Pierdzioch, Christian

Working Paper

Irreversibility, endogenous mean reversion, and the investment decision of a foreign firm

Kiel Working Papers, No. 847

Provided in cooperation with:
Institut für Weltwirtschaft (IfW)

Suggested citation: Pierdzioch, Christian (1998) : Irreversibility, endogenous mean reversion, and the investment decision of a foreign firm, Kiel Working Papers, No. 847, http://hdl.handle.net/10419/46733

Kieler Arbeitspapiere
Kiel Working Papers

Kiel Working Paper No. 847

Irreversibility, Endogenous Mean Reversion, and the Investment Decision of a Foreign Firm

by
Christian Pierdzioch

January 1998

Institut für Weltwirtschaft an der Universität Kiel
The Kiel Institute of World Economics

ISSN 0342 - 0787

The Kiel Institute of World Economics
Duesternbrooker Weg 120, D - 24105 Kiel

Kiel Working Paper No. 847

Irreversibility, Endogenous Mean Reversion, and the Investment Decision of a Foreign Firm

by
Christian Pierdzioch

January 1998

Christian Pierdzioch
The Kiel Institute of World Economics
Duesternbrooker Weg 120
24105 Kiel
Germany
Ph.: 49-0431-8814-269
Fax: 49-0431-525
E-Mail: c.pierdzioch@ifw.uni-kiel.de

Irreversibility, Endogenous Mean Reversion, and the Investment Decision of a Foreign Firm*

Abstract

The paper derives a valuation formula for the real option of a firm to undertake an irreversible investment in a foreign economy based on the endogenous dynamics of a stochastic macroeconomic framework with sluggish price adjustment. The option valuation formula is implemented to analyze the impact of macroeconomic dynamics on the attraction of an economy for foreign investors.

JEL classification:

F21 (International Investment; Long-Term Capital Movements)

F41 (Open Economy Macroeconomics)

---

* This paper has been written in the context of a research project on the integration of the reform states in Central and Eastern Europe into the international capital market. Financial support from the Volkswagen-Foundation is gratefully acknowledged. I would like to thank Claudia M. Buch, Ralph P. Heinrich, Joachim Scheide and Hubert Strauß for valuable comments. I am responsible for all remaining errors and obscurities.

1. The Setting

Recent contributions to the theoretical literature on investment decisions emphasize that the possibility to postpone an irreversible investment expenditure under uncertainty creates a positive value of waiting to invest. This value of waiting can be thought of as a real option which can be valued along the lines suggested by financial theory [see *e.g.* McDonald and Siegel (1985); Pindyck (1991); Dixit (1992); Dixit and Pindyck (1994)]. The bulk of this literature introduces untertainty into the model by assuming that returns or prices are driven by purely exogenous stochastic processes. Among others, Leahy (1993) and Dixit (1993) depart from this assumption and show how irreversible investment decisions of firms are affected when the market entry of new firms establishes an upper reflecting barrier to the price process.

The present paper develops an alternative approach of integrating endogenous price dynamics into the optimization problem of a firm and derives a formula for the valuation of the real option to invest based on the endogenous dynamics of a stochastic macroeconomic framework. The macroeconomic environment is characterized by a stochastic version of the Dornbusch (1976) sticky-price model which has first been analyzed by Miller and Weller (1988 and 1990). This setup is used to study the optimality of the investment decision of a foreign firm which has the right to realize a single irreversible investment project in the domestic economy.

Attempts to model the market entry of a foreign firm when investment is irreversible were undertaken in the literature on exchange rate hysteresis and include Dixit (1989) and Baldwin and Lyons (1994). Dixit introduces a social planner to study the market entry and exit of foreign firms when the real exchange rate follows an exogenous geometric Brownian motion.

Baldwin and Lyons utilize a discrete-time dynamic programming approach to analyze the optimal entry and exit decisions of foreign firms under perfect competition when the domestic money supply follows an exogenous stochastic process. They apply a version of the Dornbusch sticky-price framework to analyze the macroeconomic implications of hysteresis and focus attention on proving the existence of hysteresis for the case of an endogenized exchange rate.

