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Indeterminacy, intergenerational redistribution, endogenous longevity and human capital accumulation

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Abstract

This paper sets up an OLG economy with endogenous life expectancy to study how fiscal policy that redistributes between generations can open the door to sunspot equilibria. Agents invest independently in their own human capital, produce and consume output, and receive a pension upon retirement. The model produces an expectations coordination problem that can explain significant differences in growth paths followed by otherwise identical countries. In particular, we show that our economy may be characterised by local indeterminacy of dynamic equilibria, and hence feature fluctuations which are driven by extrinsic uncertainty.

Keywords: Endogenous Longevity, Human Capital, Intergenerational Redistribution, Local Indeterminacy, Sunspots.

JEL: E60, H30, J18, J24, O40.

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1 Introduction

Recent years have seen an extensive research on the possibility of indeterminate equilibria in dynamic general equilibrium models.¹ This literature is motivated by the difficulties in explaining differences in growth in different countries and accounting for various empirically observed growth paths by means of the usual economic fundamentals.²

Indeterminacy of equilibria can be either global or local. The first kind corresponds to multiple balanced growth paths (BGPs hereafter), while the second kind refers to the existence of a continuum of transition paths leading to a given BGP and a possibility of existence of sunspot equilibria.³ Multiplicity of equilibrium paths can explain why otherwise similar countries may be characterized by different per capita incomes and/or different growth rates.

In general, it has been shown that indeterminacy is the consequence of non-decreasing returns to capital, monopolistic competition or some forms of production externalities that generate non-decreasing returns at the social level - that is, of what may generate persistent growth in the first place.⁴ Here, instead, we focus on the role of *intergenerational redistribution* in opening the door to sunspot equilibria, when agents invest independently in their human capital and *longevity is increasing with aggregate human capital*.⁵

We show that intergenerational transfers may distort human capital accumulation for any given rationally anticipated longevity, and thereby affect actual longevity. If investment in human capital is indeed responsive to the tax particulars, then intergenerational transfers generate indeterminate equilibria, and in particular local indeterminacy. This implies that self-fulfilling beliefs of economic agents or sunspots determine the equilibrium path, since the initial human capital investment is freely chosen. So, when longevity is increasing with the average level of human capital, our economy can feature endogenous business cycles - i.e. business cycles which are driven by extrinsic as opposed to intrinsic uncertainty - due to the external effects of intergenerational redistribution.

As in any endogenous growth model, government policies can generate differences in the growth paths followed by otherwise similar economies. Nevertheless, our work highlights that identical economies, even with the same pension particulars (like the pension rate and the choice between funded or pay-as-you-go pensions), may feature different growth patterns and different income taxes even in the absence

¹See Benhabib and Farmer (1999) for a survey.

²See, for instance, Levine and Renelt (1992) and Benhabib and Gali (1995).

³See, for instance, Shell (1987).

⁴For an alternative model, emphasising indeterminacy of the distribution of the uniquely determined total wealth and income, see Krussel and Rios-Rull (1999).

⁵A number of studies have found human capital and its various proxies to have important impacts on adult health and longevity. For an extensive survey of the literature see Sickles and Taubman (1997).

of shocks to its fundamentals.

Our paper differs from other accounts of fiscal policy and indeterminacies in economic growth in that here tax revenues do not provide an external effect through financing public consumption goods and infrastructure.⁶ In addition, in our model labour supply is inelastic, and thereby income taxation does not affect the social returns to capital.⁷ Also, in this paper tax revenues are not used to finance subsidies for human capital accumulation.⁸ Instead, in our model tax revenues are used to finance intergenerational transfers.

Importantly, however, intergenerational redistribution is not sufficient to generate multiplicity. As we emphasize, it is *the interaction of intergenerational redistribution and the dependence of longevity on human capital that opens the door to local indeterminacy*. The reason is that, for any given rationally anticipated longevity, private decisions on human capital accumulation may depend on rationally anticipated tax-contributions, which in turn depend on actual longevity and hence private investments in human capital. So, we have self-fulfilling rational expectations equilibria.

The organization of the paper is as follows. Next Section lays out the model. Section 3 describes individual choices, while Sections 4 and 5 characterize equilibrium paths. Finally, Section 6 concludes.

2 The Model

Time is discrete and is denoted with the superscript $t \geq 1$. There is a population of agents belonging to overlapping generations with finite but uncertain lifetimes. Each agent matures safely from youth to adulthood and has a probability of surviving to old age. In the first period of her life an agent decides how much *time* to put into labour quality enhancing activities, like education, training and fitness, and how much time to devote to labour. An agent enjoys no leisure in youth and adulthood. So, a young agent supplies her net-of-education time endowment to labour, as long as the returns from doing so are positive. Similarly, as an adult, an agent allocates her time endowment to labour, if relevant returns are positive, and consumes (part) of her returns from savings when young.⁹ If she survives to old age, an agent does not work and consumes her returns from savings when adult. The endowment of time in each period is normalized to one. Generation size is assumed to increase at a non-negative rate $n - 1$ when

⁶See, for instance, Cazzavillan (1996), Park and Philippopoulos (2004).

⁷See, for instance, Schmitt-Grohe and Uribe (1997) and Guo and Lansing (1998).

⁸See Bond et. al. (1996) and Alonso-Carrera and Freire-Serén (2004).

⁹For related modeling of human capital accumulation and discussions see, for instance, Lucas (1988) and Azariadis (1993) Ch 14. Note that we abstain from parental investments towards childrens' human capital, and any associated altruism on the part of parents and/or intrafamily trade (see, for instance, Becker et.al. (1990), Ehrlich and Lui (1991) and Nordblom (2003)). Allowing for such investments would give rise to a non-autonomous dynamic system making the stability investigation of balanced growth paths intractable and the interpretation of our results cumbersome.

in adulthood; that is each adult is assumed to bear $n \geq 1$ children. All agents have identical preferences and are aware of their life expectancies. Also, in period $t = 1$ government policies are introduced in the form of income taxes/subsidies and pensions.

Let us denote with $\pi^{t+1} \in [\underline{\pi}, \bar{\pi}]$ the probability of an adult who was born in period t surviving to the third period of her life, or longevity. Let π^1 denoting the longevity of adults who are alive in period $t = 1$, i.e. of the parents of the ‘first’ generation $t = 1$. Moreover, c_o^{t+2} denotes the consumption of an old agent in period $t + 2$. In addition, consumption of an adult agent in period $t + 1$ is given by c_a^{t+1} . Furthermore, denote with c_y^t the consumption of the typical young agent born in period t . Finally, define with $\delta \in (0, 1)$ the discount factor.

We then have that the expected lifetime utility of an agent of generation t is given by

$$\phi(c_y^t) + \delta\phi(c_a^{t+1}) + \pi^{t+1}\delta^2\phi(c_o^{t+2}) \quad (1)$$

where $\phi()$ is a standard utility function.

An agent’s human capital determines, in a one-to-one way, the agent’s labour productivity. Each agent enters her first period of life with an amount of human capital, h^t , which is inherited from her parent. An adult’s human capital, however, is partly inherited from her parent and partly the result of his own educational effort when young. In particular,

$$h^{t+1} = h^t\mu(e^t), \quad (2)$$

where e^t is the time/effort spent in education when young, and $\mu(0) = 1$, $\mu' > 0$, $\mu'' < 0$ and $\mu(1) \equiv \mu > 1$ being finite. That is, the growth rate of human capital of agent t is an increasing and concave function of her training when young. We also assume the Inada condition $\lim_{e \rightarrow 0} \mu'(0) = \infty$. It ensures, in a simple manner, that time spent in education is positive in each period, regardless of policies. The inherited human capital of the typical agent born at time $t = 1$, h^1 , is exogenously given and assumed to be strictly positive.

