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A Continuous Time Cyclical Growth Model for the Federal Republic of Germany: Construction, Estimation and Analysis*

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A Continuous Time Cyclical Growth Model for the Federal Republic of Germany: Construction, Estimation and Analysis*

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This paper seeks to unify and extend a number of recent directions in macro-economic research. Firstly, there has been a tendency to introduce new factors and to unify their treatment. Thus imported raw materials together with floating exchange rates are now an established part of the literature. This has naturally led to the introduction of real wage rigidity as an essential element in the study of transmission of impulses between countries. Yet the unified treatment of such factors in a macro-econometric model is rare, the empirical work being mainly of the single equation on reduced form variety. The empirical work on real wages and employment is a good example (see Sachs, 1983, for a review). In addition most macro-econometric models are Keynesian in construction, the supply side being relatively poorly developed.

Secondly, there has been a regained interest in economic dynamics, in part due to the availability of suitable computer facilities. Thus, for example Day, 1982, Stutzer, 1980, and Dana and Malgrange, 1981, have investigated dynamics which arise from non-linear systems where solutions may be aperiodic and chaotic. However, the dimensions are extremely small and the models are regarded by the authors as illustrative of what <u>may</u> arise in more realistic systems. The interest in exchange rate dynamics has also led to a rebirth in dynamics and the introduction of factors such as the government budget constraint. Examples are Turnovsky, and Sachs and Wyplosz, 1984. Again the systems are extremely small and regarded as illustrative. Nevertheless despite their smallness, the more usual qualitative techniques prove extremely limited and must be supported by numerical simulation.

The third area of recent interest is that of rational expectations. This field has quickly moved beyond the early descriptive work (mainly associated with the simplistic monetarist models) and now heavily relies on empirical techniques and simulation for model solution (eg Buiter, 1984, Blanchard and Khan, 1980, Lipton et al. 1982).

This paper is concerned with the first two topics. The third and part of the second are covered in a separate paper. In this paper we construct, estimate and analyse a 21 equation neo-classical macro-economic model. We incorporate imported raw materials, flexible exchange rates and full flow of funds accounting identities. The model is fully dynamic and includes growth features (ie investment alters the capital stock). The model is outlined in Section I. In Section II we solve the model for its steady state growth path and discuss the problem of steady state levels. Section III reports the estimation by FIML techniques of the complete system. Following Leamer, 1983, we report alternative models in order to present evidence as to model sensitivity. Section IV is concerned with the analysis of the estimated model using eigenvalue analysis, sensitivity analysis and the steady state. Section V discusses a number of questions for future work which represent caveats on the present results.

Before proceding to the description of the model we must briefly address an important issue which is handled consistently throughout the entire paper. Being firstly a theoretical model it is naturally expressed in continuous time: the dynamics are represented by differential rather than difference equations.

Perhaps from the pure theoretical perspective a mixed differential/difference equation system is preferable. However, such a system is analytically intractable and no estimator exists. Given a choice, differential equations are preferable from both the theoretical and econometric perspectives. Although individual economic decisions may be made at discrete time intervals it is difficult to believe that they are perfectly synchronized. If they overlap in some stochastic manner then the economic phenomena may be treated as if it were continuous. Moreover, a further difficulty with discrete analysis is that there is no obvious time interval that can serve as a natural unit. The assumption of a certain fixed period length may unwittingly be the cause of misleading conclusions leading authors such as Turnovsky, 1977, to argue that all discrete models should be tested for consistency by allowing the time interval to tend to zero. A good example of this which illustrated that seemingly similar differential and difference equation systems may be as different as night and day is Graves and Telser, 1968. Closely related is the problem in discrete time models of budget identities: does equilibrium obtain at the beginning or at the end of a period? This gives rise to a confusion between stock and flow equilibria.

From the econometric perspective there are also many advantages to posing the theory in continuous time. The advantages do however relate to the need to recognize that a discrete equivalent to a continuous time system is not achieved simply by replacing differentials with a first difference (see Section III). Furthermore it is explicitly recognized that a flow variable cannot be measured instantaneously thereby allowing for correct treatment of stock flow models. With these changes the estimator is independent of the observation interval. By contrast discrete models must be explicitly formulated in relation to the data so that a quarterly model will be different from one built to be estimated with annual data (see Gandolfo, 1981). This raises serious identification problems. Dynamics are our focus and here a continuous model allows a more satisfactory treatment of distributed lag processes. In a discrete model the disturbances in successive observations are usually assumed to be independent, but this assumption can only be maintained if the size of the time unit <u>inherent</u> in the model is not too small relative to the observation period (see Gandolfo, 1981, and Bergstrom, 1976). The lags in the system are, however, not always integral mutiples of one time unit whose size is compatible with the independence assumption. Since it may happen that distributed time lags with a lower time limit of almost zero (ie nearly instantaneous equilibrium) have to be considered, a continuous time specification is more correct.

Our model emphasizes the connection between the short and long run so it is best if we clear up at once a common confusion. The difference between a short and long run model is not whether the data is quarterly or yearly. It is the presence or absence of feedback which determines the difference, time aggregation only serving to confuse identification issues (eg Sims, 1971). Our model is estimated with quarterly data. It is not thereby a quarterly model! A continuous time system can be solved for any time interval for there is a clear distinction between those parameters which are time interval dependent (ie adjustment speeds) and those which are not (eg elasticities).

Section I - The Structural Model

In describing a macro econometric model it is possible to devote pages to single equations, their history, literature etc. This we cannot do and in part it is not necessary given that many of our component equations are quite conventional. Where they are not we shall go into more details but must remain brief.

One of the most important choices in constructing a model is that of income sector and the menu of financial assets associated with the corresponding flow of funds system. These choices are illustrated in Table I. There are four income sectors: Household, Firm, Government and Overseas. In the top part of Table I the national income account relationships determine the net financial savings of each sector. These are associated with an increase or decrease in financial assets detailed in the bottom half of the Table: the flow of funds accounts. Only two types of financial assets are recognized: home denominated financial assets and overseas denominated financial assets. Hence for the home country bonds, money and equity are aggregated the focus being strictly on the substitutability between home and foreign net assets (see also Meade et al., 1984).

Aggregation in this manner can only be theoretically justified if the assets are perfect substitutes or the relative prices are independent of relative supplies. We cannot pretend our choice is based on such evidence but there again, neither is the more usual money, bonds, equity aggregation. The reason for the aggregation is to avoid an unwarranted simplification, that of the three assets only one is internationally traded or held. To make all three traded would increase dramatically the model dimensions and in any case, given data availability, could not be estimated. We therefore assume or assert that our aggregation captures the major portfolio influences[1].

The equations reflecting the flow of funds identities are detailed in Table II, equations 18-21. The latter equation reflects the adding up restriction across sectors (ie only three are defined independently). In order to reflect our data, for estimation, transfers are added to these equations. Equations 18-21 reflect a much more important feature of the model; income is defined in the national accounting sense without regard to capital gains or losses. Thus the sectoral budget identities are not defined along the lines of Turnovsky, 1977, to include revaluation effects, something particularly important for government via the inflation tax (eg Hendry, 1980). The reason for their exclusion is strictly data related but in another paper we will investigate the effects via simulation. Finally, the national income accounting identity is defined by equation 17. As final goods imports, TMY, and consumer goods, C, are measured in own prices a terms of trade effect must be incorporated.

Having defined the accounting structure we may now turn to a brief description of the behavioural or stochastic equations of Table II. For simplicity the disturbance terms are ommitted. The general form of each equation is

$$DlnX = \alpha ln(\frac{XS}{X}) \qquad (1)$$

where D is the differential operator, $\frac{d}{dt}$, and ln is the natural log. The left hand side could also be written as $\frac{DX}{X}$. The variable XS is a latent variable defining the desired or partial equilibrium level toward which X is adjusting at a speed given by α . XS is in turn defined by our theoretical model. If XS, the desired level, is greater than the current level of X then X will change, DlnX, the extent of the change given by α . It can be shown (Gandolfo, 1981) that (1) is simply a geometric lag distribution in continuous time

$$x(t) = {}_0\int^{\infty} \alpha e^{-\alpha 7} xs(t-\tau)d\tau \qquad (2)$$

It therefore follows that $\frac{1}{\alpha}$ is the mean lag of (1): the time required for 63% of the disequilibrium to be corrected. The parameter therefore depends on the time aggregation of the model whereas the parameters affecting XS are independant. The parameter α also reflects ones smoothness priors but in continuous time. It does not follow that such priors may be imposed on discrete data without alteration (eg Sims, 1971).

There is of course no reason to suppose that α is independant of disequilibrium in other markets nor on other factors. As it stands, (1) implies that DlnX is only dependant on disequilibrium in its own market. If α is so dependant then the relationship expressed by (1) is extremely nonlinear. We therefore use a linearization

$$DlnX = \alpha ln\frac{XS}{X} + \alpha' ln\frac{YS}{Y} \qquad (3)$$

where α is now influenced by the disequilibrium in market Y. $\frac{1}{\alpha'}$ does not have a mean lag interpretation.

Equations 1-3, Table II, are based upon three factors of production: capital, K, labour, EMP and imported raw materials, EN. In order to avoid the implicit assumption of separability in the production function, output Y is not defined as value added or GNP but as output gross of raw materials, EN. The partial equilibrium values of YS, EMPS and ENS are given from the usual constrained optimization solution and are therefore functions of time, real factor prices and capital stock. No explicit production function is specified although one is implicit in cross equation restructions between the elasticities, β . In contrast to Nadiri and Rosen, 1969, we do not define cross equation restrictions on the adjustment speeds for we do not assume that the production function holds at every point in time. Equations (1)-(3) are therefore comparatively unrestricted. A Keynesian spillover effect in which disequilibrium in inventories directly affects output is also included in equation (1). Equation (2) follows evidence from among others, Tinsley, 1971, that the speed of employment adjustment is influenced by the level of unemployment here given as the log of the ratio of employment, EMP, to labour force, LF.

Equation (4) reflects all the strength and weaknesses of neo-classical theory in that investment is modelled as determined by the difference between the marginal product of capital, $\beta 9 \frac{YS}{K}$, and the real interest rate. Investment refers to net investment and, given the assumption of long adjustment lags, modelled as an Almon lag distribution (ie increasing then decreasing). In continuous time this is a second order differential equation with the lag parameters α_5 and α_{51}. For simplicity the second order equation is reduced to first order by defining the identity (16). Whether the marginal product so defined in (4) has any meaning at all is of course a controversial question. We utilize it here not out of belief but because it is present in all theoretical neo-classical macro-economic models. Our intention is to empirically implement such a model and not to test for "truth" or power against a competing model. In any event we regard the latter as illusory. It should also be noted that from identity (16), the capital stock at any time is simply the integral of past net investment in real terms. Whilst this facilitates data construction it is not an ad hoc formulation. Rather it is an untestable imposed hypothesis, an implied restriction.

Equation (5) models the demand for final good imports, TMY, as a function of an activity variable, C, and relative prices. Unlike many other macro models it is derived from a well defined demand system, Armington, 1969. Home and foreign goods are assumed to be imperfect substitutes, the utility function being a CES function. Solving the maximization problem yields (5a) where β_{11} is the elasticity of substitution between home and foreign goods and β_{11a} is unity. Maximization also yields a consistent price index which in this case has the interpretation of a consumer price index, equation (15a). Identities (5b) and (6b) are discussed further in Section III. Exports are similarly modelled as a function of an activity variable, YO, and relative prices. Not being derived from a well specified demand system it does not have the same interpreation as (5).

Equation (6) models household consumption expenditures as a function of the interest rate, r, real <u>financial</u> wealth, $\frac{WI}{PC}$, and real disposable income, $\frac{YH}{PC}$. As suggested by finite-horizon optimizing behaviour of households (eg Blanchard, 1983), the coefficient β_{17} in (7a) will generally exceed the interest rate. It is important to note that household wealth, WI, does not <u>directly</u> include investment from retained earnings by firms. Household disposable income is defined by equation (7c) where δ is the average taxation rate on households, Θ the average corporate tax rate and Θ' a complex combination of the two. Equation (7c) is defined so as to avoid the need for a differential equation explaining dividends but at the same time incorporating the need for differential average tax rates.

Wage rates are represented in equation (8a) as evolving in response to real consumer wage defense, β_{38}, a time trend reflecting views about technological growth sharing, λ_4, and the terms of trade, β_{39}. The equation can therefore be seen as the reduced form of a union bargaining process (eg Pencavel, 1984, Nickell, 1983) where the capital stock has been subsumed into the time trend. The success of these wage demands is modified by the level of unemployment via α_9'. This can be seen as related to bargaining power evolution or the variation of payoffs at the threat point of a Nash cooperative game (Nickell, 1983).

Government activity is modelled by equations (9), government expenditures, (11), interest rate policy, (12), tax policy, and (13), exchange market intervention. Especially for expenditures it is usual to treat policy instruments exogenously. Whilst this may be justifiable at the theoretical level, it is less convincing at the empirical level where the simultaneous determination of the behaviour of both the public and private sector must be taken into account. Especially in a dynamic model, failure to take into account systematic feedbacks on the part of the authorities leads to inconsistent estimates of private sector behaviour.

As with wage behaviour, which in Europe should be firmly based on a model of Union behaviour, the above equations should be based on a well defined model of government behaviour such as ideology maximization subject to a re-election constraint (eg Kirkpatrick and Widmaier, 1984). Given the purposes of this paper we shall not do so here but instead specify each equation on a more or less ad hoc basis. Expenditures are a function of GNP, γ_{17}, labour market conditions and inflation while taxation reflects also unemployment conditions and the level of the government deficit relative to corporate borrowing. Taken together they define structural deficit policy in addition to its structure. The interest rate reaction function and exchange market intervention equations reflects the substitutability of the two "instruments". Interest rates may be used to influence the exchange rate or more directly for internal purposes (eg β_{40}). In both equations the authorities have in mind a targeted real exchange rate. A more formal derivation along the lines of Mutoh, 1980, and Suss, 1980, is planned.

Where the model is unique is in the specification of producer prices, equation (10). Our model is a disequilibrium one and therefore has an important role for buffer stocks such as inventories. Such stocks allow a separation between plans and actions, a feature characteristic of disequilibrium. However, in disequilibrium explicit price adjustment rules must emerge. "They must be constructed in such a way that actions can take place more or less continuously even though current prices transmit more or less erroneous information and must themselves be adjusted according to unfolding information". Day, 1984, Pg. 65.

For this model, and for Day, such unfolding information is contained in inventory flows which in turn reflect errors or information from both production and sales decisions[2]. In disequilibrium both stocks and flows have a role[3] and are reflected by parameters α_{10} and α_{11} respectively.

For those accustomed to thinking in monetarist terms our price equation may seem strange containing no nominal variables and particularly no money. However, (10) is a structural equation not an equilibrium or reduced form equation. The price level being merely a renormalized money demand function is an equilibrium condition and so cannot be compared with ours. The point of comparison can only be made for the steady state to which we shall return in the next section. On the other hand proponents of cost plus or cost based pricing will also find our form strange. However, ceteris paribus, an increase in wages will lead to a decline in output from (1), perhaps an increase in demand from (7), a decline in inventory from (17) and so a price increase.

