Monetary Policy and Key Unobservables: Evidence from Large Industrial and Selected Inflation-Targeting Countries

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In informal terms, we are uncertain about where the economy has been, where it is now, and where it is going.

—Donald Kohn

In recent years, the design of monetary policy has focused on gaps—the output gap, the interest rate gap, and the unemployment rate gap have all played a role in policy discussions. Standard models used for policy analysis are either specified in terms of such gaps or imply important roles for these gap variables in the implementation of monetary policy. In each case, the gap is defined as the difference (often in percentage terms) between an observable variable, such as output or unemployment, and an unobserved variable, such as potential output or the natural rate of unemployment.

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The presence of unobservable variables in the definitions of these gaps poses significant problems for central banks as they implement monetary policy. These problems are both conceptual in nature (what is the right definition of the output gap, potential output or the neutral real interest rate?) and practical (which of many empirical strategies for estimating unobservables should be used?). These problems are compounded by the fact that real-time data used to estimate unobservables will be revised in the future, implying that the best estimates available at the time policy decisions must be taken may, in hindsight, diverge significantly from estimates based on subsequent vintages of data.

To estimate these key unobservables, economists have drawn on a variety of methodologies. Univariate approaches based on statistical methods designed to decompose a time series into trend and cycle have been widely used to estimate variables such as potential output or the natural rate of unemployment. Multivariate approaches, in turn, employ the joint behavior of several variables whose trend or cyclical elements may be related. Multivariate strategies offer the possibility of bringing economic structure to bear on the estimation problem by incorporating the restrictions implied by an economic model. For example, Okun's Law suggests a relationship between the output gap and the gap between unemployment and the natural rate of unemployment. Thus, the joint behavior of output and unemployment may provide information that is useful for estimating both these gaps. However, the results obtained by previous researchers studying different time periods or different economies are difficult to compare across countries since estimation methodologies often differ significantly. This hinders the ability to assess how business cycles might be linked across countries, how potential output or the neutral real interest rate in different countries might be related, and how closely related the various gaps might be across a sample of countries.

While the literature on international business cycles employs common methods to estimate output gaps (Backus, Kehoe, and Kydland, 1992), this work typically uses univariate statistical techniques (such as the Hodrick-Prescott filter) to extract the cyclical component of output. A univariate approach ignores the information that is potentially available if one considers the joint behavior of several macroeconomic variables that are affected by the same set of unobservable variables. Variable definitions, sample periods, and the set of unobservables examined also vary across applications to individual countries. And while individual central banks have undertaken efforts to estimate these unobservable variables, their approaches have generally been country specific and have not provided either systematic estimation or comparison across countries.

Garnier and Wilhelmsen (2009) and Benati and Vitale (2007) adopt a joint estimation approach to uncover important unobservables for several countries. Garnier and Wilhelmsen focus on the United States, the euro area, and Germany, while Benati and Vitale study the United States, the United Kingdom, the euro area, Sweden, and Australia. However, this approach has not been extended to include a larger number of inflation-targeting economies or any emerging or developing economies.

Our objective is to provide a consistent approach to estimating potential output, the neutral interest rate, and the natural rate of unemployment, using data from ten economies: the three largest industrial economies (the United States, the euro area, and Japan) and seven inflation-targeting countries (Australia, Canada, Chile, New Zealand, Norway, Sweden, and United Kingdom). Country-by-country estimation of the three unobservables is based on a parsimonious monetary policy model, extending Laubach and Williams' (2003) sequential-step estimation procedure. This allows us to exploit our ten countries' time-series estimates of unobservables to test for commonalities and differences in their macroeconomic developments.

Section 1 provides a brief discussion of the role of unobservables in the design and implementation of monetary policy. This discussion serves, in part, to motivate the variables on which our empirical analysis focuses—namely, potential output, the neutral real interest rate, and the natural rate of unemployment. Section 2 then briefly sets out our empirical strategy. In section 3, we discuss the monetary policy model, the estimation approach, and the data, and report the country-by-country empirical results for parameter estimates and unobservables' time series. Section 4 extends the model and reports the corresponding results and robustness test results for the United States and Chile. Section 5 then uses our estimated series on the key unobservables to provide evidence of common trends, rising macroeconomic stability (the Great Moderation), comovements across our sample economies, and convergence of observables and unobservables in sample countries toward the United States and the euro area. Section 6 concludes and discusses extensions.

1. The Role and Importance of Unobservables in Monetary Policy

In this section, we discuss the role that key unobservables play in policy design. We then briefly review how errors in estimating potential gross domestic product (GDP) and the natural rate of unemployment have contributed to critical policy mistakes.

1.1 Unobservable Variables and Policy Design

The theoretical foundations both for monetary policy analysis and for the empirical models employed by central banks contain several important variables that are not directly observable. The output gap (the log difference between real GDP and an unobserved time-varying benchmark such as potential GDP) and the unemployment rate gap (the difference between the actual unemployment rate and the unobserved natural rate of unemployment) are typically the driving forces explaining inflation. Central banks may also need to monitor these unobservables out of a direct concern for macroeconomic stability. Both potential GDP and the natural rate of unemployment must be inferred from observable macroeconomic variables. Policymakers must also monitor difficult-to-measure expectations of inflation because they need to ensure that private sector expectations are consistent with the central bank's inflation targets (that is, they need to ensure that expectations are anchored) and because movements in inflation expectations can contribute to fluctuations in actual inflation. They also need to adjust policy interest rates to reflect changes in the economy's neutral real interest rate.

The critical role of these unobservable variables in designing monetary policy can be illustrated using a simple New Keynesian model. This benchmark model consists of a forward-looking Phillips Curve, an expectational IS relationship, and a specification of policy in terms of either an objective function (which the central bank is then assumed to maximize) or a decision rule (see Clarida, Galí, and Gertler, 1999).

If the central bank's objective is to minimize the volatility of inflation and the gap between output and potential output, then optimal policy (under discretion) can be described in terms of what Svensson and Woodford (2005) call a targeting rule. Such a rule involves ensuring that a weighted sum of the output gap and the inflation gap (that is, inflation minus the inflation target) is always kept equal to zero. Intuitively, the output gap should be negative when inflation is above target, as this will tend to produce a fall in inflation and thus bring inflation back to its target level. Similarly, the output gap should be positive when inflation is below target. The Bank of Norway describes such a targeting relationship between the output gap and inflation in its inflation report, in discussing the desirable properties of future interest rate paths. The discussions of interest rate projections in the Reserve Bank of New Zealand's monetary policy statements are consistent with a similar, though implicit targeting rule. In following such a rule, the central bank knows its inflation target, and it has direct measures of both inflation and output (while the latter may be subject to serious real-time measurement errors, it is directly observable in principle), but it must estimate the level of potential output.

Potential output is not the only unobserved variable the central bank must estimate as it implements policy. To actually implement an optimal targeting rule, the central bank must still determine how to move its policy interest rate to maintain the required relationship between the output and inflation gaps. Determining the nominal interest rate that will implement the optimal policy requires knowledge of the relationship between interest rates and real spending, a relationship commonly summarized in New Keynesian models by an expectational IS curve. Using a standard specification of the IS relationship, one finds that the optimal interest rate will satisfy the following relationship (see Clarida, Galí, and Gertler, 1999):

$$i_t = r_t^* + \left[1 + \frac{\sigma\kappa(1-\rho)}{\rho\lambda}\right] E_t \pi_{t+1}, \tag{1}$$

where i is the nominal interest rate, π is the inflation rate, r^* is the neutral real interest rate, the rate consistent with a zero output gap, and E is the conditional expectations operator.[1] The parameters σ, κ, λ, and ρ are, respectively, the inverse of the interest elasticity of aggregate demand, the output gap elasticity of inflation, the relative weight the policymaker places on output gap volatility relative to inflation volatility, and the degree of serial correlation in shocks to

1. There are numerous ways to write this relationship and to define the various unobservables. For example, it would be more in keeping with standard New Keynesian models to define r^* as the real interest rate consistent with output and the flexible-price equilibrium level of output being equal.

the inflation equation. Both the variables on the right-hand side of equation (1) are unobservable or measurable only indirectly—for example, via surveys, asset prices, or the term structure of interest rates.[2]

To solve for the equilibrium under the interest rate rule given by equation (1), the IS and Phillips curve relationships must also be specified. The ones underlying the derivation of equation (1) take the form

$$x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - E_t \pi_{t+1} - r^*{}_t) \tag{2}$$

and

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + e_t, \tag{3}$$

where x is the output gap and e is a zero-mean stochastic error term. The parameter β is the inflation-expectations elasticity of inflation.

It is clear from equation (1) that the neutral real interest rate will be of critical importance for getting the level of the policy rate right. Under an interest rate operating procedure for monetary policy, the level of the nominal rate when the inflation rate is equal to its target must be consistent with the economy's equilibrium real rate of return. When inflation is equal to its (constant) target level, the Fisher relationship requires that the nominal interest rate equal the neutral rate plus the target inflation rate. Thus, while most of the recent literature emphasizes the importance of the Taylor Principle—that is, the need to adjust the nominal rate more than one for one with changes in inflation—it is equally important to fully adjust the nominal rate in response to changes in the neutral real interest rate. Woodford (2003) has labeled the equilibrium real interest rate associated with the absence of fluctuations resulting from nominal distortions as the Wicksellian real rate. An optimal monetary policy that maintains zero inflation to "undo" the real distortions created by nominal rigidities would ensure that the gap between the nominal interest rate and the Wicksellian rate remains equal to zero.

2. If the inflation-adjustment relationship incorporates lagged inflation, the targeting rule would also include further terms involving forecasts of future inflation rates and output gaps.

Unfortunately, this Wicksellian or neutral real rate is unobservable. It is, however, closely related to another key unobservable—the output gap. In the context of the simple model used to derive equation (1), the neutral real interest rate is proportional to the growth rate of potential real output. Laubach and Williams (2003) use this relationship between these two unobservable variables to help them estimate the neutral real interest rate for the United States.

Equations (2) and (3) also serve to highlight the key role of unobservable variables. The output gap appears in both, as does expected future inflation, while the neutral real interest rate appears in the IS relationship. Before a central bank can actually use this simple framework for policy analysis, methods need to be developed for estimating potential output (to obtain an output gap measure), expected inflation, and the neutral real interest rate.

The difficulties in measuring the output gap go, in some sense, beyond the need to measure potential output, because the very definition of the output gap has evolved over the past twenty years. At the conceptual level, three distinct definitions have been employed. The first definition of the output gap is in terms of the relationship between actual GDP and potential GDP, where potential GDP is typically associated with the level of GDP that would be produced at full employment of labor and capital at normal utilization rates. This is the definition most commonly used in models employed by central banks.

In recent years, the development of the New Keynesian Phillips curve has focused attention on a second definition of the output gap, which the underlying theory identifies as the key variable driving inflation. This is the output gap measured as the gap between actual GDP and the level of GDP that would be produced in the absence of nominal wage and price rigidities. This flexible-price output gap provides a measure of economic fluctuations that are due to nominal rigidities. These nominal rigidities allow monetary policy to have real effects, but they also create real distortions. Standard New Keynesian models imply that monetary policy should aim at eliminating these distortions by minimizing fluctuations in the output gap.

However, stabilizing the flexible-price output gap is difficult, not least because the economy's equilibrium output that would arise if there were no nominal rigidities is clearly not observable, and it cannot be estimated using the (often) univariate statistical approaches employed to estimate potential output. Instead, any estimate must come from employing a dynamic stochastic general equilibrium (DSGE) model that can simulate the behavior of an economy that is not subject to nominal rigidities. Since the correct model of the economy is unknown, any estimate of the output gap will be subject to a great deal of uncertainty. Levin and others (2006) provide one example of a DSGE model that is estimated based on U.S. data, which they use to construct a measure of the flexible-price output level and the associated flexible-price output gap. To date, no central banks have employed such a definition of the output gap in their formal policy models.[3] Nevertheless, many central banks are working on developing DSGE models and applying them to estimate flexible-price output levels, as well as other unobservables.

Finally, a third definition of the output gap is the gap between output and the welfare-maximizing level of output. The gap defined in this manner is sometimes called the welfare gap. While this gap may be the most relevant for policy from a conceptual viewpoint, it is also the hardest to measure. The welfare gap and the flexible-price output gap move together in standard New Keynesian models, so stabilizing one is equivalent to stabilizing the other, a property that Blanchard and Galí (2007) label "the divine coincidence." In general, however, the relationship between the two gap measures holds only under very special conditions. If real wages are sticky or if there are other labor market frictions or fluctuations in distortionary taxes, the flexible-price output gap and the welfare gap will diverge.

In addition to illustrating the general point that hard-to-measure variables are conceptually relevant for policy, equations (1) through (3) highlight the variables that are the primary focus of our study. These are the neutral real interest rate, potential output, and expected inflation. For our purposes, we define the output gap as the log of real GDP minus the log of potential GDP, which is the common definition among central banks. The natural rate of unemployment, which is linked to potential output, does not appear explicitly in equation (1), but we incorporate it into our analysis.

3. A possible exception is models that have developed from the Bank of Canada's Quarterly Projections Model (QPM), such as the Forecasting and Policy System model of the Reserve Bank of New Zealand. This model distinguishes between a long-run component, a short-run equilibrium component, and a cyclical component to output. The output gap is then defined relative to the short-run equilibrium level and thus might correspond to a flexible price output gap. However, the short-run equilibrium level of output is an estimate of a slow-moving trend, based on a multivariate filter. Variables (in addition to output) included in the trend estimation procedure include capacity utilization, unemployment, and inflation. QPM was replaced recently at the Bank of Canada by a new open economy DSGE model, called the Terms-of-Trade Economic Model (ToTEM); see Murchison and Rennison (2006).

1.2 Unobservable Variables and Policy Mistakes

Unobservable variables play a critical role in the design and implementation of optimal monetary policy, but these same variables have also been center stage in a number of accounts of past policy errors.[4] For example, Orphanides (2002, 2003), Erceg and Levin (2003), Reis (2003), and Primiceri (2006) all argue that errors by either policymakers or the public in estimating key macroeconomic variables were central to an understanding of critical episodes in the inflation history of the United States over the past forty years.

Orphanides focuses on the Federal Reserve's real-time overestimation of potential (trend) output following the productivity slowdown of the early 1970s. Simply put, overestimation of potential GDP implied an underestimation of the output gap. This led to a policy stance that was, in retrospect, too expansionary and contributed to producing the Great Inflation of the 1970s. Orphanides and Van Norden (2002) document the difficulties of estimating the output gap when, for policy purposes, this must be done using real-time data.[5] McCallum (2001) draws the conclusion that policymakers should not respond strongly to movements in the estimated output gap.[6]

Primiceri (2006) argues that the Fed's failure to correctly estimate potential output is only part of the story behind the Great Inflation.[7] He argues that if that were the only mistake, inflation would not have risen so much or for so long. The second factor contributing to the persistence of high inflation was the Fed's underestimation of the persistence of inflation. Initial increases in inflation were not expected to persist, so policy did not react strongly. Because potential output was overestimated, economic slowdowns that were

4. See Sargent (2008) for an overview and discussion.

5. The Reserve Bank of New Zealand provides a figure comparing their real-time quarterly output gap estimates and estimates prepared using final data (as of November 2002) for the period 1997–2002 (Reserve Bank of New Zealand, 2004, figure 9, page 15). There are sizable differences between the two: for instance, the final series changes sign four times during the period shown, while the real time series changes sign three times *and never in the same quarter* as the final estimate series.

6. Orphanides and Williams (2002) find that policy rules that respond to the change in the unemployment rate gap or the output gap perform well. One reason might be that differencing eliminates much of the error in measuring the level of the output gap.

7. Primiceri's model is actually expressed in terms of the natural rate of unemployment rather than potential output.

thought to be associated with negative output gaps did not seem to lower inflation. Policymakers thus concluded that inflation was unresponsive to economic activity and that a major recession would be needed to lower inflation. Perceiving that they faced a large sacrifice ratio if they tried to lower inflation, policymakers hesitated to try to bring inflation down. Primiceri develops a simple general equilibrium model in which the policymaker learns about the natural rate and the degree of inflation persistence, and his model accounts for both the policy mistakes of the 1970s, as the Fed underestimated the natural rate of unemployment and overestimated the sacrifice ratio associated with lowering inflation, and the disinflationary shift in policy under Volcker. Primiceri's analysis shows that both the difficulties in estimating unobservable variables and the fact that central banks do not know the true structure of the economy can contribute to policy errors.

The public also faces the need to estimate unobservable variables. Erceg and Levin (2003) focus on shifts in the Fed's implicit inflation target when these shifts are not publicly announced. In this case, the public becomes aware of the shift in target only gradually. Erceg and Levin characterize the Volcker disinflation as the result of a fall in the Fed's target inflation rate. Since this target change was not made explicit through any public announcement, agents overestimated inflation, which led to a significant contraction in real economic activity. While our focus is on estimating unobservable variables for use in designing monetary policy, the work of Erceg and Levin provides a reminder of the consequences that can occur when the central bank's inflation target is, from the perspective of the public, an unobservable.

2. Alternative Approaches to Estimating the Neutral Real Rate, the Output Gap, and the Natural Rate of Unemployment

There is a vast literature that uses a range of empirical techniques to estimate unobservable macroeconomic variables. Our survey is therefore brief and highly selective, focusing on contributions that are the most directly relevant for our own empirical approach. For example, while a large amount of work employs univariate methods to estimate potential output or the natural rate of unemployment, we do not focus on these approaches. We follow multivariate approaches that incorporate information from other macroeconomic variables, usually employing theory to guide the relationship between the variables or employing structural equations motivated by theory. We focus on multivariate approaches that are directly relevant for the methods we use to obtain estimates of key unobservable variables. These approaches generally combine statistical representations borrowed from the literature on identifying trend and cyclical components of a time series with relationships among variables implied by an economic model.

The general methodology we employ involves a multivariate Kalman filter to extract estimates of unobserved components from observed time series. The basic framework can be represented in quite general terms of a specification for the dynamic evolution of a vector $\mathbf{Z}_t$ of unobserved factors and a vector of observed variables $\mathbf{Y}_t$ that are related to $\mathbf{Z}_t$. The evolution of the unobserved variables is given in state-space form by

$$\mathbf{Z}_{t+1} = \mathbf{A}\mathbf{Z}_t + \mathbf{u}_{t+1}. \tag{4}$$

The measurement equations linking $\mathbf{Y}_t$ to $\mathbf{Z}_t$ take the form

$$\mathbf{Y}_t = \mathbf{B}\mathbf{Y}_{t-1} + \mathbf{C}\mathbf{Z}_t + \mathbf{D}\mathbf{Z}_{t/t} + \mathbf{G}\mathbf{X}_t + \mathbf{v}_t, \tag{5}$$

where $\mathbf{Z}_{t/t}$ is the time t estimate of the state vector $\mathbf{Z}_t$ and $\mathbf{X}_t$ is a vector of exogenous and observable variables. Both $\mathbf{u}_{t+1}$ and $\mathbf{v}_t$ are zero-mean stochastic error terms. In section 3, we specify the formulations of equations (4) and (5) that we use in our empirical analysis.

Time t estimates of $\mathbf{Z}_t$ are updated using the Kalman filter. Since

$$\mathbf{Y}_t - \mathbf{B}\mathbf{Y}_{t-1} - (\mathbf{C} + \mathbf{D})\mathbf{Z}_{t/t-1} - \mathbf{G}\mathbf{X}_t$$

is the new information available from observing $\mathbf{Y}_t$ in period t, the equation for updating estimates of $\mathbf{Z}$ is given by

$$\mathbf{Z}_{t/t} = \mathbf{Z}_{t/t-1} + \mathbf{K}\,[\mathbf{Y}_t - \mathbf{B}\mathbf{Y}_{t-1} - (\mathbf{C} + \mathbf{D})\mathbf{Z}_{t/t-1} - \mathbf{G}\mathbf{X}_t]. \tag{6}$$

The basic structure given by equations (4) through (6) has been used extensively to estimate a range of unobservable variables. Data on the observables $\mathbf{Y}_t$ and $\mathbf{X}_t$ are used to estimate the parameter matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$, and $\mathbf{G}$.

An early application of the Kalman filter approach to estimating potential GDP for the United States is provided by Kuttner (1994).[8] Kuttner lets $\mathbf{Z}_t$ consist of trend and cyclical components of output, with the trend following a random walk with drift and the cyclical component described by a second-order autoregressive, or AR(2), process. The vector $\mathbf{Y}_t$ consists of real output and inflation and reflects a Phillips curve relationship. Output is the sum of its trend and cyclical components, and inflation is a function of lagged output growth and the cyclical component of output.

Basistha and Nelson (2007) take a related approach to estimating potential GDP and output in the United States. Like Kuttner, they adopt a latent variable approach and incorporate a Phillips curve relationship. They also include the unemployment rate and allow trend and cyclical components of output to be correlated.

Laubach and Williams (2003) extend the Kuttner framework to incorporate the neutral real interest rate, r^*, as an additional unobserved variable. They assume that r^* is a function of the growth rate of potential GDP and a stochastic component that follows an autoregressive process. They expand the set of measurement equations to include an IS relationship linking the output gap to the gap between the real and neutral interest rates.[9] While this specification allows for an integrated approach to estimating potential GDP and the neutral real interest rate, Laubach and Williams employ a separate univariate inflation-forecasting equation to obtain the estimate of expected inflation they need to construct the real interest rate.

Fuentes, Gredig, and Larraín (2008) further extend the approach of Laubach and Williams by incorporating the unemployment rate and Okun's Law linking the output gap and the gap between the unemployment rate and the natural rate of unemployment. The latter is assumed to follow a random walk. They compare the resulting measures of the output gap for Chile with gap estimates obtained from structural vector autoregressions (VARs) and production function approaches. Interestingly, the estimates based on the Kalman filter provided the best out-of-sample forecasts for inflation.

8. Orphanides and Williams (2002) provide an overview of the literature that estimates the natural rates of unemployment and the neutral real interest rates for the United States.

