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Spatial Competition and Equilibrium in a Circular Market

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Spatial Competition and Equilibrium in a Circular Market

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Abstract: The paper presents an analysis of spatial competition between two firms in a circular market. It is shown that the existence problem of a Bertrand-Nash-Equilibrium exists for the circular market in the same way as for the linear and bounded market. This is demonstrated by directly studying the reaction functions of the competing producers and without referring to merely technical fixed point arguments. The existence problem is solved by adding a move structure to the game, regarding market entry as sequential and taking place in historical time.

Introduction

The purpose of the paper is to study spatial competition between two firms in a spatial market. The spatial market operates over a network of geographically dispersed buyers and sellers, due to the topography of space and the location decisions of firms and households. In contrast to most other models of spatial competition the geography of the market is represented by a circle with unit circumference. This allows to avoid the characteristic of the models portraying space as a straight line that part of the market is possibly sheltered from competition.[1]

Given that transportation is costly the assumption of a dispersed market changes the character of competition between firms producing a homogeneous product: Even with many buyers and sellers each firm fins only a few rivals in its immediate neighbourhood; further away there are more competitors but their influence is lessened by the transportation costs[2]. The second subsection of this paper provides a full characterisation of potential market areas, of industries in a spatial context and of demand and supply of individual firms.

The properties of a market equilibrium depend on the way in which firms actually compete. Following the seminal work of Hotelling (1929) spatial competition is mostly modelled as a two-stage non-cooperative game with firms choosing location on the first stage and setting prices on the second. The problem of the existence of an equilibrium, when the spatial market is circular and when decisions are taken simultaneously, will be discussed by constructively referring to the firms' reaction functions.

The existence problem well be remedied by choosing the variation of the model that is closest to the original Hotelling framework , i. e. by assuming a plausible move structure. Firms are supposed to enter the market one after the other in historical time and firm one is thus a natural Stackelberg leader of the location game. Firm two is held to be the leader of the second stage game. It will be shown that firm one actually prefers to be in the followership position what the price setting game is concerned.

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1 The representation of space as a circle goes back to Samuelson (1967).

2 That in spatial markets the competitive assumptions becomes hard to maintain and that other approaches are to be adopted to describe the conscious interactions amongst few separated seller and many scattered buyers has long been advocated (cf. Sraffa 1926). It has also been the basis of Chamberlinian models of monopolistic competition since the thirties (cf. Kaldor 1935). The difference between the concepts of concentrated and dispersed markets has an analogue in the difference between a market for differentiated products in the Lancaster sense. In a market with a homogeneous product, substitutes are bunched into a single point of the space of characteristics, and sellers of this product may be numerous. In an industry with differentiated products, substitutes are dispersed in the space of characteristics, and the seller of a particular variety enjoys a quasi-monopolistic position relative the buyers who most preferred it.

Spatial markets

Competition in space is modelled as several firms interacting within the same industry. What is to be understood by an "industry" in the context of spatial markets relies on the geographical distribution of firms and consumers. In general, one might consider a finite set $N=\{1,...,n\}$ of firms producing a given homogeneous product, and a finite set $M = \{1,..,m\}$ of consumers. Each firm $j \in N$ is located at some point s_j in space S. Each consumer $i \in M$ is located at $s^i \in S$. $t(s^i,s_j)$ denotes the cost of transporting one unit of the good between consumer i's and firm j's locations, measured in terms of a given numéraire. We assume that the producers have no control over the transport sector, and hence compete on mill prices. That is, the consumers bear the transport costs, and discriminatory pricing practices, which would be possible under delivered pricing by the firms controlling the transport sector is excluded. The product is produced by any firm j at constant marginal cost c_j and fixed set up costs f, both measured in terms of the numéraire.

Consumers either don't buy the product, or buy exactly a single unit of it per period. As the number of firms is assumed to be finite, consumers are facing a choice within a finite set of mutually exclusive alternatives. π_i denotes the reservation price of consumer i.

From this follows that not all consumers are potential customers of a given firm: If the consumer is located sufficiently far away from the producer such that the reservation price is lower than the transport costs plus the marginal production costs the consumer will never buy from the respective firm, not even if its prices are reduced to ultimately imply zero profits. This leads to the definition of the *potential market area* of a firm

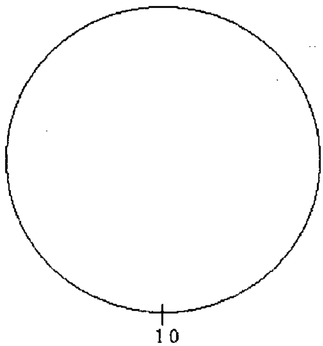
$$M_j = \left\{ i \in M: c_j + t(s^i + s_j) \leq \pi_i \right\}.$$

This definition implies that we can have more than one industry in one region if a region is defined according to the boundaries of a jurisdiction. This occurs of distances between neighbouring firms are so large that their potential markets don't overlap.