Lastly, but by no means least, is the exchange rate determination, equation (14). Given the asset aggregation, it is natural for us to focus on the role of asset substitutability: the portfolio balance model of exchange rate determination. In an earlier work, Kirkpatrick 1984, we explicitly specified a foreign asset demand function for households and then formed an excess demand function for the exchange rate - with mixed success. However, in this paper, we utilize Blundell-Wignalls', 1984, model. This is not only due to his excellent results but also out of an interest to see whether they will hold in a full macro-economic model and over a longer data period.

Very briefly the model specifies that the return from domestic assets, r, must equal the expected return from foreign assets, ro+EDlnDOLR, plus the risk premium X:

$$X = r - ro - EDlnDOLR \tag{4}$$

For rational expectations simulation EDlnDOLR may be replaced by the actual rate of depreciation thereby leading, hopefully, to an unstable root (see Buiter, 1984). For estimation this is not possible so one must form a model for EDlnDOLR. This Blundell-Wignall does by positing the real exchange rate to be a function of the cumulative partial first difference of the current account (our equation (14a)). Assuming the expected rate of change in the equilibrium exchange rate to be equal to the expected inflation differential, and substituting actual inflation rates for the latter yields.

$$EDlnDOLR = \theta(lnP-lnPF- \delta\tilde{CA}-DOLR) + DlnP-DlnPF \tag{5}$$

The parameter, θ, reflects the speed with which the expected rate of depreciation adjusts to a gap between actual and expected exchange rate levels. The parameter δ reflects the influence of the current account on the expected level of the equilibrium real exchange rate.

The risk premium is given by

$$X = X_o - \emptyset(lnBM-lnWI) \tag{7}$$

where perfect asset substitutability would imply X_o, $\emptyset = 0$. Equations (4), (5) and (6) are next solved for the equilibrium exchange rate, DOLRS, and the usual partial adjustment given by α specified as resulting from transactions costs, uncertainty etc. Equation (14) is therefore quite flexible allowing a nesting of perfect and imperfect substitutability hypotheses. We shall have more to say about potential deficiencies below. It is also clear why we have chosen the interest rate reaction function, (11), to reflect real interest differentials.

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Section II - Qualitative Analysis - The Search for the Steady State

The model specified in Table II is, despite the explicit choice of functional form, a theoretical model. As such one is naturally concerned with model solution and in particular whether an equilibrium exists. Given that the model is dynamic and intended to describe the economy over a relatively long time period, it is natural that the equilibrium to be considered be a steady state one. Not only does the analysis of the steady state give information on the dynamic behaviour of the model, it also acts as a check on the mathematical consistency of the model itself. "Implausible long run behaviour could indicate a structural defect such as the omission of an important feedback. This could seriously affect the predictive powers of the model and its usefulness for either medium term forecasting or policy analysis. If a macro-economic model ... does not have a steady state the variables will be fluctuating in some way for all t and, except in special circumstances, these oscillations will be unstable". (Wymer, 1976, Pg. 12).

For details of model solution the reader is referred to Gandolfo (1981) and an Appendix available from the author. Very briefly one assumes that exogenous variables grow at a constant proportional rate

$$Z_{i(t)} = Z_i{*}e^{\lambda it} \tag{7}$$

where the initial value is $Z_i{*}$ and λ_i may take on any value. If endogenous variables have the form

$$Y_{i(t)} = Y_i{*}e^{\rho it} \tag{8}$$

then one solves for the growth rates, ρ_i, and the initial conditions, $Y^*_{i(t)}$.

The growth rates are comparatively easy to solve and are given in Table III. On the long run reference path employment is assumed to grow at the same rate as the labour force (ie a constant <u>rate</u>

of unemployment). Real variables with the exception of raw material imports grow at a rate determined by the world growth rate and the income elasticity of demand for exports. Countries with an elasticity greater than one have a higher growth rate.

The growth rate of nominal variables is indeterminate so has been arbitrarily fixed at λ_{11}. The inflation rate is given as the difference between the nominal and real growth rates, something which should be quite acceptable to monetarists! It also illustrates the point made above when discussing the structural price equation. The exchange rate moves to offset inflation differentials or to hold a real exchange rate constant. Finally the steady state growth rate of the real wage is determined by technological progress, $\frac{\lambda 2}{\beta 4}$, the raw material terms of trade, $\frac{\beta 5}{\beta 4}(\lambda_7 - \lambda_6)$ and the difference between the real and labour force growth rates. Hence an economy which lowers the growth rate of population, ceteris paribus, can move to a path characterized by a higher growth rate of real wages. Whether it will do so and how is a question for stability analysis (see below).

The story is not yet complete for the solution implies a number of restrictions, most being detailed in Table III. The elasticity of inventory demand with respect to output, β_{41}, must be unity as indeed must a similar elasticity in equation (9a). Not unsurprisingly the elasticity of imports with respect to consumption, β_{11a}, must also be unity. It is also interesting to note that when equation (15a) defining consumer prices is replaced by a Cobb-Douglas function, a practice quite common in theoretical models incorporating a consumer real wage, then the cross equation restriction on β_{11}, equation (5a), the elasticity of imports with respect to relative price, is unity. This is exactly what we expect, from the underlying CES demand system!

Perhaps the most important restriction, for it is one immediately related to institutions and policy is that related to the wage setting equation, restriction (6) of Table III. If there is real wage rigidity, defined as $\beta_{38} = 1.0$, then the time trend for real wages must take on a quite specific value as should the parameter relating to raw material terms of trade changes. By what social and economic institutions is this brought about? We should again reiterate that the steady state path is a reference path. The economy may not be on it for any length of time and indeed it may even be unstable.

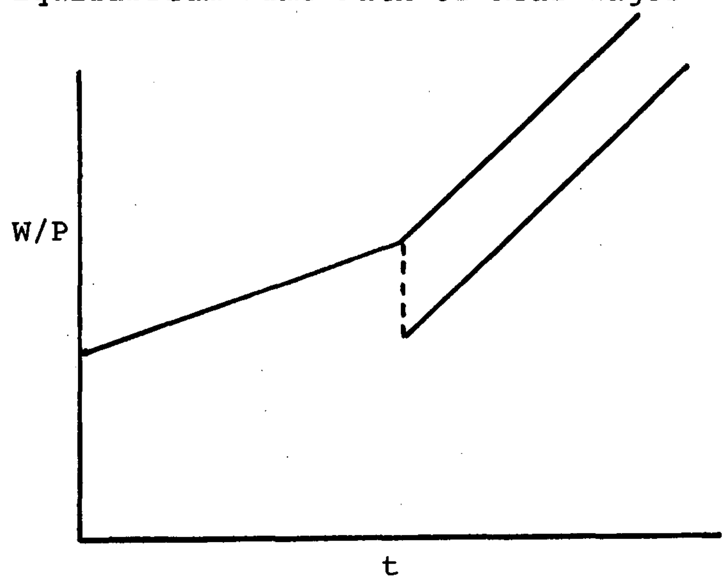
For those accustomed to thinking of steady state growth paths as determined by the sum of technological and population growth rates and not by world income, the reconcilation is in the cross equation restrictions given by (7). Taking the first two terms and rearranging we have

$$\beta_{12a}\lambda_8 = \lambda_5 + (\frac{\beta 4}{\beta 1}\lambda_1 - \lambda_2) + (\frac{\beta 4 \beta 2}{\beta 1} - \beta_5)(\lambda_7 - \lambda_6) \quad (9)$$

The first term is population growth and the second related to technological growth, related since the structural equations are not production functions but constrained maximization solutions. The third term is the raw material terms of trade. Unity is re-established but the old issues of growth still remain though in a more interesting form. The problem is usually one of describing why the technological growth rate, or population growth rate, is the way it is leading to a number of attempts at endogenisation (see Jones, 1975). (9) still has these problems but observe what happens if we take all the growth rates as constant: the income elasticity of exports is then uniquely defined. Let us assume a higher technological growth rate in the home country. This must lead to a change in <u>product quality</u> and therefore a higher income elasticity. But all this is just speculation about the linking of growth and trade theory.

Having determined the comparative dynamics of the steady state growth rates we must now turn to the second part of the solution: steady state levels or initial conditions, Y_i^*. The relationship between the two is illustrated in Figure I, which refers to real wages. Up to time t_1 real wages grow at a rate given by the solution in Table III. At time t_1 we may suppose an increase in the growth rate due, for example, to a change in technology. This may have the effect of rotating the line at that point but a

Figure I - Equilibrium Time Path of Real Wages



discrete jump may also be involved. A once and for all terms of trade change would clearly bring about a level shift, the growth rate or the slope of the curve remaining unchanged. Whether and how the economy moves to the new path is a question for stability analysis which we will defer to Section IV.

Despite their importance solving for the steady state levels is an extremely complex undertaking involving practically all parameters of the model and indeterminate signs. As simulation or numerical methods are necessary we defer discussion to another paper.

Section III - Estimation

For estimation (see Wymer, 1976, 1978 for details) the non-linear differential equation system of Table II must be transformed into an equivalent discrete time, difference equation system reflecting the discrete time sampling that our data represents. To do this the system must first be linearized. Nonlinearity in the model arises in equation (4), investment and the identities (17)-(21) so that it is quite plausible. There are many methods available for linearisation and estimation results will generally be sensitive to the method chosen. We utilize linearisation about sample means using a Taylor expansion truncated after the first order. For the identities (17)-(21) a remainder variable has also been created to make the data exact over the full data series (and not just at the mean) avoiding some of the problems of truncation and extreme non-linearity[4].

The linear differential equation system of the form

$$Dx(t) = Ax(t) + Bz(t) + v(t) \tag{7}$$

is then integrated over the interval (t- ,t) and the following approximations utilized

$$\frac{1}{\sigma}\int_0^{\sigma} dy(\tau\sigma-\theta)\,d\theta = \Delta\, y_\tau, \quad \int_0^{\sigma} y(\tau\sigma-\theta)\,d\theta = My_\tau \tag{8}$$

where $\Delta \equiv \frac{1}{\sigma}(1-L)$ and $M \equiv \frac{1}{2}(1+L)$

L being the lag operator. The disturbance term, if serially uncorrelated in (7), will still remain so. However, the approximation (8) is only valid for variables measured, in theory, at a point in time (ie stocks). If flow variables are present the model must be integrated a second time for only integrals over some period are observable. If we let y(t) be the generic flow variable, the integral

$$y^o(t) = \frac{1}{\delta}\int_0^{\infty} y(t-\Theta)\,d\Theta \tag{9}$$

is measurable. Where variables are in princple measurable at a point but are in fact period averages (eg GNP deflator) they are treated as flows. The second integration does, however, introduce a moving average error term. Fortunately this has an analytical solution (see Gandolfo, 1981), so that all data have been prewhitened with this filter.

Let us use equation (1), the aggregate supply function as an example using lower case letters to denote natural logs. In continuous time the equation is

$$Dy = \alpha_1\lambda_1 t - \alpha_1\beta_1 w - \alpha_1\beta_2 ew - \alpha_1\beta_2 dolr + \alpha_1(\beta_1+\beta_2)p$$

$$+ \alpha_1 k - \alpha_1 y + \alpha_1\gamma_1 + \epsilon_1 \quad (9)$$

Integrating the model we have

$$\Delta y = \alpha_1\lambda_1 Mt - \alpha_1\beta_1 Mw - \alpha_1\beta_2 Mew - \alpha_1\beta_2 Mdolr + \alpha_1(\beta_1+\beta_2)Mp$$

$$+ \alpha_1 Mk - \alpha_1 My + \alpha_1\gamma_1 + \mu \quad (10)$$

But as y, GNP, is a flow variable, the integration must be performed a second time and variables transformed by a MA filter. However, with the exception of capital stock, which is measured at the end of a period, all right hand side variables are measured as period averages (ie as flows). As with y the second integration only serves to bring the model into observable form, no further data transformations being necessary[5]. The right hand side variables with the exception of ew, world raw material price, are, however, endogenous to the model so that for example, Mp must be replaced by

$$\frac{1}{2}\Delta p + Lp \quad (11)$$

since $M = (L+\frac{1}{2}\Delta)$ where L is the lag operator.

The estimating equation is therefore

$$(1+\frac{1}{2}\alpha 1)\Delta y = \alpha_1\lambda_1 Mt - \alpha_1\beta_1 Lw - \frac{1}{2}\alpha_1\beta_1\Delta w - \alpha_1\beta_2 Mew$$

$$- \alpha_1\beta_2 Ldolr - \frac{1}{2}\alpha_1\beta_2\Delta dolr + \alpha_1(\beta_1+\beta_2)Lp$$

$$+ \frac{1}{2}\alpha_1(\beta_1+\beta_2)\Delta p + \alpha_1 Lko + \frac{1}{2}\alpha_1\Delta ko - \alpha_1 Ly$$

$$+ \alpha_1\gamma_1 + \mu \quad (12)$$

It is important to note that the transformation of a recursive continuous time model into its difference equation equivalent results in a <u>simultaneous</u> system with overidentifying restrictions. FIML estimation is therefore necessary.

We can restate the care needed in moving from differential to difference equations in another way emphasising dating. Consider the differential equation

$$Dy(t) = Ax(t) \tag{13}$$

where a "usual" transformation is made by replacing the differential with the first difference

$$(y_t - y_{t-1}) = Axt \tag{14}$$

But note the important transformation in the meaning for the transformation is asymmetric. The differential refers to time t but the first difference to $t-\frac{1}{2}$. A symmetric first difference would be

$$y_{t+1} - y_{t-1} = Axt \tag{15}$$

Our transformation shifts the date to $t-\frac{1}{2}$ but we are also careful to do it for the other variables, in this case x. In both symmetric transformations, it should be noted, forward information is involved, in our case half a period at t. Following Graves and Telser, 1968, this is due to the fact that a continuous slope at a point means that the right and left derivatives must be equal. This equality implies that in every small neighbourhood of a given point the future is just like the past.

For estimation the 21 equation system has been transformed along the lines of (12) above. One or two changes were however necessary these being in the investment equation (4) and the consumer price equation (15a). The former evaluates the marginal product at the actual level of output rather than at YS thereby avoiding a rather complex non-linearity problem. The consumer price equation is a more difficult problem and relates to the fact that the true consumer price index is unobservable, being dependant on the elasticity, β_{11}. The substitution of such a non-observable CES function into (7a) and (18) would have made for an extremely complex second order non-linear system. On the other hand it makes no sense to estimate (15a) given that the observed series is not so constructed. We have therefore replaced the CES equation with a Cobb Douglas function

$$PCS = \beta_{11}'\ln P + (1-\beta_{11}')\ln(DOLR.PF_1) \quad (16)$$

and omitted the cross equation restrictions.

The only other changes to note is that the zero order household disposable income equation, YH, was not estimated. This would have created problems with the identities. We have instead introduced YH as an exogenous variable.

The estimating model is therefore as close as possible the theoretical model and has been estimated over the period 1967-2/1981-4 using Wymers FIML package, RESIMUL. Use of data from the comparatively fixed exchange rate period prior to 1972 was necessary given the fact that in FIML the number of observations must be greater than the number of predetermined variables.

