subtracted each period:

$$k^{p,r}_{m,t+1} \leq k^r_{m,t} - y^r_{m,t}, \tag{4.15}$$

where $k^{p,r}_{m,t+1}$ is the planned stock at the end of period *t*. The exhaustible resource has amenity value, $y^b_{m,t}$, denoted by a superscript *b*. The amenity level is proportional to the stock level, relative to the initial stock $\overline{k}^r = k^{p,r}_{m,1}$:

$$y^b_{m,t} \leq k^r_{m,t} / \overline{k}^r . \tag{4.16}$$

Thus $y^b_{m,t}$ is measured as an index, with maximum output $y^b_{m,t} = 1$. Because of constant returns to scale in (4.45) and (4.16), profit maximizing behavior by the environmental firm can be represented through the recursive zero-profit condition:

$$\psi^r_{m,t} k^r_{m,t} = p^r_t y^r_{m,t} + p^b_t y^b_{m,t} + \psi^r_{m,t+1} k^r_{m,t+1}, \tag{4.17}$$

where p^r_t and p^b_t are the prices of the extracted resource *r* and the amenity level *b* in period *t* respectively, and $\psi^r_{m,t}$ is the price of the resource stock at the beginning of period *t*. The equation states that the value of the resource, $\psi^r_{m,t} k^r_{m,t}$, is equal to the value of its output.

A second firm, denoted by *j='n'*, uses the extracted resource and 'labor', denoted by a superscript *l*, $y^{l-}_{n,t}$, for the production of the consumer good, denoted by a superscript *c*:

$$y^c_{n,t} = y^{l-}_{n,t}\, g_t(y^{r-}_{n,t} / y^{l-}_{n,t}), \tag{4.18}$$

where the superscript minus signs denote inputs, $g_t(.)$ is a continuous, differentiable, concave production function, with $g_t(0)=1$, and $0<g_t'(0)<\infty$. The subscript t is maintained to allow for technological innovation. Because $g_t(0)=1$ and the restriction on the slope, the extracted resource is valuable, but not essential for the production. Profit maximization can, in view of constant returns to scale, be represented through the zero-profit condition:

$$p_t^c y_{n,t}^c = p_t^l y_{n,t}^{l-} + p_t^r y_{n,t}^{r-}. \tag{4.19}$$

This completes the description of producer behavior.

Generations maximize their lifetime utility $U(x_i^c, x_i^{nr,b}; n_i)$ derived from rival consumption of the consumer good, $x_i^c = (x_{i,i}^c, x_{i,i+1}^c)$, and non-rival consumption of the resource amenity, $x_i^{nr,b} = (x_{i,i}^{nr,b}, x_{i,i+1}^{nr,b})$. Here, unlike the assumption in Chapter 3, generations live only two periods, for reasons we will give below. Their utility function U(.) is assumed to be non-negative, differentiable, concave, and strictly increasing in all arguments. The generations maximize utility subject to the budget constraint:

$$max \{ U(x_i^c, x_i^{nr,b}; n_i) \mid \sum_{t=i,...,i+1} p_t^c x_{i,t}^c + \varphi_{i,t}^b x_{i,t}^{nr,b} \leq \sum_{t=i,...,i+1} p_t^l \omega_{i,t}^l + H_{i,t} \}, \tag{4.20}$$

where p_t^c, p_t^l denote the given prices of the (rival) consumer goods and labor, respectively, $\varphi_{i,t}^b$ are the given Lindahl prices for non-rival consumption of the resource amenity of generation i in period t, $\omega_{i,t}^l$ denotes the labor endowment and $H_{i,t}$ is the income from the environmental resource received by generation i in period t:

$$\sum_{i=0,...,\infty} H_i = \psi_{m,1}^r k_{m,1}^{p,r}. \tag{4.21}$$

where H_i denotes the aggregate income of generation i. The first generation not living two periods has adjusted utility functions and budget constraints in the obvious way. Notice that generation t=0 only has a claim H_0 to the resource, and has no other claims. Formally, future claims $H_{i,t}$ are equal to net transfers $\Phi_{i,t}$ as in (4.8)-(4.11), while the first generation pays the transfer $\Phi_{0,1} = H_{0,1} - \psi_{m,1}^r k_{m,1}^{p,r} \leq 0$. However, deviating from the notation of Chapter 2 and Section 4.3 of this chapter, we assume that the environmental resource was previously treated as a free good, so that the first generation had no initial claim on this resource, and its value can be distributed without the need to specify this distribution as a transfer from the first generation to future generations.

We assume that endowments converge to stationary values, and this stylized economy satisfies the standard Assumption *O1*. Utility is strictly increasing in the resource amenity, and therefore the resource satisfies Assumption *F7*, and is thus non-negligible.

Finally, for every period t, there is a commodity balance for the resource stock, the extracted resource, labor, and the consumer good:

$$k_{m,t}^{r} \leq k_{m,t}^{p,r} \perp \psi_{m,t}^{r} \geq 0, \tag{4.22}$$

$$y_{n,t}^{r-} \leq y_{m,t}^{r} \perp p_{t}^{r} \geq 0, \tag{4.23}$$

$$y_{n,t}^{l-} \leq \omega_{t,t}^{l} + \omega_{t,t}^{l} \perp p_{t}^{l} \geq 0, \tag{4.24}$$

$$x_{t-1,t}^{c} + x_{t,t}^{c} \leq y_{n,t}^{c} \perp p_{t}^{c} \geq 0. \tag{4.25}$$

Nonrivalness of the demand for the resource amenity is expressed through a commodity balances for all living generations in every period:

$$x_{i,t}^{nr,b} \leq y_{m,t}^{b} \perp \varphi_{i,t}^{b} \geq 0, \tag{4.26}$$

for t=i,i+1, which indicates that contemporary consumers should agree about the amenity level. Associated to these balances are Lindahl prices which should add up to the production price:

$$p_{t}^{b} = \varphi_{t-1,t}^{b} + \varphi_{t,t}^{b}. \tag{4.27}$$

This completes the description of the model. For convenience, we repeat the definition of the equilibrium of Chapter 3 (Definition 3.6):

DEFINITION 4.7. *A competitive equilibrium of model* (4.45)-(4.58) *is an allocation supported by prices* $p_t^c, p_t^l, p_t^e, p_t^b, \varphi_{i,t}^b, \psi_t$, *for* $p_t^c, p_t^l, p_t^e, p_t^b$ *normalized on the simplex, in which production maximizes profits subject to the technology constraints* (4.45)-(4.50*), consumption maximizes utility subject to lifetime budget constraints* (4.51), *a transfer mechanism distributes the value of the environmental resource over the generations* (4.52), *markets clear as in* (4.53)-(4.57), *and Lindahl prices satisfy* (4.58).

As was briefly noticed above, unlike the formulation in Chapter 3, we have not included the increasing life-expectancy ('aging') in the present model. Results in this section are based on calculations with ALICE 1.0 in which all generations live 2 periods of 20 years. This will simplify the steady state analysis in the next section. Moreover, we have seen in Section 3.3 that the aging of the population increased the life-cycle savings, decreased the interest rate, and in the numerical application lead to an increased optimal reduction of cumulative greenhouse gas (GHG) emissions, compared to the scenario without aging (Table 3.2). However, the results from IAMs one might compare our numerical findings with do not show substantial decreasing interest rates over time. If we include the trust fund within an economy with aging, and compare the resulting optimal emission reductions with the results from other IAMs, we have to distinguish the emission reductions resulting from aging and the emission reductions resulting from the trust fund. This makes the results less transparent. We choose to keep the reference scenario as close as possible to the other IAMs, and assume life-expectancy to remain constant.

Furthermore, because utility is homogeneous of degree one, and because population is constant over the life-cycle, consumption resulting from utility maximization is independent of the population level, and we omit the population variable in the extended utility function for convenience.

We are now in a position to describe the environmental resource policies for this economy. In Section 3.4, we specified two different policies to manage the environmental resource and distribute its value. The first (strong sustainability) policy banned all resource extraction, and exempted all generations from paying for the amenity value. The second policy grandfathered the entire resource to the first generation, and enforced payments for all generations for both extraction and amenity values of the resource. Here, we specify a third policy, in which the environmental resource accrues to a trust fund.

We recall from Chapter 3 that the strong sustainability scenario constrains the use of the environmental resource by banning all extraction:

$$y^r_{m,t} = 0. \tag{4.28}$$

Consequently, the level of the resource amenity is maximal and stationary, and $x^{nr,b}_{t-1,t} = x^{nr,b}_{t,t} = y^b_{m,t} = 1$. Moreover, the scenario exempts all generations from paying for the non-rival consumption of the resource amenity. This can be represented through an income claim that is exactly equal to the value of non-rival consumption,

$$H_{i,t} = \varphi^b_{i,t}, \tag{4.29}$$

so that the budget equation becomes:

$$p^c_t x^c_{t,t} + p^c_{t+1} x^c_{t,t+1} = p^l_t \omega^l_{t,t} + p^l_{t+1} \omega^l_{t,t+1}. \tag{4.30}$$

The strong sustainability policy treats the environmental resource as an exogenous factor, and reduces the economy to a one-good exchange economy, comparable to ALICE 0. Unlike Chapter 3, here, the strong sustainability policy is not accompanied by the issue of a non-negligible claim. The economy is therefore equivalent to a pure exchange economy without stocks, with consumption equal to endowments.

$$x^c_{t,t} = \omega^l_{t,t},\ x^c_{t,t+1} = \omega^l_{t,t+1} \tag{4.31}$$

Price dynamics follow directly. Denoting the intertemporal price ratio by $\beta_t = p^c_{t+1} / p^c_t$, we find

$$\beta_t = u_2(\omega^l_{t,t}, \omega^l_{t,t+1}, 1, 1) / u_1(\omega^l_{t,t}, \omega^l_{t,t+1}, 1, 1). \tag{4.32}$$

where the subscripts for u(.) denote the second and first derivatives. These price dynamics fully characterize the equilibrium. Since labor is measured in output units and the resource amenity is exogenous to the economy, in equilibrium the price of 'labor' will be equal to the price of 'consumer goods' in every period. Thus, without loss of generality, we can take $p^c_1 = 1$ and derive all future prices from equation (4.32). Given this price path, generations maximize utility by consuming their own endowments (converted from 'labor' into the 'consumer good'). As a strong sustainability policy can be viewed as the imposition of an additional, linear restriction on the production technology, it does not pose specific problems for existence of equilibrium, Theorem 2.2 from Chapter 2 can be applied, but since it operates like a quota, the resulting equilibrium will in general be inefficient.

The second scenario grandfathers the resource to the first generation, so that

$$H_0 = \psi^r_{m,1} k^{p,r}_{m,1} \tag{4.33}$$

and

$$H_i = 0, \tag{4.34}$$

for i=1,..., ∞. There is no further intergenerational transfer. The environmental resource becomes a 'normal' capital good whose value is equal to life-cycle savings:

$$p^l_t \omega^l_{t,t} - p^c_t x^c_{t,t} - \varphi^b_{t,t} x^{nr,b}_{t,t} = \psi^r_{m,t+1} k^r_{m,t+1}. \tag{4.35}$$

Unlike the strong sustainability policy, the extracted resource can now be bought by the firms and all generations have to pay for their non-rival use of the resource amenity. The budget constraint for all generations after the first becomes:

$$\sum_{t=i,i+1} (p^c_t x^c_{i,t} + \varphi^b_{i,t} x^{nr,b}_{i,t}) = \sum_{t=i,i+1} p^l_t \omega^l_{i,t}. \tag{4.36}$$

As the economy satisfies the standard assumptions *O1* and *F7*, Theorem 3.2 from Chapter 3 applies; the equilibrium exists and is efficient.

Finally, we add the trust fund policy developed in this chapter. The trust fund entitles every generation to the same income claim as in the strong sustainability policy, i.e., to one unit of resource amenity:

$$H_{i,t} = \varphi^{b}_{i,t}, \tag{4.37}$$

similar to the strong sustainability policy. The trust fund is endowed with the initial value of the biogeochemical system

$$A_1 = \psi^{r}_{m,1}\,\bar{k}^{r}, \tag{4.38}$$

for which we have to show that it is sufficient to meet its commitments (4.37). In every period, the transfers paid are subtracted:

$$A_{t+1} = A_t - H_{t-1,t} - H_{t,t}, \tag{4.39}$$

which is the model specific equivalent of (4.12). In contrast to the strong sustainability policy, the income claims (4.37), which are based on one unit output of the resource amenity, are not identical to the actual output of the biogeochemical system (which allows for resource extraction), and we must show that these income claims sum to the initial value of the biogeochemical system (4.21). Because there is a trade off between resource extraction and the future stock, prices must satisfy:

$$\psi^{r}_{m,t+1} = p^{r}_{t}. \tag{4.40}$$

On the other hand, the marginal value of the current stock is equal to the marginal value of extraction plus the marginal value of $1/\bar{k}^{r}$ unit of the resource amenity, and prices must satisfy:

$$\psi^{r}_{m,t} = p^{r}_{t} + p^{b}_{t}/\bar{k}^{r}. \tag{4.41}$$

Subtracting the first equation from the second, and multiplying by $\bar{k}^{r}$ gives:

$$\psi^{r}_{m,t}\,\bar{k}^{r} = p^{b}_{t} + \psi^{r}_{m,t+1}\,\bar{k}^{r}. \tag{4.42}$$

Therefore, the trust fund can meet its commitments in every period if it holds assets of value $A_t = \psi^{r}_{m,t}\,\bar{k}^{r}$, starting from $A_1 = \psi^{r}_{m,1}\,\bar{k}^{r}$. This ensures that the distribution rule is consistent with the asset policy, thus (4.21) is satisfied, and it implies that the capital budget (4.35) becomes:

$$p^{l}_{t}\omega^{l}_{t,t} + \varphi^{b}_{t,t} - p^{c}_{t}x^{c}_{t,t} - \varphi^{b}_{t,t}x^{nr,b}_{t,t} = \psi^{r}_{m,t}(k^{r}_{m,t} - \bar{k}^{r}). \tag{4.43}$$

Notice that both policies satisfy the general capital budget (4.13) for this economy which is:

$$p_t^l \omega_{t,t}^l + H_{t,t} - p_t^c x_{t,t}^c - \varphi_{t,t}^b x_{t,t}^{nr,b} + A_t = \psi_{m,t}^r k_{m,t}^r , \tag{4.44}$$

where $H_{t,t}$=0 and A_t=0 (t=1,…,∞) for the grandfathering policy. It follows from (4.43) that private life-cycle savings are negative, and in particular that

$$p_t^l \omega_{t,t}^l - p_t^c x_{t,t}^c < 0 , \tag{4.45}$$

whenever $k_{m,t}^r < \bar{k}^r$, or, equivalently, whenever $y_{m,t}^b < 1$. The budget constraint now reads:

$$\sum_{t=i,i+1} p_t^c x_{i,t}^c = \sum_{t=i,i+1} (p_t^l \omega_{i,t}^l + \varphi_{i,t}^b (1 - x_{i,t}^{nr,b})) . \tag{4.46}$$

This shows that, although all generations have to pay for their non-rival use of the resource amenity, their real income claim exceeds the actual value of the resource amenity. To apply Theorem 4.3 and prove existence of the trust fund equilibrium, the first generation should own an arbitrary small fraction (0<ε<1) of the initial stock; the remainder can be attributed to the trust fund for redistribution among future generations. In our economy, this would formally be represented by $H_o = \varepsilon \psi_{m,1}^r \bar{k}^r + (1-\varepsilon)\varphi_1^0$ and $H_t = (1-\varepsilon)(\varphi_{t,t}^b + \varphi_{t,t+1}^b)$ for t=1,…,∞. However, we abstract from the fraction ε in the equations, since it can be arbitrarily small.

4.4.2. Steady state analysis

We recall the steady state equations of Section 3.4.2. Profit maximization for the environmental resource (treated as one firm) leads to the following four steady state conditions determining the stock and output levels k_m^r, y_m^b, y_m^r, and the stock price ψ_m^r, given prices p^r, and p^b:

$$k_m^r \le k_m^r - y_m^r \quad \perp \quad \beta \psi_m^r \ge 0 , \tag{4.47}$$

$$y_m^b \le k_m^r / \bar{k}^r \quad \perp \quad p^b \ge 0 , \tag{4.48}$$

$$p^p \le \beta \psi_m^r \quad \perp \quad y_m^r \ge 0 , \tag{4.49}$$

$$\psi_m^r \le \beta \psi_m^r + p^b / \bar{k}^r \quad \perp \quad k_m^r \ge 0 . \tag{4.50}$$

Profit maximization for the firm that produces the consumer good gives three conditions that determine the input and output levels for the consumer good, given the prices p^r, p^l, and p^c:

$$y_n^c \le y_n^{l-}\, g(y_n^{r-} / y_n^{l-}) \;\perp\; p^c \ge 0\,, \tag{4.51}$$
$$p^c(g(y_n^{p-} / y_n^{l-}) - (y_n^{r-} / y_n^{l-})g'(y_n^{r-} / y_n^{l-})) \le p^l \;\perp\; y_n^{l-} \ge 0\,, \tag{4.52}$$
$$p^c g'(y_n^{r-} / y_n^{l-}) \le p^r \;\perp\; y_n^{r-} \ge 0\,. \tag{4.53}$$

Utility maximization gives the price and its dynamics for the consumer good and the relative prices of the resource amenity.

$$\partial\, u(x^c, x^{nr,b}) / \partial x_i^c \le \beta^i p^c \;\perp\; x_i^c \ge 0 \qquad i\in\{0,1\} \tag{4.54}$$
$$\partial\, u(x^c, x^{nr,b}) / \partial x_i^{nr,b} \le \beta^i \varphi_i^b \;\perp\; x_i^{nr,b} \ge 0 \qquad i\in\{0,1\} \tag{4.55}$$

where we normalized prices by taking them equal to marginal utility. The commodity balances, and summing Lindahl prices to the production price yields:

$$x_0^c + x_1^c \le y_n^c \;\perp\; p^c \ge 0\,, \tag{4.56}$$
$$y_n^{l-} \le \omega_0^l + \omega_1^l \;\perp\; p^l \ge 0\,, \tag{4.57}$$
$$y_n^{p-} \le y_m^p \;\perp\; p^p \ge 0\,, \tag{4.58}$$
$$x_i^{nr,b} \le y_m^b \;\perp\; \varphi_i^b \ge 0\,, \qquad i\in\{0,1\} \tag{4.59}$$
$$\varphi_y^b + \varphi_o^b \le p^b \;\perp\; y_m^b \ge 0\,. \tag{4.60}$$

Finally, the steady state budget constraint is given by:

$$p^c x_0^c + \beta p^c x_1^c + \varphi_0^b x_0^{nr,b} + \beta\varphi_1^b x_1^{nr,b} = p^l \omega_0^l + \beta p^l \omega_1^l + H_0 + H_1. \tag{4.61}$$

We can now derive : p^c, p^l, p^r, p^b, φ_i^b, $x_i^c, x_i^{nr,b}$, $y_n^c, y_n^{l-}, y_n^{p-}, y_m^r, y_m^b$, k_m^r, ψ_m^r, and β from the equations (4.84)-(4.83), given the parameters $\omega_{i,t}^l$, where the real claims H_i are determined by the distributive policy. We recall from Theorem 3.4 that the steady states form a continuum in the one-dimensional state space for k_m^r. Finally, we notice that because resource extraction is zero in every steady state, the consumer good and labor have the same price (4.52), p^c=p^l, and the commodity balance for the consumer good (4.73) and (4.78) reduces to:

$$x_0^c + x_1^c \le \omega_0^l + \omega_1^l \tag{4.62}$$

The restriction of the life-cycle to two periods enables us to make some clarifying figures of the steady state equilibria. Under the strong sustainability scenario, the equilibrium is fully characterized by (4.32). Hence, the convergence of the dynamic trajectory to a given steady state follows directly from convergence of the parameters. The steady state selection is illustrated in Figure 4.1.

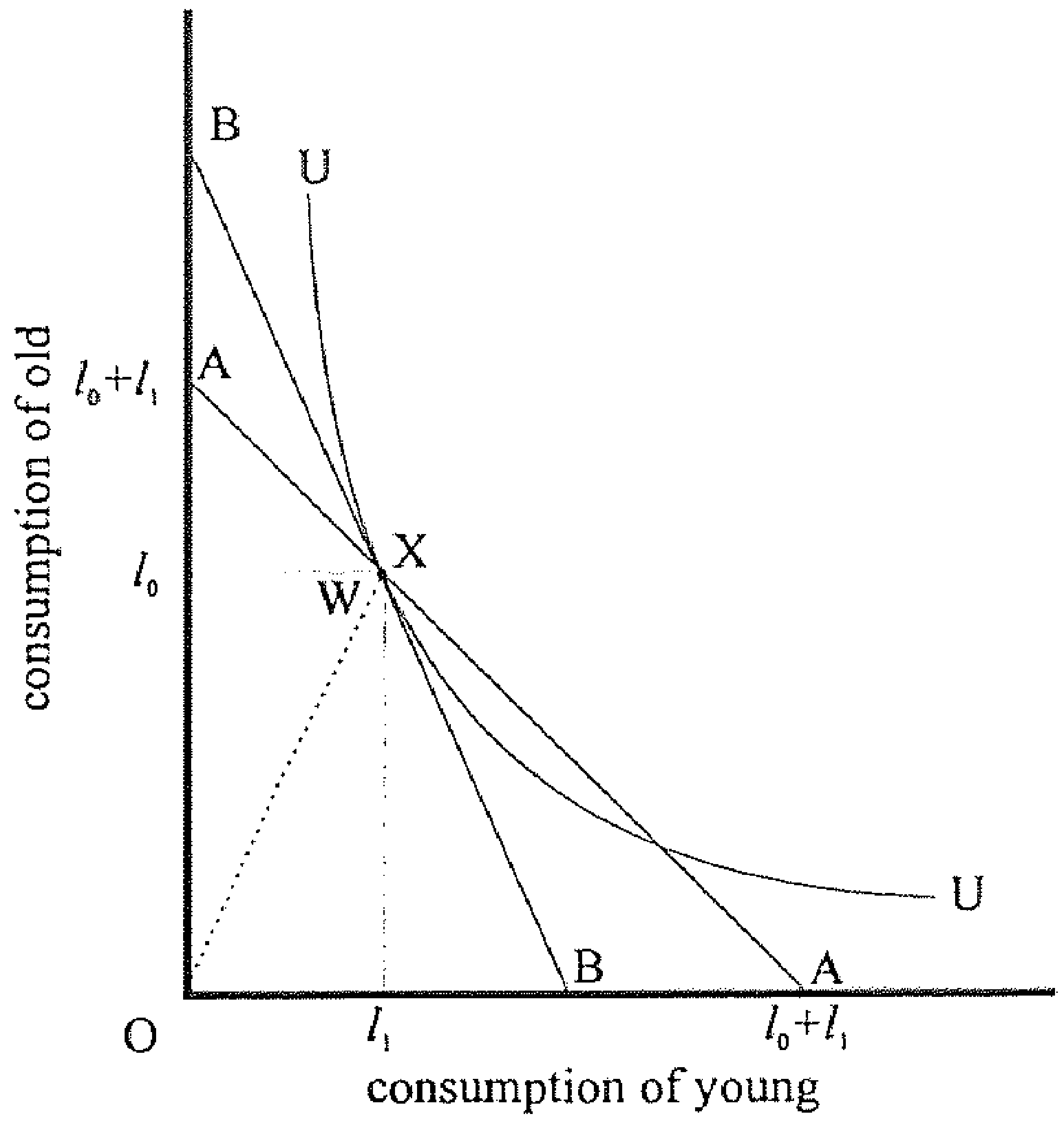

FIGURE 4.1. *Steady state selection for the strong sustainability scenario*

In this figure, UU denotes the iso-utility curve, AA the commodity balance, BB the budget constraint, l_0 and l_1 the labor endowments of the young and old respectively, W the endowments, X the equilibrium consumption. The slope of the budget constraint BB is equal to the price ratio β. The figure is the standard representation of a two-period, one-good OLG exchange steady state. This equilibrium is unique. Since it has β<1, the price of the consumer good is taken to decrease, but β≥1 is also possible. In that case the price would increase or remain constant, and though equilibrium exists, it will be dynamically inefficient. For convenience, we exclude this possibility by the following assumption:

ASSUMPTION 4.8. *For any amenity value that remains constant over a consumer's life-cycle, an autarkic consumer values consumption less when old than when young*:

$$u_2(\omega_0^l, \omega_1^l, b, b) \,/\, u_1(\omega_0^l, \omega_1^l, b, b) < 1, \textit{ for all } 0<b\leq 1. \tag{4.63}$$

where b denotes the constant amenity level: $b = k_n^r / \bar{k}^r = y_n^b = x_0^{nr,b} = x_1^{nr,b}$. The assumption is somehow 'technical', it states that the segment OB on the horizontal axis in Figure 4.1 is smaller than the segment OB on the vertical axis. Thereby, it ensures that β<1. Furthermore, to establish a property of the interest rate under the three scenarios, we use:

ASSUMPTION 4.9. *The utility function is a nested CES function with positive intertemporal elasticity of substitution and a Cobb Douglas branch for the substitution between the amenity value and the consumer good.*

The intertemporal elasticity of substitution is denoted by $\rho>0$. The assumption implies that generations pay a constant share $0<v<1$ of their income for the non-rival consumption of the resource amenity, and use the remaining share $1-v$ for rival consumption of the consumer good. The steady state selection for the grandfathering scenario is similar to the one for the strong sustainability scenario, Figure 4.1 becomes:

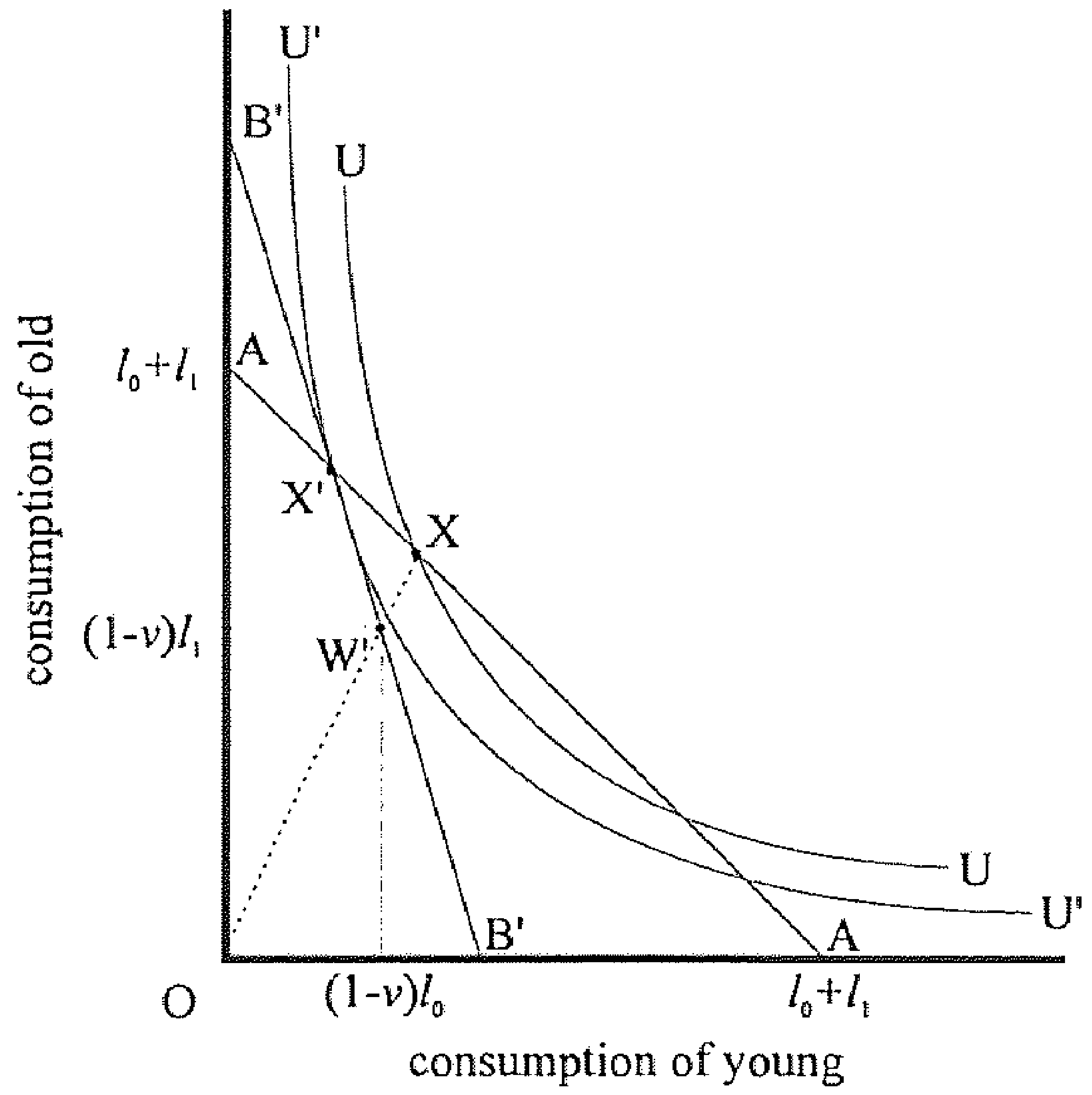

FIGURE 4.2. *Steady state selection for grandfathering scenario*

The new equilibrium values are denoted by primes. The income used for consumption of the consumer goods (measured in commodity units) drops from W to W′. The new budget constraint becomes B′B′, and the equilibrium consumption shifts to X′. The original iso-utility curve UU shifts to the lower iso-utility curve U′U′, i.e., utility derived from the consumption of the consumer goods decreases. Notice that since the economy with grandfathering has two marketed consumer goods, the uniqueness of the selected steady state is no longer guaranteed. Fortunately, there is only one strong sustainability equilibrium and any grandfathering steady state can be compared with this solution.

For welfare comparisons across the steady states selected, we measure welfare relative to its value in an autarkic economy where generations consume their own endowments in an environment with maximal supply of environmental functions:

$$w_t = \mathrm{u}(x_0^c, x_1^c, x_0^{nr,b}, x_1^{nr,b}) \,/\, \mathrm{u}(\omega_0^l, \omega_1^l, 1, 1)\,. \tag{4.64}$$

This value is unity for the strong sustainability equilibrium. Let us denote the steady state variables by superscripts '*SS*', and '*GF*' for the 'strong sustainability' and 'grandfathering' policy respectively. We can now state and prove:

THEOREM 4.4. *If Assumption* 4.8 *holds, grandfathering reduces welfare* ($w^{GF}<w^{SS}=1$), *and if Assumption* 4.9 *holds as well, grandfathering strictly increases the steady state interest rate relative to the strong sustainability scenario* ($\beta^{GF}<\beta^{SS}$).

Proof. Because of concavity of u(.), we have:

$$\begin{aligned}&\mathrm{u}(x_y^c, x_o^c, x_y^{nr,b}, x_o^{nr,b}) \le \mathrm{u}(\omega_y^l, \omega_o^l, x_y^{nr,b}, x_o^{nr,b}) \\ &+ (x_y^c - \omega_y^l)\,\mathrm{u}_1(\omega_y^l, \omega_o^l, x_y^{nr,b}, x_o^{nr,b}) + (x_o^c - \omega_o^l)\,\mathrm{u}_2(\omega_y^l, \omega_o^l, x_y^{nr,b}, x_o^{nr,b})\end{aligned} \tag{4.65}$$

It follows from equation (4.35) that $x_0^c < \omega_0^l$, and because the same commodity balance (4.62) applies, Assumption 4.8 gives:

$$\mathrm{u}(x_y^c, x_o^c, x_y^{nr,b}, x_o^{nr,b}) \le \mathrm{u}(\omega_y^l, \omega_o^l, x_y^{nr,b}, x_o^{nr,b}) \le \mathrm{u}(\omega_y^l, \omega_o^l, 1, 1)\,, \tag{4.66}$$

as the amenity level in which utility is increasing cannot exceed unity. This completes the first part of the proof.

To show that the interest rate increases, note that consumption of the consumer good by the young is decreased by the grandfathering policy. Again, we use the fact that the same commodity balance (4.62) applies for both policies, which implies that grandfathering reduces the consumption when young relative to the consumption when old. Now by Assumption 4.9, the price ratio β is increasing, $\beta^{GF}<\beta^{SS}$ and B'B' in Figure 4.2 is steeper than BB in Figure 4.1. ■

The third policy, the trust fund, improves welfare for all generations:

THEOREM 4.5. *If resource extraction is positive in period* 1, *the trust fund policy increases welfare for all generations as compared to the strong sustainability policy:* $w_t>1$ *for all* t.

Proof. If emissions occur in period 1, we have $x_{t,t}^{nr,b} < 1$, and therefore $(\omega_{t,t}^l, \omega_{t,t+1}^l, 1, 1)$ lies in the strict interior of the budget space (4.46) and relative welfare exceeds unity: $w_t>1$. ■

The proposition does not depend on Assumption 4.8 and 4.9. Next, to analyze the steady state, Assumption 4.9 is needed. For convenience, let the steady state resource level be denoted by b. The Lindahl prices satisfy:

$$\varphi_0^b b = \frac{\nu}{1-\nu} c_0, \tag{4.67}$$

and

$$\varphi_1^b b = \frac{\nu}{1-\nu} c_1. \tag{4.68}$$

The steady state budget equation (4.83) now becomes:

$$x_0^c + \beta x_1^c = \omega_0^l + \beta\omega_1^l + \frac{\nu}{1-\nu}(b^{-1}-1)x_0^c + \beta\frac{\nu}{1-\nu}(b^{-1}-1)x_1^c, \tag{4.69}$$

which reduces to

$$x_0^c + \beta x_1^c = \eta(\omega_0^l + \beta\omega_1^l) \tag{4.70}$$

where $\eta = (1 - \frac{\nu}{1-\nu}(b^{-1}-1))^{-1} > 1$. Thus, Figure 4.1 is adjusted to

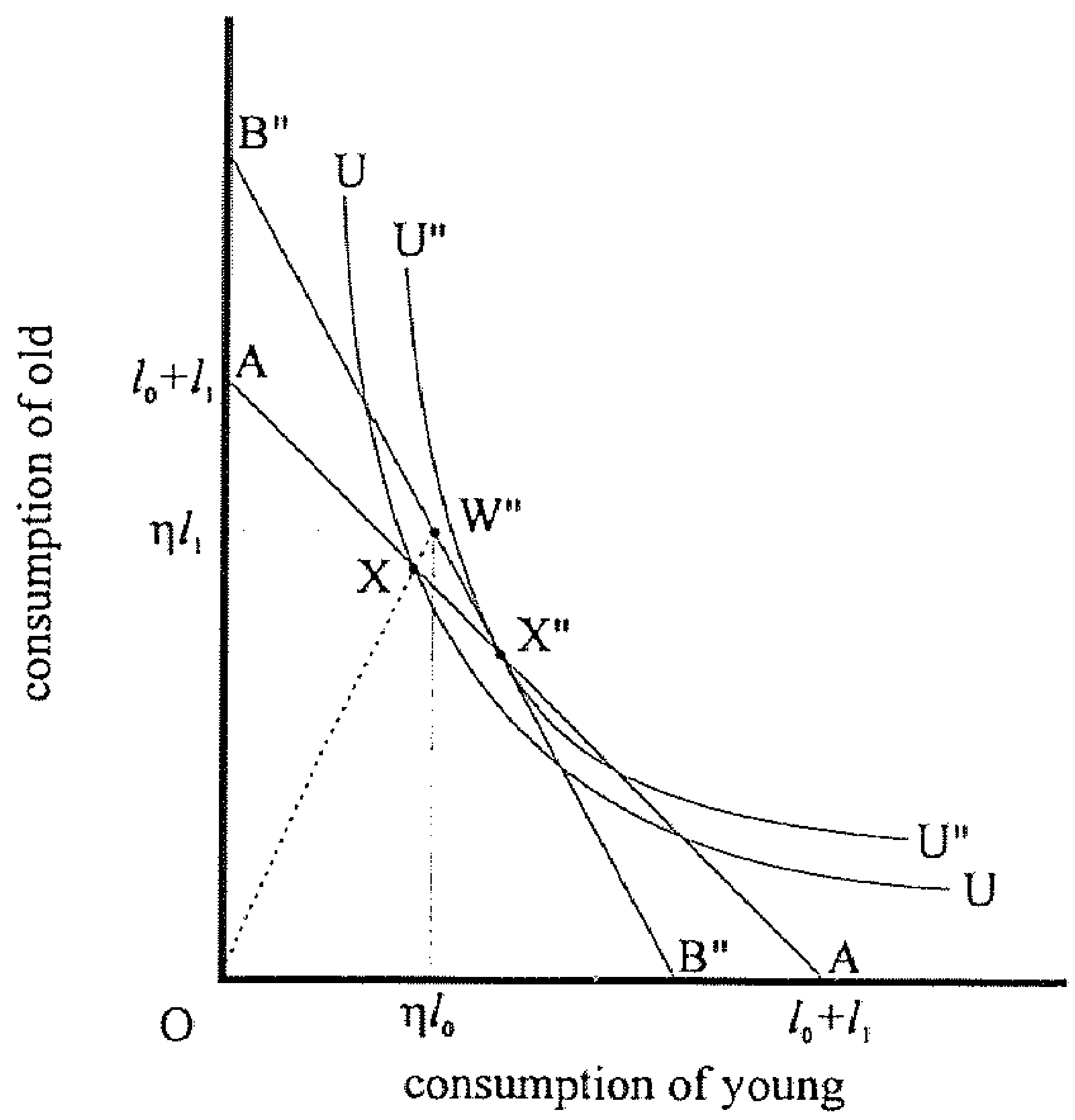

FIGURE 4.3. *Steady state selection for trust fund policy*

In contrast to Figure 4.2 for the grandfathering scenario, we now find that the income used for the consumer good increases from W to W″, the budget constraint becomes B″B″, and steady state consumption has moved to X″ on a higher iso-utility curve U″U″. Notice that η depends on the amenity level, which implies that the steady state consumption is determined in conjunction with the stock and amenity level. We can state and prove:

THEOREM 4.6. *If resource extraction is positive in period* 1, *the trust fund policy increases welfare in the steady state relative to the strong sustainability scenario* ($w^{TF}>1$), *and if Assumption* 4.9 *holds, the trust fund strictly reduces the interest rate* ($\beta^{TF}>\beta^{SS}$).

Proof. The welfare increase in the steady state follows directly from the welfare increase for all generations, as shown in the previous theorem. Life-cycle savings are negative by equation (4.45). Therefore, $x_y^c > \omega_y^l$ in the steady state selected. By the same argument as for the grandfathering steady state, $\beta^{TF}>\beta^{SS}$, even if the steady state is non-unique. ■

In short, grandfathering improves efficiency compared to the strong sustainability policy but it also favors the first generation that receives the entire environmental resource, leaving future generations with a welfare reduction, because they have to pay for a degenerated environment. The same result was found in Chapter 3, considering the 3 period OLG model. Now, we have an alternative for grandfathering: the trust fund restores efficiency without favoring any particular generation.

4.4.3. Numerical application to climate change

We continue the analysis with a numerical application to climate change, as in Section 3.4.3, and add the trust fund policy to the numerical analysis. In the general analysis, the trust fund was developed to prevent unsustainable use of environmental resources. Unfortunately, the IAMs on which the model is calibrated (see e.g., CETA [Peck and Teisberg 1992], DICE [Nordhaus 1994], and MERGE [Manne *et al.* 1995]) do not allow for unsustainability. In these models, all damages caused by climate change can be compensated by economic growth, as they are converted in monetary units and assumed to stay well below 5 per cent of income [IPCC 1996, Table 6.4], whereas 100 years of economic growth leads to a tenfold income increase. Thus, even after subtracting the monetary damage from total income, future welfare always exceeds present welfare, which means that all concerns about endangered welfare of future generations are unwarranted. We accept these benchmark values nonetheless, not because we adhere to this optimistic view, but because we want to keep two areas of debate well separated, with on the one hand the correct estimation of present costs and future benefits of climate change regulations, and on the other hand the proper rules for policy making on which the regulations should be based. Our calculations indicate that, even if sustainability is not a vital issue, proper rules can still facilitate reduction of GHG emissions. The next two sections extent the model with alternative assumptions with respect to environmental damage and substitution with man-made capital for the loss of environmental capital.

The outcomes from the scenario simulations are summarized in Table 4.2 below, in terms of three pairs of variables related to emissions, interest rates and welfare, respectively. The economy has converged to its steady state after 20 periods of 20 years, and we identify the steady state with the periods after 2400.

TABLE 4.2. *Scenario results for ALICE 1.0*[1]

	Unit	Strong sustainability	Grand-fathering	Trust Fund
Price of emission units in first period	US$/tC	400.	9.	50.
Cumulative Emissions	GtC[2]	0.	2000.	500.
Real interest rate (2020) [3,4]		0.056	0.086	0.051
Real interest rate after 2400 [3]		0.028	0.055	0.014
Welfare of first generation		1.00	1.23	1.04
Welfare of generations born after 2400		1.00	0.88	1.01

1. Without aging
2. Gigaton Carbon in CO_2 equivalents
3. Price ratio between two periods minus unity, on an annual basis
4. Between the first two periods

The first indicator is the price of emission units, and is equivalent to the CO_2 tax in other models. The table only shows the emission price for the first period (2000-2020) because this is the measure of direct relevance to the present day economy. Recall that the reduction of emissions is assumed to be linear in price: a 1 per cent point reduction per 4 US$/tC. Thus, the emission price in the strong sustainability scenario (400 US$/tC) reduces emissions by 100 per cent. The emission price of 9 US$/tC for the grandfathering scenario reduces emissions by 2 per cent, whereas the 50 US$/tC in the trust-fund scenario reduces emissions by 12 per cent. The activity of the trust fund increases the current optimal reduction of GHG emissions by factor six.

The second indicator is the cumulative amount of emissions over the whole simulation period and is meant as a measure of the cumulative environmental degradation. Optimal cumulative emissions in the grandfathering scenario sum to 2000 GtC, which implies that no substantial emission reduction is undertaken before the year 2100. However, the trust fund cuts emissions to 500 GtC, which requires substantial emission reductions, as it is considerably below the 1500 GtC cumulative emissions reached in 2100 according to the IS92a scenario [IPCC 1993].

The third and fourth indicator are 'real' interest rates, defined as the price ratio for consumer goods between two periods minus unity. Their value is expressed on an annual basis. For the grandfathering scenario, the interest rates are higher than in the other two scenarios, because there is an increased demand for savings to balance the value of the capitalized biogeochemical system. The long term interest rates are in accordance with Theorem 4.4 and 4.6: $\beta^{GF}<\beta^{SS}<\beta^{TF}$. These outcomes relate to the lively discussion on the proper use of discount rates in the current debate on policies to address climate change, cf. [IPCC 1996, Chapter 4]. Our results suggest that a sustainable and efficient policy requires a flexible discount rate that decreases substantially in the long term. This puts to question the dynastic Ramsey-type formulation in most IAMs with exogenous and constant discount rates.

The last two rows give the welfare measure (relative utility compared to the strong sustainability scenario) for the first generation and for the steady state. The trust fund increases welfare for all generations, $w_t^{TF} > 1$, confirming Theorem 4.5. Grandfathering increases welfare for the first generation, $w_1^{GF} > 1$, and decreases welfare of the future generations, $w^{GF}<1$, confirming Theorem 4.4. The welfare effects of grandfathering are impressive. The generation that capitalizes the environment receives a vast gift, while future generations have to pay the bill. Compared to these sizable effects, the introduction of the trust fund seems to have only minor impacts on the economy. Nonetheless, two comments are in order.

First, the calculations are restricted to climate change and CO_2 emissions. If other aspects of environmental degradation were included, such as acidification, soil erosion, losses of natural areas such as rain forests and temperate forests, both the costs of prevention and adaptation would obviously increase significantly.

Secondly, recall that the model shares the optimistic view of most economic IAMs that, because of substitution possibilities, emission reductions are easy and damages from climate change are minor. Consequently, the benefits from improvement in efficiency will be modest. Yet, these benefits are impressive in nominal terms. A four per cent increase in GDP amounts to more than a trillion US$ annually, and yields an enormous capitalized value.

4.5. AN EXHAUSTIBLE RESOURCE WITH CAUSE-EFFECT DELAY

4.5.1. Model specification and steady state analysis

We will now introduce some extensions to the model of the previous section, in order to check whether its results remain valid in a more general setting. In model (4.45)-(4.58), it was assumed that resource extraction has immediate impacts on the amenity value of the resource. However, ecosystems need adjustment time to accommodate to new conditions, and if the preconditions are destroyed, they only slowly disappear. We model this adjustment process by means of an auxiliary stock, which represents the amenity level,

$$y^b_{m,t+1} \le k^b_{m,t}, \tag{4.71}$$

and which adjusts at a fixed rate ξ in the direction of some long term equilibrium level, which, as in the previous model (4.16), is set equal to the relative resource stock, $k^r_{m,t}/\bar{k}^r$:

$$k^b_{m,t+1} \le (1-\xi)k^b_{m,t} + \xi(k^r_{m,t}/\bar{k}^r). \tag{4.72}$$

Consequently, the recursive zero profit condition (4.17) changes into:

$$\psi^r_{m,t}k^r_{m,t} + \psi^b_{m,t}k^b_{m,t} = p^r_t y^r_{m,t} + p^b_t y^b_{m,t} + \psi^r_{m,t+1}k^r_{m,t+1} + \psi^b_{m,t+1}k^b_{m,t+1}. \tag{4.73}$$

We now turn to the steady state analysis of this adjusted economy.

Under the strong sustainability policy, the equilibrium remains exactly the same as in the previous economy (4.45)-(4.58), because the environmental resource produces the same amount of amenity value. Under grandfathering however, the delay between cause and effect can have far reaching consequences. In the steady state, it is possible that the resource stock is strictly positive, $k^r_m > 0$, and consequently, the amenity level is constant and equal to the relative resource stock:

$$k^b_m \le k^r_{m,t}/\bar{k}^r \quad \perp \quad \psi^b_m \ge 0. \tag{4.74}$$

In this case, the analysis exactly matches the analysis of the steady state without cause-effect delay.

Another possibility is, however, a zero resource stock, and a continuously declining amenity level. Let Assumption 4.8 and Assumption 4.9 be satisfied; the latter implies that marginal utility becomes unbounded for the amenity value converging to zero, and in the previous model, this property guaranteed a strictly positive resource stock, $k^r_m > 0$. Now that the complete exhaustion of the resource stock, $k^r_{m,t} = 0$, does not lead to an immediate drop in the amenity value to zero, we cannot rule out the possibility of a zero resource stock in the steady state, $k^r_m = 0$, with a steadily declining amenity level k^b_m and y^b_m by a factor (1–ξ) in every period. In this case, eventually, the amenity level converges to zero and welfare converges to zero as well. We might have expected that treating the resource stock as valuable capital could prevent such a doom scenario, but the analysis does not suggest any check on resource depletion.

The result is similar to the findings of Mourmouras [1993] who showed that the resource stock can drop to zero if the resource is essential for production. Our analysis indicates that the same can happen for a resource whose extraction is not essential for production and is characterized by some delay between the extraction and the fall of the amenity it provides.

The remaining properties of the steady state are unaffected; Theorem 4.4 still applies after adjusting Assumption 4.8 to deal with an amenity level that drops at an exponential rate:

ASSUMPTION 4.10. *For any amenity value that decreases with rate* $(1-\xi)$ *over a consumer's life-cycle, an autarkic consumer values consumption less when old than when young*:

$$u_2(\omega_0^l, \omega_1^l, b, (1-\xi)b) \,/\, u_1(\omega_0^l, \omega_1^l, b, (1-\xi)b) < 1, \textit{for all } 0 < b \leq 1. \tag{4.75}$$

Under the trust fund policy, delayed effects do not have much consequences. Theorem 4.5 and Theorem 4.6 still apply, which implies that welfare increases for all generations because of the trust fund.

Thus, we arrive at the same conclusion for this economy as for the previous one: grandfathering increases welfare of the first generations at the expense of future generations, while the trust fund improves welfare of all.

4.5.2. Numerical application to climate change

We repeat the calculations of Section 4.4.3 for the adjusted economy. The threshold level of cumulative emissions beyond which the environment collapses decreases from 5000 GtC to 1500 GtC, and in each period of twenty years, the amenity level adjusts to its long term level by a factor ξ=0.1, so as to maintain consistency with the IS92a emission scenarios and associated climate change damages (3 per cent in 2100) used in the reference IAMs.