At this point we consider the workings of the capital market. Agents transact in the capital market through intermediaries. These intermediaries are infinite-horizon entities, and, hence, when they enter the capital market they face the risk-free interest rate. Denote with R the fixed, exogenous, rate of interest determined at the world level.¹⁰ When intermediaries lend to or borrow from individuals, the corresponding rate must incorporate the risk involved due to the agents’ uncertain lifetimes. Assuming, then, that intermediaries operate under conditions of perfect competition, and that entry is costless, we

¹⁰The small open economy assumption allows us to consider both R and the wage per unit of effective labour w as exogenous variables determined at the world level. This is a common assumption in this literature (see, for instance, Galor and Weil, 2000). Alternatively, one could postulate that the economy is endowed with a production function which is linear in capital and effective units of labour, with R and w being the corresponding coefficients of linearity (see, for instance, Ehrlich and Lui, 1991, where $R = 0$ and $w = 1$).

have that the rate of return faced by adult agents is equal to R/π^{t+1} . Adult borrowers pay more than the riskless rate of return R to compensate the intermediaries for the risk of the debtor's death. Adult lenders, on the other hand, earn a higher return than R , due to competition between intermediaries for the appropriation of profits that may result from a lender dying and hence not claiming the return to her investment. Also, as there is no risk involved with young agents, the rate of return they face by entering the capital market is R . Finally, assume that the government is an infinitely-lived entity. Thus, whenever it enters the capital market it also faces the riskless rate of return R .

Let us denote with $f \geq 0$ a net subsidy received by young workers, and financed through current tax revenues, as a proportion of their income. Let also τ^t being the income tax rate in period t . We then have that the consumption level of a young agent, born in period $t \geq 1$, is

$$c_y^t = wh^t(1 - e^t)(1 - \tau^t + f - s_y^t), \quad (3)$$

where w is the fixed wage per unit of effective labour (normalized to 1 hereafter).¹¹ Also, expressed as a proportion of the typical young agent's labour income $h^t(1 - e^t)$, we have that $s_y^t \leq 1 - \tau^t + f$ denotes savings as young. The latter inequality implies non-negative consumption. We will say that f captures the extend of *forward* intergenerational transfers between young and adult generations, i.e. from adult to young workers.¹²

Let now $b \geq 0$ being the net (effective) tax contribution by adults, as a proportion of their labour income, towards the financing of current pensions.¹³ The consumption level of an adult agent, born in period $t \geq 1$, is, then,

$$c_a^{t+1} = (1 - \tau^{t+1} - b - s_a^{t+1})h^{t+1} + Rs_y^t h^t(1 - e^t), \quad (4)$$

where $s_a^{t+1} \leq 1 - \tau^{t+1} - b + (Rs_y^t(1 - e^t)/\mu(e^t))$ denotes savings in adulthood, as a proportion of an adult worker's labour income. The latter inequality ensures non-negative consumption. We will say that b reflects the extent of *backward* intergenerational transfers between old and adult generations, i.e. from adult workers to senior citizens.

Each and every senior citizen who is alive in period $t \geq 2$ receives a net (effective) pension as a proportion p of her effective wage in adulthood.¹⁴ Define with $R(\pi) = R/\pi$ the rate of return to savings

¹¹For our assumption that w is exogenously given, see the previous footnote.

¹²Here, we should mention that allowing for backward transfers in the form of $f < 0$ would also introduce fixed education-subsidies of the form deployed in the representative agent models of Bond et. al. (1996) and Alonso-Carrera and Freire-Serén (2004). As we would like to emphasise multiplicity of dynamic equilibria for completely different reasons to the ones in those papers we choose to restrict attention to $f \geq 0$. See also footnote 24.

¹³Note that this tax contribution can be thought of as including any PAYG contributions and any tax liabilities that may arise from the returns to savings when young.

¹⁴Note that this pension can be thought of as being net of any tax liabilities that may arise from the returns to savings when adult.

on the part of adult workers who face longevity π . It follows that consumption in old age for generations $t \geq 1$ is

$$0 \leq c_o^{t+2} = [p + s_a^{t+1}R(\pi^{t+1})] h^{t+1}. \quad (5)$$

It will prove useful to derive the intertemporal budget constraint of the typical adult member of generation $t \geq 1$. We have that:

$$\begin{aligned} c_y^t R + c_a^{t+1} + \frac{c_o^{t+2}}{R(\pi^{t+1})} &= h^{t+1} \left[1 - \tau^{t+1} - b + \frac{p}{R(\pi^{t+1})} \right] + h^t [(1 - e^t)R(1 - \tau^t + f)] \\ &\equiv h^{t+1} \omega_a(\tau^{t+1}, \pi^{t+1}) + h^t (1 - e^t) \omega_y(\tau^t) \\ &\equiv h^{t+1} \omega_a^{t+1} + h^t (1 - e^t) \omega_y^t \\ &= h^t \{ \mu(e^t) \omega_a^{t+1} + (1 - e^t) \omega_y^t \}, \end{aligned} \quad (6)$$

where the last equation follows from using (2). Note that the wealth expressed in terms of labour income as young is finite.

The net income tax/subsidy rate τ^t , $t \geq 1$, is *endogenously* determined given the pension and redistributive policies. In particular, the following government budget constraint is assumed to hold for any $t \geq 1$.

$$\tau^t (1 + n(1 - e^t)) - n(1 - e^t)f = \frac{\pi^t p}{R} - \frac{nh^{t+1}b}{Rh^t} = \frac{\pi^t p}{R} - \frac{bn\mu(e^t)}{R}. \quad (7)$$

The above specification of policies and government's budget constraint supports a wide range of *government* inter-generational transfers.¹⁵ In particular, observe that in each period $t + 1$ the fiscal mechanism is in need of $h^t p$ pensions per pensioner. These revenue requirements can be financed in a number of ways. For instance, they can be financed by taxing each adult worker by $h^{t+1}b$. Alternatively, pensions can be financed by own contributions in adulthood. Or, they can be financed by taxing young workers and investing the proceeds in the capital market. In addition, in each period t the fiscal mechanism is in need of $fh^t(1 - e^t)$ subsidies per young worker, due to forward redistribution, and hence each adult worker is effectively taxed by $fh^t(1 - e^t)n$. So, in the present economy, labour incomes are taxed to (partly) finance own pensions. Labour income is also taxed to finance current and future pensions of the previous generation.

¹⁵Allowing for time variation in the policy parameters b , f and p - while maintaining the autonomous character of the dynamic system that characterises our economy - could be done by conditioning the corresponding policies on before-tax incomes. One way to justify such policies is to postulate a standard second-best taxation argument. Namely, (inherited) human capital, i.e. the worker's (inherited) productivity, and non-marketable activity e^t are not verifiable tax attributes. Before-tax labour income is, however, a verifiable tax-base. So, policies have to be conditioned on income. Allowing for income-dependent pension and transfer rates would complicate exposition considerably without altering the main insights of the paper. Also, our dynamic system under exogenous longevity would be non-autonomous making the stability analysis and the interpretation of our results cumbersome. Therefore, we have chosen to abstain from such policies.

Importantly, the ‘initial’ income tax τ^1 is not pre-determined; it depends on the human capital accumulation in the initial period $\mu(e^1)$. In other words, the investment in human capital and thereby the income tax τ - in the presence of intergenerational redistribution - is a jump variable. This observation will prove to be crucial for our forthcoming results.