In part to meet the criticisms of Leamer, 1983, we report in Table IV the results for two models. The model termed Basic is exactly the model of Table II with the above changes. The model termed Modified follows the work of Camen, 1983, in introducing an overseas wealth variable (proxied by world trade) into Blundell-Wignall' exchange rate equation. We should nevertheless say something about our general experience with the numerous runs not here reported. The first point we should note is that life at the top of the likelihood surface is especially difficult. This is not surprising given multi-collinearity problems. Thus the Basis model exhibits zero change in likelihood yet collinearity produces marked parameter changes. This is particularly so in the present model which is comparatively unrestricted (ie the restrictions from Table II are generally not imposed). The second point is closely related and concerns machine precision. The Basic model has been estimated on a PDP10 but the Modified model on a CDC 170-835. In our experience it is possible that the differences between models is primarily due to the superior precision of the CDC. In any case all estimations, whether OLS or otherwise, can only be regarded as tentative.

With these caveats in mind we can examine the coefficients reported in Table IV. For the Basic model only four parameters from fifty have incorrect signs (two are "significant" or well defined)[6] and for the Modified model nine from fifty three. In addition, the Carter Narger system R^2 is significant for both models. The levels, .43 and .48 respectively, are no cause for concern given that we are attempting to parameterise a theoretical model and not to maximize R^2.

When we consider the usually controversial supply side, equations (1)-(3), our results are indeed good. For both models real wages strongly influence employment and output although this is not found for raw material imports. Real raw material prices also exert a well defined influence on all three equations.

Particularly interesting are the adjustment speeds. Output and raw material imports have approximately the same mean lag, 1.3 quarters, but employment adjusts with an extremely slow speed, the mean lag being 10-12 quarters. These results are supported by three other model variants reported in Kirkpatrick, 1984. Only in one case is a spillover effect well defined (the inventory spillover in the Modified model) but with an incorrect sign.

The price equation (10) is a more controversial relationship and in this respect it is pleasing to observe that despite data problems both α_{10} and α_{11} are both well determined. Unfortunately the interest rate effect on desired stock holdings proved to be extremely poorly determined and was constrained to zero. Absence of the cost of stockholding effect may account for the absence of any spillover effect (or correct sign) in the aggregate supply function. For the three variants reported in Kirkpatrick, 1984, the flow effect on prices was always well defined, α_{11}, but the stock effect was not (ie insignificant).

The third controversial area is the exchange rate equation and here, not surprisingly, our results are mixed. For the Basic model the parameter θ was constrained to that estimated by Blundell-Wignall. This is because the parameter exploded, usually a sign of underidentification. Given this constraint, the adjustment speed is well determined but surprisingly long (mean lag 2.8 quarters). Interesting is the parameter ϕ which reflects asset substitutability. It is rather low and well determined. The influence of the current account on the expected equilibrium exchange rate has the right sign but not so well determined.

For the Modified model it proved possible to release the constraint. The parameter is highly significant and indicates a relatively slow adjustment of the expected depreciation to the gap between the actual and expected rates. Particularly poor is the wrong sign of δ. The world wealth proxy is highly significant and with the correct sign suggesting the importance of overseas conditions in addition to interest rates and inflation. The substitutability parameter in this case suggests perfect substitutability and must be further examined.

An obvious candidate to explain the mixed results is the change in exchange rate regime. Indeed inspection of the static simulations (see below) reveals systematic misspecification to around 1972 indicating the need for a switching function. This is the subject for future work since it is rather complex (eg Wymer, 1984). On the other hand, the poor results in the form of exploding parameters may simply reflect instability due to rational expectations or to misspecification elsewhere in the model.

Closely associated with the exchange rate equation are the reserve intervention and interest rate functions. The former indicates for both models a leaning against the wind policy and a rather poorly defined reserves level policy. Surprisingly the shift dummy proved highly variable and not well determined. The interest rate equation does by contrast exhibit sharp differences only two parameters from the five being similar to one another.

There seems to be a case for respecifying the equation explicitly in terms of real interest rates (with the implied restrictions) rather than in its comparatively unrestricted form.

As every equation is really related to one another it makes no sense to continue tracking exchange related equations.

For the investment equation not all parameters are identified. β_9 was therefore constrained to a value given by average factor shares. The interest rate parameter β_{10} is insignificant but, it should be remembered, is the influence of the short term interest rate on the cost of capital. By contrast α_{51} is well determined and reflects the effect of variations in the marginal product or cost of capital on the investment rate. An increase in the cost of capital from say 4 percent to 5 percent would reduce the partial equilibrium rate of capital accumulation from say, 6.0 percent to 4.5 percent or by about 25 percent. A one percent change in the short term interest rate has therefore only about .08 percent effect. The parameter α_5 has a mean lag of 1.7 quarters and is the time required for firms to adjust the proportional rate of change of capital stock to its desired level. Cancellations may have a role here. The use of actual rather than desired output probably heavily influences these results, in particular via the speedy effect of capital stock on output via α_1.

The export and import functions show a high degree of uniformity between model types. Imports are highly price elastic but react with a mean lag of 7 quarters, surprisingly low. Exports react with a quite fast lag, 1.29 quarters, are price inelastic but highly income elastic.

Consumption responds with a very short lag (.5 quarters) and is strongly influenced by income and real wealth. The major puzzle is the implausibly strong role of interest rates but only in the Basic model.

One of the most difficult equations in estimation proved to be the wage equation, there being a strong CPI/exchange rate/wage connection. The mean lag of around 5 quarters is compatible with "successful" yearly bargaining and the effects of real raw material prices are surprisingly similar to steady state restrictions arising from the labour demand function. However, β_{38} did have to be constrained taking on implausibly high values. In a word, data does not speak, only theory! The imposition of steady state restrictions on the entire model may improve this function.

The government expenditure function is similar between models with a mean lag of around 4 quarters. In both cases unemployment has a significant expansionary influence on expenditures (excluding transfers). By contrast tax receipts adjust very quickly (around .5 quarters) probably reflecting the importance of indirect taxes. By contrast with expenditures unemployment is associated with higher taxes. Only for the Basic model has the deficit a role in taxation policy.

Having briefly discussed the parameters we should conclude this section with a look at the equations themselves. For systems estimations this is best done by examining the Root Mean Square Error of single period forecasts, Table V. For both models the RMSE is around 3 percent the worst equation in both cases being reserves. The Modified model is substantially better in the wage equation but by and large there is not a great deal between them. The low RMSE on the inventory identity is particularly pleasing. The dynamic forecasts represent another story but one properly belonging to analysis.

Section IV - Model Analysis

As a system what exactly have we estimated? To approach this question we report the eigenvalues[7] for the linear differential equation models in Table VI. The Modified model contains three unstable roots but the Basic model is stable. The model is, however, strongly cyclical, the dominant eigenvalue (the eigenvalue with the largest ratio of the damping period to the period of the cycle) being 12 and 13. The damping period (the period required for the amplitude of a cycle to decrease by 63%) is longer than the periodicity implying almost the presence of a limit or permanent cycle.

The question naturally arises as to what parameters are responsible for the strong cycles of the Basic model and for the instability exhibited by the Modified model. More generally, which parameters dominate the dynamics of the linear differential system and in which manner? These important issues may be addressed using sensitivity analysis (Wymer, 1982, Gandolfo, 1981). This technique utilizes the eigenvalues of a model and involves the computation of the partial differential of each eigenvalue with respect to each parameter. Parameters with large partial differentials[8] can therefore be said to be important since small changes will produce comparatively large changes in dynamic behaviour.

Table VII presents the full matrix of partial differentials for the Basic model. The first question, which parameters are responsible for the dominant cycle, can now be answered by examining column 12 and the associated imaginary column. It is the latter which is responsible for cyclical behaviour. The key parameters are α_2, β_{10}, α_6, α_{10}, α_{21} and Ø, but clearly Ø and α_{10} dominate with imaginary partial differentials of .47 and .51 respectively. As both have positive signed partial differentials a decrease in the size of the parameters will decrease this particular cycle. For the real eigen value the partial differentials are also positive .05 and .63 respectively, so that a decrease in the parameter would increase the negative size of the real eigenvalue and thereby stability (in this case also speeding up the long damping factor). The parameter α_{10} is here dominant.

However, before we, for example, draw conclusions that a decrease in the sensitivity of prices to inventory disequilibrium is beneficial for model stability we should also read across the Table: examine the influence on other eigen values. When we do this for α_{10} we find a series of large positive partial differentials but a large negative sign on eigen value 16. A decline in α_{10} may well dampen the whole system but there are effects in both directions.

The economic explanation for the role of both parameters is interesting. The higher is $\emptyset$ the more imperfect are domestic and foreign assets and hence the greater is the role of relative asset supplies. The latter alter in response to the balance of payments identity and hence, among other factors, exchange rates. These in turn feed through the system. Given speeds of adjustment and elasticities elsewhere in the system, it is not hard to visualize the role of this parameter. A rather similar story applies for α_{10}.

Answers to the second question posed can similarly be found by scanning through the matrix. Rather than do so here we shall concentrate on one or two problem parameters. In this respect β_{10}, the sensitivity of the capital cost to the interest rate, is particularly important being associated with very large partial differentials. The parameter itself was insignificant or poorly identified. Put another way the standard error is very large relative to the parameter so that it could just as well take on a value of .009 as .003. Given the smallness of the parameter the change in eigenvalues is not however great despite the large differentials. The parameter clearly requires attention but not as much as some others. This is clearly not so with the exchange rate parameter δ in respect to eigen value 3. Changes which are plausible in light of the standard error can make the model unstable.

The last point we wish to make from Table VII is the problem of policy making in a dynamic system. If policy is concerned with dynamics and not just equilibrium states then it is interesting to ask whether this is at all easy. An example from Table VII concerns β_{40}, the influence of unemployment on the interest rate which we assume to be a policy decision. Suppose one is contemplating the effects of a change in government policy, what can one say. The policy has mixed effects as seen by the changing differential signs. An increase in the parameter certainly stabilizes the system by making eigen value 3 more negative but eigen value 6 moves in the other direction. Most imaginary eigenvalues increase thereby increasing the cyclical content. Clearly a better policy may be possible but tradeoffs are required and the policies bounded.

We may now return to Table V and discuss the results of dynamic simulation in the light of the eigenvalue analysis. Given its instability the Modified model exhibits higher RMSE than the stable Basic model. A notable exception is the wage equation which is suspect in the Basic version.

When one compares the static and dynamic RMSE for the Basic model some dramatic differences are apparent. These are frequently taken as a sign of dynamic misspecification. However, dynamic simulation is sensitive to initial conditions. An examination of the exchange rate panel of Figure II shows substantial disequilibrium or poorness of fit of our start up values. Given such a shock the eigenvalue analysis indicates a strong cyclical response which is dampened only gradually. This is basically all the dynamic RMSE are reporting. Further simulation analysis will be reported in another paper.

Finally, model analysis also involves examining empirically the steady state. This is also reported in a separate paper where we investigate the initial condition problem.

Sevtion V - Concluding Comments

In this paper we have constructed a log linear neo-classical macroeconomic model of a type commonly found in the literature. In contrast to this literature, the dynamics have not been constrained to one or two equations but made quite general in the form of 21 differential equations. The model has been analysed for its steady state thereby confirming its mathematical consistency in equilibrium. Steady state levels have not as yet been computed being the subject of a separate paper. On this point we diverge from the theoretical literature.

The model has been estimated using a consistent transformation to discrete time. Our results, both in terms of parameters and system behaviour are excellent. The most interesting results arise from the neo-classical factor demand and aggregate supply functions which are all quite well determined. Such neo-classical specifications are commonly used in the theoretical literature but are seldom satisfactorily implemented in econometric models. Moreover these results seem relatively robust to changes elsewhere in the model. Import, consumption, exports and investment functions are also compatible with the theoretical models. Of interest is especially the high price elasticity of demand for final imports and the role of wealth in consumption.

As to be expected the main model problems arise in the exchange rate, wage and interest rate equations results being sensitive to re-specification. The major problem appears to be the exchange rate equation and sensitivity analysis also revealed it to be crucial for overall system behaviour. This equation clearly requires re-specification to adequately reflect the shift in exchange rate regimes in 1972. At present this is only handled by a shift variable in the intervention function the theory being that the determinants of the equilibrium exchange rate did not alter, only the intervention rule.

For the Basic model the system exhibits cyclical growth and is stable. This holds for the linearized differential equation model but what is not clear are the properties of the non-linear model. This important aspect is the subject of current work.

Finally, a major contribution of the paper is once again clearly to illustrate the feasibility and usefulness of constructing, analysing and consistently estimating relatively sophisticated dynamic systems along the lines proposed by Wymer.

Footnotes

1 Another way of viewing the problem is along the lines of Armington, 1969. Over all foreign and home assets one is using a separability assumption to aggregate all home assets. One is also aggregating all foreign assets thereby avoiding the need to track bilateral financial holdings, empirically an impossible task. Across all foreign assets one is therefore assuming a constant elasticity of substitution. For a well developed global exchange rate model following this approach see Richard, 1980.

2 Totally overlooked in the Keynesian literature is the supply function of Keynes' General Theory. Stressing supply price and demand price, cash flow and the recalculation of plans on the basis of such flows it is similar to Days specification.

3 For example "the distinction between stock and flow equilibrium (is) relevant only in the analysis of what, in our definition, are disequilibrium situations". Arrow and Hahn, Pg. 50, 1971.

4 If the identity is $Y \equiv X+Z$ this holds in the data by construction. However, $\ln Y = a\ln X + a'\ln Z$, where a and a' are linearisation constants, will only hold at the linearisation point. To make the identity exact we therefore created a remainder variable equal to $\ln Y - a\ln X - a'\ln Z$.

5 If there are variables measured at the end of the data period these must be transformed with the M operator. In other words, as flows are measured at the mid point of the data period, the end of period observations must be similarly shifted.

6 The t ratio reported in Table IV is simply the ratio of the FIML estimate to the FIML standard error. The latter is calculated using the Hessian of the concentrated log-likelihood and has an asymptotically normal distribution. Given our small sample size we cannot pretend that a Student t distribution is appropriate. Even if we could there is no good reason to take 2.0 or 1.96 as in some sense a critical value. Rather one must utilize an information criteria (Sawa, 1978) in which case useful values are above 1.0.

As the t statistic interpretation is problematic we refer instead to whether a parameter is well defined or not. This derives from the likelihood surface effects of the standard error. If the t ratio is high then this means that with respect to this parameter the likelihood surface is quite peaked. By contrast a low t ratio implies that the surface is quite flat so that a number of plausible choices of parameter values are possible. In our terminology it is not well defined.

7 For the linear differential equation system

$$DX(t) = AX(t) + BZ(t)$$

the local stability depends only on the characteristic roots of the matrix A. Such roots or eigenvalues may also comprise an imaginary component which reflects the periodicity of any cycle. For differential equation models the criteria for stability is that real eigenvalues be negative. For difference equations the criteria is less than unity. Where the variables Z(t) include time the system is non-autonomous and particular care in interpretation is required. For extensive discussion on the user of eigenvalue analysis see Wymer, 1976, 1979 and Gandolfo, 1981.

