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A Dynamic Singular Equation System of Asset Demand

Abstract

The paper presents estimates of a dynamic demand system of the AIDS type for financial assets. The results suggest that dynamic behavior plays a major role in determining asset demand. Estimates on the basis of the equivalent static equilibrium models prove to be clearly inferior statistically. Also, the theoretical restrictions of homogeneity and symmetry are thoroughly rejected by the static model versions, however, not by the dynamic demand system. The cross rate elasticities between bonds and savings deposits and also between money and time deposits are found to be negligible for Germany. Time deposits turn out to be very sensitive to own and cross rates of return.

A Dynamic Singular Equation System of Asset Demand

A better understanding of the determinants of portfolio choice behavior of households is central to such issues as the influence of monetary policy on capital markets, the impact of capital-income taxation on risk bearing or of discriminatory interest rate taxation in general. Quite in contrast to the potential usefulness of knowing more about the set of own and cross-rate of return elasticities that determine asset demand functions, there is a dearth of empirical work in this area. Studies that do exist, on the other hand, such as Conrad (1980) for Germany or Taylor and Clements (1983) for Australia, employ econometric techniques which could be questioned in the light of some recent work by Anderson and Blundell (1982) on the dynamics of singular equation systems. In particular, using a model of production factor shares the latter authors demonstrate that the frequently reported failure of complete systems of demand equations to support the parameter restrictions of demand theory, i.e. symmetric cross effects and homogeneity of degree zero,[1] may have been due largely to inadequate dynamic specification rather than to inadequate theory.

The purpose of this paper is to test whether the methodology of Anderson and Blundell (1982) can also be usefully applied to the econometric analysis of portfolio choice in the framework of a complete demand system. On the basis of semi-annual data for the German

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[1] See for example the work by Barten (1969), Christensen et al. (1975), and Deaton and Muellbauer (1980) on the demand for consumer goods and Taylor and Clements (1983) for asset demand.

household sector, a general dynamic structure is superimposed on an asset demand system in the Brainard-Tobin (1968) tradition and very close in spirit to the familiar 'Almost Ideal Demand System' (AIDS) of Deaton and Muellbauer (1980). Various simpler dynamic representations of the model such as the partial adjustment model and the static equilibrium model are tested as nested hypotheses. Within each dynamic specification, tests of the theoretical restrictions of homogeneity and symmetry are performed to check for the superiority of dynamic over static model representations.

The model is set out in the next section, followed by a description of the data. The estimation and test results are reported next. The paper concludes with a summary of the main points.

The Model

A singular demand system for financial assets that is linear in parameters can be represented by

$$S(t) = \Pi Z(t) + u(t) \qquad (1)$$

where s(t) is an nxl vector of the shares in total wealth of n assets, the matrix Π consists of nxk parameters that are assumed constant over time, and where Z(t) is a kxl vector of nonstochastic variables; u(t) is an nxl vector of stochastic errors. The adding up restriction implies that for the nxl unit vector i the following restrictions hold for system (1)

$$(2) \qquad i' \Pi = (1 \quad 0 \ldots 0)$$
$$i'u(t) = 0, \qquad \text{for all } t$$

Expanding (1) into a general dynamic system implies premultiplication of s(t) and Z(t) by the polynomial expressions B(L) and Γ(L) in the lag operator L such that

$$(3) \qquad B(L)\ S(t) = \Gamma(L)\ Z(t) + \varepsilon(t)$$

where ε is an independent identically distributed random disturbance vector. Anderson (1980) has derived the parameter restrictions that imply adding up for (3).

To simplify the exposition and to increase its correspondence to the actual system that forms the maintained hypothesis of this study, the subsequent exposition is restricted to the first order form of the general dynamic system (3). If one incorporates the restrictions resulting from adding up, Anderson and Blundell (1982) have shown that the equations for estimation of a first order system can be represented by

$$(4) \quad \Delta\ S(t) = \Gamma\ \Delta\ \tilde{Z}(t) - A\ [S(t-1) - \Pi\ Z(t-1)] + \varepsilon(t)$$

where Δ is a first difference operator, Γ an nx(k-1) parameter matrix, and $\tilde{Z}(t)$ a variable vector of length k-1, with $\sim$ indicating that the constant term is lost through first differencing; matrix A is of order nxn. System (4) has nested within itself some familiar model specifications such as the static equilibrium model, the static model with autoregressive errors, or the partial adjustment model. Each of these models implies a unique set of parameter restrictions. They are summarized in Table 1.