The present paper is related to the approach suggested by Baldwin and Lyons but differs from their model in three ways. First, the continuous-time stochastic technique employed by Dixit is adopted to analyze the optimization problem of a foreign firm, second, the domestic price level replaces the money supply as the random variable of the model and evolves according to a continuous-time stochastic process with *endogenous* mean reversion and third, a valuation formula for the option to invest abroad is derived which incorporates the dynamics of the macroeconomic environoment and which can be easily implemented to analyze the impact of the various macroeconomic parameters on the decision to invest abroad.

The paper is organized as follows. In section 2, the stochastic macroeconomic framework is introduced and the dynamic adjustment path of the system is derived. Section 3 is devoted to the study of the optimal entry decision of a foreign firm which incurs sunk costs when it invests in the domestic economy. The section demonstrates how the dynamic structure of the stochastic macroeconomic system can be integrated into the firm's optimization problem. A numerical simulation of the model is provided in section 4. Concluding remarks can be found in section 5.

2. The Macroeconomic Framework

The macroeconomic model consists of the following set of logarithmic equations:[1]

Price adjustment	$dp = \phi(y - \bar{y})dt + \sigma dz$	$0 < \phi < 1$	(1)
Output demand	$y = -\delta(s + p)$		(2)
Currency arbitrage	$E_t(ds) = (r^* - r)dt$		(3)
Money market	$m - p = \kappa y - \lambda r$		(4)

where the Greek letters are assumed to be strictly non-negative and an asterisk indicates a foreign variable. Equation (1) states that the adjustment of the domestic price level consists of two parts. First, a sluggish adjustment of the price level is induced by commodity market disequilibria which can be identified as the difference between output demand y and the natural output $\bar{y}$. Second, the price level is driven by the increment of a standard Wiener process dz with instantaneous standard deviation σ. Equation (2) models output as a function of the real exchange rate. The nominal exchange rate s is defined as the foreign currency price of domestic currency. Equation (3) is the uncovered interest rate parity. According to this currency arbitrage condition, any differential between the domestic r and the exogenous foreign r^* instantaneous interest rate has to be covered by corresponding exchange rate expectations. Expectations are conditional on information available at

[1] For the remainder of the paper, the foreign price level, the natural output, and the stock of domestic money are fixed at unity.

time t (E_t). Equation (4) is the equilibrium condition for the domestic money market. The symbol m denotes the log of the domestic money stock. Money demand takes a standard form and depends on output demand as well as on domestic interest rate.

This stochastic system of differential equations can be written as:

$$\begin{bmatrix} \mathrm{d}p \\ \mathrm{E}_t(\mathrm{d}s) \end{bmatrix} = \mathbf{A}\begin{bmatrix} p \\ s \end{bmatrix}\mathrm{d}t + \begin{bmatrix} \sigma \mathrm{d}z \\ 0 \end{bmatrix} \tag{5}$$

where $$\mathbf{A} = \frac{1}{\lambda}\begin{bmatrix} -\phi\lambda\delta & -\phi\lambda\delta \\ -(1-\kappa\delta) & \kappa\delta \end{bmatrix}$$

and s and p are now interpreted as measures of the deviations from the long-run equilibrium of the deterministic system. The saddlepath stability of the system follows from $|\mathbf{A}| = -\phi\delta\lambda^{-1} < 0$.