8 One must be careful for Table VII is not normalised. Hence a very small parameter with a large partial differential will require extremely large changes to alter a given eigenvalue.

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Table I - Sectoral Relationships and Flow of Funds

	Household	Firm	Government	Overseas		Total
Income	Y_1	Y_2	Y_3		=	Y
Net Transfers	T_1	T_2	T_3	T_4	=	O
Consumption	$-C_1$		$-C_2$		=	-C
Investment	$-I_1$	$-I_2$	$-I_3$		=	-I
Bal. of Goods & Services				-X+M	=	-X+M
Financial Surplus/ Deficit	F_1	$-F_2$	$-F_3$	F_4	=	O
=						
Financial Assets						
Domestic Financial Claims	DWIH	DSIF	DSIG	-DWIO		O
Foreign Financial Claims	DBM			-DBM		O
Reserves	-		DRES	-DRES		O
=	F_1	$-F_2$	$-F_3$	F_4		

Net nominal financial claims held, household $WI = \int_{-\infty}^{t} F_1 dt$

Net nominal financial claims issued $SI = \int_{-\infty}^{t} F_2 dt + \int_{\infty}^{t} F_3 dt$

Table II - <u>Basic Model Equations</u>

1. Aggregate Supply

$$DlnY = \alpha_1 ln\frac{YS}{Y} + \alpha_1' ln\frac{VS}{V} \tag{1}$$

$$YS = \gamma_1 e^{\lambda 1t}(\frac{W}{P})^{-\beta 1}(\frac{EWPH}{P})^{-\beta 2} K \tag{1a}$$

$$EWPH = EW.DOLR \qquad \text{identity (1b)}$$

2. Labour Demand

$$DlnEMP = \alpha_2 ln(\frac{EMPS}{EMP}) + \alpha_2' ln(\frac{EMP}{LF}) \tag{2}$$

$$EMPS = \gamma_2 e^{\lambda 2t}(\frac{W}{P})^{-\beta 4}(\frac{EWPH}{P})^{-\beta 5} K \tag{2a}$$

3. Raw Material/Energy Import Demand

$$DlnEN = \alpha_3 ln(\frac{ENS}{EN}) \tag{3}$$

$$ENS = \gamma_3 e^{\lambda 3t}(\frac{W}{P})^{-\beta 7}(\frac{EWPH}{P})^{-\beta 8} K \tag{3a}$$

4. Investment Demand

$$Dk = \alpha_5\{\alpha_{51}(\beta_9\frac{YS}{K} - rc) + \beta' - k\} \tag{4}$$

$$rc = \gamma_5 + \beta_{10} r - \beta_{10}' EDlnP \tag{4a}$$

5. Final Good Import Demand

$$DlnTMY = \alpha_6 ln(\frac{TMYS}{TMY}) \quad (5)$$

$$TMYS = bC^{\beta 11a}(\frac{PM}{PC})^{-\beta 11} \quad (5a)$$

$$PM = PF1.DOLR \quad \text{identity (5b)}$$

6. Exports

$$DlnTX = \alpha_7 ln(\frac{TXS}{TX}) \quad (6)$$

$$TXS = \gamma_7 YO^{\beta 12a}(\frac{P}{PM_1})^{-\beta 12} \quad (6a)$$

$$PM_1 = PF.DOLR \quad \text{identity (6b)}$$

7. Private Consumption Demand

$$DlnC = \alpha_8 ln(\frac{CS}{C}) \quad (7)$$

$$CS = \gamma_9 e^{-\beta 13r}(\frac{YH}{PC})^{\beta 16}(\frac{WI}{PC})^{\beta 17} \quad (7a)$$

$$WI = BM+WIH \quad \text{identity (7b)}$$

$$YH = (1-\delta-\theta)W.EMP+ro.BM+\theta(Y.P-EN.EWPH)-\theta' TAX \quad (7c)$$

8. Wage Rate

$$DlnW = \alpha_9 ln(\frac{WS}{W}) + \alpha_9' ln(\frac{EMP}{LF}) \quad (8)$$

$$WS = \gamma_{14} e^{\lambda 4t} PC^{\beta 38}(\frac{EWPH}{P})^{-\beta 39} \quad (8a)$$

9. Government Consumption and Investment Expenditure

$$DlnG = \alpha_{16}ln(\frac{GS}{G})-\alpha_{17}ln(\gamma_{12}\frac{EMP}{LF})-\alpha_{18}(DlnP-\gamma_{13}) \quad (9)$$

$$GS = \gamma_{17}Y \quad (9a)$$

10. Producer Prices

$$DlnP = \alpha_{10}ln(\frac{VS}{V})-\alpha_{11}(DlnV-\gamma_{15}') \quad (10)$$

$$VS = \gamma_{15}Y^{\beta 41}e^{-\beta 42.rc} \quad (10a)$$

11. Interest Rate Reaction Function

$$Dr = \alpha_{12}(\gamma_{11}.ro-r)+\alpha_{13}DlnP-\alpha_{14}DlnPF+\beta_{40}ln(\gamma_{12}.\frac{EMP}{LF}) \quad (11)$$

12. Tax Income or Tax Rate Function

$$DlnTAX = \alpha_{4}ln(\frac{TAXS}{TAX}) \quad (12)$$

$$TAXS = \gamma_{171}P.Y.(\gamma_{172}.\frac{SIG}{SIF})^{\beta 20}(\gamma_{12}.\frac{EMP}{LF})^{\beta 21} \quad (12a)$$

13. Exchange Market Intervention

$$DlnRES = \alpha_{20}ln(\frac{\gamma 11.P}{PF.DOLR})+\alpha_{21}ln(\frac{RESS}{RES}) \quad (13)$$

$$RESS = \gamma WI \quad (13a)$$

14. Exchange Rate, DM/$

$$\mathrm{DlnDOLR} = \alpha\,[-\frac{\emptyset}{\Theta}(\mathrm{lnBM}-\mathrm{lnWI})-\frac{1}{\Theta}(\mathrm{r}-\mathrm{ro}-\mathrm{DlnP}+\mathrm{DlnPF})$$

$$+\mathrm{lnP}-\mathrm{lnPF}-\delta\overline{\mathrm{CA}}-\mathrm{lnDOLR}] \qquad (14)$$

$$\overline{\mathrm{CA}} = \mathrm{DlnBM}+\mathrm{DlnRES} \qquad (14a)$$

15. Consumer Prices

$$\mathrm{DlnPC} = \alpha_{51}\ln(\frac{\mathrm{PCS}}{\mathrm{PC}}) \qquad (15)$$

$$\mathrm{PCS} = (\mathrm{bPM}^{1-\beta_{11}}+(1-\mathrm{b})\,\mathrm{P}^{1-\beta_{11}})^{\frac{1}{1-\beta_{11}}} \qquad (15a)$$

<u>Differential Identities</u>

16. Capital Stock

$$\mathrm{DlnK} = \mathrm{k} \qquad (16)$$

17. Inventory (National Income Identity)

$$\mathrm{DV} = \mathrm{Y}-\mathrm{DK}-\frac{\mathrm{C.PC}}{\mathrm{P}}-\mathrm{G}-\mathrm{TX}+\frac{\mathrm{TMY.PM}}{\mathrm{P}} \qquad (17)$$

18. Household Budget Identity

$$\mathrm{DWI} = \mathrm{YH}-\mathrm{PC.C} \qquad (18)$$

19. Government Budget Restriction

$$\mathrm{D\ SIG} = \mathrm{G.P}-\mathrm{TAX}+\mathrm{D\ RES}+\mathrm{r.SIG} \qquad (19)$$

20. Balance of Payments Budget Restriction

$$\mathrm{D\ BM} = \mathrm{TX.P}-\mathrm{TMY.PM}-\mathrm{EN.EWPH}+\mathrm{ro.BM}-\mathrm{D\ RES} \qquad (20)$$

21. Corporate Borrowing

$$D\ SIF = DWI - DSIG - DBM \tag{21}$$

Endogenous Variables

Real

Y real gross product (i.e., GNP + imported raw materials)

K real capital stock

EMP total employment

EN real imported raw material input in raw material prices

V real inventory stock

k growth rate of capital stock

TMY -real imports of final goods in own price

C -real consumption in consumer prices

TX -real exports in Y deflator prices

DK -real investment - capital stock increment

G -real government total current expenditure in Y prices

Nominal

W -direct wage payments per man year

P -Y deflator

EWPH -imported raw material price index in domestic currency

DOLR -exchange rate in domestic currency per $

TAX -nominal tax payments

SIF -net financial claims by firms of country (i) on issue

PM -domestic price of imported final goods

PC -consumer price index

RES -overseas reserves held by government in domestic currency

YH -nominal household after tax disposable income

SIG -net financial claims issued by government

r -domestic nominal interest rate

WI -total net nominal wealth of households

SIF -net borrowing by corporate sector

Exogenous Variables

EW -world foreign currency raw material price index

PFI_1 -world foreign currency price index for imported consumer goods

PF -world foreign currency final goods price index

YO -real world income

ro -world nominal interest rate

LF -domestic labour supply

t -time

Table III - Steady state growth rates of endogenous variables

Real Variables	Steady State Growth Rate
Y, K, V, TMY	$\beta_{12a} \lambda_8$
C, TX, G	$\beta_{12a} \lambda_8$
EN	$\beta_{12a}\lambda_8 + \lambda_7 - \lambda_6$
EMP	λ_5

Nominal Variables	
W	$\frac{\lambda_2}{\beta_4} + (\lambda_{11} - \beta_{12a}\lambda_8) + \frac{\beta_5}{\beta_4}(\lambda_7 - \lambda_6) + \frac{1}{\beta_4}(\beta_{12a}\lambda_8 - \lambda_5)$
DOLR	$\lambda_{11} - \lambda_7 - \beta_{12a}\lambda_8$
P	$\lambda_{11} - \beta_{12a}\lambda_8$
PC	$\lambda_{11} - \beta_{12a}\lambda_8$
TAX, SIF, RES, YH	λ_{11}
SIG, WI	λ_{11}

where λ_8 = growth rate of world income (YO)

λ_7 = growth rate of world export price (PF)

λ_6 = growth rate of world raw material price (EW)

λ_5 = growth rate of labour force

λ_2 = growth rate of employment related to Harrod neutral technological progress

λ_{11} = exogenously given growth rate of nominal government debt.

Steady State Growth Rate Restrictions

1. $\beta_{41} = 1.0$

2. $\beta_{16} + \beta_{17} = 1.0$

3. $\beta_{11a} = 1.0$

4. $\beta_{38} = 1.0$ (for simplicity)

5. $\lambda_9 = \lambda_7$

6. $\lambda_4 = \frac{\lambda_2}{\beta_4} + \frac{1}{\beta_4}(\beta_{12a}\lambda_8 - \lambda_5)$

$$\beta_{39} = \frac{\beta_5}{\beta_4}$$

7. $\frac{\lambda_1}{\beta_1} + \frac{\beta_2}{\beta_1}(\lambda_7 - \lambda_6) = \frac{\lambda_2}{\beta_4} + \frac{\beta_5}{\beta_4}(\lambda_7 - \lambda_6) + \frac{1}{\beta_4}(\beta_{12a}\lambda_8 - \lambda_5)$

$$= \frac{\lambda_3}{\beta_7} + \left(\frac{\beta_{8+1}}{\beta_7}\right)(\lambda_7 - \lambda_6)$$

Table IV - Parameter Estimates

* Parameter signs are those of Table so that a negative sign indicates a reversal of that expected.

Equation	Basic Model		Modified Model	
	Coef.	t ratio[1]	Coef.	t ratio
Aggregate Supply				
α 1	.752	5.05	.791	4.75
λ 1	.0067	3.3	.005	2.96
β 1	.593	3.10	.643	3.60
β 2	.081	3.02	.042	1.70
α 1'	-.017	.17	-.248	2.26
constant	.49	1.43	.458	1.36
Labour Demand				
α 2	.103	1.97	.070	1.42
λ 2	.008	1.35	.01	1.06
β 4	1.250	2.47	1.570	1.69
β 5	.138	1.82	.125	1.29
α 2'	-.002	.02	-.001	.02
constant	-.435	3.39	-.341	2.91
RM Imports				
α 3	.768	4.33	.729	4.37
λ 3	.0006	.12	-.001	.33
β 7	.339	.69	.156	.35
β 8	.248	3.21	.235	3.14
constant	2.94	2.50	3.064	2.88
Investment				
α 5	.587	5.74	.563	5.17
α 51	1.55	7.61	1.739	8.23
β 9	.42	constrained	.42	constrained
β 10	.003	.19	.002	.11
β 10'	0.0	constrained	0.0	constrained
constant	-.07	4.79	-.07	4.70

Equation	Basic Model		Modified Model	
	Coef.	t ratio	Coef.	t ratio
Import Demand				
$\alpha 6$	.149	3.12	.130	2.73
$\beta 11a$	1.0	constrained	1.0	constrained
$\beta 11$	2.74	1.91	2.320	1.89
constant	.106	2.8	.090	2.34
Export Demand				
$\alpha 7$	.77	5.27	.794	5.39
$\beta 12a$	1.12	21.15	1.124	19.00
$\beta 12$	.82	4.93	.788	5.06
constant	-2.06	5.17	-2.131	5.31
Consumption				
$\alpha 8$	1.87	5.79	1.699	5.91
$\beta 13$	.70	2.68	.094	.38
$\beta 16$	.789	10.21	.756	11.28
$\beta 17$	.209	4.10	.149	3.32
constant	.667	4.47	-.152	1.46
Wage				
$\alpha 9$	.269	1.80	.162	2.21
$\alpha 9'$	.908	.98	-.071	.15
$\lambda 4$	.0001	.12	.014	1.00
$\beta 38^2$	.99	1.7	.924	12.00
$\beta 39$	.167	1.52	.291	2.90
constant	-.586	1.77	-.341	2.07
Government				
$\alpha 16$	.389	3.62	.277	2.75
$\alpha 17$	1.357	2.52	1.012	2.09
constant	.460	3.53	.326	2.66
Price				
$\alpha 10$	.046	2.06	.053	1.57
$\alpha 11$	.261	2.44	.385	3.72
$\beta 41$	1.0	constrained	1.0	constrained
$\beta 42$	0.0	constrained	0.0	constrained
constant	-.016	4.52	-.017	4.25

Equation	Basic Model		Modified Model	
	Coef.	t ratio	Coef.	t ratio
Interest				
α12	.228	2.90	.335	4.25
γ11	.575	2.76	.841	5.73
α13	-.620	2.02	-.170	2.11
α14	.085	9.50	-.002	.26
β40	.028	.90	.068	2.48
constant	-.008	2.37	-.002	2.07
Tax				
α4	1.566	4.98	1.744	5.06
β20	.018	1.61	.002	.17
β21	-.706	1.84	-1.156	3.19
constant	1.577	4.76	1.815	4.98
Reserves				
α20	.886	2.20	.576	1.52
α21	.073	1.22	.100	1.50
dummy	.081	1.09	-.028	.37
constant	-.331	1.85	-.351	1.76
Exchange Rate				
α	.349	4.44	.299	3.59
φ	.1o7	2.01	.008	.95
θ	.97	constrained	.198	3.89
δ	.267	1.32	-.610	1.97
δ'	-	-	.285	2.77
constant	.067	2.33	-.025	1.09
Consumer Price				
α51	.195	6.10	.101	3.00
β11'	.81	12.60	.98	6.98
constant	-.009	15.67	-.009	15.24

Equation	Basic Model		Modified Model	
	Coef.	t ratio	Coef.	t ratio
Carter Narger System R^2 of over-identified Model	.436		.483	
$\chi 2$	684.67		829.8	
DOF	66		68	
1 % critical value	95.6		98.0	

Notes:

1 The t ratio is the ratio of the parameter to the FIML standard error calculated from the Hessian at convergance. It should not be confused with a Students' t distribution.