Table 1. Parameter Restrictions for Various Dynamic Specifications

Model Type	Parameter Restrictions on (4)
Static Equilibrium Model	$\Gamma = \tilde{\Pi} \quad A = I$
Static Equilibrium Model with First Order Autocorrelation	$\Gamma = \tilde{\Pi}$
Partial Adjustment Model	$\Gamma = A\,\tilde{\Pi}$

Note: I is an nxn identity matrix. A tilde indicates that the first column (i.e. the constant term) has been deleted.

If one incorporates these restrictions, the static equilibrium version of system (4) reduces to

$$(5) \qquad S(t) = \Pi\, Z(t) + \varepsilon(t)$$

The corresponding equation system for the static model with an autoregressive error process of order one is given by

$$(6) \qquad (I-RL)\, S(t) = (I-RL)\, \Pi\, Z(t) + \varepsilon(t)$$

where R is an nxn matrix of autocorrelation parameters. Since R is assumed non-diagonal, cross-equation autocorrelations are allowed for. Employing the restrictions of the partial adjustment hypothesis, one can rewrite (4) as

$$(7) \qquad \Delta\, S(t) = M\, [\Pi\, Z(t) - S\,(t-1)]$$

where M is an nxn matrix of partial adjustment parameters. For the restrictions of Table 1, M equals A. Matrix M is non-diagonal like R.

The functional specification of system (4) is adapted from the 'Almost Ideal Demand System' of Deaton and Muellbauer (1980). In its static equilibrium version corresponding to (5) the demand function for a single asset can be written as

$$(8) \qquad S_i = \alpha_i + \sum_j \gamma_{ij} \log r_j + \beta_i \log W$$

where s_i is the share of asset i in real wealth W and r_j is the rate of return received on asset j. The triad of consumer theory implies the following restrictions on the parameters of (8)

Adding up : $\Sigma \alpha_i = 1 \quad \sum_i \gamma_{ij} = 0 \quad \Sigma \beta_i = 0$ for all j

Homogeneity: $\sum_j \gamma_{ij} = 0$ for all i

Symmetry : $\gamma_{ij} = \gamma_{ji}$ for all i,j

In the estimation of the system adding up is imposed by including only n-1 equations. This convenient simplification implies that the symmetry restrictions on the remaining n-1 equations have to be rewritten in terms of the estimated paramters taking into account the adding up restriction $\sum_i \gamma_{ij} = 0$.[2]

[2] See the appendix for more detail on how to implement the theoretical restrictions in a model with n equations and k independent variables.

Data

The model is estimated on semi-annual data for the private household sector of West Germany. The sample covers the years 1972-84. Four categories of financial assets are considered:

- cash and demand deposits
- time deposits of under 4 years
- savings deposits
- bonds.

Assets held in the form of deposits at building and loan associations or at capital life insurance companies are not considered since their size is to a large extent determined by legislation promoting private wealth accumulation and/or by insurance motives. Similar reasoning forms the basis for the decision to exclude those assets from the savings deposits that are government subsidized (Prämiensparen etc.). Finally, common stock is left out owing to the minimal changes that have occurred for this category over the time period covered.

All asset data are drawn from official publications of the Deutsche Bundesbank and are measured as end of period stocks. Wealth is defined as the sum of all assets considered. Real wealth is found by deflating nominal asset holdings with the general consumer price index. Savings deposits include both low interest passbook savings and high interest savings deposits with a fixed maturity date. Since stock data on savings deposits of private households are not available for the whole time span, the respective figures for private households and firms are substituted. This seems justified given the very small share of firms. A rather difficult problem arises in conjunction with semi-annual stock data on bonds. The Deutsche Bundesbank only publishes flow values on a semi-annual basis and stock data on an annual basis. Since the two time series are not entirely consistent, semi-annual stock data were obtained by assuming that the ratio of the flow for half a year to the annual flow equals the ratio of the corresponding stock changes.

