Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model

Stephanie Schmitt-Grohé
Duke University, Centre for Economic Policy Research, and National Bureau of Economic Research

Martín Uribe
Duke University and National Bureau of Economic Research

What is the optimal monetary policy, and how can the central bank implement it? Both questions have been extensively studied, but always in the context of simple theoretical structures, which by design are limited in their ability to account for actual observed business cycle fluctuations. This article seeks to characterize optimal monetary policy and its implementation using a medium-scale, empirically plausible model of the U.S. business cycle.

The model we consider is the one developed in Altig and others (2005). This model has been estimated econometrically and shown to account fairly well for business cycle fluctuations in the postwar United States. The theoretical framework emphasizes the importance of combining nominal as well as real rigidities in explaining the propagation of macroeconomic shocks. Specifically, the model features four nominal frictions (sticky prices, sticky wages, a transactional demand for money by households, and a cash-in-advance constraint on the wage bill of firms) and four sources of real rigidities (investment adjustment costs, variable capacity utilization, habit formation, and imperfect competition in product and factor markets). Aggregate

We thank Jan Marc Berk, Juan Pablo Medina, Frederic Mishkin, and Klaus Schmidt-Hebbel; seminar participants at the Central Bank of Chile, the National Bureau of Economic Research (NBER) Summer Institute, the Atlanta Federal Reserve Bank, and the European Central Bank for comments—and Anna Kozlovskaya for research assistance.

Monetary Policy under Inflation Targeting, edited by Frederic Mishkin and Klaus Schmidt-Hebbel, Santiago, Chile.

fluctuations are driven by three shocks: a permanent neutral productivity shock, a permanent investment-specific technology shock, and temporary variations in government spending. Altig and others (2005) and Christiano, Eichenbaum, and Evans (2005) argue that the model economy for which we seek to design optimal monetary policy can indeed explain the observed responses of inflation, real wages, nominal interest rates, money growth, output, investment, consumption, labor productivity, and real profits to neutral and investment-specific productivity shocks and monetary shocks in the postwar United States.

In our characterization of optimal monetary policy, we depart from the widespread practice in the neo-Keynesian literature on optimal monetary policy of limiting attention to models in which the nonstochastic steady state is undistorted. This approach usually involves assuming the existence of a battery of subsidies to production and employment aimed at eliminating the long-run distortions originating from monopolistic competition in factor and product markets. The efficiency of the deterministic steady-state allocation is assumed for purely computational reasons, as it allows the use of first-order approximation techniques to evaluate welfare accurately up to second order (see Rotemberg and Woodford, 1997).

This practice has two potential shortcomings. First, the instruments necessary to bring about an undistorted steady state (for example, labor and output subsidies financed by lump-sum taxation) are empirically uncompelling. Second, it is not clear ex ante whether a policy that is optimal for an economy with an efficient steady state will also be optimal for an economy in which the instruments necessary to engineer the nondistorted steady state are unavailable. For these reasons we refrain from making the efficient steady-state assumption and instead work with a model whose steady state is distorted.

Departing from a model whose steady state is Pareto efficient has a number of important ramifications. One is that to obtain an accurate second-order measure of welfare it no longer suffices to approximate the equilibrium of the model up to first order. We solve the equilibrium of the model up to second order using the methodology and computer code developed in Schmitt-Grohé and Uribe (2004d) for second-order accurate approximations to policy functions of dynamic, stochastic models. One advantage of this numerical strategy is that because it is based on perturbation arguments, it is particularly well suited to handle economies with a large number of state variables, such as the one studied here.

We address the question of what business cycle fluctuations should look like under optimal monetary policy by characterizing the Ramsey equilibrium associated with our model. The central policy problem faced by the monetary authority is the need to stabilize prices in order to minimize price dispersion stemming from nominal rigidities while minimizing and stabilizing the opportunity cost of holding money to avoid transactional frictions. The task of characterizing Ramsey-optimal policy is challenging, because the model is large and highly distorted. A methodological contribution of the research project to which this article belongs is the development of computational procedures to derive and characterize the Ramsey equilibrium for a general class of dynamic rational expectations models.[1]

We find that the policy tradeoff faced by the Ramsey planner is resolved in favor of price stability. In effect, the Ramsey-optimal inflation rate is -0.4 percent a year, with a standard deviation of only 0.1 percentage points. The optimality of near-zero inflation, however, is highly sensitive to the assumed degree of price stickiness. Estimates of the degree of price stickiness vary between two and five quarters. Within this range the optimal rate of inflation increases from a deflation of about 4 percent a year when prices are reoptimized every two quarters to a mild deflation of less than 0.5 percent a year when prices are reoptimized every five quarters. Depending on what estimate of price rigidity one chooses, the Ramsey-optimal policy can range from close to the Friedman rule to close to price stability.

Quite independent of the precise degree of price stickiness, the optimal inflation target is below zero. In light of this robust result, it is puzzling that all countries that self-classify as inflation targeters set inflation targets that are positive. Annual inflation targets in developed countries range from 2 percent to 4 percent. Somewhat higher targets are observed in developing countries. An argument often raised in defense of positive inflation targets is that negative inflation targets imply nominal interest rates that are dangerously close to the zero lower bound and hence may impair the central bank's ability to conduct stabilization policy. We find, however, that this argument is of no relevance in the context of the medium-scale estimated model within which we evaluate policy. The reason is that under the optimal policy regime, the mean of the nominal interest rate is about 4.5 percent a year, with a standard deviation of only 0.4

1. The Matlab code to replicate the quantitative results reported in this article is available at www.econ.duke.edu/~uribe.

percent. This means that for the zero lower bound to pose an obstacle to monetary policy, the economy must suffer an adverse shock that forces the interest rate to be more than 10 standard deviations below target. The likelihood of such an event is practically nil.

We address the question of implementation of optimal monetary policy by characterizing optimal, simple, and implementable interest rate feedback rules. We restrict attention to what we call operational interest rate rules, by which we mean an interest rate rule that satisfies three requirements. First, the operational rule prescribes that the nominal interest rate be set as a function of a few readily observable macroeconomic variables. In the tradition of Taylor (1993), we focus on rules whereby the nominal interest rate depends on measures of inflation, aggregate activity, and possibly its own lag. Second, the operational rule must induce an equilibrium satisfying the zero lower bound on nominal interest rates. Third, the operational rule must render the rational expectations equilibrium unique. This restriction closes the door to expectations-driven aggregate fluctuations.

Our numerical findings suggest that in the model economy we study, the optimal operational interest rate rule responds aggressively to deviations of price and wage inflation from the target. The price-inflation coefficient is about 5 and the wage-inflation coefficient about 2. In addition, the optimal interest rate rule prescribes a mute response to deviations of output growth from target. In this sense the implementation of optimal policy calls for following a regime of inflation targeting. The parameters of the optimized rule are robust to using a conditional or unconditional measure of welfare.

Remarkably, the optimal operational interest rate rule delivers a welfare level that is virtually identical to the one obtained under the Ramsey-optimal policy. Specifically, the welfare cost associated with living in an economy in which the monetary authority follows the optimal operational rule as opposed to living in the Ramsey economy is only 23 cents a year per person (or 0.001 percent of 2006 annual per capita consumption).

The remainder of the article is organized in five sections. The next section presents the theoretical economy and derives nonlinear recursive representations for the price and wage Phillips curves as well as for the state variables summarizing the degree of wage and price dispersion. Section 2 describes the calibration of the model and discusses the solution method. Section 3 characterizes the steady state of the Ramsey equilibrium. Section 4 examines the dynamics induced by the Ramsey monetary policy. Section 5 computes the optimal operational interest rate rule. The last section provides concluding remarks.

1. The Model

The skeleton of the model economy used here for policy evaluation is a standard neoclassical growth model driven by neutral productivity shocks, investment-specific productivity shocks, and government spending shocks. The economy features four sources of nominal frictions and five real rigidities. The nominal frictions include price and wage stickiness à la Calvo (1983) and Yun (1996) with indexation to past inflation and money demands by households and firms. The real rigidities originate from internal habit formation in consumption, monopolistic competition in factor and product markets, investment adjustment costs, and the variable costs of adjusting capacity utilization.

To evaluate monetary policy, we are forced to approximate the equilibrium conditions of the economy to an order higher than linear. Toward that end we derive the exact nonlinear recursive representation of the complete set of equilibrium conditions. Of particular interest is the recursive nonlinear representation of the equilibrium Phillips curves for prices and wages. These representations depart from most of the existing literature, which restricts attention to linear approximations to these functions. Another byproduct of deriving the exact nonlinear set of equilibrium conditions is the emergence of two state variables measuring the degree of price and wage dispersion in the economy induced by the sluggishness in the adjustment of nominal product and factor prices. We present a recursive representation of these state variables and track their dynamic behavior.

1.1 Households

The economy is assumed to be populated by a large representative family with a continuum of members. Consumption and hours worked are identical across family members. The household's preferences are defined over per capita consumption, c_t, and per capita labor effort, h_t. They are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - bc_{t-1}, h_t), \tag{1}$$

where E_t denotes the mathematical expectations operator conditional on information available at time, t, $\beta \in (0,1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing in its first argument, strictly decreasing in its second argument, and strictly concave. Preferences display internal habit formation, measured by the parameter $b \in [0,1]$. The consumption good is assumed to be a composite made up of a continuum of differentiated goods c_{it} indexed by $i \in [0,1]$ via the aggregator

$$c_t = \left[\int_0^1 c_{i,t}^{\ 1-1/\eta} di\right]^{1/(1-1/\eta)}, \tag{2}$$

where the parameter $\eta > 1$ denotes the intratemporal elasticity of substitution across different varieties of consumption goods.

For any level of consumption of the composite good, purchases of each individual variety of goods $i \in [0,1]$ in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{i,t} c_{i,t} di$, subject to the aggregation constraint (2), where $P_{i,t}$ denotes the nominal price of a good of variety i at time t. The demand for goods of variety i is then given by

$$c_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} c_t, \tag{3}$$

where P_t is a nominal price index defined as

$$P_t \equiv \left[\int_0^1 P_{i,t}^{1-\eta} di\right]^{1/1-\eta}. \tag{4}$$

This price index has the property that the minimum cost of a bundle of intermediate goods yielding c_t units of the composite good is given by $P_t c_t$.

Labor decisions are made by a central authority within the household (a union), which supplies labor monopolistically to a continuum of labor markets of measure 1 indexed by $j \in [0,1]$. In each labor market j, the union faces a demand for labor given by $(W_t^j / W_t)^{-\tilde{\eta}} h_t^d$, where W_t^j denotes the nominal wage charged by the union in labor market j at time t, W_t is an index of nominal wages prevailing in the economy, and h_t^d is a measure of aggregate labor demand by firms. We postpone a formal derivation of this labor demand function until we consider the firm's problem. In each labor market, the union takes W_t and h_t^d as exogenous.[2] Given the wage it charges in each labor market $j \in [0,1]$, the union is assumed to supply enough labor, h_t^j, to satisfy demand. That is,

$$h_t^j = \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d, \tag{5}$$

where $w_t^j \equiv W_t^j/P_t$ and $w_t \equiv W_t/P_t$. In addition, the total number of hours allocated to the different labor markets must satisfy the resource constraint

$$h_t = \int_0^1 h_t^j dj.$$

Combining this restriction with equation (5), we obtain

$$h_t = h_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} dj. \tag{6}$$

Our set-up of imperfectly competitive labor markets departs from most expositions of models with nominal wage inertia (for example, Erceg, Henderson, and Levin, 2000). These models assume that each household supplies a differentiated type of labor input. This assumption introduces equilibrium heterogeneity across households in the number of hours worked. To avoid this heterogeneity from spilling over into consumption heterogeneity, it is typically assumed that preferences are separable in consumption and hours and that financial markets exist that allow agents to fully insure against employment risk. Our formulation avoids the need to assume both separability of preferences in leisure and consumption and the existence of such insurance markets. As we explain below in more detail, our specification gives rise to a wage-inflation Phillips curve with a larger coefficient on the wage mark-up gap than the model with employment heterogeneity across households.

The household is assumed to own physical capital, k_t, which accumulates according to the following law of motion

$$k_{t+1} = (1-\delta)k_t + i_t\left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right], \tag{7}$$

2. The case in which the union takes aggregate labor variables as endogenous can be interpreted as an environment with highly centralized labor unions. Higher-level labor organizations play an important role in some European and Latin American countries. They are less prominent in the United States.

where i_t denotes gross investment and δ is a parameter denoting the rate of depreciation of physical capital. The function S introduces investment adjustment costs. It is assumed that in the steady state, the function S satisfies $S = S' = 0$ and $S'' > 0$. These assumptions imply the absence of adjustment costs up to first-order in the vicinity of the deterministic steady state.

Like Fisher (2005) and Altig and others (2005), we assume that investment is subject to permanent investment-specific productivity shocks. Fisher argues that this type of shock is needed to explain the observed secular decline in the relative price of investment goods in terms of consumption goods. More important, he shows that investment-specific technology shocks account for about 50 percent of aggregate fluctuations at business cycle frequencies in the postwar U.S. economy. (As we discuss below, Altig and others 2005 find smaller numbers in the context of the model studied here.)

We assume that investment goods are produced from consumption goods by means of a linear technology whereby $1/\Upsilon_t$ units of consumption goods yield one unit of investment goods, where Υ_t denotes an exogenous, permanent technology shock in period t. The growth rate of Υ_t is assumed to follow an AR(1) process of the form $\hat{\mu}_{\Upsilon,t} = \rho_{\mu_\Upsilon}\hat{\mu}_{\Upsilon,t-1} + \varepsilon_{\mu_{\Upsilon,t}}$, where $\hat{\mu}_{\Upsilon,t} \equiv \ln(\mu_{\Upsilon,t}/\mu_\Upsilon)$ denotes the percentage deviation of the gross growth rate of investment specific technological change and μ_Υ denotes the steady-state growth rate of Υ_t.

Owners of physical capital can control the intensity at which this factor is used. Formally, we let u_t measure capacity utilization in period t. We assume that using the stock of capital with intensity u_t entails a cost of $\Upsilon_t^{-1}a(u_t)k_t$ units of the composite final good. The function a is assumed to satisfy $a(1) = 0$, and $a'(1)$, $a''(1) > 0$.

The specification of both capital adjustment and capacity utilization costs are somewhat peculiar. More standard formulations assume that adjustment costs depend on the level of investment rather than its growth rate, as is assumed here. The costs of capacity utilization typically take the form of a higher rate of depreciation of physical capital. The modeling choice here is guided by the need to fit the response of investment and capacity utilization to a monetary shock in the U.S. economy. (For further discussion of this issue, see Christiano, Eichenbaum, and Evans, 2005 and Altig and others, 2005.)

Households rent the capital stock to firms at the real rental rate r_t^k per unit of capital. Total income stemming from the rental of capital is given by $r_t^k u_t k_t$. The investment good is assumed to be a composite good made with the aggregator function shown in equation (2). Thus the demand for each intermediate good $i \in [0,1]$ for investment purposes, $i_{i,t}$, is given by $i_{i,t} = \Upsilon_t^{-1} i_t \,(P_{i,t}/P_t)^{-\eta}$.

As in our earlier work (Schmitt-Grohé and Uribe 2004a,b), we motivate a demand for money by households by assuming that purchases of consumption goods are subject to a proportional transactions cost that is increasing in consumption-based money velocity. Formally, the purchase of each unit of consumption entails a cost given by $\ell(v_t)$. The ratio of consumption to real money balances held by the household, which we denote by m_t^h, is given by

$$v_t \equiv \frac{c_t}{m_t^h}. \tag{8}$$

The transactions cost function ℓ satisfies the following assumptions: $\ell(v)$ is nonnegative and twice continuously differentiable; there exists a level of velocity $\underline{v} > 0$, which we refer to as the satiation level of money, such that $\ell(\underline{v}) = \ell'(\underline{v}) = 0$; $(v - \underline{v})\ell'(v) > 0$ for $v \neq \underline{v}$; and $2\ell'(v) + v\ell''(v) > 0$ for all $v \geq \underline{v}$. The first assumption implies that the transaction process does not generate resources. The second assumption ensures that the Friedman rule (that is, a zero nominal interest rate) need not be associated with an infinite demand for money. It also implies that both the transactions cost and the associated distortions in the intra- and intertemporal allocation of consumption and leisure vanish when the nominal interest rate is zero. The third assumption guarantees that in equilibrium money velocity is always greater than or equal to the satiation level $\underline{v}$. As will become clear shortly, the fourth assumption ensures that the demand for money is decreasing in the nominal interest rate. This assumption is weaker than the more common assumption of strict convexity of the transactions cost function.

Households are assumed to have access to a complete set of nominal state-contingent assets. Specifically, each period $t \geq 0$, consumers can purchase any desired state-contingent nominal payment X_{t+1}^h in period $t + 1$ at the dollar cost $E_t r_{t,t+1} X_{t+1}^h$. The variable $r_{t,t+1}$ denotes a stochastic nominal discount factor between periods t and $t + 1$. Households pay real lump-sum taxes in the amount τ_t per period. The household's period-by-period budget constraint is given by

$$E_t r_{t,t+1} x_{t+1}^h + c_t[1 + \ell(v_t)] + \Upsilon_t^{-1}[i_t + a(u_t)k_t] + m_t^h + \tau_t = \frac{x_t^h + m_{t-1}^h}{\pi_t} + r_t^k u_t k_t + \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d \, dj + \phi_t. \tag{9}$$

The variable $x_t^h/\pi_t \equiv X_t^h/P_t$ denotes the real payoff in period t of nominal state-contingent assets purchased in period $t-1$. The variable ϕ_t denotes dividends received from the ownership of firms; $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of consumer price inflation.

We introduce wage stickiness in the model by assuming that each period the household (or unions) cannot set the nominal wage optimally in a fraction $\tilde{\alpha} \in [0,1)$ of randomly chosen labor markets. In these markets the wage rate is indexed to average real wage growth and to the previous period's consumer price inflation according to the rule

$$W_t^j = W_{t-1}^j(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}},$$

where $\tilde{\chi} \in [0,1]$ is a parameter measuring the degree of wage indexation. When $\tilde{\chi}$ equals 0, there is no wage indexation. When $\tilde{\chi}$ equals 1, there is full wage indexation to long-run real wage growth and past consumer price inflation.

The household chooses processes for c_t, h_t, x_{t+1}^h, w_t^j, k_{t+1}, i_t, u_t, and m_t^h so as to maximize the utility function (1) subject to expressions (6)–(9), the wage stickiness friction, and a no Ponzi game constraint, taking as given the processes $w_t, r_t^k, h_t^d, r_{t,t+1}$, π_t, ϕ_t, and τ_t and the initial conditions x_0^h, k_0, and m_{-1}^h. The household's optimal plan must satisfy constraints (6)–(9). In addition, letting, $\beta^t\lambda_t w_t\tilde{\mu}_t$, $\beta^t\lambda_t q_t$, and $\beta^t\lambda_t$ denote the Lagrange multipliers associated with constraints (6), (7), and (9), respectively, the Lagrangian associated with the household's optimization problem is

$$\begin{aligned}
L = E_0\sum_{t=0}^{\infty}\beta^t \Bigg\langle & U(c_t - bc_{t-1}, h_t) \\
& + \lambda_t\left\{h_t^d\int_0^1 w_t^i\left(\frac{w_t^i}{w_t}\right)^{-\tilde{\eta}} di + r_t^k u_t k_t + \phi_t - \tau_t\right. \\
& \left. - c_t\left[1+\ell\left(\frac{c_t}{m_t^h}\right)\right] - \Upsilon_t^{-1}[i_t + a(u_t)k_t] - r_{t,t+1}x_{t+1}^h - m_t^h + \frac{m_{t-1}^h + x_t^h}{\pi_t}\right\} \\
& + \frac{\lambda_t w_t}{\tilde{\mu}_t}\left[h_t - h_t^d\int_0^1\left(\frac{w_t^i}{w_t}\right)^{-\tilde{\eta}} di\right] \\
& + \lambda_t q_t\left\{(1-\delta)k_t + i_t\left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right] - k_{t+1}\right\}\Bigg\rangle.
\end{aligned}$$

The first-order conditions with respect to c_t, x^h_{t+1}, h_t, k_{t+1}, i_t, m^h_t, u_t, and w^i_t, in that order, are given by

$$U_c(c_t - bc_{t-1}, h_t) - b\beta E_t U_c(c_{t+1} - bc_t, h_{t+1}) = \lambda_t[1 + \ell(v_t) + v_t \ell'(v_t)], \tag{10}$$

$$\lambda_t r_{t,t+1} = \beta\lambda_{t+1}\frac{P_t}{P_{t+1}}, \tag{11}$$

$$-U_h(c_t - bc_{t-1}, h_t) = \frac{\lambda_t w_t}{\tilde{\mu}_t}, \tag{12}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1}\left[r^k_{t+1}u_{t+1} - \Upsilon^{-1}_{t+1}a(u_{t+1}) + q_{t+1}(1-\delta)\right], \tag{13}$$

$$\begin{aligned}\Upsilon_t^{-1}\lambda_t = \lambda_t q_t &\left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right)S'\left(\frac{i_t}{i_{t-1}}\right)\right] \\ &+\beta E_t \lambda_{t+1} q_{t+1}\left(\frac{i_{t+1}}{i_t}\right)^2 S'\left(\frac{i_{t+1}}{i_t}\right),\end{aligned} \tag{14}$$

$$v_t^2 \ell'(v_t) = 1 - \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}}, \tag{15}$$

$$r^k_t = \Upsilon^{-1}_t a'(u_t), \text{ and} \tag{16}$$

$$w^i_t = \begin{cases} \tilde{w}_t & \text{if } w^i_t \text{ is set optimally in } t \\ w^i_{t-1}(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}/\pi_t & \text{otherwise} \end{cases}, \tag{17}$$

where $\tilde{w}_t$ denotes the real wage prevailing in the $1-\tilde{\alpha}$ labor markets in which the union can set wages optimally in period t. Let $\tilde{h}_t$ denote the level of labor effort supplied to those markets. Because the labor demand curve faced by the union is identical across all labor markets and the cost of supplying labor is the same for all markets, one can assume that wage rates, $\tilde{w}_t$, and employment, $\tilde{h}_t$, are identical across all labor markets updating wages in a given period. By equation (5), $\tilde{w}_t^{\tilde{\eta}}\tilde{h}_t = w_t^{\tilde{\eta}} h^d_t$.

It is useful to track the evolution of real wages in a particular labor market. In any labor market j where the wage is set optimally in period t, the real wage in that period is $\tilde{w}_t$. If in period $t+1$ wages are not reoptimized in that market, the real wage is $\tilde{w}_t(\mu_z * \pi_t)^{\tilde{\chi}}/\pi_{t+1}$. This is because the nominal wage is indexed by $\tilde{\chi}$ percent of the sum of past price inflation and long-run real wage growth. In general, s periods after the last reoptimization, the real wage is $\tilde{w}_t \prod_{k=1}^{s}(\mu_z * \pi_{t+k-1})^{\tilde{\chi}} / \pi_{t+k}$. To derive the household's first-order condition with respect to the wage rate in those markets where the wage rate is set optimally in the current period, it is convenient to reproduce the parts of the Lagrangian given above that are relevant for this purpose,

$$L^w = E_t \sum_{s=0}^{\infty} (\tilde{\alpha}\beta)^s \lambda_{t+s} h_{t+s}^d w_{t+s}^{\tilde{\eta}} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{(\mu_{z^*} \pi_{t+k-1})^{\tilde{\chi}}} \right)^{\tilde{\eta}}$$

$$\times \left[\tilde{w}_t^{1-\tilde{\eta}} \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{(\mu_{z^*} \pi_{t+k-1})^{\tilde{\chi}}} \right)^{-1} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}} \tilde{w}_t^{-\tilde{\eta}} \right].$$

The first-order condition with respect to $\tilde{w}_t$ is

$$0 = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} w_{t+s}^{\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{(\mu_{z^*} \pi_{t+k-1})^{\tilde{\chi}}} \right)^{\tilde{\eta}}$$

$$\times \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s} \pi_{t+k} / (\mu_{z^*} \pi_{t+k-1})^{\tilde{\chi}}} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}} \right].$$

Using equation (12) to eliminate $\tilde{\mu}_{t+s}$, we obtain that the real wage $\tilde{w}_t$ must satisfy

$$0 = E_t \sum_{s=0}^{\infty} (\beta\tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}} \right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{(\mu_{z^*} \pi_{t+k-1})^{\tilde{\chi}}} \right)^{\tilde{\eta}}$$

$$\times \left[\frac{\tilde{\eta}-1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s} \left(\pi_{t+k} / (\mu_{z^*} \pi_{t+k-1})^{\tilde{\chi}} \right)} - \frac{-U_{ht+s}}{\lambda_{t+s}} \right].$$

This expression states that in labor markets in which the wage rate is reoptimized in period t, the real wage is set so as to equate the union's future expected average marginal revenue with the average marginal cost of supplying labor. The union's marginal revenue s periods after its last wage reoptimization is given by $\left[(\tilde{\eta}-1)/\tilde{\eta}\right]\tilde{w}_t \prod_{k=1}^{s}(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}/\pi_{t+k}$. Here $\tilde{\eta}/(\tilde{\eta}-1)$ represents the mark-up of wages over the marginal cost of labor that would prevail in the absence of wage stickiness. The factor $\prod_{k=1}^{s}(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}/\pi_{t+k}$ in the expression for marginal revenue reflects the fact that as time goes by without a chance to reoptimize, the real wage declines as the price level increases when wages are imperfectly indexed. In turn, the marginal cost of supplying labor is given by the marginal rate of substitution between consumption and leisure, $-U_{ht+s}/\lambda_{t+s} = w_{t+s}/\tilde{\mu}_{t+s}$. The variable $\tilde{\mu}_t$ is a wedge between the disutility of labor and the average real wage prevailing in the economy. Thus $\tilde{\mu}_t$ can be interpreted as the average mark-up that unions impose on the labor market. The weights used to compute the average difference between marginal revenue and marginal cost are decreasing in time and increasing in the amount of labor supplied to the market.

In order to write the wage-setting equation in recursive form, we define

$$f_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t E_t \sum_{s=0}^{\infty}(\beta\tilde{\alpha})^s \lambda_{t+s}\left(\frac{w_{t+s}}{\tilde{w}_t}\right)^{\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}, \text{ and}$$

$$f_t^2 = -\tilde{w}_t^{-\tilde{\eta}} E_t \sum_{s=0}^{\infty}(\beta\tilde{\alpha})^s w_{t+s}^{\tilde{\eta}} h_{t+s}^d U_{ht+s} \prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{(\mu_{z^*}\pi_{t+k-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}}.$$

We can express f_t^1 and f_t^2 recursively as

$$f_t^1 = \left(\frac{\tilde{\eta}-1}{\tilde{\eta}}\right)\tilde{w}_t \lambda_t \left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}-1}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}-1} f_{t+1}^1, \tag{18}$$

$$f_t^2 = -U_{ht}\left(\frac{w_t}{\tilde{w}_t}\right)^{\tilde{\eta}} h_t^d + \tilde{\alpha}\beta E_t \left(\frac{\pi_{t+1}}{(\mu_{z^*}\pi_t)^{\tilde{\chi}}}\right)^{\tilde{\eta}}\left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t}\right)^{\tilde{\eta}} f_{t+1}^2. \tag{19}$$

The wage-setting equation then becomes

$$f_t^1 = f_t^2. \tag{20}$$

The household's optimality conditions imply a liquidity preference function featuring a negative relation between real balances and the short-term nominal interest rate. To see this, first note that the absence of arbitrage opportunities in financial markets requires that the gross risk-free nominal interest rate, denoted by R_t, be equal to the reciprocal of the price in period t of a nominal security that pays one unit of currency in every state of period $t + 1$. Formally, $R_t = 1/E_t r_{t,t+1}$. This relation, together with the household's optimality condition (11), implies that

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{21}$$

which is a standard Euler equation for pricing nominally risk-free assets. Combining this expression with equations (10) and (15), we obtain

$$v_t^2 \ell'(v_t) = 1 - \frac{1}{R_t}.$$

The right-hand side of this expression represents the opportunity cost of holding money, which is an increasing function of the nominal interest rate. Given the assumptions regarding the form of the transactions cost function ℓ, the left-hand side is increasing in money velocity. Thus this expression defines a liquidity preference function that is decreasing in the nominal interest rate and unit elastic in consumption.

1.2 Firms

Each variety of final goods is produced by a single firm in a monopolistically competitive environment. Each firm $i \in [0,1]$ produces output using as factor inputs capital services, $k_{i,t}$, and labor services, $h_{i,t}$. The production technology is given by $F(k_{i,t}, z_t h_{i,t}) - \psi z_t^*$, where the function F is assumed to be homogenous of degree one, concave, and strictly increasing in both arguments. The variable z_t denotes an aggregate, exogenous, and stochastic neutral productivity shock. The parameter $\psi > 0$ introduces fixed costs of operating a firm in each period. In turn, the presence of fixed costs implies that the production function exhibits increasing returns to scale. We model fixed costs to ensure a realistic profit to output ratio in steady state. Finally, we follow Altig and others (2005) and assume that fixed costs are subject to permanent shocks, z_t^*, with $z_t^* / z_t = \Upsilon_t^{\theta/1-\theta}$. This formulation of fixed costs ensures that along the balanced growth path, fixed costs do not vanish. Let $\mu_{z,t} \equiv z_t / z_{t-1}$ denote the gross growth rate of the neutral technology shock. By assumption, in the nonstochastic steady state, $\mu_{z,t}$ is constant and equal to μ_z. Let $\hat{\mu}_{z,t} = \ln(\mu_{z,t} / \mu_z)$ denote the percentage deviation of the growth rate of neutral technology shocks. Then the evolution of $\mu_{z,t}$ is assumed to be given by $\hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \varepsilon_{\mu_{z,t}}$, with $\varepsilon_{\mu_{z,t}} \sim (0, \sigma^2_{\mu_z})$. Aggregate demand for good i, which we denote by $y_{i,t}$, is given by $y_{i,t} = (P_{i,t}/P_t)^{-\eta} y_t$, where

$$y_t \equiv c_t[1 + \ell(v_t)] + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t], \tag{22}$$

denotes aggregate absorption. The variable g_t denotes government consumption of the composite good in period t.

We rationalize a demand for money by firms by imposing that wage payments be subject to a working capital requirement that takes the form of a cash-in-advance constraint. Formally, we impose

$$m^f_{i,t} = \nu w_t h_{i,t}, \tag{23}$$

where $m^f_{i,t}$ denotes the demand for real money balances by firm i in period t and $\upsilon \geq 0$ is a parameter indicating the fraction of the wage bill that must be backed with monetary assets.

Firms incur financial costs in the amount $(1 - R_t^{-1}) m^f_{i,t}$, stemming from the need to hold money to satisfy the working capital constraint. Letting the variable $\phi_{i,t}$ denote real distributed profits, the period-by-period budget constraint of firm i can then be written

$$E_t r_{t,t+1} x^f_{i,t+1} + m^f_{i,t} - \frac{x^f_{i,t} + m^f_{i,t-1}}{\pi_t} = \left(\frac{P_{i,t}}{P_t}\right)^{1-\eta} y_t - r_t^k k_{i,t} - w_t h_{i,t} - \phi_{i,t},$$

where $E_t r_{t,t+1} x^f_{i,t+1}$ denotes the total real cost of one-period state-contingent assets that the firm purchases in period t in terms of the composite good.[3] We assume that the firm must satisfy demand at

3. Implicit in this specification of the firm's budget constraint is the assumption that firms rent capital services from a centralized market. This is a common assumption in the related literature (for example, Christiano and others, 2005; Kollmann, 2003; Carlstrom and Fuerst, 2005; and Rotemberg and Woodford, 1992). A polar assumption is that capital is firm specific, as in Woodford (2003) and Sveen and Weinke (2003). Both assumptions are clearly extreme. A more realistic treatment of investment dynamics would incorporate a mix of firm-specific and homogeneous capital.

the posted price. Formally, we impose

$$F(k_{i,t}, z_t h_{i,t}) - \psi z_t^* \geq \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} y_t. \tag{24}$$

The objective of the firm is to choose contingent plans for $P_{i,t}$, $h_{i,t}$, $k_{i,t}$, $x^f_{i,t+1}$, and $m^f_{i,t}$ so as to maximize the present discounted value of dividend payments, given by $E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \phi_{it+s}$, where $r_{t,t+s} \equiv \prod_{k=1}^{s} r_{t+k-1,t+k}$, for $s \geq 1$, denotes the stochastic nominal discount factor between t and $t + s$, and $r_{t,t} \equiv 1$. Firms are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games.

Clearly, because $r_{t,t+s}$ represents both the firm's stochastic discount factor and the market pricing kernel for financial assets and the firm's objective function is linear in asset holdings, it follows that any asset accumulation plan of the firm satisfying the no Ponzi game constraint is optimal. Without loss of generality, suppose that the firm manages its portfolio so that its financial position at the beginning of each period is nil. Formally, assume that $x^f_{i,t+1} + m^f_{i,t} = 0$ at all dates and states. Note that this financial strategy makes $x^f_{i,t+1}$ state noncontingent. In this case distributed dividends take the form

$$\phi_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{1-\eta} y_t - r_t^k k_{i,t} - w_t h_{i,t} - (1 - R_t^{-1}) m^f_{i,t}. \tag{25}$$

For this expression to hold in period zero, we impose the initial condition $x^f_{i,0} + m^f_{i-1} = 0$. The last term of the right-hand side of the expression for dividends represents the firm's financial costs associated with the cash-in-advance constraint on wages. This financial cost is increasing in the opportunity cost of holding money, $1 - R_t^{-1}$, which in turn is an increasing function of the short-term nominal interest rate R_t.

Letting $r_{t,t+s} P_{t+s} mc_{i,t+s}$ denote the Lagrange multiplier associated with constraint (24), the first-order conditions of the firm's maximization problem with respect to capital and labor services are

$$mc_{i,t} z_t F_2(k_{i,t}, z_t h_{i,t}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right], \text{ and} \tag{26}$$

$$mc_{i,t} F_1(k_{i,t}, z_t h_{i,t}) = r_t^k. \tag{27}$$

It is clear from these optimality conditions that the presence of a working capital requirement introduces a financial cost of labor that is increasing in the nominal interest rate. Moreover, because all firms face the same factor prices and have access to the same production technology, with the function F being linearly homogeneous, marginal costs, $mc_{i,t}$, are identical across firms. Indeed, because the first-order conditions hold for all firms independently of whether they are allowed to reset prices optimally, marginal costs are identical across all firms in the economy.

Prices are assumed to be sticky à la Calvo (1983) and Yun (1996). Specifically, each period $t \geq 0$, some fraction $\alpha \in [0,1)$ of randomly picked firms are not allowed to optimally set the nominal prices of the good they produce. Instead, these firms index their prices to past inflation according to the rule $P_{i,t} = P_{i,t-1}\pi_{t-1}^{\chi}$. The interpretation of the parameter χ is similar to that of its wage counterpart $\tilde{\chi}$. The remaining $1-\alpha$ firms choose prices optimally. Consider the price-setting problem faced by a firm that has the opportunity to reoptimize the price in period t. This price, denoted by $\tilde{P}_t$, is set so as to maximize the expected present discounted value of profits. That is, $\tilde{P}_t$ maximizes the following Lagrangian:

$$L = E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left\{ \left(\frac{\tilde{P}_t}{P_t} \right)^{1-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}} \right)^{1-\eta} y_{t+s} - r_{t+s}^k k_{i,t+s} \right.$$
$$- w_{t+s} h_{i,t+s} [1 + \nu(1 - R_{t+s}^{-1})]$$
$$\left. + mc_{i,t+s} \left[F(k_{i,t+s}, z_{t+s} h_{it+s}) - \psi z_{t+s}^{*} - \left(\frac{\tilde{P}_t}{P_t} \right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}} \right)^{-\eta} y_{t+s} \right] \right\}.$$

The first-order condition with respect to $\tilde{P}_t$ is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t} \right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}} \right)^{-\eta} y_{t+s}$$
$$\times \left[\frac{\eta - 1}{\eta} \left(\frac{\tilde{P}_t}{P_t} \right) \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}} \right) - mc_{i,t+s} \right] = 0. \tag{28}$$

According to this expression, optimizing firms set nominal prices so as to equate average future expected marginal revenues to average future expected marginal costs. The weights used in calculating these averages are decreasing with time and increasing in the size of the demand for the good produced by the firm. Under flexible prices ($\alpha = 0$), the above optimality condition reduces to a static relation equating marginal costs to marginal revenues period by period.

It is useful to express this first-order condition recursively. To that end, let

$$x_t^1 \equiv E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s y_{t+s} mc_{i,t+s} \left(\frac{\tilde{P}_t}{P_t} \right)^{-\eta-1} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}^{(1+\eta)/\eta}} \right)^{-\eta}$$

and

$$x_t^2 \equiv E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s y_{t+s} \left(\frac{\tilde{P}_t}{P_t} \right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}^{\eta/(\eta-1)}} \right)^{1-\eta}.$$

Express x_t^1 and x_t^2 recursively as

$$x_t^1 = y_t mc_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\eta-1} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}} \right)^{-\eta} x_{t+1}^1, \tag{29}$$

$$x_t^2 = y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\pi_t^{\chi}}{\pi_{t+1}} \right)^{1-\eta} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\eta} x_{t+1}^2. \tag{30}$$

We can then write the first-order condition with respect to $\tilde{P}_t$ as

$$\eta x_t^1 = (\eta - 1) x_t^2. \tag{31}$$

The labor input used by firm $i \in [0,1]$, denoted $h_{i,t}$, is assumed to be a composite made up of a continuum of differentiated labor services, $h_{i,t}^j$ indexed by $j \in [0,1]$. Formally,

$$h_{i,t} = \left[\int_0^1 {h_{i,t}^j}^{1-1/\tilde{\eta}} dj \right]^{1/(1-1/\tilde{\eta})}, \tag{32}$$

where the parameter $\tilde{\eta} > 1$ denotes the intratemporal elasticity of substitution across different types of activities. For any given level of $h_{i,t}$, the demand for each variety of labor $j \in [0,1]$ in period t must solve the dual problem of minimizing total labor cost, $\int_0^1 W_t^j h_{i,t}^j dj$, subject to the aggregation constraint (32), where W_t^j denotes the nominal wage rate paid to labor of variety j at time t. The optimal demand for labor of type j is then given by

$$h_{i,t}^j = \left(\frac{W_t^j}{W_t} \right)^{-\tilde{\eta}} h_{i,t}, \tag{33}$$

where W_t is a nominal wage index given by

$$W_t \equiv \left[\int_0^1 W_t^{j1-\tilde{\eta}} dj\right]^{1/1-\tilde{\eta}}. \tag{34}$$

This wage index has the property that the minimum cost of a bundle of intermediate labor inputs yielding h_{it} units of the composite labor is given by $W_t h_{i,t}$.

1.3 The Government

Each period the government consumes g_t units of the composite good. Assume that the government minimizes the cost of producing g_t. As a result, public demand for each variety $i \in [0,1]$ of differentiated goods $g_{i,t}$ is given by $g_{i,t} = (P_{i,t}/P_t)^{-\eta} g_t$.

We assume that along the balanced growth path, the share of government spending in value added is constant, that is, we impose $\lim_{j\to\infty} E_t g_{t+j}/y_{t+j} = s_g$, where s_g is a constant indicating the share of government consumption in value added. To this end we impose $g_t = z_t^* \bar{g}_t$, where $\bar{g}_t$ is an exogenous stationary stochastic process. This assumption ensures that government purchases and output are cointegrated. We impose the following law of motion for $\bar{g}_t$:

$$\ln\left(\frac{\bar{g}_t}{\bar{g}}\right) = \rho_{\bar{g}} \ln\left(\frac{\bar{g}_{t-1}}{\bar{g}}\right) + \varepsilon_{\bar{g},t}.$$

The government issues money given in real terms by $m_t \equiv m_t^h + \int_0^1 m_{i,t}^f di$. For simplicity, we assume that government debt is zero at time zero and that the fiscal authority levies lump-sum taxes, τ_t, to bridge any gap between seignorage income and government expenditures, that is, $\tau_t = g_t - (m_t - m_{t-1}/\pi_t)$. As a consequence, government debt is nil at all times. We postpone the presentation of the monetary policy regime until after we characterize a competitive equilibrium.

1.4 Aggregation

We limit attention to a symmetric equilibrium in which all firms that have the opportunity to change their price optimally at a given time choose the same price. It then follows from expression (4) that the aggregate price index can be written as $P_t^{1-\eta} = \alpha(P_{t-1}\pi_{t-1}^{\chi})^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta}$. Dividing this expression through by $P_t^{1-\eta}$, we obtain

$$1 = \alpha \pi_t^{\eta-1} \pi_{t-1}^{\chi(1-\eta)} + (1-\alpha)\tilde{p}_t^{1-\eta}. \tag{35}$$

1.5 Market Clearing in the Final Goods Market

Naturally, the set of equilibrium conditions includes a resource constraint. Such a restriction is typically of the type $F(k_t, z_t h_t) - \psi z_t^* = c_t[1 + \ell(v_t)] + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]$. In the model presented here, however, this restriction is not valid, because the model implies relative price dispersion across varieties. This price dispersion, which is induced by the assumed nature of price stickiness, is inefficient and entails output loss.

To see this, consider the following expression stating that supply must equal demand at the firm level:

$$F(k_{i,t}, z_t h_{i,t}) - \psi z_t^* = \left\{[1 + \ell(v_t)]c_t + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]\right\}\left(\frac{P_{i,t}}{P_t}\right)^{-\eta}.$$

Integrating over all firms and taking into account that the capital-labor ratio is common across firms; the aggregate demand for the composite labor input, h_t^d, satisfies $h_t^d = \int_0^1 h_{i,t}di$; and that the aggregate effective level of capital, $u_t k_t$ satisfies $u_t k_t = \int_0^1 k_{i,t}di$, we obtain

$$z_t h_t^d F\left(\frac{u_t k_t}{z_t h_t^d}, 1\right) - \psi z_t^* = \left\{[1 + \ell(v_t)]c_t + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t]\right\} \times \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di.$$

Let $s_t \equiv \int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} di.$ Then we have

$$s_t = \int_0^1 \left(\frac{P_{i,t}}{P_t} \right)^{-\eta} di$$

$$= (1-\alpha)\left(\frac{\tilde{P}_t}{P_t} \right)^{-\eta} + (1-\alpha)\alpha \left(\frac{\tilde{P}_{t-1}\pi_{t-1}^{\chi}}{P_t} \right)^{-\eta} + (1-\alpha)\alpha^2 \left(\frac{\tilde{P}_{t-2}\pi_{t-1}^{\chi}\pi_{t-2}^{\chi}}{P_t} \right)^{-\eta} + \ldots$$

$$= (1-\alpha)\sum_{j=0}^{\infty}\alpha^j \left(\frac{\tilde{P}_{t-j}\prod_{s=1}^{j}\pi_{t-j-1+s}^{\chi}}{P_t} \right)^{-\eta}$$

$$= (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_t}{\pi_{t-1}^{\chi}} \right)^{\eta} s_{t-1}.$$

Summarizing, the resource constraint in the model is given by the following two expressions

$$F(u_t k_t, z_t h_t^d) - \psi z_t^* = \left\{ [1 + \ell(v_t)]c_t + g_t + \Upsilon_t^{-1}[i_t + a(u_t)k_t] \right\} s_t, \text{ and} \quad (36)$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha \left(\frac{\pi_t}{\pi_{t-1}^{\chi}} \right)^{\eta} s_{t-1}, \quad (37)$$

with s_{-1} given. The state variable s_t summarizes the resource costs induced by the inefficient price dispersion featured in the Calvo model in equilibrium.

Three observations are in order about the price dispersion measure s_t. First, s_t is bounded below by 1. That is, price dispersion is always a costly distortion in this model. To see that s_t is bounded below by 1, let $v_{it} \equiv (P_{it}/P_t)^{1-\eta}$. It follows from the definition of the price index given in equation (4) that $\left[\int_0^1 v_{i,t}\right]^{\eta/(\eta-1)} = 1$. By definition we have $s_t = \int_0^1 v_{i,t}^{\eta/(\eta-1)}$. Taking into account that $\eta/(\eta-1) > 1$, Jensen's inequality implies that

$$1 = \left[\int_0^1 v_{it}\right]^{\eta/(\eta-1)} \leq \int_0^1 v_{it}^{\eta/(\eta-1)} = s_t .$$

Second, in an economy in which the nonstochastic level of inflation is nil (that is, $\pi = 1$) or prices are fully indexed to any variable ω_t

with the property that its deterministic steady-state level equals the deterministic steady-state value of inflation (that is, $\omega = \pi$), the variable s_t follows, up to first order, the univariate autoregressive process $\hat{s}_t = \alpha \hat{s}_{t-1}$. In these cases the price dispersion measure s_t has no first-order real consequences for the stationary distribution of any endogenous variable of the model. This means that studies that restrict attention to linear approximations to the equilibrium conditions are justified in ignoring the variable s_t if the model features no price dispersion in the deterministic steady state. But s_t matters up to first order when the deterministic steady state features movements in relative prices across goods varieties. More important, the price dispersion variable s_t must be taken into account if one is interested in higher-order approximations to the equilibrium conditions, even if relative prices are stable in the deterministic steady state. Omitting s_t in higher-order expansions would amount to leaving out certain higher-order terms while including others. Finally, when prices are fully flexible, $\alpha = 0$, $\tilde{p}_t = 1$, and thus $s_t = 1$. (Obviously, in a flexible-price equilibrium there is no price dispersion across varieties.)

As discussed above, equilibrium marginal costs and capital-labor ratios are identical across firms. Therefore, one can aggregate the firm's optimality conditions with respect to labor and capital, equations (26) and (27), as

$$mc_t z_t F_2(u_t k_t, z_t h_t^d) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right] \tag{38}$$

and

$$mc_t F_1(u_t k_t, z_t h_t^d) = r_t^k. \tag{39}$$

1.6 Market Clearing in the Labor Market

It follows from equation (33) that the aggregate demand for labor of type $j \in [0,1]$, which we denote by $h_t^j \equiv \int_0^1 h_{i,t}^j di$, is given by

$$h_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\tilde{\eta}} h_t^d, \tag{40}$$

where $h_t^d \equiv \int_0^1 h_{i,t} di$ denotes the aggregate demand for the composite labor input. Taking into account that at any point in time the nominal wage rate is identical across all labor markets at which wages are allowed to change optimally, labor demand in each of those markets is

$$\tilde{h}_t = \left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} h_t^d.$$

Combining this expression with equation (40), describing the demand for labor of type $j \in [0,1]$, and with the time constraint (6), which must hold with equality, we can write

$$h_t = (1-\tilde{\alpha})h_t^d \sum_{s=0}^{\infty} \tilde{\alpha}^s \left(\frac{\tilde{W}_{t-s} \prod_{k=1}^{s} (\mu_{z^*} \pi_{t+k-s-1})^{\tilde{\chi}}}{W_t} \right)^{-\tilde{\eta}}.$$

Let $\tilde{s}_t \equiv (1-\tilde{\alpha})\sum_{s=0}^{\infty}\tilde{\alpha}^s (\tilde{W}_{t-s}\prod_{k=1}^{s}(\mu_{z^*}\pi_{t+k-s-1})^{\tilde{\chi}} / W_t)^{-\tilde{\eta}}$ The variable $\tilde{s}_t$ measures the degree of wage dispersion across different types of labor. The above expression can be written as

$$h_t = \tilde{s}_t h_t^d. \tag{41}$$

The state variable $\tilde{s}_t$ evolves over time according to

$$\tilde{s}_t = (1-\tilde{\alpha})\left(\frac{\tilde{w}_t}{w_t}\right)^{-\tilde{\eta}} + \tilde{\alpha}\left(\frac{w_{t-1}}{w_t}\right)^{-\tilde{\eta}} \left(\frac{\pi_t}{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}\right)^{\tilde{\eta}} \tilde{s}_{t-1}. \tag{42}$$

Because all job varieties are identical ex ante, any wage dispersion is inefficient, as reflected in the fact that $\tilde{s}_t$ is bounded below by 1. The proof of this statement is identical to that offered earlier for the fact that s_t is bounded below by unity. To see this, note that $\tilde{s}_t$ can be written as $\tilde{s}_t = \int_0^1 (W_{it} / W_t)^{-\tilde{\eta}} di$. This inefficiency introduces a wedge that makes the number of hours supplied to the market, h_t, larger than the number of productive units of labor input, h_t^d. In an environment without long-run wage dispersion, the deadweight loss created by wage dispersion is nil up to first order. Formally, a first-order approximation of the law of motion of $\tilde{s}_t$ yields a univariate autoregressive process of the form

$\hat{\tilde{s}}_t = \tilde{\alpha}\hat{\tilde{s}}_{t-1}$ as long as there is no wage dispersion in the deterministic steady state. When wages are fully flexible, $\tilde{\alpha} = 0$, wage dispersion disappears, and $\tilde{s}_t$ equals 1.

It follows from our definition of the wage index given in equation (34) that in equilibrium the real wage rate must satisfy

$$w_t^{1-\tilde{\eta}} = (1-\tilde{\alpha})\tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{(\mu_{z^*}\pi_{t-1})^{\tilde{\chi}}}{\pi_t} \right)^{1-\tilde{\eta}}. \tag{43}$$

Aggregating the expression for firm's profits given in equation (25) yields

$$\phi_t = y_t - r_t^k u_t k_t - w_t h_t^d - \nu(1 - R_t^{-1}) w_t h_t^d. \tag{44}$$

In equilibrium, real money holdings can be expressed as

$$m_t = m_t^h + \nu w_t h_t^d, \tag{45}$$

and the government budget constraint is given by

$$\tau_t = g_t - (m_t - m_{t-1}/\pi_t). \tag{46}$$

1.7 Functional Forms

We use the following standard functional forms for utility and technology:

$$U = \frac{\left[(c_t - bc_{t-1})^{1-\phi_4} (1-h_t)^{\phi_4} \right]^{1-\phi_3} - 1}{1-\phi_3}, \quad \text{and} \quad F(k,h) = k^\theta h^{1-\theta}. \tag{47}$$

The functional form for the investment adjustment cost function is taken from Christiano, Eichenbaum, and Evans (2005):

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - \mu_I\right)^2,$$

where μ_I is the steady-state growth rate of investment.

Following Schmitt-Grohé and Uribe (2004a, b), we assume that the transactions cost technology takes the form

$$\ell(v) = \phi_1 v + \phi_2/v - 2\sqrt{\phi_1\phi_2}. \tag{48}$$

The money demand function implied by the above transaction technology is of the form

$$v_t^2 = \frac{\phi_2}{\phi_1} + \frac{1}{\phi_1}\frac{R_t - 1}{R_t}.$$

Note the existence of a satiation point for consumption-based money velocity, $\underline{v}$, equal to $\sqrt{\phi_2/\phi_1}$. The implied money demand is unit elastic with respect to consumption expenditures. This feature is a consequence of the assumption that transactions costs, $c\ell(c/m)$, are homogenous of degree one in consumption and real balances and independent of the particular functional form assumed for $\ell(\bullet)$. Furthermore, as the parameter ϕ_2 approaches zero, the transactions cost function $\ell(\bullet)$ becomes linear in velocity and the demand for money adopts the Baumol-Tobin square root form with respect to the opportunity cost of holding money, $(R-1)/R$. That is, the log-log elasticity of money demand with respect to the opportunity cost of holding money converges to 1/2, as ϕ_2 vanishes.

The costs of higher capacity utilization are parameterized as follows:

$$a(u) = \gamma_1(u-1) + \frac{\gamma_2}{2}(u-1)^2.$$

1.8 Inducing Stationarity

This economy features two types of permanent shocks. As a result a number of variables, such as output and the real wage, will not be stationary along the balanced growth path. We therefore perform a change of variables in order to obtain a set of equilibrium conditions that involves only stationary variables. To this end we note that the variables c_t, m_t^h, m_t, w_t, $\tilde{w}_t$, y_t, g_t, ϕ_t, x_t^1, x_t^2, and τ_t are cointegrated with z_t^*. Similarly, the variables k_{t+1} and i_t are cointegrated with $\Upsilon_t z_t^*$, the variable λ_t is cointegrated with $z_t^{*(1-\phi_3)(1-\phi_4)-1}$, the variables q_t and r_t^k are cointegrated with $1/\Upsilon_t$, and the variables f_t^1 and f_t^2 are cointegrated with $z_t^{*(1-\phi_3)(1-\phi_4)}$. We therefore divide these variables by the appropriate cointegrating factor and denote the corresponding stationary variables with capital letters.

1.9 Competitive Equilibrium

A stationary competitive equilibrium is a set of stationary processes u_t, C_t, h_t, I_t, K_{t+1}, v_t, M_t^h, M_t, Λ_t, π_t, W_t, $\tilde{\mu}_t$, Q_t, R_t^k, Φ_t, F_t^1, F_t^2, $\tilde{W}_t$, h_t^d, Y_t, mc_t, X_t^1, X_t^2, $\tilde{p}_t$, s_t, $\tilde{s}_t$, and T_t satisfying expressions (7), (8), (10), (12)–(22), (29)–(31), (35)–(39), and (41)–(46) written in terms of the stationary variables, given exogenous stochastic processes $\mu_{\Upsilon,t}$, $\mu_{z,t}$, and $\bar{g}_t$; the policy process, R_t; and initial conditions c_{-1}, w_{-1}, s_{-1}, $\tilde{s}_{-1}$, π_{-1}, i_{-1}, and k_0. A complete list of the competitive equilibrium conditions in terms of stationary variables is given in the appendix (Schmitt-Grohé and Uribe, 2006).

1.10 Ramsey Equilibrium

We assume that at $t = 0$ the benevolent government has been operating for an infinite number of periods. In choosing the optimal policy, the government is assumed to honor commitments made in the past. This form of policy commitment has been referred to as "optimal from the timeless perspective" (Woodford, 2003).

Formally, we define a Ramsey equilibrium as a set of stationary processes u_t, C_t, h_t, I_t, K_{t+1}, v_t, M_t^h, M_t, Λ_t, π_t, W_t, $\tilde{\mu}_t$, Q_t, R_t^k, Φ_t, F_t^1, F_t^2, $\tilde{W}_t$, h_t^d, Y_t, mc_t, X_t^1, X_t^2, $\tilde{p}_t$, s_t, $\tilde{s}_t$, T_t, and R_t for $t \geq 0$ that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(z_0^* \prod_{s=1}^{t} \mu_{z^*,s}\right)^{(1-\phi_4)(1-\phi_3)} \left[\left(C_t - bC_{t-1}/\mu_{z^*,t}\right)^{1-\phi_4} (1-h_t)^{\phi_4}\right]^{1-\phi_3} - 1}{1-\phi_3}$$

subject to the competitive equilibrium conditions (7), (8), (10), (12)–(22), (29)–(31), (35)–(39), and (41)–(46) written in stationary variables and $R_t \geq 1$, for $t > -\infty$, given exogenous stochastic processes $\mu_{z,t}$, $\mu_{\Upsilon,t}$, and $\bar{g}_t$; values of the variables listed above for $t < 0$; and values of the Lagrange multipliers associated with the constraints listed above for $t < 0$.

Technically, the difference between the usual Ramsey-equilibrium concept and the one employed here is that here the structure of the optimality conditions associated with the Ramsey equilibrium is time invariant. By contrast, under the standard Ramsey-equilibrium definition, the equilibrium conditions in the initial periods are different from those applying to later periods.

Our approach to analyzing the business cycle properties of Ramsey-optimal policy is comparable to that in the literature under the standard definition of Ramsey optimality (for example, Chari, Christiano, and Kehoe, 1995). The reason is that studies of business cycles under the standard Ramsey policy focus on the behavior of the economy in the stochastic steady state (that is, they limit attention to the properties of equilibrium time series, excluding the initial transition).

2. Calibration

The time unit is meant to be one quarter. For most of the calibration, we draw on Altig and others (2005) (hereafter ACEL). We assign most of the parameter values from the "high mark-up" case of the ACEL estimation results, in which the steady-state mark-up in product markets is 20 percent ($\eta = 6$) (table 1).

Following ACEL we assume that in the deterministic steady state of the competitive equilibrium, the rate of capacity utilization equals one ($u = 1$) and profits are zero ($\phi = 0$). ACEL calibrate the discount factor, β, to be $1.03^{-1/4}$; the depreciation rate, δ, to be 0.025; and the capital share, θ, to be 0.36. They assume that preferences are separable in consumption and leisure and logarithmic in habit-adjusted consumption ($\phi_3 = 1$). Their assumed functional form for the period utility function implies a unit Frisch elasticity of labor supply. ACEL assume a steady-state mark-up of wages over the marginal rate of substitution between leisure and consumption of 5 percent (or $\tilde{\eta} = 21$).

ACEL estimate the degree of nominal wage stickiness at slightly more than three quarters ($\tilde{\alpha} = 0.69$). They estimate the degree of habit formation, measured by the parameter b, at 0.69; the elasticity of the marginal capital adjustment cost, κ, at 2.79; the elasticity of the marginal cost of capacity utilization, γ_2/γ_1, at 1.46; and the annualized interest semi-elasticity of money demand by households, $(1/4)\partial \ln(m_t^h)/\partial(R_t)$, at -0.81. They estimate the parameters of the exogenous stochastic processes for the investment-specific and neutral technology shocks $\mu_{\Upsilon,t}$ and $\mu_{z,t}$ at $(\mu_\Upsilon, \sigma_{\mu_\Upsilon}, \rho_{\mu_\Upsilon})$= (1.0042, 0.0031, 0.20) and $(\mu_z, \sigma_{\mu_z}, \rho_{\mu_z})$= (1.00213, 0.0007, 0.89), respectively. They estimate the degree of price stickiness at five quarters (or $\alpha = 0.8$) when capital is not firm specific, which is the assumption maintained in this paper.

Table 1. Structural Parameters of the Model

Parameter	*Description*	*Value*
β	Subjective discount factor (quarterly)	$1.03^{-1/4}$
θ	Share of capital in value-added	0.36
ψ	Fixed cost parameter	0.25
δ	Depreciation rate (quarterly)	0.025
υ	Fraction of wage bill subject to a cash-in-advance constraint	0.6011
η	Price elasticity of demand for a specific good variety	6
$\tilde{\eta}$	Wage elasticity of demand for a specific labor variety	21
α	Fraction of firms not setting prices optimally each quarter	0.8
$\tilde{\alpha}$	Fraction of labor markets not setting wages optimally each quarter	0.69
b	Degree of habit persistence	0.69
ϕ_1	Transactions cost parameter	0.0459
ϕ_2	Transactions cost parameter	0.1257
ϕ_3	Preference parameter	1
ϕ_4	Preference parameter	0.5301
κ	Parameter governing investment adjustment costs	2.79
$\gamma 1$	Parameter of capacity utilization cost function	0.0412
$\gamma 2$	Parameter of capacity utilization cost function	0.0601
χ	Degree of price indexation	0
$\tilde{\chi}$	Degree of wage indexation	1
μ_γ	Quarterly growth rate of investment-specific technological change	1.0042
σ_{μ_γ}	Standard deviation of the innovation to the investment-specific technology shock	0.0031
ρ_{μ_γ}	Serial correlation of the log of the investment-specific technology shock	0.20
μ_z	Quarterly growth rate of neutral technology shock	1.00213
σ_{μ_z}	Standard deviation of the innovation to the neutral technology shock	0.0007
ρ_{μ_z}	Serial correlation of the log of the neutral technology shock	0.89
$\bar{g}$	Steady-state value of government consumption (quarterly)	0.2141
σ_{ε^g}	Standard deviation of the innovation to log of government consumption	0.008
ρ_g	Serial correlation of the log of government spending	0.9

Source: Altig and others (2005); Cogley and Sbordone (2005); Levin and others (2005); Chistiano and others (2005); Ravn (2005).

We do not draw on the work of ACEL to calibrate the degree of indexation in product prices or wages, because they do not estimate the parameters governing the degree of indexation but simply assume full indexation of all prices to past product price inflation. We draw from the econometric work of Cogley and Sbordone (2005) and Levin and others (2005), who find no evidence of indexation in product prices. We therefore set $\chi = 0$ Levin and others (2005) estimate a high degree of indexation in nominal wages. We therefore assume that $\tilde{\chi} = 1$, which happens to be the value assumed in ACEL.

Following Christiano, Eichenbaum, and Evans (2005), hereafter CEE, we set the steady-state share of money held by households, m^h/m, to 0.44. Using postwar U.S. data, we measure the average money to output ratio as the ratio of M1 to GDP and set it equal to 17 percent a year. Neither ACEL nor CEE impose this calibration restriction. Instead, they assume that all of the wage bill is subject to a cash-in-advance constraint—that is, $\upsilon = 1$. By contrast, our calibration implies that only 60 percent of wage payments must be held in money ($\upsilon = 0.6$).

In calibrating the model, we assume that in the deterministic steady state of the competitive equilibrium, the rate of inflation equals 4.2 percent a year. This value coincides with the average growth rate of the U.S. postwar GDP deflator.

ACEL do not consider government purchases shocks. One study that estimates the process for government purchases in the context of a model similar to ours is Ravn (2005), whose findings we use to calibrate this process. Ravn estimates $\rho_g = 0.9$ and $\sigma_{\varepsilon^g} = 0.008$. Finally, we impose that the steady-state share of government consumption in value-added is 17 percent, the average value observed in the United States over the postwar period.

3. The Ramsey Steady State

In this section we characterize the long-run state of the Ramsey equilibrium in an economy without uncertainty. We refer to this state as the Ramsey steady state. The Ramsey steady state is in general different from the allocation/policy that maximizes welfare in the steady state of a competitive equilibrium.

In most studies on optimal monetary policy in economies with neo-Keynesian features, characterizing the Ramsey steady state is trivial, because they assume the existence of a single nominal distortion—namely, sluggish adjustment in nominal product or factor prices or both. In this case the optimal rate of inflation in the

Ramsey steady state is nil. By contrast, the economy studied here features additional nominal frictions in the form of money demand by households and firms. This feature complicates the computation of the Ramsey steady state.

Two exceptions to the common practice of abstracting from money demand in analyzing optimal monetary policy in the neo-Keynesian model are Khan, King, and Wolman (2003) and Schmitt-Grohé and Uribe (2004a). In both studies the computation of the Ramsey steady state is relatively straightforward because of the simplicity of the theoretical structures considered. In particular, neither study features wage stickiness, capital accumulation, habit formation, variable capacity utilization, or factor adjustment costs. When all of these complications are added, it becomes virtually impossible to characterize the Ramsey steady-state conditions analytically. A contribution of the research project to which this paper belongs is the development of a general algorithm to characterize and numerically solve the Ramsey equilibrium in medium-scale macroeconomic models. This algorithm yields an exact numerical solution for the Ramsey steady-state equilibrium.

3.1 Price Stickiness and the Optimal Inflation Rate

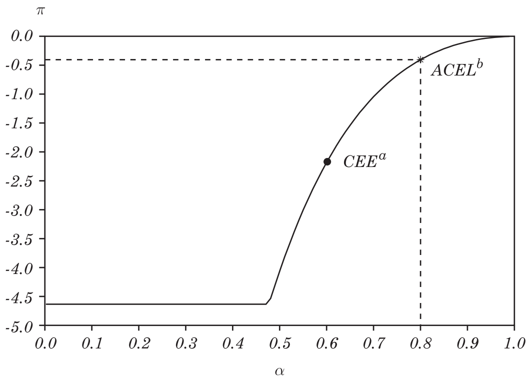
The most striking characteristic of the Ramsey steady state is the high sensitivity of the optimal rate of inflation to the parameter governing the degree of price stickiness, α, for the range of values of this parameter that is empirically relevant. Empirical estimates of the degree of price rigidity using macroeconomic data vary from two to five quarters, or $\alpha \in [0.5,0.8]$. In the context of a model similar to ours, CEE estimate α to be 0.6. By contrast, using a model identical to ours, ACEL estimate a marginal cost gap coefficient in the Phillips curve that is consistent with a value of α of about 0.8 when the market for capital is assumed to be centralized, as is maintained in our formulation.[4] Both CEE and ACEL use an impulse-response matching technique to estimate α. Bayesian estimates of this parameter include Del Negro and others (2004) and Levin and others (2005), who report posterior means of 0.67 and 0.83, respectively, and 90 percent probability intervals of (0.51, 0.83) and (0.81, 0.86), respectively. Evidence on price stickiness based on microeconomic data suggests a much higher

4. If, instead, capital accumulation is assumed to be firm specific, then ACEL's estimate of the Phillips curve is consistent with a value of α of about 0.7.

frequency of price changes than the evidence based on macro data. The findings reported in Bils and Klenow (2004) and Golosov and Lucas (2003), for example, suggest values of α of about one-third, or a degree of price stickiness of about 1.5 quarters.

Figure 1 displays the relationship between the degree of price stickiness, α, and the optimal rate of inflation in percent a year, π. When α equals 0.5, the lower range of the empirical evidence using macro data, the optimal rate of inflation is -4 percent, virtually equal to the level called for by the Friedman rule. For our baseline value of $\alpha = 0.8$, which is near the upper range of the empirical evidence using macro data, the optimal level of inflation rises to -0.4 percent, which is close to price stability. Also evident from figure 1 is the fact that values of α based on microeconomic evidence, of about one-third, imply that the Friedman rule is Ramsey optimal in the long run.

Figure 1. Degree of Price Stickiness and the Optimal Rate of Inflation



Source: Authors' computations.
* Benchmark parameter value
a. CEE = parameter values estimated by Christiano, Eichenbaum, and Evans (2005).
b. ACEL = parameter values estimated by Altig and others (2005). All parameters other than α take their baseline values, given in table 1.

In addition to the uncertainty surrounding the estimation of the degree of price stickiness, a second aspect of the apparent difficulty in reliably establishing the long-run level of inflation has to do with the shape of the relationship linking the degree of price stickiness to the optimal level of inflation. The problem resides in the fact that this relationship becomes significantly steep precisely for that range of values of α that is empirically most compelling. The problem would not arise if the steep portion of the relationship took place at values of α below 0.33 or above, say, 0.8. It turns out that an important factor determining the shape of the function relating the optimal level of inflation to the degree of price stickiness is the underlying fiscal policy regime.

3.2 Fiscal Policy and the Optimal Inflation Rate

We follow the widespread practice in the literature on optimal monetary policy in the neo-Keynesian framework of ignoring fiscal considerations by implicitly or explicitly assuming the existence of lump-sum, nondistorting taxes that balance the government budget at all times under all circumstances. This assumption is clearly unrealistic and usually maintained on the sole basis of simplicity. We argue that taking the fiscal side of the optimal policy problem explicitly into account has crucial consequences for the optimal long-run level of inflation.

Fiscal considerations fundamentally change the long-run tradeoff between price stability and the Friedman rule. To see this, we briefly consider an economy in which lump-sum taxes are unavailable and the fiscal authority must finance government purchases through proportional capital and labor income taxes. The social planner jointly sets monetary and fiscal policy in a Ramsey-optimal fashion. The details of this environment are described in Schmitt-Grohé and Uribe (2006).

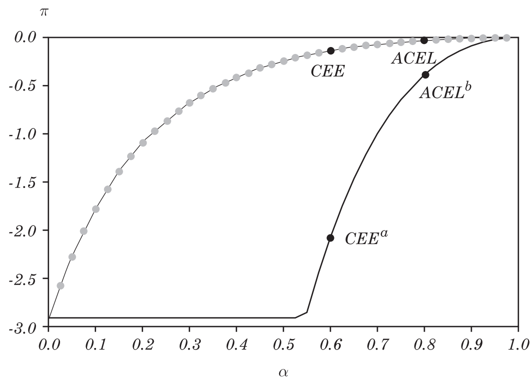
Figure 2 displays the relationship between the degree of price stickiness, α, and the optimal rate of inflation, π. The solid line corresponds to the baseline case considered here (featuring lump-sum taxes).[5] The solid-circled line corresponds to the economy with optimally chosen income taxes analyzed in Schmitt-Grohé and Uribe (2006).[6] In stark contrast to what happens under lump-sum taxation, under optimal distortionary taxation the function linking

5. In producing the solid line shown in figure 2, we assign all structural parameters the baseline values shown in table 1, except for the long-run growth rates of the two productivity shocks, which are set to zero. This deviation from the baseline calibration is necessary to preserve comparability with the model in Schmitt-Grohé and Uribe (2006), which features no long-run growth. The solid line looks essentially like the one shown in figure 1—the only difference is that at the Friedman rule, the inflation rate is -2.9 percent, whereas in figure 1 it is -4.6 percent. This difference is explained by the lack of growth in the model used to produce the solid line in figure 2.

6. In producing the solid-circled line shown in figure 2, we set all structural parameter values to those shown in table 1, except for those governing long-run growth, which are set to zero. The model economy features proportional labor, capital, and profit taxes. The profit tax rate is constrained to be equal to the capital income tax rate. Government transfers are set to zero.

π and α is flat and very close to zero for the entire range of macro data–based empirically plausible values of α, namely 0.5–0.8. In other words, when taxes are distortionary and optimally determined, price stability emerges as a prediction that is robust to the existing uncertainty about the exact degree of price stickiness. Even if one focuses on the evidence of price stickiness stemming from micro data, the model with distortionary Ramsey taxation predicts an optimal long-run level of inflation that is much closer to zero than to the level predicted by the Friedman rule.

Figure 2. Price Stickiness, Fiscal Policy, and Optimal Inflation



Source: Authors' computations.
a. CEE = parameter values estimated by Christiano, Eichenbaum, and Evans (2005).
b. ACEL = parameter values estimated by Altig and others (2005).

Our intuition for why price stability arises as a robust policy recommendation in the economy with optimally set distortionary taxation runs as follows. Consider the economy with lump-sum taxation. Deviating from the Friedman rule (by raising the inflation rate) reduces the price dispersion that originates in the presence of price stickiness. Consider next the economy with Ramsey-optimal income taxation and no lump-sum taxes. In this economy deviating from the Friedman rule reduces price dispersion. In addition, raising inflation increases seignorage revenue, allowing the social planner to lower distortionary income tax rates. The tradeoff between the Friedman rule and price stability is thus tilted in favor of price stability.

It follows from this intuition that what is essential in inducing the optimality of price stability is that at the margin the fiscal authority trades off the inflation tax for regular taxation. Indeed, it can be shown that if distortionary tax rates are fixed, even at the level that is optimal in a world without lump-sum taxes, and the fiscal authority has access to lump-sum taxes at the margin, the optimal rate of inflation is much closer to the Friedman rule than to zero. In this case increasing inflation no longer has the benefit of reducing distortionary taxes. As a result the Ramsey planner has less incentive to inflate.

3.3 Price Indexation and the Optimal Inflation Rate

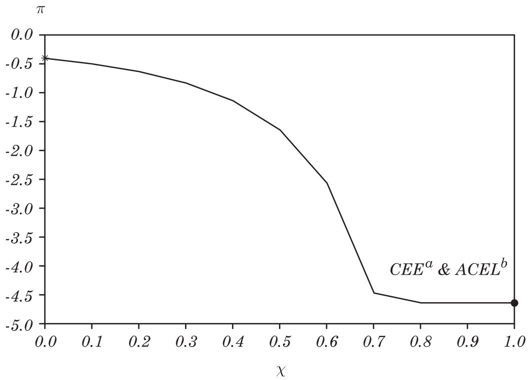
The parameter χ, measuring the degree of price indexation, is crucial in determining the optimal level of long-run inflation, because when prices are fully indexed ($\chi = 1$), price dispersion disappears in the deterministic steady state. As a result the social planner no longer faces a tradeoff between minimizing price dispersion and minimizing the opportunity cost of holding money. In such an environment the Friedman rule is Ramsey optimal. In the absence of perfect indexation ($\chi < 1$), any deviation from zero inflation will entail price dispersion; the lower the degree of indexation, the higher the price dispersion associated with a given level of inflation. Consequently, the Ramsey-optimal deflation rate is increasing in the degree of price indexation.

Figure 3 shows that the Ramsey-optimal inflation rate is indeed a decreasing function of the indexation parameter χ. CEE and ACEL assume that prices are perfectly indexed to lagged inflation—they calibrate the parameter χ to be unity. Under this assumption the Friedman rule is optimal in the deterministic Ramsey steady state. However, the few studies that attempt to estimate the indexation parameter χ find little empirical support for price indexation. For example, using Bayesian methods, Levin and others (2005) report a tight estimate of χ of 0.08. Using a different empirical strategy, Cogley and Sbordone (2005) also find virtually no evidence of price indexation in U.S. data. These two empirical studies motivate our setting $\chi = 0$.

3.4 Money Demand and the Optimal Inflation Rate

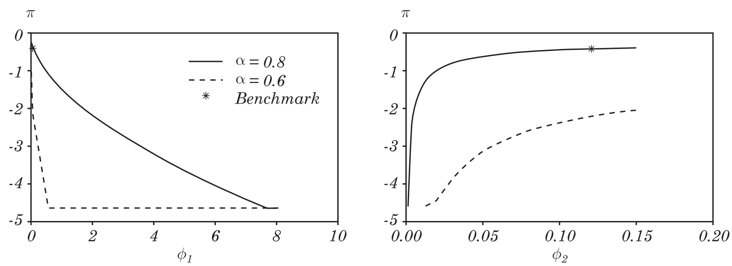
Given the long-run policy tradeoffs present in the model—namely, minimizing the opportunity cost of holding money (by setting $R_t = 1$) versus minimizing price dispersion (by setting $\pi_t = 1$)—one should expect that the larger the money demand friction, the closer the optimal rate of inflation to the one prescribed by the Friedman rule.

Figure 3. Degree of Price Indexation and the Optimal Rate of Inflation



Source: Authors' computations.
* Benchmark Parameter Value
a. CEE = parameter values estimated by Christiano, Eichenbaum, and Evans (2005).
b. ACEL = parameter values estimated by Altig and others (2005). All parameters other than χ take their baseline values, given in table 1.

Figure 4. Money Demand and the Optimal Rate of Inflation



Source: Authors' computations.
Note: In each panel all parameters other than the one shown take their baseline values, given in table 1.

Figure 4 displays the optimal rate of inflation as a function of the two structural parameters defining the demand for money by households, ϕ_1 and ϕ_2. It suggests that the optimal rate of inflation is rather insensitive to changes in these two parameters. At the baseline value of 0.05 for the parameter ϕ_1, the optimal rate of inflation is -0.4 percent a year and money demand is 17 percent of GDP. If one increases ϕ_1 by a factor of 10, to 0.5, the optimal rate of deflation is still small, at only 1 percent, but the demand for money doubles to 35 percent of GDP. One must increase ϕ_1 by a factor of more than 150, to about 8, to induce an optimal inflation rate close to the Friedman rule. At this value of ϕ_1, the demand for money is larger than annual GDP.

The reason for the implied low sensitivity of the Ramsey inflation rate to the parameters defining the demand for money is the assumed high degree of price stickiness. This distortion is so dominant in this model that optimal policy is overwhelmingly geared toward price stability. As a result, low inflation survives as the overriding goal of monetary policy, even for economically large values of the money demand distortion. If one lowers the degree of price stickiness, the optimal rate of inflation becomes much more sensitive with respect to the transactions cost parameter, ϕ_1. Figure 4 displays with a dashed line the relationship between the optimal rate of inflation and the parameters ϕ_1 and ϕ_2 when the sticky-price parameter α takes the value 0.6. In this case the optimal rate of inflation falls from near price stability to the Friedman rule much more rapidly as one increases ϕ_1 than in the baseline case, in which $\alpha = 0.8$.

A similar message emerges as one varies the other transactions cost parameter, ϕ_2. Only for economically implausible values of ϕ_2 (values implying extremely high interest rate elasticities of money demand) does the Friedman rule emerge as Ramsey optimal.

3.5 Implications for Inflation Targeting

A robust implication of the ACEL model studied here is that the central bank should target mild deflation. This implication is at odds with the observed inflation goals among the large number of industrial and emerging market countries that self-identify their monetary policy as inflation targeting. In industrial countries, inflation targets typically lie in the rage of 2–3 percent a year. Inflation targets are somewhat higher in developing countries.

It is therefore a challenge for monetary policy to square theoretically optimal inflation targets with actual ones. One reason often offered for why the inflation target should be positive is that too low an inflation target (in particular, zero or negative targets) would leave the central bank too close to the zero bound on nominal interest rates, thereby impairing the monetary authority's ability to steer the economy out of recession. Our analysis thus far is necessarily mute on this point, because we have limited attention to a characterization of the Ramsey steady state. In order to ascertain whether the zero bound will indeed be frequently visited under the Ramsey- optimal stabilization policy, a dynamic equilibrium analysis must be carried out.

4. Ramsey Dynamics

We approximate the Ramsey equilibrium dynamics by solving a first-order approximation to the Ramsey equilibrium conditions. There is evidence that first-order approximations to the Ramsey equilibrium conditions deliver dynamics that are fairly close to those associated with the exact solution. In Schmitt-Grohé and Uribe (2004b), we compute the exact solution to the Ramsey equilibrium in a flexible-price dynamic economy with money, income taxes, and monopolistic competition in product markets. In Schmitt-Grohé and Uribe (2004a), we compute the solution to the same economy using a first-order approximation to the Ramsey equilibrium conditions. We find that the solution is not significantly different from the one based on a first-order approximation. In the context of optimal taxation in the standard real-business-cycle model, Benigno and Woodford (2005) show that the first-order approximation to the Ramsey equilibrium conditions implies second moments that are similar to those computed from an approximation based on a minimum-weighted-residual method reported in Chari and others (1995).

4.1 Is the Zero Bound an Impediment to Optimal Policy?

One argument against setting a zero or negative inflation target, as recommended by our model, is that at zero or negative rates of inflation the risk of hitting the zero lower bound on nominal interest rates would severely restrict the central bank's ability to conduct successful stabilization policy. We compute the standard deviations of the nominal interest rate as well as other key macroeconomic variables under the Ramsey-optimal stabilization policy (table 2).

In computing these second moments, we assign all structural parameters of the model the values shown in table 1. We find that the annual standard deviation of the nominal interest rate is only 0.4 percentage points, while the Ramsey steady-state level of the nominal interest rate is 4.4 percent (see table 2). Taken together these figures imply that for the nominal interest rate to hit the zero bound, it must fall more than 10 standard deviations below its target level. The probability of this happening is so small that in the context of the estimated medium-scale model studied here, the zero bound on nominal interest rates does not impose an economically important constraint on the conduct of optimal monetary policy. This conclusion appears to be robust to changes in the degree of price or wage stickiness within the range of available empirical estimates for the parameters determining the degree of nominal sluggishness (see columns 2 and 3 of table 2).

4.2 Optimality of Inflation Stability

The Ramsey authority faces a three-way tradeoff in determining the optimal degree of inflation volatility. The sticky price distortion in isolation calls for minimizing inflation volatility. The money demand distortion calls for stabilizing the opportunity cost of holding money, that is, minimizing the standard deviation of R_t. The sticky wage distortion renders stabilization of wage inflation (in the absence of indexation) or stabilization of wage inflation net of lagged price inflation (under full indexation to past price inflation) Ramsey optimal. Table 2 indicates that this three-way tradeoff is resolved overwhelmingly in favor of inflation stability.

To see how sensitive the inflation stability goal is to the size of the sticky wage distortion, consider the case of $\tilde{\alpha} = 0.9$, which implies that unions reoptimize wages only every 10 quarters. The optimal volatility of price inflation increases and that of wage inflation falls. The optimal standard deviation of price inflation is now 0.4 percent a year and the optimal standard deviation of wage inflation 1.0 percent. Yet price inflation continues to be significantly smoother over the business cycle than wage inflation. We conclude that a central characteristic of optimal stabilization policy is smooth inflation rates. In this sense one could say that the Ramsey planner pursues a policy of inflation targeting.

4.3 Ramsey Optimal Impulse Responses and Variance Decomposition

Optimal stabilization policy will in general be shaped by the number and nature of exogenous shocks generating aggregate fluctuations. There is considerable debate in the empirical literature about the identification of the main sources of business cycle fluctuations. One branch of the literature uses structural vector autoregression analysis to identify specific structural shocks. Examples of this approach are Altig and others (2005) and Fisher (2005). The work of Fisher suggests that investment-specific technology shocks may explain as much as 50 percent of variations in hours worked. Altig and others identify monetary policy shocks and investment-specific as well as neutral

Table 2. Ramsey Optimal Stabilization Policy: Second Moments

Variable	$\alpha = 0.8$ $\tilde{\alpha} = 0.69$	$\alpha = 0.8$ $\tilde{\alpha} = 0.9$	$\alpha = 0.6$ $\tilde{\alpha} = 0.69$
Standard deviation (percentage points per year)			
Nominal interest rate	0.4	0.4	0.3
Price inflation	0.1	0.4	0.2
Wage inflation	1.2	1.0	1.2
Output growth	0.8	0.8	0.8
Consumption growth	0.5	0.5	0.5
Investment growth	1.3	1.5	1.3
Serial correlation			
Nominal interest rate	0.9	0.8	0.9
Price inflation	0.8	0.9	0.8
Wage inflation	0.7	0.5	0.6
Output growth	0.4	0.5	0.5
Consumption growth	0.9	0.9	0.9
Investment growth	0.8	0.7	0.8
Correlation with output growth			
Nominal interest rate	0.4	0.0	0.3
Price inflation	-0.3	-0.5	-0.4
Wage inflation	0.6	0.4	0.6
Output growth	1.0	1.0	1.0
Consumption growth	0.4	0.4	0.4
Investment growth	0.4	0.5	0.4

Source: Authors' computations.

technology shocks. They find that investment-specific shocks play a smaller role in generating business cycles. Specifically, they estimate that neutral and investment-specific technology shocks together explain only about one-third of the fluctuations in hours, output, and consumption.

Some recent literature uses Bayesian methods to estimate the entire data-generating process of a dynamic stochastic general equilibrium model. Smets and Wouters (2004) is a key example of this line of research. The authors estimate a model with 10 shocks. One might consider using all of those 10 estimated shocks in the optimal policy problem. However, in econometrically estimated versions of the model studied here (or variations thereof), many of these shocks are often difficult to interpret economically. To a large extent, these shocks represent simple econometric residuals reflecting the distance between model and data rather than true sources of business cycle fluctuations.

A case in point are shocks to Euler equations or mark-up shocks. Before incorporating this type of residual as driving forces, it is our position to first give theory a chance to get closer to the data. We therefore do not attempt to build a model that includes all sources of fluctuations. Instead, we focus on three shocks that have been shown in the empirical literature to explain a significant fraction of aggregate fluctuations: neutral shocks, investment-specific technology shocks, and government purchases shocks.

Variations in output growth are explained in equal parts by government purchases shocks and neutral technology shocks, which each account for 45 percent of output growth variance (table 3). Investment-specific productivity shocks play a minor role in driving fluctuations in output growth, but they are important in explaining movements in hours worked (47 percent), wage inflation (37 percent), and investment growth (61 percent). Fluctuations in consumption growth, the nominal interest rate, inflation, and wage inflation are driven mainly by neutral productivity shocks, with a small contribution by government purchases shocks.

Table 3. Fraction of Variance Explained by Exogenous Disturbances in the Ramsey Equilibrium

Variable	$\mu_{\Upsilon,z}$	$\mu_{z,t}$	g_t
$\ln y_t/y_{t-1}$	0.11	0.44	0.45
$\ln c_t/c_{t-1}$	0.10	0.80	0.10
$\ln I_t/I_{t-1}$	0.61	0.33	0.06
$\ln R_t$	0.21	0.62	0.17
$\ln \pi_t$	0.13	0.83	0.04
$\ln \pi_t^W$	0.37	0.63	0.00
$\ln h_t^d$	0.47	0.44	0.09

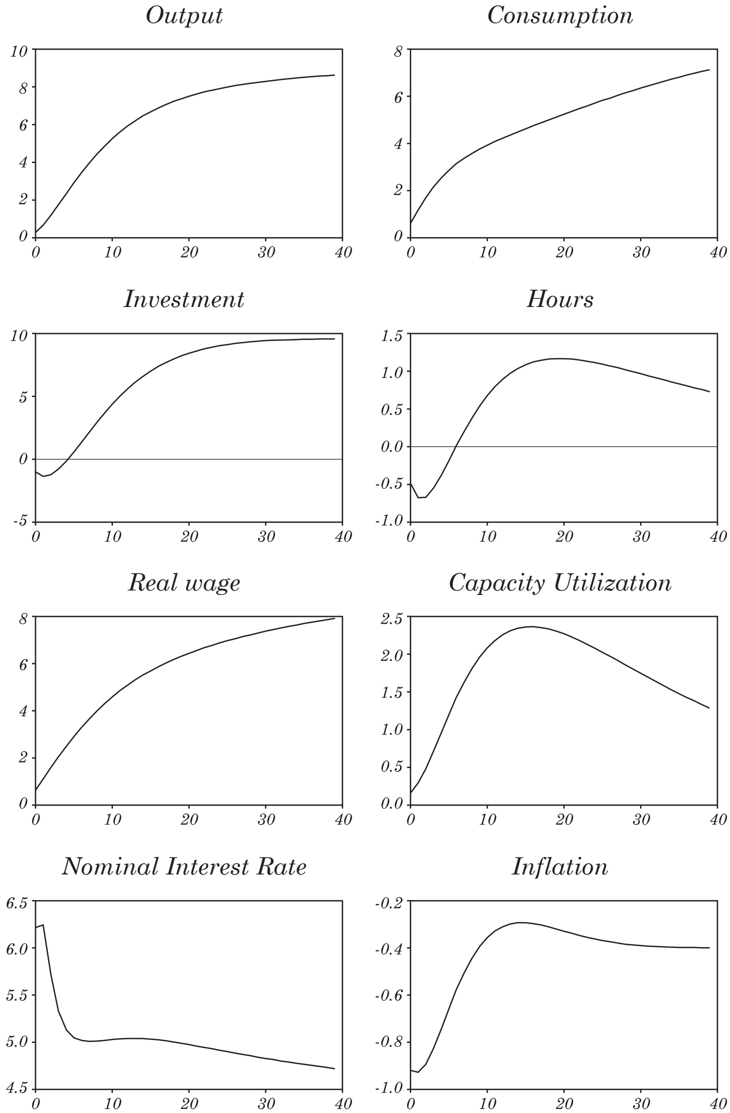
Source: Authors' computations.

In response to a 1 percentage increase in the growth rate of the neutral technology shock ($\ln(\mu_{z,0}/\mu_z) = 1\%$), the Ramsey planner raises nominal interest rates by 175 basis points on impact and allows inflation to fall 45 basis points (figure 5). This monetary tightening is short lived, however: after six quarters the nominal interest rate is back at 5 percent, or 50 basis points above its long-run target. We conjecture that the reason for this tightening is as follows. The Ramsey planner aims to replicate the real allocation associated with the flexible-price/ flexible-wage economy. In such an economy, the real interest rate would rise at least temporarily in response to a positive shock to the growth rate of technology. With sluggish nominal price adjustment, the Ramsey planner would like to induce a rise in the real interest rate without relying on costly movements in the inflation rate. Because the real interest rate equals the risk-free nominal interest rate minus the inflation rate, it follows that the Ramsey-optimal policy is to raise nominal interest rates roughly by the amount that real interest rates would rise in the flexible-price economy. Interestingly, nominal interest rates are tightened not to avoid inflation but to avoid deflation.

An active debate is ongoing over the estimated effects of neutral technology shocks on hours. Galí (1999) finds that hours decline on impact, whereas ACEL find that hours increase. Consistent with the findings of Galí, our model predicts that under the Ramsey policy, hours decline on impact in response to a positive innovation in the neutral technology shock. Our intuition for the initial decline in hours is as follows. Because monetary policy induces a sharp increase in real interest rates on impact, the wealth effect on consumption is initially muted. In addition, due to the presence of adjustment costs in investment, investment spending does not increase much on impact. As a result the positive wealth effect generated by the increase in productivity growth materializes in an expansion of the consumption of leisure.

A 1 percent increase in government consumption raises output by 0.15 percent (figure 6). Given that in the model the share of public consumption in GDP is assumed to be 17 percent, it follows that the government spending multiplier implied by the model is slightly below unity. The model predicts that the government should increase interest rates in response to a positive government spending shock, which is in line with conventional wisdom.

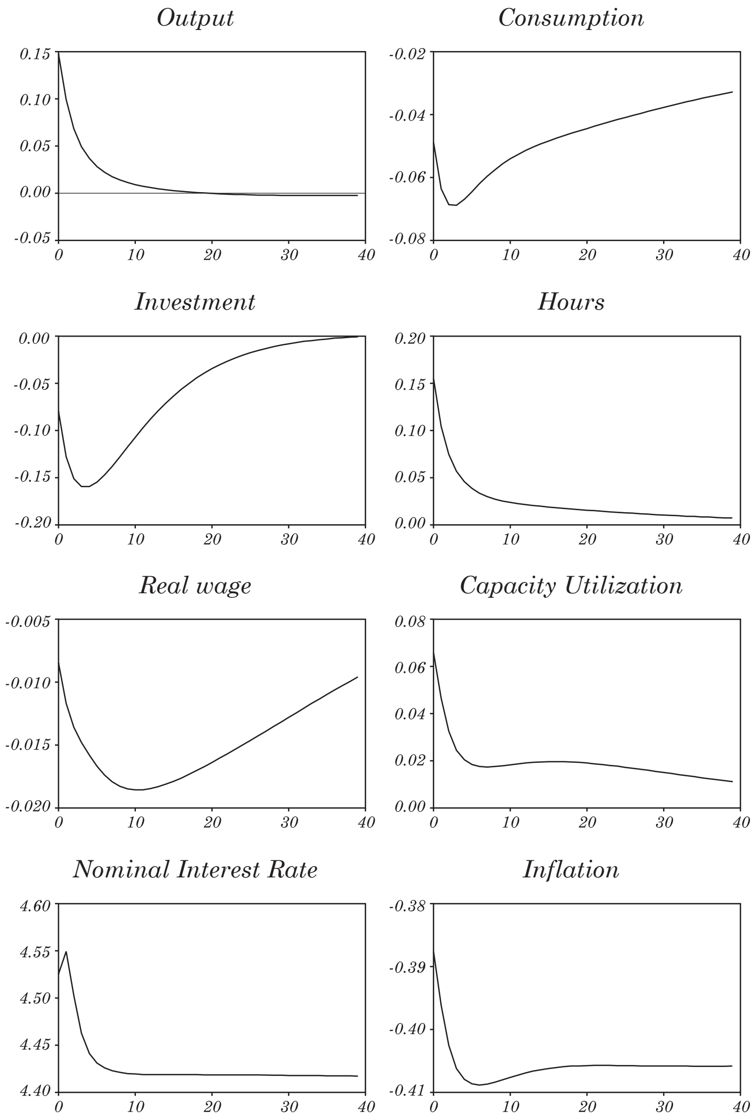
Figure 5. Ramsey Response to a Neutral Productivity Shock



Source: Authors' computations.

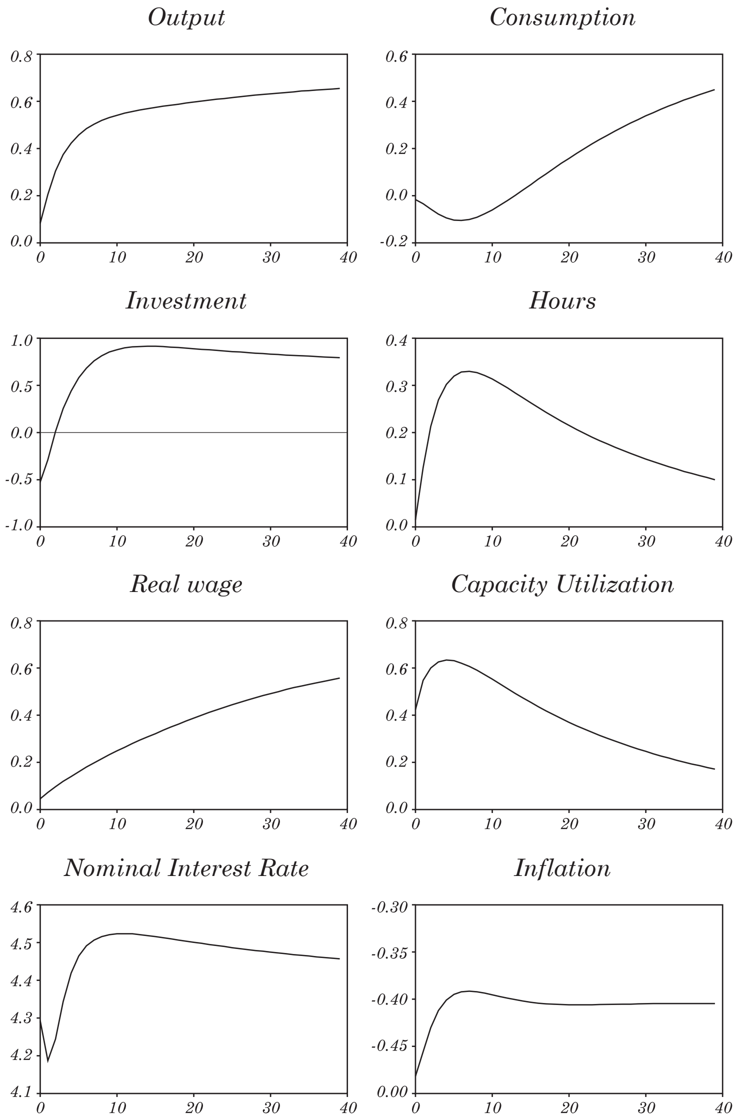
Note: The size of the initial innovation to the neutral technology shock is 1 percent, $\ln(\mu_{z,0}/\mu_z) = 1\%$. The nominal interest rate and the inflation rate are expressed in levels in percent a year. Output, wages, investment, and consumption are expressed in cumulative growth rates in percent. Hours and capacity utilization are expressed in percentage deviations from their respective steady-state values.

Figure 6. Ramsey Response to a Government Purchases Shock



Source: Authors' computations.
Note: The size of the initial innovation to government purchases is 1 percent of its steady-state value, $\ln(\bar{g}_0 / \bar{g}) = 1\%$. The nominal interest rate and the inflation rate are expressed in levels in percent at an annual rate. Output, wages, investment, and consumption are expressed in cumulative growth rates in percent. Hours and capacity utilization are expressed in percentage deviations from their respective steady-state values.

Figure 7. Ramsey Response to an Investment-Specific Productivity Shock



Source: Authors' computations.

Note: The size of the initial innovation to the neutral technology shock is one standard deviation, $\ln(\mu_{\gamma,0}/\mu_{\gamma}) = 1\%$. The nominal interest rate and the inflation rate are expressed in levels in percent a year. Output, wages, investment, and consumption are expressed in cumulative growth rates in percent. Hours and capacity utilization are expressed in percentage deviations from their respective steady-state values.

In response to a 1 percentage point increase in the growth rate of investment-specific technological change, Ramsey policy calls for an easing of money market conditions (figure 7). Our intuition is that the Ramsey planner tries to mimic the flexible-price equilibrium. In the absence of price stickiness, real interest rates would fall. Hence the Ramsey planner lowers nominal rates in order to reduce real rates without putting upward pressure on inflation.

5. Optimal Operational Interest Rate Rules

Ramsey outcomes are mute on the issue of what policy regimes can implement them. The information on policy one can extract from the solution to the Ramsey problem is limited to the equilibrium behavior of policy variables such as the nominal interest rate, information that is in general of little use for central banks seeking to implement the Ramsey equilibrium. Specifically, the equilibrium process of policy variables in the Ramsey equilibrium is a function of all of the states of the Ramsey equilibrium. These state variables include all of the exogenous driving forces and all of the endogenous predetermined variables. Among this second set of variables are past values of the Lagrange multipliers associated with the constraints of the Ramsey problem. Even if the policymaker could observe the state of all of these variables, using the equilibrium process of the policy variables to define a policy regime would not guarantee the Ramsey outcome as the competitive equilibrium. The problem is that such a policy regime could give rise to multiple equilibria.

A simple interest rate feedback rule implements the Ramsey equilibrium in the medium-scale model under study. Specifically, we focus on finding parameterizations of interest rate rules that satisfy the following four conditions: they are simple, in the sense that they involve only a few observable macroeconomic variables; they guarantee local uniqueness of the rational expectations equilibrium; the associated path of the nominal interest rate does not violate the zero bound;[7] and they maximize the expected lifetime utility of the representative household conditional on the initial state of the economy being the deterministic steady state of the Ramsey economy. We refer to rules that satisfy the first three criteria as operational

7. We approximate this constraint by requiring that in the competitive equilibrium, two standard deviations of the nominal interest rate be less than the steady-state level of the nominal interest rate.

rules. We refer to operational rules that satisfy the fourth criterion as optimal operational rules.

The family of rules that we consider consists of interest rules whereby the nominal interest rate depends linearly on its own lag, the rates of price and wage inflation, and the growth rate of output. Formally, the interest rate rule is given by

$$\ln\left(\frac{R_t}{R^*}\right) = \alpha_\pi \ln\left(\frac{\pi_t}{\pi^*}\right) + \alpha_W \ln\left(\frac{\pi_t^W}{\pi^{W^*}}\right) + \alpha_y \ln\left(\frac{y_t}{y_{t-1}\mu_y^*}\right) + \alpha_R \ln\left(\frac{R_{t-1}}{R^*}\right). \qquad (49)$$

The target values R^*, π^*, π^{W^*} and μ_y^* are assumed to be the Ramsey steady-state values of their associated endogenous variables. (The steady-state growth of output is indeed exogenous and given by μ_{z^*}.) The variable π_t^W denotes nominal wage inflation; in the nonstochastic steady state $\pi^{W^*} \equiv \mu_{z^*}\pi^*$. It follows that in our search for the optimal operational policy rule, we choose the four policy parameters (α_π, α_W, α_y, α_R) so as to maximize welfare, $V_t \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - bc_{t-1}, h_t)$, where expectations are taken conditional on the initial state being the nonstochastic steady state of the Ramsey equilibrium. Given the complexity of the model, an exact numerical solution does not exist. We therefore approximate our conditional welfare measure to second-order accuracy using the numerical method developed in Schmitt-Grohé and Uribe (2004d).

5.1 The Optimal Operational Rule

The optimal operational interest rate is given by

$$\ln\left(\frac{R_t}{R^*}\right) = 5.0 \ln\left(\frac{\pi_t}{\pi^*}\right) + 1.6 \ln\left(\frac{\pi_t^W}{\pi^{W^*}}\right) - 0.1 \ln\left(\frac{y_t}{y_{t-1}\mu_y^*}\right) + 0.4 \ln\left(\frac{R_{t-1}}{R^*}\right).$$

It is active in both price and wage inflation, because both coefficients are greater than unity. In addition, the rule prescribes virtually no response to output growth. In this sense the optimized interest rate rule can indeed be interpreted as a pure inflation targeting rule. According to the rule, policymakers react positively to lagged nominal interest rates. Because the interest rate coefficient is less than unity, the rule is inertial but not superinertial. Thus policymakers are backward looking in their response to inflation deviations from target.

To quantify the difference between the level of welfare under the Ramsey policy and the optimal operational rule, we compute the welfare costs of the optimal operational interest rate rules relative to the time-invariant equilibrium process associated with the Ramsey policy. We assume that at time zero all state variables of the economy equal their respective Ramsey steady-state values. Because the nonstochastic steady state is the same across all policy regimes we consider, computing expected welfare conditional on the initial state being the nonstochastic steady state ensures that the economy begins from the same initial point under all possible polices.

We denote the contingent plans for consumption and hours under the Ramsey policy by c_t^r and h_t^r. We denote the contingent plans under the alternative policy regime by c_t^a and h_t^a. Let λ^c denote the welfare cost of adopting policy regime a instead of the Ramsey policy conditional on a particular state in period zero. We define λ^c as the fraction of regime r's consumption process that a household would be willing to give up to be as well off under regime a as under regime r. It follows that λ^c is implicitly defined by[8]

$$E_0 \sum_{t=0}^{\infty} \beta^t U((c_t^a - bc_{t-1}^a), h_t^a) = E_0 \sum_{t=0}^{\infty} \beta^t U((1-\lambda^c)(c_t^r - bc_{t-1}^r), h_t^r).$$

One can derive an unconditional welfare cost measure in a similar manner. That is, one can ask what fraction of consumption under the Ramsey policy are agents willing to give up to attain the same unconditional expectation of lifetime utility as under the alternative policy. Let λ^u denote this unconditional welfare cost measure. Then λ^u is implicitly given by

$$E\left\{\sum_{t=0}^{\infty} \beta^t U((c_t^a - bc_{t-1}^a), h_t^a)\right\} = E\left\{\sum_{t=0}^{\infty} \beta^t U((1-\lambda^u)(c_t^r - bc_{t-1}^r), h_t^r)\right\}.$$

We restrict attention to approximations of λ^c and λ^u that are accurate up to second order (see the appendix for a derivation).

The welfare costs of following the optimal operational interest rate rule rather than the Ramsey policy are virtually zero; agents are willing to give up less than 0.001 percent of the Ramsey consumption

8. For analytical convenience we apply the factor $(1 - \lambda^c)$ to c_{-1}, even though this variable is predetermined at the time of the policy evaluation. In Schmitt-Grohé and Uribe (2004d), we show that if one were not to apply the factor $(1 - \lambda^c)$ to c_{-1}, one would obtain a welfare cost measure that is slightly smaller than the one obtained here. However, because the alternative welfare cost measure is proportional to the one used here, the welfare rankings would be unchanged. Our conclusion that the optimal operational rule yields virtually the same level of welfare as the Ramsey policy would only be strengthened.

Table 4. Welfare under the Optimal Operational Rules

Parameterization	α_π	α_W	α_y	α_R	$(100\times\lambda^c)$	$(100\times\lambda^u)$	$c_{2006}\lambda^c$	$c_{2006}\lambda^u$
Optimized rules (equation 48)								
Baseline calibration	5.0	1.6	-0.1	0.4	0.001	0.001	$0.23	$0.19
High wage stickiness ($\tilde{\alpha}$ = 0.9)	0.4	1.9	0.1	2.3	0.008	0.005	$2.50	$1.41
Ad hoc rule								
Taylor rule—output level	1.5	0.0	0.5	0.0	0.14	0.16	$41.81	$48.06

Source: For consumption in 2006, Bureau of Economic Analysis (www.bea.gov).
Note. The variable $c_{2006} \equiv \$30{,}441$ denotes nominal U.S. per capita personal consumption expenditures seasonally adjusted at annual rates in the first quarter of 2006.

stream (less than 23 cents a year) to be as well off under the optimal operational rule as under the Ramsey policy (table 4).

A central characteristic of the optimal rule is that its response to output is mute. Forcing the output coefficient, α_y, to be zero, increases the welfare cost by less than 1 cent a year. This finding has an important policy implication. Central banks need not respond to a measure of output in order to implement an equilibrium that provides virtually the same level of welfare as the Ramsey policy.

While it is true that responding to output has virtually zero welfare gains, it may have significant welfare costs. In table 4 we consider a Taylor rule with a coefficient of 0.5 on deviations of output from trend ($\ln(Y_t/Y)$) and an inflation coefficient of 1.5. This rule is associated with welfare costs of almost \$50 a year per person (\$200 a year per four-person household).

Interest rate smoothing is not essential from a welfare point of view in this economy. Under the optimal rule, the interest rate smoothing coefficient is 0.4. If one eliminates interest rate smoothing by setting $\alpha_R = 0$ while keeping the other rule coefficients at $\alpha_\pi = 5$, $\alpha_y = 0$, and $\propto_{\pi W} = 1.6$, the welfare costs of the rule increase by 3 cents a year to 26 cents a year, which we regard as negligible.[9]

Next we address the question of how important it is for the central bank to respond to both wage and price inflation rather than to just price inflation. Setting $\propto_{\pi W} = \alpha_y = \alpha_R = 0$ and leaving α_π at the optimized value of 5 increases welfare costs to 81 cents a year per person (0.003 percent of annual consumption). This is still a fairly small number, which leads us to conclude that a simple policy prescription—namely, responding aggressively to price inflation only—can bring about an equilibrium in which agents are virtually as well off as under the Ramsey policy. In this sense we can interpret our findings as supportive of inflation targeting policies.

Table 4 also presents the optimal operational rule coefficients when wages are reoptimized every 10 quarters ($\tilde{\alpha} = 0.9$). In the baseline calibration, we draw from the work of ACEL and assume that wage contracts are reoptimized about every third quarter ($\tilde{\alpha} = 0.69$). ACEL adopt the Erceg, Henderson, and Levin (2000) model of nominal wage stickiness. Under this formulation, wage dispersion generates heterogeneity in work intensity across households. In our formulation,

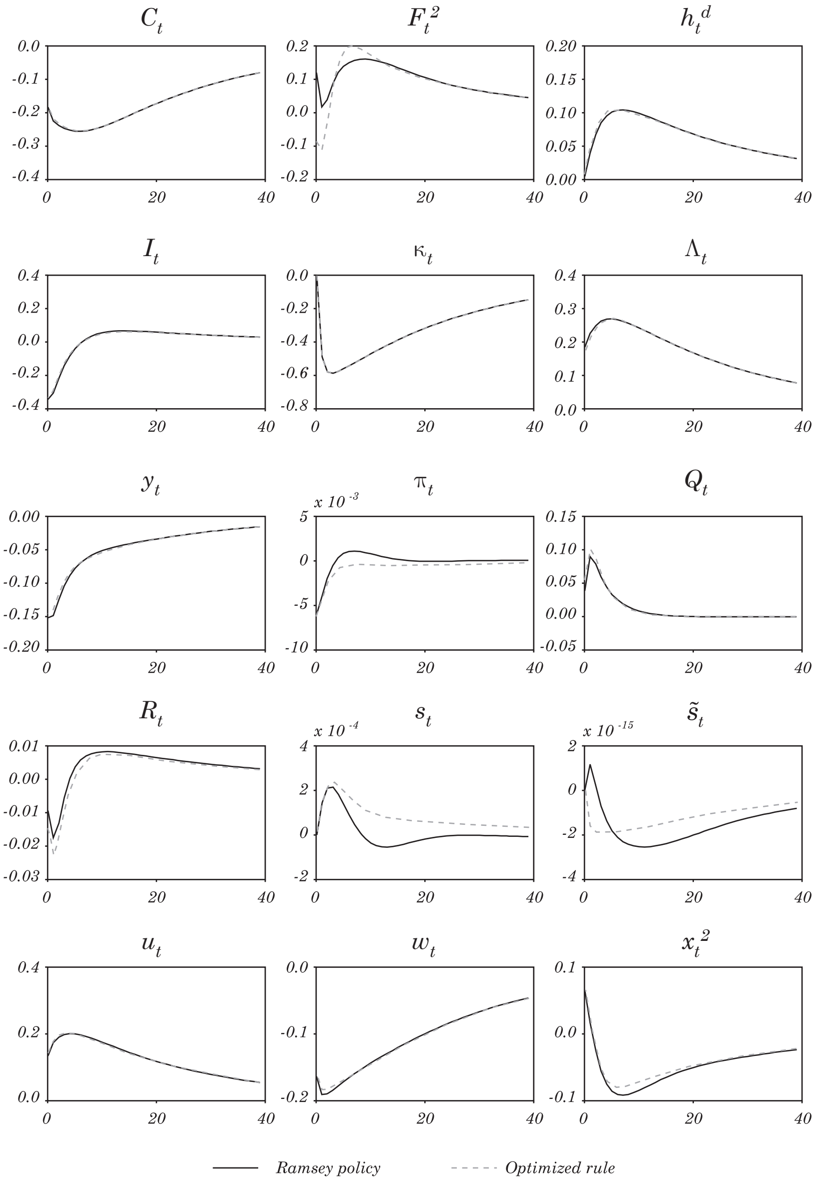
9. In Schmitt-Grohé and Uribe (forthcoming), we study a simpler model without nominal wage rigidity or growth. We find that the optimal interest rate rule delivers virtually the same level of welfare as the Ramsey policy, that the optimal response to output is nil, that responding to output can entail significant welfare costs, and that the welfare gains from interest rate smoothing are negligible.

all households supply the same amount of labor. In equilibrium these two modeling strategies result, up to first order, in a different labor mark-up coefficient in the wage Phillips curve. Specifically, the log-linear approximation to the wage inflation Phillips curve in the ACEL model can be written as $\hat{\pi}_t^W - \hat{\pi}_{t-1} = \beta(E_t\hat{\pi}_{t+1}^W - \hat{\pi}_t) - \gamma\hat{\hat{\mu}}_t$, where $\gamma = \left[1/(1-\tilde{\eta})\right] \times \left[(1-\tilde{\alpha})(1-\beta\tilde{\alpha})/\tilde{\alpha}\right]$. In our model, under the assumption of full wage indexation, $\tilde{\chi} = 1$ (as maintained in ACEL and in our baseline calibration), the wage Phillips-curve is given by $\hat{\pi}_t^W - \hat{\pi}_{t-1} = \beta(E_t\hat{\pi}_{t+1}^W - \hat{\pi}_t) - (1+\tilde{\eta})\gamma\hat{\hat{\mu}}_t$. This means that the coefficient on the labor market mark-up differs in the two models by a factor of $(1+\tilde{\eta})$. Given the estimated value for γ reported by ACEL and given our baseline values for $\tilde{\eta}$ and β of 21 and $1.03^{-0.25}$, respectively, the implied value of $\tilde{\alpha}$ in the context of our model is about 0.9. With this degree of wage stickiness, the optimized interest rate rule calls for a more aggressive response to wage inflation and a less aggressive response to price inflation. In addition, the optimal rule now displays a superinertial response to lagged interest rates. The rule continues to call for a mute response to output variations. The welfare differences between the optimal operation rule and the Ramsey policy are still small, at 0.005 percent of the Ramsey consumption stream.

In computing the coefficients of the optimized policy rule, we have restricted attention to maximizing lifetime utility of the representative household conditional on a particular initial the initial state of the economy being the nonstochastic Ramsey steady state. Alternatively, one could pick policy-rule coefficients so as to maximize an unconditional measure of lifetime utility. Our results are robust to adopting this alternative. Specifically, under the unconditional welfare objective, we obtain $\alpha_\pi = 5.1$, $\alpha_W = 1.6$, $\alpha_y = -0.1$, $\alpha_R = 0.4$, $100 \times \lambda^c = 0.001$, and $100 \times \lambda^u = 0.001$.

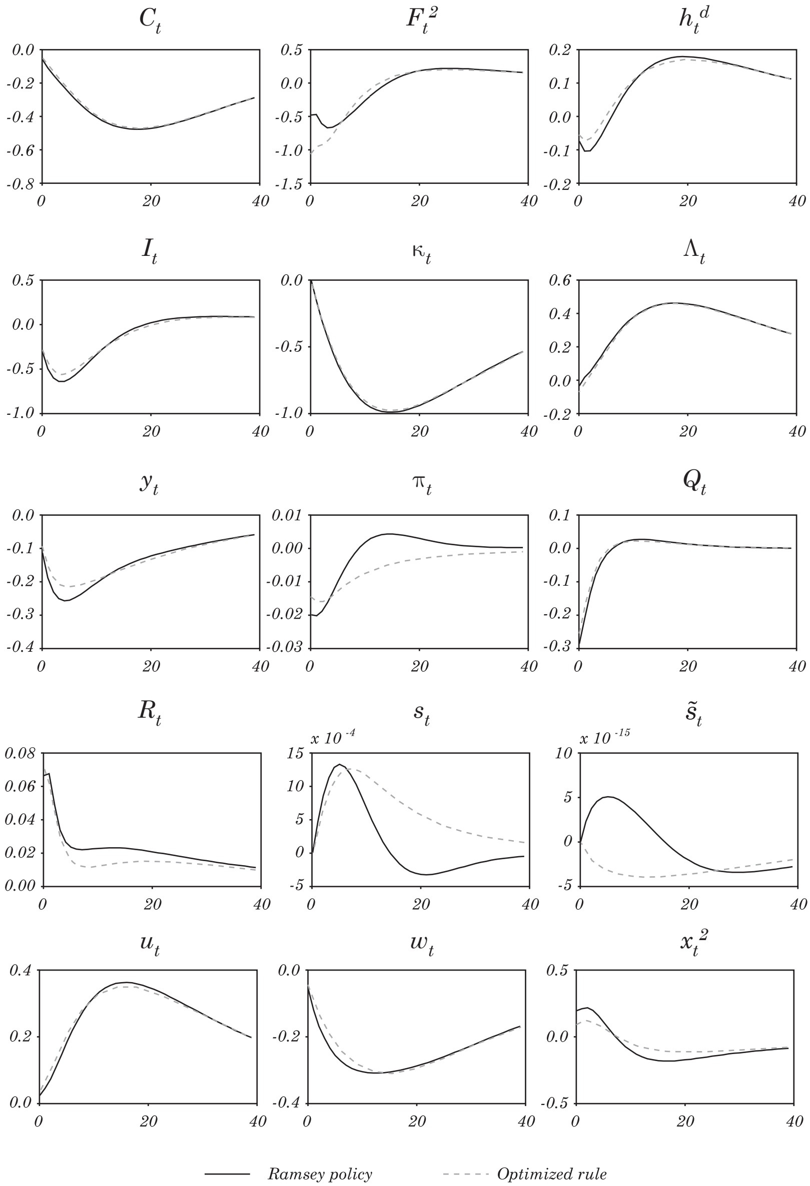
Figures 8, 9, and 10 compare the impulse responses of all variables of the model to the three shocks driving aggregate fluctuations under the Ramsey-optimal policy (solid lines) and under the optimized operational interest rate rule (broken lines). Inflation and the nominal interest rate are shown in percent per quarter deviations from their steady-state values. All other variables are expressed in percent deviations from their deterministic steady state. Variables in capital letters are stationarity-inducing transformations of the corresponding variables in lowercase letters. The figures suggest a remarkable match between the Ramsey responses and the impulse responses associated with the optimized operational interest rate rule.

Figure 8. Ramsey and Optimized Responses to an Investment-Specific Productivity Shock



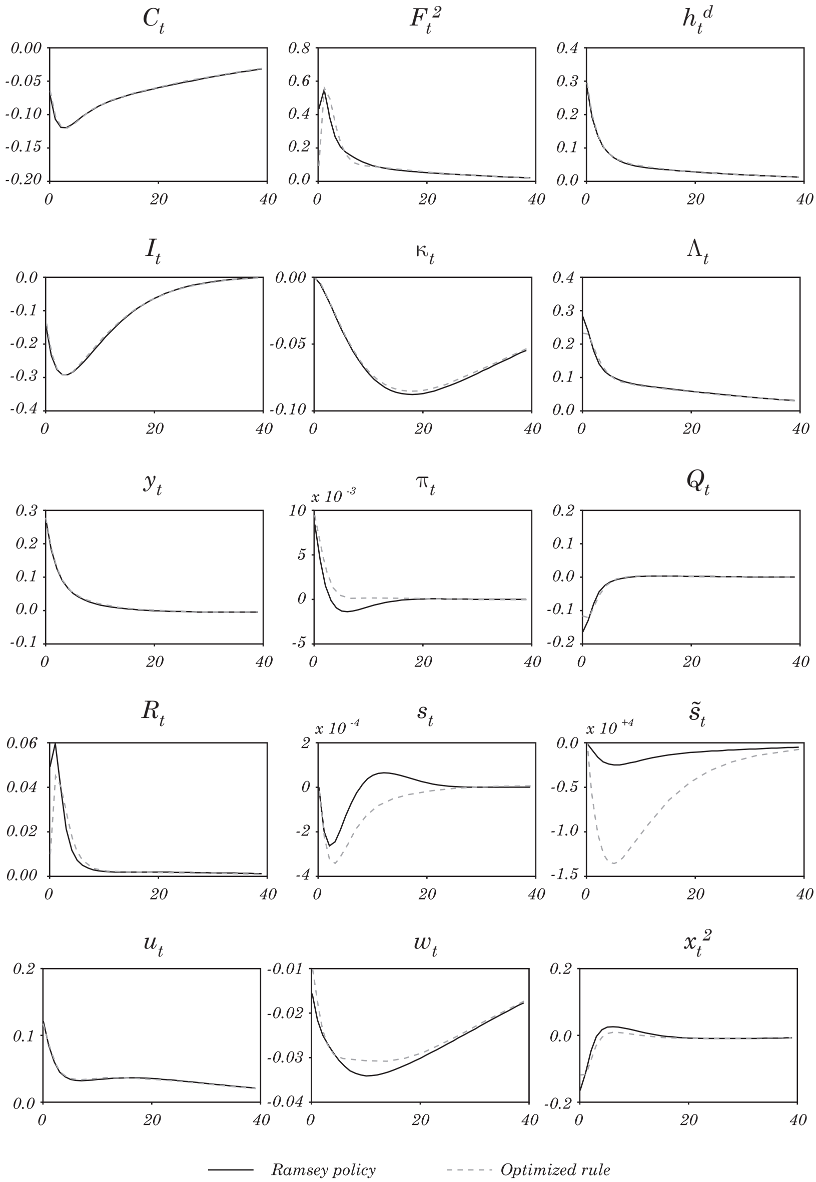
Source: Authors' computations.

Figure 9. Ramsey and Optimized Responses to a Neutral Productivity Shock



Source: Authors' computations.

Figure 10. Ramsey and Optimized Responses to a Government Purchases Shock



Source: Authors' computations.

5.2 Interest Rate Rules and Equilibrium Determinacy

For an interest rate feedback rule to be operational, we require that it induce a locally determinate rational expectations equilibrium. A natural question is what restrictions this requirement imposes on the values that the parameters defining the interest rate rule can take.

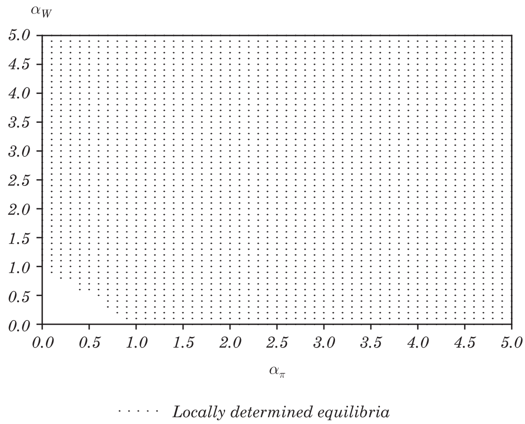
Figure 11 displays the values of the price- and wage-inflation coefficients (α_π and α_W) in the interest rate rule (49) for which the equilibrium is locally determinate. The remaining two policy parameters, α_y and α_R, associated with output growth and the lagged interest rate, are set to zero. To a first approximation, a condition for determinacy is that the sum of the price- and wage-inflation coefficients be greater than unity. The result that the inflation coefficient must be greater than unity for the equilibrium to be unique is easily established in small models with few frictions (see, for example, Leeper, 1991). It is of interest that the same principle applies to a much richer theoretical structure, such as the one studied here. Also noteworthy is the apparent perfect substitutability at the margin between the price- and wage-inflation coefficients in ensuring local uniqueness. In effect, at the southwest frontier of the uniqueness area the coefficients satisfy $\alpha_\pi + \alpha_W \approx 1$.

Local uniqueness of equilibrium is related to the long-run values of the inflation coefficients of the interest rate rule. In the example, the inertial term of the policy rule, α_R, is assumed to be nil. As a result, the short- and long-run values of the price- and wage-inflation coefficients coincide and are equal to α_π and α_W, respectively. Increasing the value of α_R to its optimal level of 0.4 results in a local-determinacy area defined by the relation $\alpha_\pi + \alpha_W > 0.6$. This result appears to generalize to other values of the interest rate coefficient. Thus the pattern that appears to emerge implies roughly a determinacy area defined by the relation $[\alpha_\pi + \alpha_W]/(1 - \alpha_R) > 1$. In other words, the long-run value of the price- and wage-inflation coefficients of the interest rate rule must add up to a number larger than unity for the equilibrium to be locally unique.

6. Discussion and Conclusion

The central focus of this study is the characterization and implementation of optimal monetary policy in the context of a rich model of the macroeconomy with parameters and sources of uncertainty estimated to fit observed fluctuations at business cycle frequency. The

Figure 11. Interest-Rate Feedback Rules and Equilibrium Determinacy



Source: Authors' computations.
Note: The policy parameters α_y and α_R are set to zero. All structural parameters take their baseline values, given in table 1.

central recommendation that emerges from the solution of the Ramsey optimization problem is that the central bank should aim at a low and highly stable rate of inflation. This prescription is very much in line with those proposed by advocates of inflation targeting.

At a deeper level, however, the inflation predictions of the Ramsey equilibrium are neither robust nor coincidental with inflation targeting principles. With respect to robustness, the Ramsey-optimal inflation target varies widely with the parameter determining the degree of price stickiness. For empirically plausible values of this parameter, the optimal inflation target ranges from the Friedman rule (that is, minus the real interest rate) to price stability. This apparent hypersensitivity of the optimal rate of inflation calls for an increased effort at obtaining tighter estimates of the amount of nominal sluggishness present in the economy.

An important difference between the predictions of the Ramsey equilibrium and the observed behavior of central banks adhering to inflation targeting regimes is that the Ramsey-optimal rate of inflation is negative (although possibly close to zero) whereas inflation targeters around the world set targets for the inflation rate that are significantly above zero. In the context of the estimated medium-scale model studied here, fear of confronting the zero bound on nominal interest rates can hardly represent an impediment to adopting the Ramsey-optimal rate of inflation. In effect, in the Ramsey equilibrium the nominal interest rate takes an average value of about 4.5 percent a year, with a standard deviation of about 0.05 percent. It follows that the chances that a shock would push the nominal interest rate to zero are negligible.

This result poses a challenge for future researchers to find a theoretical explanation for the optimality of positive inflation targets. Some have argued that the presence of downward inflexibility in nominal prices and wages may provide a justification for setting positive inflation targets. Formalizations of this idea have been limited to highly stylized models. It remains to be seen whether medium-scale models incorporating a realistic degree of nominal downward rigidities can generate optimal inflation targets similar in magnitude to those observed across inflation-targeting countries.

The hypersensitivity of the optimal inflation target to the degree of price stickiness may disappear under certain fiscal arrangements. This is the case, for instance, when fiscal policy is also set optimally and the fiscal authority has access only to distortionary income taxes. Under alternative fiscal scenarios, however, the hypersensitivity may be exacerbated. This is the case, for instance, when the fiscal authority has access to a combination of distortionary and nondistortionary taxes but distortionary taxes are fixed (even if at the level prescribed by the Ramsey steady state), so that lump-sum taxes are used at the margin to achieve intertemporal solvency. The interaction between optimal fiscal and monetary policy in the context of medium-scale models requires much more research.

We limit attention to an economy driven by three shocks that have been shown to account for a sizable fraction of business cycles in the U.S. economy: neutral productivity shocks, investment-specific productivity shocks, and government spending shocks. Ideally, the study of optimal monetary policy would incorporate all of the sources of uncertainty that are important drivers of business cycles in the real world. This study is far from this theoretical desideratum.

Better models are needed, but there are no clear guidelines on how to create them. We are skeptical of the approach—recently adopted in some studies—of using the estimation residuals obtained from econometric estimations of the dynamic general equilibrium model as structural economic sources of uncertainty. In many instances, these estimation errors are hardly interpretable as structural economic shocks and are more likely a reflection of the fact that theory lags behind business cycles. The dimension of the challenge that the presence of these "nonstructural" errors poses for macroeconomic theory is demonstrated by the fact that in most of the estimates of relatively large macroeconomic models, this class of shocks explains the majority of observed business cycle fluctuations.

Appendix
Deriving the Welfare Costs Measure

Consider the Ramsey policy, denoted by r, and an alternative policy regime, denoted by a. We define the welfare associated with the time-invariant equilibrium implied by the Ramsey policy conditional on a particular state of the economy in period 0 as $\tilde{V}_0^r = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^r - bc_{t-1}^r, h_t^r)$, where c_t^r and h_t^r denote the contingent plans for consumption and hours under the Ramsey policy. Using the particular functional form for the period utility function given in equation (47) and setting ϕ_3 to its baseline value of one, we can express the above expression in terms of the stationary transformation of consumption, C_t^r,

$$\tilde{V}_0^r = E_0 \sum_{t=0}^{\infty} \beta^t (1-\phi_4) \ln z_t^* + E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^r - bC_{t-1}^r / \mu_{z^*,t}, h_t^r).$$

Similarly, we define the conditional welfare associated with policy regime a as

$$\tilde{V}_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^a - bc_{t-1}^r, h_t^a),$$

which can be written in terms of the stationary transformation of consumption as follows

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-\phi_4) \ln z_t^* + E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^a - bC_{t-1}^a / \mu_{z^*,t}, h_t^a).$$

Let $V_0^r \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^r - bC_{t-1}^r / \mu_{z^*,t}, h_t^r)$ so that

$$\tilde{V}_0^r = E_0 \sum_{t=0}^{\infty} \beta^t (1-\phi_4) \ln z_t^* + V_0^r \text{, and let}$$

$$V_0^a \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(C_t^a - bC_{t-1}^a / \mu_{z^*,t}, h_t^a) \ ,$$

so that $\tilde{V}_0^a = E_0 \sum_{t=0}^{\infty} \beta^t (1-\phi_4) \ln z_t^* + V_0^a$.

Let λ^c denote the welfare cost of adopting policy regime a instead of the Ramsey policy, conditional on a particular state in period zero. We define λ^c as the fraction of regime r's consumption process that a household would be willing to give up to be as well off under regime a as under regime r. It follows that λ^c is implicitly defined by

$$\tilde{V}_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U((1-\lambda^c)(c_t^r - bc_{t-1}^r), h_t^r).$$

Using the definitions given above, this expression can be written as

$$V_0^a = V_0^r + \frac{(1-\phi_4)}{(1-\beta)} \ln(1-\lambda^c). \tag{50}$$

We restrict attention to an approximation of λ^c that is accurate up to second order. In equilibrium, V_0^a and V_0^r are functions of the initial state vector x_0 and the parameter σ_ε scaling the standard deviation of the exogenous shocks (see Schmitt-Grohé and Uribe, 2004c). Therefore, we can write $V_0^a = V^{ac}(x_0, \sigma_\varepsilon)$ and $V_0^r = V^{rc}(x_0, \sigma_\varepsilon)$. This implies that λ^c must be a function of x_0 and σ_ε as well $\lambda^c = \Lambda^c(x_0, \sigma_\varepsilon)$.

Consider a second-order approximation of the function Λ^c around the point $x_0 = x$ and $\sigma_\varepsilon = 0$, where x denotes the deterministic Ramsey steady state of the state vector. Because we wish to characterize welfare conditional on the initial state being the deterministic Ramsey steady state, in performing the second-order expansion of Λ^c only its first and second derivatives with respect to σ_ε have to be considered. Formally, we have

$$\lambda^c \approx \Lambda^c(x,0) + \Lambda^c_{\sigma_\varepsilon}(x,0)\sigma_\varepsilon + \left(\frac{\Lambda^c_{\sigma_\varepsilon \sigma_\varepsilon}(x,0)}{2}\right)\sigma_\varepsilon^2.$$

Because the deterministic steady-state level of welfare is the same across all monetary policies belonging to the class defined in equation (49), it follows that λ^c vanishes at the point $(x_0, \sigma_\varepsilon) = (x,0)$. Formally, $\Lambda^c(x,0) = 0$. Totally differentiating equation (50) with respect to σ_ε, evaluating the result at $(x_0, \sigma_\varepsilon) = (x,0)$, and using the result derived in Schmitt-Grohé and Uribe (2004c) that the first derivatives of the policy functions with respect to σ_ε evaluated at $(x_0, \sigma_\varepsilon) = (x,0)$ are nil ($V^{ac}_{\sigma_\varepsilon} = V^{rc}_{\sigma_\varepsilon} = 0$), it follows immediately that $\Lambda^c_{\sigma_\varepsilon}(x,0) = 0$. $(x,0) = 0$. Totally differentiating (50) twice with respect to σ_ε and evaluating the result at $(x_0, \sigma_\varepsilon) = (x,0)$ yields $\Lambda^c_{\sigma_\varepsilon\sigma_\varepsilon}(x,0) = (1-\beta/1-\phi_4)[V^{rc}_{\sigma_\varepsilon\sigma_\varepsilon}(x,0) - V^{ac}_{\sigma_\varepsilon\sigma_\varepsilon}(x,0)]$. Thus the conditional welfare cost measure is given by

$$\lambda^c \approx \left(\frac{1-\beta}{1-\phi_4}\right)[V^{rc}_{\sigma_\varepsilon\sigma_\varepsilon}(x,0) - V^{ac}_{\sigma_\varepsilon\sigma_\varepsilon}(x,0)]\frac{\sigma_\varepsilon^2}{2}. \tag{51}$$

References

Altig, D., L.J. Christiano, M. Eichenbaum, and J. Lindé. 2005. "Firm-Specific Capital, Nominal Rigidities, and the Business Cycle. NBER Working paper 11034." Cambridge, Mass.: National Bureau of Economic Research.

Benigno, P. and M. Woodford. 2005. *Optimal Taxation in an RBC Model: A Linear-Quadratic Approach.* NBER Working paper 11029. Cambridge, Mass.: National Bureau of Economic Research.

Bils, M. and P. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112(5): 947–85.

Calvo, G.A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12(3): 383–98.

Carlstrom, C.T. and T.S. Fuerst. 2005. "Investment and Interest Rate Policy: A Discrete Time Analysis." *Journal of Economic Theory* 123(1): 4–20.

Chari, V.V., L.J. Christiano, and P.J. Kehoe. 1995. "Policy Analysis in Business Cycle Models." In *Frontiers of Business Cycle Research,* edited by T.F. Cooley. Princeton, N.J.: Princeton University Press.

Christiano, L.J., M. Eichenbaum, and C.L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113(1): 1–45.

Cogley, T. and A.M. Sbordone. 2005. "A Search for a Structural Phillips Curve." Staff Report 203. New York: Federal Reserve Bank of New York.

Del Negro, M., F. Schorfheide, F. Smets, and R.Wouters. 2004. "On the Fit and Forecasting Performance of New-Keynesian Models." November. Philadelphia, Pa.: University of Pennsylvania.

Erceg, C.J., D.W. Henderson, and A.T. Levin. 2000. "Optimal Monetary Policy with Staggered Wage and Price Contracts." *Journal of Monetary Economics* 46(2): 281–313.

Fisher, J.D.M. 2005. "The Dynamic Effects of Neutral and Investment-Specific Technology Shocks." May. Chicago, Ill.: Federal Reserve Bank of Chicago.

Galí, J. 1999. "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" *American Economic Review* 89(1): 249–71.

Golosov, M. and R.E. Lucas. 2003. "Menu Costs and Phillips Curves." NBER Working paper 10187. Cambridge, Mass.: National Bureau of Economic Research.

Khan, A., R.G. King, and A. Wolman. 2003. "Optimal Monetary Policy." *Review of Economic Studies* 70(4): 825–60.

Kollmann, R. 2003. "Welfare Maximizing Fiscal and Monetary Policy Rules." March. Bonn, Germany: University of Bonn.

Leeper, E.M. 1991. "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies." *Journal of Monetary Economics* 27(1): 129–47.

Levin, A.T., A. Onatski, J.C. Williams, and N. Williams. 2005. "Monetary Policy under Uncertainty in Micro-Founded Macroeconometric Models." Paper presented at the National Bureau for Economic Research's 20th Annual Conference on Macroeconomics, 31 March, Cambridge, Mass.

Ravn, M. 2005. "Labor Market Matching, Labor Market Participation and Aggregate Business Cycles: Theory and Structural Evidence for the United States." October. Florence, Italy: European University Institute.

Rotemberg, J.J. and M. Woodford. 1992. "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity." *Journal of Political Economy* 100(6): 1153–207.

————. 1997. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." In *NBER Macroeconomics Annual 1997,* edited by B.S. Bernanke and J.J. Rotemberg. Cambridge, Mass.: MIT Press

Schmitt-Grohé, S. and M. Uribe. 2004a. "Optimal Fiscal and Monetary Policy under Imperfect Competition." *Journal of Macroeconomics* 26(2): 183–209.

————. 2004b. "Optimal Fiscal and Monetary Policy under Sticky Prices." *Journal of Economic Theory* 114(2): 198–230.

————. 2004c. "Optimal Operational Interest-Rate Rules in the Christiano-Eichenbaum-Evans Model of the U.S. Business Cycle." August. Durham, N.C.: Duke University.

————. 2004d. "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function." *Journal of Economic Dynamics and Control* 28(4): 755–75.

————. 2005. "Optimal Inflation Stabilization in a Medium-Scale Macroeconomic Model: Technical Appendix." Manuscript. Durham, N.C.: Duke University.

————. 2006. "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model." In *NBER Macroeconomics Annual 2005,* edited by M. Gertler and K. Rogoff. Cambridge, Mass.: MIT Press.

————. Forthcoming. "Optimal Simple and Implementable Monetary and Fiscal Rules." *Journal of Monetary Economics*.

Smets, F. and R. Wouters. 2004. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." April. Frankfurt, Germany: European Central Bank.

Sveen, T. and L. Weinke. 2003. "Inflation and Output Dynamics with Firm-Owned Capital." Barcelona, Spain: Universitat Pompeu Fabra.

Taylor, J.B. 1993. "Discretion versus Policy Rules in Practice." *Carnegie Rochester Conference Series on Public Policy* 39(December): 195–214.

Woodford, M. 2003. *Interest and Prices*. Princeton, N.J.: Princeton University Press.

Yun, Tack. 1996. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics* 37(2–3): 345–70.