2 Following implausibly high values of around 3.0, this parameter was constrained to ≤1.0.

Table V - Ex post root mean square errors[1]
1967(2) - 1981(4) - 59 quarters

Variable	Root Mean Square Error[2] of Single Period Forecasts		Root Mean Square Error[2] of Dynamic Forecasts	
	BASIC	MODIFIED	BASIC	MODIFIED
Y	2.02	1.94	4.60	5.56
EMP	.47	.34	4.60	8.96
EN	4.67	4.60	6.15	5.27
k	.0008	.0008	.002	.003
TMY	4.28	4.23	14.54	24.9
TX	4.00	3.78	9.04	9.61
C	1.91	1.38	8.88	5.33
W	5.22	2.17	12.63	4.98
G	3.04	3.02	11.88	21.71
P	1.25	1.18	10.24	10.33
r	.003	.002	.034	.022
TAX	4.05	4.04	10.58	9.31
RES	10.76	10.59	42.56	49.82
DOLR	4.67	2.69	14.01	21.31
PC	.50	.44	8.60	5.50
K	.04	.04	2.07	2.31
V	.96	.98	23.58	22.94
WI	.16	.10	3.01	1.42
SIG	2.95	2.65	7.01	50.21
BM	8.73	8.43	98.68	147.62
SI	.64	.56	18.34	21.76

[1] Simulation has been carried out using the restricted reduced form of the linear, discrete time, estimating model. The PREDIC program of Wymers RESIMUL package was utilized.

[2] With the exception of the natural growth rate variables, k and r, the root mean sqare errors are reported as percentages. For the two variables the root mean square error refers to points of that variable.

Table VI

BASIC MODEL

EIGENVALUES AND EIGENVECTORS

EIGENVALUES REAL PART IMAGINARY PART MODULI DAMPING PERIOD PERIOD OF CYCLE
THE DAMPING PERIOD AND PERIOD OF CYCLE ARE CALCULATEDFOR A DIFFERENTIAL EQUATION SYSTEM.

EIGENVALUES	REAL PART	IMAGINARY PART	MODULI	DAMPING PERIOD	PERIOD OF CYCLE
1	-0.02436534	0.00000000	0.02437	41.042	
2	-0.04852000	0.00000000	0.04852	20.610	
3	-0.05574788	0.00000000	0.05575	17.938	
4	-0.06049973	0.00000000	0.06050	16.529	
5	-0.06683517	0.00000000	0.06684	14.962	
6	-0.17007178	0.00000000	0.17007	5.880	
7	-0.41582936	0.00000000	0.41583	2.405	
8	-0.74656898	0.00000000	0.74657	1.339	
9	-0.76781208	0.00000000	0.76781	1.302	
10	-1.55059617	0.00000000	1.55060	0.645	
11	-1.90990017	0.00000000	1.90990	0.524	
12	-0.01388054	0.16700832	0.16758	72.043	37.622
13	-0.01388054	-0.16700832	0.16758	72.043	37.622
14	-0.14335740	0.28178883	0.31616	6.976	22.297
15	-0.14335740	-0.28178883	0.31616	6.976	22.297
16	-0.18345332	0.05219624	0.19073	5.451	120.376
17	-0.18345332	-0.05219624	0.19073	5.451	120.376
18	-0.28131205	0.20599491	0.34867	3.555	30.502
19	-0.28131205	-0.20599491	0.34867	3.555	30.502
20	-0.61562089	0.27443171	0.67402	1.624	22.895
21	-0.61562089	-0.27443171	0.67402	1.624	22.895

MODIFIED MODEL

EIGENVALUES AND EIGENVECTORS

EIGENVALUES REAL PART IMAGINARY PART MODULI DAMPING PERIOD PERIOD OF CYCLE
THE DAMPING PERIOD AND PERIOD OF CYCLE ARE CALCULATED FOR A DIFFERENTIAL EQUATION SYSTEM

EIGENVALUES	REAL PART	IMAGINARY PART	MODULI	DAMPING PERIOD	PERIOD OF CYCLE
1	.00809676	0.00000000	.00810		
2	-.03294242	0.00000000	.03294	30.356	
3	-.04654173	0.00000000	.04654	21.486	
4	-.04852000	0.00000000	.04852	20.610	
5	-.05122912	0.00000000	.05123	19.520	
6	-.09117180	0.00000000	.09117	10.968	
7	-.12602273	0.00000000	.12602	7.935	
8	-.19490794	0.00000000	.19491	5.131	
9	-.72998969	0.00000000	.72999	1.370	
10	-.92457117	0.00000000	.92457	1.082	
11	-1.16076760	0.00000000	1.16077	.861	
12	-1.69128485	0.00000000	1.69128	.591	
13	-1.74215827	0.00000000	1.74216	.574	
14	.04315140	.15377436	.15971		40.860
15	.04315140	-.15377436	.15971		40.860
16	-.09435611	.06933901	.11709	10.598	90.615
17	-.09435611	-.06933901	.11709	10.598	90.615
18	-.38322458	.15891367	.41487	2.609	39.538
19	-.38322458	-.15891367	.41487	2.609	39.538
20	-.39976588	.03318777	.40114	2.501	189.322
21	-.39976588	-.03318777	.40114	2.501	189.322

Table VII - BASIC MODEL

EIGENSYSTEM SENSITIVITY ANALYSIS

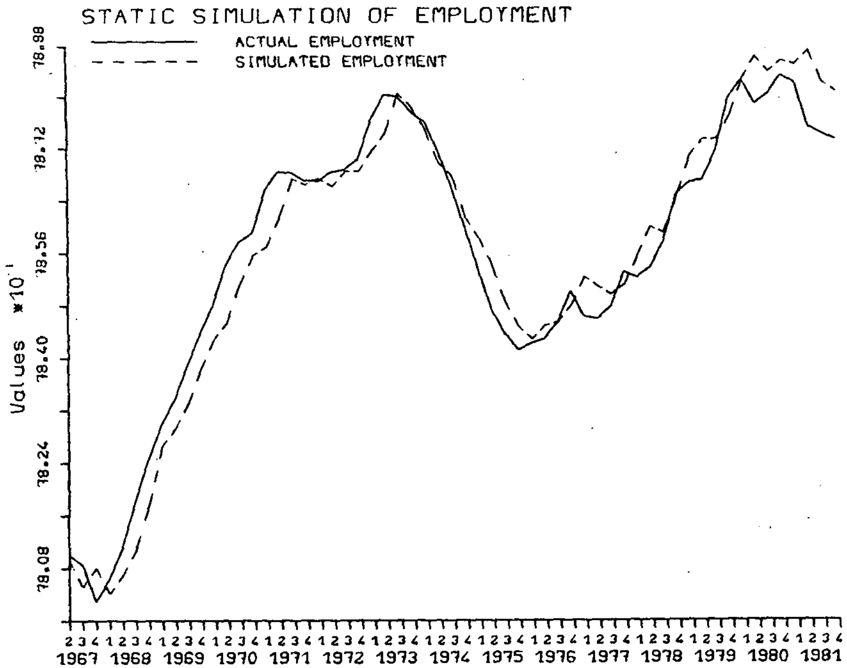
SENSITIVITY MATRIX OF EIGENVALUES WITH RESPECT TO PARAMETERS OF SYSTEM

Eigenvalues			1 REAL	2 REAL	3 REAL	4 REAL	5 REAL	6 REAL	7 REAL	8 REAL
PARAMETER		VALUE	-0.0244	-0.0485	-0.0557	-0.0605	-0.0668	-0.1701	-0.4158	-0.7466
1	α1	0.75218	-0.0004	0.0000	-0.0004	0.0000	0.0002	0.0025	-0.0002	-0.1198
2	λ1	3.35908	0.0000	0.0000	0.0000	-0.0588	0.0000	0.0000	0.0000	0.0000
3	β1	0.59276	-0.0398	0.0000	-0.0004	-0.0358	-0.0001	-0.0088	0.0028	-0.0267
4	β2	0.08126	-0.0338	0.0000	-0.0023	0.0384	0.0037	0.0192	0.0253	-0.0556
5	α1'	-0.01702	0.0595	0.0000	-0.0180	-0.0365	0.0105	0.0770	0.0453	-0.0575
6	α2	0.10290	0.0163	0.0000	-0.0036	0.0990	0.0031	0.0566	-0.0811	0.0321
7	α2'	-0.00210	0.0635	0.0000	-0.0063	0.0000	0.0047	0.0338	-0.0200	0.0044
8	λ2	1.69329	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	β4	1.25023	0.0163	0.0000	0.0002	0.0000	0.0000	0.0038	-0.0037	0.0036
10	β5	0.13820	0.0138	0.0000	0.0012	0.0000	-0.0020	-0.0083	-0.0329	0.0075
11	α3	0.76781	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0004	0.0003
12	λ3	0.13079	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	β7	0.33907	0.0001	0.0000	0.0000	0.0000	0.0000	-0.0001	0.0001	-0.0001
14	β8	0.24788	0.0001	0.0000	-0.0001	0.0000	-0.0001	0.0002	0.0011	-0.0001
15	α5	0.58758	0.0018	0.0000	0.0001	0.0000	0.0000	-0.0037	-0.0744	0.0506
16	α51	1.55494	-0.0159	0.0000	-0.0002	0.0000	0.0001	0.0034	0.0116	0.0041
17	β10	0.00338	0.0992	0.0000	-0.0483	0.0000	0.0264	0.0948	0.6484	-0.0693
18	α6	0.14932	-0.0003	0.0000	0.0019	0.0000	0.0539	-0.1581	0.0120	-0.0023
19	β11	2.74399	0.0005	0.0000	0.0005	0.0000	-0.0005	0.0058	0.0004	0.0000
20	α7	0.77044	0.0000	0.0000	0.0001	0.0000	0.0002	-0.0009	0.0029	-0.8989
21	β12a	1.12648	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	β12	0.82830	0.0005	0.0000	-0.0007	0.0000	-0.0017	0.0031	-0.0023	0.0267
23	α8	1.87322	0.0000	0.0000	-0.0002	0.0000	0.0004	0.0017	-0.0005	0.0006
24	β13	0.70133	0.0000	0.0000	0.0041	0.0000	-0.0010	0.0053	0.0042	-0.0016
25	β16	0.78997	0.0000	0.0000	0.0351	0.0000	-0.0091	-0.0353	-0.0001	-0.0004
26	β17	0.20979	0.0000	0.0000	-0.0745	0.0000	-0.0567	-0.0343	0.0019	-0.0010
27	α9	0.26933	0.0162	0.0000	0.0001	0.0000	-0.0009	0.0191	-0.0174	0.0033
28	α9'	0.90812	-0.0052	0.0000	0.0000	0.0000	0.0004	-0.0047	0.0110	0.0030
29	λ4	0.32074	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	β38	20.35700	0.0010	0.0000	0.0000	0.0000	-0.0001	0.0005	0.0000	0.0002
31	β39	0.16767	0.0030	0.0000	0.0000	0.0000	0.0004	-0.0030	-0.0472	-0.0133
32	α16	0.38887	-0.0001	0.0000	-0.0025	0.0000	0.0019	-0.0102	-0.8135	-0.0066
33	α17	1.35692	0.0000	0.0000	-0.0244	0.0000	-0.0004	0.0016	-0.0121	0.0001
34	α10	0.04611	0.0255	0.0000	-0.0485	0.0000	-0.0060	1.9881	0.2081	-0.0073
35	α11	0.26151	-0.0001	0.0000	-0.0557	0.0000	0.0004	-0.3534	-0.0822	0.0168
36	α12	0.22845	-0.0017	0.0000	-0.0605	0.0000	0.0033	-0.0809	-0.0324	0.0091
37	α13	-0.62044	-0.0009	0.0000	-0.0668	0.0000	0.0010	-0.0004	0.0102	-0.0076
38	α14	0.08562	0.0000	0.0000	-0.1701	0.0000	0.0000	0.0000	0.0000	0.0000
39	β40	0.02808	-0.0067	0.0000	-0.4158	0.0000	0.0042	0.1586	0.0083	-0.0011
40	γ11	0.57454	0.0000	0.0000	-0.7466	0.0000	0.0000	0.0000	0.0000	0.0000
41	α4	1.56650	0.0000	0.0000	0.0072	0.0004	0.0000	0.0000	0.0000	0.0000
42	β20	0.01860	0.0000	0.0000	0.0611	-0.8638	0.0000	0.0000	0.0000	0.0000
43	β21	-0.70654	0.0000	0.0000	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000
44	α20	0.88590	0.0001	0.0000	0.0000	0.0000	0.0054	0.0026	0.0118	-0.0016
45	α20	0.07336	-0.0009	0.0000	0.0000	0.0000	-0.7339	0.0239	0.0305	-0.0021
46	dummy	0.08154	0.0000	0.0000	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
47	α	0.34995	0.0000	0.0000	-0.6123	0.0000	0.0006	-0.0041	-0.1225	0.0297
48	φ	0.10767	0.0006	0.0000	-0.0096	0.0000	-0.0116	0.0037	0.0728	-0.3846
49	δ	0.26692	0.0000	0.0000	-2.1280	0.0000	0.0029	0.0018	0.0046	0.1472
50	β11'	2.02053	-0.0033	0.0000	0.0430	0.0000	0.0040	0.1495	-0.0346	-0.0034
51	α51	0.19507	-0.0070	0.0000	0.0015	0.0000	-0.0148	-1.2139	-0.0018	0.0117

