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Working Paper

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Kiel Working Papers, No. 777

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Institut für Weltwirtschaft (IfW)

Suggested citation: Stähler, Frank (1996) : The market entry paradox, Kiel Working Papers, No. 777, http://hdl.handle.net/10419/46836

Kieler Arbeitspapiere
Kiel Working Papers

Kiel Working Paper No. 777

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November 1996

Institut für Weltwirtschaft an der Universität Kiel
The Kiel Institute of World Economics

ISSN 0342 - 0787

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THE MARKET ENTRY PARADOX*

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November 1996

Key words. Market entry, mixed strategy equilibrium.

JEL classification: D43, L13.

Abstract. *This paper discusses a simultaneous market entry game between two firms with different fixed costs. This case is a typical application of mixed strategy equilibria. Conventional wisdom would suggest that the low-cost firm is more likely to enter the market. This presumption is wrong. Instead, the paper demonstrates a market entry paradox: the equilibrium probability of entry is higher for the high-cost firm than the equilibrium probability of entry of the low-cost firm.*

* This paper benefited substantially from discussion with my colleague Christian Scholz. The usual disclaimer applies.

1. Introduction

Explaining the evolution of industry structures has become a focus of modern industrial organization theory. One field of interest is the endogenous determination of oligopolistic market structures. If a certain number of oligopolists, say N firms, may enter the market, but due to fixed entry costs the market carries only a real subset of them, say M (< N) firms, the pure strategy equilibria are asymmetric. In every pure strategy equilibrium, M out of N enter with probability one, and N - M out of M do not enter. This result gives the pure strategy equilibrium for two different entry games, the sequential entry game and the simultaneous entry game. In the sequential entry game, firms decide on entry one after each other being aware of the previous entry decisions, in the simultaneous entry game, firms decide on entry being unaware of the decisions of their rivals.

As the information sets differ substantially between both games, one may find that the results should differ as well. In recent papers, simultaneous entry games have been therefore predominantly modelled by employing strictly mixed strategy equilibria.[1] Strictly mixed strategies mean that each firm chooses an entry probability strictly between 0 and 1. Although this equilibrium concept is not without difficulty itself and subject to criticism as well, its merits are that it is capable of explaining the uncertainty surrounding entry decisions, that is able to mirror organizational behavior inside a firm which may be not observable for other firms, and that it leads to expected profits of zero in equilibrium (Dixit, Shapiro, 1986). Additionally, it gives the notion of potential competition sense because the number of rivals determines the expected market performance in this setting (see e.g. Dasgupta, Stiglitz, 1988, Dixit, Shapiro, 1986, Nti, 1988, Elberfeld, Wolfstetter, 1996). When employing pure strategy

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1 This paper uses the notion of *strictly* mixed strategies when equilibrium probabilities are neither zero nor one.

equilibria, the number of rivals plays no role for the market performance as the performance is given by the number of firms carried by the market.

The models which employ strictly mixed strategy equilibria are exclusively models of identical firms such that all firms are risk-neutral, and profits and entry costs do not differ among firms. These assumptions might be quite reasonable for some markets, and this paper will assume identical profits and risk neutrality as well. However, the entry costs might be also different among firms. For example, when different firms consider to enter a world market, their entry costs might depend on national regulations or on their home country's infrastructure. Different entry costs might also be due to different degrees of experience in similar, already existing markets. These differences may be non-negligible but not too substantial to make a high-cost firm refrain from market entry in every case. It is this case of non-negligible, but not too substantial entry costs which this paper considers. It will demonstrate a market entry paradox by employing a duopolistic market entry model. Employing a duopolistic model simplifies deriving equilibrium probabilities and avoids serious difficulties which would arise when discussing a general asymmetric model. The basic result, however, has a straightforward intuitive implication which makes this result hold for the general case as well.

The paper is organized as follows. Section 2 will demonstrate the market entry paradox in a static entry game, and section 3 will demonstrate the market entry paradox in a dynamic entry game. Section 4 will present some simulations which show that the basic result holds under more general conditions as well. The last section will give some concluding remarks.