9. They also allow the growth rate of potential GDP to follow a random walk.

Each of these examples from the literature focuses on a single country; the United States in the cases of Kuttner (1994), Basistha and Nelson (2007), and Laubach and Williams (2003) and Chile in the case of Fuentes, Gredig, and Larraín (2008). The closest formulation to our approach is by Benati and Vitale (2007). They, too, focus on multiple unobservables (namely, potential output, the natural unemployment rate, the neutral real interest rate, and expected inflation), and they obtain estimates of each unobservable for five economies (Australia, the euro area, Sweden, the United Kingdom, and the United States). Benati and Vitale allow for time variation in the model parameters. We restrict our attention to constant coefficient models.

Björksten and Karagedikli (2003) report estimates of the neutral real interest rate for seven countries (namely, Australia, Canada, New Zealand, Sweden, Switzerland, the United Kingdom, and the United States), using a methodology based on long- and short-term interest rates. To extract real interest rates, however, they assume that expected inflation is equal to actual inflation. They find a marked decline since 1998 in neutral real rates for all seven countries.[10] Similarly, Fuentes and Gredig (2008) find evidence of a trend decline in Chile's neutral interest rate.

3. Empirical Results

Our approach, following the preceding literature, is based on a parsimonious New Keynesian specification. We use the core relationships in the New Keynesian model to guide our specification of the linkages between observable variables and the key unobservables as summarized in equation (5). The two relationships from the New Keynesian model that we draw on are the IS equation and the Phillips curve. We also use a Taylor rule to represent monetary policy and Okun's Law to link the unemployment gap and the output gap.

3.1 The Model

We start with a simple backward-looking IS relationship, as in Rudebusch and Svensson (1999), where the output gap (x) is determined by its own lag, the lagged real interest rate gap (the

10. See also Basdevant, Björksten, and Karagedikli (2004).

difference between the observed ex ante real interest rate, r, and the unobserved neutral real interest rate, r^*), and a serially uncorrelated error term (ε_1):

$$x_t = \alpha_1 x_{t-1} + \alpha_2 (r_{t-1} - r^*_{t-1}) + \varepsilon_{1,t}. \tag{7}$$

The output gap is defined as the difference between actual output (y) and unobserved potential output or the natural level of output (y^*), both in logs:

$$x_t = y_t - y^*_t. \tag{8}$$

The second relationship is a standard Phillips curve specification for inflation. We specify this equation in terms of the inflation gap rather than the level of inflation, where the inflation gap, $\overline{\pi}_t$, is the difference between actual inflation and either trend inflation (in the case of non-inflation-targeting countries) or between actual inflation and the target inflation rate (for inflation targeters). The inflation gap is determined by its own lag, the expected inflation gap, the lagged output gap, and a serially uncorrelated error term (ε_2):

$$\overline{\pi}_t = \beta_1 \overline{\pi}_{t-1} + \beta_2 \overline{\pi}^e_t + \beta_3 x_{t-1} + \varepsilon_{2,t}. \tag{9}$$

The inflation gap is an observable variable, given by

$$\overline{\pi}_t = \pi_t - \pi_t^T, \tag{10}$$

where π_t is actual inflation and π_t^T is the trend or target rate. Similarly, the inflation expectations gap is defined as the difference between observed (estimated) inflation expectations and trend or target inflation:

$$\overline{\pi}^e_t = \pi^e_t - \pi_t^T. \tag{11}$$

We specify a standard Taylor rule that relates the observed ex ante real interest rate to the ex ante real natural rate, the real interest rate lag, the inflation expectations gap, the lagged output gap, and a serially uncorrelated error term (ε_3):

$$r_t = r^*_t + \delta_1 (r_{t-1} - r^*_{t-1}) + \delta_2 \overline{\pi}^e_t + \delta_3 x_{t-1} + \varepsilon_{3,t}. \tag{12}$$

Equations (7) through (12) comprise our basic model. As an extension of this model, we add Okun's Law that relates the observed unemployment rate (u) to the unobserved natural rate of unemployment (u^*), the lagged gap between the observed unemployment rate and the natural rate of unemployment, the output gap, and a serially uncorrelated error term (ε_4):

$$u_t = u_t^* + \gamma_1(u_{t-1} - u_{t-1}^*) + \gamma_2 x_{t-1} + \varepsilon_{4,t}. \tag{13}$$

Now we turn to the transition equations of the model corresponding to equation (4) in the schematic formulation of section 2. As in Laubach and Williams (2003), potential output is taken to follow a second-order integrated, or I(2), process and unobserved potential output growth (g) follows a random walk:

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_{5,t} \tag{14}$$

and

$$g_t = g_{t-1} + \varepsilon_{6,t}, \tag{15}$$

where ε_5 and ε_6 are serially uncorrelated error terms.

To close the model, we specify random-walk processes for both the neutral real interest rate and the natural rate of unemployment:

$$r_t^* = r_{t-1}^* + \varepsilon_{7,t} \tag{16}$$

and

$$u_t^* = u_{t-1}^* + \varepsilon_{8,t}, \tag{17}$$

where ε_7 and ε_8 are serially uncorrelated error terms.

3.2 Estimation Method

We closely follow Laubach and Williams' (2003) procedure in estimating our model, adapting it to our specification. As they note, maximum-likelihood estimates of the standard deviations of the innovations to the transition equations of the unobservables, as in equations (14) through (17), are likely to be biased toward zero because of the pile-up problem discussed by Stock (1994). We therefore also use the Stock and Watson (1998) median unbiased estimator to obtain estimates of the signal-to-noise ratios reflected by the ratios of the corresponding residual variances $\lambda_g = \sigma_6/\sigma_5$, $\lambda_r = (1 - \delta_1)\,\sigma_7/\sigma_3$, and $\lambda_u = (1 - \gamma_1)\,\sigma_8/\sigma_4$, where σ_i ($i = 1,\dots 8$) denote the corresponding variances of the error terms, ε_i. We impose the latter ratios when estimating the remaining model parameters by maximum likelihood.

We also follow Laubach and Williams (2003) closely in the subsequent sequential-step estimation procedure. In the first step (following Kuttner, 1994), we apply the Kalman filter to estimate jointly the IS relationship—after substituting equation (8) into (7)—and the Phillips curve—after substituting equations (10) and (11) into (9). In this stage we omit the real interest rate gap from the IS equation and assume that potential output growth (g) is constant. From the latter preliminary estimation, we obtain a preliminary potential output level series from which we compute an estimate of the (preliminary) constant potential output growth. We then estimate equation (14) to test for structural breaks in the level of g. Using Stock and Watson (1998, table 3), we determine a positive value for λ_g when the null of no structural break is rejected.

In the second step, we apply the Kalman filter to estimate jointly the IS relationship, the Phillips curve, the Taylor rule (equation 12), and the transition equations for potential output level (equation 14) and potential output growth (equation 15). At this stage, we impose a preliminary constant neutral interest rate (r^*) in the IS relation and the Taylor rule. We also impose the λ_g estimate obtained in the first step. From the latter preliminary estimation, we obtain an estimate of the (preliminary) constant neutral rate interest rate. We then estimate equation (12) to test for structural breaks in the level of r^*. Using Stock and Watson (1998, table 3), we determine a positive value for λ_r when the null of no structural break is rejected.

In step 3, we estimate jointly the IS relationship, the Phillips curve, the Taylor rule, and Okun's Law (equation 13), in addition to transition equations (14), (15), and (16). We impose a preliminary constant natural unemployment rate in Okun's Law. We also impose the λ_g and λ_r estimates obtained in the first and second steps. From the latter preliminary estimation, we obtain an estimate of the (preliminary) constant neutral unemployment rate. We then estimate equation (13) to test for structural breaks in the level of u^*. Using Stock and Watson (1998, table 3), we determine a positive value for λ_u when the null of no structural break is rejected.

Final step 4 comprises Kalman filter estimation of the full model, imposing the estimates for λ_g, λ_r, and λ_u obtained sequentially in the preceding steps. This yields the final estimates for our model coefficients and time series of unobservables. As in Laubach and Williams, we compute confidence intervals and standard errors for the parameters and unobservables applying Hamilton's (1986) Monte Carlo method.

3.3 Data

Our sample covers ten economies: the three largest industrial economies (namely, the United States, the euro area, and Japan), all of which have central banks that do not explicitly or exclusively target inflation; a group of six industrial countries with inflation-targeting central banks, comprised of New Zealand, Canada, United Kingdom, Australia, Sweden, and Norway; and Chile, an emerging economy with an inflation-targeting central bank.[11]

Time coverage of each country sample is determined by availability of quarterly data. Our standard sample covers the 1970–2006 period. One exception on the long side is the United States (1960–2007) and on the short side exceptions are New Zealand (1974–2006), Norway (1979–2006), and, in particular, Chile (1986–2006).[12] Data sources and definitions are reported in a data appendix.

3.4 Estimation Results

Here we report estimation results for our state-space model in its basic version (without Okun's Law) for all countries. This implies omitting step 3 of the estimation method described above and modifying step 4 accordingly. The model thus consists of equations (7) through (12) and (14) through (16). In section 4 below, we report empirical results based on the extended model that includes equations (13) and (17) for the United States and Chile and the corresponding full four-step estimation procedure.[13]

11. We attempted to include Israel (with 1986–2006 data), but we were not able to attain convergence of our estimation model.

12. We were restricted to using smaller samples owing to the lack of data on monetary policy rates or short-term deposit rates for New Zealand (before 1974) and Norway (before 1979) and the lack of quarterly data for most series for Chile before 1986.

13. We have experimented with two alternative specifications. The first includes one additional lag in both the IS and Phillips curves. In the second, we impose the restriction that the coefficients associated with inflation expectations and lagged inflation sum to unity. We did not obtain successful results applying either of these changes. In the first, we were not able to run the third step, while in the second, we encountered numerical problems.

Tables 1 through 5 report country estimates for the two key ratios of the standard deviations of the residuals (λ_g and λ_r), all structural model parameters, and standard deviations of the equation residuals. We report results for the full sample available for each country and a shorter sample extending from 1986 to 2006 for nine countries, except the United States, where it extends through 2007:2. Figures 1–10 depict the estimated time series of observables and unobservables for each country, consistent with the full-sample estimations.

Our estimation strategy is the following. When obtaining estimation results from the last step (that is, the modified fourth stage of the generalized model), we report them directly. If estimation results were not obtained at either the second or third stages, we conduct a grid search over an interval of values for the standard deviation ratios (λ_g and λ_r), as reported in the footnotes of the tables. We therefore report a varying number of results for each country. For example, for

Figure 1. Inflation, Output, and the Interest Rate in the United States, 1960:1–2007:2 and 1986:1–2007:2[a]

A. Inflation, inflation forecast, and trend inflation

B. Actual output growth and trend output growth

C. Output gap

D. Actual interest rate and natural interest rate

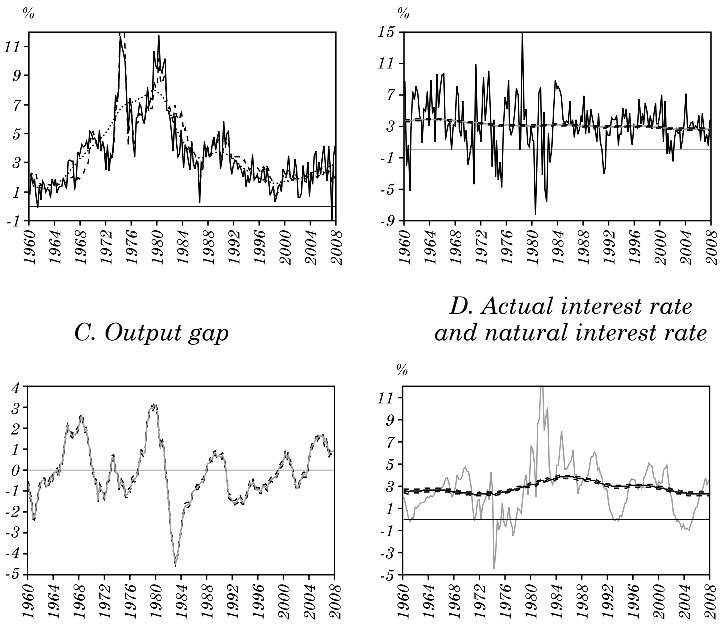
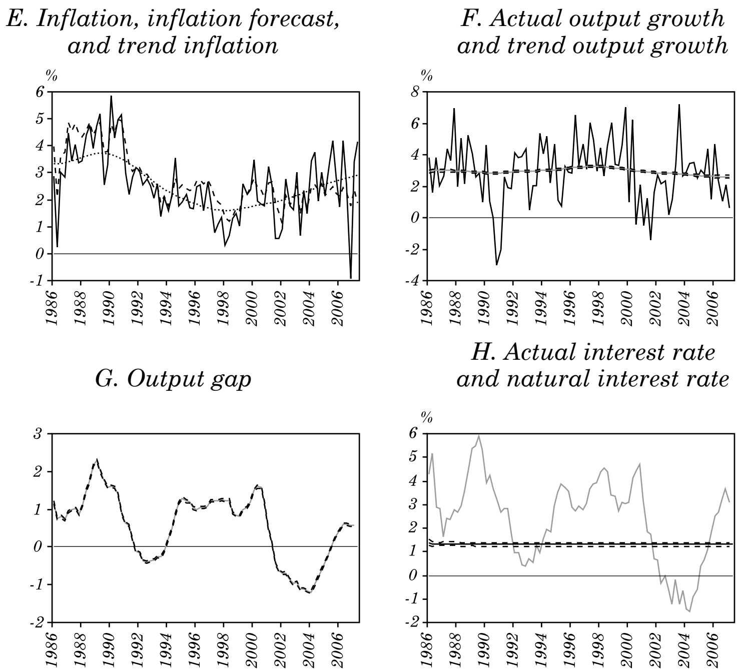


Figure 1. (continued)

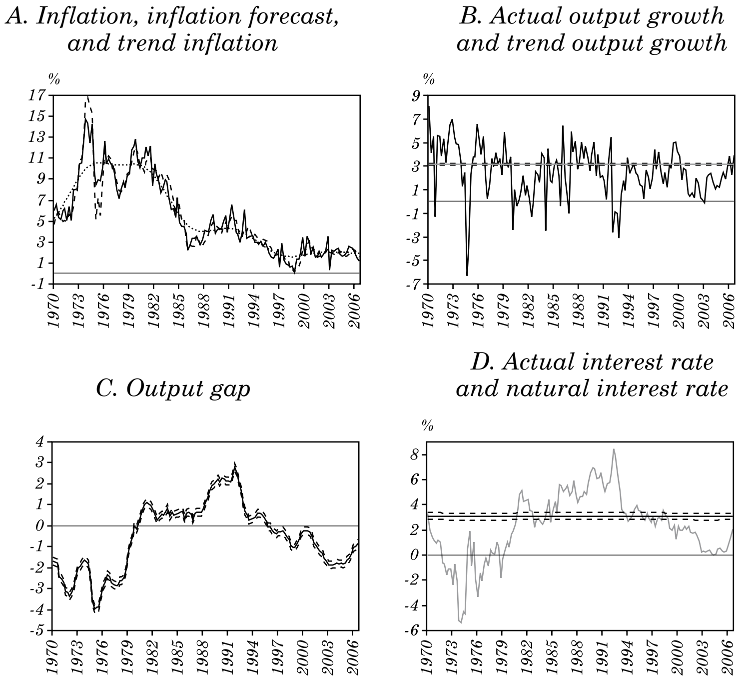


Source: Authors' calculations.
a. In panels A and E actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line. Panels A through D correspond to data from 1960:1−2007:2 and panels from E through H correspond to data from 1986:1−2007:2.

the United States (table 1), we report only one set of results for each sample period, as we obtained estimates for all model parameters. In contrast, we experienced estimation problems in the case of Japan (table 1), so we report a second set of results for each sample period, based on predetermined median values for λ_g and λ_r, corresponding to an interval of values over which we conducted a grid search.

While estimation results differ in significant ways across the ten countries, we point out the following general findings (abstracting from country-specific exceptions), reported in tables 1–5 and figures 1–10. First, the potential growth rate and the neutral real interest rate are typically not constant—not even for the shorter 1986–2006 sample—as reflected by nonzero values of λ_g and λ_r reported in the tables and depicted in the figures. This has implications for the

Figure 2. Inflation, Output, and the Interest Rate in the Euro Area, 1970:2–2006:4[a]



Source: Authors' calculations.
a. In panel A actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line.

construction of output gap measures as well as for the specification of Taylor rules.

Second, point values and significance levels of structural parameter estimates vary from country to country and sometimes from sample to sample for a given country. For example, most parameter estimates conform to our priors in the full-sample estimations for Canada, Chile, and the United States. At the other extreme is Japan, where parameter estimates were hard to obtain and, when estimated over a grid search, often did not conform to expected signs or significance levels.

Third, the IS equation generally reflects very large output gap inertia (reflected in the large and significant parameter estimate of its own lag). However, the sensitivity of the output gap to the lagged real interest rate gap ranges from negative and significant to positive and significant.

Fourth, the Phillips curve generally reflects small but significant inflation gap reversion, suggesting partial reversal of quarterly inflation shocks. (The exception is Chile, which reflects positive inflation gap persistence.) Expected inflation shocks affect inflation gaps positively, significantly, and by a large magnitude in many countries. The lagged output gap raises inflation significantly, positively, and by a sizable magnitude in most countries.

Fifth, the Taylor rule reflects significant inertia in central bank real interest rate innovations in all countries, with the exception of Japan. Most central banks raise nominal interest rates in response to a lagged inflation shock ($\delta_2 \geq -1$), but not enough to satisfy the Taylor principle. (Because we have specified the Taylor rule for the real interest rate, the Taylor principle requires that $\delta_2 \geq 0$.) The exception is Chile, where the coefficient estimate was found to be not

Figure 3. Inflation, Output, and the Interest Rate in Japan, 1970:2–2006:4[a]

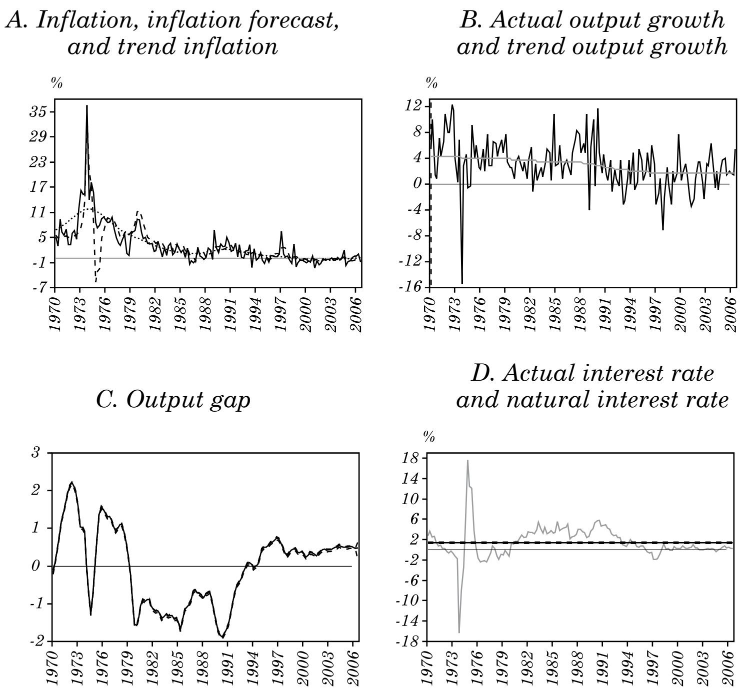
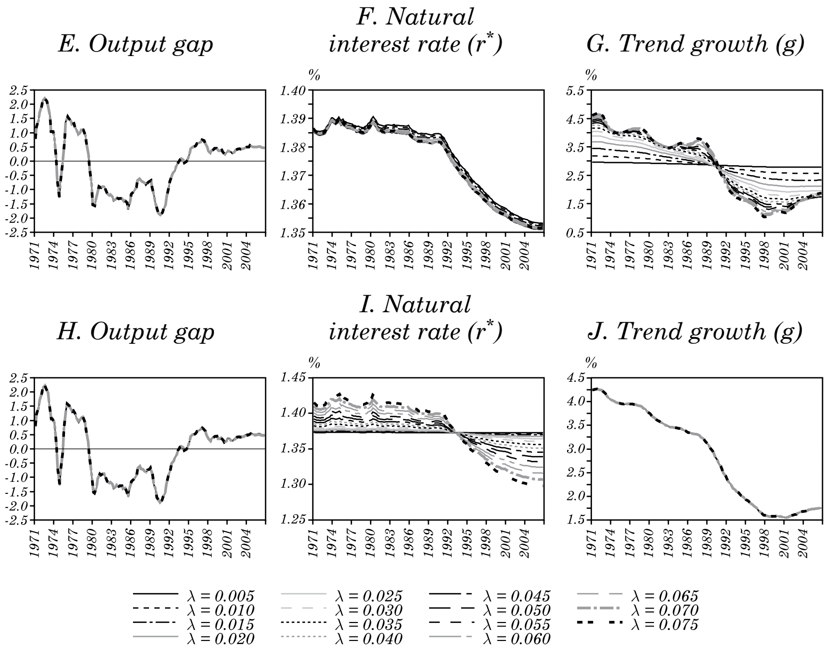


Figure 3. (continued)



Source: Authors' calculations.

a. In panel A actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line. Panels E, F, and G show the unobservables for different grid values for λ_g, while panels H, I, and J show the unobservables for different grid values for λ_r.

significantly different from zero.[14] We obtain a wide range for the interest rate gap response to a lagged output gap shock: monetary policy ranges from countercyclical (United States) to acyclical (Sweden) and to procyclical (Japan).

Finally, judging by conformity of parameter point estimates and significance levels to priors, the best country results were obtained for the United States (1960–2007) and Chile (1986–2006).

