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Christopher Otrok

Panayiotis M. Pourpourides

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Address
80 Kennedy Avenue
CY-1076 Nicosia, Cyprus

Postal Address
P. O. Box 25529
CY-1395 Nicosia, Cyprus

E-mail
publications@centralbank.gov.cy

Website
http://www.centralbank.gov.cy

Fax
+357 22 378153

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On the Cyclicality of Real Wages and Wage Differentials

Christopher Otrok*[*] and Panayiotis M. Pourpourides[**]

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Abstract

In this paper we investigate the cyclicality of real wages. The approach we take is to search for the largest possible common cyclical component in a statistical sense. This contrasts with the existing literature which uses observable variables to proxy for a common cycle. We do so by using a Bayesian dynamic latent factor model and longitudinal microdata. We find that the comovement of real wages can be related to a common factor that exhibits a significant but far from perfect correlation with the national unemployment rate. Our findings indicate that (i) the common factor explains, on average, no more than 9% of wage variation, (ii) the common factor accounts for 20% or less of the wage variability for 88% of the workers in the sample and (iii) roughly half of the wages move procyclically while half move countercyclically. These facts are inconsistent with claims of a strong systematic relationship between real wages and the business cycle. We show that these results are inconsistent with models of Walrasian labor markets typically used in DSGE models. We also confirm findings of previous studies in which skilled and unskilled wages exhibit roughly the same degree of cyclical variation.

Keywords: Wages, wage differentials, business cycles, Bayesian analysis.

JEL Classification: C11, C13, C22, C23, C81, C82, J31.

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[*] University of Missouri – Columbia and Federal Reserve Bank of St. Louis, [**] Cardiff Business School, Cardiff University and Central Bank of Cyprus. We thank Julie Yates (Bureau of Labor Statistics) and Steve McClaskie (National Longitudinal Surveys user services) for information and help with the data. The views in this paper are those of the authors and should not be attributed to the Central Bank of Cyprus, the Eurosystem or the Federal Reserve Bank of St Louis.

Correspondence: Christopher Otrok, University of Missouri, Department of Economics, 909 University Avenue, Columbia, MO 65211, USA, E-mail: otrokc@missouri.edu, Panayiotis M. Pourpourides, Economic Research Department, Central Bank of Cyprus, 80 Kennedy Avenue, P.O.Box 25529, 1395 Nicosia, Cyprus, E-mail: PanayiotisPourpourides@centralbank.gov.cy or Cardiff Business School, Cardiff University, Aberconway Building, Colum Drive, Cardiff, CF10 3EU, UK, E-mail: pourpouridesp@cardiff.ac.uk

1 Introduction

The cyclicality of real wages is a crucial element that allows us to differentiate between competing theories of the labor market. Pissarides (2009) provides a recent survey of the literature and documents that no consensus on cyclicality has emerged. Previous econometric studies impose structure on the relationship between real wages and an indicator of the business cycle (the unemployment rate) prior to estimation. In fact, the findings from the literature suggest that the estimated wage cyclicality depends on this structure: real wages are estimated to be mildly to strongly procyclical when wages are assumed to depend only on the current unemployment rate [e.g. Bils (1985), Keane et al. (1988), Shin (1994), Solon et al. (1994), Shin and Solon (2006)] whereas they are estimated to be weakly cyclical or acyclical when wages are assumed to also depend on 'appropriate' lagged values of the unemployment rate [e.g. Beaudry and DiNardo (1991)].

In this paper we estimate wage cyclicality without imposing any *a priori* structure on the relationship between real wages and indicators of the business cycle. In particular, we provide new evidence using longitudinal microdata in conjunction with a new econometric approach, that of a dynamic factor model. The dynamic factor model searches directly for the largest common cycle in wage data, alleviating the problem of defining the cycle as any particular macroeconomic variable. The use of individual micro data allows us to determine whether the cyclicality of wages is specific to a certain subset of individuals, which alleviates the problem of composition bias. Our main objective is to investigate whether real wages comove substantially over the business cycle, and to disentangle the cyclical properties of wages for skilled (college) and unskilled (no college) workers. To do so, we employ a dynamic latent factor model in which real wages respond to common as well as skill-specific factors.

Our dynamic factor model is motivated by the fact that if real wages exhibit a systematic relationship with the business cycle, then there should be a common factor accounting for a considerable amount of their variability. In an appendix we show that this property follows directly from a popular class of DSGE models. Furthermore, from a macroeconomic perspective, wage cyclicality would imply that the common factor drives the majority of real wages in the same direction. We provide evidence that real wages do not comove substantially and do not exhibit strong comovement related to the aggregate business cycle. This is despite the fact that we employed an econometric approach that finds the maximal amount of cyclicality. The common factor present in wage data that we estimate does turn out to be correlated with the national unemployment rate. However, we show that no more than 9% of wage variability is attributed to the common factor. We also show that real wages do not exhibit a distinct pattern as regards to their response to the common factor. Finally, we confirm findings of previous studies in which the wages of skilled and unskilled workers exhibit roughly the same degree of cyclical variation.

The cyclical behavior of real wages enable us to draw inferences about different theories of the labor market. In most macroeconomic models, business cycles are driven by aggregate shocks and endogenous variables can be expressed as functions of the state variables in the model so that their solution is expressed in the form of an approximate dynamic factor model. Competing theories (models) impose a structure on the relationship between real wages and the business cycle which depends on the evolution of state variables. For instance, in Walrasian models real wages coincide with marginal productivity. Since marginal productivity (usually driven by an aggregate shock) is highly correlated with the business cycle, real wages appear to be strongly procyclical. On the other hand, a labor contracting model implies that real wages depend not only on marginal productivity, but also on an insurance component that results from bargaining between worker and firm. These two components offset each other in such a way that real wages become acyclical.[1] Moreover, in models of this class, equilibrium real wages also depend on market conditions the time labor contracts are signed. In this case, real wages depend not only on the current state

[1] The theoretical backround of implicit contracts lies in the work of Bailey (1974), Azariadis (1975, 1976) and Gordon (1974). Other theories developed to address real wage acyclicality include efficiency wage and insider-outsider models.

of the economy but also on previous states of the economy. In the appendix of the paper, we briefly describe the relevant feature of neoclassical and labor contracting models which determines how wages behave and demonstrate that the models have different implications for both the relationship between individual wages and the common factor, as well as the quantitative importance of the factor itself. The dynamic factor analysis in this paper can then be a direct test of the neoclassical and the labor contracting model or, more generally, any model which imposes structure on the relationship between real wages and the business cycle. It has two advantages over a direct estimation of structural models. First, we can consider a large panel of workers of different types to see if the neoclassical implications hold for 'most' indviduals, or for at least a subset of workers. Second, our test of the model will not lead to a rejection simply because some other feature of the structural model (such as the consumption Euler equation) rejects the RBC model. It must be noted that the appendix of the paper is not exhaustive across all the relevant models in the literature but rather an indicative reference of two major theories.

Our findings suggest that real wages behave in a manner inconsistent with Walrasian models of the labor market. On the other hand, one class of models that is consistent with these results are models of labor contracting. This is in the line of the findings presented by Cooley and Ogaki (1996) who show that the time series properties of real wages are compatible with Walrasian models only in the long-run, whereas in the short-run they are better explained by an optimal labor contract model.[2] Finally, our evidence confirms the findings of Keane and Prasad (1993) in that the behavior of real wages of skilled and unskilled workers do not exhibit substantial differences over the business cycle. We reach this conclusion from the fact that the skill-specific factors we estimate are not quantitatively important.

The remainder of the paper is organized as follows. The next section provides a brief survey of the empirical literature on the cyclicality of real wages. Section 3 presents a

[2] Similar results to Cooley and Ogaki are reported by Osano and Inoue (1991), Beaudry and DiNardo (1991,1995) and Ham and Reilly (2002) who contrast and test Walrasian and labor contacting models. While the Walrasian models perform poorly in testing, the contracting models cannot be rejected by the data.

description of our dataset which is extracted from the National Longitudinal Surveys (NLS). Section 4, introduces the factor model and presents our econometric methodology. Section 5 presents our results and section 6 concludes.