Eigenvalues

PARAMETER		VALUE	9 REAL	10 REAL	11 REAL	12 REAL	12 IMAGINARY	14 REAL	14 IMAGINARY
			-0.7678	-1.5506	-1.9099	-0.0139	0.1670	-0.1434	0.2818
1	α1	0.75218	0.0001	0.0000	0.0078	-0.0002	0.0006	0.0211	0.0047
2	λ1	3.35908	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	β1	0.59276	0.0000	0.0000	0.0057	-0.0025	0.0057	0.0211	-0.0771
4	β2	0.08126	0.0000	0.0000	0.0053	0.0041	-0.0079	0.0007	-0.0350
5	α1'	-0.01702	0.0001	0.0000	0.0298	0.0219	0.0289	-0.0832	-0.0810
6	α2	0.10290	0.0000	0.0000	-0.0016	-0.0521	-0.1160	-1.0372	1.1161
7	α2'	-0.00210	0.0000	0.0000	-0.0001	0.0678	-0.0386	-0.4762	-0.1330
8	λ2	1.69329	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	β4	1.25023	0.0000	0.0000	-0.0001	-0.0110	-0.0098	-0.0531	0.1079
10	β5	0.13820	0.0000	0.0000	-0.0001	0.0148	0.0149	-0.0110	0.0515
11	α3	0.76781	-0.9996	0.0000	0.0000	0.0000	-0.0001	-0.0006	-0.0001
12	λ3	0.13079	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	β7	0.33907	0.0000	0.0000	0.0000	0.0016	-0.0011	-0.0009	0.0031
14	β8	0.24788	0.0000	0.0000	0.0000	-0.0024	0.0014	-0.0001	0.0014
15	α5	0.58758	0.0000	0.0000	0.0016	-0.0064	0.0065	0.0507	0.0737
16	α51	1.55494	0.0000	0.0000	0.0004	0.0068	0.0101	0.0491	-0.0273
17	β10	0.00338	0.0000	0.0000	-0.0399	0.6424	0.1030	0.1130	-0.5570
18	α6	0.14932	0.0000	0.0000	-0.0152	0.0800	0.2473	-0.1241	0.2010
19	β11	2.74399	0.0000	0.0000	0.0000	0.0176	0.0143	-0.0033	0.0104
20	α7	0.77044	0.0000	0.0000	0.0022	-0.0071	-0.0007	-0.0095	-0.0092
21	β12a	1.12648	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	β12	0.82830	0.0000	0.0000	0.0012	-0.0071	0.0298	-0.0177	0.0201
23	α8	1.87322	0.0000	0.0000	-0.9971	-0.0015	0.0001	0.0005	0.0033
24	β13	0.70133	0.0000	0.0000	-0.0876	0.0131	0.0036	0.0124	-0.0129
25	β16	0.78997	0.0000	0.0000	-0.0118	-0.0052	0.0273	0.0197	-0.0068
26	β17	0.20979	0.0000	0.0000	0.1663	-0.0110	0.0388	0.0300	-0.0048
27	α9	0.26933	0.0000	0.0000	0.0000	0.0238	0.0453	-0.1245	-0.3601
28	α9'	0.90812	0.0000	0.0000	-0.0002	0.0094	-0.0295	-0.1011	0.1029
29	λ4	0.32074	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	β38	20.35700	0.0000	0.0000	0.0000	-0.0019	-0.0012	0.0022	0.0005
31	β39	0.16767	0.0000	0.0000	0.0005	-0.0219	-0.0010	-0.0304	-0.0264
32	α16	0.38887	0.0000	0.0000	0.0005	0.0267	-0.0508	-0.0942	0.0101
33	α17	1.35692	0.0000	0.0000	-0.0001	-0.0140	0.0122	0.0248	0.0269
34	α10	0.04611	0.0000	0.0000	0.1440	0.6377	0.5179	0.6800	-0.1296
35	α11	0.26151	0.0000	0.0000	-0.3010	-0.0841	0.0979	0.0221	0.1424
36	α12	0.22845	0.0000	0.0000	-0.0360	-0.0973	0.0546	0.0163	-0.0147
37	α13	-0.62044	0.0000	0.0000	0.0978	-0.0515	-0.0037	0.0096	0.0029
38	α14	0.08562	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
39	β40	0.02808	0.0000	0.0000	0.0039	-0.0693	0.0788	0.0159	-0.0534
40	γ11	0.57454	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
41	α4	1.56650	0.0000	-1.0004	0.0000	0.0000	0.0000	0.0000	0.0000
42	β20	0.01860	0.0000	0.8638	0.0000	0.0000	0.0000	0.0000	0.0000
43	β21	-0.70654	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
44	α20	0.88590	0.0000	0.0000	-0.0001	-0.0235	0.0068	-0.0026	0.0133
45	α20	0.07336	0.0000	0.0000	0.0000	0.0075	-0.1218	-0.0412	0.0022
46	dummy	0.08154	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
47	α	0.34995	0.0000	0.0000	-0.0024	-0.0020	0.0684	-0.1394	0.0309
48	φ	0.10767	-0.0023	0.0000	-0.0023	0.0589	0.4718	0.1570	0.1953
49	δ	0.26692	0.0009	0.0000	-0.0003	-0.0390	0.0194	-0.0283	0.0215
50	β11'	2.02053	0.0000	0.0000	0.0102	-0.0075	-0.0140	0.0047	-0.0143
51	α51	0.19507	0.0000	0.0000	-0.0690	-0.0737	-0.0081	0.0678	0.0209

Eigenvalues			16 REAL	IMAGINARY	18 REAL	IMAGINARY	20 REAL	IMAGINARY
PARAMETER		VALUE	-0.1835	0.0522	-0.2813	0.2060	-0.6156	0.2744
1	α1	0.75218	-0.0126	0.0119	0.0252	-0.0529	-0.4784	0.0272
2	λ1	3.35908	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	β1	0.59276	0.0311	-0.0252	-0.1355	0.0197	0.1195	0.1765
4	β2	0.08126	-0.0217	0.0279	0.2029	0.0305	-0.1669	0.3878
5	α1'	-0.01702	-0.4014	0.2901	0.4182	-0.4499	0.4711	2.0938
6	α2	0.10290	-0.3518	0.1194	0.5563	0.8526	0.3739	-0.4775
7	α2'	-0.00210	-0.1980	0.0105	-0.0151	0.2986	0.0815	-0.0433
8	λ2	1.69329	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	β4	1.25023	-0.0205	0.0155	0.0673	0.0635	0.0072	-0.0320
10	β5	0.13820	0.0145	-0.0174	-0.0690	-0.1203	0.0611	-0.0222
11	α3	0.76781	0.0000	-0.0002	-0.0002	0.0001	0.0006	0.0008
12	λ3	0.13079	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
13	β7	0.33907	0.0008	-0.0013	-0.0015	-0.0006	0.0000	-0.0003
14	β8	0.24788	-0.0004	0.0013	0.0019	0.0015	0.0005	-0.0004
15	α5	0.58758	-0.0009	0.0262	0.0809	-0.1226	-0.6123	0.2799
16	α51	1.55494	0.0090	-0.0192	-0.0571	-0.0138	-0.0096	0.1037
17	β10	0.00338	-1.2016	-0.3873	2.2186	0.9246	-2.1280	0.2286
18	α6	0.14932	-0.1344	-0.0741	-0.3105	-0.0505	0.0430	-0.0239
19	β11	2.74399	-0.0081	-0.0039	-0.0110	-0.0017	0.0015	-0.0002
20	α7	0.77044	-0.0003	-0.0011	0.0279	-0.0005	-0.0636	0.0693
21	β12a	1.12648	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
22	β12	0.82830	-0.0003	0.0033	-0.0209	-0.0335	0.0326	0.0247
23	α8	1.87322	0.0031	-0.0015	-0.0113	-0.0011	0.0067	0.0072
24	β13	0.70133	-0.0209	-0.0059	0.0399	0.0414	-0.0060	-0.0313
25	β16	0.78997	-0.0375	0.0149	0.0338	0.0378	0.0001	-0.0105
26	β17	0.20979	-0.0627	0.0021	0.0426	0.0718	0.0003	-0.0220
27	α9	0.26933	0.1734	-0.1813	-0.7136	0.4280	0.1306	-0.0470
28	α9'	0.90812	-0.0558	0.0285	0.1051	0.1020	0.0402	-0.0539
29	λ4	0.32074	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
30	β38	20.35700	0.0056	-0.0024	-0.0084	0.0030	0.0017	-0.0021
31	β39	0.16767	-0.0051	0.0180	0.1743	0.0351	-0.0871	0.0880
32	α16	0.38887	-0.1295	0.1392	0.1405	-0.1311	-0.0283	0.0646
33	α17	1.35692	0.0246	-0.0138	-0.0253	-0.0245	-0.0049	0.0030
34	α10	0.04611	-3.7800	-1.0698	1.4593	1.5250	-0.1758	-1.4872
35	α11	0.26151	0.7871	-0.0042	-0.8325	-0.1753	0.2621	0.8302
36	α12	0.22845	-0.0812	0.2208	-0.2301	0.0816	-0.0297	-0.0205
37	α13	-0.62044	-0.0220	0.0156	-0.0112	-0.0873	0.0000	0.0000
38	α14	0.08562	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
39	β40	0.02808	0.0540	0.1426	-0.0826	-0.0872	0.0072	0.0068
40	γ11	0.57454	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
41	α4	1.56650	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
42	β20	0.01860	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
43	β21	-0.70654	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
44	α20	0.88590	-0.0071	0.0152	0.0177	0.0170	0.0053	-0.0033
45	α20	0.07336	-0.0941	0.0781	0.0019	0.0744	0.0090	-0.0008
46	dummy	0.08154	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
47	α	0.34995	0.0164	-0.0184	-0.2182	0.0312	-0.0588	0.1399
48	φ	0.10767	-0.3806	0.1743	0.3659	-0.1168	-0.0358	-0.1149
49	δ	0.26692	0.0336	-0.0131	-0.0076	0.0659	0.0384	0.0200
50	β11'	2.02053	-0.0382	-0.0011	0.0154	0.0969	-0.0365	0.0137
51	α51	0.19507	0.5068	0.3752	-0.4440	-0.1067	0.0990	-0.1175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ACTUAL EMPLOYMENT
SIMULATED EMPLOYMENT
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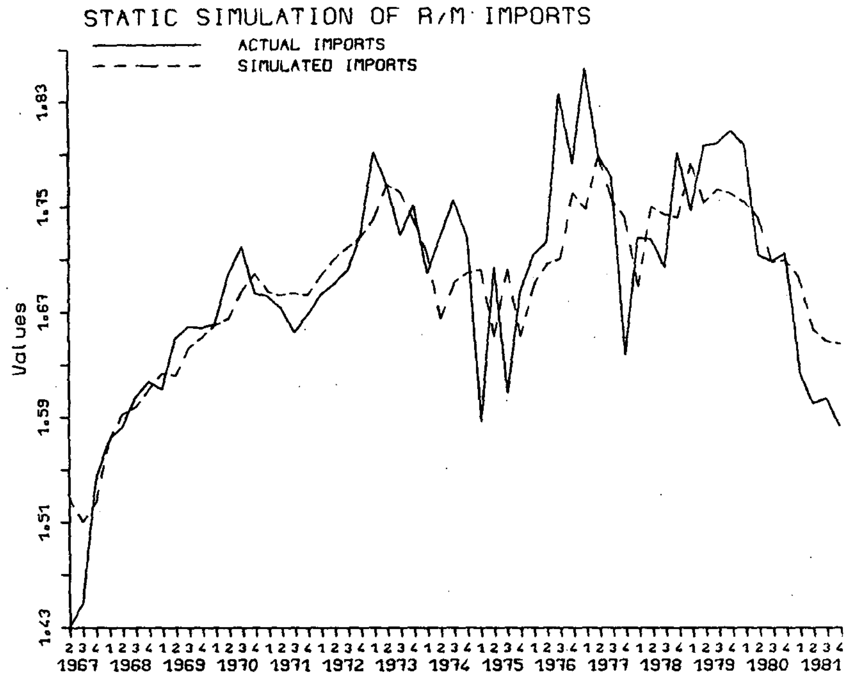
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ACTUAL IMPORTS
SIMULATED IMPORTS
Values
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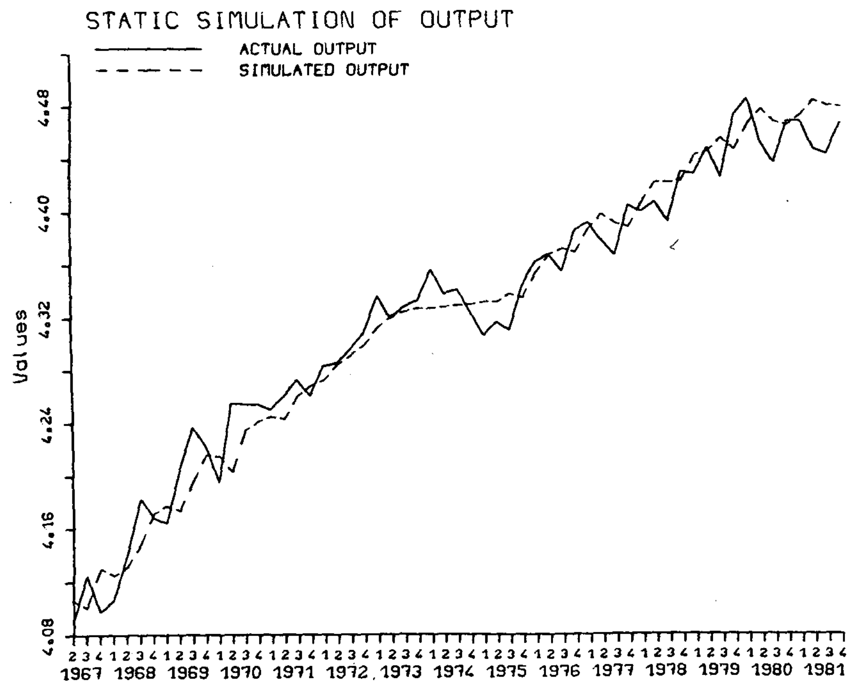
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ACTUAL OUTPUT
SIMULATED OUTPUT
Values
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4.32
4.40
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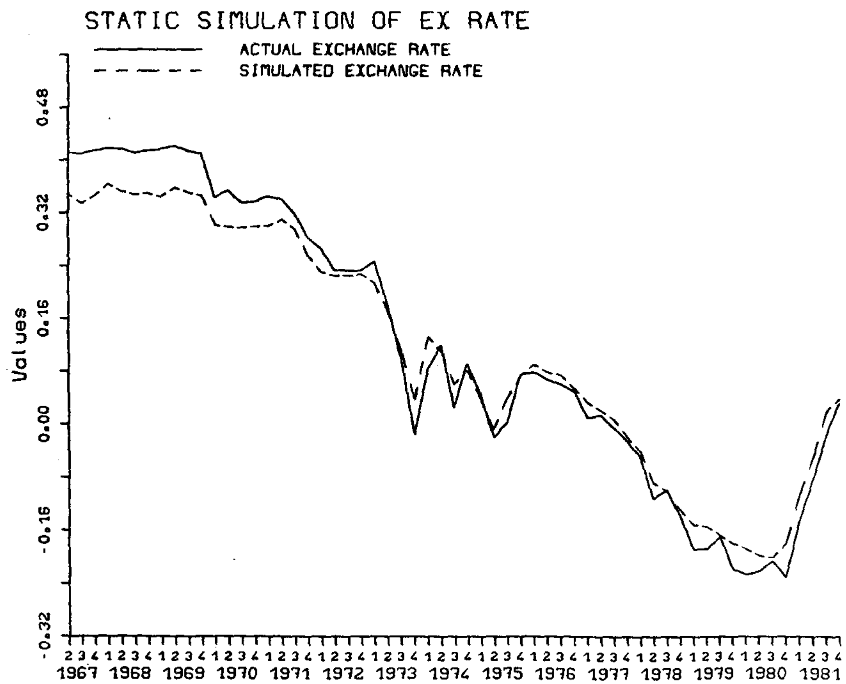
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ACTUAL EXCHANGE RATE
SIMULATED EXCHANGE RATE
Values
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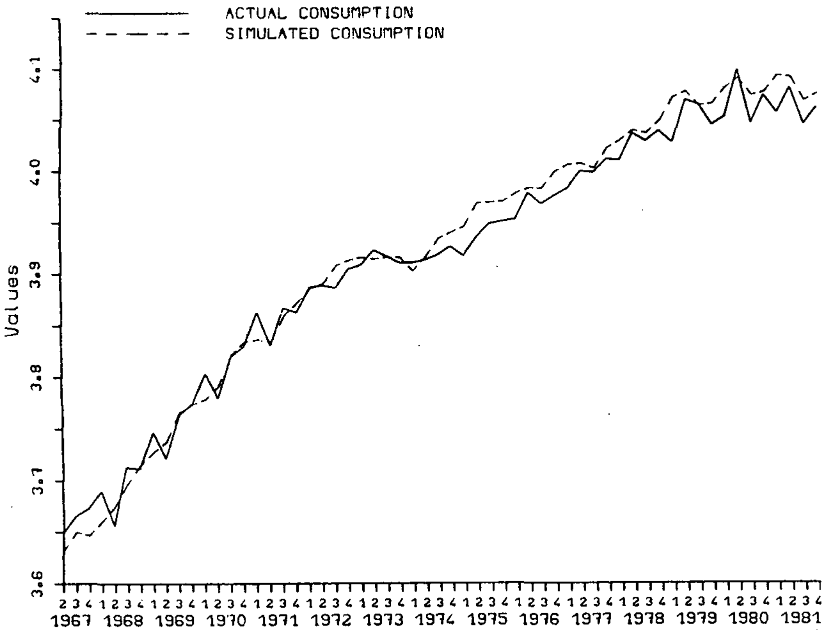
0.00
0.16
0.32
0.48
1967 1968 1969 1970 1971 1972 1973 1974 1975 1976 1977 1978 1979 1980 1981

STATIC SIMULATION OF CONSUMPTION



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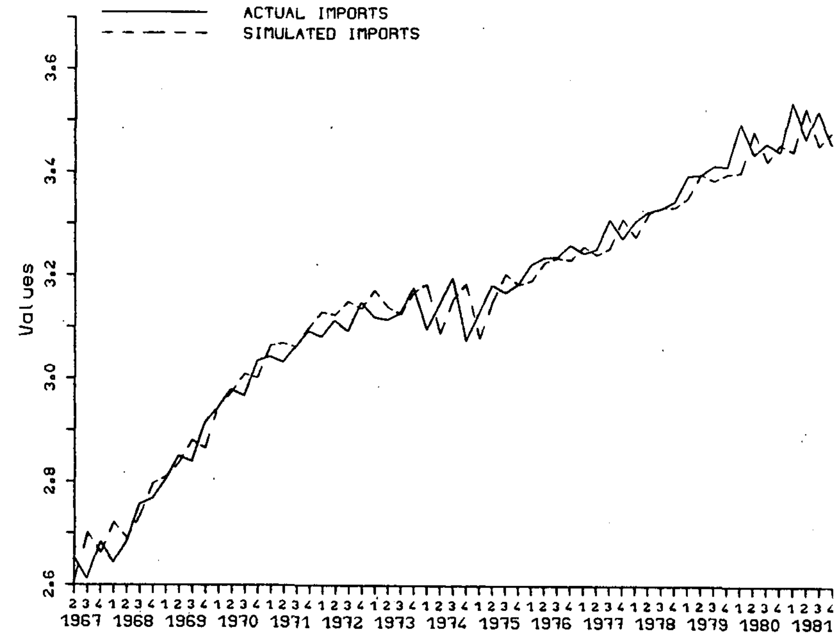
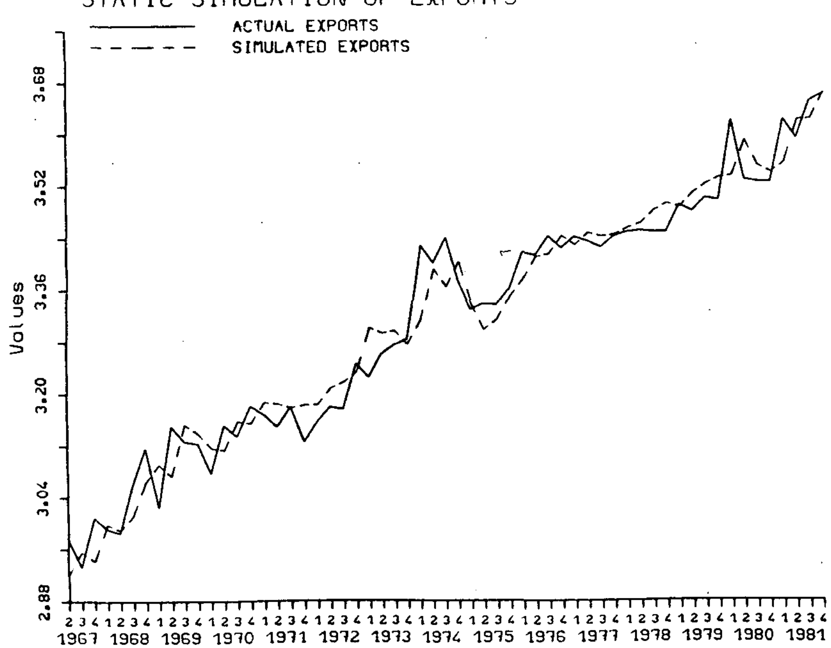
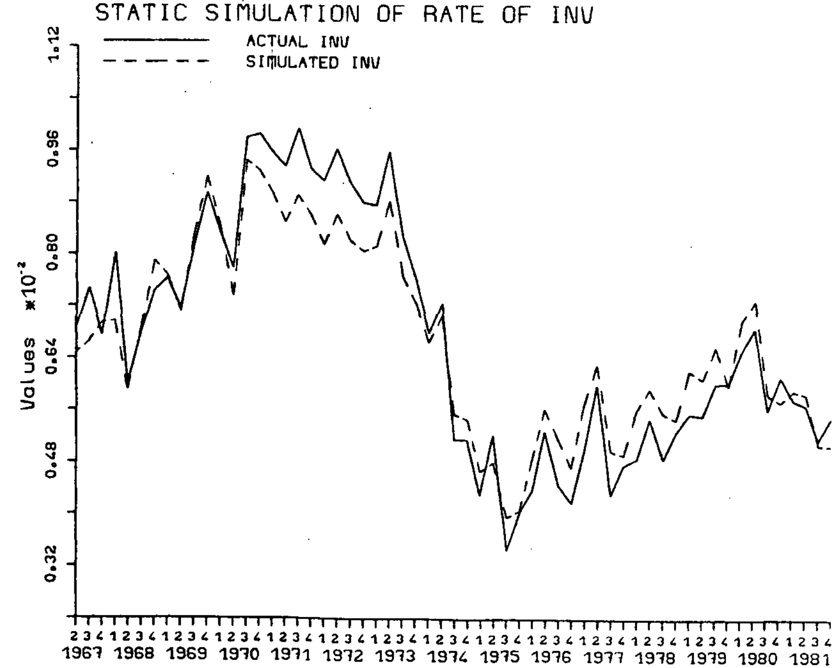


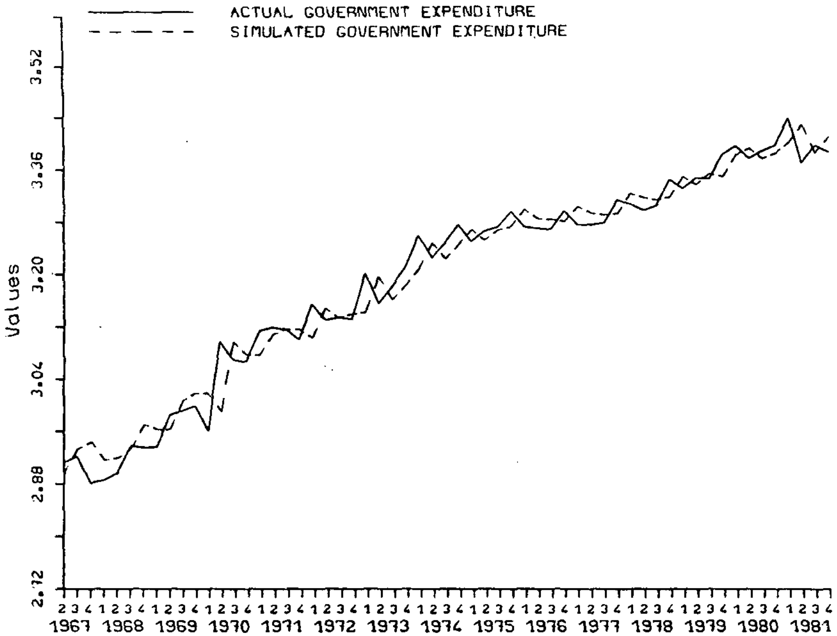
STATIC SIMULATION OF EXPORTS



STATIC SIMULATION OF RATE OF INV



STATIC SIMULATION OF GOVERNMENT EXPENDITURE



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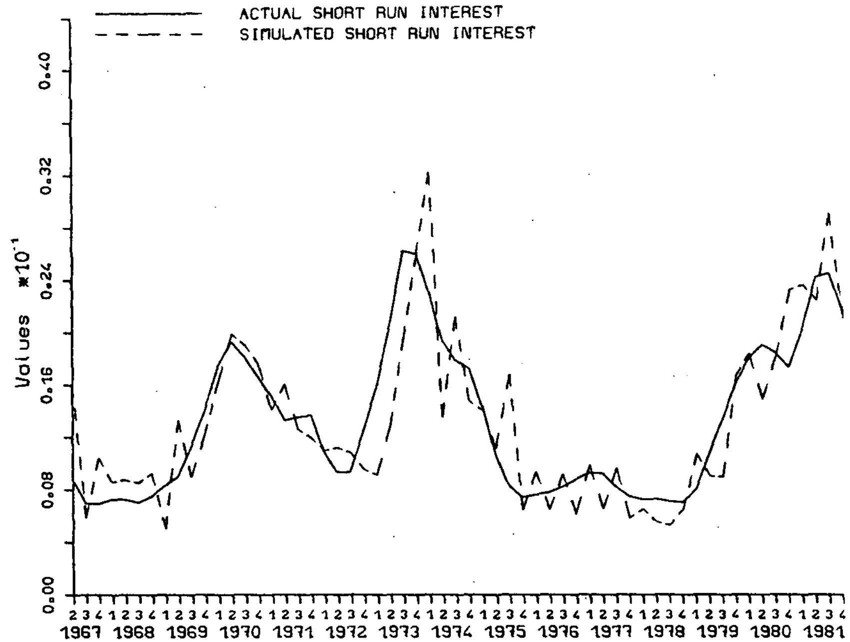
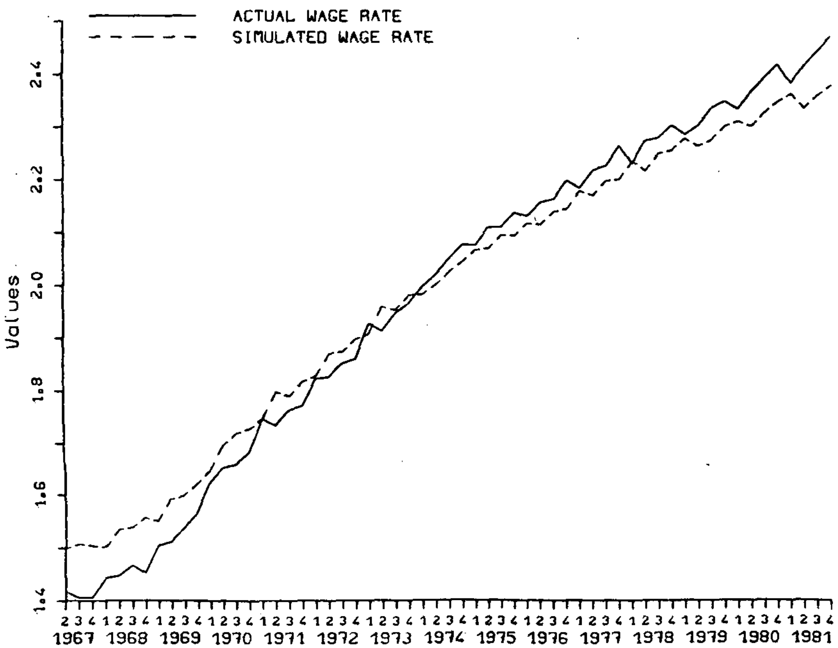
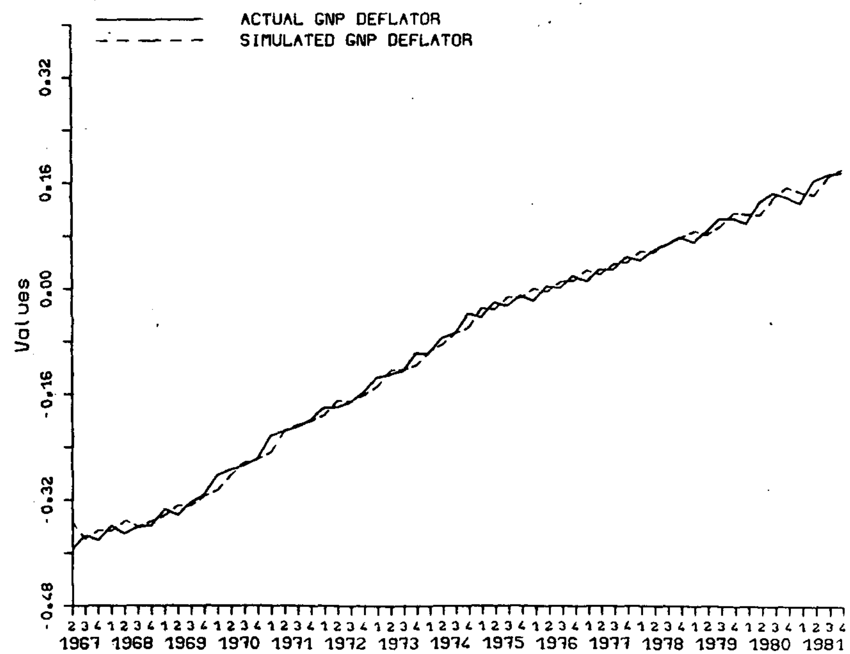
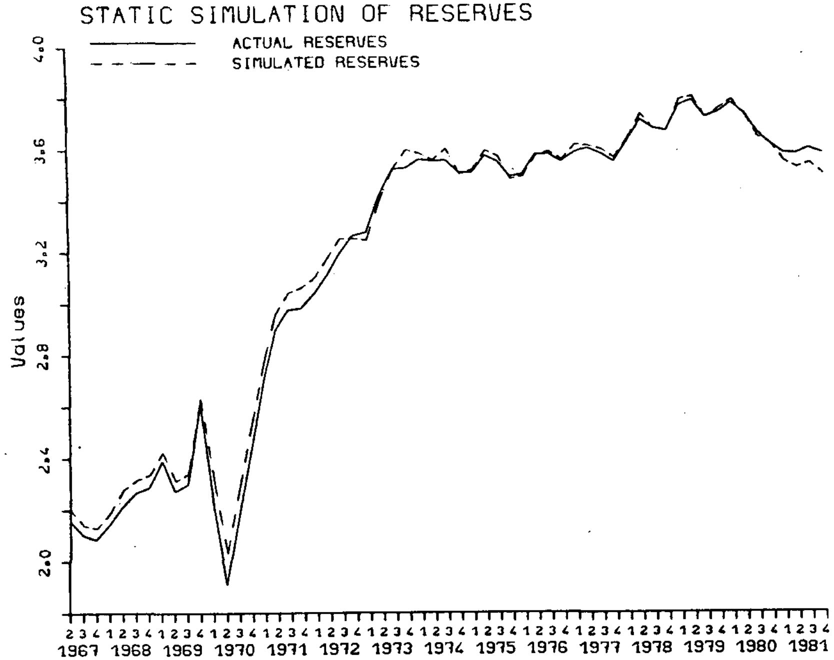