The solution of the model can be obtained by defining a deterministic, continuous, and twice differentiable function $s = g(p)$. Applying the rules of stochastic calculus on this mapping yields the following nonlinear ordinary second-order differential equation:

$$\frac{\sigma^2}{2} g_{pp}(p) = a_{21}p + a_{22}g(p) - [a_{11}p + a_{12}g(p)]g_p(p) \tag{6}$$

where the a_{ij} denote the corresponding elements of the matrix $\mathbf{A}$. Equation (6) is valid in the deterministic as well as in the stochastic case. In the deterministic case ($\sigma = 0$), the stable linear saddlepath $s/p = \theta$ can be established as the unique solution of equation (6) by ruling out extrinsic bubbles. In the stochastic case, however, both the stable saddlepath and an infinite number of locally stable nonlinear trajectories pass through the origin. This implies that the model possesses multiple solutions in the absence of additional boundary conditions [see Miller and Weller (1990)]. A unique solution coinciding with the stable linear saddlepath can be obtained by imposing the requirement of global stability on the system.

3. The Maximization Problem of the Foreign Firm

In this section we consider the optimal investment strategy of a single risk-neutral foreign firm which has the right to realize one investment project in the domestic economy. The firm faces the choice between carrying out the investment plan today or at some point in the future. We assume that the firm incurs a lump-sum cost i to invest in the project. This cost is already measured in terms of foreign currency. It is convenient to assume that the investment yields one unit of domestic output and that production has no operating costs. The foreign firm can sell the output of the investment at the domestic price level.[2] The firm's operating profits in terms of foreign currency are given by $\pi = s + p$. Using the results of section 2, this can also be expressed as $\pi = (1+\theta)p$. Let capital letters denote antilogs. Then the antilog of the operating profits $\Pi = P^{1+\theta}$ evolves according to the following geometric stochastic process:

$$d\Pi = \Pi(1+\theta)\left[-\phi\delta\pi + 0.5\sigma^2(1+\theta)\right]dt + \Pi(1+\theta)\sigma dz \tag{7}$$

2 Since the analysis is restricted to the case of a single foreign firm producing one unit of output, the effect of foreign investment on the path of the real exchange rate can be neglected. This rules out the nonlinearities discussed in Baldwin and Lyons (1994).

The foreign firm has the right to invest in the domestic economy. At every moment in time, however, the management of the foreign firm can arrive at the decision to defer undertaking investment and to observe further realizations of the stochastic domestic price level. Hence, the investment opportunity of the foreign firm is equivalent to a perpetual call option with strike price I. The value $F(\Pi)$ of this call option at time t is the expected present value of the investment payoffs:

$$F(\Pi) = \max \mathrm{E}_t\left[(\Pi_T - I)exp(-r^*T)\right] \tag{8}$$

where T denotes the unknown future time at which the investment is made. The Bellman equation for this maximization problem is given by:

$$r^*\mathrm{d}t = \mathrm{E}_t(\mathrm{d}F) \tag{9}$$

Applying Ito's lemma on $F(\Pi)$ and taking expectations results in:

$$\frac{\mathrm{E}_t(\mathrm{d}F)}{\mathrm{d}t} = \Pi(1+\theta)\left[-\phi\delta\pi + 0.5(1+\theta)\sigma^2\right]F_\Pi(\Pi) + 0.5\sigma^2\Pi^2(1+\theta)^2 F_{\Pi\Pi}(\Pi) \tag{10}$$

Substitution into equation (9) yields:

$$0.5\sigma^2\Pi^2(1+\theta)^2 F_{\Pi\Pi}(\Pi) + \Pi(1+\theta)\left[-\phi\delta\pi + 0.5(1+\theta)\sigma^2\right]F_\Pi(\Pi) - r^*F(\Pi) = 0 \tag{11}$$

The model is closed by the following set of boundary conditions:

$$F(0) = 0 \tag{12}$$

$$F(\hat{\Pi}) = \hat{\Pi} - I \tag{13}$$

$$F_{\Pi}(\hat{\Pi}) = 1 \tag{14}$$

The stochastic process given in equation (6) implies that zero is an absorbing barrier for Π. That is, the option to invest will be worthless if the operating profits take on the value $\Pi = 0$. This is formalized in (12). Equation (13) is a value-matching condition and states that at the investment trigger $\hat{\Pi}$ the operating profits have to cover the full costs of realizing the investment which consist of a lump-sum component I and the option to invest $F(\Pi)$ [see Dixit and Pindyck (1994)]. Equation (14) is known as the smooth-pasting condition and rules out arbitrage opportunities by stipulating that the option valuation function become tangent to $\Pi - I$ at the boundary $\hat{\Pi}$.