The adjusted model is named ALICE 1.1, and its results are presented in Table 4.3.

TABLE 4.3. *Scenario results for ALICE 1.1*[1]

	Unit	Strong sustainability	Grand-fathering	Trust Fund
Price of emission units in first period	US$/tC	400.	4.	140.
Cumulative Emissions	GtC[2]	0.	1500.	170.
Real interest rate (2020) [3,4]		0.056	0.086	0.046
Real interest rate after 2400 [3]		0.028	0.055	0.014
Welfare of first generation		1.00	1.22	1.02
Welfare of generations in the long run		1.00	0.00	1.01

1. Without aging
2. Gigaton Carbon in CO_2 equivalents
3. Price ratio between two periods minus unity, on an annual basis
4. Between the first two periods

If we compare these outcomes with those of Table 4.2, we find that, under a strong sustainability policy, the competitive equilibrium remains unchanged; the delayed climate change damage does not occur if there are no emissions. However, the delayed damages affect the competitive equilibria for the other two policies.

Under grandfathering, the emission price in the first period decreases from 9 US$/tC to 4 US$/tC. Because of discounting, the increased damage per unit of emissions is outweighed by the delay of the damages, which causes a decrease of the net present value of these damages. All available emission units (1500 GtC) are used eventually. Yet because of the delay, during the first centuries increased consumption will compensate for the decrease of the environmental functions and welfare will increase. Once, the environment has deteriorated completely, there comes a period during which increased consumption cannot compensate for the collapse of the environmental functions, and welfare decreases to zero irreversibly. This result of grandfathering is a good illustration of the fears of ecological economists that economic models support efficient resource use but sacrifice the future for the pleasures of present day.

If the trust fund is implemented, the price of emissions permits increases from 50 US$/tC to 140 US$/tC, which conforms with the cumulative emissions that reduce by nearly 70 per cent, from 500 GtC to 170 GtC. Since, this reduction of cumulative emissions implies that the long term amenity level under the trust fund policy is about the same in both models (the initial resource stock also differs by 70 per cent), the long term welfare level and interest rate are also the same. The trust fund is able to transfer the required price signals from the future to the present day, to reach the sustainable allocation, and the short term allocation is adjusted to ensure sustainability in the long term.

To summarize, the trust fund establishes efficient use of the environmental resource, and attains sustainability at the same time. If sustainability requires a stringent reduction of emissions, the trust fund directs the competitive equilibrium to an allocation with substantial reductions independently of the short term costs and the time between cause and effect. Grandfathering, on the other hand, tends to sacrifice the future benefits of environmental regulations in favor of short term interests, and might not be able to prevent the eventual break down of the environmental system.

4.6. An exhaustible and a renewable resource

4.6.1. Model specification and steady state analysis

The previous section analyzed the implications of a pessimistic perspective in which resource extraction leads to a delayed decrease of amenity values that are easily neglected by current generations. Here, we analyze the opposite optimistic perspective in which a reduced amenity level can be compensated by an increased stock of a renewable (man-made) resource. We abstract from the cause-effect delay of model (4.71)-(4.73) and return to the original model (4.45)-(4.58).

Let us assume that the second firm, denoted by j='n', not only uses the extracted resource and 'labor' for the production of the consumer good, but also a capital stock which is itself produced by the same sector. For convenience, we assume that the capital stock is made up of the consumer good, and that it has to be replaced after in use in one period. The production structure can thus be represented by

$$y_{n,t}^{c} + k_{n,t+1}^{p,c} = \mathrm{h}_t(k_{n,t}^{c}, y_{n,t}^{r-}, y_{n,r}^{l-}), \tag{4.76}$$

where the superscript minus signs denote inputs, $\mathrm{h}_t(.)$ is a continuous, differentiable, and concave production function, linearly homogeneous, strictly increasing in the first and third argument, and with bounded derivative for the second argument, $0<\mathrm{h}_{t,2}(.,0,.)<\infty$, which implies that, as in the original economy, the extracted resource is valuable, but not essential for production. The subscript t is maintained to allow for technological innovation. The stock satisfies the commodity balance:

$$k_{n,t}^{c} \leq k_{n,t}^{p,c} \perp \psi_{n,t}^{c} \geq 0. \tag{4.77}$$

Profit maximization can, in view of constant returns to scale, be represented through the zero-profit condition:

$$p_t^c y_{n,t}^c + \psi_{n,t+1}^c k_{n,t+1}^{p,c} = p_t^l y_{n,t}^{l-} + p_t^r y_{n,t}^{r-} + \psi_{n,t}^c k_{n,t}^c . \tag{4.78}$$

This completes the modification of producer behavior. The policies are as for the original economy. The main difference is that the value of capital that balances with life-cycle savings and the trust fund includes the man-made capital, and the general capital budget (4.44) becomes

$$p_t^l \omega_{t,t}^l + H_{t,t} - p_t^c x_{t,t}^c - \varphi_{t,t}^b x_{t,t}^{nr,b} + A_{t+1} = \psi_{m,t+1}^r k_{m,t+1}^r + \psi_{n,t+1}^c k_{n,t+1}^c . \tag{4.79}$$

The steady state equations (4.84)-(4.83) apply to all agents in this economy except the producer of the consumer good, whose equations (4.73)-(4.53) are adjusted to

$$y_n^c + k_n^{p,c} \le \mathrm{h}(k_n^c, y_n^{l-}, y_n^{l-}) \perp p^c \ge 0 , \tag{4.80}$$
$$k_n^{p,c} \le k_n^c \perp \psi_n^c \ge 0 , \tag{4.81}$$
$$\psi_n^c \le \beta^{-1} p^c \perp k_n^{p,c} \ge 0 , \tag{4.82}$$
$$p^c \mathrm{h}_1(k_n^c, y_n^{r-}, y_n^{l-}) \le \psi_n^c \perp k_n^c \ge 0 , \tag{4.83}$$
$$p^c \mathrm{h}_2(k_n^c, y_n^{r-}, y_n^{l-}) \le p^r \perp y_n^{r-} \ge 0 , \tag{4.84}$$
$$p^c \mathrm{h}_3(k_n^c, y_n^{r-}, y_n^{l-}) \le p^l \perp y_n^{l-} \ge 0 . \tag{4.85}$$

Consequently, the price for the consumer good is not necessarily equal to the price for labor, commodity balance (4.62) is replaced by

$$x_y^c + x_o^c \le \mathrm{h}(k_n^c, 0, \omega_y^l + \omega_o^l) - k_n^c \perp p^c \ge 0 \tag{4.86}$$

The welfare properties of the original economy, Theorem 4.4, 4.5 and 4.6, which state that grandfathering decreases welfare while the trust fund increases welfare do not apply to this economy for two reasons. First, the reference strong sustainability policy does not result in a simple 'autarkic' exchange economy. Therefore, the critical utility level that serves as a lower bound for the trust fund scenario does not coincide with the strong sustainability scenario, which implies that application of Theorem 4.3 does not straightforwardly result in Theorem 4.5 and 4.6. There might exist multiple steady states, and for comparison of two scenarios, one has to compare all mutual pairs, as noted earlier in connection with Theorem 4.4. Secondly, if there is man-made capital, this leads to an ambiguous relation between price dynamics, life-cycle savings and the value of the capital stock. Recall that comparing grandfathering and the trust fund, the latter can be understood as a transfer from the first generation to the future generations. We therefore expect a welfare increase for future generations. In general, however, the transfer paradox cannot be excluded, which states that these transfers decrease welfare of future generations. See Modigliani

[1961] and Diamond [1965] for classic discussions on the transfer paradox in OLG economies. Donsimoni and Polemarchakis [1994] show for a general economy that the transfer paradox is not an irregular phenomenon. Therefore, we cannot prove that there will be welfare gains and we restrict the comparative analysis to the numerical application.

4.6.2. Numerical application to climate change

The model with man-made capital represents a more optimistic perspective than the original single resource model, because the loss of the environmental resource can (partially) be compensated by an increase of man-made capital. Indeed, this is confirmed by the numerical results discussed below.

The parametric form of the production function $h_t(.)$ is shown in equations (4.87)-(4.89) of Annex 4A. Man-made capital and labor are combined in a Cobb Douglas production structure, similar to other IAMs such as DICE [Nordhaus 1994]. The parameters were calibrated so that (i) the value of man-made capital to be balanced by life-cycle savings is approximately three times the annual GDP, and, (ii) as in the previous model, the maximal production and associated emissions follow the IS92a scenario [IPCC 1992], and (iii) emissions decrease by one per cent point for every 4 US$/tC increase in the price of emission units. The model is named ALICE 1.2, and the scenario results are listed in the table below.

TABLE 4.4. *Scenario results for ALICE 1.2*[1]

	Unit	Strong sustainability	Grand-fathering	Trust Fund
Price of emission units in first period	US$/tC	400.	7.	12.
Cumulative Emissions	GtC[2]	0.	2150.	1400.
Real interest rate (2020) [3,4]		0.080	0.094	0.079
Real interest rate after 2400 [3]		0.047	0.065	0.023
Welfare of first generation		1.00	1.18	1.05
Welfare of generations born after 2400		1.00	0.83	1.08
Consumption level after 2400		1.00	0.93	1.07

1. Without aging
2. Gigaton Carbon in CO_2 equivalents
3. Price ratio between two periods minus unity, on an annual basis
4. Between the first two periods

If we compare the numerical outcomes in Table 4.4 with the initial model (Table 4.2), we find that the welfare properties of the different policies as stated in Theorem 4.4, 4.5 and 4.6 still hold in numerical terms.

However, there is a remarkable difference in the effects of the trust fund policy compared to the original economy. In this economy, the trust fund policy cannot be reduced to an environmental policy, because it affects the economy in a broader way. While the emission price and cumulative emissions for the grandfathering and the trust fund policy have become relatively similar, the long term welfare effects of the trust fund versus grandfathering have not diminished, but increased from 1.01 vs. 0.88 in Table 4.2 to 1.08 vs. 0.83 in Table 4.4. In short, the trust fund has not so much prevented environmental degradation, but has compensated this degradation by an increase in man-made capital. The last row entry shows that the consumption of the consumer good has increased with 14 per cent if we compare the trust fund with grandfathering.

Summarizing the findings of this section, we cannot claim in general that the trust fund always improves welfare of future generations as compared to a strong sustainability or a grandfathering policy. However, if we use common functional forms that correspond to other IAMs, we find that grandfathering increases the well-being of the present at the cost of future generations, while the trust fund increases well-being of all generations. However, the emission reductions achieved by the trust fund are ambiguous, and it might happen that environmental degradation is compensated by an increase in man-made capital.

4.7. CONCLUSION

It is the task of environmental policy to stimulate the efficient as well as the sustainable use of environmental resources. Both objectives have to be treated separately because efficient use of the resources does not guarantee sustainability, and strict (physical) sustainability criteria might be effective but inefficient, causing unnecessary welfare loss. Indeed, the illustrations for climate change in Sections 4.4, 4.5 and 4.6 showed that the welfare effects of efficient but unsustainable and sustainable but inefficient policies can be substantial. In the worst case, grandfathering can lead to an efficient allocation in which the environment deteriorates so much that future generations are being sacrificed to present interests (Section 4.5).

We have first analyzed efficiency and sustainability in the context of the general model developed in Chapter 2, for both dynastic and OLG economies. We used the WCED [1987, Ch.2] definition of sustainability, which states that every generation has to be capable of meeting its own needs, without constraining other generations in meeting theirs.

Within the dynastic approach, it is straightforward and sufficient to include an explicit sustainability criterion in the welfare program (Theorem 4.1). This is preferable to an adjustment of the so called discount rate in the welfare function, which is rather arbitrary, because it is impossible to predict the level of the discount rate that leads to a sustainable solution. Within a dynastic economy, sustainability might also be guaranteed through explicit physical boundary conditions, but if these boundaries are not related to welfare, they impose distortionary constraints and (if binding) lead to an inefficient allocation.

As one is shifting the perspective from the representative agent to a competitive equilibrium in which every generation has its own budget, it is no longer sufficient to include a sustainability condition in the welfare program, since it has to be indicated how every generation can receive a sufficient income to meet its basic needs. Property rights are critical in this respect and can make the difference between a sustainable and an unsustainable economy (Theorem 4.2 and Example 4.6). In case the prevailing distribution of property rights fails to guarantee sustainability, transfers are required, and there is a need for an institution that mobilizes these transfers. For this purpose, we proposed a trust fund as an alternative to grandfathering. It receives the property rights over the environmental resource, and uses the income to give all present and future generations a claim to services from the environmental resources. Together with the income future generations receive from their (flow) endowments, the claim guarantees a minimal income level that suffices to meet the basic needs (Theorem 4.3). By the same token, it creates the foundation for a social security system.

The trust fund imposes no direct constraint on the use of the environmental resource, and one might doubt whether it can actually prevent environmental deterioration. The answer given in our analyses is that it indeed ensures that the competitive prices make sustainable use of the environmental resources profitable. Intuitively, this is because the trust fund reverses the relation between the present and future generations with respect to the environmental resource. Under grandfathering, the present generation will only leave a clean environment to future generations if they pay for it. With the trust fund, present polluters have to pay because the resource they use is not their property, and the pollution is only accepted if the payments are sufficient to compensate future generations for the damage caused.

After the general analysis, we applied the theory to a more specific economy, and subsequently introduced two modifications. First, in Section 4.4, it was shown that the trust fund substantially improves the environmental quality and welfare of future generations, even if sustainability itself is not at stake (Theorem 4.4, 4.5 and 4.6). Secondly, in Section 4.5, it was shown that the effectiveness of the trust fund is independent of assumed time delays between cause and effect, in contrast to grandfathering, which appears to sacrifice the welfare of future generations. The trust fund generates the necessary price signals in early periods to ensure sustainability. Finally, in Section 4.6, the flexibility of the trust fund was highlighted when it was shown that the trust fund does not lead to a rigid cut in environmental resource use, but allows to produce more man-made capital to compensate for the loss of an exhaustible resource (Table 4.4).

The single exhaustible resource economy and its modifications were applied to the issue of climate change, in a model that treats this phenomenon comparable with other IAMs. The numerical findings made clear that the optimal price for emission units and the optimal amount of cumulative emissions lie within a broad range, and depend on assumptions with respect to the magnitude of damages caused by emissions, the time-delay between emissions and damages, and the possibility to compensate climate change with increased man-made capital. These characteristics have been known to affect optimal emission levels in IAMs.

In addition, the numerical findings showed that the trust fund could substantially reduce (optimal) cumulative GHG emissions. Although the model specification neglects many features such as uncertainty, economies of scale and market failures, and the numerical illustrations were highly simplified, the results have a transparent interpretation and advocate a trust fund policy. It is especially clear that the intergenerational distribution of property rights for the environmental resources (grandfathering vs. the trust fund) has substantial consequences for production. This in contrast to the famous paper by Coase [1960] where it is implicitly suggested that the production allocation is independent of the distribution of property rights over environmental resources, and more recently, to the results from an OLG/IAM analysis by Stephan *et al.* [1997].

Yet the generation that unveils the value of the environmental resources gains more if it keeps the value of the resources for its own, than if it establishes a trust fund. The trust fund can only come into play through empathy of the present generation with future generations. If it is not concerned with environmental degradation, it has no reason to accept the trust fund. However, once the objective of sustainability is accepted, the trust fund is, within a general equilibrium context, the natural institution to ensure sustainability without sacrificing efficiency and decentralization. It relieves the living generations of the painstaking task of bringing their use of the environmental resource in agreement with their empathy for their unborn successors.

ANNEX 4A. MODEL SPECIFICATION FOR ALICE 1.2

In ALICE 1.2, the production function for the consumer good is a straightforward extension of the production structure in ALICE 1.0 by taking a Cobb Douglas composite of capital and labor, $y^{a}_{n,t}$, instead of labor in the production function $g_t(.)$:

$$h_t(k^{c}_{n,t}, y^{r-}_{n,t}, y^{l-}_{n,t}) = y^{a}_{n,t}\, g_t(y^{r-}_{n,t} / y^{a}_{n,t}), \tag{4.87}$$

where

$$y^{a}_{n,t} = (k^{c}_{n,t})^{a} (y^{l-}_{n,t})^{1-a}, \tag{4.88}$$

and

$$g(y^{r-}_{n,t} / y^{a}_{n,t}) = 1 + \eta(y^{r-}_{n,t} / y^{a}_{n,t})(1 - y^{r-}_{n,t} / 2\zeta_t y^{a}_{n,t}). \tag{4.89}$$

The parameters a, ζ_t and η are chosen so that the value of man-made capital is about three times the annual GDP, and the maximum production and associated emissions follow the IS92a scenario [IPCC 1992], and emissions decrease by one per cent point for every 4 US$/tC increase in the price of emission units.

5. ENVIRONMENTAL RESOURCES AND DEMOGRAPHIC CHANGE

5.1. INTRODUCTION

In previous chapters, we analyzed efficiency and sustainability within the robust formal framework developed in Chapter 2. However, the formal model has some limitations, and in this chapter, we broaden the analysis and address two issues that were raised in Chapter 1, but not dealt with in the preceding chapters, namely, first, the physical characteristics of environmental resource systems in relation to the abstract economic characteristics, and secondly, the joint determination of population and environmental resource dynamics.

The chapter proceeds as follows. In Section 5.2, we deal with the first aspect, continuing the illustrative integrated assessment exercises with our model ALICE, and incorporate explicit physical variables such as the atmospheric CO_2 concentration and the global average temperature. The extension makes the model comparable with the mainstream integrated assessment models (IAMs) with climate change, such as CETA [Peck and Teisberg 1992], DICE [Nordhaus 1994], and MERGE [Manne *et al.* 1995]. The numerical findings are presented in Section 5.3. In qualitative terms, the numerical outcomes are very similar to those of the previous chapters, but they are possibly easier to understand with this more realistic representation of climate change.

In Section 5, we discuss the relation between physical characteristics and economic characteristics of environmental resource systems in more general terms. Natural resource systems possess some typical characteristics which we have neglected till now, since we have taken the classic competitive economy [Arrow and Debreu 1954] as our point of departure, with the market as a central concept where demand and supply of homogeneous economic goods are matched. Typical exhaustible resources such as minerals can reasonably be treated as homogeneous goods, and in most cases property rights can be established that make it possible to incorporate these resources in the market economy. However, complex environmental resource systems with substances that diffuse through various media are not so easily incorporated in the Arrow-Debreu framework. We will show that these physical characteristics can often be represented through a combination of rival and non-rival goods, though they could in some cases also require the introduction of indivisibilities.

Section 5.5 turns to the second aspect and studies the joint determination of population dynamics and environmental resource use which is at the root of the sustainability discussion. Malthus [1798] expressed the opinion that in the long run linear increasing food production would be incapable of supporting the exponentially increasing population. In this doom scenario, food shortage will lead to vice and misery, and this provides the unpleasant means to check population growth. Modern literature has a more optimistic view on these joint dynamics. It takes the number of children as part of a rational decision process by parents. If the environmental resources become relatively scarce, the number of children will fall endogenously, long before vice and misery begins [Eckstein *et al.* 1988]. So far, we neglected this mechanism, and assumed an exogenous population path (Assumption *N1*), with environmental resources that are sufficient for the production of basic needs (Assumption *F8*). As for the long term in which we are interested, the exogeneity of population would seem too naive; we have to pay more attention to joint dynamics of population growth and scarce environmental resources, and we argue that in models with endogenous fertility and empathy for the offspring, overpopulation and unsustainability is unlikely to occur, but human extinction is well possible. Of course, complete human extinction seems rather improbable as an outcome, but the analysis will make the point that present environmental resource use is at the expense of the number of people that the future environment can support. We argue that the trust fund ensures 'non-extinction' of the equilibrium, which seems to be an interesting extension of the sustainability result from Chapter 4.

5.2. AN IAM WITH CLIMATE CHANGE

5.2.1. Introduction

In the previous chapters, we developed several versions of the ALICE model to illustrate the policy implications of the formal analysis in numerical terms. Here, we continue these illustrations, by extending the version of Chapter 4, ALICE 1.2, including an explicit specification of some physical processes underlying potential climate change, more specifically, the change of atmospheric CO_2 concentrations and the global mean surface temperature. We refer to this version as ALICE 2.0. In contrast to the previous chapters, this numerical analysis is not meant to illustrate the formal analysis but to highlight some problems that arise if we attempt to include explicitly realistic relationships in a stylized model. The extension serves to improve the comparability of the numerical results with those of other well-known IAMs. Qualitatively, the results do not differ from those of earlier chapters. Quantitatively, the issue of climate change is covered with too many uncertainties to warrant any use of the model for calculating the 'true' optimal price of greenhouse gas emissions or any other typical policy relevant variable. This problem is common, however, to all IAMs that incorporate climate change, and it should not prevent us from sketching different scenarios, as these can reveal differences in inputs in various strategies that are robust under changes in specification.

We begin with a brief sketch of the relevant physical variables. Solar radiation is of major importance for all life on earth. It is common knowledge that the biological system feeds on solar energy by means of photosynthesis in plants, which is then transferred to other life forms through the food chain. In addition, solar radiation is driving the earth's climate system that provides a relatively stable and favorable environment for the prosperous diversity of life forms. About two-thirds of the solar radiation reaching the earth is absorbed by the atmosphere, the ocean, ice, land and biota. The atmosphere prevents the heat to escape immediately into space, and so it works as a greenhouse that maintains the temperature of the bodies inside. It is estimated that without the atmospheric insulation, the global average temperature of the earth's surface would decrease by about 33 °C [IPCC 1990, p. xiv].

The greenhouse capacity of the atmosphere is due to so called greenhouse gases (GHGs) that allow solar radiation to pass, while reflecting the infra-red radiation emitted by the earth's surface. Human activities in the recent past, industrialization in particular, have led to an increase of several of these gases, such as carbon dioxide (CO_2), chlorofluorocarbons (CFCs), methane (NH_3), and nitrous oxide (NO_x). In principle, as the insulation capacity of the atmosphere increases, the global temperature increases as well. If not controlled, future GHG emissions are expected to increase further, and the resulting potential future 'global warming' has become a threatening perspective as it might cause an unprecedented disturbance of the environmental functions on a global scale.

It must be added that the environmental functions are threatened in many ways other than GHG emissions, and that the relation between GHG emissions and environmental functions is poorly understood [IPCC 1996, Section 6.2.13]. We recollect from Chapter 1 that the subject is still a matter of great controversy. Some studies suggest that measured (historic) climate change is not caused by human activity [Lassen and Friis-Christensen 1995], some argue that the problem will be of short duration since there will anyway in the near future be an endogenous shift towards fossil free fuels [Chakravorty *et al.* 1997], while others claim that the potential damages are insubstantial compared to present day poverty issues and therefore do not warrant a reduction in economic growth [Schelling 1992].

Nonetheless, it is generally believed that climate change will cause damages that outweigh potential benefits, and this understanding has led to the development of an extensive number of IAMs that attempt to strike a balance between present costs of GHG emission reductions and future damages if these reductions do not take place. In the previous chapters, we have developed ALICE 1, which can be used as an IAM, containing a simple one-dimensional exhaustible resource with amenity value to represent the basic features of climate change. We argued that such an OLG model provides relevant additional insights into the issue of climate change by linking intergenerational distribution of environmental resources with sustainability and efficiency.

Notwithstanding its usefulness for illustrative numerical analyses, ALICE 1 did not contain an explicit description of the biogeochemical processes that control climate change. We believe that this lack of specificity did not impair on the validity of the findings. Nonetheless, the results might be difficult to understand, because they do not allow to express the outcomes in terms of climate change variables. Here, we extend the model with explicit climate change variables such as CO_2 concentrations and global mean temperatures of other IAMs: CETA, DICE, MERGE, FUND, RICE. To preempt unjustified conclusions, let us from the outset remark that the process of climate change and its consequences are subject to so many uncertainties, that the numerical results presented in this chapter are not more or less realistic than the numerical results of ALICE 1 presented in earlier chapters, or than any of the numerical results of the other IAMs mentioned above.

5.2.2. Modeling the biogeochemical system

Peck and Teisberg [1992] and Nordhaus [1994] have much contributed to the development of stylized economic IAMs by providing highly simplified representations of geophysical interactions to make these applicable in macro-economic models. The typical simplified aggregate representation that has thereafter evolved links emissions to concentrations, concentrations to temperatures, and temperatures to damages. We follow this literature.