2 Related Empirical Literature

Our results are distinct from the existing literature in the use of the individual level data coupled with the dynamic factor model. At the same time our work is part of a long history of studying the cyclical behavior of wages and it is useful to briefly review some of the main contributions in the literature. The literature begins with Dunlop (1938) and Tarshis (1939), who conducted the earliest empirical studies on real wage cyclicality. They found that Keynes's view, in the *General Theory*, that real wages move countercyclically is not borne statistically. A simple average measure of real wages does not appear to move systematically over the business cycle. Thus, many leading macroeconomists have accepted the acyclical behavior of real wages as a stylized fact of the business cycle.[3]

Several studies, beginning from Stockman (1983), questioned the validity of the average measure of wages and stressed the importance of controling for composition bias in obtaining accurate measurements of wage cyclicality. The idea is that during recessions workers in the lower tale of the skill distribution are more likely to be laid off and thereby the average wage might be countercyclicaly biased.[4] This argument implies that "true" and "spurious" movements in real wages may not be disentangle by a simple average measure. Since then, several studies estimate econometric models using disaggregated data to control for composition and aggregation effects. In particular, real wages are regressed on the unemployment rate, as an indicator of the business cycle, and other worker-specific characteristics. Among others,

[3]For instance, the acyclicality of real wages is reported in Lucas (1977), Greenwald and Stiglitz (1988), Mankiw (1989), Blanchard and Fischer (1989), Hall and Taylor (1991), Christiano and Eichenbaum (1992), Gomme and Greenwood (1995), Boldrin and Horvath (1995), Rebelo and King (2000).

[4]Heckman and Sedlacek (1985) find evidence of composition bias in the manufacturing sector.

these studies, include the work of Bils (1985), Keane, Moffitt and Runkle (1988), Beaudry and DiNardo (1991), Solon, Barsky, and Parker (1994), Shin (1994), Ziliak, Wilson and Stone (1999) and Shin and Solon (2006). Bils, Solon et al., Shin, Ziliak et al. and Shin and Solon, find that wage acyclicality is simply a statistical illusion and that real wages are strongly procyclical. Bils however, finds that the impact of composition bias is not particularly large and argues that wage procyclicality is due to the inclusion of overtime earnings. Contrary to the previous studies, Keane et al. report that real wages are mildly procyclical after controling for sample selection bias. Beaudry and DiNardo (1991) extend the wage equation employed by Bils (1985) and Solon et al. (1994) by adding lagged values of the cyclical indicator to test their theory of implicit contracts. Their finding is that when appropriate lagged values of the cyclical indicator are taken into consideration the contemporaneous correlation between real wages and the business cycle decreases substantially.[5]

Previous studies have found that the real wages of skilled workers exhibit different low frequency variation than that of the real wages of unskilled workers. Katz and Murphy (1992), find that this behavior can be explained by different demand shifts for skilled and unskilled labor. Motivated by those findings, Acemoglu (1998), develops a theoretical framework to show that the wage gap between skilled and unskilled workers as well as the changes in the demand for skills are due to skilled-biased technological change which is determined endogenously. Krusell, Ohanian, Rios-Rull and Violante (2000), report empirical evidence showing that wage differentials are due to the existence of capital-skill complementarity which is present in the production process. These theoretical and empirical arguments have direct implications in building alternative theories of the labor market. These theories must also be consistent with the cyclical behavior of wage differentials and thus, knowledge of cyclical facts of skilled versus unskilled wages is essential. Within the context of our framework, if the cyclical properties of real wages for skilled and unskilled workers are not alike, then

[5]A number of empirical studies isolate job changers and job stayers in estimating wage cyclicality (e.g. Bils,1985; Shin, 1994; Deveraux, 2001; Barlevi, 2001; Shin and Solon, 2006). Pissarides (2009), provides a survey of this literature. In this paper, we do not make this distinction.

there should exist different skill-specific factors explaining a considerable portion of wage variability. We find that there is little evidence of significantly distinct cycles for these two groups of workers.

The Bayesian dynamic factor model we employ is part of an emerging literature on developing techniques to estimate factor models on large datasets (ours has a cross-section of over two thousand workers). We make a technical contribution to this literature by developing a method to apply large scale factor models to unbalanced panel-time series datasets. Following Otrok and Whiteman (1998), Kose, Otrok and Whiteman. (2003) we proceed with an explicitly Bayesian approach for estimating the parameters and the factors.[6]

3 The Data

Our data on hourly wages are taken from the National Longitudinal Survey, which is a nationally representative sample of 12,686 men and women born in the years 1957 through 1964. All respondents were interviewed annually from 1979 to 1994. We use the time series from 1979 to 1993 and collect information from the survey on employment, wages and sociodemographic characteristics.[7]

The advantage of the NLS panel data set is that it avoids problems related to having a changing work force and enables us to control for various worker characteristics. Unlike the Michigan's Panel Study of Income Dynamics (PSID), where the hourly wage in a given year is the ratio of the annual income to the annual hours of work, in the NLS the respondents directly report their hourly rate of pay in the week of the interview. Thus, the advantage

[6] Doz, Giannone and Reichlin (2011) and Forni and Reichlin (1998) are examples of alternative estimation approaches for large scale factor models, a literature too large for us to survey here.

[7] The text of question for the years 1979 to 1993 asks the respondents to report amount earned that includes tips, overtime and bonuses before deductions. The hourly rate of pay in survey year 1994 is calculated a little differently. Respondents are first asked if they are paid hourly; if so, then that reported hourly wage is used in the created hourly rate. Presumably, this hourly wage does not inlcude tips, overtime and bonuses. Otherwise, if the respondents report other than an hourly wage, then they are asked for earnings that include tips, overtime, and bonuses (just as in the years 1979-1993) from which hourly rate of pay is created. Given that there is a difference in methodology for 1994 we exclude this year from our sample.

of using NLS over PSID is that hourly wages are less contaminated by recall bias.[8] [9] We accept only those respondents that meet the following restrictions: 1) Must be at least 18 years old at the interview date; 2) Are not self-employed; 3) There must be at least 7 years of available time series observations; 4) Are not enrolled in school the last 2 years of the sample period.

After removing the respondents who do not meet our criteria our sample contains 2,123 individuals and 31,845 person-year observations. We provide further analysis of our sample by classifying individuals into 8 broadly defined categories on the basis of skills, gender and race. We define skilled workers as those having at least a college degree and unskilled workers as the remainder of the sample. Race is defined based on the information provided by NLS, which classifies the respondents into three race groups, Hispanic, black and non-black/non-Hispanic. We group the sample into two main categories. One category consists of blacks and Hispanic and the other one consists of the remainder of the sample, which is assumed to be largely non-minority. A detailed description of the composition of our sample can be found in Table 1. The wage measure is deflated by the Consumer Price Index (CPI) to provide a real wage measure normalized in terms of 1983 CPI dollars. The data are log-first-differenced and demeaned before estimation.[10]

One potential issue that we face is that our dataset is an unbalanced panel as missing observations constitute 27.7% of the sample. Missing observations arise in the NLS because either the respondent is not interviewed or he/she is enrolled at school or he/she is unemployed. Wage observations where respondents are enrolled at school but at the same time report a positive wage rate are treated as missing observations. (Information about missing observations for each category can be found in Table 1.) One approach to solving this prob-

[8]The reported hourly wage refers to the respondent's current or most recent job at the time of the interview. In the NLS survey the current or most recent job is refered to as job #1 which, after 1982, is nearly always the CPS job.

[9]We do not use the newest NLS survey of 1997 because it is still in progress and a shorter sample period is currently available.

[10]This treatment of the data is the same form as the log deviations from steady-state that would come from a RBC model.

lem is to simply drop the time series containing missing observations. Since this signifcantly reduces the sample size, and may induce a selection bias, we take an alternative approach. We treat the missing oberservations as random variables and estimate them as part of our econometric model. Our methodology for estimating the missing observations is described in section 5.1.

4 The Dynamic Factor Model

To estimate the cyclical properties of real wages we use a dynamic factor model along the lines of Sargent and Sims (1977), Stock and Watson (1989) and Kose et al. (2003). This statistical model differs from the models traditionally employed to estimate wage cyclicality. In previous work, wages are associated with cyclical indicators (e.g. the unemployment rate). Of course, if one chooses the 'wrong' cyclical indicator the results will be biased towards finding acyclical wages. The factor model, by definition, extracts the largest common cycle(s) in the wage data. Hence, we are finding the maximum possible amount of cyclicality in the wage data. Our model then gives the best possible chance to the theories in favor of cyclical wages.