Our estimates for unobservables reveal the following results. First, the estimated time series for potential output growth displays

14. This may reflect that Chile's Central Bank responded to a rise in inflation expectations by maintaining its indexed policy rate when it was indexed to past inflation (1986–2000) and raising its nominal rate by the same magnitude of the shock in inflation expectations when the policy rate was set in nominal terms (2001–06).

smooth behavior, but *g* changes over time in most countries (except the euro area and Australia), consistent with positive country estimates for λ_g. Second, with relatively stable potential output growth, the variance of country output gaps is largely determined by the variance in actual output growth rates. Third, similar to potential output growth, the neutral real interest rate follows a smooth pattern in all countries, in line with positive country estimates for λ_r. Fourth, we generally obtained precise estimates for our three unobservables, as reflected by the narrow confidence intervals depicted in the figures. Fifth, we obtain similar estimates for potential output growth and the neutral real interest rates across the long and short samples for most countries. The exceptions are Australia and Norway, for which we obtain neutral interest rates well above actual levels in the shorter samples. Finally, we also obtain similar estimates for output gaps across the long and short samples in many countries. However,

Figure 4. Inflation, Output, and the Interest Rate in New Zealand, 1974:2–2006:4 and 1986:2–2006:4[a]

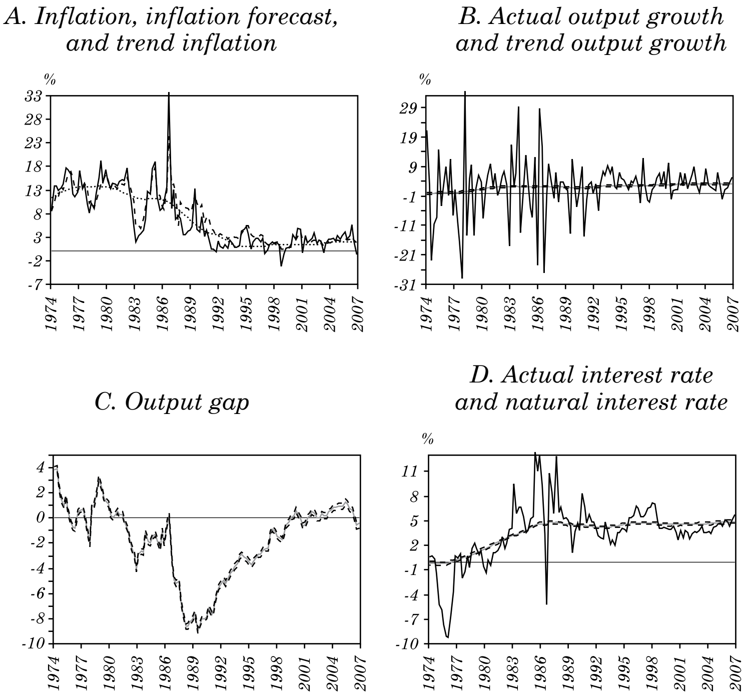
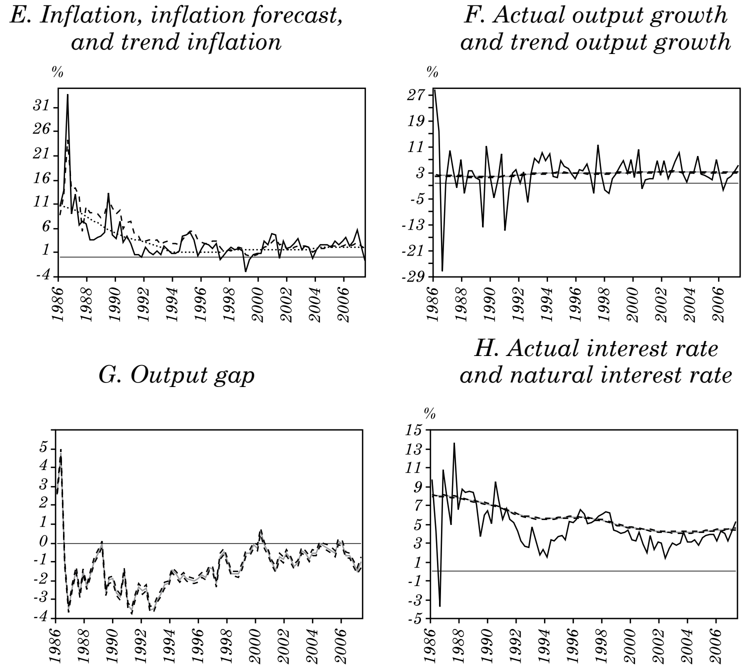


Figure 4. (continued)



Source: Authors' calculations.

a. In panels A and E actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line. Panels A through D correspond to data from 1974:2−2006:4 and panels from E through H correspond to data from 1986:2−2006:4.

in Australia, New Zealand, Sweden, and the United Kingdom, the dynamic pattern, sign, and/or magnitude of output gap estimates differ significantly in the 1986–2006 sample from those obtained for the larger samples. This may reflect small-sample bias. We thus conduct our tests of the Great Moderation, comovements, and convergence across countries based on our large-sample estimates of unobservables.

Figure 5. Inflation, Output, and the Interest Rate in Canada, 1970:2–2006:4 and 1986:2–2006:4[a]

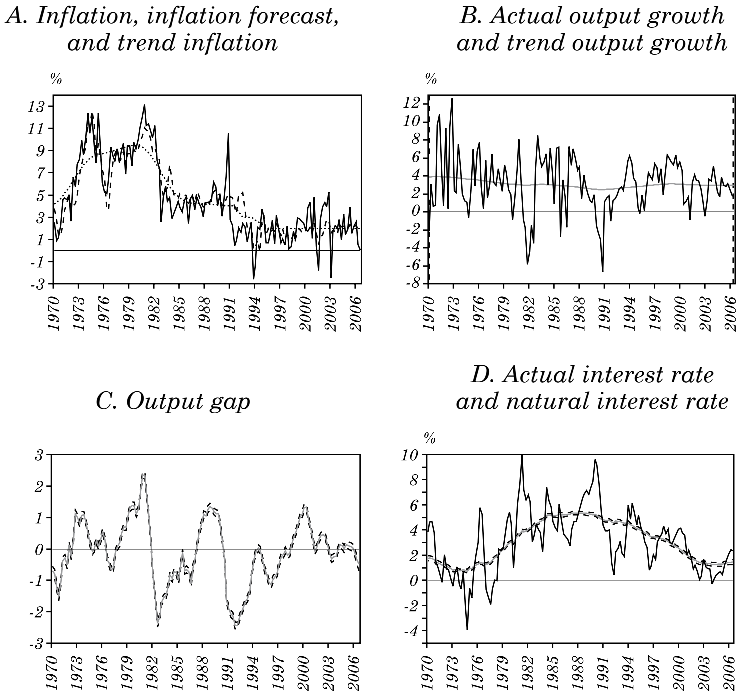
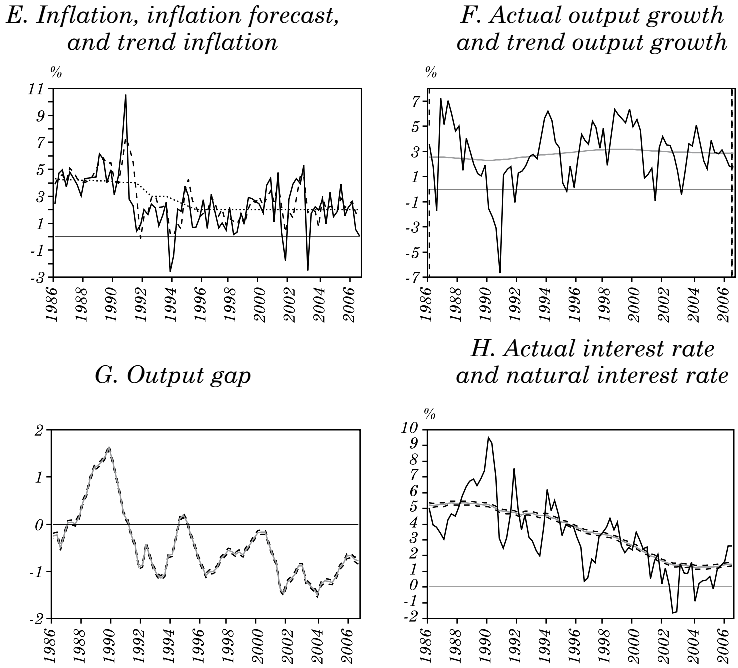


Figure 5. (continued)



Source: Authors' calculations.

a. In panels A and E actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line. Panels A through D correspond to data from 1970:2−2006:4 and panels from E through H correspond to data from 1986:2−2006:4.

Figure 6. Inflation, Output, and the Interest Rate in the United Kingdom, 1970:2–2006:4 and 1986:2–2006:4[a]

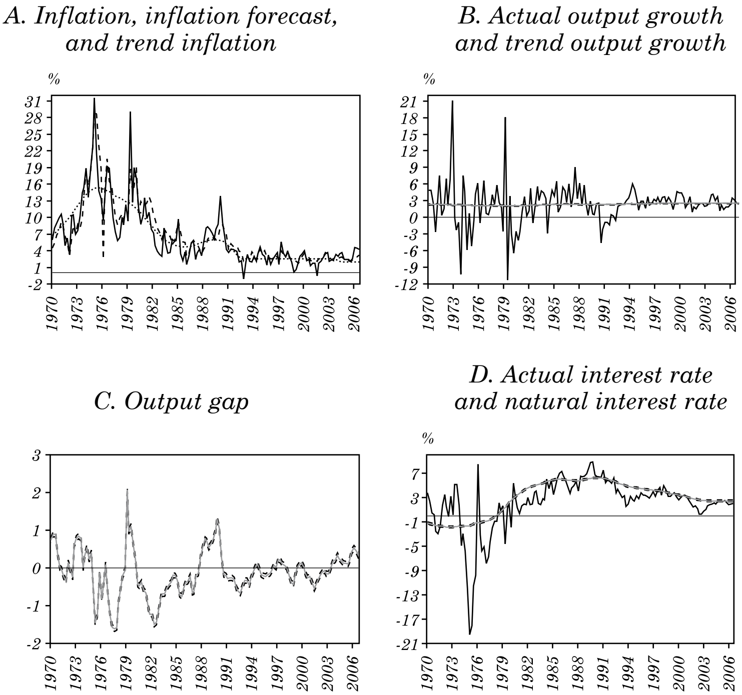
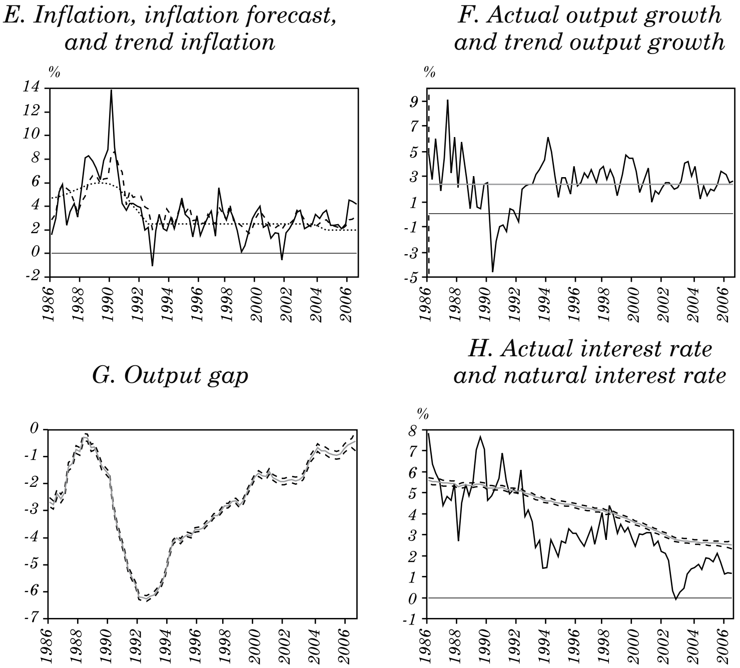


Figure 6. (continued)



Source: Authors' calculations.

a. In panels A and E actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line. Panels A through D correspond to data from 1970:2−2006:4 and panels from E through H correspond to data from 1986:2−2006:4.

Figure 7. Inflation, Output, and the Interest Rate in Australia, 1970:2–2006:4 and 1986:2–2006:4[a]

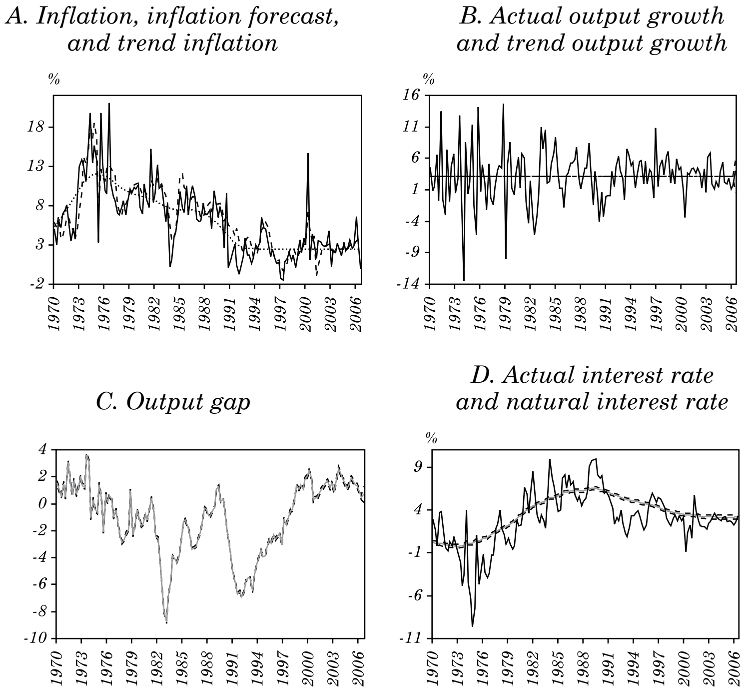
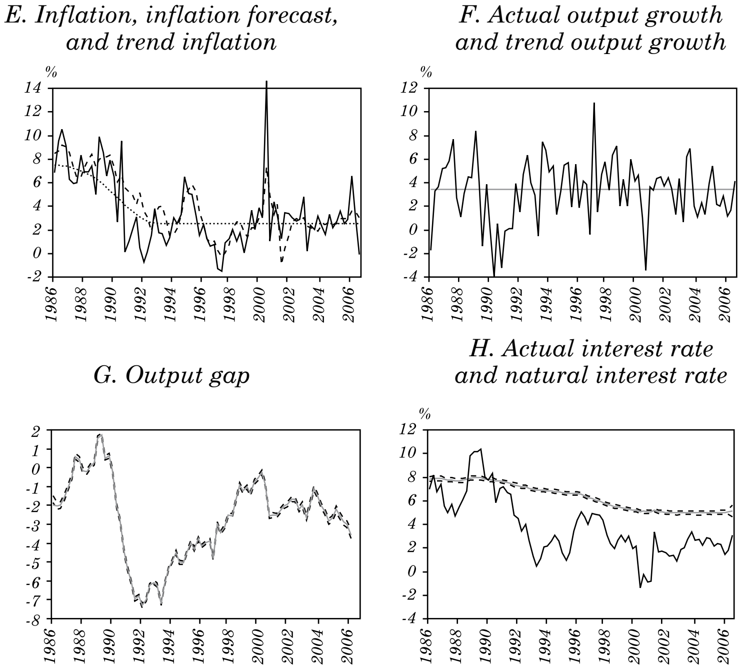


Figure 7. (continued)



Source: Authors' calculations.

a. In panels A and E actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line. Panels A through D correspond to data from 1970:2−2006:4 and panels from E through H correspond to data from 1986:2−2006:4.

Figure 8. Inflation, Output, and the Interest Rate in Sweden, 1970:2–2006:4 and 1986:2–2006:4[a]

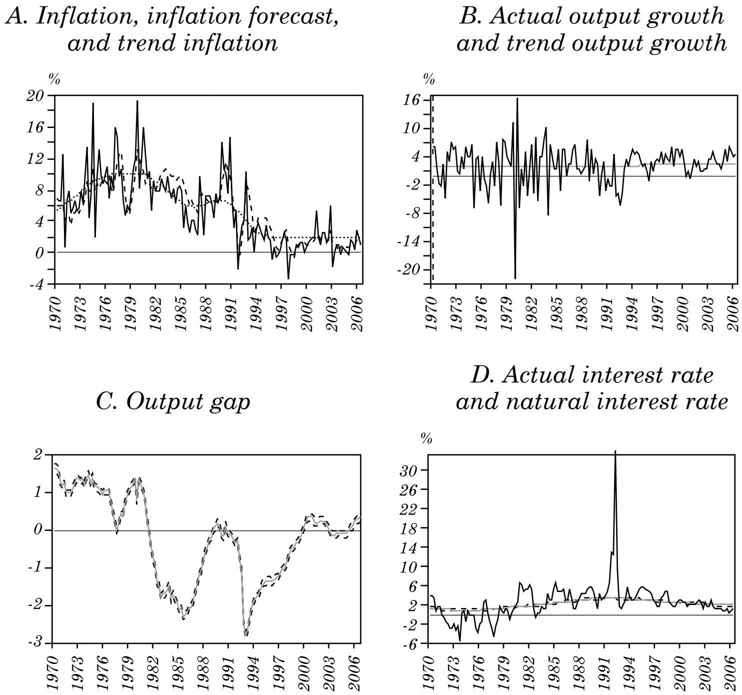
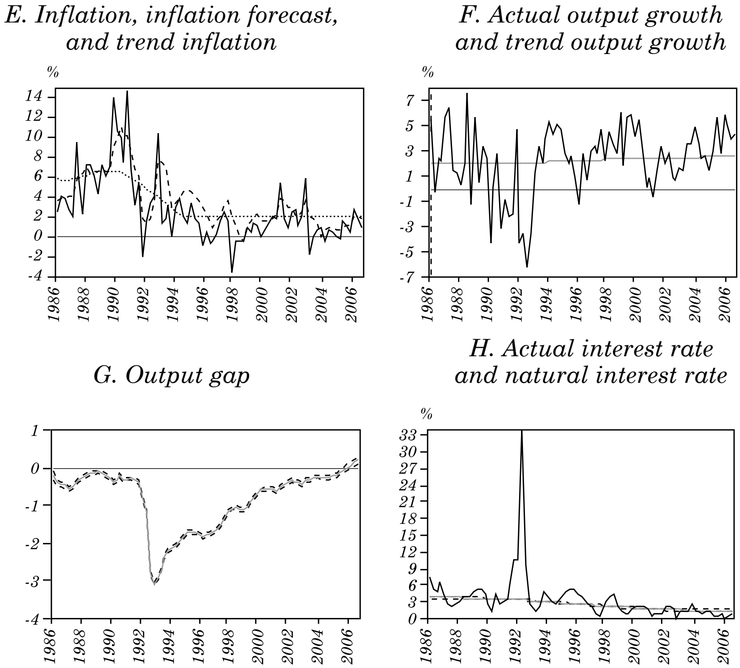


Figure 8. (continued)



Source: Authors' calculations.

a. In panels A and E actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line. Panels A through D correspond to data from 1970:2−2006:4 and panels from E through H correspond to data from 1986:2−2006:4.

Figure 9. Inflation, Output, and the Interest Rate in Norway, 1979:2–2006:4 and 1986:2–2006:4[a]

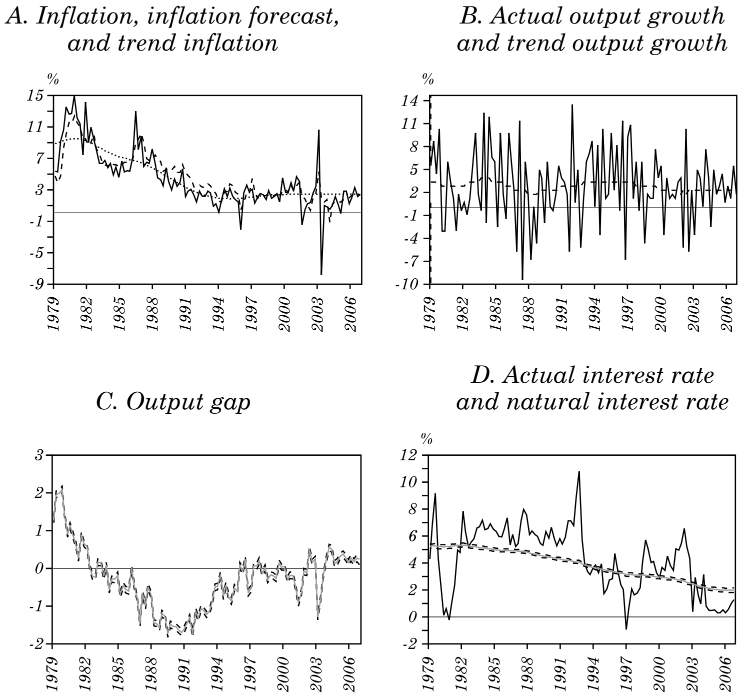
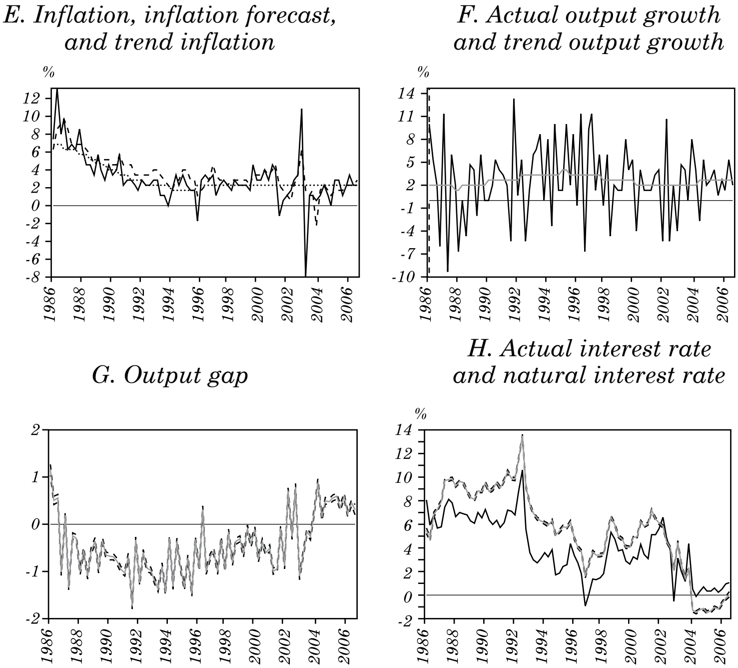


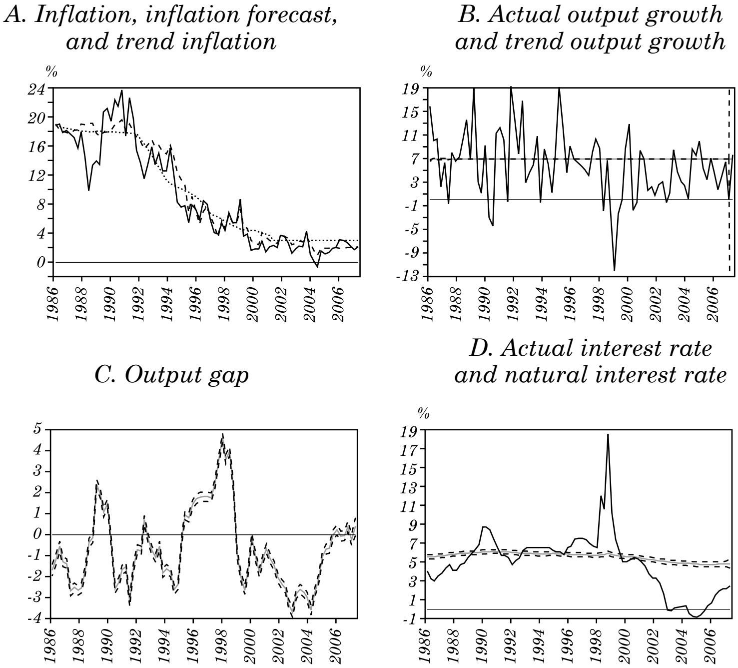
Figure 9. (continued)



Source: Authors' calculations.

a. In panels A and E actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line. Panels A through D correspond to data from 1979:2−2006:4 and panels from E through H correspond to data from 1986:2−2006:4.