The rates of return of the assets included in the study are approximated by nominal weighted interest rates or yields. In particular, the rate on savings deposits is a weighted average of low and high interest savings deposits. For high interest savings deposits, we employ a weighted average of the interest paid on deposits with a maturity of one and four years. For low interest savings deposits, the standard bank rate is used. The interest rate for 3-months time deposits for amounts up to one million DM substitutes for the rate of return on assets held as time deposits. This short-term rate can be considered a good approximation for the actual yield because short-term time deposits are clearly dominant as measured by market value. The average yield on all outstanding bonds is used as our bond rate.[3] The yield on outstanding bonds is preferred over the issue yield because private households have the option to buy and sell marketable bonds from and to other sectors of the economy and are, hence, not limited to the market for security issues.

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[3]It does not reflect the actual capital yield an investor can secure because bond price fluctuations are not incorporated. These price changes, however, are not relevant if one assumes that the market interest rate correctly represents the expectations of the market.

Results

The limited number of available observations does not allow estimation of a dynamic four equation model in unrestricted form. Hence, to be able to analyze the contribution of dynamics to asset demand estimation, the model has to be reduced in dimension. Since earlier studies on portfolio choice for Germany[4] seem to indicate that cash and demand deposits are largely determined by transactions motives, we first estimate a three equation model excluding money (Model A). Subsequently, the maintained hypothesis that the utility function that spans the four assets is separable between money and the remaining assets is put to a test. For that purpose, a four equation model is estimated that incorporates parameter restrictions relating to both model dynamics and theoretical model structure that appear to be justified on the basis of the prior modeling exercise (Model B).

The results for the three equation model encompassing savings deposits, time deposits, and bonds are summarized in Tables 2 to 4. What may be of primary interest from an econometric perspective is the evidence collected on the usefulness of a dynamic vis-a-vis a static specification of an asset demand system. Two questions require answering in this respect. First, how does the static model representation compare to the dynamic ones in terms of explanatory power? Second, can the theoretical restrictions of symmetrical cross effects and linear homogeneity in rates of return be rejected for both the dynamic and the static model specifications? Table 2 presents the statistical evidence

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[4]See, for example, Dieckheuer (1985) and Conrad (1980).

Table 2. Likelihood Ratio Tests of Theoretical and Empirical Restrictions - Three Equation Model

Model Type	Tests of Simplified vs. General Dynamic Specification	Tests of Theoretical Restrictions	
		Homogeneity	Symmetry
General Dynamic	-	3.4 (2)	3.4 (3)
Partial Adjustment	21.8 (10)	1.7 (2)	2.0 (3)
Autocorrelation of First Order	21.5 (10)	16.0 (2)	16.3 (3)
Static Equilibrium	69.5 (14)	21.04 (2)	23.5 (3)

Note: The degrees of freedom of the χ^2 tests are given in parenthesis below the test statistics. The model incorporates the three assets savings deposits, time deposits, and bonds.

that will help answer these questions. Two types of tests are reported, each relating to one of the questions posed. The first column of Table 2 provides the likelihood ratio statistic for the test of each of the three simplified dynamic specifications of Table 1 versus the general dynamic one. No theoretical restrictions of demand theory are applied apart from adding-up. The test results indicate that, at the 1 percent level of significance, both the partial adjustment and the autocorrelated error model can not be rejected by the data. The static equilibrium model, however, can be clearly discarded on the basis of the same criteria. Columns 2 and 3 of Table 2 provide a test of the restrictions of demand theory for each of the four model specifications. Interestingly enough, the restrictions of linear homogeneity in rates of return and symmetrical cross effects are thoroughly rejected for the static equilibrium version of the model. This result is familiar from many other studies that employ complete demand systems.[5] The restrictions of demand theory are also rejected for the autocorrelated error model. On the other hand, the test statistics support both theoretical restrictions, i.e. homogeneity and symmetry, for the general dynamic model specification and the partial adjustment hypothesis. At a minimum, this indicates that adjustment lags as they are captured in the dynamic model specifications play an important role for asset demand systems and should, therefore, not be ignored in econometric modeling. Going somewhat further, the test results seem to confirm the conclusion of Anderson and Blundell (1982) as well as Veall and Zimmermann (1984)

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[5]For asset demand systems, a similar outcome was recently reported by Taylor and Clements (1983).

that the failure of previous studies employing static demand systems to find support for the restrictions of demand theory is likely to arise from inadequate model specification rather than from a basic deficiency of the theory itself.