The potential market of firm j, denoted by M_j is the set of all consumers $i \in M$ for which the full price of the firm j $c_j+t(s^i,s_j)$ does not exceed the maximal willingness to pay of the customer π_i. Firms are said to be potential direct competitors if their potential markets intersect. For potential direct competitors there exist non-negative prices at which consumers consider buying the product from one or the other of these firms at these prices. When firms are not potential direct competitors indirect ways of mutually influencing each others' market shares exist if there exists a chain of firms in N which are potential direct competitors. Although no consumer ever considers to buy the product from one or the other of these (indirectly competing) firms they can influence the market share of the other potential indirect competitor through a change in their own price and the resulting price reactions of the intermediate firms in the chain.

On this background, an industry is defined as any subset I of firms in N such that any pair of firms in I are potential direct or indirect competitors, while no firm in I is a potential direct or indirect competitor of a firm in N \ I. Graphically, an industry is the set of firms belonging to the same connected component of a graph. Adjacent vertices correspond to potential direct competition, while vertices linked through some intermediate vertices correspond to potential indirect competitors. The graph chosen here to represent a continuum of possible locations in geographical space is a circle with a circumference of unit length, and 0 and 1 identified. As mentioned above, this specification is chosen to avoid complications that result from the existence of market boundaries.

Fig. 1: Circle with unit circumference representing a spatial market



More formally, an industry of a spatial market is defined as subsets

$$I = \{I \subset N\text{: } i, j \in I \equiv \text{direct or indirect competitors, } i \neq j\}$$

of the total of firms which are either direct or indirect competitors as defined above. No member of an industry can be a direct or indirect competitor of a firm which is not a member of I but of N. This definition of an industry implies that even within one jurisdiction several industries producing a homogeneous good might coexist. This would result if two neighbouring firms are sufficiently far apart that the full price exceeds the willingness to pay of the consumers for one unit of the good regardless of which pricing strategy the firms choose. Any change in the geographical distribution of the firms and the consumers, in the number of firms and transport or production costs will change the internal structure of the industry and change the intensity of competition.

Demand of individual firms

The n firms charge the mill prices $(p_1,...,p_j,...,p_n)$. Then necessary and sufficient conditions for consumer i buying from firm j are:

a. $p_j + t(s^i,s_j) \le \pi_i$ for all $j \in N$ and all $i \in M$,

b. $p_j + t(s^i,s_j) = \mathrm{Min}_{k=1}^{n}\{p_k + t(s^i,s_k)\}$ for all $j,k \in N$ and all $i \in M$, $j \ne k$, and

c. if there is a $k \ne j$ such that $p_j + t(s^i,s_j) = p_k + t(s^i,s_k)$ then $t(s^i,s_j) < t(s^i,s_k)$, or $j < k$.

Condition (a) ensures that the consumer benefits from buying one unit of the good supplied by firm j. The second condition makes sure that the consumer maximises his utility for the given mill prices, and the third condition is a rule of mutually exclusive choices and avoids the difficulty that a consumer may be indifferent between any two suppliers.

These conditions allow for the definition of the *market area* A_j of firm j:

$$A_j(p_1,...,p_j,...,p_n) \equiv \{i \in N | p_j + t(s^i,s_j) \le \pi_i\}$$

That is, the market area of a particular firm depends on the other firms' mill prices, the transportation costs and the willingness to pay of the consumers. Given the transportation cost function and the willingness to pay of the consumers any vector of mill prices induces a new partitioning of the set of all consumers. As we assume that each consumer demands price inelastically one unit of the good, the total demand of firm j, denoted D_j is equal to the cardinality of A_j.

A necessary condition for the existence of continuous demand functions of the individual firms is a nonatomic distribution of consumers over space (Gabszewicz/Thisse 1986, pp. 15-19). Therefore the demand side of the model is modified as follows. There is a continuum of consumers with all consumers $s \in S$ having the same reservation price $\pi(s)$ and for which

a. $p_j + t(s,s_j) \le \pi(s)$ for all $j \in N$ and all $i \in M$,

b. $p_j + t(s,s_j) = \mathrm{Min}_{k=1}^{n}\{p_k + t(s,s_k)\}$ for all $j,k \in N$, $j \ne k$, and

c. if there is a $k \ne j$ such that $p_j + t(s,s_j) = p_k + t(s,s_k)$ then $t(s,s_j) < t(s,s_k)$, or $j < k$.

It is assumed that consumers are identical, i.e. $\pi = \pi(s)$ for all $s \in S$, and evenly spread out along the unit circle. The demand to every firm is then given by the (Lebesgue) measure of its market area at prices $(p_1,...,p_j,...,p_n)$.

The demand of each firm still depends on the mill prices of the direct competitors only. This dependence need not to hold for all direct competitors in the same way. It rather hinges upon the relative position of firms in the transportation network and the relevant transportation cost functions. To ascertain whether any two firms belong to the same industry requires the analysis of some "chain effects" linking together firms selling a homogeneous product. The factors that determine the industry structure influence the demand of that industry as well. Hence, it should be expected that the entry of a new firm into a given industry should lead to a variation of demands to existing firms after entry.