The next step of the solution procedure is to introduce a continuous, twice-differentiable function $F(\Pi) = F(exp[(1+\theta)p]) \equiv G(p)$. First, this allows us to express the option valuation function in terms of the underlying p.[3] Second, this change of variables transforms equation (11) into a standard form and thus permits to find a solution by a simple look up technique.

Computing the derivatives

$$F_{\Pi}(\Pi) = [(1+\theta)\Pi]^{-1} G_p(p) \tag{15}$$

---

3 Dumas *e.a.* (1993) and (1995) apply this technique in the context of the valuation of currency options in exchange rate target zones in order to express the boundary conditions of their model in terms of an underlying fundamental.

$$F_{\Pi\Pi}(\Pi) = \left[(1+\theta)^2 \Pi^2\right]^{-1} \left[G_{pp}(p) - (1+\theta) G_p(p)\right] \tag{16}$$

and substituting these expressions into equation (11) yields an ordinary homogeneous second-order linear differential equation:

$$0.5\sigma^2 G_{pp}(p) - \phi\delta(1+\theta) p G_p(p) - r^* G(p) = 0 \tag{17}$$

Finally, the boundary conditions (12) - (14) can be expressed in terms of p as follows:

$$\lim_{p \to \infty} G(p) = 0 \tag{18}$$

$$G(\hat{p}) = exp\left[(1+\theta)\hat{p}\right] - I \tag{19}$$

$$G_p(\hat{p}) = (1+\theta) exp\left[(1+\theta)\hat{p}\right] \tag{20}$$

where $\hat{p}$ denotes the investment trigger in terms of the domestic price level. The general solution to (17) can be expressed in terms of the Confluent Hypergeometric Function (CHF) [see *e.g.* Sutherland (1996)]:

$$\begin{aligned} G(p) = C_1 \frac{\sqrt{\phi\delta(1+\theta)}p}{\sigma} H\left[0.5 + \frac{r^*}{2\phi\delta(1+\theta)}, 1.5, \frac{\phi\delta(1+\theta)p^2}{\sigma^2}\right] \\ + C_2 H\left[\frac{r^*}{2\phi\delta(1+\theta)}, 0.5, \frac{\phi\delta(1+\theta)p^2}{\sigma^2}\right] \end{aligned} \tag{21}$$

Here, $H[\cdot]$ denotes the CHF, and C_1 and C_2 are constants. The set of unknown parameters $\{C_1, C_2, \hat{p}\}$ can be evaluated by invoking the boundary conditions (18) - (20).

The CHF plays a prominent role in the literature on irreversible investment under uncertainty when prices or returns follow an *exogenous* mean reversion process [see *e.g.* Metcalf and Hassett (1995)]. Therefore it is interesting to note that the specification of the present model allows to express the solution of the firms maximization problem in terms of the CHF even though the process which drives the operating profits is enriched with an element of *endogenous* mean reversion. The mean reversion of the model is represented in the formula stated in equation (21) by the expression $\phi\delta(1+\theta)$. The parameter δ reflects how movements of the real exchange rate translate into fluctuations of aggregate demand, the parameter ϕ denotes the price adjustment coefficient, and the expression $(1+\theta)$ indicates how the adjustment of the price level affects operating profits. Note that this term contains the slope of the stable trajectory of the macroeconomic system and thus represents all the information about the other structural parameters of the host economy in a condensed form.

4. Simulation of the Model

It is interesting to examine how the strike price I of the real option to invest has to change in order to guarantee that the firm will find it optimal to exercise the option at a given investment trigger price even though a change in the structural parameters of the macroeconomic model has taken place.

Therefore, this section offers the results of a numerical simulation of the model.