Some of the economically relevant evidence from model A is collected in Tables 3 and 4. The results pertain to the general dynamic version of model A. For ease of interpretation, the original parameter estimates are converted into demand (not share) elasticities. The standard errors are computed on the basis of the variance-covariance matrix of the estimated parameters. All elasticities are evaluated at their sample mean, i.e. the years 1972 to 1984. The point estimates of the rate of return (e_{ij}) and wealth elasticities (e_{iw}) are computed according to the formulas

$$e_{ij} = \frac{\gamma_{ij}}{S_i} \qquad e_{iw} = 1 + \frac{\beta_i}{S_i}$$

respectively, where the variable and parameter definitions equal those of (8). Tables 3 and 4 differ in that the cross rate elasticity between bonds and time deposits is set to zero. Based on the value of the log likelihood function, the two model versions are indistinguishable. Also, the parameter values are very similar. However, reducing the dimension of the model leads to a perceptible improvement in the standard errors of the estimated parameters of Table 4.

Regardless of whether one takes the results of Table 3 or 4, the wealth elasticities for model A indicate that bonds are strong and time deposits weak luxuries whereas savings deposits qualify as necessities.

Table 3. Rates of Return and Wealth Elasticities - Model A
General Dynamic Version with no Theoretical Restrictions

Model Equation	e_{i1}	e_{i2}	e_{i3}	e_{iw}
(1) Savings Deposits	0.46	-0.22	-0.48	0.67
	(.24)	(.12)	(.24)	(.10)
(2) Time Deposits	-2.21	1.69	1.63	1.15
	(1.0)	(.51)	(1.0)	(.44)
(3) Bonds	-0.41	-0.04	0.67	1.82
	(.32)	(.15)	(.32)	(.13)

Note: Standard errors are reported in parenthesis. They are computed on the basis of the variance-covariance matrix of the estimated parameters. e(ij) is the rate of return elasticity, and e(iw) the wealth elasticity. All elasticities are evaluated at their sample mean for the period 1972-84 and have to be interpreted as demand rather than share elasticities.

Table 4. Rates of Return and Wealth Elasticities - Model A
General Dynamic Version with Restriction $e_{32} = 0$

Model Equation	e_{i1}	e_{i2}	e_{i3}	e_{iw}
(1) Savings Deposits	0.50	-0.25	-0.47	0.69
	(.20)	(.05)	(.24)	(.08)
(2) Time Deposits	-2.34	1.78	1.59	1.09
	(.98)	(.38)	(1.0)	(.39)
(3) Bonds	-0.47	.	0.65	1.79
	(.25)		(.31)	(.09)

Note: See Table 3 for explanations.

This corresponds to a priori reasoning. Investment in bonds and time deposits is not quite risk free and usually involves minimum transactions of a magnitude in excess of those common for savings deposits. Hence, these financial instruments are generally not suitable for the small saver. In a similar vein, there is good reason to believe that the larger elasticities reflect the growing knowledge of investment opportunities in the case of growing wealth: larger wealth holdings increase the opportunity cost of not obtaining the additional information necessary to invest above the minimum interest level of, say, low interest passbook accounts.

The own rate elasticities are all positive for model A, as one would expect for assets. The cross rate elasticities are generally negative, as they should be in the case of substitutes. One exception can be found in the time deposits equation. Similar to the findings of Conrad (1980), the cross rate elasticity with bonds (e_{23}) is positive. This suggests that an increase in the bond rate leads to an increase not only in the demand for bonds but also for time deposits. An explanation may be that a good part of the potential investors in the bond market associate rising rates of return with capital losses and hence switch to time deposits, an alternative asset largely without this risk.[6] Conversely, changing rates of return for time deposits apparently do not influence investors in the bond market. Somewhat surprising, at first, are the very large own and cross rate elasticities for time deposits. They reveal that, over the estimation period of model A (1972-1984),

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[6] Statistically, however, the positive cross elasticity is not well determined and should therefore not be overinterpreted.

households were very responsive to the interest rate differential between traditional assets such as savings deposits and newly established investment opportunities such as time deposits. Given the historical development of time deposits in Germany this appears to be reasonable. Initially introduced by banks to offer customers an attractive alternative to the low yield on short-term savings deposits during times of inflation, time deposits managed to attract a sizable share of total savings as inflation was rising to unprecedented levels in the seventies. Their large own and cross rates of return can be interpreted to imply that they have come to serve as a kind of buffer in the portfolio of households that bridges over the uncertainty in the bond market as well as the slow reaction of savings accounts to rising interest rates. As such they are rather volatile. This characteristic is also borne out by the historical time path of time deposits, which shows significant ups and downs around a steadily rising trend.