As the emissions of CO_2 account for the main antropogenic contribution to the greenhouse effect, we will focus on the carbon-cycle and its relation to climate change. Let us assume that the atmospheric GHG accumulation can be represented by a 'linear box' model in which each of the boxes i=1,...,5 is well mixed as described by Maier-Reimer and Hasselman [1987]. If CO_2 is emitted, it is distributed over the boxes for given shares a_i. Within a box, CO_2 concentrations exponentially adjust to their 'natural' levels at annual adjustment rate $1/\tau_i$, where τ_i is the so called 'e-folding time' or turnover time, which is the expected period that a gas particle will remain in the box. Let M_t^i denote the accumulated antropogenic emissions in box i at the beginning of period t and E_t the emissions during period t, we then have:

$$M_{t+1}^i = e^{-N\tau_i} M_t^i + a_i E_t \tag{5.1}$$

for $\Sigma_i a_i = 1$, where N is the period length of the discrete time model. Maier-Reimer and Hasselman [1987] have estimated the following parameters a and τ for a 5-box model:

TABLE 5.1. *Parameter values for the linear Maier-Reimer and Hasselman 5-box model*

e-folding time (τ_i) (yr)	∞	313.8	79.8	18.8	1.7
share (a_i)	0.14	0.24	0.32	0.21	0.09

Remarkably, 14 per cent of emissions remains in the atmosphere for the infinite horizon, which implies that the absorption capacity of the biogeochemical system is exhaustible.

The accumulation of GHGs causes an increase of the equilibrium global mean temperature (see [IPCC 1990, Table 2.2] for a list of GHGs and their potential contribution). For CO_2, the temperature increase is expected to be of approximate logarithmic nature,

$$T_t^{eq} = \overline{\mathrm{T}}\ {}^2\log(1 + \sum_i M_t^i / \overline{\mathrm{M}}),^{17} \tag{5.2}$$

where T_t^{eq} is the long term equilibrium temperature for given concentrations $\sum_i M^i$, and $\overline{\mathrm{T}}$ is the benchmark equilibrium temperature associated with a doubling of total accumulated antropogenic GHGs in the atmosphere compared with the 'natural' level $\overline{\mathrm{M}}$.

The earth surface and atmosphere have a warmth capacity, and therefore the temperature slowly adjusts to the long term equilibrium level:

$$T_{t+1} = e^{-Ne}T_t + (1 - e^{-Ne})T_{t+1}^{eq}. \tag{5.3}$$

where e is the annual adjustment rate, which we set to 2 per cent per year following Peck and Teisberg [1992].

So far, the above equations relate to complex geophysical relationships, and though present knowledge of these relationships is limited, the approximations are based on the firm foundation of a quantitative understanding of physical processes. However, when calculating impacts of climate change, the scientific understanding is grossly insufficient to warrant even something like a 'best guess'. In general, it is assumed that damages caused by climate change will outweigh the benefits, but the lack of knowledge is unmistakably revealed by many sensitivity analyses that are carried out with a variety of so called damage functions. These damage functions are supposed to provide a reduced form for the many complex damages associated with climate change, such as loss of coastal zones due to sea level rise, loss of biodiversity, vector borne diseases, and extreme events. Some damage functions take the global temperature as arguments, others take the rate of increase of global temperature as an argument, some damage functions are quadratic, other are of higher or lower order, cf. [Tol 1996a]. The lack of understanding is

[17] We use the notation that ${}^2\log x = y$ is equivalent to $2^y = x$.

also recognized by the IPCC [IPCC 1996a, Section 6.2.13]. Yet, in the same report, several damage estimates are listed [IPCC 1996a, Table 6.4]. The use of these figures for our analysis does not mean that we consider them reliable, but reflects our wish to maintain compatibility with the prevalent IAMs. Typically, the IAMs include a reduced damage function h(.):

$$D_t = \mathrm{h}(T_t), \tag{5.4}$$

where D_t is the damage, usually a function of actual temperature increase, expressed in monetary units or as a percentage of GDP.

The IAMs have different ways of incorporating the damage functions, either by subtracting damages from production or consumption, or directly from utility. We find this practice misleading, because environmental damage is better understood as a decrease in the quality or quantity of environmental functions, than as a reduction in a flow of man made goods. For lack of a better term, we call the fulfilling of these environmental functions 'biodiversity'. Environmental degeneration can lead to a reduction both in biomass and in biodiversity, with the former measuring the quantity and the latter the quality of the environmental functions [IPCC 1996b, Section 1.3.2]. We abstract from the biomass aspect, and focus on biodiversity. Let the 'biodiversity' level be given by:

$$B_{t+1} \le e^{-Nd} B_t + (1 - e^{-Nd})(1 - \mathrm{f}(T_{t+1})). \tag{5.5}$$

where d is the annual adjustment rate, and f(.) is the reduced form function that describes long term losses of biodiversity which we take to be increasing on its positive domain, and for which f(0)=0, see [Tol 1996a] for various functions. To summarize, the biogeochemical system produces emission units E_t, and biodiversity B_t.

This completes the description of the biogeochemical system in ALICE 2.0; the remainder of the economy is kept the same as in ALICE 1.2 of Chapter 4. Emission units are used for the production of man-made consumer goods, biodiversity is consumed as a non-rival good that represents the amenity value of the biogeochemical system. In the next section, we give a brief formal explanation of this assumption. A more extensive discussion follows in Section 5.4.1 on the distinction between quality and quantity aspects of environmental functions, and the relation to rival and non-rival use of environmental resources.

5.2.3. Biodiversity: variety as an aggregate (non-rival) good

One particular aspect of the quality of environmental systems is the variety of organisms it contains. Biological diversity, often referred to as 'biodiversity', is believed to be indispensable for the resilience of ecosystems. Though it is difficult to measure biodiversity, as there is no obvious unit of measurement, its importance is unquestionable, and attempts have been made to estimate its contribution to welfare [Pearce and Moran 1994]. In this section, we conduct a simple formal analysis to show that biodiversity can be represented as a quality variable, more specifically as a non-rival good, and it can be understood as an aspect of the amenity value of ecosystems.

In the economic literature, there exists a substantial body of analysis on varieties, its production by different producers, its effect on consumers' utility, and its effects on trade. The concepts in the present literature have been initialized by Dixit and Stiglitz [1977], who focused on the divergence between optimal and actual variety in a partial equilibrium. The main conclusion from the literature is that those who benefit from variety tend not to pay for its production. Therefore, there is insufficient stimulus to increase variety, and under specific assumptions, this implies sub-optimality (see comments by Yang and Heijdra [1993] and a reply by Dixit and Stiglitz [1993]). This conclusion seems natural, once we recognize that variety can be viewed as a non-rival good for which the coordination problems are well-known [Mäler 1985]. Applying the analysis to biodiversity, it implies that there is a tendency not to pay for the value of biodiversity, resulting in sub-optimal levels. Recognition of the value of biodiversity and its incorporation in development plans could be a step forward towards sustainable development [Panayotou 1994], [Van Soest 1998].

Let us now proceed with a stylized formal analysis. Suppose that there are b identical species (b for 'biodiversity') with total biomass x. Furthermore, for convenience, assume that biomass is homogeneously distributed, implying that every species has biomass x/b, assume that consumption of every species is proportional to its biomass which we therefore take as its approximation (up to a constant), and assume that utility is separable and additive:

$$\mathrm{u}(x,b) = b\,\tilde{\mathrm{u}}(x/b) \tag{5.6}$$

where $\tilde{\mathrm{u}}(.)$ denotes the utility per species. Finally, assume that the utility function $\tilde{\mathrm{u}}(.)$ is continuous, concave, strictly increasing, and $\tilde{\mathrm{u}}(0) = 0$. It follows by construction that 'aggregate utility' $\mathrm{u}(x,b)$ is continuous, linearly homogeneous, and concave[18].

Now, we introduce several consumers to reveal non-rivalry. Assume that there are n identical consumers, each consuming $1/n$-th of the available biomass. We have:

$$\mathrm{U}(x,b;n) = nb\,\tilde{\mathrm{u}}(x/nb), \tag{5.7}$$

where U(.) denotes aggregate welfare. This equation can be rewritten as:

[18] See Annex 2B.

$$U(x,b;n) = n\,u(x/n,b), \tag{5.8}$$

and this makes clear that *per capita* consumption of biomass x/n is reduced if the (human) population n increases, while the *per capita* consumption of biodiversity b is not reduced. Hence, to represent aggregate demand, biodiversity can be treated as a non-rival good.

This discussion abstracts from the production and the use of biodiversity by others than consumers. Moreover, it draws on several assumptions, noticeably the homogeneity of species and consumers. Nonetheless, we claim that the main argument carries through under more general assumptions: the diversity of ecological systems is a part of its amenity value which, in the meta-models such as the IAMs, can properly be described as a non-rival good. If environmental systems deteriorate, both biomass and biodiversity will be affected, but some species are better equipped to adjust to a changing environment [IPCC 1996b, Section 1.3]. In fact, the most relevant effect of environmental degradation could be the decline in biodiversity, not in biomass.

5.2.4. Welfare maximization, equilibrium and specification of three policies

If we compare the production structure (5.1)-(5.5) with the standard convex transformation function F(.) that satisfies Assumptions *F1-F6*, used in the illustrations of previous chapters, two differences can be noticed. First, we have not specified stocks to determine the state of the environment, but state variables, and therefore, because the state variables do not represent homogeneous commodities, it is not possible to multiply the state variables with any price. Consequently, there is no direct rule for determination of the value of the biogeochemical system, other than by calculating the net present value of its output.

Secondly, this biogeochemical system might correspond to a non-convex production set. Increasing emissions cause temperatures to increase, but on the margin, the additional temperature increase diminishes because of the logarithmic temperature function (5.2). Thus, if damages were linear in the temperature increase, marginal damages would decrease with rising emission levels. On the other hand, if the convexity of the damage function h(.) in (5.4) is sufficiently strong, this might compensate for the non-convexity in the temperature function (5.2). For the chosen specification of h(.), we cannot prove analytically the convexity of the resulting system, and have to rely on a numerical analysis.

Both the pricing and the non-convexity are addressed in this section, in relation to the specification of the three policies of Chapter 4, i.e. strong sustainability, grandfathering, and the set up of a trust fund. The first policy bans all GHG emissions as soon as possible, and exempts all generations from paying for the non-rival use of biodiversity. To ensure dynamic efficiency, a small non-negligible claim is issued (γ=0.01, one per cent of income is paid to the owner of the claim), and given to the first generation. Because we abstracted from any interactions between the biogeochemical system and the production and consumption of man-made goods, the explicit specification of the biogeochemical cycles does not affect the latter and the economy is the same as in Chapter 4. For this scenario, the added value of ALICE 2.0 compared to ALICE 1.2 lies in its capacity to make clear that even if emissions stop immediately, global warming will continue for a while, stressing the inertia of the biogeochemical system.

The second policy grandfathers the environmental resources to the present generations, which receive full ownership. By selling the property rights to future generations, they indirectly receive the revenues from the biogeochemical cycles' capacity to absorb emissions and the production of biodiversity. However, because the biogeochemical system is possibly non-convex, welfare maximization is not guaranteed by profit maximization, and requires supporting public regulation instead. We assume that a public authority specifies a time path for the biodiversity levels, or more generally: for the environmental quality, which the owners of the environmental resources have to conform to. Moreover, because we specified the biogeochemical system through state variables rather than stocks, we will simply assume that the first generation receives all revenues from the output of the biogeochemical system as income, instead of specifying income from an environmental resource stock. Now, the budget constraint for the first generation reads:

$$p_0^c x_{0,1}^c + \varphi_{0,1}^b x_{0,1}^{nr,b} \leq p_0^l \omega_{0,1}^l + \psi_{n,1}^c k_{n,1}^c + \sum_{t=1,\dots,\infty} (p_t^e y_{m,t}^e + p_t^b y_{m,t}^b), \tag{5.9}$$

requiring the value of rival consumption of the consumer good plus the value of non-rival consumption of biodiversity not to exceed income from labor endowments, plus income from the stock of man-made capital, plus revenues from the output of the environmental resource.

The generations t=1,...,∞ live three periods and maximize utility subject to the budget constraint:

$$\sum_{i=0,1,2} p_{t+i}^c x_{t,t+i}^c + \varphi_{t,t+i}^b x_{t,t+i}^{nr,b} \leq \sum_{i=0,1,2} p_{t+i}^l \omega_{t,t+i}^l , \tag{5.10}$$

i.e., they have only income from labor flow endowments.

The third policy creates the trust fund. Future generations receive a claim to the maximum biodiversity output whose level is derived from the strong sustainability policy. Recall from Chapter 4 that the trust fund might have a surplus in every period that is distributed between the generations alive. However, because the environmental resource is not expressed through stock variables, we cannot calculate the surplus for every separate period, and instead, the entire surplus of the trust fund is given to the first generation. As in the strong sustainability scenario, to ensure dynamic efficiency, the first generation receives the non-negligible claim, $\chi_1 = \gamma \sum_{t=1,\ldots,\infty} p_t^l \omega_t^l$, and, consequently, income from the labor endowments is reduced by a factor (1−γ). Formally, this is represented by:

$$p_0^c x_{0,1}^c + \varphi_{0,1}^b x_{0,1}^{nr,b} \leq (1-\gamma) p_0^l \omega_{0,1}^l + \psi_{n,1}^c k_{n,1}^c + \sum_{t=1,\ldots,\infty} (p_t^e y_{m,t}^e + p_t^b (y_{m,t}^b - \bar{y}_{m,t}^b)) + \chi_1, \quad (5.11)$$

where $\bar{y}_{m,t}^b$ is the claim of future generations. If the feasible outputs of the biogeochemical cycles form a (locally) convex set, we have:

$$\sum_{t=1,\ldots,\infty} (p_t^e y_{m,t}^e + p_t^b y_{m,t}^b) \geq \sum_{t=1,\ldots,\infty} p_t^b \bar{y}_{m,t}^b , \quad (5.12)$$

that is, the actual allocation maximizes profits and hence actual revenues exceed the value of the claims so that the trust fund surplus is positive. If there are (local) non-convexities, the inequality may not hold, and the surplus of the trust fund may be negative, and has to be financed by the first generation. As in the grandfathering policy, and because of the possible non-convexity, the optimal allocation has to be supported by a publicly announced time path for the environmental quality which the owners of the environmental resources have to supply.

The budget constraint for future generations reads:

$$\sum_{i=0,1,2} p_{t+i}^c x_{t,t+i}^c + \varphi_{t,t+i}^b x_{t,t+i}^{nr,b} \leq (1-\gamma) \sum_{i=0,1,2} p_{t+i}^l \omega_{t,t+i}^l + \sum_{i=0,1,2} \varphi_{t,t+i}^b \bar{y}_{t+i}^b . \quad (5.13)$$

While we seem unable to investigate convexity of the biogeochemical system analytically, it is also nearly impossible to do so numerically, because the output production vector is of infinite dimension. Nonetheless, we attempt to give a graphical representation of the production set that convincingly reveals the convexity in a neighborhood of the trust fund equilibrium. This implies that the biogeochemical system can be supposed to maximize profits, as will be discussed in Section 5.4.2.

The numerical exercise proceeds as follows. We project the production possibility set for the biogeochemical system on a two-dimensional plane, to obtain a visual impression of its (non-)convexity: for several production vectors on the frontier of the production possibility curve, we project the value of emissions, $\sum_{t=1,\ldots,\infty} p_t^e y_{m,t}^e$, valued at the prices of the trust fund equilibrium, against the value of biodiversity, $\sum_{t=1,\ldots,\infty} p_t^b y_{m,t}^b$. To generate the production vectors on the frontier. Consider the welfare program that maximizes the weighted sum of utilities of generations:

$$w = \sum_{t=0,\dots,\infty} \alpha_t \upsilon_t \,, \tag{5.14}$$

where υ_t is the utility of generation t, and α_t is the welfare weight that supports the trust fund equilibrium. Because the issue of climate change essentially boils down to a trade off among the welfare of the early generations t=0,...,3 living during the 20[th] and the 21[st] century, who have to reduce GHG emissions, and the later generations t=4,...,∞ who face the consequences, we define a homotopy for $s \in [0,1]$ by adjusting the welfare function to:

$$w = s \sum_{t=0,\dots,3} \alpha_t \upsilon_t + (1-s) \sum_{t=4,\dots,\infty} \alpha_t \upsilon_t \,. \tag{5.15}$$

For s=½, the welfare function supports the trust fund scenario. For small s, future generations are given higher welfare weights which implies that emissions will be less and future biodiversity will increase. For large s, emissions increase and future levels of biodiversity decrease. Maximizing welfare for different values of $s \in [0,1]$ generates outputs on the production frontier. We scaled both values of emissions and biodiversity such that they sum to unity for the trust fund scenario. The result is given in Figure 5.1. The trust fund scenario, with s=½, is marked by the crossing of the perpendicular lines.

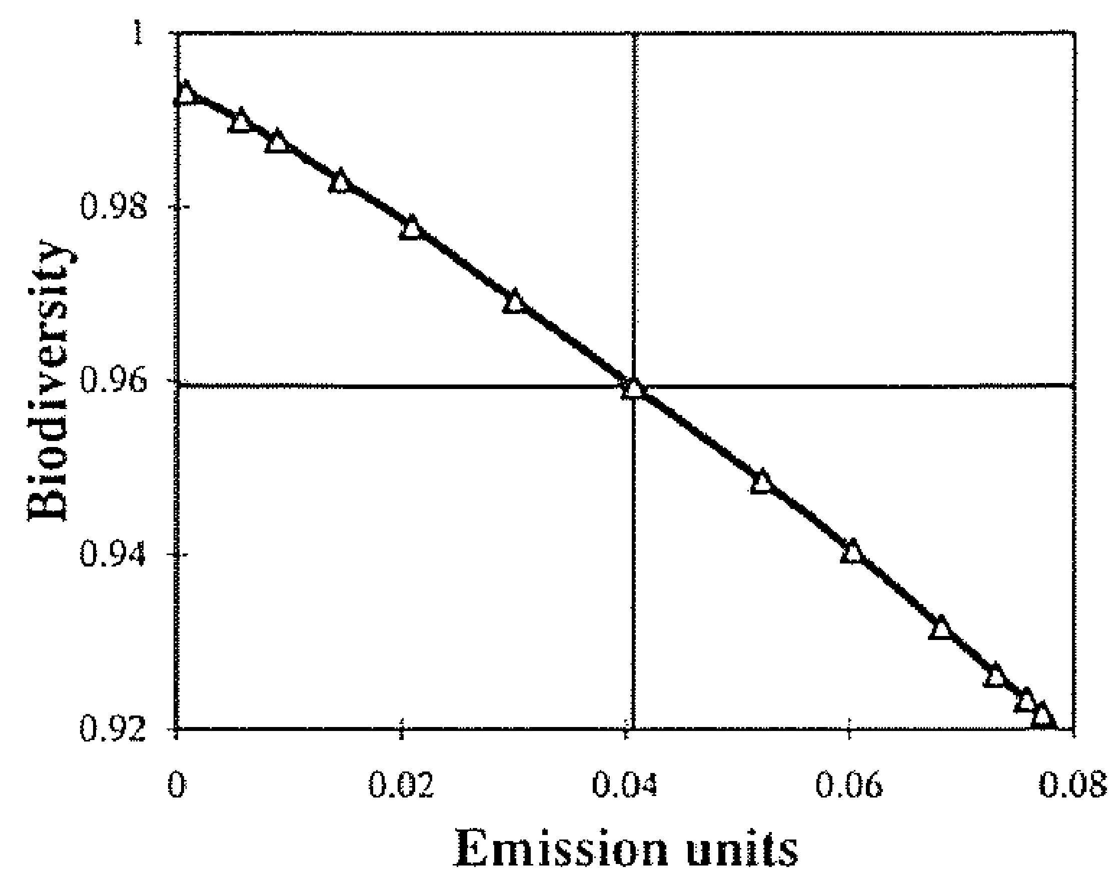

FIGURE 5.1. *Frontier of biogeochemical production possibility set*

The production set is the surface at the lower left side of the frontier. The production set seems convex. The production bundle underlying the trust fund, $\bar{y}^b_{m,t}$, is represented by the left upper part of the frontier line where it crosses the y-axis. The figure shows that the value of this bundle does not exceed the value of actual output, thus (5.12) holds. It follows that the first generation does not have to pay for a negative surplus and receives at least a non-negligible share γ of the value of the actual output, and hence, prices decrease sufficiently fast to ensure that the aggregate value of consumption is bounded and equal to the aggregate value of endowments. This is, however, not sufficient to ensure that the equilibrium will be a global welfare optimum.

Though Figure 5.1 suggests that the production set is convex, it is almost linear and, moreover, this is only a two-dimensional projection. A sensitivity analysis with the model suggests that a small change in model specification can result in a non-convexity. Therefore, we present a similar analysis for the utility possibility set. For the same homotopy used above, we project the average welfare levels of the first four generations against the average welfare levels of future generations in Figure 5.2, scaled by the trust fund values. The utility possibility set is more likely to be convex, since the welfare functions are strongly convex.

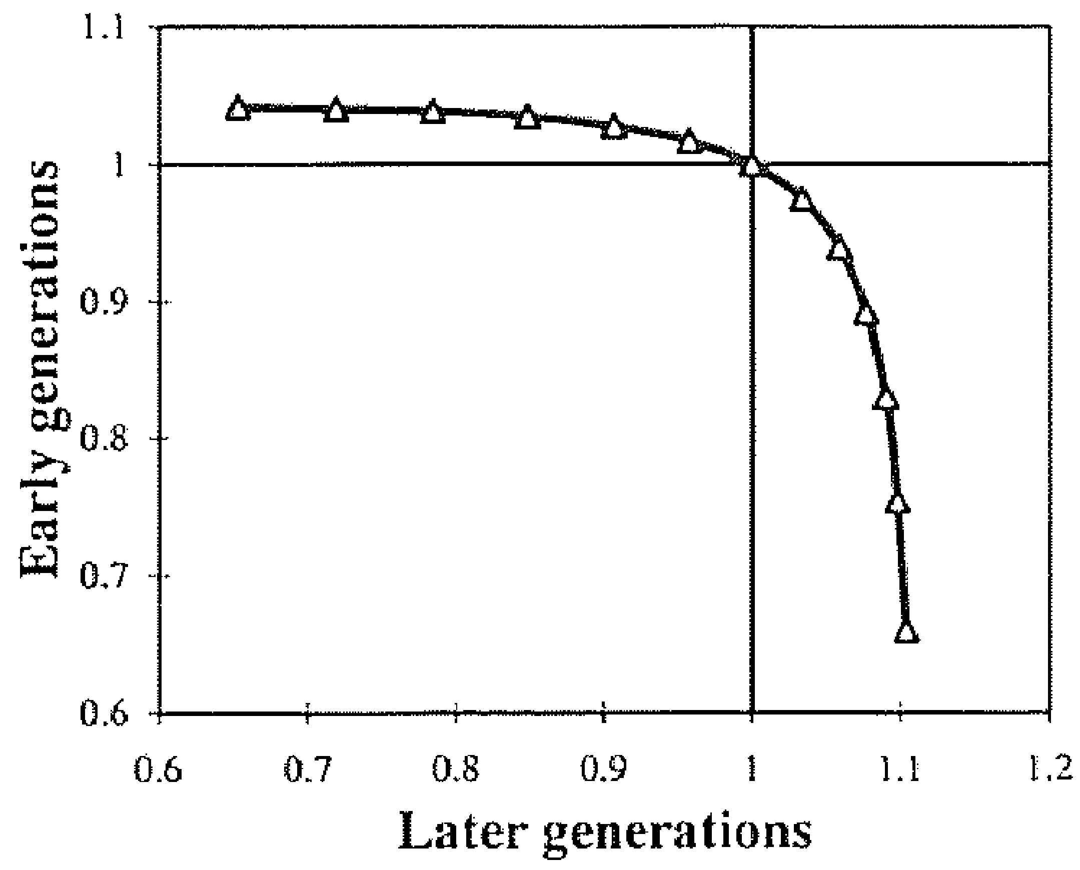

FIGURE 5.2. *Frontier of utility possibility set*

This figure unambiguously reveals the convexity of this projected utility possibility set. It appears that the possibility to transfer welfare from early generations to later generations is rather limited. On the margin, a reduction of emissions increases future welfare at the costs of present welfare, and the inverse holds for an increase in emissions. But a substantial increase of present emissions does not add much to present welfare because of decreasing returns to scale for the production of man-made consumer goods. Generations 3 and 4 can also transfer welfare by adjusting consumption when old and young respectively, and by transferring capital, but this does not change the overall perspective very much.

To summarize, the welfare program of ALICE 2.0 is most likely to be convex. Though, it exhibits modest non-convexities with regard to parts of the production structure, these are 'convexified' by other parts of the welfare program. We thus proceed with the numerical analysis and assume the equilibria to be Pareto optimal.

5.3. NUMERICAL ILLUSTRATION

5.3.1. Introduction

Now that we have included the climate variables in the ALICE model of Chapters 3 and 4, it is possible to include numerical results for the climatic consequences of the three policy scenarios: the strong sustainability, grandfathering, and the trust fund policy. In the figures that follow, state variables, such as CO_2 concentrations, are defined for the years between two periods: 2000, 2020, 2040, etc., while flow variables, such as consumption and emissions, describe a flow during a period, e.g. 2000-2020, and these periods are identified with the central year, e.g. 2010.

Section 5.3.2 deals with the outcomes for production and consumption of man-made commodities, and the emission of GHGs that are produced in this process. We start the presentation with the optimal level of the emission price, or 'carbon tax', which is the variable of most interest for policy makers. Next, we show how these prices affect the emission levels. GHG emissions can be considered an extractive exploitation of a valuable environmental resource and we will study whether this decrease is compensated by an increase of the man-made capital stock.

Section 5.3.3 continues with a presentation of outcomes on the main geophysical variables contained in the model: the atmospheric CO_2 concentrations and the global mean temperatures. Finally, Section 5.3.4 concludes with a discussion of welfare levels.

5.3.2. Consumption, production and emissions

We have assumed that CO_2 emissions decrease by one per cent point for each 4 US$/tC increase in the emission price, and this implies that we abstract from endogenous technological improvement and transition costs associated with a shift towards a 'backstop' energy technology. Under these assumptions, the strong sustainability policy with zero net emissions is supported by a constant emission price of 400 US$/tC. Though in the model no emissions occur under this policy, in reality the situation is to be thought of as an equilibrium in which gross emissions balance with gross uptake. We can think of measures that remove CO_2 from the atmosphere, such as afforestation, to implement a policy of zero net emissions.

Grandfathering leads to a modest carbon emission price that slowly increases from nearly zero in 2000 to 100 US$/tC in 2100 and 400 US$/tC in 2200. The value of marginal damages associated with one unit of emissions rise with cumulative emissions. In the 22nd century, the rise in emission prices outweighs the autonomous increase in emissions due to economic growth. This leads to a steady decrease of emissions, and by

2200, net emissions are zero (Figure 5.4). In 2100, cumulative emissions amount to 1450 GtC, which is comparable with the IPCC IS92a scenario [1992], but it should be understood that these estimates are highly dependent on the assumptions made. Chakravorty *et al.* [1997] for example estimate an autonomous decline of fossil fuel energy use before 2100 because of competitive solar energy supply.

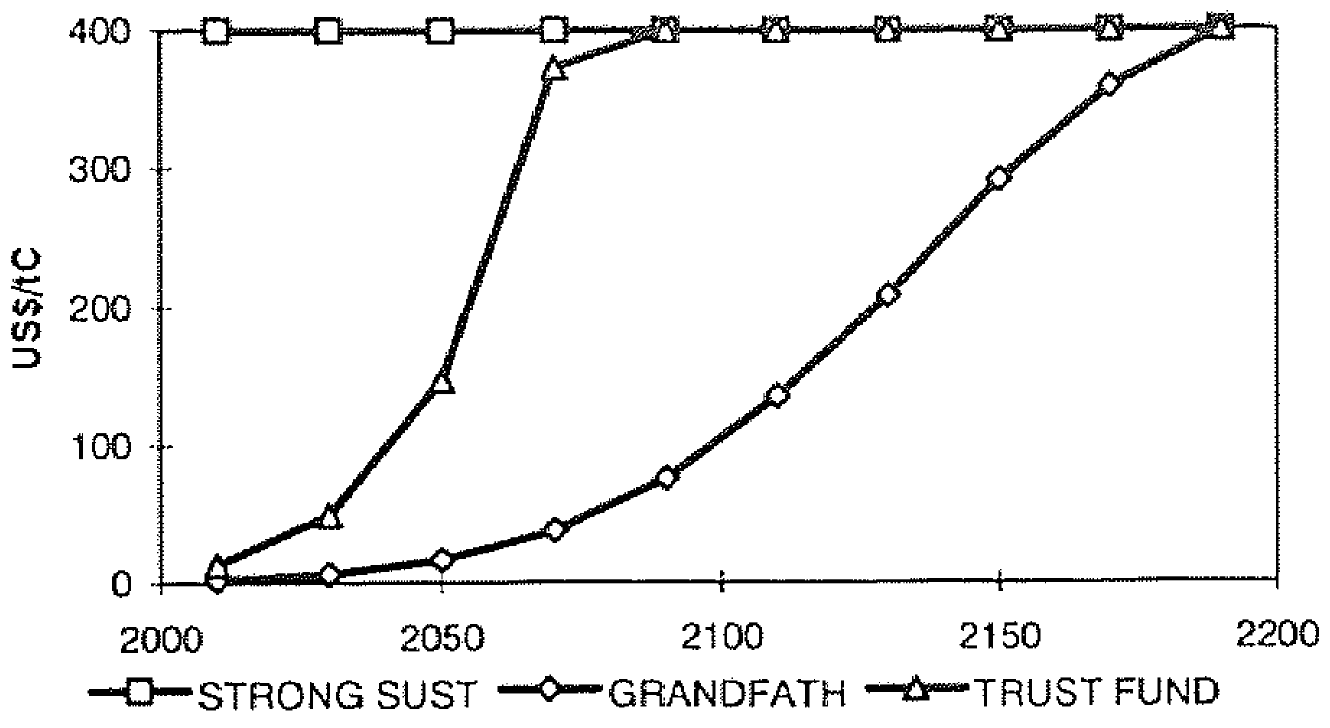

FIGURE 5.3. *CO_2 emission prices*

The trust fund substantially increases the emission price already in the first period, and the price further increases rapidly towards 400 US$/tC in 2080, which by assumption implies a 100 per cent reduction of net CO_2 emissions. The policy does not limit the expansion of net emissions in the medium term, leaving present generations the possibility to adapt and to develop alternative energy sources. After 2050, net emissions decrease rapidly, and by 2100 a complete substitution of fossil fuel energy carriers by fossil free energy carriers has taken place.

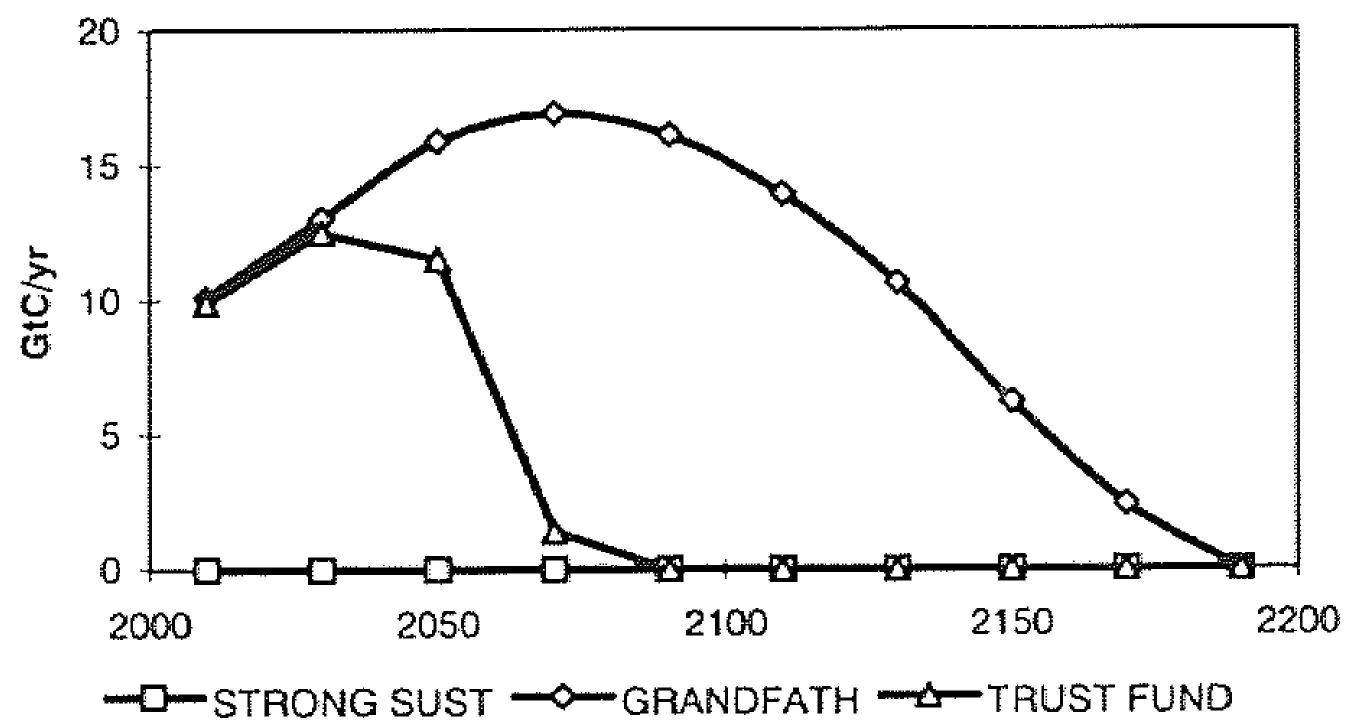

FIGURE 5.4. *Net CO_2 emissions*

Taking the strong sustainability scenario as reference path, net GHG emissions can be considered an extractive exploitation of a valuable resource that decreases the value of the resource, as it decreases the amenity level of the resource in future periods. This is not a problem as long as revenues of the exploitation of the environmental resource are being compensated by investments in man-made capital.

The model has been calibrated so that the man-made capital stock in 2000 is valued at three times annual production. In the reference (strong sustainability) scenario, this ratio increases to a factor 4.5 by 2100 and then stabilizes (Figure 5.5). This is largely explained by the higher life-expectancy, which induces increasing life-cycle savings for the retirement, and are balanced by an increased capital stock. This mechanism was discussed earlier in Section 3.3.

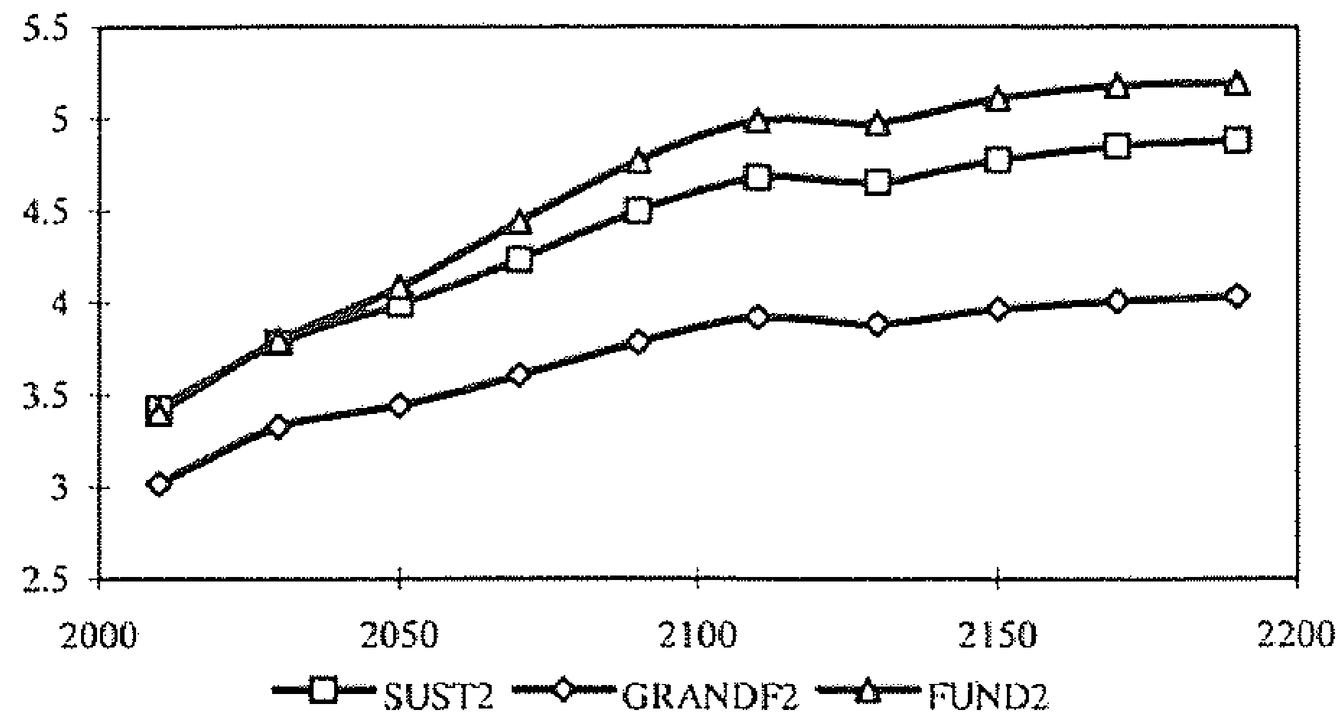

FIGURE 5.5. *man-made capital/GDP ratios*

Grandfathering leads to a substantial decline in the stock of man-made capital. The reason is that the environmental resource competes with man-made capital in balancing with life-cycle savings. This implies that the revenues of the resource exploitation, that is the value of emissions, are not invested but are immediately consumed. Consequently, the increased consumption compared to the sustainability scenario in the first two periods (Figure 5.6) feed on the exploitation of the environmental resource. But as these emissions have to decrease eventually, consumption cannot be sustained at this high level, and even drops below the reference level from 2050 onwards.

Contrary to this, under the trust fund policy, which compensates future generations for environmental losses, revenues from the sale of emission units are indeed used to increase the man-made capital stock (Figure 5.5) and this makes it possible to maintain the increased consumption level after net emissions have decreased to zero (Figure 5.6).

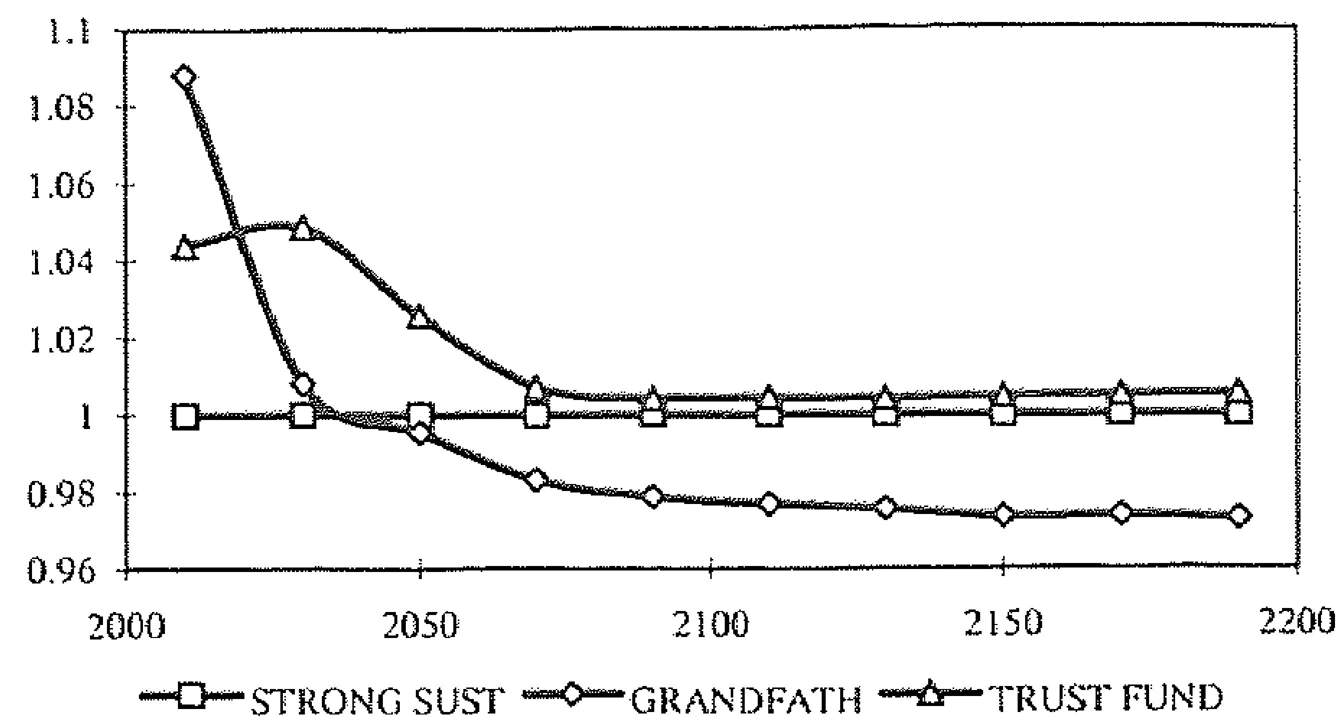

FIGURE 5.6. *Relative consumption*

However, because the supply and use of man-made capital is subject to decreasing returns to scale, and because the maintenance costs are assumed to be proportional to the level of the stock, future consumption does not increase as much as might be suggested by the substantial increase in the capital stock.

5.3.3. Climate change

The cumulative emissions used for the production of the consumer good lead to a build up of CO_2 in the atmosphere. The immediate ban on net emissions under the strong sustainability policy causes a slow decline of CO_2 concentrations to a level close to the pre-industrial value of 280 ppmv (Figure 5.7). In the climate model, the net emissions under the trust fund policy increase the atmospheric CO_2 concentrations until 2050 and after net emissions have been phased out, the concentration decreases. As part of the net emissions is not absorbed by the biogeochemical system, the CO_2 concentrations do not converge to the pre-industrial levels but stabilize at a higher level. The grandfathering policy leads to a steady increase in CO_2 concentrations, and it takes up to 2140, seventy years after the start in emission reductions, until the concentrations show a downward bending. This is because the positive net emissions still cumulate and outweigh the slow decay of atmospheric CO_2.

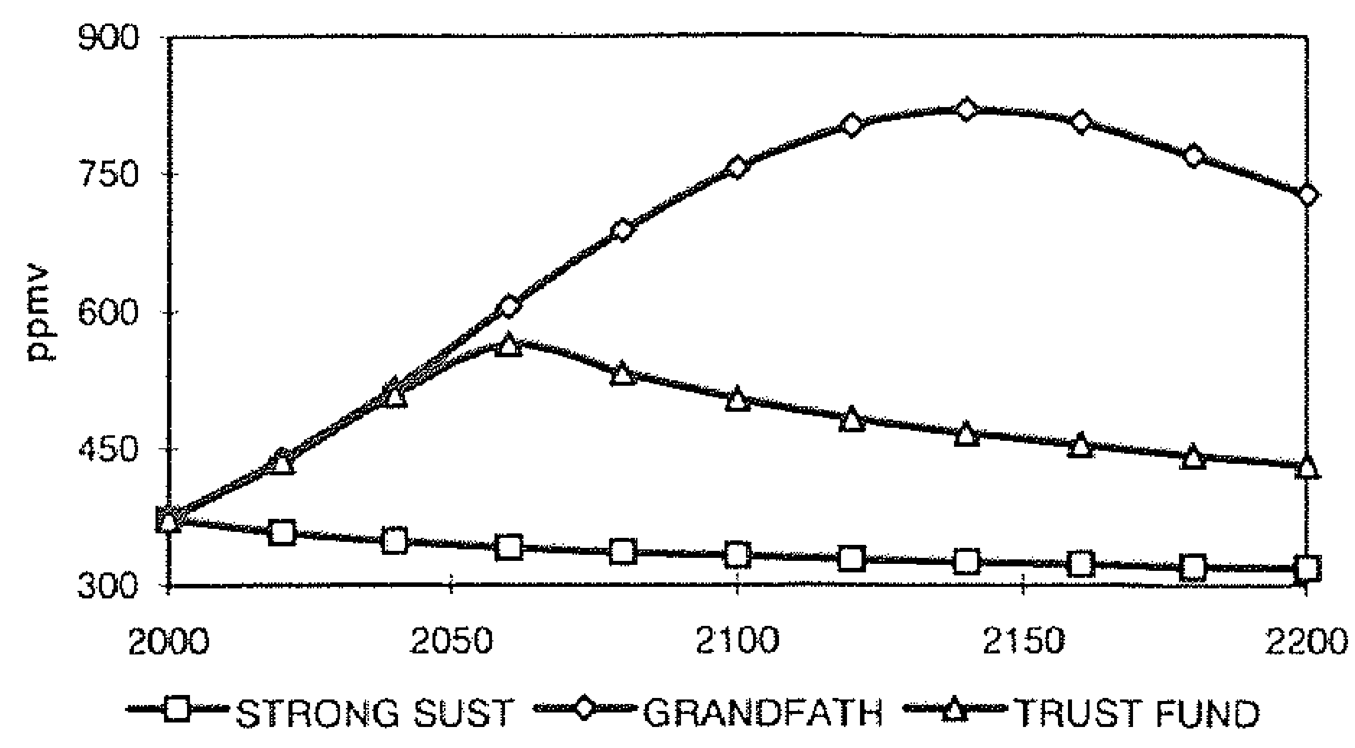

FIGURE 5.7. *Atmospheric CO_2 concentrations*

The global mean temperatures lag behind the CO_2 concentration levels in all scenarios. Figure 5.8 shows temperatures over the interval 2000-2200; the delay becomes visible as the figure nearly coincides with the left lower corner of Figure 5.7 which only represents the first century 2000-2100.

A strict ban on net GHG emissions cannot prevent the global average temperature from increasing further by an estimated 0.5 degrees Celsius in 2000, relative to the pre-industrial or 'natural' level, towards nearly one degree Celsius in the second half of the 21st century. As concentrations decrease further, the temperature also drops towards the pre-industrial level, but this takes rather long; the temperature does not decrease below its current level until 2200.

Grandfathering checks the CO_2 concentrations only after 2140 when they have reached thrice its pre-industrial level, and the temperatures stabilize another 50 years later, at a level of 4.5 degrees Celsius. The time lag between stabilization of emissions and temperature thus amounts to more than hundred years.

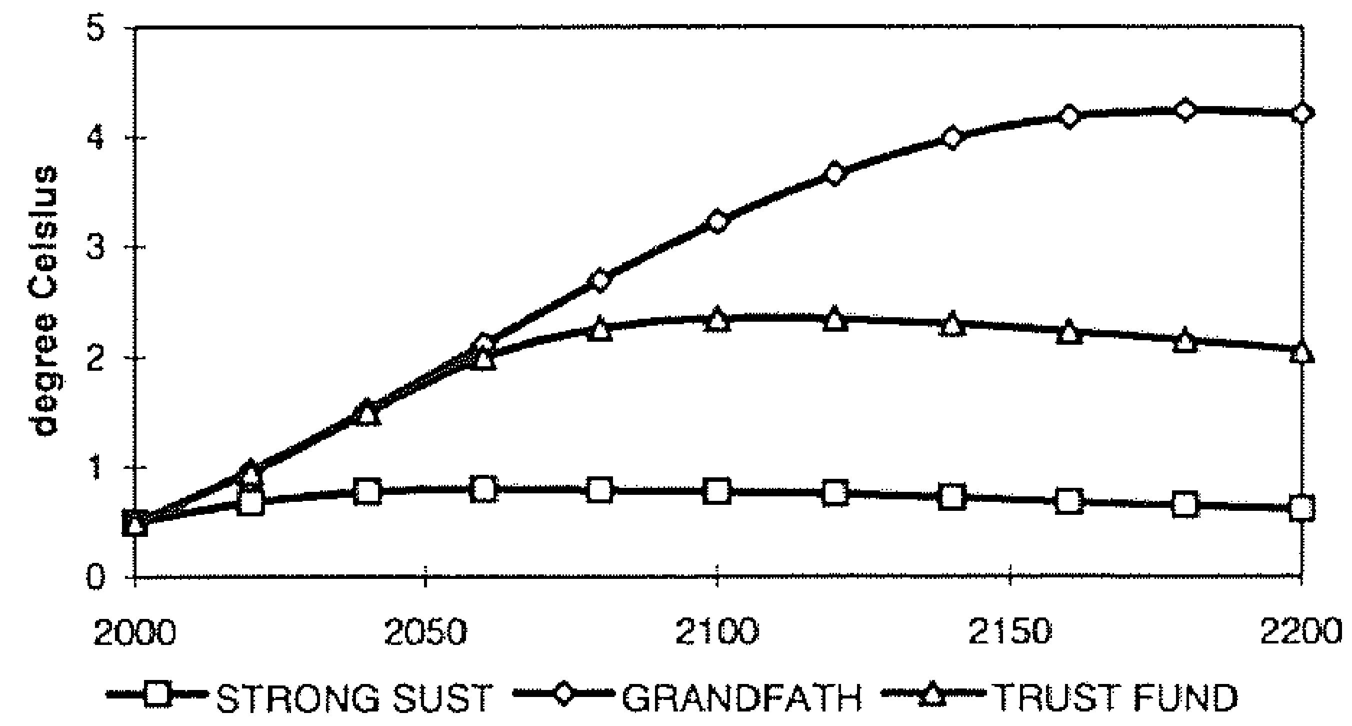

FIGURE 5.8. *Global mean temperatures*

The temperature figure suggest clearly that a climate change policy has to consider the long term. As we have seen in Figure 5.4, under the trust fund policy, emissions reach their maximum during the first half of the 21st century, and the atmospheric CO_2 concentration reaches its maximum several decades later in the second half of the 21st century. The temperature lags still further behind and reaches its maximum by 2100 after a steady increase during the 21st century. Only after 2050, we find a small deviation between the trust fund and the grandfathering policy, which, however, increases rapidly (Figure 5.8). A policy that focuses on the short term only will easily lead to emission levels that harm generations for centuries to come. This emphasizes the need of present day actions though effects might not be witnessed by the present decision makers.

5.3.4. Welfare

Ultimately, welfare is our main indicator variable, as it combines the value of man-made and natural production into one measure. Relative to the strong sustainability scenario, we have seen that grandfathering increases the consumption in the short term, but that the increase diminishes over time (Figure 5.6). The calculations show that only the present generations born before 2000 benefit from the grandfathering policy. All future generations are better off under a strong sustainability policy (Figure 5.9).

The trust fund policy has led to a build up of man-made capital to compensate for the loss of the environmental resource, and indeed welfare levels do not decrease below the reference strong sustainability levels.

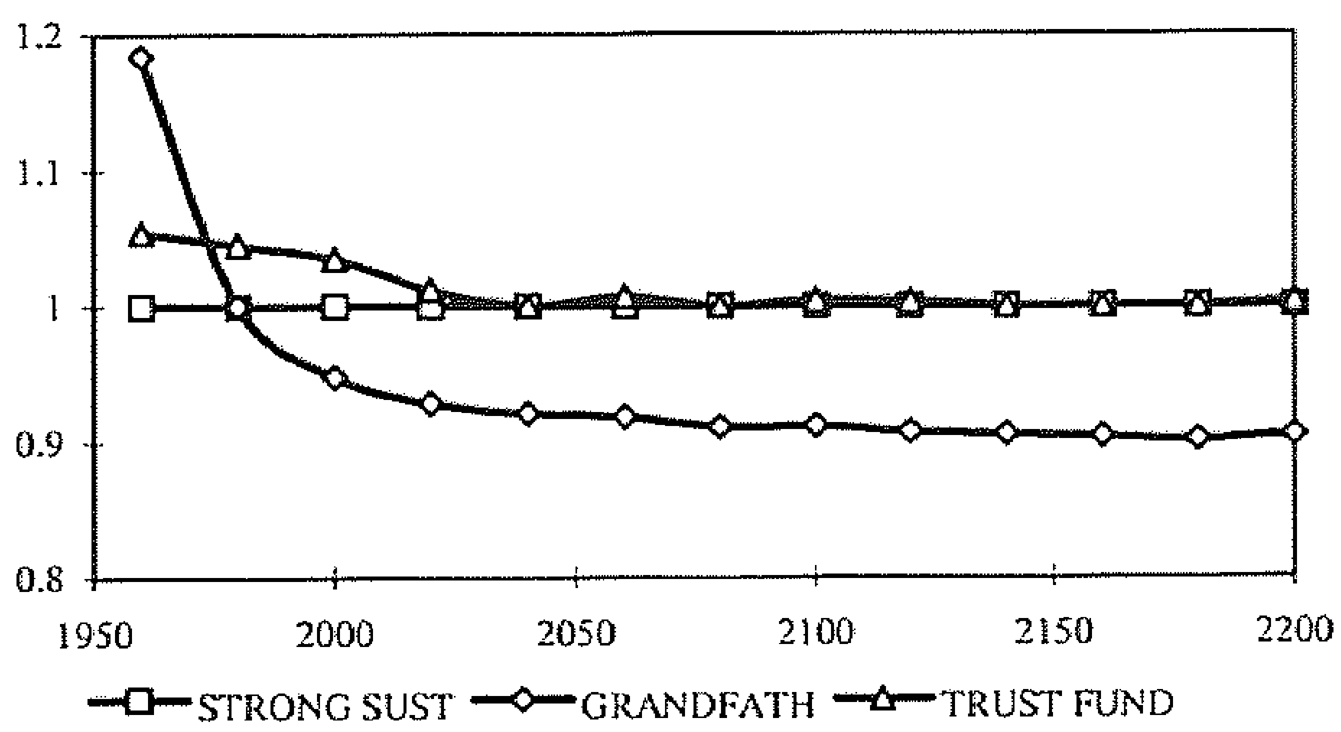

FIGURE 5.9. *Relative welfare*

Summarizing the numerical findings, the inclusion of explicit climate change variables gives the same results as the simple model we employed in Chapters 3 and 4. The trust fund will increase welfare for all generations compared to the strong sustainability policy, whereas grandfathering the environmental resource increases welfare only for the early generations. Thus, if we want to combine efficiency and sustainability, it is advisable not to grandfather environmental resources to present generations, but to set up a trust fund as caretaker for future generations.

Grandfathering leads to sustained emissions and a global temperatures rise by 4.5 degrees Celsius in the next two centuries. This result is comparable with the scenario results from other dynastic IAMs [Peck and Teisberg 1992], [Nordhaus 1994, [Manne *et al.* 1995], [Nordhaus and Yang 1996]. These models, as well as the model we employed, are also based on the expectation that such a substantial temperature rise will have severe consequences such as induced sea level rise and floods that threaten most of the current coastal zones. Yet the sustained emissions are efficient because of the immense time delay which makes it unprofitable to cut present emissions for future benefits.

In the trust fund scenario, the optimal emission path is subject to the same profit maximization principle, but because of the different property rights distribution, it now becomes profitable to cut present emissions in order to reduce future temperature rises. The trust fund triggers an economic transition towards zero-emission technologies in the next century. The temperature rise is checked at 2.5 degree Celsius, and though we cannot claim that this causes no substantial environmental and economic damages, they are undoubtedly of a different order of magnitude.

5.4. Environmental production

5.4.1. Quantity and quality; rival versus non-rival use

The exercise with the ALICE model indicated that several difficulties might arise if we include explicit physical variables such as temperature in a competitive equilibrium model, because it is not clear how the formal framework of transformation functions specified in Chapter 2 (*F1-F6*) can include environmental processes. On the one hand, we found in Section 5.2.3 that, though biodiversity is not a homogeneous good with an objective measurable quantity, it is compatible with the framework as it can meaningfully be included as a non-rival good. On the other hand, in Section 5.2.4, it became clear that there might be non-convexities, and had to check numerically that the calculated equilibrium represented a Pareto optimum. In this section, both issues are discussed in more general terms. First, we consider the distinction between quality and quantity of environmental services and the associated distinction between rival and non-rival use in the formal framework. Thereafter, in Section 5.4.2, we discuss the emergence of non-convexities and link these with diffusion processes that play a central role in environmental dynamics.

In Chapter 2, we assumed that the state of the environment and the services it renders can be represented through homogeneous stocks and flows, following the common practice in economic modeling. Indeed, many aspects of environmental production can be captured within this framework. There are fertile soils, and barren soils, and if used for food production, the total area of fertile soil (a stock) is a useful measure. In a more sophisticated set up, it is possible to classify soils further into numerous sub-types which still fit well in the formal framework.

Yet not all environmental services are usefully represented through quantities of distinct stocks and flows. Consider pollution that spreads homogeneously through a given medium. Rather than transforming a part of the clean stock into a dirty stock, leaving the remainder clean, the contamination transforms the entire clean stock into a less-clean stock. The question is whether the change in quality (as opposed to a change in the quantity of a clean stock) can meaningfully be represented as a change in an environmental stock. To illustrate this distinction between quantity and quality, let us think about the case in which the quantity of the stock is abundantly available, as is the case for air. If the contamination was restricted within a compartment, no-one would need to suffer from it, as there would remain a sufficient quantity of clean air. Unfortunately, the contamination spreads all over and the quality of all the environmental services decreases. Consider the following example.

EXAMPLE 5.1. *Rival and non-rival use of a lake.*

Assume that there are n consumers who make use of the same environmental resource, say, a lake that is used for both waste disposal (emissions) and for its amenity value (swimming). The emissions are an unavoidable consequence of the consumption of man-made consumer goods, and, therefore, we use the level of emissions as a 'proxy' for the consumption level, which we denote by x. The enjoyment from swimming is assumed to be proportional to the water quality, q, which is taken to decrease linearly with the emission level:

$$q = \overline{q} - x , \tag{5.16}$$

where $\overline{q}$ is the 'natural' quality without emissions. Because we have expressed water quality and emissions in the same units, we can also interpret (5.16) in the following way. The total resource endowment is given by $\overline{q}$, and can be used for emission absorption, x, or for the production of amenity, q. The total amount of services, x+q, is constrained by the resource endowment. The interesting point is that x represents a rival good, and q represents a non-rival good, and that the resource produces both.

To substantiate this point, we follow the same reasoning as for biodiversity in Section 5.2.3. Assume that, for every individual consumer, utility depends on the consumption of the man-made consumer good, and on the quality of the swimming water. We assume that swimming causes no externality, and that there is no alternative swimming pool. Aggregate utility is then given by:

$$U(x,q;n) = n\,u(x/n,q), \tag{5.17}$$

where U(.) is the aggregate utility function, u(.) is the individual utility function, and where we assume that the aggregate quantity of the consumer good is divided equally among the number of consumers.

In analogy to equation (5.8) that we used for biodiversity, (5.17) reveals that the use of the environmental resource for waste disposal is a rival service. Its *per capita* quantity decreases if the number of people increases, whereas the quality of the resource is a non-rival good, since the use for swimming water is not reduced by the number of consumers. ■

In this example, the same environmental resource produces both a rival and a non-rival service. It much resembles the stylized environmental resource of the numerical illustrations in previous chapters. This is no surprise if we realize that the production of rival and non-rival services represents a general feature of many complex environmental resource systems [Siebert 1987, Ch.5]. Instead of a lake, we could have described a forest that produces timber, and that filters the air of, say, dust. These services are rival. In addition, forests produce aesthetic values for tourists, and as long as the forests do not become congested, the quality of the forest produces a non-rival good (e.g. diversity of trees [Van Soest 1998]). The air, in its relation to ecosystems, also produces rival as well as non-rival services. Not only with respect to climate change, but also in relation to local problems of air pollution, we find that the use of air for emission absorption is a rival service, while the use of air for breathing is non-rival. The multitude of possible examples suggests a common character. Indeed, they all concern environmental media in which emitted substances readily mix and they appear at all different scales of environmental processes, and it is the speed and spread of this diffusion process that is decisive in determining whether the environmental functioning can be represented through pairs of rival and non-rival goods.

5.4.2. Non-convexities, indivisibilities and diffusion processes

Besides non-rivalry, environmental production processes might also be characterized by non-convexities. This can be a source of inefficiency, since the pure decentralization through prices as it exists in the competitive model can lead the economy towards local welfare optima that have substantially lower welfare levels than the global optimum, or to so called 'saddle points', or even to local minima [Baumol 1964].

A simple example of a non-convex environmental resource system is given by Forster [1975]. Consider a lake, as in Example 5.1, and assume that its emission absorption capacity is not limited by $\bar{q}$, but extends beyond the point where the amenity value has become zero. This represents a situation in which, say, the lake is biologically dead and unfit for swimming, so that increased pollution does not affect the amenity level. The production set of feasible outputs, (x,q), has become non-convex. Forster shows that the non-convexity results in a complex optimization program in which the first-order conditions are insufficient to determine the welfare optimum, in particular in a dynamics analysis. After Forster, several authors have analyzed examples of welfare optimization under non-convexities associated to environmental resources [Lewis and Schmalensee 1977], [Tahvonen 1996].

Here, it is not our objective to discuss comprehensively welfare optimization and decentralization under non-convexities, for which we refer to [Ginsburgh and Keyzer 1997, Ch. 10], but we follow the typology introduced there to discuss briefly the nature of the environmental non-convexities, their cause, and at which levels we can expect them to emerge. Let us relate the non-convexities to the level at which the underlying indivisibilities occur. At the lowest, process level, indivisibilities are most common. A single wheat plant is indivisible, and hence, its yield can have increasing returns to scale with respect to inputs such as fertilizer use for low levels, and decreasing returns for higher fertilizer use levels. However, at the level of a farm, the farmer can freely divide his area and fertilizer among different crops. Consequently, production becomes divisible, and at farm level, indivisibilities can be neglected, and the production set is convex.

More in general, at a level above the physical scale of a process, the production set becomes convex. Thus, many non-convexities that can be expected to emerge at the scale of environmental processes, will vanish at a higher level. The issue is therefore to indicate the level at which environmental processes operate, and this depends largely on the relevant diffusion processes. For the example of the lake above, if the emitted pollution readily mixes through the lake, but does not leak downstream or to groundwater reservoirs, then the size of the lake determines the scale of the process. At this level, non-convexities can emerge. If at a higher, regional level, there are many lakes that have no leakage, then the production set becomes approximately divisible and convex. Clearly, such a 'convexification' through aggregation is impossible for truly global processes. Since CO_2 emissions rapidly spread through the atmosphere, it is impossible to keep them within the, say, northern hemisphere. The scale of the process is necessarily global. Consequently, if non-convexities emerge with respect to climate change, this happens at the global level, consistently with the findings in Section 5.2, and has to be accounted for in a global model.

Yet in most cases, the scale of environmental processes is not only global or local. Consider the use of persistent pesticides that diffuse through both biotic and abiotic media [Pethig 1991]. In the short term, these pesticides only affect the local flora, fauna and people directly exposed. In the medium term, the pesticides diffuse locally and regionally through the food chain and possibly pollute water reservoirs. In the long term, traces are found to have spread globally, possibly affecting ecosystems everywhere.

If one aims at including such heterogeneous spatial distributions of causes and effects in environmental resource systems, one has to represent the economy in a spatial continuum. This is not easy, and requires advanced techniques for the computation of optimal allocations [Ermoliev and Norkin 1997] [Ermoliev *et al.* 1997], [Keyzer and Ermoliev 1998].

5.5. DEMOGRAPHY, SUSTAINABILITY AND NON-EXTINCTION

5.5.1. Demand and supply of essential resources

Finally, at the end of this study, we return to the issue of demography, that was considered by the early economists to be at the heart of the sustainability issue. So far, we neglected endogenous demography, and in this section, we attempt to compensate this by a brief discussion.

The relation between environmental resource scarcity and population dynamics has been subject of scientific inquiry from the 18^{th} century on. Adam Smith [1776] takes a 'liberal' perspective as he argues that 'every species of animals naturally multiply in proportion to the means of their subsistence and no species can ever multiply beyond it.' The argument suggests that population growth will adapt to the environmental resource supply without requiring public guidance. Abundance of environmental resources will be manifest through relative high wages and these induce population growth, while environmental resource scarcity will result in relative low wages which induce a check on the population level. Recall from Section 1.5, that Malthus [1798] was not convinced that such adjustment processes are effective for population growth. He feared that 'the passion between the sexes' would lead to an exponential population growth that could only be checked by 'vice and misery'. To prevent future poverty and starvation, he advocated strong regulative measures. Godwin [1820] fiercely rejected the hypothesis of autonomous population growth, which in his opinion was based on unscientific reasoning, in sharp contrast with empirical evidence that showed great diversity of growth rates of population around the world and fast changes in these rates. Godwin held the view that intellectual and moral improvement and good governance would be capable of establishing perfection and happiness on earth, including the realization of an optimal level of population.

These three perspectives are still represented in present day concerns about sustainability. We summarize them as three hypotheses: (i) Population and economic production will automatically adjust to sustainable levels if environmental resources are well incorporated in the market economy. (ii) Population and economic production grow autonomously until scarcity leads to a sudden decline in welfare. Finally, (iii) increased social awareness will lead to the use of either economic or other instruments that direct the society towards sustainability. The second hypothesis falls outside the scope of this study, as it precludes the use of normative economic analysis for studying the issue. There is an intermediate hypothesis between the first two, which states that (ii') population and production growth rates adjust to environmental resource scarcity, but this adjustment is not by itself sufficient to prevent unsustainability. In this section, we will confine ourselves to a comparison between the optimistic hypothesis (i) and the more pessimistic hypothesis (ii'). In Section 5.5.3, we extent the trust fund to take care of population growth, which falls in the third hypothesis.

In Chapter 4, we considered sustainability in relation to economic production for given population dynamics. Here, we focus on economic processes behind endogenous demographic transitions that might direct the economy towards sustainability, or fail to do so. For this, we reflect on the concept of optimal population levels in relation to available environmental resources, and discuss the effects of (future) environmental resource scarcity on current fertility and the way this directs population dynamics. Finally, we have to investigate whether the adjustment of fertility to environmental resource scarcity is sufficiently fast, to ensure sustainability and non-extinction in the long run.

5.5.2. Welfare and the optimal population

Given limited resources, there is a trade off between the number of people and per capita resource availability, and this affects per capita welfare. There is a difference of opinion in the literature whether maximization of average welfare is the social objective to be pursued in normative analysis, or maximization of total welfare [Nerlove *et al.* 1982]. The Millian (average) social welfare function is maximized if the number of people with which the resources has to be shared is minimal. The alternative Benthamian (total) welfare maximizes aggregate welfare, and in the current literature, this seems to be the preferred criterion [Meade 1955, Ch.6]; see [Ng 1983] for a choice-theoretic argument.

However, welfare maximization with perfectly efficient use of environmental resources cannot, under regular assumptions, prevent "the repugnant conclusion" [Parfit, 1984, Ch. 17]. Assume that per capita welfare is a continuous concave function of the per capita available environmental resource, and that welfare is supposed to be zero if no resources are available. Then, it follows that welfare increases if the number of people increases, while average welfare falls. We can in this way imagine an 'optimal' allocation in which every individual is nearly starving. To prevent this repugnant conclusion Parfit [1981], Meade [1955, Ch.6], Blackorby and Donaldson [1984] and Blackorby *et al.* [1995] specify a subsistence welfare level. If welfare of an additional person lies below this subsistence level, then social welfare is supposed to decrease. In practice, this amounts to subtracting of a critical utility level from individual welfare before aggregation to social welfare. Alternatively, we might think of a basic needs consumption vector that is necessary for raising children and for subsistence [Dasgupta 1969]; only if the available (per capita) resource level exceeds the basic needs, is it desirable for a person to be born.

The choice between aggregate welfare as the objective and individual utility as the objective remains unsettled, because the criteria reflect different ethical principles [Cowen 1989]. However, the discussion so far was based on a static perspective that ignores the dynamic relation between parents and children. Recently, an alternative 'laissez faire' perspective has come to the fore in which the parents decide upon the optimal population level for the next generation. The number of children and their welfare becomes an argument in the welfare function of the parents. Parents choose the number of children and the wealth they bequest in order to maximize a dynastic recursive welfare function [Razin and Ben-Zion 1975], [Dasgupta 1969], [Lane 1975], [Kemp and Kondo 1986], and [Becker and Barro 1988]. Under certain assumptions, the parents' decision coincides with the population level that children would choose themselves [Keyzer and Merbis 1995]. Some explicitly distinguish overlapping generations [Gigliotti 1983], but because parents and children are not bound to their own budget constraint, the resulting equilibrium remains a dynastic optimum. The relevance of this omission becomes clear in Section 5.5.3 where we give future generations claims for a minimum level of environmental resources.

We will not give a comprehensive analysis, and refer for a detailed overview to [Ehrlich and Lui 1997]. We restrict ourselves to economies in which parents have empathy for their children. This implies a recursive welfare function, which can readily be included in the dynastic welfare Program 2.8 of Chapter 2. Typically, the dynastic framework implies intergenerational transfers, for example, because parents pay for the education of their children [Barro and Becker 1989]. It is possible to include costs of living, or a (convex) efficiency wage relation [Rosenzweig 1988, section 2.1.3], [Keyzer and Merbis 1995] without altering the main properties of the program. If welfare per capita is a concave function of consumption per capita, then aggregated welfare remains a concave function of total consumption and population size, and if the program is feasible and has a strict interior, there exists an optimal solution. The question is whether the optimal solution is sustainable, and whether this is sufficient.

Recollect from Chapter 4 that we defined sustainability as a set of conditions that the equilibrium should satisfy. Every consumer should be able to meet his basic needs. This condition will also have to hold for an equilibrium with endogenous population. If parents have sufficient empathy for their children, they will, also in an OLG framework, ensure that each next generation satisfies the sustainability condition. The question is whether additional sustainability conditions should be imposed for the population level itself, as there might be a 'last generation' which decides to have no children [Parfit 1981, p.170]. A reason for this decision might be that the environment has lost its carrying capacity due to past over-exploitation. This is the opposite of the classic 'repugnant' conclusion with overpopulation.

If we are only concerned with sustainability as the guarantee for a minimum quality of life for existing people, a 'last generation' phenomenon is no reason to reject an equilibrium. However, we might argue that such an equilibrium is too meager, and demand non-extinction, that is a strictly positive population for the indefinite future, or 'strong non-extinction' if the population level has to be bounded away from zero. We can now say that the 'last generation' situation violates the non-extinction condition as it prevents future generations from coming into existence. To facilitate the discussion, it may be useful to define an 'extinguished' allocation as one in which the total number of people that live over an infinite horizon is bounded, and a 'non-extinguished' allocation as one where it is unbounded.

The question whether one should strive for non-extinction is a complex ethical issue, that has received only limited attention in the economic literature. Parfit [1981] concludes that, though there is no clear-cut ethical argument against human extinction, it should be avoided. The issue is in some ways comparable with the distinction between maximal average and maximal total welfare. In an extinguished equilibrium, there is room for a finite number of people to live, and therefore aggregate welfare is bounded. If maximization of unweighted and undiscounted aggregate welfare is the aim, any non-extinguished sustainable equilibrium will be preferable to the extinguished one. On the other hand, if maximal average welfare is the objective, an extinguished equilibrium might be preferable in case resource exhaustion leads to a substantial increase in current welfare.

One reason for the 'last generation' problem to be so rarely discussed in the economic literature might be the specification of environmental resources, which are typically reduced to constant stocks (e.g. land [Eckstein *et al.* 1988]) that induce decreasing returns to scale for the production factors which they are combined with. These models abstract from at least two important characteristics of environmental resources that were mentioned in Chapter 1: the possibility of irreversible damages by over-exploitation, and the time delays between cause and effects. In such models, every generation can adjust its population level, or the number of its offspring, to bring it in line with the fixed amount of environmental resources. Overpopulation in a given period has no ever lasting consequences for future periods. Consequently, these models will possess sustainable steady states with fixed positive population levels, and often converge to it.

Once we include irreversible damage and time delay, a 'last generation' problem can arise. A stylized example may illustrate the point.

EXAMPLE 5.2. *The last generation*

Consider an economy with discrete time periods $t=1,\ldots,\infty$, endogenous population, n_t, one exhaustible resource, $k_{t+1} \leq k_t$, which is used for the production of a consumer good, denoted by y_t. There is a given time delay of s periods between changes in the resource level and changes in output: $y_t \leq k_{t-s}$ (and $y_t \leq k_1$ for $t=1,\ldots,s$). The output can temporarily be increased, but this leads to over-exploitation in the sense that it exhausts the resource: $y_t \leq k_{t-s}+k_t-k_{t+1}$. The over-exploitation cannot be undone, since the resource is exhaustible: $k_{t+1} \leq k_t$. The per capita basic needs consist of one unit of the consumer good, and the surplus of consumption is denoted by x_t, so that the commodity balance reads: $n_t+x_t \leq y_t$. Per capita welfare is given by the square root of surplus consumption, and total welfare is linear recursive. For convenience, we abstract from overlapping generations, but this does not affect the analysis. The resulting welfare program is given by:

PROGRAM 5.3. *Dynastic welfare program with endogenous population*

max w_1

$w_t, n_t, x_t, k_{t+1}, y_t \geq 0$ for $t=1,\ldots,\infty$

subject to

$$w_t \leq \sqrt{x_t n_t} + \beta w_{t+1} \qquad t=1,\ldots,\infty$$

$$x_t + n_t \leq y_t \qquad t=1,\ldots,\infty$$

$$y_t \leq k_1 + (k_t - k_{t+1}) \qquad t=1,\ldots,s$$

$$y_t \leq k_{t-s} + (k_t - k_{t+1}) \qquad t=s+1,\ldots,\infty$$

$$k_{t+1} \leq k_t \qquad t=1,\ldots,\infty$$

for given initial resource stock $k_1>0$ and time factor $\beta<1$.

It is obvious that the program is feasible, convex, has a strict interior solution, and that in the optimum, half of the resource is used for basic needs and the other half for surplus consumption: $x_t=n_t$. From this, it follows that welfare in period t is linear proportional to the available resource: $\sqrt{x_t n_t} = \frac{1}{2}(y_t + k_t - k_{t+1})$, that the additional welfare derived from extraction is equal to the half of the extracted amount, and that the opportunity costs are equal to the integral of the indefinite aggregate non-extractive use from period s on. For s sufficiently large (or stated the other way around, for β sufficiently small), it is optimal to exhaust the resource immediately, and to reduce fertility to zero after period s. ■

As all people living along the equilibrium path described above are able to satisfy their basic needs, the equilibrium is sustainable. There is no regret by future generations that do not come into existence. Nonetheless, we do not accept the extinction, because there exists a feasible alternative that is sustainable and non-extinguished. Though the example is very simple, the conclusion stands that there is no inherent mechanism in the economy to prevent extinction.

The conclusion is not restricted to dynastic models; budget constraints for every generation will not prevent extinction either. For example, as mentioned before, the OLG model by Mourmouras [1993] has an equilibrium with resource levels converging to zero. It is possible to have endogenous population in this model such that the population levels remain positive, but converge to zero, and a sustainable equilibrium results in which every consumer has a utility level that exceeds the critical level. This equilibrium violates the strong non-extinction condition.

5.5.3. Extending the trust fund to ensure non-extinction

Suppose that we accept strong non-extinction as an additional requirement for an equilibrium to be considered equitable. Then, we consider it the responsibility of the present generation to ensure that all future (unborn) generations have the right to come into existence and to enjoy a minimal quality of life.

Under this requirement, it follows from the previous section that it is insufficient to let the parents choose the optimal level of their offspring, even if they base these levels on the welfare of the children themselves. Moreover, if environmental services have non-rival characteristics, it is even less probable that endogenous population adjustments automatically lead to non-extinction, because in this case, population reduction does not relieve the remaining individuals from the scarcity of the environmental resource. The issue is now to specify rights for the unborn generations, and the trust fund can be helpful to implement it.

Extension of the trust fund to an economy with endogenous population is not a trivial matter. Recall that the trust fund endows present and future generations with claims for environmental services that act like pseudo-endowments in the competitive equilibrium. These claims are based on a feasible production path, which guarantee that they can be paid out of the initial value of the environmental resource. With endogenous population, in every period, the aggregate claim has to be shared among the people alive. The per capita claim is therefore endogenous, and two distinct distribution schemes can be thought of.

First, the aggregate claim for period t can be divided equally among all the individuals alive in period t. Given perfect foresight, every parent knows in advance the claim his offspring will be entitled to. If the population level decreases, per capita wealth of the individuals alive increases, and this could create an incentive for increased fertility, resulting in strictly positive populations throughout the future. The mechanism that drives demographic changes is essentially the same as for an economy with a constant environmental resource flow (independently of the population level), and we conjecture that existence and sustainability of the equilibrium can be proven in a relatively straightforward manner. However, this approach has the major drawback that the distribution rule produces an externality, because all parents consider the effect of their fertility on the total offspring negligible. As a result, population will tend to exceed its optimal level.

Efficiency is restored in the second distribution scheme, that distributes present and future claims to both parents and offspring, i.e. to dynasties. Because the parents incorporate the relation between the size of their offspring and the *per capita* claims, the externality of the first distribution scheme is avoided.[19] In this economy, if the size of the dynasty increases, per capita wealth of its members decreases, and vice versa. However, in contrast to the previous scheme, there is the possibility of an uneven intra-generational distribution of wealth, a poverty trap. If, for some reason, rich people tend to have less children than the poor, wealthy parents with relatively large claims will have low fertility rates which implies that their offspring have relatively even larger per capita claims, while poor parents with relatively small claims will raise many children with even less per capita claims. If parents take account of subsistence welfare for their children, poverty will not be extreme, but such an uneven distribution of wealth goes against the basic spirit behind the trust fund.

At any rate, the trust fund will increase the size of future generations because it increases future wealth. In this way, the intergenerational distribution of entitlements is an instrument that affects the population dynamics. Moreover, if an increased per capita

[19] For the same reason, Nerlove *et al.* [1986, p.18], argue that in an economy with endogenous population, optimal taxes necessary to finance public goods should be levied per dynasty, instead of an equal levy per head for every generation.

wealth of the offspring increases fertility (a usual feature of most models), then the trust fund will ensure a strongly non-extinguished sustainable equilibrium.

6. CONCLUSIONS

6.1. SUMMARY OF MAIN RESULTS

Chapter 1 emphasized four characteristics of environmental resources that call for a treatment different from that of man-made resources. First, they are essential to support human life and thus, in abstract terms, they provide an essential contribution to welfare, which, secondly, can be continued indefinitely. Thirdly, if overexploited, the resources become irreversibly damaged and their production capacity decreases. Fourthly, there are significant time delays between the economic cause and the environmental effects. The present economic institutions do not safeguard the efficient, or the sustainable exploitation of environmental resources, and this implies both a waste of welfare and the possibility of welfare of future generations becoming unacceptably low.

In Chapters 2, 3, 4 and 5, we have developed a formal framework in which we analyzed this issue, and in which we distinguished two different approaches. The first, dynastic one, assumes a benevolent dynastic planner, who owns the endowments of all generations, and distributes income according to his welfare function. This implies a kind of slavery, since the central planner has at its disposal the income of all consumers, who might in a certain generation be left with even less than their income from labor. This structure could sustain rather inequitable consumption patterns, as was seen in Section 2.6.2. Alternatively, we adopt the possibly more realistic framework of an overlapping generations economy, in which property rights over endowments fully determine the competitive equilibrium. The outcomes were as follows.

(i) *In OLG economies,* (a) *dynamic efficiency is restored by a non-negligible environmental resource if grandfathered to the first generation, or alternatively,* (b) *by a non-negligible claim.*

The illustration with ALICE 0 showed that the predicted aging in the next century could cause an increase in life-cycle savings, and a downwards pressure on the real interest on these savings. Ultimately, aging is capable of shifting the economy towards a dynamically inefficient equilibrium with negative interest rates. This decreases life-cycle income because consumers have to save more when young to obtain the same retirement payments when old. The presence of a 'non-negligible environmental resource', or the issuing of a non-negligible claim by a public authority restores efficiency and increases life-cycle income.

(a) The fact that many environmental resources are capable of producing an indefinite flow of valuable services without any maintenance costs (the first two characteristics) makes these resources into 'non-negligible' assets, whose value constitutes a strictly positive share of the total endowment value. The incorporation of these resources restores efficiency in economies with overlapping generations if given as endowments to the first generation.

(b) Alternatively, if there are no non-negligible resources, or if these resources are not given to the first generation, efficiency can be restored by issuing a 'non-negligible' claim. The non-negligible claim is a tradable asset that entitles in every period the owner to revenues that are equal in value to a fixed (small) share of the endowments in that period. This revenue is obtained from lump-sum taxes. The non-negligible claim is an implicit public debt, as it implies a commitment on future payments. Thus, the analysis suggests that a positive public debt can sometimes be preferable to a zero debt, as it can ensure dynamic efficiency. The advantage of the non-negligible claim is that it automatically generates the public debt level that is necessary and sufficient to ensure dynamic efficiency.

(ii) (a) *In dynastic economies, sustainability can be established through utility constraints.* (b) *In OLG economies, it can be established through a trust fund.* (c) *In economies with endogenous population, sustainability is ensured by the empathy of parents for their children, but the trust fund is needed to guarantee non-extinction.*

In the past, Pigovian taxes have figured prominently in the environmental economics literature which was mainly concerned with the efficiency of resource use. More recently, the attribution of property rights over environmental resources has received increased attention. The idea is that if the present generation receives environmental resources in private ownership (grandfathering), this generation will look after these resources with due care as these resources represent an enormous value. Giving environmental resources their proper price should presumably be sufficient to guarantee sustainable development. This optimistic view has not been shared by ecological economists who do not want to rely on efficiency improvements, and as an alternative, propose strong sustainability measures that are based on the secure but inefficient physical control of environmental resource use. These concerns find further support in the literature which shows that efficiency does not imply sustainability.

For practical environmental policies, these findings have far-reaching consequences. They imply that 'giving the environment its price' is only a first step towards a sustainable development. Indeed, the price of the environment and, hence, its optimal use will be sensitive to the distribution of property rights.

In this study, we looked for policy measures that can combine efficiency with sustainability, and non-extinction as an extension of the latter for economies with endogenous population. This led to the following results.

(a) In dynastic economies, sustainability can straightforwardly be implemented through explicit criteria in the welfare program. The efficient and sustainable optimum can afterwards be decentralized as a competitive equilibrium with income transfers. There is no need for imposing an exogenous zero or low rate of discount, an often proposed alternative that unfortunately generates unrealistic allocation patterns which do not seem to represent consumer preferences. Another alternative, the imposition of constraints on the use of environmental resources, has the disadvantage that it distorts the economy and thereby leads to inefficient resource use.

(b) In an overlapping generations economy, it becomes impossible to impose exogenous sustainability criteria, since property rights over endowments fully determine the competitive equilibrium. For these economies, we proposed the creation of a trust fund that protects the interests of future generations, and at the same time, secures efficiency, as it allows for trade in ownership titles over environmental resources. The trust fund receives the initial property rights over these resources, which it uses to entitle all present and future generations to claims, or pseudo property rights, for the maximal sustainable income that can be produced from these environmental resources. Loosely stated, these 'claims for a clean environment' ensure that future generations can pay for a clean environment. This set up guarantees that the proper price signals are sent, from the future to the present, to keep the environment clean. The trust fund is an institution that, once the claims are recognized as pseudo-endowments and set on equal footing with the standard property rights, relieves the present and future generations from the recurring task of bringing their use of environmental resources in agreement with their empathy for future generations. It is worthwhile to add that the entitlements from this trust fund provide a minimum income for everyone, and can thus serve as the basis for a world-wide social security system.

(c) The initial set up of the trust fund as formulated in Chapter 4 was based on exogenous population dynamics. If we take account of parents' decision on the number of children, and if we assume that parents have empathy for their offspring, the fertility decision will most likely prevent unsustainability, in the precise sense that parents will not bring forth children if these are unable to meet their basic needs. However, this cannot rule out the possibility of over-exploitation of environmental resources with parents in some generation deciding not to raise children any longer, a situation that leads to human extinction. A trust fund can prevent this from happening, because all future generations will have the entitlements to sufficient environmental resources.

(iii) *The environmental resources produce a continuum of steady states, and this results, both in dynastic and OLG economies, in path dependency.*

As shown in Chapter 2, the property that environmental resources can be damaged irreversibly (the second characteristic) causes the steady states to form continuums. This in turn implies that different equilibrium paths do not converge to the same state in the long term, even if, initially, these equilibria are arbitrarily close to one another. This means that there is path dependency, and that policies in early periods affect the economy in the indefinite future and that these effects do not dampen out.

The decrease in welfare caused by over-exploitation of an exhaustible resource in the early periods, i.e. exploitation that exceeds the optimal level and irreversibly deteriorates the resource, does not diminish. Therefore, if environmental resources can be damaged irreversibly, there is an urgent need for policy intervention, because any delay might lead to an indefinite welfare loss.

(iv) *A practical way of representing the deterioration of environmental quality is through a decrease in the supply of non-rival environmental goods.*

The application of the theory to practical problems is not without difficulties. However, one important problem, that environmental degradation is a loss of quality, not only a loss of 'environmental quantity', can be handled by our formal model. We found that environmental resources typically provide both rival and non-rival services where the former are associated with the capacity to absorb pollutants, and the latter with the amenity value. Thus, within the formal model, it becomes possible to represent these basic characteristics of many environmental processes.

(v) *Finally, with respect to climate change, the intergenerational distribution of resource entitlements is expected to have substantial consequences for the optimal emission levels.*

The problem of climate change makes clear that sustainability is not a hypothetical issue. Research in natural sciences suggests that climate change can indeed be a serious threat for future generations. Even a strong sustainability policy that bans GHG emissions as soon as possible cannot prevent continued warming of the global temperature in the next century. Assuming that the 'pre-industrial' atmospheric conditions were optimal, this implies that some welfare loss has already become inevitable.

Though damages from climate change can be severe, many studies with IAMs suggest that it is efficient to delay emission reductions, because the discounting of future costs reduces their net present value so much that they become negligible. This result is consistent with our findings in the grandfathering scenarios, which lead to (efficient but) modest emission reductions only. Given the exponential increase in energy-related emissions, the emissions continue to rise. In 2100, the global temperature will have risen by 3 degrees Celsius, while the trend continues with an increase of nearly 5 degrees Celsius in 2200.

However, the price of emission units and the optimal use of the biogeochemical system depends on the actual distribution of the property rights. In many IAMs, a GHG-tax is levied the revenues from which are redistributed to the current generations, and this implicitly assumes that these generations acquire the rights to use these emission units. If we want to ensure sustainability as well as efficiency, future generations should receive a claim to 'a clean environment without climate change'. This can be achieved through the trust fund. In this scenario, revenues from selling GHG emission units are transferred to future generations. The numerical illustrations show that, under this policy, the emission price rises gradually, and that this triggers an economic transition during the 21st century towards energy technologies with zero emission. The CO_2 concentration peaks shortly after 2050, and the global temperature increase is checked at the level of 2.5 degrees Celsius by 2100. Because of the immense time delay (the fourth characteristic), the benefits from the trust fund only appear after 2100, but, remarkably, the effect of the trust fund is robust against time delays.

6.2. FURTHER ANALYSIS WITH ALICE

As regards the formal and applied analysis with the ALICE model, several issues need further attention.

(i) *Further formal analysis of ALICE 1*

In the formal analysis of ALICE 1, three issues were left uncovered. First, we have shown that the steady states in ALICE 1 form continuums (Theorem 3.4), but we did not characterize these, nor did we analyze the equilibrium paths towards them. For a stylized economy such as ALICE 1, it is probably possible to give a more complete characterization of both the continuums of steady states and the equilibrium paths. This could improve our understanding of these phenomena.

Secondly, we did not extend the formal welfare analysis to ALICE 1.2 which has man-made capital as a partial substitute for environmental resources. It might be possible to prove the same theorems as for ALICE 1.0 and 1.1 without the man-made capital stock (Theorems 3.5, 4.4, 4.5, and 4.6), or alternatively, to point out under which conditions these theorems do not hold. This would provide a test of the robustness of the numerical results on welfare gains through the trust fund (Table 4.4).

Finally, the numerical findings (Table 4.3) suggest that time delays are a major source of potential unsustainability of resource use. Further analysis of the stylized model ALICE 1.1 could help clarifying this issue.

(ii) *Optimal climate change policy and aging*

ALICE contributes to the existing IAMs, as used for optimal climate change policy analysis, in two ways. First, it enables us to implement an explicit intergenerational distribution of property rights, whereas other IAMs use the dynastic perspective or limit the distribution analysis to a small part of the environmental resource only, e.g. the production of emission units. This characteristic of ALICE has fully been exploited in this study. Secondly, ALICE contains an explicit specification of aging, and it enables us to analyze the effects of this phenomenon on the equilibrium dynamics.

However, throughout the illustrations, we paid limited attention to the second characteristic of ALICE. The numerical results suggest that aging leads to a lower interest rate and, because this favors the maintenance of a high-quality environment, to a reduction of present GHG emissions. The calculations were based on a fully-funded system of social security, and presumably, a pay-as-you-go system will give other results. This is also an interesting issue for further research, since most IAMs assume that the future real interest rates are only linked to economic growth, and this assumption is critical for most of their quantitative results.

6.3. Issues Outstanding

As noted in the outlook in Section 1.4.2, this study has abstracted from properties that cannot be neglected once the theory has to be put into practice. It is clearly impossible to discuss all of these comprehensively, but we will return to the issues that were mentioned in Section 1.4.2 and add some new ones for which further analysis is suggested on the basis of the current results: (i) distorted markets, (ii) endogenous technology, (iii) endogenous preferences, (iv) uncertainty, (v) the representation of the production set and the production decision for environmental resources, (vi) dynamic efficiency and path-dependency in monetary economies, and (vii) sustainability, non-extinction and endogenous fertility.

(i) *Market distortions*

Throughout the study, we have assumed that commodity markets were functioning optimally, though we allowed for intertemporal inefficiency. This implies that, within every period, the economy makes efficient use of its resources. Real world economies suffer from market imperfections of various kinds that lead to Pareto inefficiencies. This will affect the optimal environmental pricing.

For example, if environmental resources are priced through Pigovian taxes, these will interact with pre-existing distortionary taxes [Bovenberg and Goulder 1996]. It might even be possible to reap a 'double dividend', a situation in which eco-taxes are less distortionary than other existing taxes, and therefore both the economy and the environment would benefit from a shift.

Market imperfections have several implications for our results. When distortions prevail, the non-negligible claim no longer ensures efficiency in the infinite horizon OLG economy. Yet it might be part of an efficient tax policy, being preferred to other distributive instruments that are more distortionary. As regards the trust fund, we notice that it can only function if some kind of markets exists for the environmental resources considered. If this is the case, and if one is willing to disregard the trade offs between intra- and intergenerational equity, the trust fund specified in the earlier chapters can function without much adjustment. The claim received ensures that every generation can meet its basic needs.

(ii) *Endogenous technology, economic growth, and sustainability*

As Mill [1848] already noticed, technological innovation increases the opportunities to use the environmental resources more effectively, and thereby potentially provides the means for offsetting a rising resource scarcity. Much of the recent literature takes such an optimistic view. It does not question the capacity of overcoming resource scarcity and restricts the analysis to the effects of environmental policy on economic growth taking its positive sign for granted, see e.g. [Bovenberg and De Mooij 1994]. Within this optimistic perspective, it is natural to question the need for redistribution of environmental resource use, as one believes that technological innovation will reduce the resource exploitation even in the absence of enforcing policies [Chakravorty *et al.* 1997]. As mentioned in Chapter 1, this optimistic perspective has a psychological dimension and does not rest on scientific grounds. Clearly, if technology can overcome all problems 'automatically', there is no need for a trust fund.

Nonetheless, ecological economists will argue that technology will not be able to substitute for ecosystem and species extinction, and therefore, it will be necessary to safeguard these by use of economic instruments such as the trust fund. Moreover, even in cases where technological innovation can offer solutions, the trust fund could stimulate the development of these technologies, since it sends the proper price signals to ensure a transition of economic production towards sustainability.

Another aspect of technological innovation is its contribution to economic growth. In this study, we assumed an upper bound on production and consumption levels. If, however, economic growth is sustained for several decades, it can be argued that welfare will increase so much that this growth offsets any welfare loss caused by environmental damages. In this case, there is no reason to worry for future generations, because their welfare derived from man-made consumer goods will exceed our welfare anyway.

This line of reasoning is highly questionable. If marginal welfare derived from additional consumption of man-made consumer goods diminishes as the consumption level rises, it will be impossible to compensate for environmental damages by any increase of man-made consumer goods. Usually, this issue is stated as the question whether man-made consumer goods are a good substitute for environmental goods. A more precise analysis of this issue could prove helpful to clear up the difference of opinions between ecological economists who do not believe in the substitution possibilities of man-made consumer goods, and other economists who consider the concerns for future generations exaggerated.

(iii) *Endogenous preferences*

As most welfare economic analyses, this study was based on the questionable presumption of rational behavior and exogenous preferences of consumers [Duesenberry 1949]. Some critics of the free market have argued that consumption only serves to satisfy the wants that are created by the producers themselves [Koopmans 1957] [Mishan 1967], others that consumption is used to increase the social position, rather than to satisfy the individual needs [Easterlin 1974]. These critics argue that economic growth is an inadequate yardstick for measuring welfare growth and happiness [Scitovsky 1976], and that certain taxes on 'observable or positional goods' might compensate for distortions instead of creating them, and thus actually increase welfare [Frank 1985] [Howarth 1996]. If one adopts this viewpoint, the value of non-rival natural services will be underestimated relative to the value of rival man-made goods by any market-oriented valuation technique [Jaeger 1995]. The consequence would be that higher priority to nature conservation should be given than most models, based on the low estimates, suggest. Though these arguments cannot be rejected completely, it is nearly impossible to correct for induced preferences in order to recover 'authentic' preferences, as the psychological and social process of preference formation is highly complex and does not lead to tractable formulations. Therefore, we must accept the revealed preferences as if they were exogenous, since the alternative easily leads to paternalism with the researcher deciding for others what is best for them.

(iv) *Uncertainty*

The abstraction from uncertainty might be one of the major limitations of this study. To avoid confusion, let us distinguish the idiosyncratic uncertainty at the individual level from uncertainty at the aggregate level. Starting with the former, to ensure sustainability in the sense that every consumer can meet his basic needs, every individual should have an opportunity to insure himself against unacceptable income loss because of private circumstances. This requires a complete insurance market for idiosyncratic risks, but unfortunately, all insurance related problems are not resolved in this way, since some people are born with valuable talents, while others are born without. Thus, uncertainty at the individual level calls for some intra-generational social security, to bring about sustainability. Otherwise, the arguments of previous chapters hold.

Uncertainty at the aggregate level is different in several respects. In Chapter 4, we have shown that in a model with perfect foresight, unsustainability could persist, but the problem is worsened in case of aggregate risk. The trust fund might be accepted without much opposition if it were obvious that the present use of environmental resources ultimately leads to a deteriorated environment in which future generations cannot meet their own needs. On the other hand, if it was obvious that future generations will be capable of finding their own solutions to environmental resource scarcities, there would be no need for this trust fund. Because of uncertainty, pessimists ask for strong sustainability constraints to protect future generations, and optimists are unwilling to accept such 'needless' constraints on present-day welfare.

In standard competitive models, complete asset markets provide an insurance mechanism against aggregate risk. If these markets exist, the trust fund might be implemented along the same lines as in the economy with perfect foresight. The asset markets provide the means for hedging against future states in which environmental deterioration lead to extreme costs, and the trust fund helps to send the appropriate price signals through the asset markets to ensure sustainability in all possible future states. In short, the trust fund would not only engage in savings, but also buy insurance to safeguard its future commitments.

Unfortunately, as was pointed out in Chapter 1, the most important uncertainty arises at the fundamental level of understanding of environmental and economic processes. It is due to incomplete knowledge, and both pleasant and unpleasant 'surprises' can emerge. When the probabilities themselves are unknown, asset markets cannot provide hedging mechanisms. Moreover, there are extreme events with low probability and high risks for which it is difficult to develop insurance contracts, and as long as scientific research does not offer an improved understanding of the probabilities, policies have to rely on direct interventions that are based on alternative scenarios which represent accepted perspectives on global economic and environmental development. There simply is no better option.

(v) *Representation of the production set and the production decision*

While it has become clear from ALICE 2.0 that some physical aspects of environmental resource systems can be captured in the general equilibrium framework, namely the distinction between the quality and quantity of environmental services, it remains difficult to include other aspects, such as the explicitly spatial processes. This is mainly because the general equilibrium framework does not allow for a spatial continuum with various feed back relations, and the non-convexities that might emerge cause problems as well.

The spatial aspects and the numerous interactions between environmental variables at various spatial scales require considerable coordination to decentralize the optimal resource use. In other words, they make it difficult to partition environmental resource systems in sub-systems that can be managed separately. One might say that the standard assumptions *F1-F6* can be used to describe environmental resources, but that they are not fit to describe the spatial elements of environmental resource *systems*. Incorporation of the latter in a competitive economy is an important subject for further research.