The analysis focuses on the effect of different degrees of mean reversion as well as of alternative volatility parameters of the domestic price process on the value of the option to invest. The study is based on the following set of parameter values: $\delta = 0.25$, $\kappa = 1$, $\lambda = 2$,[4] $r^* = 0.05$. Since this constellation of parameters implies $\kappa\delta < 1$, it follows that the adjustment path of the macroeconomic model has a positive slope (i.e., $\theta > 0$).[5] The smooth-pasting point has been fixed at $\hat{p} = 2$ for all options. Table 1 reports the results of the analysis.

p	I	$\sigma = 0.5; \phi = 0.5$	$\sigma = 0.4; \phi = 0.5$	$\sigma = 0.4; \phi = 0.4$
		13.8831	23.2248	22.5985
-1.5		1.04635	0.126619	0.312739
-1		1.36763	0.14946	0.38818
-0.5		1.59138	0.16856	0.44965
0	$F(\Pi)$	1.87044	0.19889	0.54193
0.5		2.35320	0.26476	0.72962
1		3.4473	0.47292	1.24869
1.5		6.83518	1.57945	3.43114

Table 1: Sunk costs and the value of the option to invest for $\hat{p} = 2$

4 Sutherland (1994) uses this set of parameter values for the numerical simulation of a stochastic sticky-price exchange rate target zone model.

5 In the case of $\kappa\lambda > 1$, a positive price shock is accompanied by a devaluation of the domestic currency. This devaluation dampens the effect of an increase in the domestic price level on the operating profits of the foreign firm which are denominated in foreign currency. It follows that the increase (decrease) in the value of the option to invest is dampened for realizations oı the domestic price level above (below) its steady-state level.

The first column of the table depicts selected realizations of the domestic price level p. The second row shows the realizations of the strike price I for the values of σ and ϕ given in the first row of the table. A comparison of the second and the third column shows that a reduction of the instantaneous standard deviation σ of the price process from 0.5 to 0.4 produces remarkably lower values of the option to invest. The lower values of the real option correspond to lower opportunity costs of realizing the investment project. Since the investment trigger has been fixed at $\hat{p} = 2$, a rise of the sunk costs I from 13.8831 to 23.2248 is necessary to offset the reduction in the value of the option to invest. These results indicate that a reduction of the volatility of the domestic price process reduces the value of waiting to invest and enhances the foreign firms incentive to invest abroad.

The fifth column of the table displays the sunk costs and the option values for $\sigma = 0.4$ and a price adjustment coefficient of $\phi = 0.4$. A comparison with the results plotted in the fourth column shows that a reduction in the degree of mean reversion reduces the sunk costs required for an investment trigger of $\hat{p} = 2$ to $I = 22.5985$ and enhances the value of the option to invest. A reduced degree of mean reversion has two effects [see also Kempa *et.al.* (1997)]. First, the slope of the stable trajectory θ is set on the increase and this amplifies the effect of price fluctuations on the operating profits Π. Hence, a lower degree of mean reversion increases the implied volatility of operating profits and this raises the option premium. Second, reducing the degree of mean reversion dampens expectations of convergence of the domestic price level to its steady-state. This, in turn, induces a higher (lower) option premium for $p > 0$ ($p < 0$). Consequently, table 1 shows that the magnitude of the effect of a change in mean reversion on the option value increases as p takes on higher realizations.

5. Conclusion

This paper has derived a valuation formula for the option of a foreign firm to undertake an irreversible investment in the domestic economy. A macroeconomic framework has been utilized to generate endogenous mean reversion of the domestic price level which arises from temporary commodity market disequilibria. The option valuation formula incorporates the endogenous mean reversion and can be implemented to analyze the implications of macroeconomic dynamics on the optimality of the investment decision of a foreign firm. The results of a numerical analysis of the model suggest that both a reduction of the volatility of the domestic price process and a higher degree of mean reversion generate a lower value of the option to undertake an irreversible investment in the domestic economy. Thus, controlling inflation as well as reducing the persistence of commodity market disequlibria should play a prominent for increasing the attraction of the domestic economy for investors located abroad.

Of course, the profitability of an investment in a foreign economy is in reality not only a function of the dynamics of the price level of the host economy. Therefore, it would be interesting to develop an extension of the present model which allows to analyze the impact of several stochastically evolving variables on the decision to invest in a foreign economy simultaneously. Clark (1997), for example, applies the real options approach to investment in order to compute the value of waiting to invest abroad when the political framework of the host economy evolves according to an exogenous Brownian motion-jump process. Incorporating these ideas into the present setup would allow to develop a model of international investment which can be employed to examine the influence of both economic and political factors on the international allocation of capital.

References

Baldwin Richard E. and Richard K. Lyons, (1994), "Exchange Rate Hyteresis? Large Versus Small Policy Misalignmnents," *European Economic Review*, 38, 1 - 22.

Clark, Ephraim, (1997), "Valuing Political Risk", *Journal of International Money and Finance*, 16, 477 - 490.

Dixit, Avinash K, (1989), "Hysteresis, Import Penetration, and Exchange Rate Pass-Through", *Quarterly Journal of Economics*, 104, 205 - 228.

Dixit, Avinash K. (1992), "Investment and Hysteresis," *Journal of Economic Perspectives* , 6, 107 - 132.

Dixit, Avinash K. (1993), "Irreversible investment and Competition Under Uncertainty," in: Kaushik Basu, Mukul Majumdar and Tapan Mitra (eds.): *Capital, Investment, and Development*, Cambridge, Mass. Basil Blackwell, 56 - 74.

Dixit, Avinash K. and Robert S. Pindyck (1994), *Investment Under Uncertainty*, Princeton, NJ: Princeton University Press.

Dornbusch, Ruediger (1976), "Expectations and Exchange Rate Dynamics," *Journal of Political Economy*, 84, 1116 - 1176.

Dumas, Bernard, L. Peter Jennergren and Bertil Näslund (1994) "Currency Option Pricing in Credible Exchange Rate Target Zones", *The Review of Future Markets*, 12, 323 - 340.

Dumas, Bernard, L. Peter Jennergren and Bertil Näslund (1995) "Realignment Risk and Currency Option Pricing in Target Zones", *European Economic Review*, 39, 1523 - 1544.

Kempa, Bernd and Michael Nelles and Christian Pierdzioch, 1997, Bond Option Pricing and Endogenous Mean Reversion, Mimeo. (Department of Economics, University of Essen, Germany).

Leahy, John J. (1993), "Investment in Competitive Equilibrium: The Optimality of Myopic Behavoir", *Quarterly Journal of Economics*, 108, 1103 - 1133.

Mc Donald, Robert L. and Daniel Siegel (1986), "The Value of Waiting to Invest," *Quarterly Journal of Economics*, 101, 707 - 728.

Metcalf, Gilbert E. and Kevin A. Hassett (1995), "Investment Under Alternative Return Assumptions: Comparing Random Walks and Mean Reversion," *Journal of Economic Dynamics and Control*, 19, 1471 - 1485.

Miller, Marcus and Paul Weller (1989), "Solving Stochastic Saddlepoint Stable Systems: A Qualitative Treatment with Economic Applications," Centre for Economic Policy Research, Discussion Paper No. 308, London.

Miller, Marcus and Paul Weller (1990), "Currency Bubbles Which Affect Fundamentals: A Qualitative Treatment," *Economic Journal*, 100, 170 - 179.

Pindyck, Robert S. (1991), "Irreversibility, Uncertainty, and Investment," *Journal of Economic Literature*, 29, 1110 - 1148.

Sutherland, Alan (1994), "Target Zone Models with Price Inertia: Solutions and Testable Implications" *Economic Journal*, 104, 96 - 112.

Sutherland, Alan (1996), "Intrinsic Bubbles and Mean Reverting Fundamentals," *Journal of Monetary Economics*, 37, 163 - 173.