2. The market entry paradox in the static entry game

The model assumes two firms i and j which consider to enter a market which carries only one of them. Entry requires to carry costs which are denoted by c_i or c_j, respectively. After entry, each firm which has entered yields profits which depend on the number of firms which have entered. Its outside profits are zero. The inside profits which are net of operating costs are denoted by Π and its subscript gives the number of firms:

(1) $\forall k \in \{i,j\}: \ \Pi_1 > c_k > \Pi_2 \geq 0.$

Π_1 gives the monopolistic profits, Π_2 gives the duopolistic profits. (1) assumes that the equilibrium in the duopolistic market is unique. (1) is also capable of modelling a market in which a certain number of incumbents exists, and further market entry is profitable only for one additional firm.

The objective of the firm in the static entry game is to maximize its expected profits by the choice of an entry probability:

(2) $\forall k \in \{i,j\}: \ \max_{p_k}\left\{p_k\left[(1-p_{-k})\Pi_1 + p_{-k}\Pi_2 - c_k\right]\right\} \text{ s.t. } \quad 0 \leq p_k \leq 1.$

p denotes the probability of entry, k denotes either firm i or firm j, and -k denotes the firm which is not k. In (2), $\left[(1-p_{-k})\Pi_1 + p_{-k}\Pi_2 - c_k\right]$ gives the expected profits if the firm enters which depends on the entry probability of the other firm. Multiplication by its own probability gives the expected profits because the expected profits of not entering are zero. As the maximand of (2) is linear in its maximizing argument, the optimal probability (denoted by a star) is either zero, one or the whole range of possible probabilities:

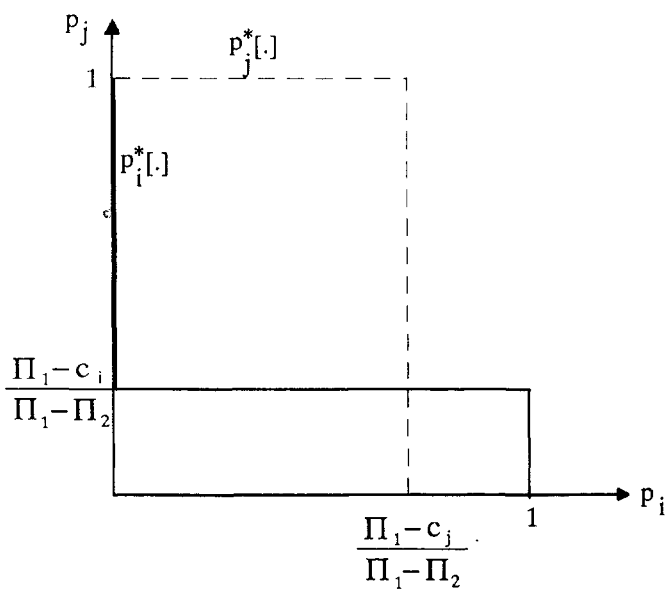
(3) $$\forall k \in \{i,j\}: \ p_k^* = \begin{cases} 0 & \text{if } (1-p_{-k})\Pi_1 + p_{-k}\Pi_2 - c_k < 0 \\ \in[0,1] & \text{if } (1-p_{-k})\Pi_1 + p_{-k}\Pi_2 - c_k = 0 \\ 1 & \text{if } (1-p_{-k})\Pi_1 + p_{-k}\Pi_2 - c_k > 0 \end{cases}$$

(3) shows that a firm enters with probability zero (one) if the expected profits of entry are strictly negative (positive). In the case of the critical probability (denoted by a prime) which makes expected profits of entry zero (see (4)), every probability maximizes expected profits.

$$(4) \qquad \forall k \in \{i,j\}: \; p'_{-k} = \frac{\Pi_1 - c_k}{\Pi_1 - \Pi_2}.$$

(3) defines the reaction function of firm k dependent on the probability of firm -k. (3) and (4) allow to depict both firms' reaction curves in Figure 1:

Figure 1: Reaction curves in the simultaneous entry game



The broken (unbroken) line depicts the reaction curve of firm j (i). Figure 1 shows all equilibria of the simultaneous entry game: the pure strategy equilibria make one firm enter and the other stay away, and the strictly mixed strategy equilibrium makes both firms enter with a certain probability. Note that the strictly mixed strategy equilibrium is not stable because any unilateral deviation of a firm leads to a pure strategy equilibrium. However, stability raises no problem in this setting because the determination of entry probabilities cannot result from a tâtonnement process since factual entry decisions are irreversible. As the critical values of (4) define the mixed strategy equilibrium, equilibrium probabilities are:

$$(5) \qquad \forall k \in \{i,j\}: \; p_k^E[c_{-k},\cdot] = \frac{\Pi_1 - c_{-k}}{\Pi_1 - \Pi_2} \qquad \Rightarrow \qquad \frac{\partial p_k^E[\cdot]}{\partial c_{-k}} < 0.$$

The superscript E denotes the strictly mixed strategy equilibrium. (5) shows the *market entry paradox: a firm's entry probability decreases with the entry costs of the other firm which means that the high-cost firm enters with higher probability than the low-cost firm.* This result can also be seen from Figure 1 in which entry costs are higher for firm i and firm i enters with a higher probability. This is obviously an unexpected and completely counter-intuitive result which deserves further discussion. The reason for this result has to be found in the nature of strictly mixed strategy equilibria. A firm which chooses a strictly mixed strategy such that the expected profits of market entry are strictly positive (negative) will imply that the other firm will enter with probability one (zero). In this case, a probability strictly between zero and one was not optimal for this firm because its best reaction to the implied pure strategy of its opponent is a pure strategy as well. Only in the case of making the rival indifferent between entry and non-entry, the firm may expect that its rival employs a strictly mixed strategy as well. The same line of reasoning holds for its rival such that each firm chooses its own entry probability in order to make the expected profits of its rival zero. If its rival has to carry higher entry costs, it must choose a lower probability in order to make its rival indifferent. In strictly mixed strategy equilibria, the entry probabilities are a function of the rival's entry costs (and not of its own entry costs), and higher (lower) entry costs require lower (higher) entry probabilities. This feature has been obviously overlooked in the literature when dealing with the case of identical firms for which all firms' entry probabilities fall together in equilibrium.

3. The market entry paradox in the dynamic entry game

It is easy to demonstrate that the market entry paradox holds also for the dynamic entry game. For this case, (1) must be substituted by

$$(1') \quad \forall k \in \{i,j\}: \ \frac{\Pi_1}{1-\delta} > c_k > \frac{\Pi_2}{1-\delta} \geq 0$$

in which δ (<1) denotes the discount factor assumed to be identical for both firms. The dynamic game assumes an infinite number of stages in which both firms decide on entry unless they have not yet entered the market.[2] The entry decisions are observable before the next stage is reached. (1') ensures that entry with probability one of each firm is no equilibrium strategy because the discounted sum of duopolistic profits falls short of entry costs.

In a dynamic game, each firm randomizes over entry only if it and its opponent have not yet entered. If firm k has entered, firm -k will stay away for the rest of the game unless it has already entered simultaneously with firm k. Hence, the entry decision is on the agenda again only in the case of both firms having not entered in previous periods. Subgame-perfection requires that the strategies chosen in different stages should not differ when the game does not differ. Consider two subsequent stages and assume that actually no firm has entered in the first stage. Then, subgame-perfection requires that the entry probabilities chosen in the second stage should be equal to those in the first stage and, more generally, that the equilibrium entry probabilities in every

2 In order to avoid confusion, it should be stressed that the notion of stages of a dynamic game refers to the time structure. The assumption still holds that each stage game comprises the two stages of randomizing over entry under complete ignorance and the market game.

stage should be identical until at least one firm has entered. Consequently, the expected profits of a firm in every stage in which entry has not yet occurred are

$$(6)\quad p_k\left\{\frac{1-p_{-k}}{1-\delta}\Pi_1+\frac{p_{-k}}{1-\delta}\Pi_2-c_k\right\}$$
$$+\delta p_k[1-p_k][1-p_{-k}]\left\{\frac{1-p_{-k}}{1-\delta}\Pi_1+\frac{p_{-k}}{1-\delta}\Pi_2-c_k\right\}$$
$$+\delta^2 p_k[1-p_k]^2[1-p_{-k}]^2\left\{\frac{1-p_{-k}}{1-\delta}\Pi_1+\frac{p_{-k}}{1-\delta}\Pi_2-c_k\right\}\quad+\quad\ldots$$
$$=\frac{p_k\left[(1-p_{-k})\Pi_1+p_{-k}\Pi_2-(1-\delta)c_{-k}\right]}{\{1-\delta\}\{1-\delta[1-p_k][1-p_{-k}]\}}.$$