Subsequent to model A a second model was constructed consisting of the three equations of model A plus a fourth equation for cash and demand deposits, that is money in its narrow definition. Since the small sample prevents estimation of a general dynamic model, we utilized a partial adjustment framework. As the test statistics of Table 2 have shown, this model specification can be regarded as statistically equivalent to the general dynamic model for the present data set yet saves a considerable number of degrees of freedom. To insure comparability between models A and B, no theoretical restrictions other than adding-up were imposed. The estimated demand elasticities are presented in Table 5. Overall, the elasticity values are rather close to

Table 5. Rates of Return and Wealth Elasticities - Model B
Partial Adjustment Version without Theoretical Restrictions

Model Equation	e_{i1}	e_{i2}	e_{i3}	e_{iw}
(1) Savings Deposits	0.35	-0.23	-0.46	0.68
	(.28)	(.16)	(.26)	(.14)
(2) Time Deposits	-2.16	1.70	2.22	1.39
	(1.1)	(.64)	(1.1)	(.57)
(3) Bonds	0.01	-0.13	0.71	2.18
	(.56)	(.32)	(.53)	(.29)
(4) Money	-0.18	0.15	-0.41	0.35
	(.27)	(.15)	(.26)	(.14)

Note: Standard errors are reported in parenthesis. They are computed on the basis of the variance-covariance matrix of the estimated parameters. e(ij) refers to the rate of return elasticity, and e(iw) is the wealth elasticity. All elasticities are evaluated at their sample mean (1972-84) and have to be interpreted as demand rather than share elasticities. Money is defined as the sum of cash and demand deposits.

Table 6. Rates of Return and Wealth Elasticities - Model B
Partial Adjustment Version without Theoretical Restrictions
but with $e_{31} = e_{32} = e_{41} = e_{42} = 0$

Model Equation	e_{i1}	e_{i2}	e_{i3}	e_{iw}
(1) Savings Deposits	0.36	-0.30	-0.33	0.70
	(.11)	(.06)	(.16)	(.09)
(2) Time Deposits	-2.55	2.11	1.86	1.16
	(.78)	(.41)	(.76)	(.46)
(3) Bonds	.	.	0.44	2.15
			(.28)	(.13)
(4) Money	.	.	-0.33	0.45
			(.14)	(.07)

Note: See Table 5 for explanations.

the ones reported in Tables 3 and 4 considering that one equation was added and a simplified specification used. Except for the value of e_{32}, which happens to be not well determined statistically in model A to begin with, only the wealth elasticity of bonds does not stay within one standard error of the values given in Table 4.

To increase the precision of the estimates reported in Table 5, model B was reestimated with several elasticities constrained to zero a priori. The corresponding demand elasticities are presented in Table 6.[7] As expected the precision of the resulting estimates has improved considerably over Table 5. Again, the elasticity values only differ marginally between Tables 5 and 6, with no change being in excess of one standard error. Also, a comparison of the estimates with those reported in tables 3 and 4 shows that the results are hardly influenced by the addition of an equation for money holdings, even if the demand for money itself does not seem to be totally independent of interest rates, as the results of table 6 demonstrate. A likely reason for the negative interest elasticity of money demand may be that the holding of money really reacts to the inflation rate incorporated in the nominal interest rates that are being use in the model.

Money and time or savings deposits, on the other hand, are unrelated. This corroborates the findings of Conrad (1980) for Germany. Similarly, the results of Table 6 confirm the suspicion one may have had based on the results of Tables 3 and 4 that bonds do not seem to react to rate

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[7] A likelihood ratio test of the parameter restrictions yielded a value of 3.0 at four degrees of freedom which means that the restrictions can not be rejected at any common level of significance.

changes in savings or time deposits. The demand for bonds appears to depend only on its own rate of return and on wealth. In contrast, Table 6 suggests that there exists a significant influence of the bond rate on the demand for other assets. A possible reason for this apparent nonsymmetric behavior of investors could be that bonds are held mainly for the purpose of capital investment. If, under these conditions, investment prospects deteriorate, one may conjecture that risk aversion leads households to prefer liquid assets, independent of the actual difference in rates of return.