Figure 10. Inflation, Output, and the Interest Rate in Chile, 1986:2–2006:4[a]



Source: Authors' calculations.

a. In panel A actual inflation is the solid line, inflation forecast the dashed line and inflation trend the dotted line.

Table 1. Parameter Estimates for the Euro Area, the United States, and Japan[a]

	United States		*Euro Area*		*Japan*			
	1960:01–2007:02	*1986:01–2007:02*	*1970:02–2006:04*	*1986:02–2006:04*	*1970:02 – 2006:04*		*1986 – 2006:04*	
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
λ_g	0.0475	0.0612	0.0000	0.0000	0.0000	0.0400	0.0000	0.0400
λ_r	0.0215	0.1399	0.0214	–	–	0.0400	–	0.0400
α_1	0.9492	1.2285	0.9365	0.9740	0.8227	1.0603	0.9753	1.0784
	(0.0351)	(0.1193)	(0.0582)	(0.0183)	(0.0707)	(0.0285)	(0.0077)	(0.0446)
α_2	–0.0710	–0.1355	0.0264	–	–	0.0562	–	0.1030
	(0.0226)	(0.0844)	(0.0325)	(–)	(–)	(0.0282)	(–)	(0.0494)
β_1	–0.0838	–0.0502	0.0144	–0.2482	–0.2137	0.0557	–0.4258	–0.0802
	(0.0565)	(0.0849)	(0.0650)	(0.0794)	(0.0478)	(7711.2291)	(0.0920)	(1557.7195)
β_2	0.8039	1.2426	0.6498	1.1070	0.6607	0.1374	1.3892	–0.0728
	(0.0486)	(0.1173)	(0.0459)	(0.0899)	(0.0309)	(0.0672)	(0.1317)	(0.1139)
β_3	0.4172	–0.3384	–0.0279	0.0481	2.2984	0.5016	0.0563	0.4485
	(0.1189)	(0.1346)	(0.0272)	(0.0593)	(0.4361)	(0.0583)	(0.0308)	(0.1613)
δ_1	0.8632	0.0251	0.3652	–	–	0.0236	–	0.0616
	(0.0233)	(0.1427)	(0.0490)	(–)	(–)	(0.0238)	(–)	(0.0761)

Table 1. (continued)

	United States		*Euro Area*		*Japan*			
	1960:01–2007:02	*1986:01–2007:02*	*1970:02–2006:04*	*1986:02–2006:04*	*1970:02 – 2006:04*		*1986 – 2006:04*	
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
δ_2	−0.1329	−0.9141	−0.5706	–	–	−0.7107	–	−0.8420
	(0.0289)	(0.1119)	(0.0506)	(–)	(–)	(0.0336)	(–)	(0.0616)
δ_3	0.1272	2.2387	1.0071	–	–	−2.2838	–	−1.2997
	(0.0752)	(0.5900)	(0.1251)	(–)	(–)	(0.9590)	(–)	(0.3804)
σ_y	0.4831	0.1947	0.3581	0.4267	0.4647	0.2167	0.7196	0.2091
	(0.0951)	(0.0462)	(0.0498)	(0.3034)	(0.1000)	(0.0924)	(0.4655)	(0.0762)
σ_π	0.6790	0.7292	0.7362	0.4680	1.3389	2.2620	1.0289	1.3858
	(0.0319)	(0.0406)	(0.0468)	(0.0401)	(0.1248)	(0.1502)	(0.0859)	(0.1207)
σ_r	1.1502	0.0000	0.6101	–	–	0.3874	–	0.1678
	(0.0317)	(5081.2000)	(0.0384)	(–)	(–)	(0.0688)	(–)	(0.0396)
σ_{y^*}	0.6543	0.4367	0.4776	0.1833	0.8164	0.8946	0.3170	0.8532
	(0.1044)	(3687.0000)	(0.1334)	(0.1583)	(0.1592)	(6510.0068)	(0.6622)	(1304.8673)

Source: Authors' calculations.

a. The estimations presented in columns 1, 2, and 3 are from the third step. Column 4 estimations are from the first step; we did not obtain estimations after the first step due to the matrix singularity problem. For Japan, the estimations in columns 5 and 7 are from the first step, since λ_r is not estimated in the second step when we impose $\lambda_g = 0$ due to the matrix singularity problem. In columns 6 and 8, the estimations are from the third step, where λ_r and λ_g are obtained across a grid search in the interval [0.005; 0.075]. Standard errors are in parentheses.

Table 2. Parameter Estimates for New Zealand and Canada[a]

	New Zealand				*Canada*			
	1974:02 – 2006:04		*1986:02 – 2006:04*		*1970:02 – 2006:04*	*1986:02 – 2006:04*		
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
λ_g	0.0544	0.0544	0.0757	0.0757	0.0484	0.0000	0.0484	0.0484
λ_r	0.0000	0.0544	0.0871	0.0757	0.0698	–	0.0698	0.8198
α_1	0.914	0.9462	0.7153	0.6256	0.9598	0.9916	0.8788	0.8773
	(0.0643)	(0.0505)	(0.1345)	(0.0927)	(0.0582)	(0.0187)	(0.0946)	(0.0813)
α_2	–0.0091	0.0203	0.2821	0.2577	–0.0790	–	0.0305	0.0369
	(0.0281)	(0.0396)	(0.0729)	(0.0643)	(0.0291)	(–)	(0.0342)	(0.0464)
β_1	–0.1923	–0.1983	–0.2158	–0.1067	0.0844	–0.3020	–0.2260	–0.2260
	(0.0703)	(0.0685)	(0.221)	(44246.3385)	(14868.7)	(0.0759)	(10298.66)	(10274.93)
β_2	1.4305	1.4288	1.2403	–0.1816	0.0223	1.2527	–0.1199	–0.2318
	(0.0897)	(0.0834)	(0.214)	(0.2006)	(0.0747)	(0.1191)	(0.0886)	(0.0878)
β_3	0.5697	0.5743	0.9306	1.1411	0.6890	0.0739	0.7680	0.8807
	(0.2459)	(0.2219)	(0.2942)	(0.1346)	(0.1242)	(0.2246)	(0.1301)	(0.1433)
δ_1	0.7038	0.5875	0.1262	0.1475	0.7370	–	0.2825	0.1968
	(0.0491)	(0.0472)	(0.0621)	(0.0651)	(0.0420)	(–)	(0.0697)	(0.0684)

Table 2. (continued)

	New Zealand				Canada			
	1974:02 – 2006:04		1986:02 – 2006:04		1970:02 – 2006:04	1986:02 – 2006:04		
Parameters	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
δ_2	−0.3204	−0.3742	−0.6219	−0.5968	−0.2602	–	−0.9390	−0.9290
	(0.0857)	(0.0779)	(0.1614)	(0.1567)	(0.0635)	(–)	(0.0883)	(0.0796)
δ_3	−0.2211	−0.1838	−0.1313	−0.2096	0.3684	–	2.2223	1.5811
	(0.142)	(0.1383)	(0.1412)	(0.1815)	(0.1469)	(–)	(0.4015)	(0.3244)
σ_y	1.1969	1.183	1.0281	1.0015	0.4408	0.5978	0.2605	0.2982
	(0.3918)	(0.3701)	(0.1749)	(0.1928)	(0.0978)	(0.9679)	(0.0624)	(0.0724)
σ_π	1.5029	1.4946	1.5014	1.5179	1.3423	1.1695	1.3798	1.2553
	(0.1417)	(0.1309)	(0.2658)	(0.2073)	(0.0707)	(0.0833)	(0.1163)	(0.1032)
σ_r	2.1501	2.0427	1.5995	1.6071	1.0691	–	0.4273	0.3548
	(0.0847)	(0.0697)	(0.1426)	(0.1417)	(0.0576)	(–)	(0.0557)	(0.0415)
σ_{y^*}	1.9964	2.0137	0.9803	0.9577	0.5649	0.0000	0.5019	0.4724
	(0.7595)	(0.6157)	(0.2825)	(37739.0760)	(11505.45)	(185845.55)	(7749.17)	(7613.58)

Source: Authors' calculations.

a. For New Zealand, the estimations in column 1 are from the second step; we did not obtain estimations in the third step due to the matrix singularity problem. The estimations in column 2 are from the third step, where λ_r is obtained across a grid search in the interval [0.0444; 0.1244]. The estimations in column 3 are from the third step. The estimations in column 4 are from the third step, where λ_r and λ_g are obtained across a grid search in the interval [0.0275; 0.9775]. For Canada, the estimations in column 5 are from the third step. The estimations in column 6 are from the first step, since λ_r is not estimated in the second step when we impose $\lambda_g = 0$, due to the matrix singularity problem. The estimations in column 7 are from the third step, where λ_g and λ_r are obtained in the estimation with the sample 1970-2006. The estimations in column 8 are from the third step, where λ_r is obtained across a grid search in the interval [0.0098; 2.0198]. Standard errors are in parentheses.

Table 3. Parameter Estimates for the United Kingdom and Australia[a]

	United Kingdom				*Australia*			
	1970:02 – 2006:04		*1986:02 – 2006:04*		*1970:02–2006:04*	*1986:02 – 2006:04*		
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
λ_g	0.0275	0.0275	0.0000	0.0275	0.0000	0.0069	0.0069	0.0569
λ_r	–	0.0900	0.0000	0.0600	0.0522	0.0000	0.0522	0.0522
α_1	0.8796	0.6669	0.9776	0.9854	0.9363	0.9669	0.9906	0.9291
	(0.0575)	(0.1249)	(0.0345)	(0.0156)	(0.0415)	(0.05)	(0.0432)	(0.1031)
α_2	–	0.0407	−0.036	−0.0490	0.0022	−0.0237	−0.0036	0.0062
	(–)	(0.0195)	(0.0388)	(0.0427)	(0.0321)	(0.0303)	(0.0453)	(0.0247)
β_1	−0.1142	−0.169	−0.2266	−0.2245	−0.2231	−0.4366	−0.4316	−0.2872
	(0.0601)	(0.0759)	(0.1021)	(0.1017)	(0.0553)	(0.1275)	(0.1214)	(15262.2512)
β_2	0.9532	0.8837	1.3391	1.3271	1.0026	1.3629	1.3581	−0.4366
	(0.0391)	(0.0545)	(0.1148)	(0.0984)	(0.0979)	(0.117)	(0.1111)	(0.1165)
β_3	1.0792	2.4103	0.2045	0.2063	0.3114	0.3246	0.3191	1.2311
	(0.3806)	(0.7842)	(0.1227)	(0.0891)	(0.114)	(0.1883)	(0.1497)	(0.1387)
δ_1	–	0.4519	0.8953	0.7431	0.7168	0.8507	0.7758	0.7554
	(–)	(0.0331)	(0.0555)	(0.0694)	(0.0481)	(0.0526)	(0.0706)	(0.0773)

Table 3. (continued)

	United Kingdom				*Australia*			
	1970:02 – 2006:04		*1986:02 – 2006:04*		*1970:02–2006:04*	*1986:02 – 2006:04*		
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
δ_2	–	−0.7096	−0.1097	−0.0995	−0.3327	−0.2668	−0.2945	−0.3612
	(–)	(0.046)	(0.0935)	(0.0805)	(0.0496)	(0.1081)	(0.0932)	(0.0946)
δ_3	–	0.6368	0.0282	0.0523	0.0438	0.1157	0.1345	0.6577
	(–)	(0.2713)	(0.0403)	(0.0497)	(0.0752)	(0.0976)	(0.0931)	(0.3732)
σ_y	0.6381	0.4554	0.1404	0.4713	1.0046	0.616	0.6615	0.3051
	(0.1161)	(0.1017)	(0.159)	(0.1190)	(0.1178)	(0.1922)	(0.1262)	(0.0995)
σ_π	1.77	1.5288	0.8628	0.8644	2.0193	1.4473	1.4495	1.4616
	(0.1337)	(0.1767)	(0.0611)	(0.0599)	(0.1177)	(0.1283)	(0.126)	(0.1383)
σ_r	–	1.6097	0.788	0.7557	1.6796	0.9827	0.9362	0.8986
	(–)	(0.0818)	(0.0568)	(0.0563)	(0.0757)	(0.083)	(0.0731)	(0.0756)
σ_{y^*}	0.6383	0.7789	0	0.0000	0.0000	0.2168	0.0008	0.5638
	(0.1174)	(0.284)	(4092)	(3802.0530)	(12158.6)	(0.3725)	(157.37)	(11804.8755)

Source: Authors' calculations.

a. For the United Kingdom, the estimations in column 1 are from the first step, since λ_r is not estimated in the second step when we impose $\lambda_g = 0.0275$ due to the matrix singularity problem. The estimations in column 2 are from the third step, where λ_r is obtained across a grid search in the interval [0.0444; 0.1244]. The estimations in column 3 are from the second step. We did not obtain estimations in the third step due to the matrix singularity problem. The estimations in column 4 are from the third step, where λ_g is obtained in the estimation with the sample 1970-2006 and λ_r is obtained across a grid search in the interval [0.055; 0.065]. For Australia, the estimations in column 5 are from the third step. The estimations in column 6 are from the second step. We did not obtain estimations in the third step due to the matrix singularity problem. The estimations in column 7 are from the third step, where λ_r is obtained in the estimation with the sample 1970-2006. The estimations in column 8 are from the third step, where λ_r and λ_g are obtained across a grid search in the interval [0.0275; 0.9775]. Standard errors are in parentheses.

Table 4. Parameter Estimates for Sweden and Norway[a]

	Sweden			*Norway*			
	1970:02–2006:04	*1986:02 – 2006:04*		*1979:02 – 2006:04*		*1986:02 – 2006:04*	
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
λ_g	0.0262	0.0000	0.0262	0.0677	0.0677	0.1186	0.1186
λ_r	0.0315	–	0.0315	0.000	0.040	0.000	0.040
α_1	0.9177	0.9913	0.9403	0.9236	0.9375	0.0072	–0.7573
	(0.0478)	(0.0274)	(0.0522)	(0.0405)	(0.0613)	(0.2289)	(0.1780)
α_2	–0.0452	–	–0.0558	–0.0958	–0.0050	–0.1925	0.5243
	(0.0190)	(–)	(0.0110)	(0.0658)	(0.0208)	(0.1273)	(0.2039)
β_1	–0.1680	–0.3390	–0.0646	–0.3339	–0.1700	–0.3609	–0.3064
	(16775.9)	(0.0888)	(11442.1623)	(0.0444)	(16845.7489)	(0.0572)	(25668.8387)
β_2	–0.3429	1.3353	–0.2998	1.4904	–0.2921	1.2578	–0.3553
	(0.0594)	(0.1098)	(0.1031)	(0.0928)	(0.0500)	(0.0891)	(0.0531)
β_3	1.3183	0.2620	1.3436	0.3326	1.5101	0.2158	1.1926
	(0.1133)	(0.3898)	(0.1289)	(0.1267)	(0.0997)	(0.2943)	(0.0766)
δ_1	0.5615	–	0.3929	0.7958	0.6415	0.8777	0.9868
	(0.0292)	(–)	(0.0581)	(0.0485)	(0.0615)	(0.0708)	(0.0115)

Table 4. (continued)

	Sweden			*Norway*			
	1970:02–2006:04	*1986:02 – 2006:04*		*1979:02 – 2006:04*		*1986:02 – 2006:04*	
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*
δ_2	−0.4751 (0.1683)	– (–)	−0.7365 (0.4081)	−0.3852 (0.0842)	−0.5778 (0.0919)	−0.4053 (0.1509)	−0.4583 (0.1064)
δ_3	−0.4555 (0.3784)	– (–)	−0.5290 (1.0947)	−0.1139 (0.0560)	−1.1227 (0.2599)	−0.2346 (0.2826)	−0.4790 (0.1677)
σ_y	0.3447 (0.1196)	0.6823 (0.4974)	0.1191 (0.1642)	0.9041 (0.2227)	0.2402 (0.0890)	0.7054 (0.1376)	0.3312 (0.1086)
σ_π	1.9639 (0.1272)	1.6336 (0.1287)	1.7076 (0.1579)	1.3759 (0.0770)	1.4408 (0.0894)	1.4839 (0.1064)	1.2810 (0.0977)
σ_r	2.6759 (0.0620)	– (–)	3.1712 (0.1470)	1.1974 (0.0727)	1.0270 (0.0693)	1.2259 (0.0822)	0.3762 (0.2991)
σ_{y^*}	0.9841 (14365.4)	0.0000 (106856.8391)	0.5951 (8956.6283)	0.7633 (0.4370)	1.1710 (16930.1425)	0.5428 (3.2854)	0.8557 (21516.3255)

Source: Authors' calculations.

a. For Sweden, the estimations in column 1 are from the third step. The estimations in column 2 are from the first step, since λ_r is not estimated in the second step when we impose $\lambda_g = 0$ due to the matrix singularity problem. The estimations in column 3 are from the third step, where λ_r is obtained in the estimation with the sample 1970-2006. For Norway, the estimations in column 4 are from the second step. We did not obtain estimations in the third step due to the matrix singularity problem. The estimations in column 5 are from the third step, where λ_r is obtained across a grid search in the interval [0.0050; 0.0750]. The estimations in column 6 are from the second step. We did not obtain estimations in the third step due to the matrix singularity problem. The estimations in column 7 are from the third step, where λ_r is obtained across a grid search in the interval [0.0050; 0.0750]. Standard errors are in parentheses.

Table 5. Parameter Estimates for Chile[a]

	Chile	
	1986:02 – 2006:04	
Parameters	*(1)*	*(2)*
λ_g	0.0000	0.0820
λ_r	0.0000	0.0800
α_1	1.0771 (0.0540)	0.9412 (0.1074)
α_2	−0.2461 (0.1245)	-0.1076 (0.0961)
β_1	0.4639 (0.0697)	0.4325 (0.0946)
β_2	0.5078 (0.1612)	0.5940 (0.1959)
β_3	0.0142 (0.0251)	0.2756 (0.2216)
δ_1	0.6996 (0.1242)	0.6552 (0.0861)
δ_2	−0.0151 (0.2658)	0.1188 (0.2049)
δ_3	0.0733 (0.0809)	0.3680 (0.2525)
σ_y	1.2847 (0.9877)	1.0436 (0.2924)
σ_π	1.8274 (0.1110)	1.7188 (0.1230)
σ_r	1.3993 (0.0750)	1.2777 (0.0833)
σ_{y^*}	0.0001 (8810.1)	0.7456 (0.3177)

Source: Authors' calculations.
a. The estimations in column 1 are from the second step; we did not obtain estimations in the third step due to the matrix singularity problem. The estimations in column 2 are from the third step, where λ_g and λ_r are obtained across a grid search in the intervals [0.062; 0.102] and [0.06; 0.10], respectively. Standard errors are in parentheses.

4. Extensions for the United States and Chile

In this section, we extend our basic model to include the unemployment gap (Okun's Law) and apply it to the United States and Chile, for which we obtained the best results for the basic model. We also test for robustness of the basic model results for the United States by replacing four-step-ahead inflation forecasts with eight-step-ahead forecasts.[15]

4.1 Results for the United States

For the extended model with Okun's Law for the United States, we proceed in the following way. When freely estimating all parameter values and unobservables, λ_u was estimated in the fourth step at a value of zero, implying a constant 5.6 percent natural rate of unemployment for the United States in 1960–2007. Following the approach adopted for countries in section 3, we next pursue a grid search over alternative preset values of λ_u. The model parameter estimates consistent with $\lambda_u = 0$ and $\lambda_u = 0.4$ (the median value of our grid search) are reported in columns 1 and 2 of table 6. Figure 11 depicts the grid-search results for the unobservables. The findings can be summarized as follows. The parameter estimates are generally similar for the extended model (in both columns 1 and 2 of table 6) to those reported for the basic model (column 1, table 1). In the IS curve, the output gap becomes more sensitive to the lagged interest rate gap, while the coefficient of lagged inflation in the Phillips curve turns positive, with a corresponding reduction in size of the two other Phillips curve coefficients. For the newly introduced Okun's Law, the parameter estimates exhibit the expected signs and are highly significant. The parameter estimate for the lagged unemployment gap reflects large unemployment inertia. The coefficient estimate of the lagged output gap is very large (–0.95) when the natural unemployment rate is estimated as constant and declines to –0.35 when the natural unemployment rate is variable, consistent with a value of λ_u set at 0.4.