The above constraint can be rewritten as

$$\tau^t = \Upsilon(\pi^t, e^t) \equiv \frac{(\pi^t p/R) + n(1 - e^t)f - (bn\mu(e^t)/R)}{1 + n(1 - e^t)} \quad (8)$$

Note that $\Upsilon_1 = p/R(1 + n(1 - e))$. The effect on tax burden of current longevity has the sign of the *net* pension rate.¹⁶ So, intuitively, if the government provides pensions, longevity increases the tax rate faced by current workers. Note also that $1 + n(1 - e^t)$ is the total endowment supplied in the labour market of period t per-adult. Thus, current investment in human capital decreases the tax base. However, it also increases the contributions of next period’s adult workers towards their parents’ pensions for any given backward transfer rate b , and it also decreases the forward subsidies for any given rate f . In fact, $\Upsilon_2 = nd/(1 + n(1 - e))$: the net effect of human capital investment in period t on current tax burden has the sign of $d^t \equiv \tau^t - f - (b\mu'(e^t)/R) = \Upsilon(\pi^t, e^t) - f - (b\mu'(e^t)/R) \equiv d(\pi^t, e^t, f, p, b)$

The model is completed by specifying one important feature which is the endogenous determination of the survival probability, π . By endogenising longevity the paper distinguishes itself from most of the existing literature. Like in Blackburn and Cipriani (2002) and in Lagerlöf (2000) we assume that longevity depends on human capital.¹⁷ In particular we assume that the longevity of generation t depends on the average human capital level of that generation to reflect the fact that better educated individuals are more likely to adopt healthy life-styles:¹⁸

$$\pi^{t+1} = \pi(\bar{h}^{t+1}) \quad (9)$$

where \bar{h}^{t+1} is the average level of human capital of generation t , $\pi'(\cdot) > 0$, $\pi(0) = \underline{\pi}$ and $\lim_{\bar{h} \rightarrow \infty} \pi(\bar{h}) = \bar{\pi} \leq 1$. Notice that $\pi' > 0$ and the finiteness of longevity implies that for large levels of human capital the longevity returns to human capital must be decreasing. In particular, it must be that there is ν such that $\pi''(h) < 0$ for any $h > \nu$, $\lim_{h \rightarrow \infty} \pi''(h) = 0$ and $\lim_{h \rightarrow \infty} \pi'(h) = 0$. However, there is no a priori reason for assigning a particular concavity on the longevity function π for low levels of human capital. It is equally plausible that $\pi''(h) < 0$ for any h , or that $\pi''(h) > 0$ for any $h < \nu$. In other words, it is equally plausible that the longevity returns to knowledge are decreasing for any level of human capital, or that for low (only) longevity the returns to better quality of life are increasing. Accordingly, in what

¹⁶Hereafter, a subscript i denotes the partial derivative of a multi-variable function with respect to its i^{th} argument, and a prime denotes the derivative of a single-variable function.

¹⁷For a model where longevity depends on medical research financed by a labour tax see Chakraborty (2004).

¹⁸For relevant evidence see the extensive survey of the literature by Sickles and Taubman (1997).

follows we allow in general for the function π to have an inflection point ν such that $\pi'' < 0$ to the right of it. Let also $\pi^1 = \pi(h^1)$, and recall that h^1 , and hence π_1 , is pre-determined.

The typical agent born in period $t \geq 1$ is faced with the problem of maximizing (1) with respect to $e^t, c_y^t, c_a^{t+1}, c_o^{t+2}$, subject to (6) and $c_y^t \geq 0, c_a^{t+1} \geq 0, c_o^{t+2} \geq 0, e^t \in [0, 1]$, taking as given inherited human capital, prices, policies and \bar{h}^{t+1} . Note that in equilibrium $\bar{h}^{t+1} = h^{t+1}$ and that the equilibrium paths for $e^t, c_y^t, c_a^{t+1}, c_o^{t+2}, \pi^{t+1}$, follow from the solution to the above problem, (2), (9) and $\bar{h}^{t+1} = h^{t+1}$ as a function of prices, policies and inherited human capital.

To facilitate our analysis, at this point we assume that the wealth of adult workers are positive; otherwise, adult agents would not supply any labour. This amounts to $1 - \tau^{t+1} - b + (p\pi^{t+1}/R) > 0$ for any generation t . Define $F^u(f; b, p, \bar{\pi}) \equiv \max_{e \in [0, 1]} \Upsilon(\bar{\pi}, e) - 1$ and $B^l(b; f, p, \underline{\pi}) \equiv 1 - \max_{e \in [0, 1]} \Upsilon(\underline{\pi}, e) + p(\underline{\pi}/R)$. Using the period $t+1$ counterpart of (8), one can easily see that the left-hand-side of the resulting inequality has lower bound at $B^l(b; f, p, \underline{\pi}) - b$. Clearly, then, to ensure positive supply of labour by adults, we assume that

$$\mathbf{A1: } b < B^l(b; f, p, \underline{\pi}).$$

We are now ready to proceed in the characterization of equilibria.

3 Individual Decisions

As the consumption choices are orthogonal to the issue of indeterminacy of dynamic equilibria this paper focuses on, we refrain from discussing them.¹⁹ What is important for our purposes is the decision over educational effort. Given the envelope theorem and the constraint of non-negative consumption, we have that optimal effort maximizes total wealth subject to wealth being positive. That is, $e^t = \arg \max_{e \in [0, 1]} \{h^t W(e, \omega_a^{t+1}, \omega_y^t) \mid W(e, \omega_a^{t+1}, \omega_y^t) \geq 0\}$ where $W(e, \omega_a^{t+1}, \omega_y^t) \equiv \mu(e) \omega_a(\tau^{t+1}, \pi^{t+1}) + (1 - e) \omega_y(\tau^t)$ is total wealth as a proportion of parental labour income. The trade-off is obvious. Given inherited human capital, investing in labour quality increases wealth as an adult worker at the cost of lower income as a young worker. The resolution of this trade-off gives a solution for investment in human capital in period $t \geq 1$ $e^t = e(\pi^{t+1}, \tau^{t+1}, \tau^t, p, f, b)$.

As $\omega_a^{t+1} > 0$ and $\mu'' < 0$, the objective function is strictly concave, and hence $e()$ is a well-defined function. Also, due too the Inada condition on the human capital accumulation function μ , we have that optimal level of investment is always positive, and so the economy is characterized by persistent growth. Full-time education is optimal if $\lim_{e \rightarrow 1} W_1(e, \omega_a^{t+1}, \omega_y^t) \geq 0$.

At an interior solution, on the other hand, the young worker's trade-off is resolved when the net marginal gain of an extra unit of human capital investment is zero. Equivalently, when the marginal

¹⁹Clearly, for any given investment in human capital, an agent born in period t faces the standard problem of intertemporal allocation of consumption with price of consumption as young R , price of consumption when old π^{t+1}/R , wealth $h^t \{\mu(e^t) \omega_a(\tau^{t+1}, \pi^{t+1}) + (1 - e^t) \omega_y(\tau^t)\}$ and patience parameters δ and $\delta\pi^{t+1}$.

increase in the human capital growth equals the associated marginal loss of income when young in terms of wealth when adult:

$$\mu'(e^t) = Q(\pi^{t+1}, \tau^{t+1}, \tau^t, p, f, b) \equiv \frac{\omega_y(\tau^t)}{\omega_a(\tau^{t+1}, \pi^{t+1})}. \quad (10)$$

Note that $Q(\pi^{t+1}, \tau^{t+1}, \tau^t, p, f, b)$ is the private rate of return to investment in human capital.²⁰ Clearly, then, $Q_2 = Q_6 > 0$, $Q_4 < 0$ and $Q_3 = -Q_5 < 0$. In particular, $Q_2 = Q/\omega_a(\tau^{t+1}, \pi^{t+1})$, $Q_4 = -Q_2/R(\pi^{t+1})$, and $Q_3 = -R/\omega_a(\tau^{t+1}, \pi^{t+1})$. That is, the price of human capital investment is increasing, and so educational effort is decreasing, with forward transfers, backward transfers and the anticipated income tax rate in adulthood. On the other hand, the price of education is decreasing, and so labour-enhancing activities are increasing, with the net pension rate and the income tax rate in youth. Note also that $Q_1 = -Q_2p/R$: the effect of anticipated longevity on the price of labour-enhancing activities has the opposite sign of the net pension rate p . So, if senior citizens receive positive pensions, investment in human capital increases with anticipated longevity.