Equilibrium

To determine equilibrium prices and quantities it has to be specified in which way firms compete. In a setting as defined in the foregoing subsections, a large number of firms in the industry is not sufficient to guarantee perfect competition: As the demand to each firm depends on the mill prices of its direct competitors only, perfect competition would require a very large number of firms in each (possibly infinitesimally small) place where any one of them is located. As long as set-up costs are not strictly non-negative competition necessarily entails conscious interaction among firms. In other words, space endows each firm with some degree of discretion with respect to determining market prices and/or quantities produced. The more potential markets of several firms intersect the more is the relative monopoly position weakened by the competition of the direct competitors. It is assumed that firms behave non-cooperatively in setting prices.[3]

It is further assumed that each firm in the industry knows how its demand depends on its own price and on those of its competitors. Looking at Nash equilibria it is postulated that an equilibrium price system should satisfy the condition that no firm can increase its profit by a unilateral price change. This condition of internal consistency is the basis of the definition of an equilibrium:

A *price equilibrium* is a n-tuple $\left(p_1^*,\ldots,p_j^*,\ldots,p_n^*\right)$ of prices such that for all j (j=1,...,n) and for all $p_j \geq 0$

$$\begin{aligned} P_j(p_1^*,\ldots,p_j^*,\ldots,p_n^*) &\equiv (p_j^* - c_j)D_j\left(p_1^*,\ldots,p_j^*,\ldots,p_n^*\right) \\ &\geq (p_j - c_j)D_j\left(p_1^*,\ldots,p_j,\ldots,p_n^*\right) \equiv P_j(p_1^*,\ldots,p_j,\ldots,p_n^*) \end{aligned}$$

That is, a price equilibrium is by definition a Nash equilibrium of a game whose players are firms, strategies are prices, and payoffs are profits.

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3 As long as the consumers don't have any possibility of strategic behaviour the non-cooperative analysis retains its validity even if there is collusion among firms. If collusion is total, then we are led to the problem of spatial monopoly. The case of partial collusion raises similar problems like those treated in the sequel insofar as coalitions of producers interact non-cooperatively, and the noncooperative static equilibrium can be considered the threat point of a dynamic collusive equilibrium.

If firms are able to choose both price and location, they do not only determine profit maximising prices but build their network of potential direct and indirect competitors. Price-location decisions can then be taken either simultaneously or sequentially. In the former case a *price-location equilibrium* can be defined in analogy to the above price equilibrium: A price-location equilibrium is a set of pairs (p_j^*, s_j^*) such that for all $p_j \geq 0$ and all $s_j \in S$, j = 1,...,n such that

$$\begin{aligned} P_j(p_1^*,...,p_j^*,...,p_n^*;s_1^*,...,s_j^*,...,s_n^*) &\equiv (p_j^* - c_j)D_j\left(p_1^*,...,p_j^*,...,p_n^*;s_1^*,...,s_j^*,...,s_n^*\right) \\ &\geq (p_j - c_j)D_j\left(p_1^*,...,p_j,...,p_n^*;s_1^*,...,s_j,...,s_n^*\right) \\ &\equiv P_j(p_1^*,...,p_j,...,p_n^*;s_1^*,...,s_j,...,s_n^*) \end{aligned}$$

As long as products which are produced in different locations are homogeneous and production and location decisions are taken simultaneously, no Nash equilibrium in pure strategies exist (Lerner and Singer 1937; for the case of differentiated products see de Palma et al. 1985). To see this assume that such an equilibrium exists. At this equilibrium both firms must earn strictly positive profits. This implies (excluding non-positive set-up costs) that for any two firms $i,j \in N$, $i \neq j$ $(p_j - c_j) > 0$ and $(p_i - c_i) > 0$. Two possible locations of these firms then have to be distinguished. In the first case $s_j \neq s_i$. Without loss of generality it can then be assumed that i's payoff exceeds that of firm j. Then firm j can increase its profits by locating at $\tilde{s}_j = s_i^*$ and by charging a price $\tilde{p}_j = p_i^* - \varepsilon$, with ε being arbitrarily small. Then $P_j(p_1^*,...,\tilde{p}_j,...,p_n^*;s_1^*,...,\tilde{s}_j,...,s_n^*) > P_j(p_1^*,...,p_j^*,...,p_n^*;s_1^*,...,s_j^*,...,s_n^*)$ as firm j now captures the whole market, establishing a contradiction to the definition of the Nash price-location equilibrium. The second case consists of assuming $s_i = s_j$. Then, each player has an incentive to undercut its competitor and capture the whole market, again a contradiction. (Gabszewicz and Thisse 1992, pp. 291-292).

Alternatively, the model of spatial competition can be reformulated as a *sequential game*. In the sequential game, price and location strategies are assumed to be played one at a time in a two-stage process. At the first stage firms take location decisions, at the second they choose their prices. Maintaining the Nash-equilibrium concept for each of the stages the relevant solution concept for the sequential game as a whole is the subgame perfect price location equilibrium. The equilibrium payoffs of the sequential game are well defined whenever the price equilibrium (in pure strategies) in the subgame of the second stage exists and is unique. The equilibrium prices depend solely on the locations chosen at the first stage. Consequently the payoffs of the second subgame can be used as payoff functions in the first-stage game in which the strategies are the firms' locations only.

The *subgame perfect equilibrium* of the sequential price location game is an n-tuple $(s_1^*,...,s_j^*,...,s_n^*) \in S_1 x S_2 x ... x S_n$ and an n-tuple of price functions $\left[p_1^*(s_1,...,s_j,...,s_n),...,p_j^*(s_1,...,s_j,...,s_n),...,p_n^*(s_1,...,s_j,...,s_n)\right]$ such that

i. for any $(s_1,\ldots,s_j,\ldots,s_n) \in S_1 x \ldots x S_n$

$$P_j\left[p_1^*(s_1,\ldots,s_j,\ldots,s_n),\ldots,p_j^*(s_1,\ldots,s_j,\ldots,s_n),\ldots,p_n^*(s_1,\ldots,s_j,\ldots,s_n);s_1,\ldots,s_j,\ldots,s_n\right]$$
$$\geq P_j\left[p_1^*(s_1,\ldots,s_j,\ldots,s_n),\ldots,p_j,\ldots,p_n^*(s_1,\ldots,s_j,\ldots,s_n);s_1,\ldots,s_j,\ldots,s_n\right]$$

for all $p_j \geq 0$, $i \in N$ and $i \neq j$

ii.

$$P_j\left[p_1^*(s_1^*,\ldots,s_j^*,\ldots,s_n^*),\ldots,p_j^*(s_1^*,\ldots,s_j^*,\ldots,s_n^*),\ldots,p_n^*(s_1^*,\ldots,s_j^*,\ldots,s_n^*);s_1^*,\ldots,s_j^*,\ldots,s_n^*\right]$$
$$\geq P_j\left[p_1^*(s_1^*,\ldots,s_j,\ldots,s_n^*),\ldots,p_j^*(s_1^*,\ldots,s_j,\ldots,s_n^*),\ldots,p_n^*(s_1^*,\ldots,s_j,\ldots,s_n^*);s_1^*,\ldots,s_j,\ldots,s_n^*\right]$$

for all $s_j \in S_j$ and all $j \in N$,

The concept of subgame perfect equilibrium is based on the idea that, when firms choose their locations, they each anticipate the consequences for the price competition. They are aware of the fact that price competition will be the more intense the closer they locate to each other. For the concept of a subgame perfect price-location equilibrium to be operational, there must be a unique corresponding price equilibrium to any choice of locations. The restrictiveness of the assumptions required to ensure existence and uniqueness of such an equilibrium poses a major problem of modelling spatial competition. The existence problem for the second stage price subgame will be illustrated in the sequel:[4]

Given that we have a linear transportation cost function $t(s,s') = c\,|\,s-s'\,|$ and that the market is circular with unit length it can be shown that no price equilibrium exists if the shortest distance between two firms gets smaller than 1/4 if location and price decisions are each taken simultaneously.

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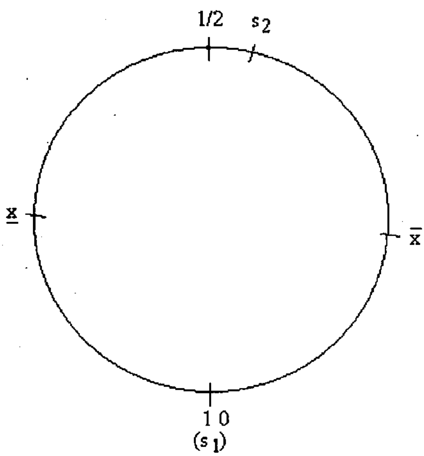
4 The concept of a subgame perfect equilibrium had already been employed in Hotelling's (1929) prototype model of spatial competition. Only fifty years later (and four years after Selten's (1975) seminal paper on the subgame perfect equilibrium) it was discovered that Hotelling's argument was flawed (d'Aspremont et al. 1979); or a restriction of the strategy set of the competing firms hidden in a footnote (Neven 1985). In contrast to what is discussed here the demonstration of the non-existence of the equilibrium in Hotelling's original model refers to a linear and *bounded* market with two firms

Equilibrium in a single market with two firms

Reaction functions and Bertrand-Nash-equilibrium

We consider two firms contemplating location and price decisions in the circular market. The location chosen by firm one will be treated without loss of generality as point 0 (1) of the circle and firm two in the first half of the circular market as shown in figure 2.

Fig. 2: Locations and market boundaries of two firms located in a circular market



Following the logic of backward induction to solve for the subgame perfect equilibrium we start by analysing the pricing decisions of the two firms deciding simultaneously. Before doing so we have to identify the individual profit functions for the case that both firms coexist. This consists of determining the demand faced by the individual firms and multiplying it by the respective mill prices.

From the equality of the full prices at the market boundaries of the two firms, the expression for these market boundaries $\overline{x}$ and $\underline{x}$ can be determined:

$p_1 + c\overline{x} = p_2 + c(s_2 - \overline{x})$, which implies

$\overline{x} = \frac{1}{2c}\{p_2 - p_1 + cs_2\}$. For the market area boundaries in the second half of the circular market we have similarly

$$\underline{x} = \frac{1}{2c}\{p_1 - p_2 + c(1+s_2)\}.$$

As consumers price-inelastically demand one unit of the consumption good, demand of the individual firms - prices being below the consumers' reservation price - are

$$D_1 = 1 + \overline{x} - \underline{x} = \frac{1}{2c}\{2p_2 - 2p_1 + c\}, \text{ and}$$

$$D_2 = \underline{x} - \overline{x} = \frac{1}{2c}\{2p_1 - 2p_2 + c\}.$$

Turning now to the strategic possibilities of firm one in the price subgame we observe three basic options to react to any price demand by firm two:

$$\pi_1 = 0 \quad \text{for } p_1 > p_2 + cs_2, \tag{1}$$

$$\pi_1 = p_1\left(1 + \frac{1}{2c}(2p_2 - 2p_1 - c)\right) \quad \text{for } |p_1 - p_2| \leq cs_2, \tag{2}$$

$$\pi_1 = p_1 \quad \text{for } p_1 < p_2 - cs_2. \tag{3}$$

In section one of the profit function firm two quotes such a low price that firm one is undercut at its own location. Profits will then be zero as firm two captures the whole market. For higher prices of firm two such that the difference between prices is smaller than the unit transportation costs between both locations a market area boundary will lie between the two firms and they will share the whole market. The demand of firm one and hence its profits will then depend on both mill prices and the slope of the (linear) transportation cost function. If, finally, the mill price of firm two is higher than the delivered price of firm one at s_2 the former will be priced out of the market. The different section of the individual demand function firm one is confronted with is shown in figure 3.

Given these outcomes the reaction functions of the two firms are determined. For low prices of firm two firm one will choose a p_1 such that the market boundary lies between the two firms (strategy M). For sufficiently high prices of firm two, it will demand a mill price that lies just below the latter's mill price (strategy U). The reaction function is then

$$p_1(p_2)\big|_U = p_2 - cs_2 - \varepsilon,, \text{ where } \varepsilon > 0 \text{ and } \varepsilon \to 0.$$

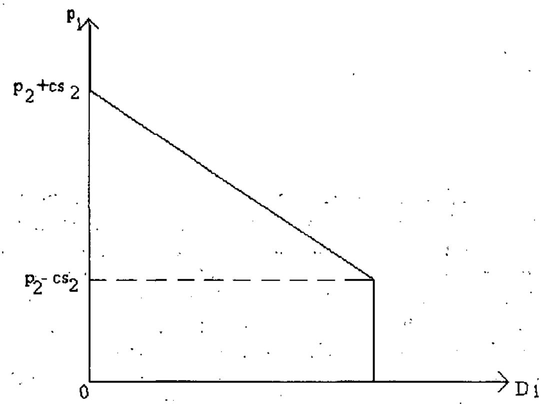
The reaction function for the second section of the profit function follows from the first order conditions for profit maximisation, i. e.

$$p_1(p_2)\big|_M = \frac{1}{4}c + \frac{1}{2}p_2.$$

To determine when firm one will adopt the undercutting strategy and when the market sharing strategy we have to compare the profit functions. If the firm can price its competitor out of the market the profit is $\pi_1|_U = p_2 - cs_2 - \varepsilon$, as it captures the whole market normalised to one. Pursuing this strategy the profit of firm one is hence a linear function of p_2. In case of adopting strategy M the profit function is rather

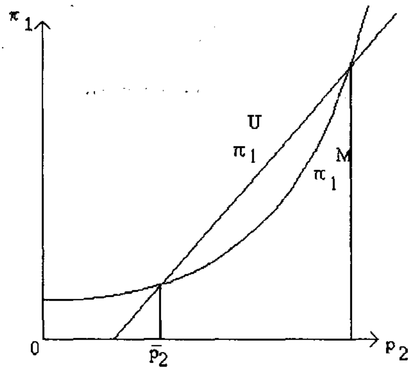
$$\pi_1|_M = \left(\frac{1}{4}c + \frac{1}{2}p_2\right)\left(1 + \frac{1}{2c}\{2p_2 - 2p_1 - c\}\right)$$
$$= \frac{1}{2c}\left(\frac{1}{2}p_2^2 + \frac{1}{2}cp_2 + \frac{1}{8}c^2\right)$$

Fig. 3: Individual demand of firm one given the supply price of firm two



That is, the profit function in case of strategy M is increasing and convex in p_2. Convexity of $\pi_1|_M$ implies that the set $\{p_2 : \pi_1|_M \le \pi_1|_U\}$ is connected. It is obvious that for very low values of p_2 $\pi_1|_M > \pi_1|_U$, as $\pi_1|_U$ becomes negative. If both profit functions intersect the set of prices of the second firm where the undercutting strategy is superior to market sharing is non-empty.

Fig. 4: Profit functions of firm one for alternative pricing strategies



The profit functions corresponding to the reaction functions U and M have two intersecting points if s_2 is restricted to be less than one half, and prices of firm two take the values:

$$\overline{p}_2 = \frac{3}{2}c - c(2-4s_2)^{\frac{1}{2}} \text{ and}$$

$$p_2^{*} = \frac{3}{2}c + c(2-4s_2)^{\frac{1}{2}}.$$

This means that for $p_2 \in [\overline{p}_2, p_2^{*}]$ strategy M is no longer feasible. From the discussion above we know that the smaller value is the relevant value at which firm one will switch to the undercutting strategy U. Firm one will adopt the strategy M at values of p_2 which are lower than $\overline{p}_2$.

The existence of a Bertrand-Nash equilibrium then hinges upon whether we can expect that firm two will choose a price that is compatible with firm one choosing strategy M. The profit function of firm two is

$$\pi_2 = p_2 \frac{1}{2c}\{2p_1 - 2p_2 + c\} \text{ which implies the reaction function}$$

$$p_2 = \frac{1}{4}c + \frac{1}{2}p_1.$$

The Bertrand Nash price equilibrium then implies that $p_1 = p_2 = 1/2\ c$. Now, firm one would turn to the strategy of mill price undercutting if 1/2 c is greater than $\overline{p}_2$. The inequality

$\frac{1}{2}c > \frac{3}{2}c - c(2-4s_2)^{\frac{1}{2}}$ is true if s_2 is smaller than 1/4. An identical argument applies for firm two. Precisely for this reason a Bertrand-Nash equilibrium of spatial competition does not exist. Intuitively, the result says that at a distance of less than 1/4 the competition between both firms becomes so intense that both try to undercut the competitor precluding a market equilibrium.

We can summarise the preceding in the following

Proposition 1: In the Hotelling model of spatial competition with firms using Bertrand-Nash strategies, firm one's reaction function is given by:

(I) for $s_2 \leq 1/4$,

$$p_1(p_2) = p_1(p_2)\big|_U = [p_2 - cs_2] - \varepsilon \text{ for all } p_2.$$

(II) for $s_2 > 1/4$

$$p_1(p_2) = p_1(p_2)\big|_U = [p_2 - cs_2] - \varepsilon \text{ for } p_2 > \overline{p}_2$$

$$p_1(p_2) = p_1(p_2)\big|_M = \frac{1}{4}c + \frac{1}{2}p_2 \text{ for } p_2 \leq \overline{p}_2.$$

That is, for $s_2 < 1/4$ the reaction function that corresponds to a sharing of the market is always dominated. If s_2 is greater than a quarter both firms will stay in the market if firm 2 charges a mill price that is lower than $\overline{p}_2$. If p_2 is greater, firm one will price the second firm out of the market.

More technically, the solution of the existence problem is usually based on a fixed point argument. According to Glicksberg's (1952) generalisation of Kakutani's fixed point theorem (cf. also Debreu 1952 and Fan 1952) a strategic-form game whose strategy spaces are non-empty compact convex subsets of an Euclidean space has a pure-strategy Nash equilibrium if payoff functions are continuous in strategy vectors and quasi-concave in strategies.

As discussed above, with transportation costs linear, the demand function of any producer is discontinuous at a supply price that is sufficiently low to capture the market area of a direct competitor, given the price charged by this neighbouring firm. This discontinuity destroys the quasi-concavity of the profit function. The non-existence of the price equilibrium is the more likely the more firms have incentives to adopt the "mill price undercutting" strategy. As shown above, these incentives depend on the form of the transportation cost function and the shortest distance between direct competitors. In a circular market of unit length with linear transportation costs and two firms the mill price undercutting strategy is a best reply strategy if the distance between the two firms is less than a quarter. The non-existence of the equilibrium indicates that the model is, at the least, either incompletely specified or misspecified. Four amendments of the non-existence problem are particularly prominent:

a. The price equilibrium of the sequential price-location game is restored by simply excluding the mill price undercutting strategy from the individual sets of strategies (Eaton 1976, Eaton and Lipsey 1978, Novshek 1980). Indeed, in the case of linear transportation costs a price equilibrium - the "modified zero conjectural variation price equilibrium" - does exist independently of the locations of the two firms. However, in view of the basic assumptions on firms' behaviour there is no reason why they should disregard the strategy to price its competitor out of business. Furthermore, a sufficient departure from the linear specification of the transportation cost function invalidates the existence result on the modified zero conjectural variation price equilibrium" (Gabszewicz/Thisse 1986, p. 68).

b. The most frequently employed "fix" of the Hotelling-equilibrium of spatial competition consists of assuming a quadratic transportation cost function (d'Aspremont et al. 1979). However, the dependence of the price equilibrium on the peculiar specification of the transportation cost function appears to be unsatisfactory. Deviations from the assumption of an exponential transportation cost function, e. g. by assuming a linear-quadratic form (Gabszewicz/Thisse 1986, pp. 26-30), again lead to non-existence.

c. A third suggestion to solve this problem has been to give up the insistence on a pure strategy Nash equilibrium and to allow for mixed strategy equilibria instead. Employing results of Dasgupta and Maskin (1986) Osborne and Pitchik (1987) have shown that the price equilibrium of spatial competition in mixed strategies does exist, given the discontinuities of the demand function mentioned above.

   If the use of mixed strategies is interpreted as an actual randomisation of strategies it is hard to justify in terms of firms' behaviour with respect to spatial price competition. To quote Rubinstein (1988): "The reason for the criticism (of using mixed strategies, A. K.) is that the naive interpretation of a mixed strategy, as an action which is conditioned on the outcome of a lottery executed by the player before playing the game, is intuitively ridiculous. We are reluctant to let our decisions be made at random, we prefer to be able to point at a reason for each action we take, we prefer to believe that "God does not play with dice" and outside Las Vegas we do not spin roulettes." (Cf. also Radner and Rosenthal 1982 and Aumann 1985)

   However, mixed strategies are not necessarily associated with this naive interpretation. Under what is called a purification idea a player's mixed strategy is considered a plan of action which depends on the player's private information which is not specified in the model. That is, the player's behaviour is taken to be deterministic although it appears as if it were based on randomisation. If this additional information structure were added to the model, the mixed strategy would be described as a pure strategy with the action depending on the extra information.

Describing an equilibrium in price competition, as a subgame of spatial competition, as an equilibrium with mixed strategies relying on the purification concept the explanation if the equilibrium entirely hinges upon the unmodelled factors which should rather be indicated. Hence, the defence of mixed strategies as plans of action implies that there are unmodelled factors which the players perceive as payoff relevant and which render the model incomplete.

d. The existence of the price equilibrium in pure strategies can be established by assuming that consumers' tastes are sufficiently heterogeneous (de Palma et al. 1985). If the consumers' preferences among firms are dispersed, a unilateral reduction in a firm's price no longer brings about a complete change of the allocation of the consumers among firms. A price equilibrium may then exist for any configuration of locations. This fact may further lead to the consequence that firms tend to agglomerate (Anderson et al. 1992, ch. 9). Price competition is then relaxed because of the implicit differentiation among vendors, which gives the firms market power even if they choose the same location. However, the existence problem then depends on the variance of the error term of stochastic demand.

e. The existence of a spatial price equilibrium could be explained by discriminatory pricing, a possibility which is here excluded by assuming mill pricing at the outset.

Alternatively, the existence problem of the models of spatial competition may be interpreted as an unsuitable equilibrium concept. Without resorting to a different equilibrium concept altogether (cf. Bester 1989, MacLeod et al. 1992) the equilibrium of price competition in spatial markets exists if we introduce a move structure instead of assuming simultaneous moves (Anderson 1987). Such an assumption acknowledges the fact that the firms' decisions are taken in real time. It then becomes essential whether firms can costlessly relocate. If relocations were costless, any entrant would rationally anticipate the relocations of the incumbents and we would return to the model of simultaneous moves. It is assumed here that firms make their location decisions once and for all. If firms are immobile once location decisions have been taken, each firm will take account of the consequences of its location upon the location decisions of further entrants. That is, in the (first stage) location subgame an entrant is a (Stackelberg) follower with respect to incumbents and a leader with respect to subsequent entrants (Prescott/Visscher 1977, Hay 1976, Rothschild 1976).

What the (second stage) price subgame is concerned it is assumed that the entrant is a (Stackelberg) leader in the price subgame. It will be shown that this is itself an equilibrium assignment of roles: The leader in the location game (the incumbent) prefers to be a follower in the price subgame and the follower of the location game has no alternative to be the leader of the price subgame.[5] Consequently, the adoption of the Stackelberg model for the price subgame of the two-stage model of spatial competition avoids the instability of the leadership/followership struggle of the original Stackelberg model. These relationships will be discussed in detail in the next subsection.

Stackelberg leadership model for the price subgame

We now assume that firm two, located at s_2 in the first half of the circular market assumes the role of the Stackelberg leader in the (second stage) price subgame. Let p_2^L denote the leadership price of the second firm. Knowing firm one's reaction function (as in section 2 of proposition I), firm 2 will never choose a price where firm one's optimal response is strategy U. As is known from the above discussion the latter is the case if p_2^L is greater than $\bar{p}_2$.

The second firm's leadership profit function is given by

$$\pi_2(p_1(p_2),p_2) = p_2 \frac{1}{2c}\left\{\frac{3}{2}c - p_2\right\}.$$

Setting the first derivative

$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2c}\left\{\frac{3}{2}c - 2p_2\right\}$, equal to zero we obtain the optimal supply price of firm two if both firms use the strategy M: $p_2^L = 3/4c$.

Next we ask under which condition this pricing rule entails a price that is higher than $\bar{p}_2$. To have

$\frac{3}{4}c > \frac{3}{2}c - c(2-4s_2)^{\frac{1}{2}}$, s_2 must be smaller than 23/64. That is, if the location of firm two is such that the distance between both firms is smaller than 23/64 firm two will charge the mill price $\bar{p}_2$ to avoid that firm one switches to undercutting. For values of s_2 of 23/64 or greater firm two's optimal price is 3/4c, as is obvious from setting the first derivative of the profit function with respect to its own price equal to zero. The price of firm one is determined by inserting firm two's leadership price into the reaction function of firm one.

That is, in the Hotelling model of spatial competition with a Stackelberg move structure the equilibrium of the price subgame always exists regardless of how close the competitors are together.

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5 The incumbent can be considered a Stackelberg leader of the game determining who gets the role of the follower in the price subgame. Firm two will adjust to the price quoted by the entrant according to its reaction function.

These results are summarised in Proposition 2.

Proposition 2: In the Hotelling model of spatial competition, firm two's Stackelberg leadership price values are given by

(a) $p_2^L = \overline{p}_2$ for $s_2 < 23/64$,

(b) $p_2^L = 3/4\,c$ for $23/64 \le s_2 \le 1/2$.

(a') The optimal supply price of the follower is given by

$$p_1^F = c - \frac{1}{2}c(2-4s_2)^{\frac{1}{2}} \text{ for } s_2 < 23/64, \text{ as } \overline{p}_2 = \frac{3}{2}c - c(2-4s_2)^{\frac{1}{2}}, \text{ and}$$

(b') for $23/64 \le s_2 \le 1/2$

$$p_1^F = \frac{5}{8}c.$$

For the latter range of distances between the two firms this means that the supply price of the leader is always higher than the price of the follower. However, this does not hold in general. For s_2 smaller than a quarter the price of the follower becomes higher than that of the leader.

Firm two's leadership price of 3/4c implies that its demand amounts to

$D_2 = \frac{1}{2c}\left\{2p_1^F - 2p_2^L + c\right\} = \frac{3}{8}$. Hence firm two's profits π_2 amount to 9/32c.

Similarly, the demand of firm one D_1 is equal to 5/8, and profits are 25/64c. That is, for larger distances between the two firms the profits of the leader in the location subgame and follower in the price subgame are higher than those of the leader of the price subgame and the follower of the location game despite the fact that the price of the leader of the price subgame is higher.

Equilibrium of the location leadership game

It has long been noted that simple Nash equilibria of a location game with given prices involve a multiplicity of equilibria. (Lerner and Singer 1937, Eaton and Lipsey 1975) Having assumed that firms enter the market sequentially in historical time it is natural to consider firm one as the leader of the first stage location (sub-) game. Firm one will choose a location arbitrarily and this location is defined as point 0 (1) on the circle with unit circumference.

If firm two locates at a larger distance from firm one than 23/64 (and not at a point larger than 1/2) the profits of both firms will be independent of the particular locations. Prices and profit functions are then independent of s_2. If s_2 is smaller than 23/64 the leadership price of firm two is a function of its location. As the price leader's profit function depends on its own price only it will hence be a function of the location in the range $0 \leq s_2 < 23/64$. Evaluating the profit function of the second firm at $\overline{p}_2$ we get

$$\pi_2 = \frac{3}{4}c(2-4s_2)^{\frac{1}{2}} - \frac{1}{2}c(2-4s_2).$$

From the first order condition of the profit maximum follows the optimal location of firm two, $s_2 = 23/64$. That is, for the sequential price-location equilibrium the range of distances between the two firms of less than 23/64 is irrelevant. This leads to

Proposition 3: Given that firm one is the leader of the location game of the sequential price-location game and assuming without loss of generality that firm two will locate in the first half of the circular market, firm two will choose a location between 23/64 and 1/2. At all these locations prices and profits will be independent of the distance between the two firms.

This means that for the price-location equilibrium only the price equilibria (b) and (b') of proposition 2 and the corresponding profit values are of importance.

The last question to be answered is whether the assignment of the roles of leader and follower in the two subgames is arbitrary. In particular, given that firm one is the leader in the location game and initially the only firm on the market one might wonder whether it is not natural to assume that firm one is the leader of the (second stage) price subgame as well. To see whether firm one has an interest in embracing the role of the leader of the price subgame the profit of the game as developed above is compared to the profit of the price location game where firm one is leader of both the location and the price game.

If firm one were the leader of both subgames it would maximise its profits with respect to firm two's reaction function and would hence take pricing decisions with respect to the following profit function:

$$\pi_1 = p_1\left\{1 + \frac{1}{2c}\left(-\frac{1}{2}c - p_1\right)\right\}$$

Maximisation then leads to the supply price $p_1^L = 3/4c$ and maximal profits of 9/32c. Comparing this to the profits of firm one being the follower of the price game, it transpires that firm one has no interest in being the leader of the price subgame as this role implies lower profits. Hence both firms are interested in having the role of the follower in the price subgame. As firm one is the first on the market and firm two is contemplating entry it cannot avoid that firm one adjusts its price after firm two has quoted its mill price. In this sense we might consider firm one as the leader of a game on the assignment of roles in the price subgame. This can be summarised in

Proposition 4: Firm one as the leader of the first stage location game has no incentive to contest firm two being the leader of the price subgame. Regarding market entry as sequential in historical time it follows that firm two is the leader of the second stage price game.

Conclusions

In this paper spatial competition between two firms in a circular market has been analysed. It has been shown that the existence problem of a Bertrand-Nash-equilibrium exists for the circular market in the same way as for the linear and bounded market. This has been done by directly studying the reaction functions of the competing producers and without referring to merely technical fixed point arguments.

It has further been shown that the existence problem can be solved by adding a move structure to the game, regarding market entry as sequential and taking place in historical time. Proceeding from this assumption it has been demonstrated that contrary to what may be expected it is natural to assume that the first firm in the market is the follower of the second stage price game.

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