Figure 11 depicts estimation ranges for unobservables for λ_u varying between 0.08 and 0.72. The estimates for both potential

15. We did not obtain model convergence when using eight-step-ahead inflation forecasts for Chile. We also conducted sensitivity analyses for the Phillips curve in both countries, by replacing one-period inflation lags with four-quarter lags; the results were almost unchanged.

Table 6. Parameter Estimates for Alternative Models for the United States and Chile[a]

	United States 1960:1 – 2007:2			*Chile 1986:2 – 2006:4*	
	Extended Model (with Okun's law)		*Eight-step-ahead inflation forecasts*	*Extended Model (with Okun's law)*	
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*
λ_g	0.0475	0.0475	0.0586	0.0000	0.0820
λ_r	0.0215	0.0215	0.0304	0.0000	0.0800
λ_u	0.0000	0.4000	–	0.0000	0.4000
α_1	0.9539	0.9558	0.9503	1.0033	1.0329
	(0.0302)	(0.0331)	(0.0441)	(0.0515)	(0.0433)
α_2	−0.0252	−0.0681	−0.0546	−0.0644	−0.1583
	(0.0100)	(0.0213)	(0.0216)	(0.0425)	(0.0685)
β_1	0.1097	0.0602	0.0680	0.4501	0.4533
	(0.0599)	(0.0593)	(0.1031)	(0.0803)	(0.0842)
β_2	0.6525	0.7032	0.4514	0.5191	0.5182
	(0.0525)	(0.0474)	(0.0482)	(0.1703)	(0.1687)
β_3	0.3926	0.2820	0.4337	0.1474	0.1173
	(0.1876)	(0.0968)	(0.1427)	(0.1614)	(0.1420)
γ_1	0.4956	0.5635	–	0.2501	0.2045
	(0.0999)	(0.0879)	(–)	(0.1791)	(0.2190)
γ_2	−0.9466	−0.3523	–	−0.6591	−0.5356
	(0.3234)	(0.1010)	(–)	(0.3348)	(0.2237)

Table 6. (continued)

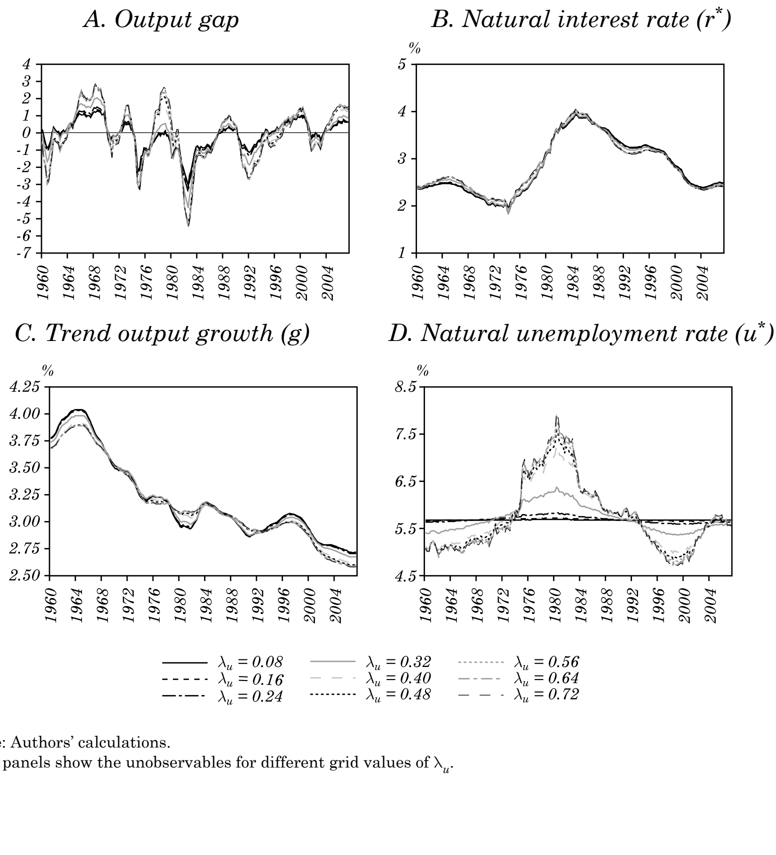
	United States 1960:1 – 2007:2			*Chile 1986:2 – 2006:4*	
	Extended Model (with Okun's law)		*Eight-step-ahead inflation forecasts*	*Extended Model (with Okun's law)*	
Parameters	*(1)*	*(2)*	*(3)*	*(4)*	*(5)*
δ_1	0.8756 (0.0316)	0.8697 (0.0256)	0.7880 (0.0262)	0.7821 (0.0600)	0.6996 (0.0724)
δ_2	–0.1478 (0.0286)	–0.1353 (0.0298)	–0.2193 (0.0201)	0.0205 (0.2750)	–0.0073 (0.2139)
δ_3	0.1731 (0.1587)	0.1250 (0.0825)	0.1910 (0.1075)	0.3329 (0.2328)	0.2654 (0.1804)
σ_y	0.2411 (0.0780)	0.4731 (0.1053)	0.5176 (0.1060)	0.5644 (0.2246)	0.5899 (0.1810)
σ_π	0.8223 (0.0385)	0.7750 (0.0340)	0.8250 (0.0411)	1.8052 (0.1135)	1.8071 (0.1175)
σ_u	0.0442 (0.0643)	0.1253 (0.0144)	– (–)	0.1935 (0.0971)	0.2151 (0.0671)
σ_r	1.1552 (0.0283)	1.1498 (0.0316)	1.2768 (0.0352)	1.3852 (0.0743)	1.3135 (0.0704)
σ_{y^*}	0.7969 (0.3020)	0.6656 (0.1485)	0.6293 (0.1288)	1.2730 (0.6356)	1.1429 (0.5518)

Source: Authors' calculations.

a. For the United States, the estimations in column 1 are from the fourth step of the extended model with Okun's Law. The estimations in column 2 are from the fourth step, where λ_u is obtained from a grid search in the interval [0.08; 0.72]. The estimations in column 3 are from the third step of the modified standard model with eight-step-ahead inflation forecast. For Chile, the estimations in column 4 are from the fourth step. The estimations in column 5 are from the fourth step, where λ_g, λ_r and λ_u are obtained from a grid search in the intervals [0.062; 0.102], [0.06; 0.10], and [0.08; 0.72], respectively. Standard errors are in parentheses.

output growth (which declines from 3.8 percent in the early 1960s to 2.8 percent in the early 2000s) and the natural interest rate (which varies between 2 percent and 4 percent between 1960 and 2006) are robust to changes in λ_u, reflected in their narrow ranges. Moreover, the estimated values and dynamics of both potential growth and the natural interest rate for the extended model are very close to those depicted for the basic model (upper panel, figure 1). However, the range of estimates for the output gap for different values of λ_u is larger. In addition, the median value for the new output gap estimate is not as close to the estimate for the basic model. This should not come as a surprise, since the extended model imposes a close relation between the output gap and the unemployment

Figure 11. Grid-Search Results for the Extended Model for the United States, 1960:1–2007:2[a]



Source: Authors' calculations.

a. The panels show the unobservables for different grid values of λ_u.

gap. Okun's Law implies that the latter gaps are almost a mirror image of each other.

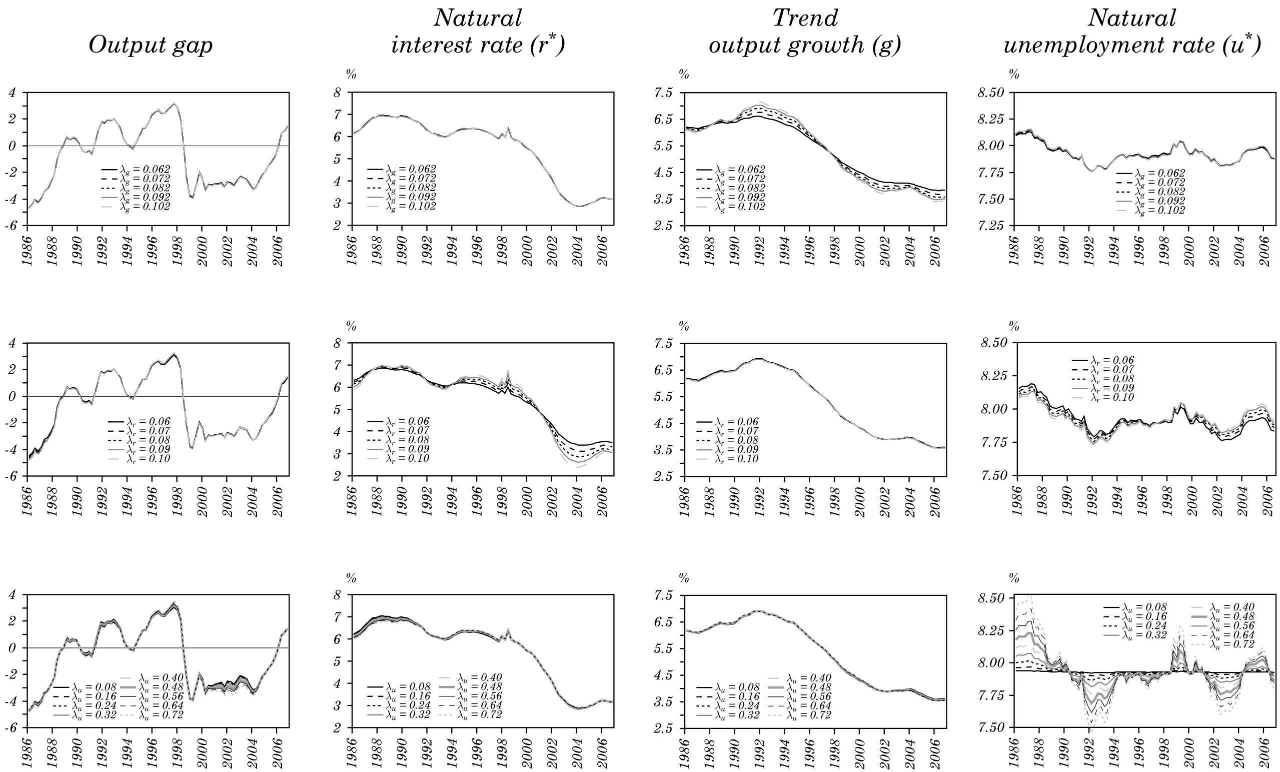
The largest range of estimates depicted in figure 11 is the one for the newly estimated natural rate of unemployment. For the median value of λ_u, the natural rate varies over time between 5.1 percent and 7.2 percent. Over the full range of λ_u values, the natural rate varies over time between 4.8 percent and 8.1 percent. This is consistent with recent findings of King and Morley (2007), who estimate the natural rate as the steady-state of a VAR and attribute most of the volatility in observed unemployment to movements in the natural rate.

We now return to the parsimonious model, replacing the four-step-ahead inflation forecast for the United States with an eight-step-ahead forecast. This change affects the measurement of inflation expectations in the three structural model equations. We obtain the following results for parameter estimates (column 3, table 6). First, the IS curve parameter estimates are not modified much (for comparison, see column 1, table 1). The parameter estimate for the inflation expectations gap in the Phillips curve declines almost by half, but it remains significant. The parameter estimate for the inflation forecast gap in the Taylor rule is still significant, but it is somewhat more negative, implying a corresponding decline in the nominal interest rate reaction to an inflation expectations shock, from +0.87 to +0.78. Both results—for the Phillips curve and the Taylor rule—may suggest that four-quarter-ahead inflation expectations describe inflation and interest rate setting better than eight-quarter-ahead inflation expectations. Finally, with regard to unobservables, the output gap, the neutral interest rate, and potential output growth exhibit similar patterns and values as those based on four-step-ahead inflation forecasts.

4.2 Results for Chile

For the extended model with Okun's Law for Chile, we proceed in a way similar to our approach with the United States. However, the difference is that when freely estimating all parameter values and unobservables, λ_g, λ_r, and λ_u are estimated at zero in the fourth-stage estimation. Therefore, we conduct separate grid searches over alternative preset values of the three signal-to-noise coefficients. The model parameter estimates consistent with $\lambda_g = \lambda_r = \lambda_u = 0$, and with $\lambda_g = 0.082$, $\lambda_r = 0.080$, and $\lambda_u = 0.4$ (the median values of

Figure 12. Grid-Search Results for the Extended Model for Chile, 1986:2–2006:4[a]



Source: Authors' calculations.

a. The panels show the unobservables for different grid values of λ_g (first row), λ_r (second row), and λ_u (third row).

our grid searches) are reported in columns 4 and 5, respectively, of table 6. Figure 12 depicts the corresponding grid-search results for the unobservables. We find that the parameter estimates for the extended model (columns 4 and 5 in table 6) are generally very similar to those reported for the basic model (corresponding columns 1 and 2 in table 5). The one important exception is the IS curve, where the output gap becomes more sensitive (and significant) to the lagged interest rate gap in the extended model (that is, the lambdas are set at positive values). The coefficient of lagged inflation in the Phillips curve now turns positive, with a corresponding reduction in size of the two other Phillips curve coefficients. For the newly introduced Okun's Law, parameter estimates exhibit the expected signs and are highly significant. The parameter estimates for the lagged unemployment gap reflect moderate unemployment inertia, while the coefficient estimate of the lagged output gap is large (close to -0.6).

The estimation ranges depicted in the three rows of figure 12 are relatively narrow for all unobservable variables. The widest range in each row is for the unobservable over which the grid search is conducted. The general dynamic pattern of three unobservables (namely, potential output growth, the output gap, and the neutral interest rate) estimated for the extended model are similar to those obtained for the basic model. Potential output growth is estimated to have declined from 6.5 percent in the late 1980s and early 1990s to 3.5 percent in the early 2000s. The neutral interest rate follows a very similar pattern, falling from 6.5 percent in the late 1980s and early 1990s to 3 percent in the early 2000s.

As in the case of the extended model applied to the United States, the differences in output gap estimates are not surprising, as the extended model imposes a close relation between the output gap and the unemployment gap. Again, Okun's Law implies that the latter gaps are almost a mirror image of each other. However, in contrast to the United States, the range for the new estimates of the natural rate of unemployment is not as large in Chile. For the median value of λ_u, the natural rate varies over time between 7.7 percent and 8.1 percent. Over the full range of λ_u values, the natural rate varies over time between 7.5 percent and 8.5 percent. This is consistent with recent findings by Restrepo (2008) based on different models of estimation for the NAIRU in Chile.

5. The Great Moderation, Comovements, and Convergence in Industrial Economies

The period of low inflation and low volatility in key macroeconomic variables beginning in the late 1980s, following the high inflation and real instability of the mid-1970s and early 1980s, is sometimes called the Great Moderation. It has been documented fairly extensively in academic research and policy evaluations.[16] At the same time, there is a presumption that rising world trade and financial integration should lead to stronger business cycle comovement across countries, as well as stronger convergence in real variables, like growth and real interest rates, particularly among industrial countries. In this section, we exploit our country time-series estimates of unobservables, in addition to the series of selected observables, to test for the Great Moderation, comovements, and convergence in our sample of nine industrial countries, using quarterly data for 1970–2006.[17]

5.1 Common Trends in Key Unobservables

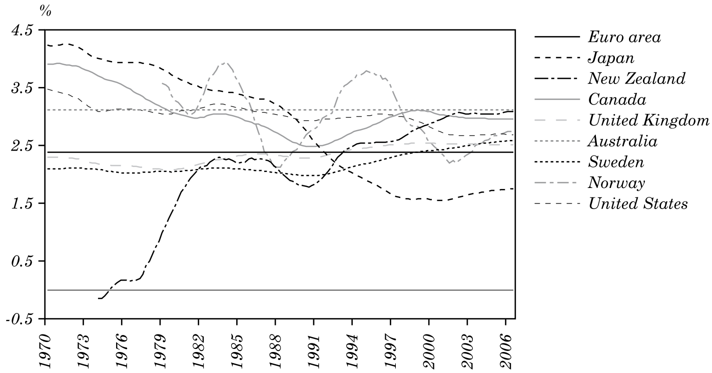
We start by describing the trends in potential output growth rates (figure 13) and neutral real interest rates (figure 14) across the nine countries. The most striking feature of the potential output growth estimates is the large reduction in cross-country variation observed between 1970 and 2006. Leaving out Japan, country point estimates of potential growth ranged from zero (New Zealand) to 4 percent (Canada) in the early 1970s. In contrast, the range of potential growth estimates for 2006 was quite narrow, delimited by Japan's low potential growth rate (1.8 percent) and Australia's constant rate (3.2 percent). The most striking increase in potential growth is New Zealand, where potential growth rose from zero to 3.2 percent in the last four decades; this stands in sharp contrast to Japan's reduction from 4.5 percent to 1.8 percent. Sweden and the United Kingdom exhibit a slight trend rise in potential growth, with the opposite pattern observed in Canada, Norway, and the United States.

Similar to the case of growth, the cross-country dispersion in neutral real interest rates has declined strongly in the last four

16. For example, the International Monetary Fund's October 2006 W*orld Economic Outlook* devotes a well-documented chapter to the Great Moderation.

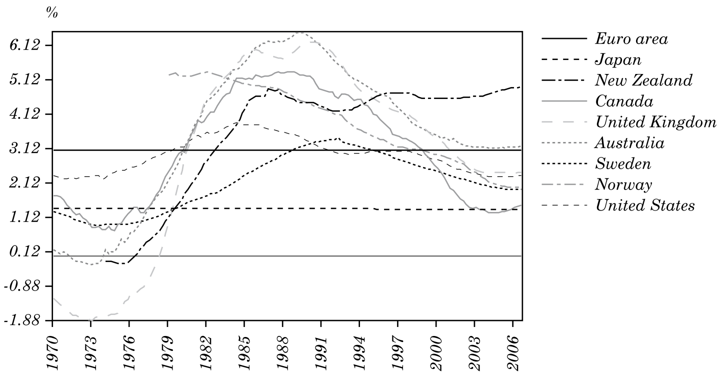
17. We use our shorter time series for New Zealand and Norway, and we drop Chile due to the lack of quarterly data before 1986.

Figure 13. Potential Output Growth in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.
a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway.

Figure 14. Neutral Real Interest Rate in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.
a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway.

decades (figure 14). In the early 1970s, neutral real rates ranged from –1.9 percent (United Kingdom) to 3.1 percent (euro area). By 2006, the range had narrowed to an interval from 1.5 percent (Japan) to 3.1 percent (euro area), except for New Zealand. Six countries exhibit an inverted-U-shaped pattern of their neutral real interest rates. This reflects strong monetary adjustment in response to the

Great Inflation of the late 1970s, with real policy rates peaking in the 1980s and early 1990s at levels of up to 6.5 percent (Australia in 1990). The stabilization success of the 1980s and 1990s that led to the Great Moderation allowed for subsequent lower neutral rates in the 1990s and 2000s. The exception to the latter trend is New Zealand, where the neutral real interest rate continued to rise, reaching 4.8 percent in 2006.

5.2 The Great Moderation

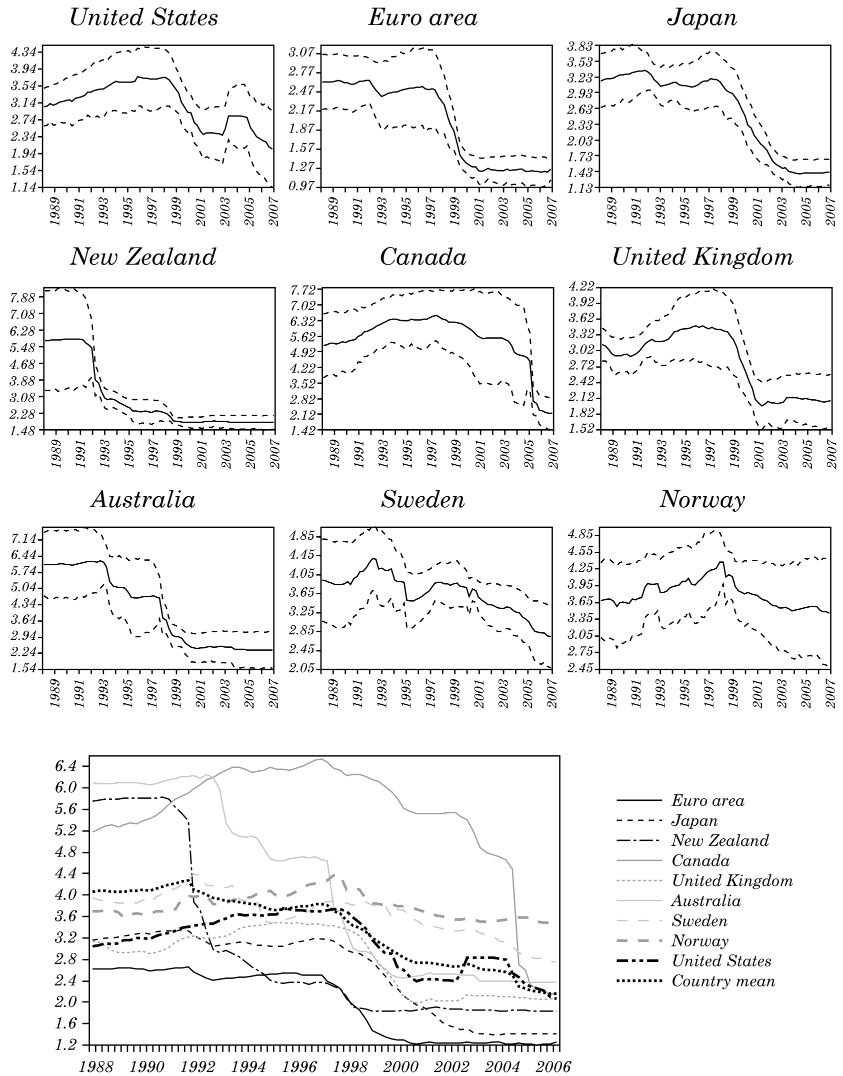
To investigate the Great Moderation, we focus on volatility trends of seven key variables in our nine sample countries. Three variables are observables (inflation, output growth, and the real interest rate) and four are unobservables (potential output growth, the output gap, the natural real interest rate, and the interest rate gap). We compute rolling standard deviations for the latter variables using a window of seventy-four quarters.[18] We then report the associated confidence intervals obtained by bootstrap techniques.[19]

This approach is informative about the Great Moderation, reflected in increased stability of key macroeconomic variables. We focus on both the level of the rolling standard deviation and the varying width of the confidence interval. The results are depicted separately for each variable in figures 15 through 21. The nine smaller panels in each figure show rolling point estimates of the standard deviation and their estimated time-varying confidence intervals for each country, while the larger bottom panel depicts the nine point estimates for each country and the corresponding country mean to better represent the common volatility trend across our sample countries. We find that the volatility of inflation has declined in all countries, except Norway; the mean volatility of inflation fell from 4.0 percent in 1970–87 to 2.2 percent

18. We use a window size of seventy-four quarters (or eighteen and a half years), which is half our thirty-seven-year sample coverage from 1970 to 2006. We choose this rather large window to show more clearly long-term volatility trends, avoiding excessive noise in standard deviations that shows up when using conventional forty-quarter (ten-year) rolling windows.