Therefore, if government policies aim at increasing growth, forward and/or backward redistribution must be low. Also, a *rationally anticipated* increase in the income tax rate has ambiguous growth effects, as it decreases human capital investment by current young workers, while it will be increasing educational effort on the part of their children. Finally, an increase in the pension rate increases growth, while it also affects positively the growth effect of longevity. Thus, an increase in net pensions has an unambiguously positive growth effect in an ageing society.

Before leaving this Section let us consider the implications of human capital accumulation being independent of policies. This could, for instance, be the case if policies are such that $\mu'(1) \geq \bar{Q}$, where \bar{Q} is the maximum possible price of investment given policy parameters f, b, p .²¹ Introduction of government in such an environment would result in maximum education and growth, in each period.²² Suppose that $e^t = \varepsilon$ for any $t \geq 1$ where $\varepsilon \in (0, 1]$ is a scalar. It follows directly then from the government's budget constraint (8) that the income tax rate in each period t is uniquely determined given any current longevity π^t and the constant human capital accumulation $\mu(\varepsilon)$. Thus, if the path of longevity is uniquely determined, so is the path for income taxes. Note now that, as the initial human capital is pre-determined and $e^t = \varepsilon$, the path of human capital is uniquely determined. Therefore, so is the path of longevity. Thus, the economy is characterized by a unique dynamic path that leads to

²⁰We suppress the dependence of Q on the interest rate R for expositional clarity.

²¹Note, after eliminating τ^t and τ^{t+1} from the definition of Q by using (8), that, given policy parameters, b, f, p , the price of investment has a lower upper bound at $R \frac{f - F^l(f; b, p, \underline{\pi})}{B^l(b; f, p, \underline{\pi}) - b}$, where $F^l(f; b, p, \underline{\pi}) \equiv \min_{e \in [0, 1]} \Upsilon(\underline{\pi}, e) - 1$. Observe also, here, that ensuring positive growth in each period in the absence of the Inada condition only requires $\mu'(0) > \bar{Q}$.

²²Alternatively, we could postulate a continuous but non-differentiable at $\varepsilon \in (0, 1)$ function $\mu(e)$ with $\mu'(\varepsilon^+) \leq \underline{Q}$ and $\mu'(\varepsilon^-) \geq \bar{Q}$, where \underline{Q} is the minimum possible price of human capital investment (see next footnote). In this case, time spend in education is constant over time and equal to ε . Note that our model could also support zero growth in each period. All that would be needed was to assume away the Inada condition and have, instead, that $\mu'(0) \leq \underline{Q}$.

a uniquely determined BGP. This BGP is described by a human capital growth rate $\mu(\varepsilon)$, longevity $\pi^* = \bar{\pi}$, and income tax rate given by (8) with $e^t = \varepsilon$ and $\pi^t = \pi^*$.

Note now that without government the price of investment is R . Thus, in the absence of government, the economy would be characterized by a unique positive level of education $e^N \equiv \min\{1, \mu'^{-1}(R)\}$. The above discussion implies then directly that our economy is characterized by a locally determinate and unique BGP, described by maximum longevity $\bar{\pi}$ and human capital growth rate $\mu(e^N)$. Thus, a necessary condition for indeterminacy is the existence of government and that investment in human capital is responsive to income taxes.

To ensure in a simple manner sensitive to policy changes human capital formation, we assume hereafter a positive labour supply by young agents, and thereby interior levels of education. That is, after defining with \underline{Q} the minimum price of investment given policy parameters f, b, p , we assume that²³

$$\mathbf{A2:} \quad \underline{Q} > \mu'(1).$$

It follows that young agents supply some of their time endowment as labour and the rest towards labour-enhancing activities, with the optimal investment in period $t \geq 1$, $e(\pi^{t+1}, \tau^{t+1}, \tau^t, p, f, b)$, given implicitly by (10). Also, after recalling (8), we have that, given any path of longevity, the equilibrium path of income taxes must satisfy

$$\tau^t = \Upsilon(\pi^t, e(\pi^{t+1}, \tau^{t+1}, \tau^t, p, f, b)), \text{ for any } t \geq 1. \quad (11)$$

The latter determines implicitly a well-defined difference equation for income taxes, given policy parameters b, f, p and longevity path if and only if either $\Upsilon_2(\cdot, e) > 0$ for any $e \in [0, 1]$ or $\Upsilon_2(\cdot, e) < 0$ for any $e \in [0, 1]$.²⁴ Define $d^u = \max_{e \in [0, 1]} \{\Upsilon(\bar{\pi}, e) - f - \mu'(e)(b/R)\}$ and $d^l = \min_{e \in [0, 1]} \{\Upsilon(\underline{\pi}, e) - f - \mu'(e)(b/R)\}$. We assume hereafter that

$$\mathbf{A3:} \quad \text{Either (i) } d^u < 0 \text{ or (ii) } d^l > 0.$$

A3(i) implies that $d^t < 0$ for any $t \geq 1$ and thereby $\Upsilon_2(\cdot, e) < 0$: education decreases the equilibrium current income tax rate. Under A3(ii), on the other hand, we have that $d^t > 0$ for any $t \geq 1$ and hence $\Upsilon_2(\cdot, e) > 0$: education increases the equilibrium current income tax rate. It follows that,

²³Note, after eliminating τ^t and τ^{t+1} from the definition of Q by using (8), that, given policy parameters, b, f, p , the price of investment has a lower minimum bound at $\underline{Q} = R \frac{f - F^u(f; b, p, \bar{\pi})}{B^u(b; f, p, \bar{\pi}) - b}$ with $B^u(f; b, p, \bar{\pi}) \equiv 1 - \min_{e \in [0, 1]} \Upsilon(\bar{\pi}, e) + p(\bar{\pi}/R)$ and $F^u(f; b, p, \bar{\pi}) \equiv \max_{e \in [0, 1]} \Upsilon(\bar{\pi}, e) - 1$.

²⁴The first partial derivative of the function $\Upsilon(\pi^t, e(\pi^{t+1}, \tau^{t+1}, \tau^t, p, f, b))$ with respect to τ^{t+1} is $\Upsilon_2 e_2$, which vanishes if and only if $\Upsilon_2 = 0$. Note, after using (8) to eliminate τ^t , that $\Upsilon_2 = n(\Upsilon - f - (b\mu'/R)) / (1 + n(1 - e))$. Also, we have that $e_2 = \mu' / [\mu''(1 - \Upsilon - b + (p\pi/R))]$. Thus, the second own partial derivative of Υ with respect to τ^{t+1} is $(\partial(\Upsilon_2 e_2) / \partial e) e_2$, which, in turn, is equal to $\Upsilon_2 e_{22} + e_2 \{(\partial \Upsilon_2 / \partial e) e_2\}$, which equals $\Upsilon_2 e_{22} + \{\Upsilon_2 (2ne_2 / (1 + n(1 - e))) - (e_2 n b \mu'' / R(1 + n(1 - e)))\} e_2$. The latter is strictly positive whenever $\Upsilon_2 = 0$. Due to continuity of $\Upsilon(\cdot, e(\cdot, \tau^{t+1}, \cdot))$ and its first two derivatives, we then have that a vanishing point is also a local minimum. So, τ^{t+1} is not uniquely determined by $\tau^t = \Upsilon(\cdot, e(\cdot, \tau^{t+1}, \cdot))$ in the neighbourhood of vanishing points. Accordingly, to ensure a well-defined solution with respect to τ^{t+1} we must have no vanishing points.

given A3, the equilibrium path of income taxes, for any path of longevity, satisfies

$$\tau^{t+1} = T(\pi^{t+1}, \pi^t, \tau^t), \text{ for any } t \geq 1, \quad (12)$$

with

$$T_1(\pi^{t+1}, \pi^t, \tau^t) = -\frac{e_1(\pi^{t+1}, T(), \tau^t, p, f, b)}{e_2(\pi^{t+1}, T(), \tau^t, p, f, b)},$$

$$T_2(\pi^{t+1}, \pi^t, \tau^t) = -\frac{p}{Rnd^t e_2(\pi^{t+1}, T(), \tau^t, p, f, b)}, \quad (13)$$

$$T_3(\pi^{t+1}, \pi^t, \tau^t) = \frac{1 + n(1 - e^t)}{nd^t e_2(\pi^{t+1}, T(), \tau^t, p, f, b)} - \frac{e_3(\pi^{t+1}, T(), \tau^t, p, f, b)}{e_2(\pi^{t+1}, T(), \tau^t, p, f, b)}. \quad (14)$$

We turn to the dynamic analysis of the economy in question.