Conclusion

The paper has presented some estimates of a dynamic demand system of the AIDS type for a selection of financial assets. The estimates suggest that dynamic behavior plays a major role in determing asset demand. Estimates on the basis of the equivalent static equilibrium models prove to be clearly inferior statistically. Also, as has been reported in many other studies using complete demand systems, the theoretical restrictions of homogeneity and symmetry are thoroughly rejected by the static model versions. On the other hand, introducing dynamics into the model fully resurrects demand theory.

The rate of return elasticities derived on the basis of the dynamic model specifications support Conrad's (1980) conclusion that the cross rate elasticities between bonds and savings deposits and also between money and time deposits are negligible for Germany. In fact, the desire of households to hold money, defined as M1, seems to be mainly determined by transactions motives. However, it appears that an increase in long-term interest rates has some negative effect on money holdings, even though this link is somewhat weak statistically. Time deposits are found to be very sensitive to own and cross rates of return. It seems they largely serve as a buffer in the portfolio of households thereby bridging over the uncertainty in the bond market as well as the slow reaction of savings accounts to rising interest rates. As such the rather volatile behavior of their historical time path can be explained.

APPENDIX

The estimating form of equation system (4) in the text consisting of n equations and k independent variables can be written as

$$\begin{bmatrix} \Delta s_1 \\ \Delta s_2 \\ \vdots \\ \Delta s_{n-1} \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \cdots & \Gamma_{1,n} & \cdots & \Gamma_{1,k} \\ \Gamma_{21} & \Gamma_{22} & & & & \\ \vdots & & & & & \\ \Gamma_{n-1,1} & \cdots & & \Gamma_{n-1,n} & \cdots & \Gamma_{n-1,k} \end{bmatrix} \times \begin{bmatrix} \Delta z_1 \\ \Delta z_2 \\ \vdots \\ \Delta z_n \\ \vdots \\ \Delta z_k \end{bmatrix}$$

$$- \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1,n-1} \\ a_{21} & a_{22} & & \\ \vdots & & & \\ a_{n-1,1} & \cdots & & a_{n-1,n-1} \end{bmatrix} \times \left(\begin{bmatrix} s_1(t-1) \\ s_2(t-1) \\ \vdots \\ s_{n-1}(t-1) \end{bmatrix} - \right.$$

$$\left. \begin{bmatrix} \Pi_{10} & \Pi_{11} & \cdots & \Pi_{1,n} & \cdots & \Pi_{1,k} \\ \Pi_{20} & \Pi_{21} & & & & \\ \vdots & & & & & \\ \Pi_{n-1,0} & & \cdots & \Pi_{n-1,n} & \cdots & \Pi_{n-1,k} \end{bmatrix} \times \begin{bmatrix} z_0(t-1) \\ z_1(t-1) \\ \vdots \\ z_n(t-1) \\ \vdots \\ z_k(t-1) \end{bmatrix} \right)$$

Note that the above system can easily accomodate k-(n+2) non-symmetric variables, i.e. variables other than rates of return and wealth. Analogous to the symmetric variables, they would appear in each equation and be subject to the adding up constraint $i'\Pi^s = (0...0)$, where Π^s is the corresponding coefficent sub-matrix of Π in (4) of size $n \times [k-(n+2)]$.

Imposing the theoretical restrictions of homogeneity and symmetry on the estimating form of the general dynamic system implies the following parameter restrictions:

1. Homogeneity Restrictions

$$\pi_{1,n} = -(\pi_{11} + \pi_{12} + \ldots + \pi_{1,n-1})$$

$$\pi_{2,n} = -(\pi_{21} + \pi_{22} + \ldots + \pi_{2,n-1})$$

$$\vdots$$

$$\pi_{n-1,n} = -(\pi_{n-1,1} + \pi_{n-1,2} + \ldots + \pi_{n-1,n-1})$$

2. Symmetry Restrictions

$$\pi_{12} = \pi_{21}$$

$$\pi_{13} = \pi_{31}$$

$$\vdots$$

$$\pi_{1,n} = -(\pi_{11} + \pi_{21} + \ldots + \pi_{n-1,1})$$

$$\pi_{2,n} = -(\pi_{12} + \pi_{22} + \ldots + \pi_{n-1,2})$$

$$\vdots$$

$$\pi_{n-1,n} = -(\pi_{1,n-1} + \pi_{2,n-1} + \ldots + \pi_{n-1,n-1})$$

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