In (6), $[1-p_k][1-p_{-k}]$ defines the probability that the dynamic game has a further relevant stage. Since $\delta[1-p_k][1-p_{-k}]<1$ by definition, the expected profits converge. Any strictly mixed strategy in this dynamic game is only sustainable if the expected profits are zero. As the strictly mixed strategy equilibrium probabilities are

$$(7)\quad \forall k\in\{i,j\}:\ p_k^E[c_{-k},\cdot]=\frac{\Pi_1-(1-\delta)c_{-k}}{\Pi_1-\Pi_2}\qquad\Rightarrow\qquad\frac{\partial p_k^E[\cdot]}{\partial c_{-k}}<0$$

in this case, the dynamic game leads to the same qualitative result as the static game.

4. Simulation results

The feature that a firm does only choose an entry probability strictly between zero and one if its rivals make it indifferent between entry and non-entry holds in general. Hence, it is obvious that the basic result carries over on the general case of N firms. However, the general case is not easy to deal with when all firms carry different entry costs because determining equilibrium probabilities is tremendously involved (and existence as well as uniqueness cannot be proven using the standard techniques).[3] Additionally, modelling entry among N firms in a dynamic game must take into account that different states exist in which entry is still an option if the market carries more than one firm. In order to demonstrate that the market entry paradox holds under general conditions as well, this section will present some simulation results for three potential entrants. Furthermore, the simulation results for three potential entrants will reveal that one firm may even stay away from the market although it would carry the least entry costs.

Table 1 shows simulation results for a static entry game.

Table 1: Equilibrium probabilities in a static entry game for $\Pi_1 = 3$, $\Pi_2 = 1$, $\Pi_3 = 0.5$

$\{c_1, c_2, c_3\}$	{1.1, 1.2, 1.3}	{1.1, 1.3, 1.5}	{0.7, 0.8, 0.9}	{0.7, 0.8, 1.1}
p_1	0.2584	0.0553	0.6399	0.5268
p_2	0.3923	0.3545	0.7885	0.6996
p_3	0.5072	0.663	0.8847	0.9476

[3] For generalization of the static game, one may consider the symmetric case for N identical firms in a static game and a marginal change away from symmetry such that a single firm's fixed costs are increased marginally. The identical equilibrium probabilities are

$$\forall i \in \{1,\ldots,N\}: \qquad \sum_{k=0}^{N-1} \binom{N-1}{k} \left[1-p^*\right]^{N-k-1} p^{*k}\, \Pi_{k+1} = c_i .$$

It is well-known that an increase in p leads to first-order stochastic dominance and — as Π decreases with k — a decrease in p must therefore increase the value of the term on the LHS (see e.g. Wolfstetter, 1993). If a certain firm, say i, faces marginally higher entry costs, all other firms must lower their entry probabilities marginally in order to make firm i indifferent between entry and non-entry whereas the equilibrium probability of firm i remains unchanged.

The equilibrium probabilities are determined in the usual fashion by making every firm's expected profits zero. In the case of three potential entrants, each firm's zero expected profit condition depends on the other two firms' entry probabilities. Table 1 demonstrates that the market entry paradox holds because firms enter with probabilities which increase with the entry costs they have to carry. The first and the second simulation show the results for a market which carries only one out of three firms, the third simulation shows the result for a market which carries two out of three firms. The last simulation demonstrates that the market entry paradox holds even in the case that the market carries either only the high-cost firm or the other two firms. In this case, the high-cost firm enters almost with certainty. The efficient solution requires firms 2 and 3 to enter. According to Table 1, the chances for an efficient solution are only 0.5268 * 0.6996 * (1 - 0.9476) = 0.0193, i.e. less than two percent!

For a dynamic game, some preliminaries are necessary. The simulations model the behavior of three firms 1, 2 and 3 for which

(8) $$\Pi_1 > \Pi_2 > (1-\delta)c_3 > (1-\delta)c_2 > (1-\delta)c_1 > \Pi_3$$

holds. (8) specifies that firm 3 is the high-cost firm, firm 2 is the medium-cost firm, and firm 1 is the low-cost firm. Furthermore, (8) ensures that the dynamics of the game are not trivial: when only one firm out of three firms has entered the market, the remaining two potential entrants will randomize over entry again in the next stage. Let the entry probabilities when one firm is already in the market be denoted by q. The superscript of q will denote the firm which is already in the market, and the subscript will denote the firm which chooses the probability. Three cases have to be distinguished for which determination of entry probabilities is straightforward:

- Only firm 1 is already in the market with ex ante probability $p_1(1-p_2)(1-p_3)$:

(9) $$q_2^1 = \frac{\Pi_2 - (1-\delta)c_3}{\Pi_2 - \Pi_3}, \qquad q_3^1 = \frac{\Pi_2 - (1-\delta)c_2}{\Pi_2 - \Pi_3}.$$

- Only firm 2 is already in the market with ex ante probability p2(1-p1)(1-p3):

(10) $q_1^2 = \frac{\Pi_2 - (1-\delta)c_3}{\Pi_2 - \Pi_3}, \qquad q_3^2 = \frac{\Pi_2 - (1-\delta)c_1}{\Pi_2 - \Pi_3}.$

- Only firm 3 is already in the market with ex ante probability p3(1-p2)(1-p3):

(11) $q_1^3 = \frac{\Pi_2 - (1-\delta)c_2}{\Pi_2 - \Pi_3}, \qquad q_2^3 = \frac{\Pi_2 - (1-\delta)c_1}{\Pi_2 - \Pi_3}.$

(9), (10) and (11) result from (7). These three cases and the case that no firm has entered (which has the ex ante probability (1-p1)(1-p2)(1-p3)) imply a further relevant stage in the dynamic game. The expected profits of firm 1 are

(12) $$p_1p_2p_3\Pi_3 + p_1[p_2(1-p_3) + p_3(1-p_2)]\Pi_2 + p_1(1-p_2)(1-p_3)\Pi_1 - p_1c_1$$

$$+\delta(1-p_1)(1-p_2)(1-p_3)$$
$$\{p_1p_2p_3\Pi_3 + p_1[p_2(1-p_3) + p_3(1-p_2)]\Pi_2 + p_1(1-p_2)(1-p_3)\Pi_1\}$$

$$+\delta p_1(1-p_2)(1-p_3)$$
$$\{(1-q_2^1)(1-q_3^1)\Pi_1 + [(1-q_2^1)q_3^1 + (1-q_3^1)q_2^1]\Pi_2 + q_2^1q_3^1\Pi_3\}$$

$$+\delta(1-p_1)p_2(1-p_3)[q_1^2(1-q_3^2)\Pi_2 + q_1^2q_3^2\Pi_3 - q_1^2c_1]$$

$$+\delta(1-p_1)p_3(1-p_2)[q_1^3(1-q_2^3)\Pi_2 + q_1^3q_2^3\Pi_3 - q_1^3c_1]$$

$$+\delta^2(1-p_1)^2(1-p_2)^2(1-p_3)^2$$
$$\{p_1p_2p_3\Pi_3 + p_1[p_2(1-p_3) + p_3(1-p_2)]\Pi_2 + p_1(1-p_2)(1-p_3)\Pi_1\}$$

$$+\delta^2p_1(1-p_2)(1-p_3)(1-q_2^1)(1-q_3^1)$$
$$\{(1-q_2^1)(1-q_3^1)\Pi_1 + [(1-q_2^1)q_3^1 + (1-q_3^1)q_2^1]\Pi_2 + q_2^1q_3^1\Pi_3\}$$

$$+\ldots..$$

The first line of (12) mirrors the events which do not imply a further relevant stage of the dynamic game. The second and the third line mirror the case that no firm has entered in the previous stage (and the game is repeated). The fourth and the fifth line mirror the case that only firm 1 has entered, and firms 2 and 3 randomize over entry in the next stage. The sixth (seventh) line mirrors the case that firm 1 has not yet entered and only firm 2 (3) is in the market. As the equilibrium probabilities between the remaining firms are determined by making expected profits zero, both the sixth and the seventh line are zero by definition (and have not to be considered any more). Hence, the expected profits hinge only on the chances that no firm has entered and that only the firm under consideration has already entered. The eighth and the ninth line mirror another round of no entry before, and the last lines mirror the case that firm 1 has entered in the first stage, but no further firm has entered in the second stage such that firms 2 and 3 randomize again in the third stage.

Since the sums converge, the expected profits of firm 1 are

$$(13)\quad \frac{p_1p_2p_3\Pi_3 + p_1[p_2(1-p_3)+p_3(1-p_2)]\Pi_2 + p_1(1-p_2)(1-p_3)\Pi_1}{1-\delta(1-p_1)(1-p_2)(1-p_3)}$$

$$+\frac{\delta p_1(1-p_2)(1-p_3)}{1-\delta(1-q_2^1)(1-q_3^1)}$$

$$\left\{(1-q_2^1)(1-q_3^1)\Pi_1 + \left[(1-q_2^1)q_3^1 + (1-q_3^1)q_2^1\right]\Pi_2 + q_2^1q_3^1\Pi_3\right\}$$

$$-p_1c_1 = 0$$

which should be zero in a strictly mixed strategy equilibrium. Note that the zero sign is also given if firm 1 sets p_1 zero. In equilibrium, no entry cannot be the strategy for each firm. But a single firm may choose no entry when it is not able to drive the expected profits of its rivals down to zero because their entry costs are high. In this case, (13) and the respective conditions for firm 2 and firm 3 imply no strictly positive solution for p_1, p_2 and p_3. If a certain firm stays away, the remaining firms randomize, and the equilibrium probabilities are given by (9), (10) or (11), respectively. In fact, the simulation results lend support to such an outcome, and the fact that the low-cost firm opts for no entry reveals a substantial inefficiency result in dynamic games.

Table 2: Equilibrium probabilities in a dynamic entry game for $\Pi_1 = 3$, $\Pi_2 = 1$, $\Pi_3 = 0.5$ and $\delta = 0.9$

$\{c_1, c_2, c_3\}$	{5.5, 6, 6.5}	{5.5, 6, 7}	{5.5, 6, 6}	{5.5, 6.5, 6.5}
p_1	0.0186	0	0.0664	0
p_2	0.1481	0.6	0.1904	0.7
p_3	0.2647	0.8	0.1961	0.7

Table 2 gives simulation results in which the low-cost firm carries only entry costs of 5.5 which exceed the discounted sum of profits when all enter only by 0.5. However, in two cases the low-cost firm enters with an extremely low probability, and in the other two cases, it does not enter at all. If it does not enter at all, the remaining two firms play a duopolistic entry game the equilibrium probabilities of which can be determined by applying (7). In all cases, the high-cost firm enters with the highest probability.

5. Concluding remark

This paper has shown that the chances for high-cost firms to enter the market conflict with efficiency requirements. Although ex ante expected profits are zero for all potential entrants, ex post efficiency requires that the entry probability of low-cost firms does not fall short of the probability of high-cost firms. The market entry paradox has shown that exactly the opposite is true. This result raises the question whether there is significant empirical evidence that the high-cost firms are more likely to enter a market than the low-cost firms. As potential entry costs are hardly observable for potential entrants which have not entered the market, however, this question is likely to remain open.

But even if there were no empirical evidence, this result would only suggest that entry decisions must not be modelled by fixed entry costs. Another strand of the industrial organization literature has endogenized the entry costs by making entry dependent on the success of innovating investment. However, the results are also not encouraging from the viewpoint of efficiency because it is unknown in general whether such investment is too little or excessive. One example are patent races in which two different effects are observable: on the one hand, the competitive threat that one of the competitors could win the race induces a firm to increase its investment, on the other hand, the profit incentive drives its investment down if imitation or inventing-around is very easy (Beath, Katsoulacos and Ulph, 1989). Hence, it seems that equilibria of entry games are not likely to belong to the realm of efficient laissez faire outcomes.

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