4 Benchmark Case: Exogenous Longevity

We start our dynamic analysis by considering in this Section the benchmark case of exogenously given longevity. In particular assume that the longevity path is given by $\pi^{t+1} = F(\pi^t)$ for any $t \geq 1$, with F being continuously differentiable. Assume that $\lim_{\pi \rightarrow \bar{\pi}-} F(\pi) = \bar{\pi}$ and $\bar{\pi} > F(\pi) > \pi$ for any $\pi < \bar{\pi}$. That is, suppose that longevity is strictly increasing over time, and is bounded from above by and approaches $\bar{\pi}$. Note that these properties of F imply also that $\lim_{\pi \rightarrow \bar{\pi}-} F'(\pi) \equiv F'(\bar{\pi}) \geq 0$.

As we shall see shortly, the model with endogenous longevity produces a similar path for π . That is, and after recalling our discussion in the previous Section, we have that in our model long-run longevity is $\bar{\pi}$, whether there is a government or not and whether longevity is exogenous or depends on average human capital. Bearing this in mind, the analysis for the benchmark case is the following.

Given that longevity in the long-run is equal to $\bar{\pi}$, the income tax rate in the BGP $\bar{\tau}$ is given implicitly by $\bar{\tau} = \Upsilon(\bar{\pi}, \epsilon(\bar{\tau}, \bar{\pi}))$, where $\epsilon(\bar{\tau}, \bar{\pi}) \equiv \epsilon(\bar{\pi}, \bar{\tau}, \bar{\tau}, p, f, b) \equiv \bar{e}$ is the BGP education level. Note, from (10), that the sign of $R(f+b) - \bar{\pi}p$ determines whether the price of investment, under a government, in a BGP with maximum longevity is greater or not than the *laisser-faire* price R . In particular, if $R(f+b) > \bar{\pi}p$ investing in human capital is more costly under intergenerational transfers, and vice versa. So, if $R(f+b) > \bar{\pi}p$ we have that $\bar{e} < e^N$, and vice versa. That is, if pensions are low relatively to intergenerational transfers, i.e. $p < R(f+b)/\bar{\pi}$, long-run human capital accumulation and growth rate are lower than their *laisser-faire* counterparts, and vice versa. Therefore, an interesting and straightforward implication is that introducing a pension scheme where income tax revenues are solely used to finance the pensions of current adult workers (i.e. $f = b = 0$), leads to higher long-run education and growth rate.

In our model investing in human capital provides a positive external effect, as total wealth of a generation is proportional to inherited human capital (see (6)). Thus, the *laisser-faire* level of education e^N will in general be inefficiently low. Accordingly, if (part of) the objective of introducing intergenerational redistribution is to improve *long-run* growth, the policy parameters p, b, f must be

such that $R(f + b) < \bar{\pi}p$. That is, correcting in the BGP the positive externality inherent in human capital investment requires a sufficiently high net pension rate relative to the extent of intergenerational redistribution. We assume this to be the case hereafter:

B1: $R(f + b) < \bar{\pi}p$.

Given B1, we turn to the analysis of the BGP. From (10), we have that the BGP investment in human capital $\epsilon(\bar{\tau}, \bar{\pi})$ is such that $\epsilon_1 = (R(f + b) - \bar{\pi}p) / \mu''(\bar{e})\omega_a(\bar{\tau}, \bar{\pi})^2 > 0$. Thus, the BGP education and income tax rate are positively related. The reason behind this counterintuitive result is simple. An increase in the long-run income tax rate has two effects on human capital investment. The first is positive, and arises from the decrease of the after-tax wage when young. The second effect is due to the decrease of the after-tax wage when adult. This effect is negative, and B1 implies, in effect, that is also dominated by the former effect.

Furthermore, global indeterminacy is a possibility. To see this, recall that the BGP income tax rate is given by $\bar{\tau} = \Upsilon(\bar{\pi}, \epsilon(\bar{\tau}, \bar{\pi}))$. Define now $\bar{d}(\tau) = d(\bar{\pi}, \epsilon(\tau, \bar{\pi}), f, p, b)$. Obviously then, due to $\mu'' < 0$ and B1, if A3(i) holds, and thereby $\bar{d}(\tau) < 0$ for any τ , we have that $\Upsilon(\bar{\pi}, \epsilon(\tau, \bar{\pi}))$ is strictly decreasing with τ and hence a unique BGP income tax rate. If however A3(ii) holds, and thereby $\bar{d}(\tau) > 0$ for any τ , we have that $\Upsilon(\bar{\pi}, \epsilon(\tau, \bar{\pi}))$ is strictly increasing with τ , and the emergence of multiple BGP tax rates (and hence education levels and growth rates) depends on the curvature of $\Upsilon(\bar{\pi}, \epsilon(\tau, \bar{\pi}))$ when $\Upsilon_2(\bar{\pi}, \epsilon(\tau, \bar{\pi}))\epsilon_1(\tau, \bar{\pi}) = 1$. In particular, if the tax τ defined by $\Upsilon_2(\bar{\pi}, \epsilon(\tau, \bar{\pi}))\epsilon_1(\tau, \bar{\pi}) = 1$ is a local extremum of the function $\tau - \Upsilon(\bar{\pi}, \epsilon(\tau, \bar{\pi}))$ then for some parameter values there are two BGP taxes in the neighbourhood of τ .

We turn to the investigation of local indeterminacy around a given BGP. Under an increasing longevity path towards $\bar{\pi}$, *regardless of the tax path*, the stability properties of our economy around a BGP depend solely on the tax function $T()$ in (12). Note, that the income tax rate path is given by $\tau^{t+1} = T(F(\pi^t), \pi^t, \tau^t)$, or $\tau^{t+1} - \tau^t = T(F(\pi^t), \pi^t, \tau^t) - \tau^t \equiv G(\pi^t, \tau^t)$. Obviously, given an equilibrium $\{\pi^t, \tau^t\}$, taxes are strictly increasing over time if $G(\pi^t, \tau^t) > 0$, and vice versa. Consider, now, the case of $\bar{d}(\tau) < 0$ for any τ , and, hence, recall from above, of a unique BGP. In this case, and after recalling that $e_2 < 0$, $e_3 > 0$ and noting - due to B2 and hence $Q < R$ - that $e_2 + e_3 > 0$, we have that $T_3 > 1$. Thus, $G_2 > 0$. Define with $\theta(\pi)$ the well-defined solution of $G(\pi, \theta(\pi)) = 0$. Note that $\theta(\pi^t)$ gives the level of income tax rate that would imply no change in the equilibrium income tax rate given a current longevity π^t . Clearly, we have that if $\tau^t > \theta(\pi^t)$ then $\tau^{t+1} > \tau^t$, while if $\tau^t < \theta(\pi^t)$ then $\tau^{t+1} < \tau^t$. Notice also that $\theta'(\pi)$ has the opposite sign of G_1 , with the latter being of an ambiguous sign (as $T_1 > 0$ and, in this case, $T_2 < 0$). Nevertheless, it follows, after a standard phase-diagram analysis, that, regardless of the sign of $\theta'(\bar{\pi})$, the unique BGP is locally saddle-path stable. Figures 1 and 2 demonstrate for the cases of a (locally) increasing and decreasing, respectively, $\theta(\pi)$.