To be concrete, let $\mathbf{y}_t$ be a vector of real wages for N individuals at time t. Then, $\mathbf{y}_t$ can be explained by a vector $\mathbf{f}_t$ of K common factors and a vector $\boldsymbol{\varepsilon}_t$ of N individual-specific noise terms. We assume that $\mathbf{f}_t$ and $\boldsymbol{\varepsilon}_t$ evolve according to the following autoregressions:

$$\mathbf{f}_t = \boldsymbol{\phi}^f (L) \mathbf{f}_{t-1} + \mathbf{u}_t^f \tag{4.1}$$

and

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\phi} (L) \boldsymbol{\varepsilon}_{t-1} + \mathbf{u}_t \tag{4.2}$$

where $\boldsymbol{\phi}^f (L)$ and $\boldsymbol{\phi} (L)$ are K x Q and N x P matrices of polynomials in the lag operator, respectively. The vectors of disturbances $\mathbf{u}_t^f$ and $\mathbf{u}_t$ are assumed to be zero mean and normally distributed with

$$E\left(\mathbf{u}_t^f \mathbf{u}_\tau^{f\prime}\right) = \begin{cases} \mathbf{M}^f & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise} \end{cases} \quad \text{and} \quad E\left(\mathbf{u}_t \mathbf{u}_\tau^{\prime}\right) = \begin{cases} \mathbf{M} & \text{for } t = \tau \\ \mathbf{0} & \text{otherwise} \end{cases}$$

where $\mathbf{M}^f$ and $\mathbf{M}$ are diagonal matrices. In other words, the factors are independent from each other and the individual-specific noise terms are independent across individuals. The statistical model for $\mathbf{y}_t$ is

$$\mathbf{y}_t = \mathbf{b}\mathbf{f}_t + \boldsymbol{\varepsilon}_t \tag{4.3}$$

where $\mathbf{b}$ is a N x K matrix of factor loadings.

We focus our attention in characterizing the dynamic effects of three factors. The common dynamics of real wages across all individuals are captured by the common factor f^c. The factors f^s (where s= skilled or unskilled) drive the wages of a subset of individuals with the same skill level. Thus, having panel data on N individuals, each observed for T time periods, our model for the real wage of individual i is

$$\begin{gathered} y_{i,t} = b_{c,i} f_t^c + b_{s,i} f_t^s + \varepsilon_{i,t} \\ \text{for } i = 1, 2, .., N; \text{s=skilled or unskilled}; \ t = 1, .., T \end{gathered} \tag{4.4}$$

where $b_{j,i}$ is the 'factor loading' that captures the sensitivity of the wage of worker i to factor j. The corresponding idiosyncratic error $\varepsilon_{i,t}$ follows a p_i-order autoregression:

$$\varepsilon_{i,t} = \phi_{i,1}\varepsilon_{i,t-1} + \phi_{i,2}\varepsilon_{i,t-2} + ... + \phi_{i,p_i}\varepsilon_{i,t-p_i} + u_{i,t} \tag{4.5}$$

where $\phi_{i,j}$ represents the exposure of the idiosyncratic error to its jth lag and $u_{i,t}$ ~iid$N(0, \sigma_i^2)$.

Likewise, the law of motion of factor j is given by the $AR(q_i)$ process:

$$f_t^j = \phi_{fk,1} f_{t-1}^j + \phi_{fk,2} f_{t-2}^j + ... + \phi_{fk,q_i} f_{t-q_i}^j + u_{k,t}^f \qquad (4.6)$$
$$\text{for } k = c,\, s$$

where $\phi_{fk,j}$ represents the exposure of factor k to its jth lag and $u_{k,t}^f \sim \text{iid}N(0, \sigma_{f,k}^2)$.

4.1 Estimation

We estimate the factors and the parameters of the econometric model 4.4 − 4.6 using the Bayesian approach developed in Otrok and Whiteman (1998).[11] We simulate from the joint posterior of the parameters and factors using a Markov Chain Monte Carlo alogorithm. The main part of their procedure is a Gibbs sampler that sequentially draws the parameters conditional on the factors, and then the factors conditional on the parameters.[12]

Since the covariance matrix $\mathbf{M}$ is diagonal, conditional on the factors, the system 4.4 consists of N independent regression models. Hence, conditional on the factors, we use Chib and Greenberg's (1994) procedure to draw the regression parameters separately for each equation. Since the model has 2,123 equations this feature of their procedure makes the estimation feasible for our dataset. A full derivation and description of the relevant conditional densities can be found in Otrok and Whiteman (1998).

The (conjugate) prior densities for $\mathbf{b}_i$, $\boldsymbol{\phi}_i$, $\boldsymbol{\phi}_{fk}$ and σ_i^2 are chosen to be the same as those used in Otrok and Whiteman (1998). Specifically, the prior for the factor loadings $\mathbf{b}_i$ is Gausian with zero mean and precision (1/variance) equal to 0.01. The persistence parameters of the innovation and factor processes $\boldsymbol{\phi}_i$ and $\boldsymbol{\phi}_{fk}$ are also Gausian with zero mean and precision equal to 0.85 for all lags. The prior of the idiosyncratic innovation variance σ_i^2 is an *inverted gamma* $\sim (\alpha/2, \beta/2)$ with $\alpha = 6$ and $\beta = 0.001$. These priors are

[11] Other estimation approaches are available for these models (e.g. Doz, Giannone and Reichlin 2011). The focus of our technical contribution is a tractable way to deal with missing observations.

[12] The scales of the factor loadings are separately identified from those of the factors by normalizing the variances of the factors to a constant, as is common in the literature

fairly diffuse and the main results are not very sensitive to values of prior parameters around the ones chosen.

4.2 Missing Observations

Our dataset poses a technical problem due to missing observations for wages in some years for many of the survey respondents. Instead of ommiting the time series we assume that the missing observations are random variables and we estimate these missing observations as part of our econometric model.[13] We do so by first deriving the distribution of the missing data points conditional on the parameters and factors. This distribution depends on both cross-sectional information as well as the time series data before and after the missing observation. Intuitively, the distribution depends on both a 'forecast' and 'backcast' of the missing observation using the univariate time series data itself, and the parameters governing the dynamics of the time series. It also includes cross-sectional information: the factor loading is used along with the factor itself to 'predict' the missing value. Our procedure combines both types of information. A direct way to do this is by applying the Kalman filter and then smoothing the means and the variances by backward induction. Details of the procedure are in the Appendix.

Our Gibbs sampler then has three blocks. In block one we condition on factors and model parameters to draw the missing observations (for those time series with missing data). Then, in block two we treat the missing data drawn in block one as data and draw the model parameters. Finally, conditional on the drawn missing data and parameters we draw the factors. The procedure is repeated 5000 times after an initial burnin of 500 draws.

[13] In related work, Jungbacker et al. (2011) introduce and study the properties of a maximum likelihood estimator of a high dimensional dynamic factor model with missing data.

5 Empirical Results

Our primary interest is to provide answers to two questions: First, do real wages exhibit a systematic relationship with the business cycle? Second, are the wages of skilled and unskilled workers subject to a significantly different degree of cyclical variation? To answer the first question we focus on the importance of the common factor in equation 4.4. To answer the second question we focus on the relative contribution of the skill factors in equation 4.4 in accounting for real wage fluctuations. Since the factors (common and skill specific) are estimated simultaneously, the skill factors are capturing how much comovement there is for a specific skill group conditional on comovement already accounted for by the factor common to all wages. That is, skilled (or unskilled) wages may comove simply because all wages comove. Our model determines instead how much comovement there is in skilled wages that is not common to wages of all skill levels. This conditioning is important, as it alleviates the danger of looking only at, say, the wages of skilled workers, and mistakenly concluding that skilled wages have a common cycle, when that cycle is in fact common to a wider array of individuals.