(vi) *Dynamic efficiency and path dependency in monetary economies*

The applicability of the 'non-negligible claim' in monetary economies is a further matter of concern. We have purposely used the term 'non-negligible claim', instead of, say, 'welfare-linked bond', to emphasize that we only considered the redistributional effects of the instrument, and abstracted from the monetary effects. In a monetary economy, the non-negligible claim would be issued as a perpetual (welfare-linked) bond and function as a liquid asset. This will create several monetary side-effects. It will affect the interest rate as well as the inflation. If the non-negligible claim is a useful instrument to prevent dynamic inefficiency, then its monetary effects are an obvious subject for further research.

Moreover, in a more complex economy, a welfare-linked bond could serve other purposes than restoring dynamic efficiency. For example, in an economy with uncertainty, it can be an alternative for the standard bond in the portfolio of a fully funded social security system. The welfare-linked bond protects the pensioners against (positive) shocks in economic growth, which, in case of a bond with fixed interest, leave the pensioners with a lower income relative to employees who benefit from the shocks. Another example is that in a monetary union, governments can buy welfare-linked bonds from other countries to hedge against differences in productivity shocks between countries [Jacques Drèze, personal communication].

(vii) *Sustainability, non-extinction, and endogenous fertility*

The inclusion of the trust fund in an economy with endogenous population needs further analysis. Since Malthus, the interaction between population dynamics and resource scarcity lies at the heart of the sustainability problem. In this study, we only have suggested ways to extend the formal analysis to economies with endogenous population. The arguments suggest that empathy of parents for their children can prevent over-population, whereas the trust fund will ensure non-extinction. However, at the present stage, it is not precisely clear how to avoid that the trust fund creates an externality with respect to the fertility decision of parents and thereby interferes with the efficiency of the equilibrium.

Finally, there is still a long way from the theoretical sustainability policies as suggested in this study to practical implementation. And, since distributional issues are at stake, there will be much hesitance and resistance from many quarters. Fortunately, as regards climate change, political support for various new initiatives has grown recently. The Kyoto protocol [UNFCCC 1998], the carbon trading initiative of the World Bank [WB 1998], and the Global Environment Facility [GEF 1994], established after the Rio 1992 World Conference on Sustainable Development, illustrate the step-by-step shaping of a global regime, and all seek to price greenhouse gas emissions, whereby emitters will have to pay, while those who plant new forests or create other sinks to absorb greenhouse gases will receive a payment.

Yet these initiatives require an arbitrary decision about the acceptable level of annual emissions. Building on the principles of the trust fund that were analyzed in the present study, one could go one step further, and extend the mandate of the new international institutions, beyond the interregional sharing of annual carbon emission budgets, to the safeguarding of both intertemporal efficiency and fair sharing among generations. This would create for the current users of emission units the possibility of 'renting' their permits from the future owners.

However, we have also seen that it would be difficult to specify unambiguously the quantities of environmental resources to which property rights should be attributed, because the resource system one has to deal with is not fully representable in terms of resource stocks. Moreover, present day knowledge of the functioning of this system is most incomplete and subject of much controversy. Consequently, the trust fund should perform a task far more complex than that of a portfolio manager. It would have to keep on updating its regulatory activities on the basis of the latest scientific insights. This is a field to which environmental economics should make a major contribution.

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SAMENVATTING IN HET NEDERLANDS (SUMMARY IN DUTCH)

HET EFFICIENT EN DUURZAAM GEBRUIK VAN MILIEUHULPBRONNEN

Verscheidene mondiale milieuproblemen vragen onze aandacht, onder andere ontbossing, verlies aan biodiversiteit, aantasting van de ozonlaag, en het versterkte broeikaseffect door de uitstoot van zogenaamde broeikasgassen. Deze milieuproblemen worden veroorzaakt door menselijk ingrijpen. Hoewel deze milieuproblemen verstrekkende gevolgen kunnen hebben voor de welvaart van toekomstige generaties, ontbreekt het aan adequaat beleid. Door sommigen wordt 'sterke duurzaamheid' nagestreefd, gericht op de absolute bescherming van het milieu, vaak door middel van fysieke regulering. Zulk beleid kan tot onnodige beperkingen van milieugebruik leiden, en daarmee tot inefficiëntie. Anderen bepleiten de privatisering van milieuhulpbronnen of markt-conform beheer, om het efficiënt gebruik te bevorderen. Als men zich alleen op efficiëntie richt kan dat echter tot onherstelbare schade aan het milieu leiden, en daardoor komt de welvaart van toekomstige generaties in het gedrang.

In deze studie verkennen we de economische principes die gebruikt kunnen worden om optimaal gebruik van het milieu te bevorderen. Daarbij letten we zowel op het efficiënte gebruik van de hulpbronnen voor de productie van welvaart, als op een billijke verdeling hiervan tussen mensen. De *intra*generationele verdeling van welvaart (tussen mensen die in dezelfde tijd leven) is niet onbelangrijk, maar valt buiten het bestek; we beperken ons tot *inter*generationele verdelingsaspecten.

Als basis voor de welvaartsanalyse gelden de volgende vooronderstellingen en (Pareto) waarde oordelen:

(i) Ieder individu handelt rationeel en heeft zelf het beste oordeel over zijn eigen welvaart.

(ii) De sociale welvaart (dat is de welvaart van de maatschappij als geheel) hangt af van de welvaart van de individuen, en deze hangt alleen af van de uiteindelijke allocatie - verdeling van goederen -, niet van de manier waarop deze bereikt is.

(iii) Er is geen animositeit tussen individuen, en als de welvaart van de één stijgt, zonder dat de welvaart van een ander daalt, dan stijgt de sociale welvaart.

Op basis van deze vooronderstellingen en principes is het mogelijk om verschillende economische allocaties met elkaar te vergelijken. In het algemeen zal een allocatie die gebaseerd is op rationele beslissingen en vrije keuzen van de individuen tot een Pareto optimum leiden, dat is een allocatie waarin het niet mogelijk is de sociale welvaart te verhogen zoals in (iii) weergegeven.

De Pareto principes zijn echter niet in staat om verschillende allocaties te rangschikken indien de welvaart van sommige individuen stijgt en die van anderen daalt, en dus zijn ze onvoldoende indien we verdelingsvraagstukken willen analyseren. Het is voor verdelingsvraagstukken nodig om een expliciet waardeoordeel toe te voegen, bijvoorbeeld dat een verdeling alleen billijk is als het laagste inkomen voldoende is om in de basisbehoeften te voorzien. Voorbeelden van basisbehoeften zijn voeding, huisvesting, onderwijs, zorg, en een schoon milieu.

In deze studie wordt duurzaamheid geanalyseerd als een verdelingsvraagstuk:

Een duurzame ontwikkeling betekent dat iedere generatie voldoende inkomen heeft om in haar basisbehoeften te voorzien.

Deze definitie van duurzaamheid is gebaseerd op het rapport van de 'World Commission on Environment and Development' die voor een duurzame ontwikkeling pleit "that meets the needs of the present, without compromising the ability of future generations to meet their own needs". Deze definitie is niet vanzelfsprekend.

Enerzijds vinden zogenaamde 'ecologische economen' dat duurzaamheid directer moet worden gedefinieerd als het behoud van het milieu, en zij pleiten voor sterke duurzaamheid, zoals boven beschreven. Onze formulering in termen van inkomen laat de mogelijkheid open dat milieudegradatie wordt afgekocht met extra consumptie van door mensen gemaakte goederen. Het eerste Pareto principe maakt duidelijk waarom wij de voorkeur geven aan onze eigen formulering. De toekomstige generaties zullen zelf bepalen of zij een schoon milieu of extra consumptiegoederen prefereren. De huidige generatie beperkt zich ertoe om het huidige milieugebruik in overeenstemming te laten zijn met de verwachte toekomstige voorkeuren.

Anderzijds is het onder economen die vertrouwd zijn met inkomen als indicator voor welvaart gebruikelijk om duurzame ontwikkeling te definiëren als een pad waarin het inkomen en de welvaart niet dalen. Een belangrijk nadeel van deze definitie is dat zij zich niet eenvoudig laat uitbreiden tot intragenerationele verdelingsvragen. Onze eigen keuze heeft een duidelijke generalisatie: een verdeling van inkomen, over en binnen generaties, is duurzaam als alle consumenten in hun basisbehoeften kunnen voorzien.

We hebben nu aangegeven wat we onder (Pareto-)efficiëntie en duurzaamheid verstaan, nu moeten we nog verklaren waarom het gebruik van milieuhulpbronnen onze aandacht behoeft. De intergenerationele verdeling van welvaart hangt nauw samen met de hoeveelheid kapitaal die wordt nagelaten aan volgende generaties, en dat gebruikt kan worden voor de productie van welvaart. We onderscheiden daarbij milieukapitaal, zoals bossen en schone rivieren, van door mensen gemaakt kapitaal, zoals machines. De milieuhulpbronnen die samen het milieukapitaal vormen hebben enkele eigenschappen die van belang zijn voor de analyse van duurzame ontwikkeling. We beperken ons hierbij tot zogenaamde vernieuwbare hulpbronnen.

(i) Ze zijn essentieel voor de productie van welvaart.
(ii) Ze zijn vernieuwbaar, in de zin dat ze de capaciteit hebben om zich te regenereren en een oneindige stroom van waardevolle diensten te leveren.
(iii) Ze zij uitputbaar, in de zin dat een overexploitatie tot een onomkeerbare reductie van de productie-capaciteit kan leiden.
(iv) Er zijn substantiële vertragingen in de dynamica van de milieusystemen.

De eerste eigenschap maakt duidelijk dat het ons niet alleen om 'lokale' milieuproblemen gaat, maar om de aantasting van de draagkracht van mondiale systemen, zoals de verminderde biodiversiteit en het versterkte broeikaseffect. De tweede eigenschap geeft weer dat het milieu een (in menselijke termen) oneindige bron van welvaart is, en dus zeer waardevol. De derde eigenschap benadrukt dat een zorgvuldig beheer nodig is. De vierde eigenschap, ten slotte, betekent dat maatregelen uit voorzorg moeten worden genomen, voordat milieuproblemen urgent zijn.

De laatste drie eigenschappen onderscheiden milieukapitaal van door mensen gemaakt kapitaal. Dit vergaat maar is goed vervangbaar door nieuw kapitaal, en de tijd die tussen de investeringen en het gebruik van het kapitaal ligt is verwaarloosbaar in verhouding tot de vertragingen in de milieudynamica. Daarom richten we ons vooral op het milieukapitaal. Indien dit wordt aangetast kan dat verstrekkende gevolgen hebben, die niet eenvoudig te corrigeren zijn.

In deze studie wordt het versterkte broeikaseffect als illustratie gebruikt. Het onderliggende biogeochemische systeem - dat bestaat uit in elkaar grijpende biologische, geologische en chemische cycli - heeft alle vier de boven beschreven eigenschappen. (i) Het huidige mondiale klimaat biedt een geschikte omgeving voor diverse ecosystemen en daarmee voor de voorziening in menselijke basisbehoeften. (ii) Het huidige klimaatsysteem lijkt, binnen de *natuurlijke* variatie, ook op de langere termijn, stabiel. (iii) De huidige klimaatmodellen geven aan dat de antropogene emissies van broeikasgassen tot onomkeerbare klimaatveranderingen (kunnen) leiden. (iv) De gevolgen van huidige emissies zullen naar verwachting pas over tientallen tot honderden jaren volledig duidelijk worden.

De analyse vindt plaats binnen de context van de welvaartseconomie. We formuleren een algemeen dynamisch competitief evenwichtsmodel en werken dit uit voor zowel een zogenaamd dynastisch (Ramsey) model, waarin een centrale planner de sociale welvaart maximaliseert voor alle generaties tegelijk, als voor een zogenaamd overlappende generaties (OLG) model, waarin iedere generatie haar eigen inkomsten en uitgaven met elkaar in overeenstemming brengt. Daarnaast ontwikkelen we een reeks van gestileerde modellen, ALICE, dat staat voor 'Applied Long-term Integrated Competitive Equilibrium model', waarin een verdergaande analyse mogelijk is van de milieu- en welvaartseffecten van verschillend milieubeleid. Deze modellen worden ook gebruikt om voor het klimaatprobleem te berekenen wat de mogelijke gevolgen zijn van verschillend klimaatbeleid. Uit de analyse komen de volgende conclusies.

(i) *Dynamische efficiëntie*

De illustratie met ALICE 0 laat zien dat door de toenemende levensverwachting in de volgende eeuw de spaartegoeden voor het (langere) pensioen mogelijk zullen toenemen. Hierdoor komt de (reële) rente op de besparingen onder druk te staan, en kan deze uiteindelijk negatief worden. Dit betekent dat de allocatie 'dynamisch inefficiënt' is en dat een deel van de besparingen beter geconsumeerd kan worden. Zowel de aanwezigheid van milieuhulpbronnen in een economie als van een hieronder te specificeren claim kan deze inefficiëntie voorkomen.

Het feit dat milieuhulpbronnen een oneindige stroom van waardevolle diensten kunnen produceren betekent dat deze een niet te verwaarlozen waarde hebben. En indien een deel van de milieuhulpbronnen in eigendom wordt gegeven aan de eerste generatie betekent dit dat het economische evenwicht in een OLG economie dynamisch efficiënt is.

Als er geen milieuhulpbronnen zijn, of als deze niet in eigendom worden gegeven, is er een alternatief om dynamische efficiëntie te garanderen door de overheid verhandelbare claims uit te laten geven aan de dan levende generaties. Deze claims zijn vergelijkbaar met obligaties. Ze geven de eigenaar in iedere periode het recht op een betaling die gelijk is aan een vast (klein) aandeel van het totale inkomen. De betalingen aan de eigenaars van de claims worden gefinancierd uit belastingen. Het komt in zekere zin neer op een positieve overheidsschuld waarbij de interestbetalingen een vast deel van het totale inkomen uitmaken.

(ii) *Duurzaamheid*

De toegepaste milieu-economie heeft tot nu toe vooral geijverd voor invoering van zogenaamde Pigou-belastingen op het milieu, om de kosten van het milieugebruik door te laten werken in de prijzen van de geproduceerde goederen. Recentelijk is er ook aandacht voor de mogelijkheid om milieuhulpbronnen verhandelbaar te maken en in eigendom te geven, zodat de eigenaars zorg dragen voor het efficiënt gebruik. De economische literatuur laat echter zien dat deze maatregelen, die er voor zorgen dat - verwachte - toekomstige preferenties in de huidige prijzen worden opgenomen, niet voldoende zijn om duurzaamheid te garanderen. In deze studie hebben we naar extra milieumaatregelen gekeken, zowel vanuit het dynastisch als vanuit het OLG perspectief.

Binnen het dynastisch perspectief is het gebruikelijk dat de dynastische welvaartsfunctie die gemaximaliseerd wordt bestaat uit de gewogen som van de welvaartsfuncties van de opeenvolgende generaties, waarbij de gewichten exponentieel afnemen. De snelheid van de afname heet de pure tijdspreferentie. Een populaire methode om een duurzaam welvaartsmaximum te bevorderen bestaat eruit dat de tijdspreferentie wordt gereduceerd, zodat toekomstige generaties een hoger gewicht krijgen in de geaggregeerde welvaartsfunctie. Dit is echter geen goede oplossing, omdat het tot niet realistische resultaten leidt, en daarmee de huidige preferenties klaarblijkelijk niet correct weergeeft. Het is eenvoudig mogelijk om expliciete duurzaamheidscriteria op te nemen in die zin dat de welvaart niet onder een kritiek minimum (geassocieerd met de basisbehoeften) mag komen. Een ander alternatief, het opleggen van fysieke beperkingen aan het milieugebruik, is ook niet wenselijk omdat dit tot onnodige inefficiëntie kan leiden.

Binnen het OLG-perspectief is er geen ruimte voor exogene duurzaamheidscriteria. In plaats daarvan formuleren we een 'milieubeheerfonds', dat is een instelling die het initiële eigendom krijgt over de milieuhulpbronnen, en de inkomsten hieruit gebruikt om de huidige en toekomstige generaties voldoende inkomen te geven om in de basisbehoeften te voorzien. (In het Engels gebruiken we het woord 'trust fund'. Dat verwijst in het gangbare taalgebruik naar een fonds waarin de erfenis van een minderjarige erfgenaam wordt gestort. Deze wordt beheerd door een vertrouwenspersoon, en komt pas beschikbaar bij meerderjarigheid van de erfgenaam.) Het beheerfonds heeft als opdracht om op ieder moment een vermogen aan te houden dat gelijk is aan de waarde - bij de huidige prijzen - van de milieuhulpbronnen die nodig zijn om in de basisbehoeften van alle toekomstige generaties te voorzien. Het beheerfonds hoeft deze milieuhulpbronnen niet zelf als activa aan te houden, het is dus geen 'groenfonds' dat alleen in natuur en milieuvriendelijke activiteiten belegt. Ook worden er geen fysieke beperkingen aan het milieugebruik opgelegd. Het milieubehoud wordt dus niet geforceerd, maar het beheerfonds zorgt ervoor dat de - verwachte - preferenties van toekomstige generaties een voldoende gewicht krijgen in de huidige prijzen.

Het beheerfonds is gebaseerd op de veronderstelling van een gegeven pad voor de bevolkingsgroei. Als de bevolkingsgroei binnen het model verklaard moet worden, en als we veronderstellen dat ouders empathie hebben voor hun kinderen, krijgt de duurzaamheidsdiscussie een ander karakter. Ouders zullen er zelf zorg voor dragen dat hun kinderen in de basisbehoeften kunnen voorzien. Indien dit niet mogelijk is door ernstige milieuvervuiling zullen zij, in het uiterste geval, geen kinderen krijgen. (Hierbij gaan we voorbij aan de wenselijkheid van kinderen op zich. Wel laat het model de mogelijkheid open dat mensen om andere redenen geen kinderen willen.) In plaats van duurzaamheid, is de mogelijke extinctie een probleem. Het beheerfonds zal deze situatie voorkomen, omdat het er voor zorgt dat elke generatie voldoende inkomen heeft om in de basisbehoeften van een aantal mensen te voorzien.

(iii) *Continua van 'steady states' en pad-afhankelijkheid*

Een 'steady state' is een economisch evenwicht dat constant blijft in de tijd. In de economische literatuur wordt er meestal vanuit gegaan dat er afzonderlijke steady states zijn. Stel dat er twee economieën zijn met een klein verschil in de beginvoorwaarden, dan zal dit verschil, tenzij het leidt tot de keuze voor andere steady states, verdwijnen in de loop der tijd als de economieën naar dezelfde steady state convergeren. Dit betekent bijvoorbeeld dat de welvaart op de lange termijn niet wordt beïnvloed als milieubeleid op de korte termijn wordt uitgesteld.

Uit de studie blijkt dat in een economie met milieuhulpbronnen de steady states niet afzonderlijk zijn, maar dicht tegen elkaar aanliggen en continua vormen. Men kan denken aan een lijn waarvan elk punt een steady state is, in tegenstelling tot losse punten van afzonderlijke steady states. (Zie figuren 2.1 en 2.2 in het proefschrift.) Het gevolg is dat verschillen in beginvoorwaarden niet verdwijnen, maar leiden tot de selectie van een andere steady state (binnen hetzelfde continuüm). Dit betekent dat het huidige milieubeleid blijvende invloed heeft op de welvaart in de toekomst; uitstel van milieubeleid kan tot een blijvende daling van de welvaart lijden.

(iv) *De representatie van milieukwaliteit in het formele model*

De toepassing van de theorie op de praktijk is niet zonder moeilijkheden. Eén van de belangrijke problemen is dat milieudegradatie een kwaliteitsverlies van het milieu betekent en geen vermindering van de 'hoeveelheid milieu'. Het blijkt echter dat het onderscheid tussen kwantiteit en kwaliteit formeel goed weergegeven kan worden door een onderscheid te maken tussen rivaliserende goederen en niet-rivaliserende goederen, waarbij bepalend is of het gebruik door de één het gebruik door een ander uitsluit. Bijvoorbeeld, ten aanzien van emissies is de kwantiteit belangrijk, en het gaat hier om het rivaliserend gebruik van de absorptiecapaciteit van het milieu. Bij een daling van biodiversiteit, of bij klimaatveranderingen, is de kwaliteit belangrijk, en het gaat hierbij om het niet-rivaliserend gebruik van de gesteldheid van het milieu. Dit betekent dat het formele model in principe aanknopingspunten biedt om de essentie van het gebruik van milieuhulpbronnen in de economie goed weer te geven, namelijk als een uitruil tussen de productie van rivaliserende en niet-rivaliserende goederen.

(v) *Klimaatveranderingen*

De mogelijke klimaatveranderingen en andere gevolgen (zoals een zeespiegelstijging) van de emissies van broeikasgassen maken duidelijk dat duurzaamheid een praktisch probleem is. Ondanks dat de gevolgen ernstig kunnen zijn en tot hoge kosten kunnen leiden, suggereren de meeste economisch georiënteerde beleidsmodellen dat het optimaal is om de emissiereductie zoveel mogelijk uit te stellen. Volgens dezelfde modellen zal dat leiden tot een stijging van de gemiddelde mondiale temperatuur met 3 graden Celsius in 2100, en met 6 graden Celsius in 2200. Vanwege de verdiscontering van de toekomstige kosten wegen deze niet op tegen de (lagere) kosten van emissiereducties nu.

Echter, de berekeningen met ALICE laten zien dat de prijs van emissies, en het optimale gebruik van het biogeochemische systeem sterk afhangt van de verdeling van eigendomsrechten. Als de huidige generaties het milieu als hun eigendom mogen beschouwen is het optimaal om de emissies van broeikasgassen 'gewoon' verder te laten stijgen. Dit komt erop neer dat we het milieu als afvaldepot gebruiken. Als we, in tegenstelling hiermee, toekomstige generaties een claim geven op een 'schoon milieu' - dat betekent hier, per definitie, een milieu van een kwaliteit die bereikt wordt als er vanaf nu geen netto emissies van broeikasgassen meer plaatsvinden - en een beheerfonds opzetten om deze claim te implementeren, dan komt uit de berekeningen met ALICE naar voren dat dit tot een significante reductie van de emissies leidt. De bereikte duurzaamheid blijkt niet afhankelijke te zijn van eventuele tijdsvertragingen in de milieudynamica. Deze berekeningen zijn gebaseerd op velerlei veronderstellingen, maar de conclusie lijkt gerechtvaardigd dat optimaal klimaatbeleid niet los kan worden gezien van de intergenerationele verdeling van rechten op het milieugebruik.

Er zijn nog velerlei obstakels op weg naar een internationaal gecoördineerd duurzaam milieubeleid. De internationale politiek lijkt gematigd positief te staan tegenover initiatieven om het klimaatprobleem effectief en efficiënt aan te pakken, en laat stap voor stap de vorming van een internationaal regime zien. In de toekomst moet betaald worden voor de uitstoot van broeikasgassen, en wordt geld ontvangen voor de absorptie, bijvoorbeeld door bossen aan te planten. Tot nu toe zijn de initiatieven nog voornamelijk gericht op de *intra*generationele verdelingsaspecten, en ze worden gebaseerd op een 'willekeurig' jaarlijks netto emissieplafond. De principes van het beheerfonds kunnen worden gebruikt om ook de *inter*generationele verdeling mee te nemen. Dit kan betekenen dat de benodigde emissierechten voor de huidige uitstoot 'gehuurd' kunnen worden van de toekomstige eigenaren. Dit zal niet eenvoudig zijn, omdat er geen eenduidige milieuhulpbron is waarvoor we eigendomsrechten kunnen specificeren. Daarbij komt dat het precieze functioneren van het klimaatsysteem nog onduidelijk is. Derhalve zal een beheerfonds constant de activiteiten moeten aanpassen aan de laatste wetenschappelijke inzichten. De milieueconomie zal een belangrijke bijdrage hieraan moeten leveren, om zo de ontwikkeling van een internationaal regime mogelijk te maken dat een efficiënt en duurzaam milieugebruik bevordert.

At the end of the 20th century, the world is confronted with several global environmental problems, such as deforestation, stratospheric ozone depletion, and climate change, that are generally attributed to human intervention. Yet current environmental policies seem inadequate to ensure an efficient and sustainable use of impacted environmental systems. The present study aims at contributing to the theoretical basis of environmental policy. It formulates economic rules and possible institutions to stimulate an optimal use of environmental resources, a use which specifically addresses both efficiency and sustainability.

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