Consider, now, the case of $\bar{d}(\tau) > 0$ for any τ . In this case, we have $T_2 > 0$ and hence, $G_1(\bar{\pi}, \bar{\tau}) > 0$ (as $T_1 > 0$ and $F'(\bar{\pi}) \geq 0$). Define with $\rho(\tau)$ the well-defined, in the neighbourhood of a BGP, solution of

$G(\rho(\tau), \tau) = 0$. Note that $\rho(\tau^t)$ gives the level of longevity that would imply no change in the equilibrium income tax rate given a current tax τ^t . Clearly, we have that if $\pi^t > \rho(\tau^t)$ then $\tau^{t+1} > \tau^t$, while if $\pi^t < \rho(\tau^t)$ then $\tau^{t+1} < \tau^t$. Notice also that $\rho'(\tau)$ has the opposite sign of G_2 , with the latter being, in this case, of an ambiguous sign. It follows, after a standard phase-diagram analysis, that a (local) BGP is locally a sink, and thereby indeterminate, if the function $\rho(\tau)$ is increasing in the neighbourhood of the BGP. If, on the other hand, $\rho(\tau)$ is decreasing in the neighbourhood of a BGP, the latter is locally saddle-path stable and hence determinate. Figures 3 and 4 demonstrate for the cases of a (locally) decreasing and increasing, respectively, $\rho(\tau)$.²⁵

Interestingly, the above discussion emphasises that if the tax burden decreases with current human capital accumulation, the dynamic equilibrium under exogenous longevity is well-determined, while if the tax burden increases with current human capital accumulation, the long-run equilibrium may be indeterminate (globally and/or locally). Specifically, we have shown that

Proposition 1: *Under A1, A2, A3(i) and B1, the BGP in the presence of intergenerational transfers is unique and locally determinate.*

Assume hereafter policies that ensure $\bar{d}(\tau) < 0$ for any τ and thereby a unique and locally determinate BGP. This amounts to restricting attention to policies that satisfy A3(i), which we re-write for convenience as:

B2: $d^u < 0$.

Summarizing our discussion so far, we have that, for any (rationally anticipated) increasing and stable path of longevity which is uniquely characterized by its initial longevity, and under intergenerational transfers that satisfy A1, A2, B1 and B2, higher education decreases current income taxes, human capital investment is higher than its *laissez-faire* counterpart, and the economy is characterized by a unique and locally determinate BGP.²⁶ Also, if $G_1(\bar{\pi}, \bar{\tau}) > 0$, the economy converges to the BGP with decreasing income taxes (see Figure 2). The latter, in conjunction with increasing longevity and the fact that income taxes and education are negatively related, implies also (see (8)) that education is increasing along the equilibrium path. If, on the other hand, $G_1(\bar{\pi}, \bar{\tau}) < 0$, the economy converges to the BGP with increasing income taxes (see Figure 1) and education which may be increasing or decreasing. We now turn to the case of endogenous longevity.

²⁵One can also see, after a phase diagram analysis, that if $\bar{\tau}$ is a local extremum of $\rho(\tau)$ then, there is a well-defined stable path which separates the space $\{\pi \leq \bar{\pi}, \tau\}$ into a stable manifold and an unstable manifold. So, the BGP is again locally indeterminate.

²⁶Note that while under B1 intergenerational redistribution subsidises, in effect, human capital accumulation in the long-run, B2 ensures determinacy of dynamic equilibria. See Bond et. al. (1996) and Alonso-Carrera and Freire-Serén (2004) for models where (short- and long-run) human capital investment subsidies create multiplicity of equilibria.

5 Equilibrium Characterization

Suppose that government policies aim at intergenerational redistribution ($f \geq 0, b \geq 0$) and that they are designed in such a way to ensure that young and adult workers supply labour (see A1 and A2). Suppose also that policies are designed to ensure a well-defined and unique long-run income tax, and to improve long-run education (see B1 and B2). In this Section we study if the equilibrium around the BGP is well-defined or if the economy is prone to animal spirits, due to endogenous longevity.

To analyse this question, note that in equilibrium, as members of any given generation are identical, longevity is determined by the human capital of the typical member, i.e. $\pi^{t+1} = \pi(h^{t+1})$. Equivalently, as $\pi' > 0$, we have that $h^{t+1} = \kappa(\pi^{t+1})$ where $\kappa = \pi^{-1}$: in equilibrium (rationally anticipated) longevity determines human capital and thereby time spent in education $e^t = \mu^{-1}(\frac{h^{t+1}}{h^t})$. In more detail, given optimal investment $e()$, the human capital accumulation function and the definition of the inverse function κ , we have that the equilibrium path of longevity must satisfy $\pi^{t+1} = \pi(\kappa(\pi^t)\mu(e(\pi^{t+1}, \tau^{t+1}, \tau^t, p, f, b)))$ for any $t \geq 1$.

Note that, for any fixed pair of income taxes τ^{t+1} and τ^t , the equilibrium longevity function $\pi(\kappa(\pi)\mu(e()))$ is a member of the family of longevity functions defined by $F(\pi)$ in the previous Section. So, one can very easily see that the path of longevity is again increasing, bounded from above by and converging to $\bar{\pi}$. Also, the BGP income tax is unique, due to B2, and given by $\bar{\tau}$. Furthermore, the unique BGP education is $\bar{e} > e^N$. In addition, the local determinacy of the unique BGP again depends solely on the behaviour of the difference equation which determines the income tax in any period $t + 1$ as a function of the state $\{\pi^t, \tau^t\}$.

Nevertheless, now, as longevity depends on education, the longevity path is not independent of the tax path. The dependence of longevity on income taxes has, in turn, major implications for the equilibrium path of income taxes. It is the interaction between the *jointly* determined paths of longevity and income taxes that may open the door to animal spirits and local indeterminacy, even under our assumptions so far.

In more detail, the first implication of endogenising longevity is that the equilibrium longevity in period $t + 1$ may not be unique for any given state π^t ; that is, the function $\pi^{t+1} = \pi(\kappa(\pi^t)\mu(e(\pi^{t+1}, .)))$ may not imply a well-defined difference equation for longevity. The reason is that, given the state of the economy, the anticipated longevity depends on the evolution of human capital, which in turn depends on anticipated longevity. This cyclicity is a direct consequence of the fact that individual educational choice (and thereby individual human capital) in each period $t \geq 1$ is determined by economic agents, simultaneously and independently, after taking as given average human capital and thereby the probability of survival in that period. This possibility has been emphasized and investigated further in a similar model in Cipriani and Makris (2004a).

Here, instead, we assume away such multiplicity of equilibria in the neighbourhood of the BGP

$\{\bar{\pi}, \bar{\tau}\}$ by denoting with $k(h) \equiv \pi'(h)h/\pi(h)$ the elasticity of longevity with respect to human capital, and focusing on an environment where

$$\mathbf{C1:} \lim_{h \rightarrow \infty} k(h) \neq \mu(\bar{e}) / [\bar{\pi}\mu'(\bar{e})e_1(\bar{\pi}, \bar{\tau}, \bar{\tau}, p, f, b)] \equiv \varpi.$$

Note that ϖ is the product of the inverse elasticities of human capital accumulation (w.r.t. time spent in education), and of education w.r.t. anticipated longevity, evaluated in the long-run. This critical value depends solely on the technology $\mu(e)$, maximum longevity $\bar{\pi}$, and the government policies that satisfy the budget constraint (8) and A1, A2, B1 and B2. Given C1 we have that longevity, around the BGP $\{\bar{\pi}, \bar{\tau}\}$ is driven by a difference equation

$$\pi^{t+1} = \Pi(\pi^t, \tau^{t+1}, \tau^t) \text{ for any } t \geq 1. \quad (15)$$

Note from $\mu() > 0$, $e() > 0$ and the properties of $\pi()$ that $\pi < \Pi(\pi, \cdot) < \bar{\pi}$ for any $\pi < \bar{\pi}$, and, crucially, for any pair of income taxes τ^{t+1}, τ^t . That is, if inherited human capital is finite, we have that, as educational effort is exerted in equilibrium, the rationally anticipated longevity (and human capital) is higher than the longevity (and human capital) of the parent generation. We also have that $\lim_{\pi \rightarrow \bar{\pi}^-} \Pi(\pi, \cdot) = \bar{\pi}$ for any path of income taxes. That is, if the parent generation have had maximum longevity then the only rationally anticipated longevity is the maximum one. These properties of Π imply also that $\lim_{\pi \rightarrow \bar{\pi}^-} \Pi(\pi, \cdot) \equiv \Pi_1(\bar{\pi}, \cdot) \geq 0$.