5.1 The Dynamic Factors

Figure 1 presents the mean of the posterior distribution of the factors along with corresponding 95 percent posterior coverage intervals. The bounds of the confidence intervals are tight which shows that the factors are estimated quite precisely. The common factor is characterized by the peaks of 1983 and 1990 and the trough of 1987. The peaks occur at roughly the same time that NBER recessions occur. In particular, the peak of the 1983 lags the NBER recession of the 1982 whereas the peak of 1990 leads the NBER recession of 1991. The variable used by the previous studies as an indicator of the business cycle is the national unemployment rate. In fact, our common factor exhibits a significant correlation (0.69) with the national unemployment rate (in levels) which indicates that the unemployment indicator captures a good portion of real wage cyclicality. However, the correlation is not perfect so assuming that unemployment is the common wage cycle underestimates the degree of wage cyclicality to a modest extent. It is the case that our estimates suggest that current macroeconomic conditions are relevant, at least to some extent, for the cyclical behavior of real wages.

The skill-specific factors appear less cyclical than the aggregate factor and have distinct dynamics from each other. The correlation coefficient between the skilled and the unskilled factors is 0.26 which signifies that real wages embody a distinct component which is specific to skills.[14] The correlation coefficient between the skill factors and the unemployment rate is almost zero. Both factors exhibit substantial variation until 1985 and relatively smooth fluctuations afterwards.

To examine whether common fluctuations are more persistent than skill specific fluctuations we report the first-order autocorrelation coefficients of the factors. Our estimates indicate that aggregate common fluctuations are highly persistent just like the unemployment rate. The common fluctuations of unskilled wages are also highly persistent with an autocorrelation coefficient of 0.68. Contrary to the common and the unskilled factors, the skilled factor exhibits a negative autocorrelation of -0.21 which suggests that it is weakly mean reverting. The differing dynamics of the skilled factor suggests that there are forces unique to skilled workers driving their wages. If we interpret this in light of the theoretical models presented in the appendix, then this would suggest skill-specific productivity shocks. We do not push this interpretation very hard though, since we will see that these factors are not quantitatively important. Next we examine the direction to which a change in each of the factors affects real wages. Figure 2 displays the cumulative distribution functions (CDFs) of the factor loadings. The CDFs illustrate that roughly half of real wages in our sample respond positively to the factors while the other half respond negatively. Thus, there

[14] The assumption in the econometric model is that the innovations between the two skill factors is zero. However, this assumption is not imposed in the estimation so the skill factor can be correlated if the data so indicate. We do impose that that aggregate factor is orthogonal to the two skill factors.

is no distinct pattern of the responses of real wages to the common factors.

Macroeconomic models of the labor market can be classified based on the mechanism according to which real wages are being determined and the implied cyclicality of the latter. Surveying all relevant models is not the objective of this paper. However, in the appendix we show that the factor model can be employed as a test of neoclassical features of the labor market. Testing via the factor model enable us to consider a large panel of longitudinal micro data on real wages to check whether the neoclassical implications hold for "most" individuals or a subset of them. The factor model also eliminates the possibility of rejecting the theory simply because some irrelevant feature of the theoretical model is problematic. As shown in the appendix, a neoclassical model of the labor market where marginal productivities depend on aggregate shocks (commonly, an aggregate technology shock) would imply that all wages respond with the same sign to the common factor. For instance, an improvement in technology would increase the wages of all workers. Therefore, in such model, we expect that real wages would exhibit a positive and strong correlation with the business cycle. On the other hand, a wage contracting model with some heterogeneity in preferences (e.g. in risk aversion) may exhibit responses with different signs for a single change in market conditions (which may reflect improvement or deterioration of the level of technology). The latter is consistent with factor loading coefficients that differ in sign and magnitude. Our results indicate that the common factor exhibits responses with different signs which suggests that the data are inconsistent with the predictions of a Walrasian model but consistent with predictions of a labor contracting model.

5.2 Variance Decompositions

To examine the quantitative significance of the cyclical factors we estimate the contribution of each of them to the overall variability of observables. Since the factors and the idiosyncratic component are orthogonal to each other it is straightforward to partition the variance of each observable into the fraction that is due to each of the underlying factors and the idiosyncratic component. The variance of observable i can be written as (by applying the Var operator to equations 4.1 and 4.2)

$$var\left(y_{i,t}\right) = \left(b_i^c\right)^2 var\left(f_t^c\right) + \left(b_i^s\right)^2 var\left(f_t^s\right) + var\left(\varepsilon_{i,t}\right) \tag{5.1}$$

Then, the fraction of the volatility explained by factor j is

$$\frac{\left(b_i^j\right)^2 var\left(f_t^j\right)}{var\left(y_{i,t}\right)} \tag{5.2}$$

Reporting the full posterior distributions of all 2,123 posteriors is infeasible, so instead we report information on the distribution of the posterior means of the 2,123 variance decompositions. (In most cases that we examined the posterior coverage intervals were tighly distributed about the mean.) Figure 3 displays frequencies and CDFs of variance decompositions across the skilled, the unskilled and the whole sample. Table 2 presents analytically the number of individuals falling in each interval of variance shares attributable to each of the factors and the idiosyncratic component.

The common factor explains, on average, no more than 9% of the variance of real wages. We obtain similar results when we examine the impact of the factor separately on skilled and unskilled wages. Overall, the common factor accounts for 20% or less of the wage variability for 88% of the workers in the sample. The share of variance attributable to the common factor exceeds 50% for only 1% of the workers. In other words, the wages of only 1% of the respondents are overwhelmingly influenced by common economic conditions, as reflected through the common factor. These results show that the factor plays a relatively minor role in accounting for wage movements over the business cycle. Consequently, the explanatory power of the common factor is inadequate to justify claims for strong procyclical or countercyclical movements of real wages. As shown in the appendix, these results are more consistent with models of labor contracting where wages incorporate a procyclical productivity component as well as a countercyclial insurance component. As a result, a change in market conditions causes offsetting effects which induce a small change in equilibrium wages in response to a given change in market conditions. Likewise, the skill factor explains, on average, no more than 10% of wage variability and accounts for 20% or less of wage variability for 84% of the workers in the sample. Those findings reinforce the evidence of previous studies which show that skilled and unskilled wages face essentially the same degree of cyclical variation.

These results are also inconsistent with a fixed nominal wage contract model. If we augment the neoclassical model in appendix A.I. with a model where nominal wages are set for a fixed number of periods, then we would find that at least half, or a quarter of the wages, depending on the nominal wage contract length, would depend almost completely on the common factor. For example, if we have nominal wages fixed for 1 period, and half of workers get to change wages in a given period, then our common factor would find that more than half of the workers respond to the common factor.[15]

Notably, the idiosyncratic component is an important factor of wage fluctuations. It can explain more than 70% of wage variability for 78% of the workers. It is possible that this residual may include the effects of characteristics such as gender and race. To examine the robustness of our main results we extend our model by including gender and race factors. Specifically, we assume that there is a specific factor driving the wages of male workers and a separate factor driving the wages of female workers. As for the race characteristics we follow the NLS classification and assume two broadly defined race factors, one driving the wages of blacks and hispanics and another driving the wages of the remainder. We call the latter group nonminority and the former group minority. For instance, in this setting, the real wage of a skilled female worker who belongs to a minority group is driven by five factors, one

[15] If productivity were *iid* then exactly 50 percent of the indivudals would be driven by the common factor, but since there is serial correlation in productivity, wages in adjacent periods would be related to each other, which would be picked up by the dynamics in the factor. This would lead to more than 50 percent of the sample having a quantitatively important response to the common factor .

that drives the wages of all workers, one that drives the wages of all skilled workers, one that drives the wages of all female workers, one that drives the wages of all minority workers and finally a factor that is specific to the worker. We find that the gender and the race factors have little to no explanatory power and do not change our main results. Thus, they are not retained in the final statistical model.

6 Concluding Remarks

The cyclical behavior of real wages has long been a central issue in macroeconomics. Our contribution to this literature is to use a dynamic factor model with longitudinal data to find the largest possible common cycle in real wages. The advantage of this approach over previous approaches is that it does not impose structure on the relationship between real wages and observed indicators of the business cycle prior to estimation. We show in an appendix that the factor model itself is motivated directly from theoretical models by providing examples of two RBC models with alternative theories of the labor market. The virtue of the dynamic factor framework is that we need not subject the full range of implications of the RBC model to a test, rather we focus on implications for the labor market.

Our approach also allows us to use longitudinal micro data from the NLS to control for composition and aggregation effects. Our model allows us to analyze the degree and the nature of the comovement of real wages across the entire population as well as separately for skilled and unskilled workers. It also enables us to quantify the contribution of each factor in wage variability.