19. We apply a bootstrap technique for estimating time-varying confidence intervals because of its superior asymptotic properties in small samples, in comparison with standard confidence intervals. Hall's confidence intervals are calculated using the stationary bootstrap method of Politis and Romano (1994). This technique guarantees stationary artificial series by allowing a random block size (indeed, it follows a geometric distribution) when resampling the data. We set the mean of the block size at three and perform 2,000 replications.

Figure 15. Inflation Volatility Trends in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

in 1988–2006 (figure 15).[20] Moreover, this trend is also significant as reflected by the narrowing confidence intervals. The exception is again Norway, where point estimates decline while confidence intervals rise after 1988. The largest reductions in inflation volatility are observed in Australia, Canada, and New Zealand, roughly from 6.0 percent to about 2.2 percent. The euro area exhibits the lowest inflation volatility during most of the sample period.

The reduction of the volatility of output growth in all nine countries is remarkable, reflected by both declining point estimates and narrowing confidence intervals (figure 16). The average country level of output growth volatility fell roughly by half, from 5.0 percent in 1970–87 to 2.7 percent in 1988–2006. The largest growth stabilization was recorded in New Zealand, where growth volatility fell from 14 percent in the 1970s and 1980s to 5 percent in the 1990s and 2000s. Australia, Sweden, and the United Kingdom also exhibit large reductions in growth volatility. Again the euro area exhibits the highest level of stability throughout the last thirty-seven years.

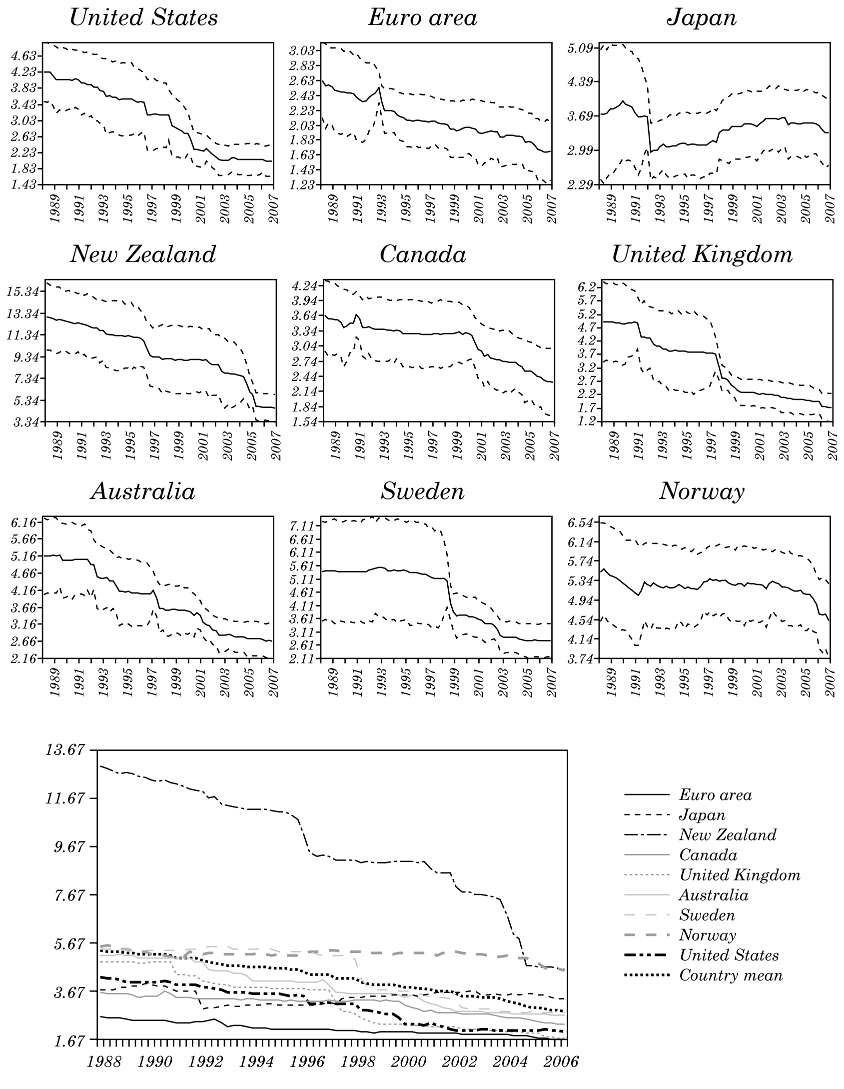
We now turn to our first unobservable, potential output growth.[21] Like all estimated unobservables, potential growth either is estimated as a constant (in the euro area and Australia) or, if variable (in the other countries), exhibits a smooth pattern over time, without high-frequency volatility (figure 17). Therefore, its volatility—like that of the neutral interest rate, reported below—is lower by an order of magnitude than the volatilities exhibited by observable variables. The average country volatility (for the seven countries where potential output varies over time) declines only marginally over time. Opposite trends are observed in different countries. For example, New Zealand records a strong trend decline in potential growth volatility, while a growing trend is observed in Japan up to 2000, which is partially reversed thereafter.

The average country volatility of the output gap (our second unobservable) falls slightly, from 1.6 percent in 1970–87 to 1.4 percent in 1988–2006 (figure 18). There are moderate to large reductions in the volatility of the output gap in six countries, no clear trends in two countries, and a slight trend rise in one country (Australia). The

20. The correlation between the first and second moments of inflation is known to be very large. Hence, the declining trends in inflation volatility described here are matched by declining trends in inflation levels.

21. The descriptive statistics discussed below for our estimates of unobservable are conditional on our estimates and should thus be taken with caution, in comparison with those reported for observables like inflation, actual growth, and actual interest rates.

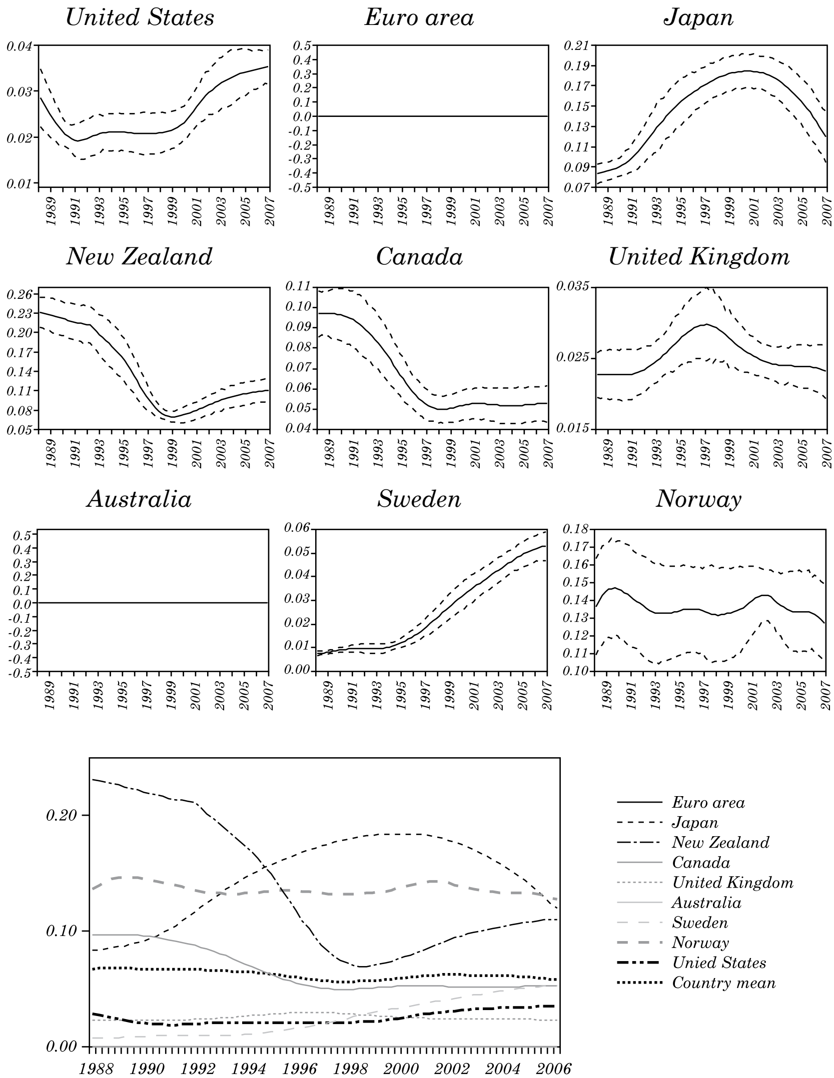
Figure 16. Actual Output Growth Volatility Trends in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

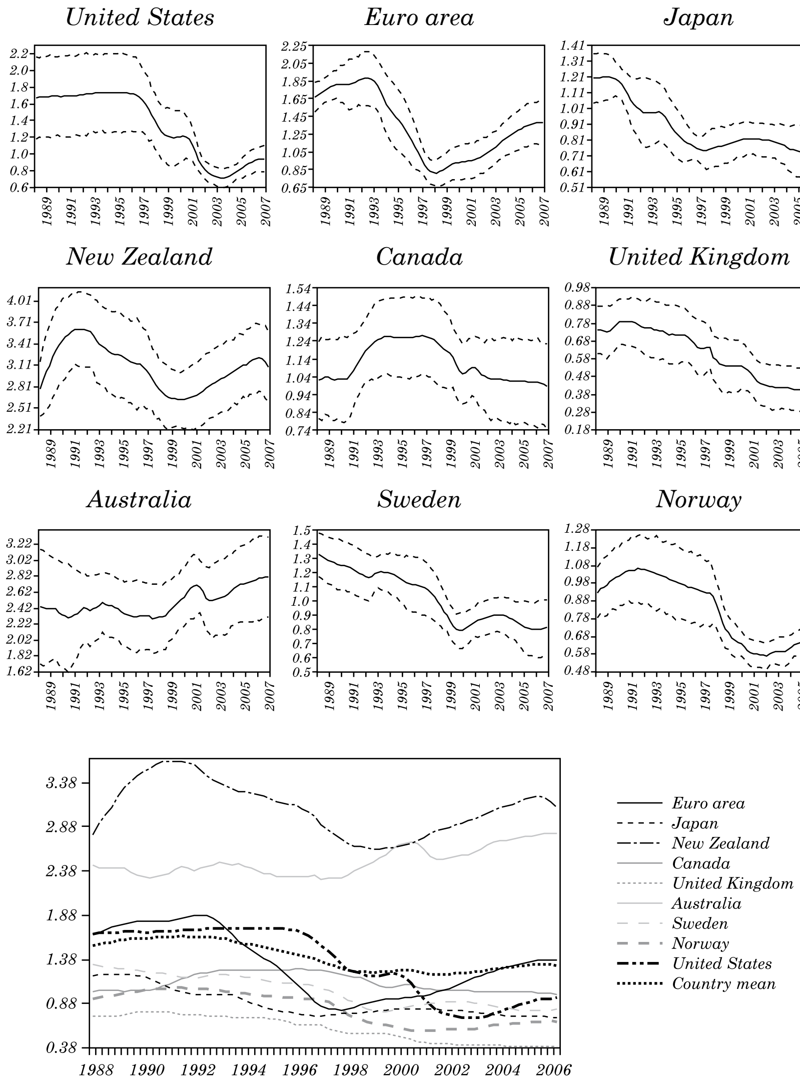
Figure 17. Potential Output Growth Volatility Trends in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

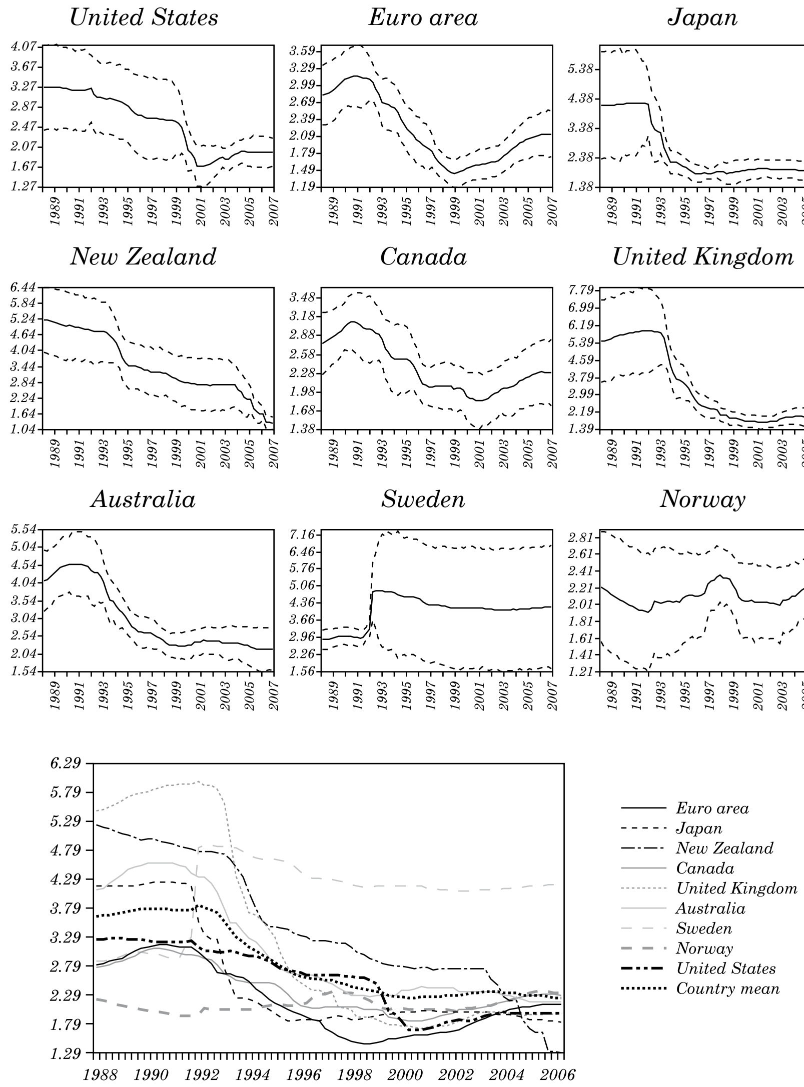
Figure 18. Output Gap Volatility Trends in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

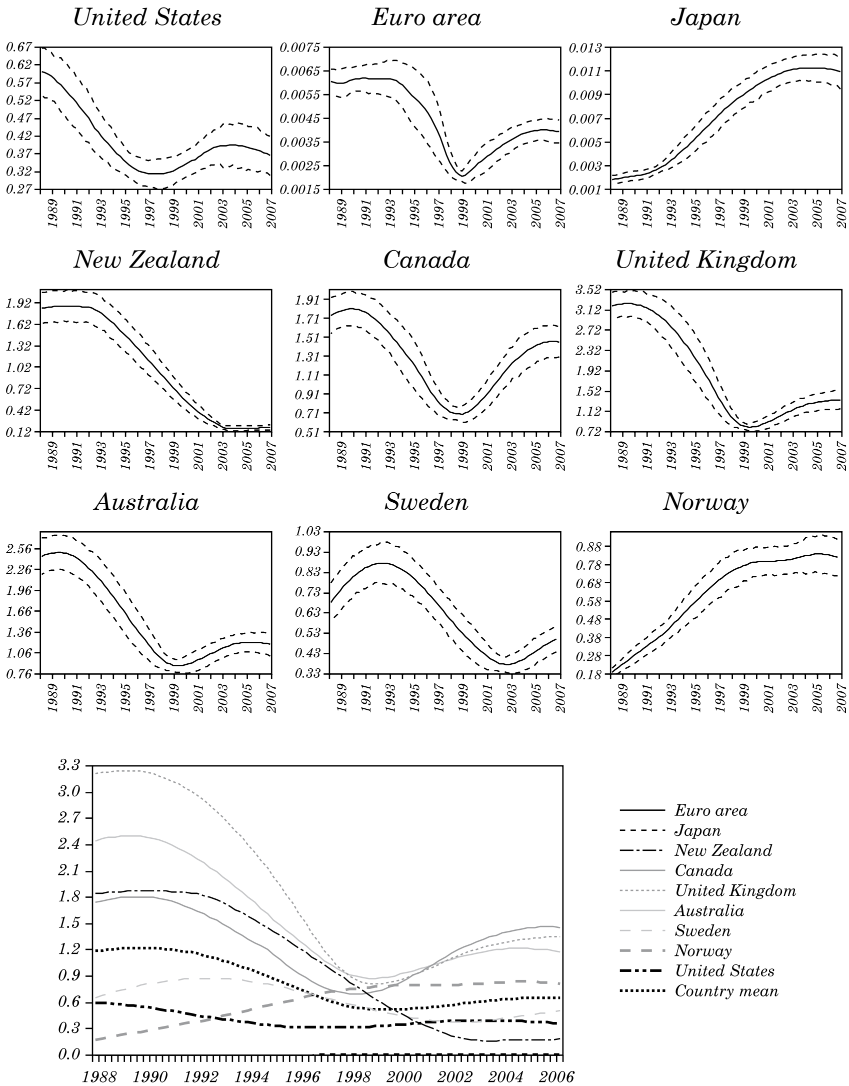
Figure 19. Actual Interest Rate Volatility Trends in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

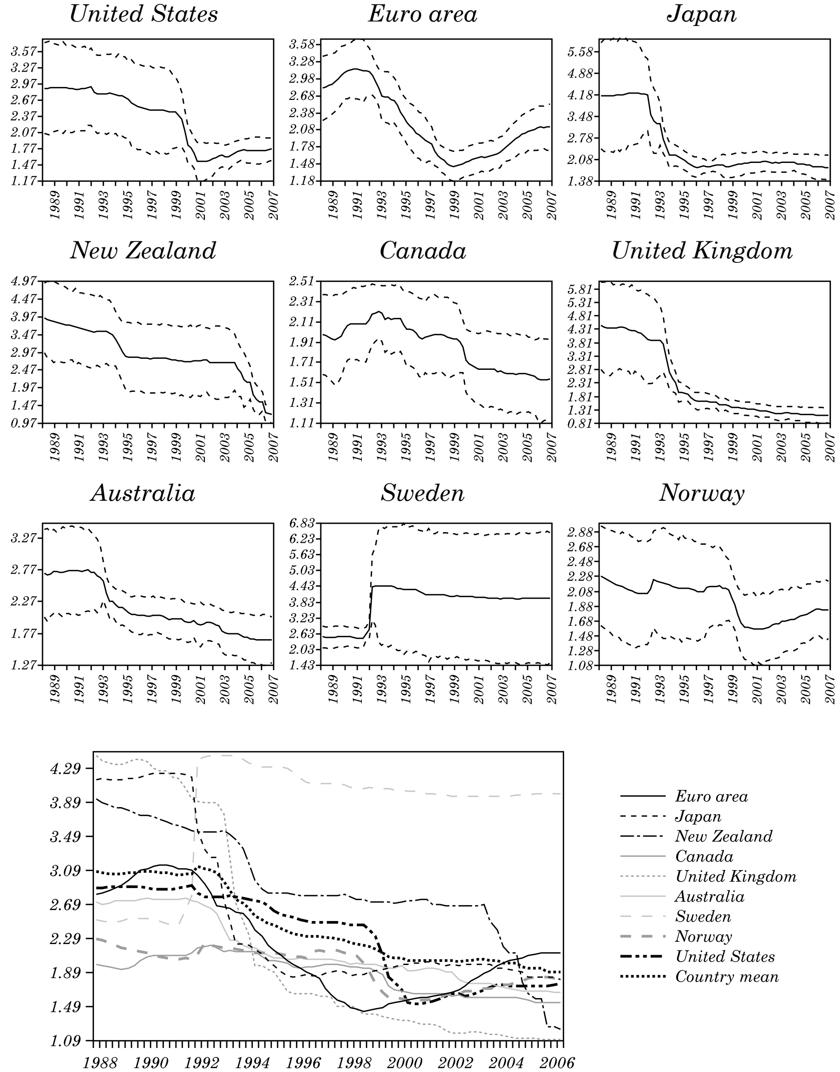
Figure 20. Neutral Interest Rate Volatility Trends in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

Figure 21. Interest Rate Gap Volatility Trends in Nine Economies, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

United Kingdom exhibits the most stable output gap throughout the full 1970–2006 period.

A general pattern of declining volatility is also found for the real interest rate: the average country volatility falls from 3.8 percent to 2.3 percent (figure 19). The largest reductions in interest rate volatility are recorded in New Zealand and the United Kingdom. Norway does not exhibit a trend reduction because its interest rate volatility is low from the start. The exception is Sweden, which experienced a sharp rise in interest rate volatility in the third quarter of 1992, as a result of a very short but very high interest rate spike.

As with potential output growth, the results for the volatility of our estimated neutral real interest rate are mixed (figure 20). Nevertheless, the average country volatility of the neutral rate declines by half, from 1.2 percent in 1970–87 to 0.6 percent in 1988–2006. The largest decline in neutral rate volatility is recorded by the United Kingdom, while volatility rises in Norway. Japan records the lowest neutral rate volatility, close to zero, throughout the full sample period.

Finally, the results for the interest rate gap largely mimic those of the real interest rate because the natural interest rate exhibits very low variability relative to the real rate (figure 21).

The evidence presented here is strongly supportive of the Great Moderation in key macroeconomic variables in industrial countries. The strong trend reduction in volatilities of three observed variables (namely, inflation, output growth, and the real interest rate) and the moderate decline in volatilities of the unobservable neutral interest rate and the two unobservable gap measures (the output gap and the interest rate gap), as well as the narrowing of the corresponding confidence intervals, are proof of the gains attained in macroeconomic stability during the period from 1988 to 2006. The narrowing of country differences in volatilities that came about with the reduction in country volatilities during the last four decades also suggests stronger comovements across countries, which is our next topic.

5.3 Comovements

This section focuses on comovements of key variables across countries. We look at the same variables as above, less inflation. Cross-country correlations are reported for each variable for the full sample period (the 1970s to 2006) in tables 7 and 8. We focus on pairwise regional patterns. Output growth correlations among the three largest economies are low but significantly different from

Table 7. Cross-Country Correlations of Key Output Variables in Nine Industrial Economies, 1970:2–2006:4[a]

Variable and country	*United States*	*Euro area*	*Japan*	*New Zealand*	*Canada*	*United Kingdom*	*Australia*	*Sweden*	*Norway*
A. Actual output growth									
United States	1.00	**0.24**	**0.19**	**0.24**	**0.50**	**0.29**	**0.30**	0.18	**0.15**
Euro area		1.00	**0.31**	**0.25**	**0.32**	**0.37**	0.10	**0.32**	**0.30**
Japan			1.00	-0.07	0.13	**0.28**	-0.02	-0.01	-0.08
New Zealand				1.00	**0.20**	0.08	0.12	**0.24**	**0.26**
Canada					1.00	**0.27**	**0.31**	0.11	0.08
United Kingdom						1.00	0.05	**0.28**	-0.01
Australia							1.00	0.08	0.01
Sweden								1.00	**0.22**
Norway									1.00
B. Potential output growth[b]									
United States	1.00	0.00	**0.82**	**-0.61**	**0.55**	**-0.64**	0.00	**-0.73**	**0.66**
Euro area		1.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
Japan			1.00	**-0.83**	**0.58**	**-0.90**	0.00	**-0.75**	0.27
New Zealand				1.00	-0.56	**0.85**	0.00	**0.70**	-0.30
Canada					1.00	**-0.34**	0.00	-0.05	0.16
United Kingdom						1.00	0.00	**0.80**	-0.31
Australia							1.00	0.00	0.00
Sweden								1.00	-0.34
Norway									1.00

Table 7. (continued)

Variable and country	*United States*	*Euro area*	*Japan*	*New Zealand*	*Canada*	*United Kingdom*	*Australia*	*Sweden*	*Norway*
C. Output gap									
United States	1.00	**-0.28**	**0.27**	0.29	**0.66**	**0.50**	**0.53**	**0.47**	**0.32**
Euro area		1.00	**-0.77**	**-0.75**	-0.20	0.04	**-0.56**	**-0.60**	**-0.57**
Japan			1.00	**0.48**	0.11	0.02	**0.48**	**0.42**	0.21
New Zealand				1.00	0.26	-0.12	**0.42**	**0.44**	**0.80**
Canada					1.00	**0.38**	**0.66**	**0.52**	**0.34**
United Kingdom						1.00	**0.40**	**0.34**	0.04
Australia							1.00	**0.65**	**0.27**
Sweden								1.00	**0.38**
Norway									1.00

Source: Authors' calculations.

a. The sample period is 1974:2–2006:4 for New Zealand and 1979:2–2006:4 for Norway. Figures in bold indicate significant correlation coefficients based on Hall's confidence intervals calculated using the stationary bootstrap technique, while underlined figures indicate significant correlation coefficients based on the *t* distribution.

b. The potential output growth estimate is constant for the euro area and Australia.

Table 8. Cross-Country Correlations of Key Interest Rate Variables in Nine Industrial Economies, 1970:2–2006:4[a]

	United States	*Euro area*	*Japan*	*New Zealand*	*Canada*	*United Kingdom*	*Australia*	*Sweden*	*Norway*
A. Actual interest rate									
United States	1.00	<u>**0.49**</u>	<u>0.26</u>	<u>0.24</u>	<u>**0.71**</u>	<u>**0.39**</u>	<u>**0.52**</u>	<u>0.22</u>	0.13
Euro area		1.00	<u>**0.52**</u>	<u>**0.48**</u>	<u>**0.63**</u>	<u>**0.55**</u>	<u>**0.69**</u>	<u>**0.65**</u>	<u>**0.60**</u>
Japan			1.00	0.06	<u>0.22</u>	-0.09	<u>0.18</u>	<u>0.25</u>	<u>**0.62**</u>
New Zealand				1.00	<u>**0.27**</u>	<u>**0.61**</u>	<u>**0.57**</u>	<u>0.29</u>	0.16
Canada					1.00	<u>**0.53**</u>	<u>**0.60**</u>	<u>0.32</u>	<u>**0.39**</u>
United Kingdom						1.00	<u>**0.66**</u>	<u>**0.37**</u>	<u>**0.40**</u>
Australia							1.00	<u>**0.38**</u>	<u>**0.30**</u>
Sweden								1.00	<u>**0.33**</u>
Norway									1.00
B. Natural interest rate									
United States	1.00	<u>**0.64**</u>	<u>**0.37**</u>	0.17	<u>**0.90**</u>	<u>**0.76**</u>	<u>**0.78**</u>	<u>**0.48**</u>	<u>**0.91**</u>
Euro area		1.00	0.14	<u>**0.60**</u>	<u>**0.83**</u>	<u>**0.74**</u>	<u>**0.73**</u>	<u>**0.68**</u>	<u>**0.60**</u>
Japan			1.00	<u>**-0.63**</u>	<u>0.21</u>	<u>-0.20</u>	-0.15	<u>**-0.38**</u>	<u>**0.99**</u>
New Zealand				1.00	<u>**0.45**</u>	<u>**0.78**</u>	<u>**0.74**</u>	<u>**0.82**</u>	<u>**-0.63**</u>
Canada					1.00	<u>**0.90**</u>	<u>**0.91**</u>	<u>**0.77**</u>	<u>**0.77**</u>
United Kingdom						1.00	<u>**0.99**</u>	<u>**0.89**</u>	<u>**0.57**</u>
Australia							1.00	<u>**0.89**</u>	<u>**0.54**</u>
Sweden								1.00	-0.05
Norway									1.00

Table 8. (continued)

	United States	Euro area	Japan	New Zealand	Canada	United Kingdom	Australia	Sweden	Norway
C. Interest rate gap									
United States	1.00	<u>**0.41**</u>	<u>0.21</u>	<u>0.18</u>	<u>**0.58**</u>	0.16	<u>**0.39**</u>	<u>0.17</u>	<u>-0.24</u>
Euro area		1.00	<u>**0.52**</u>	<u>**0.31**</u>	0.13	0.04	<u>**0.26**</u>	<u>**0.54**</u>	<u>**0.42**</u>
Japan			1.00	0.07	-0.04	<u>-0.39</u>	-0.05	<u>**0.23**</u>	<u>**0.39**</u>
New Zealand				1.00	-0.10	<u>**0.24**</u>	<u>**0.23**</u>	0.11	0.14
Canada					1.00	<u>**0.22**</u>	<u>**0.27**</u>	0.01	-0.11
United Kingdom						1.00	<u>**0.34**</u>	0.01	0.09
Australia							1.00	0.06	-0.09
Sweden								1.00	<u>**0.27**</u>
Norway									1.00

Source: Authors' calculations.
a. The sample period is 1974:2–2006:4 for New Zealand and 1979:2–2006:4 for Norway. Figures in bold indicate significant correlation coefficients based on Hall's confidence intervals calculated using the stationary bootstrap technique, while underlined figures indicate significant correlation coefficients based on the *t* distribution.

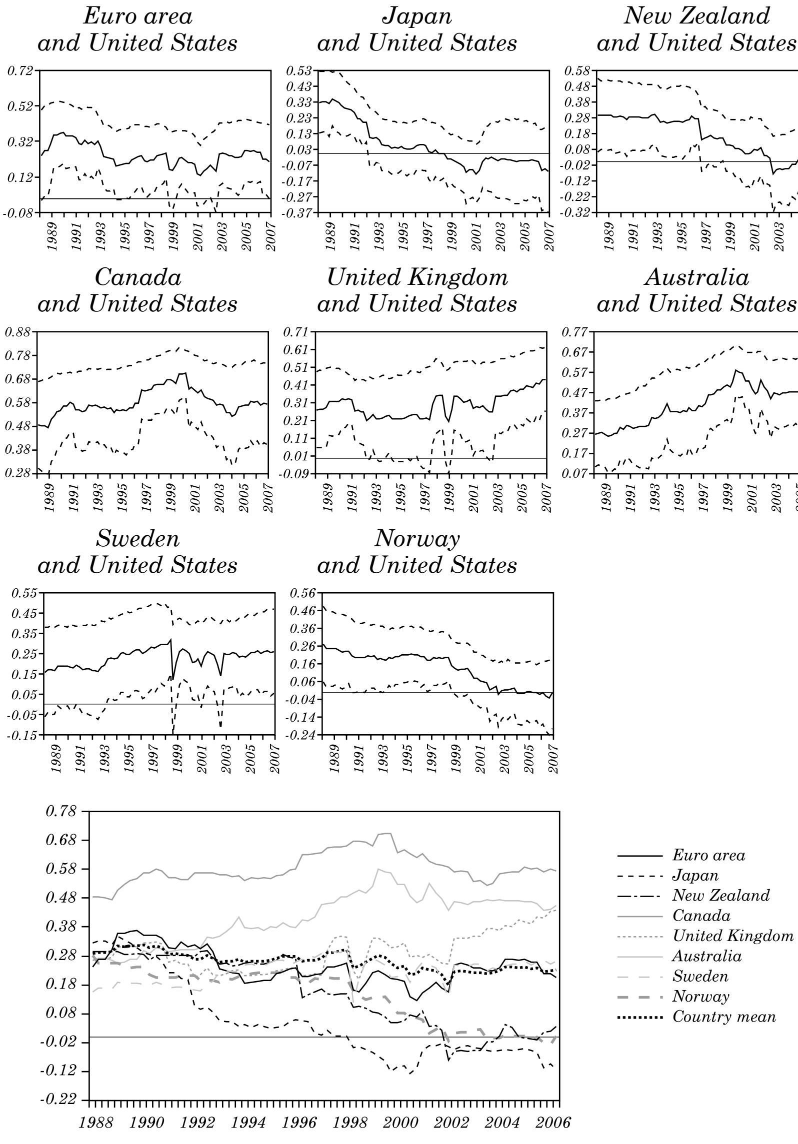
zero. The correlations between the three larger economies and some smaller countries (Canada and European economies) are somewhat larger. Our estimates for potential output growth in the euro area and Australia are constant, so we focus on correlations of third countries with the United States. Canada, Japan, and Norway display large positive correlations with the United States, whereas we find large negative correlations with the United States in New Zealand, Sweden, and the United Kingdom.

Output gap correlations between the euro area and every included country are either largely negative or zero, reflecting highly nonsynchronous business-cycle conditions of the euro area with other industrial countries. This stands in stark contrast to the United States, whose output gap is highly and positively correlated with all economies, except the euro area.

Among the three big economies, real interest rates are positively correlated. The same is true for most pairwise correlations, except Japan's. This reflects the common, long cycle of low-high-low real interest rates observed in most countries during the last four decades. Even stronger correlations are observed in the case of neutral real interest rates, again except Japan, reflecting the common world trend in monetary policy observed in most industrial countries. Cross-country interest rate gap correlations are similar to actual interest rate correlations, but they are often smaller and less significant.

To describe cross-country comovements, we follow the approach adopted above in documenting volatility trends. Here we focus on rolling correlations of key variables between the United States and the eight industrial economies. We report point estimates of correlation coefficients and their confidence intervals for seventy-four-quarter windows during 1970–2006, using the stationary bootstrap technique mentioned above. We find no common trend in output growth correlations with the United States (figure 22). While output growth correlations with the United States rise in Australia, Canada, Sweden, and the United Kingdom, they decline in Japan, New Zealand, and Norway. Potential output growth correlations turn from positive (and mostly significant) to negative (and significant) in New Zealand, Canada, United Kingdom, and Sweden (figure 23). Except for the euro area and Japan, output gap correlations of all other countries with the United States rise over time, confirming increasing cyclical synchronization between small and medium-sized industrial economies and the U.S. economy (figure 24).

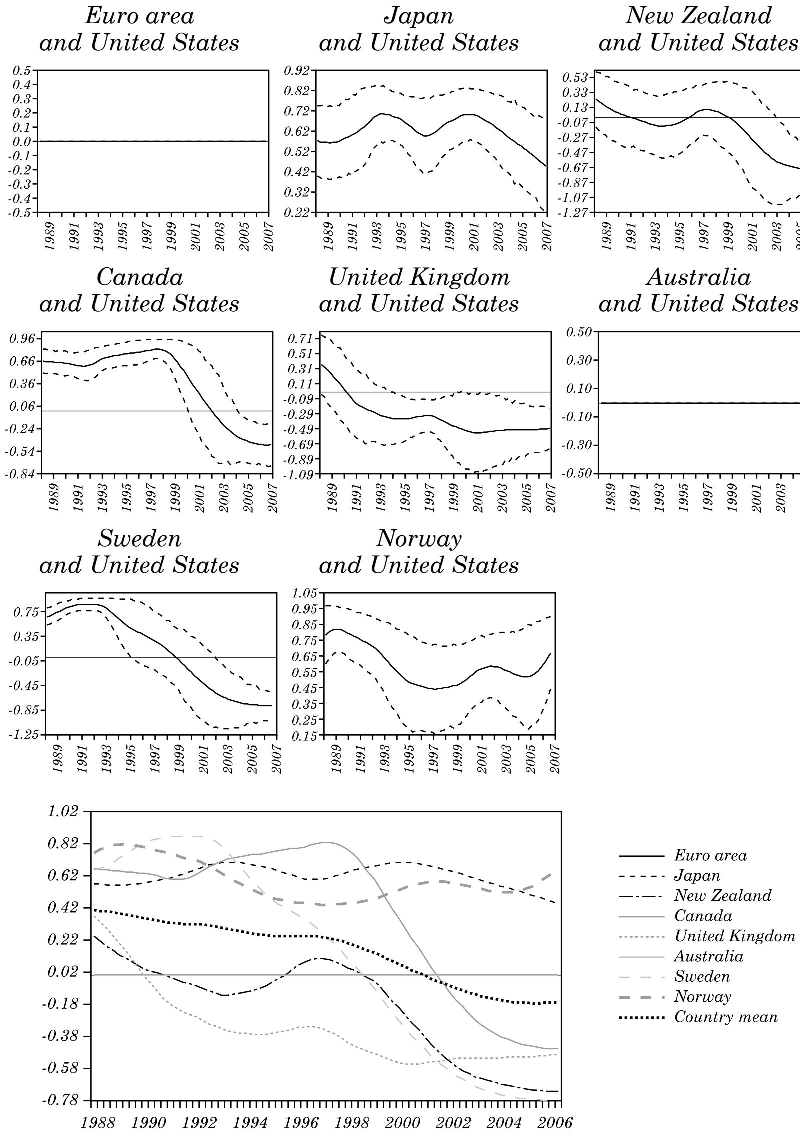
Figure 22. Actual Output Growth Correlation of Eight Industrial Economies with the United States, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

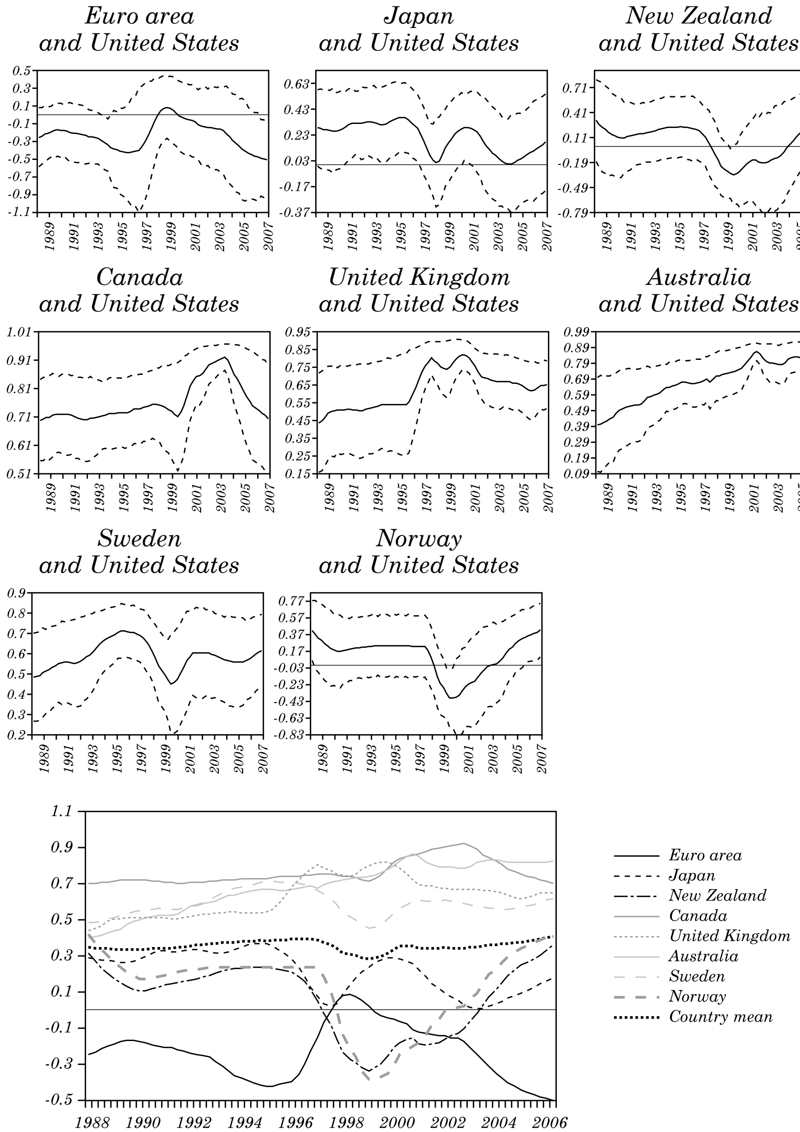
Figure 23. Potential Output Growth Correlations of Eight Industrial Economies with the United States, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

Figure 24. Output Gap Correlations of Eight Industrial Economies with the United States, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

Real interest rate correlations with the United States display a U-shaped pattern over the last four decades, reaching their lowest values during the 1980s and early 1990s and rising to high levels again in the late 1990s and early 2000s. This suggests rising monetary integration (or declining monetary independence) in the last decade (figure 25). Regarding neutral real interest rate correlations with the United States, the U-shaped pattern is confirmed in most economies, while in Japan and Norway correlations turn from negative to positive (figure 26). New Zealand displays the opposite pattern, from positive to negative. The country pattern of interest rate gap correlations with the United States replicates that of actual interest rate correlations, reflecting the smoothness of neutral rates (figure 27).

Summing up, country averages of the rolling correlation coefficients of country variables with those of the United States display slightly rising trends for the output gap, the actual interest rate, the neutral interest rate, and the interest rate gap (the lower panels in figures 22 through 27). The opposite is observed regarding average trends in actual and potential output growth with the United States, which decline over time.

5.4 Convergence

In this section, we test for cross-country convergence with the United States and the euro area in key variables for our full sample of eight countries. Because rising correlations over time do not imply convergence in levels, we carry out this final set of exercises on convergence to complement the previous evidence on increasing comovements.

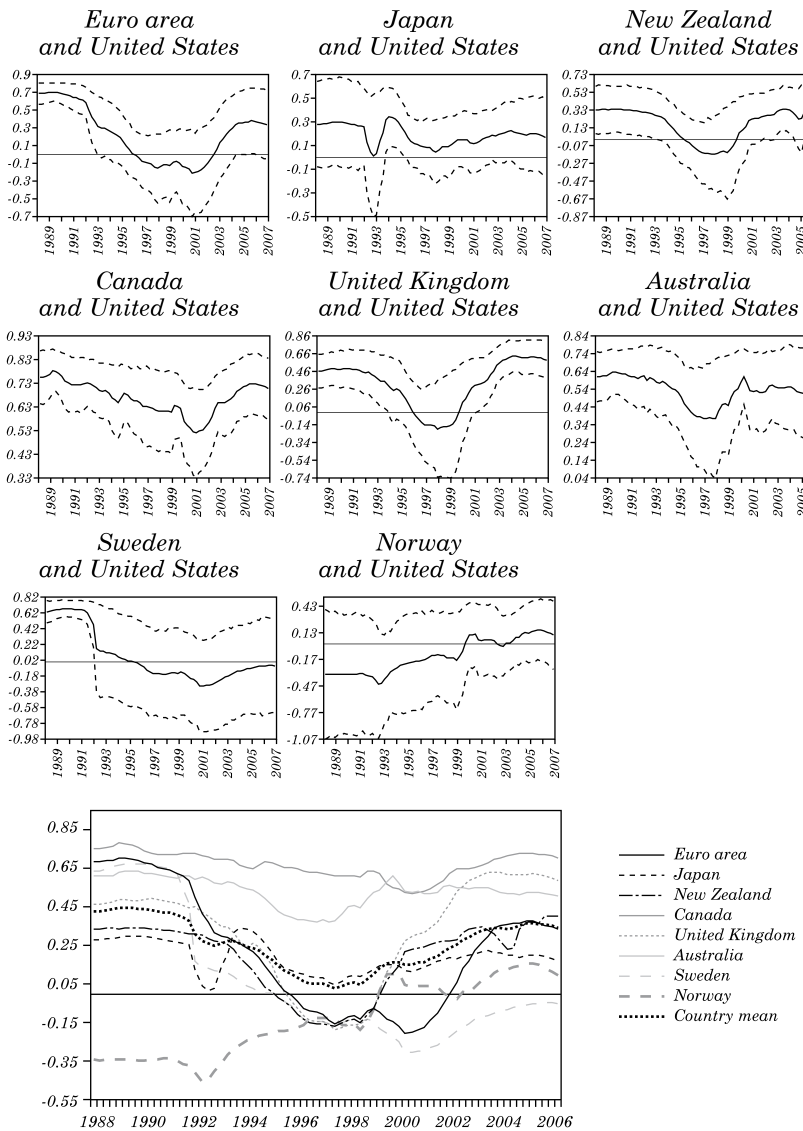
We test for convergence across countries using the following simple autoregressive models for the difference in country *j*'s variable v with respect to that of the United States or the euro area:

$$v_{j,t} - v_{us,t} = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(v_{j,t-i} - v_{us,t-i}\right) + u_{j,us,t} \tag{18}$$

or

$$v_{j,t} - v_{euroarea,t} = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left(v_{j,t-i} - v_{euroarea,t-i}\right) + u_{j,euroarea,t},$$

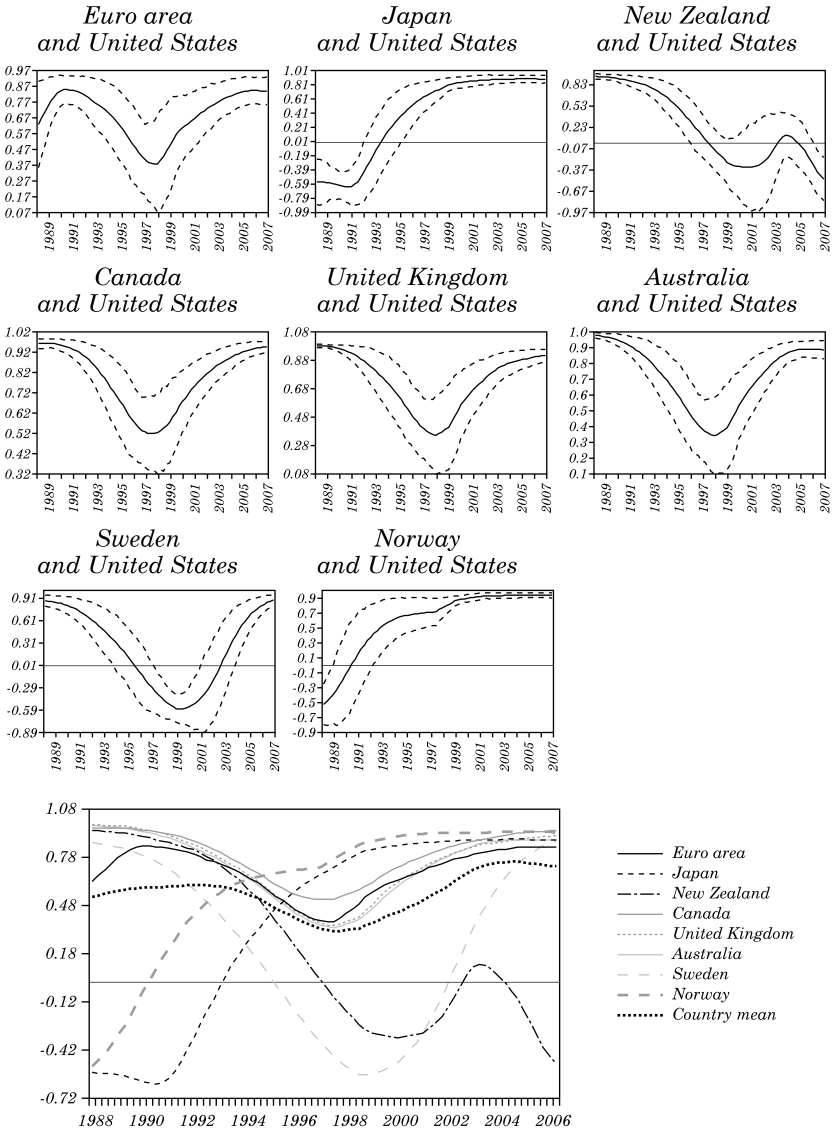
Figure 25. Actual Interest Rate Correlations of Eight Industrial Economies with the United States, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

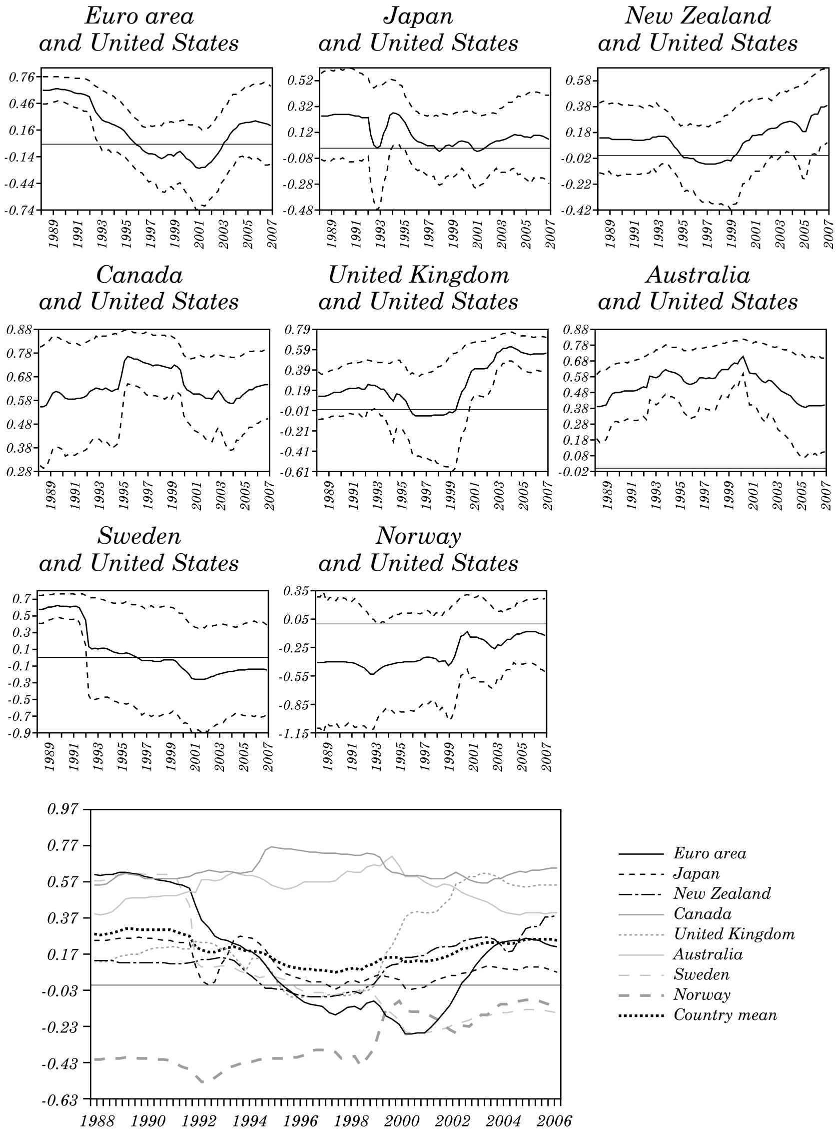
Figure 26. Neutral Interest Rate Correlations of Eight Industrial Economies with the United States, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

Figure 27. Interest Rate Gap Correlations of Eight Industrial Economies with the United States, 1970:2–2006:4[a]



Source: Authors' calculations.

a. The sample period starts in 1974:2 for New Zealand and 1979:2 for Norway. The window size for the rolling estimations is seventy-four quarters. For instance, the first point estimate corresponds to 1988:3, which is based on the period 1970:2–1988:3.

where v_j (v_{us}, $v_{euroaera}$) is an observable variable or an unobservable estimate for country *j* (for the United States, for the euro area), u_j (u_{us}, $u_{euroarea}$) is a zero-mean stochastic error term for country *j* (for the United States, for the euro area), and α_0 and α_i ($i = 1, \ldots p$) are the autoregressive coefficients of the AR(p) process.

For the AR(p) model, we obtain convergence across countries if the AR polynomial is stationary.[22] To test for stationarity, we use a grid bootstrap method to estimate confidence intervals for the parameters of interest (Hansen, 1999).[23]

The variable v represents observable variables (output growth and the interest rate), our estimates for unobservables (potential output growth and the neutral interest rate), and our estimated unobservable gaps (the output gap and the interest rate gap). We do not test for convergence in levels of cross-country gap measures, however, as they tend to zero by construction.

The convergence tests for actual output growth (table 9) and interest rates (table 10) reveal the following results. For actual growth convergence with the United States, we find that all countries are characterized by an AR(1) model, except Sweden with an AR(2) process. We find (weak) evidence of convergence with the United States for all countries, although α_j is only significant in Chile, New Zealand, Norway, and Sweden. For the remaining countries we are not able to reject a white-noise process.[24] For all countries, we obtain small half-lives of shocks (HLS) of only 0.6 quarters, on average.

When we examine actual growth convergence with the euro area, the relationships are characterized by higher-order AR processes in Japan, Norway, Sweden, and the United Kingdom. We find evidence of convergence with the euro area for all countries. The smallest HLS is 0.2 quarters (Australia) and the largest is 2.3 quarters (United Kingdom); the average HLS is 1.1 quarters.

22. For example, convergence of an AR(1) model requires that $|\alpha_1| < 1$; convergence of an AR(2) model requires that $\alpha_1 + \alpha_2 < 1$, $\alpha_2 - \alpha_1 < 1$, and $\alpha_2 > -1$. Hamilton (1994) provides a more detailed discussion of stationarity conditions.

23. The bootstrap method works as follows. Pick a grid over the parameters of interest and calculate the confidence interval by bootstrap at each parameter value, then smooth the estimated function for the confidence interval using a kernel regression, and finally obtain the confidence interval estimated by the kernel for a given value of the parameter. Lag lengths (p lags) are determined using the Akaike information criterion (AIC), the Hannan-Quinn criterion (HQC), and the Bayesian information criterion (BIC).

24. All autocorrelations and partial correlations are not significantly different from zero.

Table 9. Convergence of Actual Output Growth in Eight Countries with the United States and the Euro Area, 1970:2–2006:4[a]

Country	*I(0)* *(1)*	*Order* *(2)*	*Drift* *(3)*	*AR coefficients* *(4)*		*HLS* *(5)*
A. Convergence with the United States						
Euro area	Yes	1	0.0000	0.1260		0.3
Japan	Yes	1	0.0000	0.1105		0.3
New Zealand	Yes	1	0.0000	**−0.1836**		0.6
Canada	Yes	1	0.0000	−0.0273		0.5
United Kingdom	Yes	1	**−0.6998**	−0.1092		0.3
Australia	Yes	1	0.0000	−0.0684		0.5
Sweden	Yes	2	0.0000	−0.0162	**0.1742**	1.6
Norway	Yes	1	0.0000	**−0.3124**		0.7
Chile	Yes	1	**2.6974**	**0.2233**		0.5
				Average HLS		0.6

Table 9. (continued)

B. Convergence with the euro area								
Japan	Yes	4	0.0000	0.0797	0.1253	**0.2402**	**-0.1871**	1.7
New Zealand	Yes	1	0.0000	**−0.2010**				0.6
Canada	Yes	1	**0.6894**	**0.2240**				0.5
United Kingdom	Yes	3	0.0000	−0.0377	**0.1720**	**0.1801**		2.3
Australia	Yes	1	0.0000	0.0244				0.2
Sweden	Yes	3	0.0000	**−0.1829**	**0.2448**	**0.1837**		1.6
Norway	Yes	3	0.0000	**−0.2135**	**0.3031**	0.1768		1.1
Chile	Yes	1	**3.3966**	**0.2991**				0.6
				Average HLS				1.1

Source: Authors' calculations.

a. The sample period for Chile is 1986–2006. Significant estimates are in bold. In column 1, we use the grid bootstrap (Hansen, 1999) for autoregressive models to compute confidence intervals for all AR coefficients. In column 2, we use AIC, BIC and HQC criteria to determine lag lengths. Column 3 reports the value of the constant in the AR model. Column 4 presents the estimated AR coefficients. Column 5 reports the half-life of a unit shock (HLS) coefficient, which is defined as HLS = abs(log(1/2)/log(α)) for AR(1) model (with $\alpha \geq 0$). The HLS for AR(p) models can be calculated directly from the impulse response functions. We did not find convergence for the unobservables (natural rate of interest and potential output growth) in either case (with the United States or the euro area), since the series are not I(0) (stationary). In these cases, the HLS coefficients are explosive (∞ or a large number).

Table 10. Convergence of the Actual Interest Rate in Eight Countries with the United States and the Euro Area, 1970:2–2006:4[a]

Country	*I(0)* *(1)*	*Order* *(2)*	*Drift* *(3)*	*AR coefficients* *(4)*		*HLS* *(5)*
A. Convergence with the United States						
Euro area	Yes	1	0.0000	**0.8650**		4.8
Japan	Yes	1	0.0000	**0.8274**		3.7
New Zealand	Yes	1	0.0000	**0.7494**		2.4
Canada	Yes	1	0.0000	**0.7571**		2.5
United Kingdom	Yes	1	0.0000	**0.7625**		2.6
Australia	Yes	1	0.0000	**0.7107**		2.0
Sweden	Yes	1	0.0000	**0.6806**		1.8
Norway	Yes	1	0.0000	**0.8826**		5.6
Chile	Yes	2	0.0000	**0.7066**	**0.2182**	7.5
				Average HLS		3.6
B. Convergence with the euro area						
Japan	Yes	1	0.0000	**0.7554**		2.7
New Zealand	Yes	1	0.0000	**0.7060**		2.0
Canada	Yes	2	0.0000	**1.0074**	**-0.2645**	3.0
United Kingdom	Yes	1	0.0000	**0.6695**		1.7
Australia	Yes	1	0.0000	**0.5953**		1.3
Sweden	Yes	1	0.0000	**0.4365**		0.8
Norway	Yes	1	0.0000	**0.8115**		3.3
Chile	Yes	1	0.0000	**0.8813**		5.5
				Average HLS		2.6

Source: Authors' calculations.

a. The sample period for Chile is 1986–2006. Significant estimates are in bold. In column 1, we use the grid bootstrap (Hansen, 1999) for autoregressive models to compute confidence intervals for all AR coefficients. In column 2, we use AIC, BIC and HQC criteria to determine lag lengths. Column 3 reports the value of the constant in the AR model. Column 4 presents the estimated AR coefficients. Column 5 reports the half-life of a unit shock (HLS) coefficient, which is defined as HLS = abs(log(1/2)/log(α)) for AR(1) model (with $\alpha \geq 0$). The HLS for AR(p) models can be calculated directly from the impulse response functions. We did not find convergence for the unobservables (natural rate of interest and potential output growth) in either case (with the United States or the euro area), since the series are not I(0) (stationary). In these cases, the HLS coefficients are explosive (∞ or a large number).

Turning to convergence of actual interest rates with U.S. interest rates, we estimate an AR(1) process for almost all countries, except Chile with an AR(2) process (table 10). We find that all countries converge to the United States (and all estimated parameters are significant). As above, we also estimate HLS coefficients, which are much larger than those obtained for growth convergence. HLS coefficients range from 1.8 quarters (Sweden) to 7.5 quarters (Chile), with an average HLS of 3.7 quarters.

For interest rate convergence with the euro area, we estimate an AR(1) process for all countries, less Canada with an AR(2). All countries' interest rates converge to the euro area's. Our HLS estimates range from 0.8 quarters (Sweden) to 5.5 quarters (Chile), with an average HLS of 2.6 quarters.

We did not find country convergence of our two key estimated unobservables (that is, the potential output growth rate and the neutral real interest rate) with either the United States or the euro area. This reflects the fact that country differentials in unobservables—with either the United States or the euro area—are not stationary in the 1970–2006 sample.

6. Conclusions and Possible Extensions

The conduct of monetary policy is crucially dependent on several key unobservables. The output gap, the neutral real interest rate, the natural rate of unemployment, and expected inflation are the most critical for central bank models, forecasts, and policy decisions. Individual central banks have developed methodologies for estimating unobservable variables. Many researchers have derived estimates for single countries (usually the United States) or for a small number of developed economies. We have extended this literature by providing new estimates of key unobservables for ten economies, including the world's three largest economies and seven inflation-targeting countries. In addition, we have exploited our time-series estimates of unobservables for ten economies to test for common trends, more macroeconomic stability, comovements, and convergence across economies.

We adopted a very parsimonious monetary policy model comprising an IS relation, a Phillips curve, a Taylor rule, and transition equations for key observables and unobservables. This model was applied to all sample countries. An extended version, including Okun's law, was also applied to the United States and Chile. Our estimation model, which closely follows Laubach and Williams' (2003) sequential-step estimation procedure, yields country estimates for model parameters and unobservable-variable time series for each country.

Structural parameter estimates vary from country to country and sometimes from sample to sample for a given country. The results conform to our prior assumptions in the case of the United States, Canada, and Chile, less so for six other economies, and the least for Japan.

We also obtain reasonable and precise estimates for unobservable variables and for all countries. The evidence points to time variation in trend output growth, the neutral real interest rate, and (for the United States and Chile) the natural rate of unemployment. This time variation has important implications for the conduct of monetary policy. For example, if trend growth of potential output were constant, then policy rules that focus on the growth rate of output relative to the growth rate of potential (such as speed limit policies of the type analyzed in Walsh, 2003) might serve to eliminate (or at least significantly reduce) measurement problems in estimating the level of potential output. But if the growth rate of potential output is also subject to stochastic variation, as we find it to be, then the problem of estimating the level of potential cannot be eliminated by simply focusing on growth rates. Similarly, time variation in the neutral real interest rate implies that simple Taylor rules for the policy interest rate, which very commonly assume that the equilibrium real interest rate is constant, may lead to policy errors.

Finally we have used our estimates of unobservables and the data on observables to test for common trends and comovements across countries, the time trend toward more macroeconomic stability, and convergence in variable levels toward those observed in the United States and the euro area.

Consistent with the notion of a Great Moderation over the 1988–2006 period, measures of inflation volatility showed a marked common decline over the past decade. Output growth also declined in volatility. However, little of this decline in output growth volatility seems due to a decline in the volatility of the growth rate of potential output. The volatility of the latter has fallen slightly over the past twenty years, but this decline is small relative to the overall reduction in output growth volatility. Given these results, it is perhaps surprising that the volatility of the output gap displays only a modest decline over the sample. This reflects, in part, a rise in the average output gap volatility among our sample countries over the past decade. This is an interesting finding since it offers evidence, consistent with standard theoretical models, that greater inflation stability should come at the cost of some increase in output gap volatility. The failure of output gap volatility to fully reflect the decline in output growth volatility suggests that there may have been an increase in the volatility of the *level* of potential output over this period.

We find evidence that the volatility of the neutral real interest rate has declined when we look at the average across the sample economies. However, this masks significant differences among the individual economies.

Interestingly, we find neutral real interest rates to be more highly correlated across countries than either actual real rates or Wicksellian interest rate gaps. The notable exception to this finding is Japan. While neutral real rates were highly correlated across countries, this did not reflect a common pattern of convergence to the level of the U.S. or euro area neutral real rates. In fact, the neutral real rate differentials were nonstationary, indicating no long-run tendency to converge.

There are several extensions of the analysis that would be interesting to pursue. We would like to extend the approach to allow for richer and potentially different dynamics across the set of countries. Undoubtedly, one reason for some of our mixed results for individual countries arises from our use of a common specification of dynamics across all countries, particularly since our parsimonious model incorporated a fairly simple dynamic structure. It would also be useful to extend the sample to include more emerging market and developing economies. Many of these economies have adopted inflation-targeting frameworks in which the output gap and the neutral real interest rate are central to the design of policy. They are generally small open economies, making them candidates for exploring issues of convergence and comovements among these countries and the large industrialized economies.

Data Appendix[a]

Variable	Description	Source	Countries
Inflation measure	Consumer price index	IFS	Australia, Canada, Japan, New Zealand, Norway, Sweden, and United Kingdom
	Consumer price index	INE and CBC	Chile
	Price index for personal consumption expenditures	LW	United States
	Consumption deflactor	ECB	Euro area
Inflation targets	A composite measure that joins the HP-filtered inflation rate and the observed inflation targets for inflation targeters. For nontargeters (the euro area, Japan, and the United States) we use the HP-filtered series for the inflation measure.	Authors' construction	All countries
Inflation expectation	Calculations based on four step-ahead forecasts stemming from an AR(4) for the actual inflation rate.	Authors' construction	All countries
Gross domestic product	Seasonally adjusted real gross domestic product	OECD	Australia, Canada, Japan, New Zealand, Norway, Sweden, United Kingdom, and United States
		ECB	Euro area
		CBC	Chile
		LW	United States
Unemployment rate	Seasonally adjusted unemployment rate	OECD	Australia, Canada, Japan, New Zealand, Norway, Sweden, United Kingdom, and United States
		ECB	Euro area
		INE	Chile
Interest rate	Short-term interest rate. The real interest rate is calculated as the difference of the nominal interest rate and our estimation of the inflation expectations.	OECD	Australia, Canada, Japan, New Zealand, Norway, Sweden, and United Kingdom
		ECB	Euro area
	Real monetary policy rate. Previous to 1994 indexed CBC's 90-day bond rate. Since 2001, official nominal MPR less expected inflation from inflation reports.	CBC	Chile
	Monetary policy rate	LW	United States

a. IFS: International Financial Statistics; INE: National Statistics Institute of Chile; CBC: Central Bank of Chile; LW: Laubach and Williams (2003); ECB: European Central Bank; OECD: Organization for Economic Cooperation and Development.

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