Furthermore, we have that

$$\begin{aligned} \Pi_2(\pi^t, \tau^{t+1}, \tau^t) &= \frac{e_2(\Pi(), \tau^{t+1}, \tau^t, p, b, f)/e_1(\Pi(), \tau^{t+1}, \tau^t, p, b, f)}{D(\pi^t, \tau^{t+1}, \tau^t)} \\ \Pi_3(\pi^t, \tau^{t+1}, \tau^t) &= \frac{e_3(\Pi(), \tau^{t+1}, \tau^t, p, b, f)/e_1(\Pi(), \tau^{t+1}, \tau^t, p, b, f)}{D(\pi^t, \tau^{t+1}, \tau^t)} \end{aligned} \quad (16)$$

where

$$D(\pi^t, \tau^{t+1}, \tau^t) = \frac{1}{\pi'(\kappa(\pi^t)\mu(e()))\kappa(\pi^t)\mu'(e())e_1()} - 1. \quad (17)$$

As $e_2 < 0$ and $e_3 > 0$, we have that Π_2 and Π_3 have opposite signs. Also, as $e_1 > 0$ - due to B1, Π_3 has the sign of D . In other words, despite higher investment implying higher future human capital and thereby longevity, education and anticipated longevity may not be correlated after a change in the current or future income tax rates. This will indeed be the case if $D < 0$, i.e. if, given inherited human capital, longevity and/or human capital accumulation and/or education as a function of anticipated longevity are sufficiently elastic (note that $\pi'(h\mu(e))h\mu'(e)e_1/\mu(e)$ is equal to the product of the elasticities in question).²⁷

The above discussion implies that the dynamic behaviour of our economy, close to the BGP, is determined by the system of equations (12) and (15), with π^1 being pre-determined by the initial stock

²⁷Notice that if $D < 0$ then $\Pi_3 < 0$, $e_3 > 0$, $\Pi_2 > 0$ and $e_2 < 0$. If, on the other hand, $D > 0$ then $\Pi_3 > 0$, $e_3 > 0$, $\Pi_2 < 0$ and $e_2 < 0$.

of human capital $h^1 = \kappa(\pi^1)$, and τ^1 being free. Note, however, that according to $T(\cdot)$ the income tax in period $t + 1$ depends on current longevity. Alternatively, we have, given (15), that income taxes must satisfy $\tau^{t+1} = T(\Pi(\pi^t, \tau^{t+1}, \tau^t), \pi^t, \tau^t)$ for any $t \geq 1$. Interestingly, the temporal equilibrium of our economy may still not be uniquely determined. In fact, due to self-fulfilling prophecies, there may be multiplicity of equilibrium income tax rate in period $t + 1$ for any given past tax rate τ^t . The reason is similar to the one above: individual decisions on educational effort depends on rationally anticipated longevity π^{t+1} and income taxes τ^{t+1} , with the latter depending on anticipated longevity and thereby individual decisions. This possibility has been highlighted and analysed further in a similar model in Cipriani and Makris (2004b). Here, instead, we assume away such multiplicity in the neighbourhood of the BGP. We do so by assuming hereafter that

C2: $\lim_{h \rightarrow \infty} k(h) < \infty$

Given C2 we have that the dynamic equilibrium, close to the BGP, is characterized by (15) and

$$\tau^{t+1} = \hat{T}(\pi^t, \tau^t) \text{ for any } t \geq 1, \quad (18)$$

with

$$\begin{aligned} \hat{T}_1 &= \frac{T_1 \Pi_1 + T_2}{\Delta} \\ \hat{T}_2 &= \frac{T_1 \Pi_3 + T_3}{\Delta}, \end{aligned} \quad (19)$$

where $\Delta \equiv \Delta(\hat{T}(\pi^t, \tau^t), \pi^t, \tau^t) = 1 - T_1 \Pi_2 = (1 + D)/D$. Therefore, under C1 and C2, current longevity and income tax rate define uniquely the next period's income tax rate, and all these, in turn, determine uniquely next period's longevity.

Nevertheless, once next period arrives, bygones are bygones. True, longevity is determined by the past, but the income tax rate is not pre-determined. Given longevity, the income tax rate depends solely on investment in human capital, and not on past taxes and longevities (see (8)). In fact, the past will have a bite on current education and income tax rate only if the long-run equilibrium is saddle-path stable, as then the BGP is approached by a unique path $\{\pi^t, \tau^t\}$. If the BGP is locally stable, then the economy will be open to self-fulfilling prophecies and multiple equilibrium paths towards the BGP will exist.

To determine the local stability properties of the BGP, define $\hat{G}(\pi^t, \tau^t) \equiv \hat{T}(\pi^t, \tau^t) - \tau^t$. Following the steps in the previous Section we then have in a straightforward manner that the unique BGP is locally saddle-path stable and hence determinate if $\hat{G}_2(\bar{\pi}, \bar{\tau}) > 0$. Consider, here, the case of $\hat{G}_2(\bar{\pi}, \bar{\tau}) < 0$. Define with $\hat{\theta}(\pi)$ the well-defined solution of $G(\pi, \hat{\theta}(\pi)) = 0$. We then have that if $\tau^t > \hat{\theta}(\pi^t)$ then $\tau^{t+1} < \tau^t$, while if $\tau^t < \hat{\theta}(\pi^t)$ then $\tau^{t+1} > \tau^t$. It follows, after a standard phase-diagram analysis, that the unique BGP is locally a sink, and thereby indeterminate. Figure 5 demonstrates for the case of increasing $\hat{\theta}(\pi)$.²⁸

²⁸One can also see, after a phase diagram analysis, that if $\hat{G}_2(\bar{\pi}, \bar{\tau}) = 0$ and $\hat{G}_1(\bar{\pi}, \bar{\tau}) \neq 0$ then the stability properties

It turns out that $\hat{G}_2(\bar{\pi}, \bar{\tau}) < 0$ can, in principle, be the case. That is, policies that satisfy A1, A2, B1 and B2 can create local indeterminacy when longevity is endogenous, even if they induce a unique equilibrium path under any autonomous, increasing and stable longevity path. In particular, we have:

Proposition 2: *Under A1, A2, B1, B2, C1 and C2, there is a critical elasticity of longevity \bar{k} such that the BGP is locally indeterminate if longevity is sufficiently elastic at the limit ($\infty > \lim_{h \rightarrow \infty} k(h) > \bar{k} > \varpi$), while the BGP is locally determinate if longevity is sufficiently inelastic at the limit ($\lim_{h \rightarrow \infty} k(h) < \bar{k}$).*