STATIC SIMULATION OF WAGE RATE



STATIC SIMULATION OF GNP DEFLATOR

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ACTUAL RESERVES
SIMULATED RESERVES
Values
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3.2
3.6
4.0
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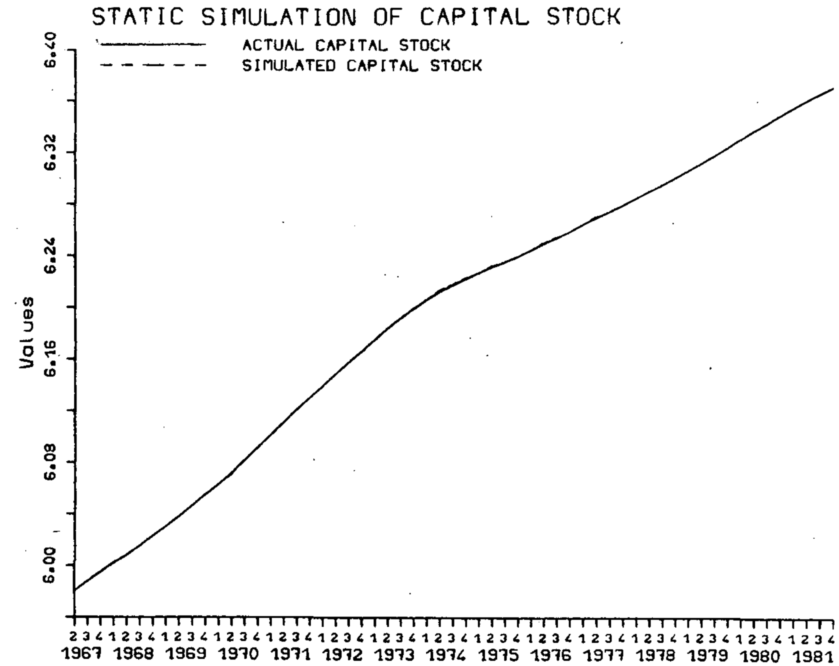
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ACTUAL CAPITAL STOCK
SIMULATED CAPITAL STOCK
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6.32
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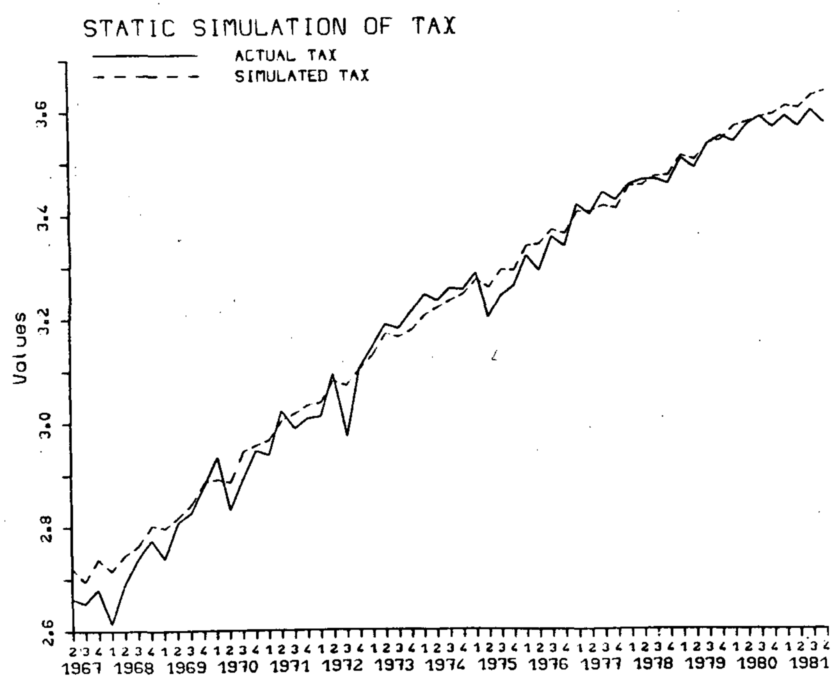
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ACTUAL TAX
SIMULATED TAX
Values
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3.4
3.6
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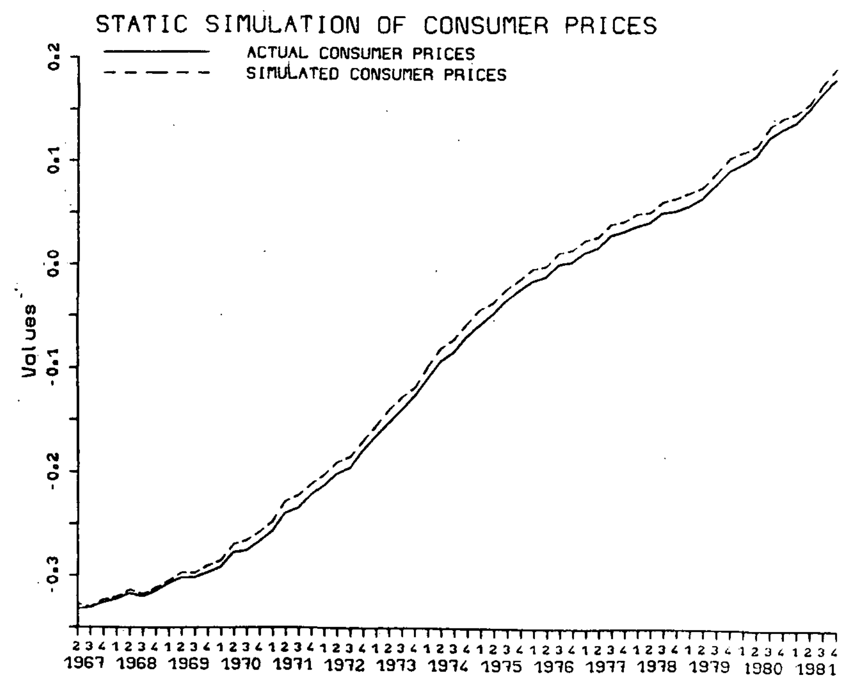
STATIC SIMULATION OF CONSUMER PRICES
ACTUAL CONSUMER PRICES
SIMULATED CONSUMER PRICES
Values
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-0.1
0.0
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0.2
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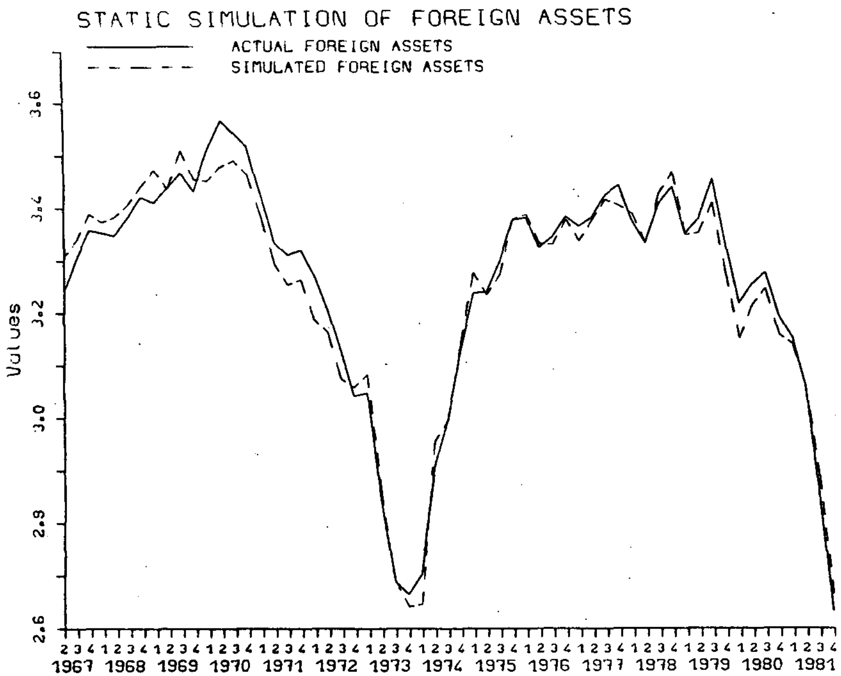
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ACTUAL FOREIGN ASSETS
SIMULATED FOREIGN ASSETS
Values
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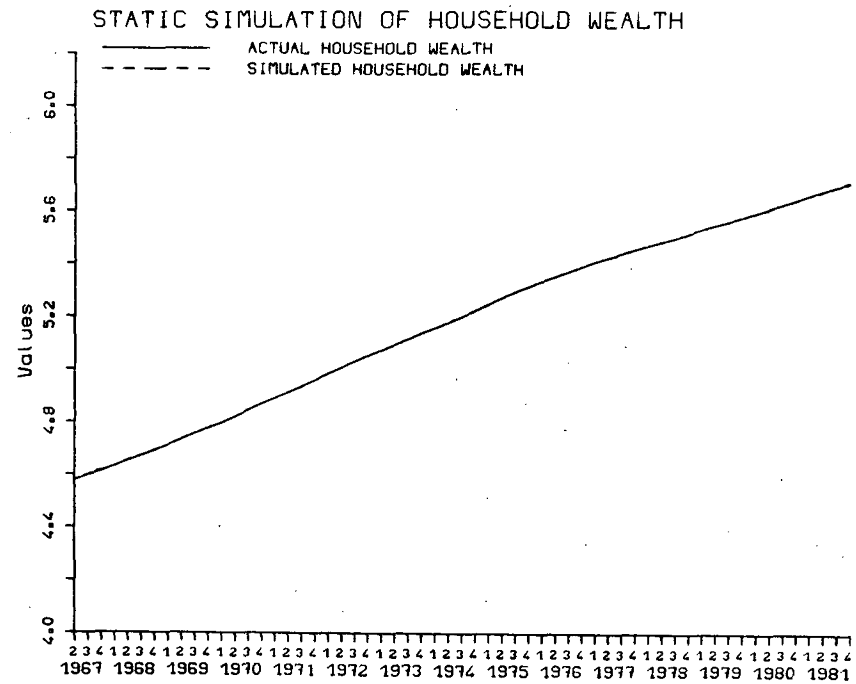
STATIC SIMULATION OF HOUSEHOLD WEALTH
ACTUAL HOUSEHOLD WEALTH
SIMULATED HOUSEHOLD WEALTH
Values
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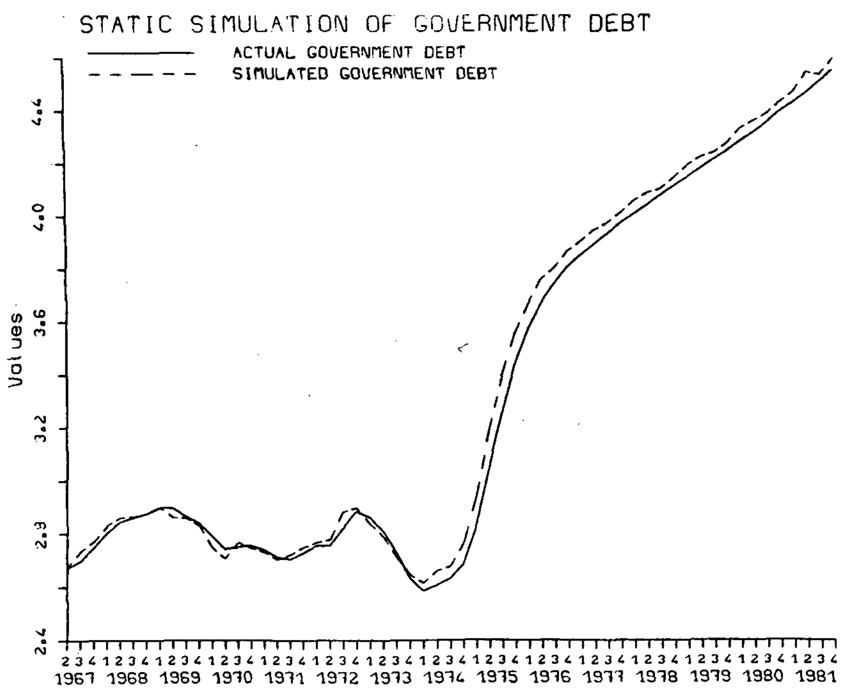
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ACTUAL GOVERNMENT DEBT
SIMULATED GOVERNMENT DEBT
Values
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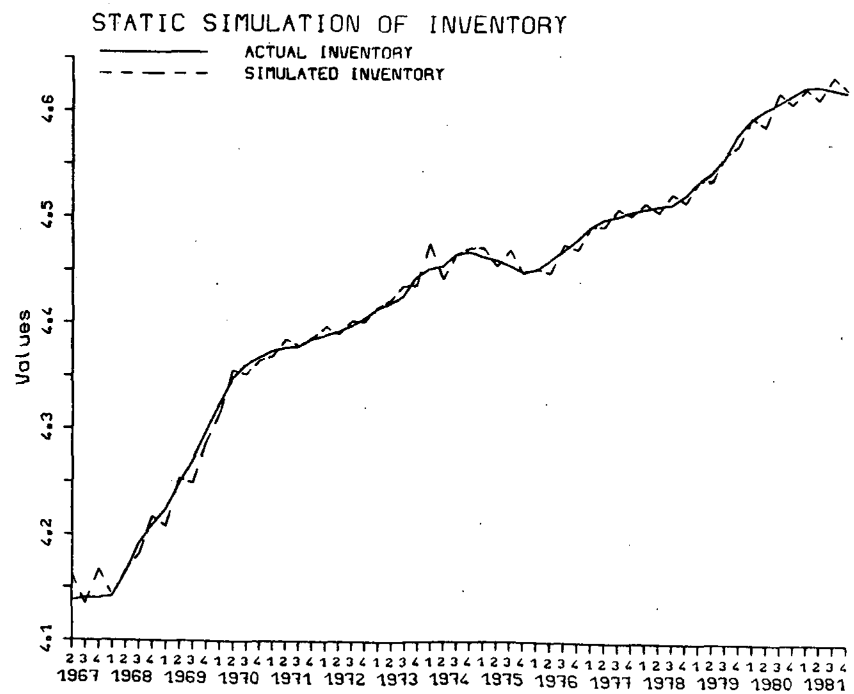
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SIMULATED INVENTORY
Values
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