We find that the result of little cyclicality is inconsistent with Walrasian labor markets. There are likely to be a number of labor markets models consistent with these results, and we provide one example here. An important point, though, is that the model we are able to reject is at the core of much of the DSGE literature.

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Appendix A: The Factor Model and Two Theoretical Models

I. A Neoclassical Model

The dynamic factor model of section 4 can be motivated by a standard real business cycle model augmented with a model of measurement error induced by the agency gathering data. This motivation follows directly from the work of Sargent (1989). We start with a 'textbook' real business cycle model, that of King, Plosser and Rebelo (1988), which specifies preferences, technology and budget constraints. Using standard parametric functional forms for preferences and technology the model can be log-linearized and solved.[16] As is well known the solution of this model takes the form of a state law of motion and set of decision rules for observable variables:

$$\mathbf{S}_{t+1} = \mathbf{\Phi}\mathbf{S}_t + \mathbf{E}_{t+1} \tag{A.1}$$

$$\mathbf{Y}_t = \mathbf{H}\mathbf{S}_t \tag{A.2}$$

The first system of equations describes the dynamic evolution of the vector of state variables and exogenous shocks, such as capital and technology. The second system of equations are the decision rules, linking the vector of endogenous choices, $\mathbf{Y}_t$, to the current state vector, $\mathbf{S}_t$. Typical decision variables are labor effort and consumption. Of course, the real wage would appear in $\mathbf{Y}_t$ as well.

As is well known, the real wage of the representative agent in this model is highly procyclical as the wage is equal to the marginal product of labor. To clarify this implication we follow the conventional way to decentralize the Pareto optimal equilibria of the model by assuming spot-competitive labor markets. Let the utility of agent i, U^i, be defined over consumption, C_{it}, and work effort, H_{it} such that $U^i_C > 0$, $U^i_{CC} < 0$, $U^i_H < 0$ and $U^i_{HH} < 0$,

[16] Typically one assumes CRRA utility, Cobb-Douglass production, AR(1) technology shocks and a linear capital accumulation equation

where subscripts denote derivatives. Let θ_t be an exogenous state variable which is the driving force of business cycle fluctuations. Without loss of generality, since in RBC models, θ_t commonly denotes the level of neutral technology, we will simply call θ_t technology.[17] Let $\psi_i(\theta_t)$ denote the agent's marginal productivity which is an increasing function of technology θ_t. The intratemporal efficiency condition derived from an RBC model is

$$\frac{-U_H^i(C_{it}, H_{it})}{U_C^i(C_{it}, H_{it})} = \psi_i(\theta_t) \tag{A.3}$$

This condition results from the agent equating his marginal rate of substitution between consumption and leisure to the real wage, while firms choose labor such that the marginal product of labor equals the real wage. The spot-market equilibrium then implies that real wages equal marginal productivities. Consequently, under spot-competitive labor markets we expect that over the business cycle there is a common (macro) component, θ, driving the real wages of all agents, and that these wages move in the same direction.

Our extension of this model assumes that we do not get to observe the 'true' real wage. Instead, we have many noisy observations on individual wages from this competitive spot labor market. The noise is induced by a data-gathering agency which must survey individuals to find out their wages. These survey data are riddled with errors, both recall errors from the agents and statistical errors from the agency itself. Our second system of equations then becomes:

$$\mathbf{Y}_t = \mathbf{H}\mathbf{S}_t + \mathbf{U}_t \tag{A.4}$$

where $\mathbf{U}_t$ represents the measurement error and the $\mathbf{Y}_t$ vector contains the full set of indivudals surveyed.

The empirical model we will use in this paper, a dynamic factor model, is motivated directly from equations A.1 and A.4. These equations take the same general form as a

[17] For the sake of simplicity we omit shocks other than θ_t from our notation; θ_t can also be viewed as a composition of different shocks.

dynamic factor model. To make this link concrete consider the dynamic factor representation for a vector of wage data $\mathbf{y}_t$:

$$\mathbf{y}_t = \mathbf{b}\mathbf{f}_t + \boldsymbol{\varepsilon}_t \tag{A.5}$$

where $\mathbf{b}$ is a N x K matrix of factor loadings. The factor $\mathbf{f}_t$ is assumed to follow an autoregressive process:

$$\mathbf{f}_t = \boldsymbol{\phi}^f(L)\,\mathbf{f}_{t-1} + \mathbf{u}_t^f \tag{A.6}$$

It is clear from comparing equations A.1 and A.4 with equations A.5 and A.6 that the dynamic factor model takes the same form as the linearized solution to the real business cycle model with measurement error. Were one to simulate data from the RBC model and estimate a factor model on the simulated data, the estimated dynamic factor would then be the common state variable (e.g. technology shock) in the business cycle model. When we turn to actual data, if the neoclassical labor market embodying this model is largely correct, then when we estimate the factor model on wage data we should have two key results. First, as long as the wage data are not dominated by measurement errors, the common factor should be quantitatively important for explaining real wage dynamics. Second, the wages should all respond with the same sign to this common factor since in the business cycle model all wages respond positively to changes in aggregate productivity (in this class of models real wages appear to be strongly procyclical because marginal productivities are strongly procyclical).

II. A Wage Contracting Model

Our second labor market model is based on an alternative way to decentralize the Pareto optimal equilibria by considering a model where agents trade labor contracts. In such a model, wages and employment are specified in a contract which is the outcome of dynamic bargaining between workers and firms. The contract, $\{w^i(\theta_t), H^i(\theta_t)\}$, consists of an hourly wage rate and hours of work that are contingent on the future state of technology. The contract is such that the efficiency condition A.3 holds, but the hourly wage rate is not necessarily equal to $\psi_i(\theta_t)$. The hourly wage not only responds to changes in productivity but also provides insurance to risk averse agents against business cycle fluctuations.[18] Contrary to the spot market case, under reasonable assumptions, in equilibrium the wage will not be strongly correlated with productivity. This is due to the fact that the wage embodies an insurance component which minimizes their fluctuations. Furthermore, a given change in θ may induce the wages of some agents to increase while others to decrease. Hence, responses of different signs to a given change in the common component are consistent with the theory of implicit contracts. To illustrate these two points, we provide a simple example where consumption equals labor earnings that is, $C_{it} = w_t^i H_{it}$, and the agents differ in terms of their aversion toward risk. Assuming separable CRRA preferences, condition A.3 can be solved for the equilibrium wage (see Boldrin and Horvath (1995)):[19]

$$w_t^i = \delta_i \left[\psi_i(\theta_t)\right]^{\frac{1}{\lambda_i}} \left[\frac{1 - H_{it}}{H_{it}}\right] \tag{A.7}$$

where $\delta_i > 0$ and λ_i is the agent's coefficient of risk aversion. (Note that the linearized version of equation A.7 would enter the decision rules A.2 or A.4 in the state space system describing the model dynamics.) In this case, the equilibrium wage is comprised of two components, productivity and insurance (which is the ratio of leisure to labor). Productivity is strongly procyclical whereas the insurance component is countercyclical because hours of work are procyclical. The latter offsets the increases (decreases) in productivity and thus, wages do not appear to respond strongly to technology shocks. Notice that parameter λ_i controls the elasticity of the hourly wage to the marginal product of labor fluctuations. Depending on the value of λ_i, for some individuals the effect of the insurance component may dominate the effect of productivity and thereby, the change in their wage, in response to an increase

[18] The idea is based on the assumption that capital markets are inadequate to fully buffer the agents' consumption against adverse shocks.

[19] The same condition for the equilibrium wage can be derived when preferences are nonseparable. In that case however, parameter λ_i is the within period elasticity of substitution between consumption and leisure (see Pourpourides (2011)).

in θ, will have a negative sign. The more risk averse an agent is the more likely she/he is to have a negative wage response to an increase in θ. To summarize, the contracting model first implies that real wages will not exhibit a strong commmon cycle, implying that any common dynamic factor should have little explanatory power for real wage fluctuations. Second, if there is heterogeneity in preferences then the model predicts that the factor loading coefficients in the dynamic factor model will have both positive and negative signs.[20]

Appendix B: Factor Models with Unbalanced Panels

In this appendix we describe the procedure for estimating the missing observations. This procedure forms one block of our Gibbs sampler. In block one we draw the parameters conditional on factors and missing data. In block two we draw the factors conditional on parameters and missing data. In block three we draw the missing data conditional on parameters and factors. In essence, we fill in the missing observations of the unbalanced panel using information in both the model and available data. It is this last block that we describe in this appendix. The first two blocks are described in Otrok and Whiteman (1998).

Let $\xi_{i,t} = \phi_{i,1}\xi_{i,t-1} + ... + \phi_{i,p_i}\xi_{i,t-p_i} + u_{i,t}$ where $\xi_{i,t} = y_{i,t} - b_{c,i}f_t^c - b_{s,i}f_t^s$. Then, the

[20] To further motivate our empirical work, consider the simple example where consumption equals labor earnings expressing equilibrium wages and work effort as functions of relevant factors. Then, (A.3) is written as

$$\frac{-U_H^i\left(w_i\left(f_t,\mathbf{f}_t^i\right)H_i\left(f_t,\mathbf{f}_t^i\right),H_i\left(f_t,\mathbf{f}_t^i\right)\right)}{U_C^i\left(w_i\left(f_t,\mathbf{f}_t^i\right)H_i\left(f_t,\mathbf{f}_t^i\right),H_i\left(f_t,\mathbf{f}_t^i\right)\right)} = \psi_i\left(\theta_t\right)$$

where f_t is a common factor across all individuals and $\mathbf{f}_t^i$ is a set of factors specific to individual i. It can be shown that this model can be reduced to models previously considered in the literature. For instance, if $f_t = \nu\left(\theta_t, \theta_{t-m}\right)$ for $m \geq 1$, the model reduces to a version of the implicit contracts model of Beaudry and DiNardo (1995). If $f_t = \theta_t$, the model can be reduced to a standard Walrasian model where $\psi_i\left(\theta_t\right) = w_i\left(\theta_t\right)$. In general, the intratemporal condition (A.3) may correspond to a class of implicit contracts models or a class of Walrasian models. Since each class of models implies a particular relationship between wages and the business cycle, the relationship between f_t and w_t can be used to draw inferences about the validity of each of these theories.

following state space system is obtained:

$$\mathbf{y}_{i,t} = \mathbf{A}_i'\mathbf{x}_t + \mathbf{H}'\ \boldsymbol{\xi}_{i,t} + \mathbf{w}_{i,t} \tag{B.1}$$

$$\boldsymbol{\xi}_{i,t+1} = \mathbf{F}_i\boldsymbol{\xi}_{i,t} + \mathbf{v}_{i,t+1} \tag{B.2}$$

where

$$\mathbf{y}_{i,t} = y_{i,t},\ \boldsymbol{\xi}_{i,t} = \begin{bmatrix} \xi_{i,t} & \xi_{i,t-1} & \cdots & \xi_{i,t-p_i+1} \end{bmatrix}',\ \mathbf{x}_t = \begin{bmatrix} 1 & f_{t-1}^c & \cdots & f_{t-q_i}^c & f_{t-1}^s & \cdots & f_{t-q_i}^{sk} \end{bmatrix}'$$

$$\mathbf{w}_{i,t} = b_{c,i}\ u_{c,t}^f + b_{s,i}u_{s,t}^f,\ \mathbf{v}_{i,t} = \begin{bmatrix} u_{i,t} & \mathbf{0}_{1x(p_i-1)} \end{bmatrix}',\ \mathbf{A}_i' = \mathbf{B}_i{\cdot}\boldsymbol{\Phi},\ \mathbf{B}_i = \begin{bmatrix} b_{c,i} & b_{s,i} \end{bmatrix}$$

$$\mathbf{H}' = \begin{bmatrix} 1 & \mathbf{0}_{1x(p_i-1)} \end{bmatrix},\ \boldsymbol{\Phi} = \begin{bmatrix} \phi_{c,1} & \cdots & \phi_{c,q_i} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \phi_{s,1} & \cdots & \phi_{s,q_i} \end{bmatrix},\ \mathbf{F}_i = \begin{bmatrix} \phi_{i,1} & \cdots & \phi_{i,p_i-1} & \phi_{i,p_i} \\ & \mathbf{I}_{(p_i-1)x(p_i-1)} & & \mathbf{0}_{(p_i-1)x1} \end{bmatrix}$$

The variance matrix of $\mathbf{v}_{i,t}$ is

$$E\left(\mathbf{v}_{i,t}\mathbf{v}_{i,\tau}'\right) = \mathbf{Q}_i = \begin{cases} \begin{bmatrix} \sigma_i^2 & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix} & \text{for } t = \tau \\ \mathbf{0}_{p_i \text{ x } p_i} & \text{otherwise} \end{cases}$$

Consequently, the system B.1 - B.2 satisfies the following conditions:

1. $E\left(\mathbf{w}_{i,t}^2\right) = b_{c,i}^2\sigma_{f,c}^2 + b_{s,i}^2\sigma_{f,s}^2 = \mathbf{R}_i$

2. $E\left(\mathbf{w}_{i,t}\mathbf{w}_{i,\tau}\right) = \mathbf{0}$, and $E\left(\mathbf{v}_{i,t}\mathbf{w}_{i,\tau}\right) = \mathbf{0}$ for all t and τ

Equations B.1 and B.2 are the observation and state equations, respectively. The recursion of the Kalman filter begins with $\widehat{\boldsymbol{\xi}}_{i,0|0}$ which denotes the unconditional mean of $\boldsymbol{\xi}_{i,1}$, where $\widehat{\boldsymbol{\xi}}_{i,0|0} = E\left(\boldsymbol{\xi}_{i,1}\right) = \mathbf{0}$, The asssociated Mean Square Error (MSE) is $\mathbf{P}_{i,0|0} = \boldsymbol{\Sigma} = E\left(\boldsymbol{\xi}_{i,1}\boldsymbol{\xi}_{i,1}'\right)$

where $\mathbf{\Sigma} = \mathbf{F\Sigma F}' + \mathbf{Q}$. To enable the recursion steps we replace missing observations with values drawn from the distribution of the data,[21]

$$L\left(y_{i,t}/\boldsymbol{\phi},\xi_{i,t},...,\xi_{i,t-p}\right) = \left(2\pi\sigma_i^2\right)^{-1/2}\exp\left\{-\frac{1}{2\sigma_i^2}\left(y_{i,t}-\widehat{y}_{i,t/t-1}\right)^2\right\}$$

where $\widehat{y}_{i,t/t-1} = y_{i,t} - \xi_{i,t} + \phi_{i,1}\xi_{i,t-1} + ... + \phi_{i,p}\xi_{i,t-p}$. The transition from $\widehat{\boldsymbol{\xi}}_{i,t-1|t-1}$ and $\mathbf{P}_{i,t-1|t-1}$ to $\widehat{\boldsymbol{\xi}}_{i,t|t}$ and $\mathbf{P}_{i,t|t}$ is given by the following set of equations[22]

$$\widehat{\boldsymbol{\xi}}_{i,t|t-1} = \mathbf{F}_i\widehat{\boldsymbol{\xi}}_{i,t-1|t-1}$$

$$\mathbf{P}_{i,t|t-1} = \mathbf{F}_i\mathbf{P}_{i,t-1|t-1}\mathbf{F}_i' + \mathbf{Q}_i$$

$$\widehat{\mathbf{y}}_{t|t-1} = \mathbf{A}_i'\mathbf{x}_t + \mathbf{H}'\widehat{\boldsymbol{\xi}}_{i,t|t-1}$$

$$\widehat{\boldsymbol{\xi}}_{i,t|t} = \widehat{\boldsymbol{\xi}}_{i,t|t-1} + \mathbf{P}_{i,t|t-1}\mathbf{H}\left(\mathbf{H}'\mathbf{P}_{i,t|t-1}\mathbf{H} + \mathbf{R}_i\right)^{-1}\left(\mathbf{y}_t - \widehat{\mathbf{y}}_{t|t-1}\right)$$

$$\mathbf{P}_{i,t|t} = \mathbf{P}_{i,t|t-1} - \mathbf{P}_{i,t|t-1}\mathbf{H}\left(\mathbf{H}'\mathbf{P}_{i,t|t-1}\mathbf{H} + \mathbf{R}_i\right)^{-1}\mathbf{H}'\mathbf{P}_{i,t|t-1}$$

Since our goal is to form an inference about the value of $\boldsymbol{\xi}_{i,t}$ based on the full set of time series we compute the smoothed estimate $\widehat{\boldsymbol{\xi}}_{i,t|T}$ and the corresponding MSE, $\mathbf{P}_{i,t|T}$, by conditioning on next period's observation that is, $\widehat{\boldsymbol{\xi}}_{i,t|T} = \widehat{\boldsymbol{\xi}}_{i,t|t} + \mathbf{J}_{i\tau}\left(\widehat{\boldsymbol{\xi}}_{i,t+1|T} - \widehat{\boldsymbol{\xi}}_{i,t+1|t}\right)$ and $\mathbf{P}_{i,t|T} = \mathbf{P}_{i,t|t} + \mathbf{J}_{it}\left(\mathbf{P}_{i,t+1|T} - \mathbf{P}_{i,t+1|t}\right)\mathbf{J}_{it}'$ where $\mathbf{J}_{it} = \mathbf{P}_{i,t|t}\mathbf{F}_i'\mathbf{P}_{i,t+1|t}^{-1}$.[23] Wherever there is a missing observation, in each loop of the Markov chain, we replace it with $y_{i,t}^{*} = \boldsymbol{\xi}_{i,t}^{*1} + b_{c,i}f_t^c + b_{s,i}f_t^s$ where $\boldsymbol{\xi}_{i,t}^{*1}$ is the first element of the drawing $\boldsymbol{\xi}_{i,t}^{*}$ from $N\left(\widehat{\boldsymbol{\xi}}_{i,t|T}, \mathbf{P}_{i,t|T}\right)$. The values for the missing observations are drawn right after the completion of steps 1 and 2 of the estimation procedure.

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[21] Alternatively, instead of drawing a value from $L(\cdot)$, we can merely skip the updating equations by assuming that $\widehat{\boldsymbol{\xi}}_{i,\tau|\tau} = \widehat{\boldsymbol{\xi}}_{i,\tau|\tau-1}$ and $\mathbf{P}_{i,\tau|\tau-1} = \mathbf{P}_{i,\tau|\tau-1}$. The results do not change significantly under this alternative.

[22] The formulas were directly taken from Hamilton's (1994) time series textbook. For more details concerning the algorithm refer to Hamilton pp. 377-381.

[23] Refer to Hamilton (1994) pp.394-397.

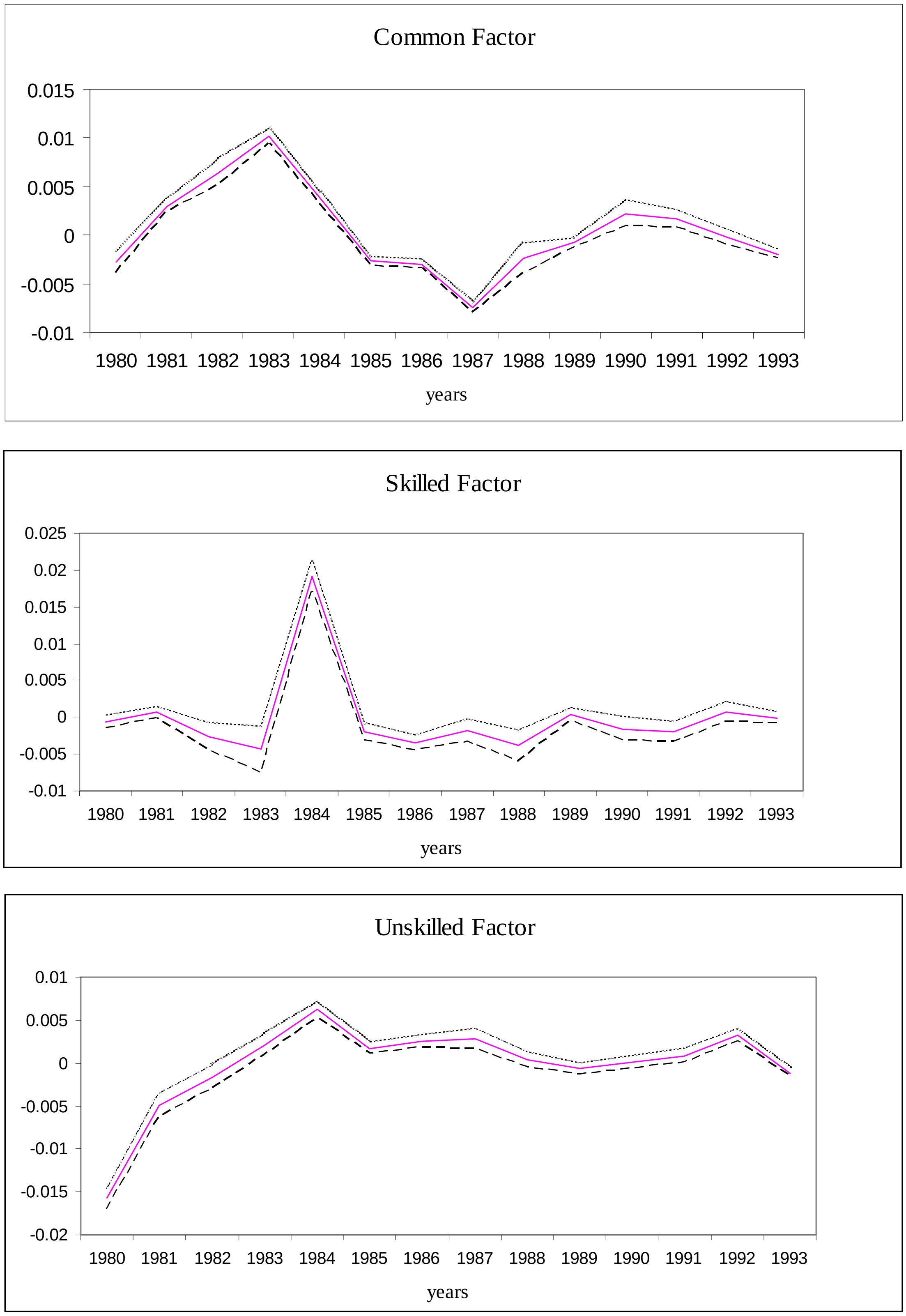

Figure 1 Dynamic Factors (means, upper and lower bounds), 1980-1993

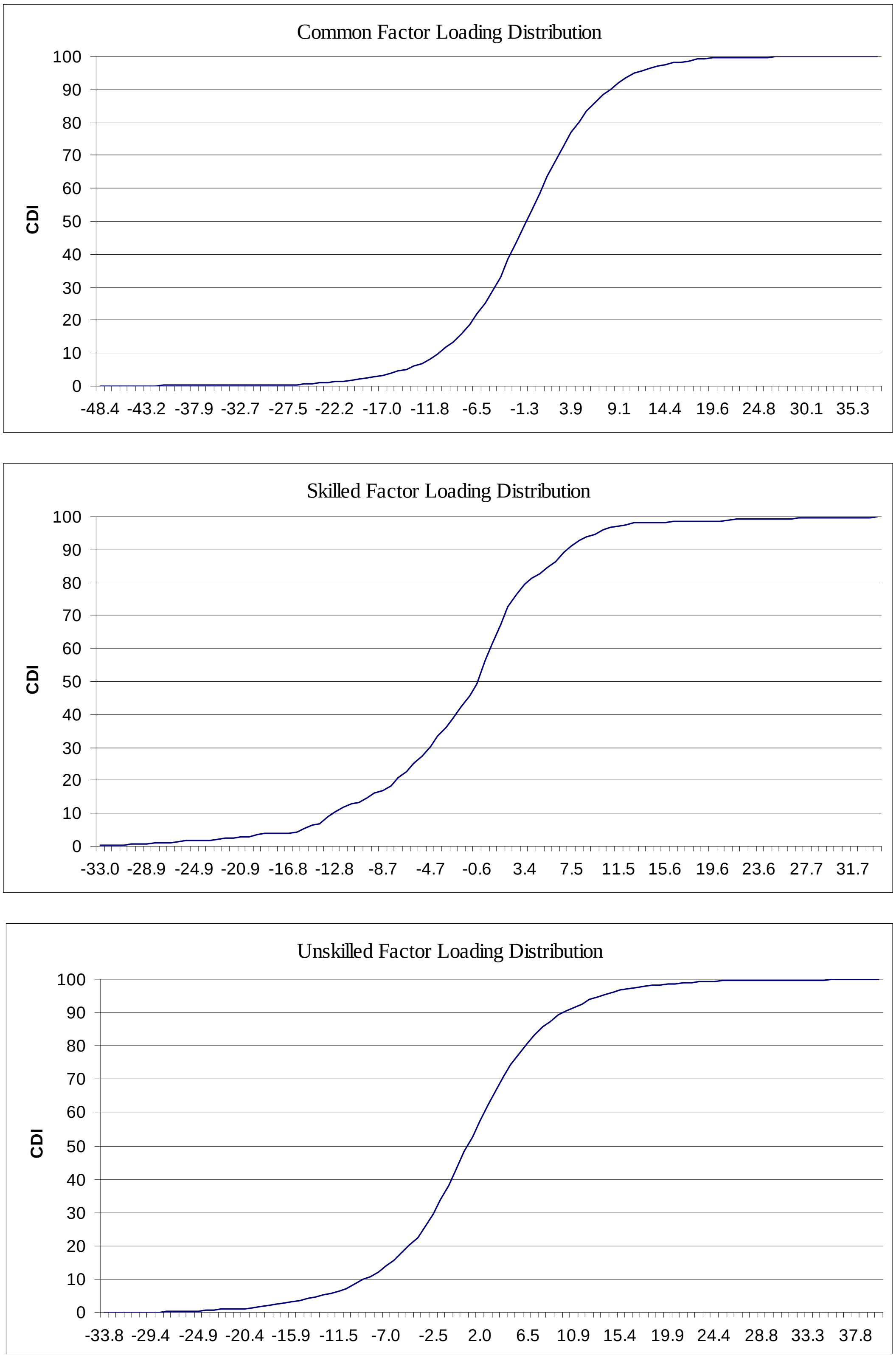

Figure 2 Factor Loading Distributions

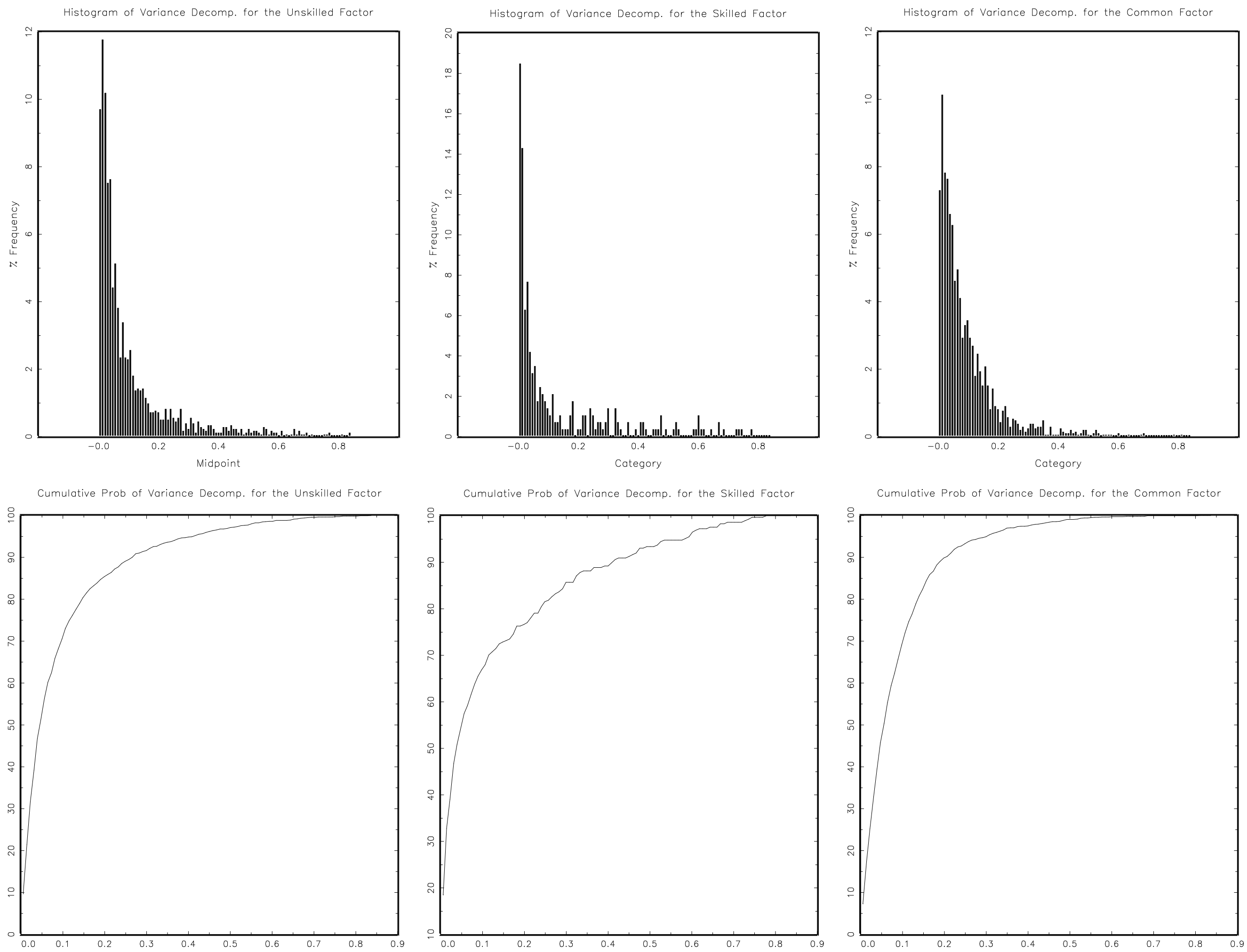

Figure 3 Variance Decompositions for the Factors

Table 1: Composition of the Sample

category	# of people	% in sample	% of missing obs.
skilled males minority	40	1.90	0.86
skilled males nonminority	89	4.19	1.80
skilled females minority	60	2.82	1.16
skilled females nonminority	98	4.61	1.99
unskilled males minority	528	24.90	6.30
unskilled males nonminority	452	21.30	4.88
unskilled females minority	428	20.14	5.70
unskilled females nonminority	428	20.14	5.08
aggregate	2123	100.0	27.77
males	1109	52.23	13.84
females	1014	47.77	13.93
skilled	287	13.52	5.81
unskilled	1836	86.48	21.96
minority	1056	49.74	14.02
nonminority	1067	50.26	13.75

Table 2: Variance Decompositions*

decomp.	common factor						skill factor						idiosyncratic factor					
	# of workers			% in sample			# of workers			% in sample			# of workers			% in sample		
(%)	sk	un	tot	sk	un	tot	sk	un	tot	sk	un	tot	sk	un	tot	sk	un	tot
0-10	194	1262	1456	67.60	68.74	68.58	190	1285	1475	66.20	69.99	69.48	0	5	5	0	0.27	0.24
10-20	48	394	442	16.72	21.46	20.82	30	282	312	10.45	15.36	14.70	6	8	14	2.09	0.44	0.66
20-30	24	92	116	8.36	5.01	5.46	25	115	140	8.71	6.26	6.59	10	31	41	3.48	1.69	1.93
30-40	9	46	55	3.14	2.51	2.59	11	58	69	3.83	3.16	3.25	6	43	49	2.09	2.34	2.31
40-50	10	23	33	3.48	1.25	1.55	12	41	53	4.18	2.23	2.50	19	55	74	6.62	3	3.49
50-60	2	13	15	0.70	0.71	0.71	8	30	38	2.79	1.63	1.79	26	80	106	9.06	4.36	4.99
60-70	0	4	4	0	0.22	0.19	7	17	24	2.44	0.93	1.13	21	145	166	7.32	7.90	7.82
70-80	0	1	1	0	0.05	0.05	4	5	9	1.39	0.27	0.42	30	256	286	10.45	13.94	13.47
80-90	0	1	1	0	0.05	0.05	0	3	3	0	0.16	0.14	44	518	562	15.33	28.21	26.47
90-100	0	0	0	0	0	0	0	0	0	0	0	0	125	695	820	43.55	37.85	38.62

*sk = skilled, un = unskilled, tot = total