Proof. If $\lim_{h \rightarrow \infty} k(h) < \varpi$ then $D(\bar{\pi}, \bar{\tau}, \bar{\tau}) > 0$, while if $\infty > \lim_{h \rightarrow \infty} k(h) > \varpi$ then $D(\bar{\pi}, \bar{\tau}, \bar{\tau}) \in (-1, 0)$. As $\Pi_1(\bar{\pi}, \cdot) \geq 0$, and $T_1 > 0$, $T_2 < 0$ we thus have that in the former case $\Delta > 0$, while in the latter case $\Delta < 0$. Note that $-\hat{G}_2 = 1 - \hat{T}_2$. So, the BGP is locally determinate if $\hat{T}_2(\bar{\pi}, \bar{\tau}) > 1$ and locally indeterminate if $\hat{T}_2(\bar{\pi}, \bar{\tau}) < 1$. Note, after using the definition of Δ , that $1 - \hat{T}_2 = \frac{1 - T_3 - T_1(\Pi_2 + \Pi_3)}{(1 + D)/D}$. Clearly, as $T_3 > 1$, $e_1 > 0$ and $e_2 + e_3 > 0$, if $\lim_{h \rightarrow \infty} k(h) < \varpi$ we have that $\Pi_2(\bar{\pi}, \bar{\tau}, \bar{\tau}) + \Pi_3(\bar{\pi}, \bar{\tau}, \bar{\tau}) > 0$ and $1 < \hat{T}_2(\bar{\pi}, \bar{\tau})$. If, on the other hand, $\lim_{h \rightarrow \infty} k(h) > \varpi$ we have that $\Pi_2(\bar{\pi}, \bar{\tau}, \bar{\tau}) + \Pi_3(\bar{\pi}, \bar{\tau}, \bar{\tau}) < 0$ and the sign of $1 - \hat{T}_2(\bar{\pi}, \bar{\tau})$ depends on the size of $D(\bar{\pi}, \bar{\tau}, \bar{\tau})$ and hence of $\lim_{h \rightarrow \infty} k(h)$. It follows in a straightforward manner, after noting $\hat{T}_2 = \frac{D[T_1\Pi_3 + T_3]}{(1 + D)}$, that $1 > \hat{T}_2(\bar{\pi}, \bar{\tau})$ (resp. $1 < \hat{T}_2(\bar{\pi}, \bar{\tau})$) and the BGP is locally indeterminate (resp. determinate) if $D < \frac{1 + (e_3/e_2)}{(T_3 - 1)}$ (resp. $D > \frac{1 + (e_3/e_2)}{(T_3 - 1)}$), evaluated at the BGP. As $\frac{1 + (e_3/e_2)}{(T_3 - 1)} \in (-1, 0)$ and $D(\bar{\pi}, \bar{\tau}, \bar{\tau})$ is strictly decreasing with $\lim_{h \rightarrow \infty} k(h)$, there is a unique critical value $\bar{k} \in (\varpi, \infty)$ such that if $\lim_{h \rightarrow \infty} k(h) > \bar{k}$, then $D < \frac{1 + (e_3/e_2)}{(T_3 - 1)}$ and if $\lim_{h \rightarrow \infty} k(h) < \bar{k}$, then $D > \frac{1 + (e_3/e_2)}{(T_3 - 1)}$ - evaluated at the BGP. ■

6 Conclusion

This paper is an attempt to study the interaction of ageing and intergenerational redistribution in a model where longevity and policies are determined jointly within a dynamic general equilibrium model. Our analysis has been based on a simple OLG model in which life expectancy increases with human capital and agents invest in education, produce and consume output, and receive a pension upon retirement. In our model, government policies aim at intergenerational redistribution. Policies also ensure that all potential workers supply labour. Finally, redistributive and pension policies are designed to ensure a well-defined and unique long-run income tax, and long-run growth higher than its *laisser-faire* level.

Our model produces an expectations coordination problem between the private and fiscal sectors, that can explain significant differences in growth paths followed by otherwise identical countries. In particular, we show that if longevity is sufficiently elastic when human capital is very high, then our

depend on the monotonicity of $\hat{\rho}(\tau)$ in the neighbourhood of the BGP, where $\hat{\rho}(\tau)$ is the well-defined solution of $\hat{G}(\hat{\rho}(\tau), \tau) = 0$. Notice, that local indeterminacy will be the case if $\hat{\rho}'(\tau) > 0$ for any $\tau < \bar{\tau}$ in the neighbourhood of the BGP. If, on the other hand, $\hat{G}_2(\bar{\pi}, \bar{\tau}) = \hat{G}_1(\bar{\pi}, \bar{\tau}) = 0$, the stability properties of the BGP cannot be determined.

economy will be characterized by local indeterminacy of dynamic equilibria and hence the possibility of sunspot equilibria. This raises the issue of how agents can coordinate their expectations. This issue points to future research for criteria of equilibrium selection.

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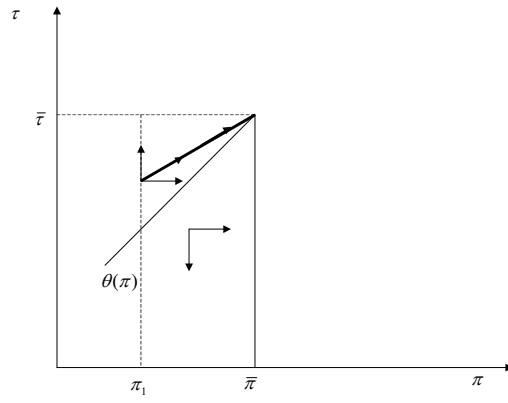


Figure 1:

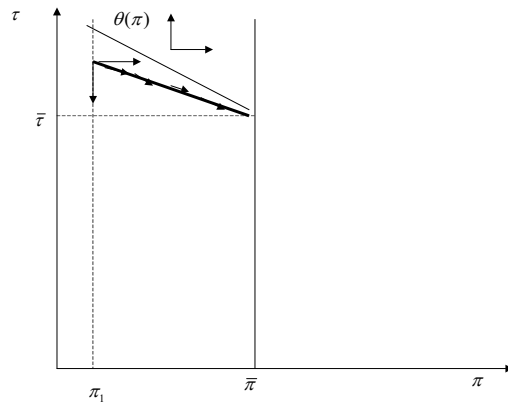


Figure 2:

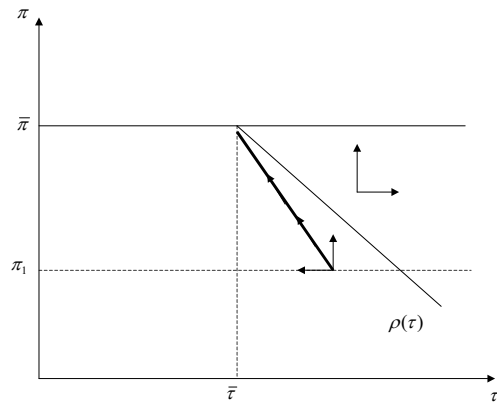


Figure 3:

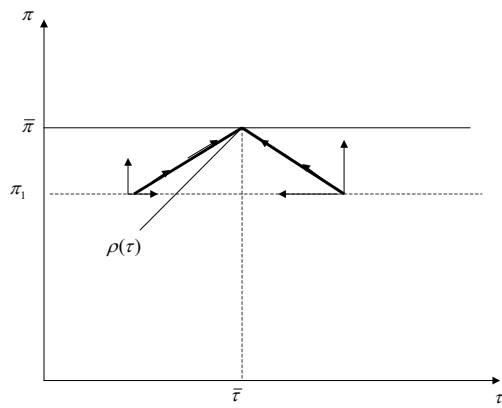


Figure 4:

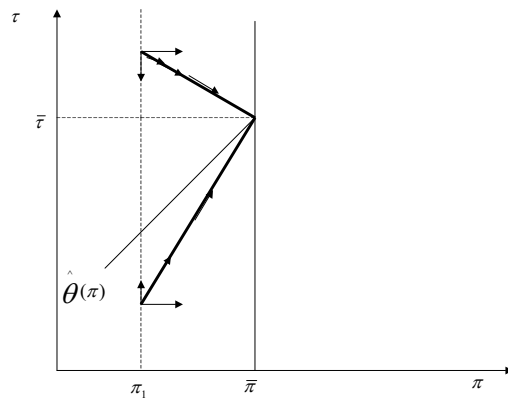


Figure 5: