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Can education be good for both growth
and the environment?

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Abstract

We develop an overlapping generations model of growth and the environment with public policy on education. Beyond the traditional mechanisms through which knowledge, growth and the environment interplay, we stress out the role played by education on environmental awareness. Assuming first that environmental awareness is constant, we show the existence of a balanced growth path along which environmental quality increases continually. Then, if education enhances environmental awareness, the equilibrium properties are modified: the economy can reach a steady state or converge to an asymptotic balanced growth path. Therefore, education does not necessarily promote sustained and sustainable growth.

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1 Introduction

By ratifying the United Nations Framework Convention on Climate Change, almost all countries of the world agreed to 'promote education, training and public awareness related to climate change" (UNFCCC (1992), Article 6). [1] Education is thus treated as a policy instrument which has a role to play in combating climate change. Even though the rational behind this is not indisputable, the macroeconomic consequences and environmental effectiveness of educational policies have not yet been explored from a theoretical perspective. Can countries really shape economic growth by greening' agents' preferences? In what direction would such changes lead the economy? Could they be beneficial to both economic growth and environmental quality?

Empirical evidence confirms the idea that knowledge and awareness about environmental issues improve as the agent's education level increases. A recent survey by the European Commission reveals that the longer the respondent has spent in full-time education, the better informed he or she feels.[2] The higher the education level, the more strongly the relationship between environmental quality and quality of life is perceived. Furthermore, as environmental awareness increases, so does the acceptability of regulation: "the higher the respondent's level of education, the more he or she values the importance of making stricter regulations" (EC (2005), p. 37).[3] Education, by enhancing awareness, modifies agents' trade-offs towards environmental-friendly decisions. Higher education is also related to higher income, and it is well-established that concern for the environment increases with wealth. Hence, a natural candidate for improving environmental awareness is raising the level of education.

On the other hand, education is today recognized as a key determinant of economic growth (see Krueger and Lindhal 2001 for a review of theoretical and empirical evidence). In Lucas's (1988) seminal paper, agents can improve their skills by devoting time to education. The accumulation of human capital enhances the productivity of labor, and allows the economy to experience sustained growth. In an overlapping generations model, Azariadis and Drazen (1990) explain the emergence of poverty trap by the existence of threshold effects in education. In these papers, time spent in education depends on private agents' decisions. However in a similar framework, Glomm and Ravikumar (1997) assume that education is financed by public expenditures. The authors emphasize the impact of education policy on growth. Their main result is that taxing revenues to finance education and the accumulation of knowledge stimulates long-term growth as long as the tax rate does not exceed the share of human capital in production.

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[1] The Convention on Climate Change set out an overall framework for intergovernmental efforts to tackle climate change. This recognizes that the climate system is a shared resource whose stability can be affected by anthropogenic emissions of greenhouse gases. Ratified by 192 countries, the Convention entered into force on 21 March 1994.

[2] This survey was conducted on 20,000 citizens in the 25 Member States. See EC (2005). It was updated in 2008.

[3] The survey also shows that increasing environmental awareness is considered to be as effective as stringer regulation and better enforcement at solving environmental problems.

On that basis, the question may be raised as to whether public education policy can promote both environmental quality and economic growth. Is education good for *green growth*?

Over the last twenty years, the literature on growth and the environment has increased substantially (see Xepapadeas 2005 and Brock and Taylor 2005 surveys). Naturally, some authors have attempted to dissect the link between education, growth and the environment. Gradus and Smulders (1993) review the consequences of introducing environmental concern into both exogenous and endogenous growth models. In particular, by adding a flow of pollutant and abatement into Lucas' model, they showed that sustained growth remains the rule in the long run. It turns out that the constant growth rate of the economy is in fact independent of the degree of environmental concern. Even though care for the environment crowds out investment in physical capital (the polluting input), this effect is exactly offset by the substitution of physical capital by human capital (the clean input). Vellinga (1999) gets the same result with a model extended by a stock of pollutants and with separability in utility between consumption and the environment. Within an overlapping generations model *à la* Blanchard (1985), Pautrel (2008) asks whether environmental policy may have a positive impact on long term growth. In his setting the growth rate of the economy depends on the environmental tax. Then, under certain conditions a *win-win* situation occurs, in which increasing the tax rate also benefits growth by providing households with greater incentives for education.

In sum, there are many studies that deal with education, growth and the environment. However the interdependence between education, environmental awareness and growth has not been explored in the literature. The purpose of our paper is to deal with this issue.

We consider an overlapping generations model (OLG) of public expenditure on education *à la* Glomm and Ravikumar (1997). We extend this framework by introducing the environment. Economic activity generates, as a by-product, polluting emissions that degrade the quality of the environment. Private agents, whose welfare depends on environmental quality and consumption, have the opportunity to devote part of their resources to environmental maintenance. In this respect, our model also extends the OLG models developed by John and Pecchenino (1994) and John *et al.* (1995).

As regards preferences about the environment, we follow Ono (2003) by defining environmental awareness as the elasticity between consumption and environmental quality. Two cases are then investigated. In the first, we analyze the properties of an economy composed of agents with *constant environmental awareness*. The literature on endogenous growth and the environment states that constant awareness is a necessary condition for sustainable balanced growth.[4] Within this benchmark framework we show that, under general and usual conditions, there exists a balanced growth path where physical capital, human capital and the environmental quality grow at a constant rate. In contrast to the earlier studies, we find that environmental

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[4]See notably Bovenberg and Smulders (1995, 1996) and Bovenberg and de Mooij (1997).

awareness does influence the growth rate of the economy: the stronger the environmental awareness, the lower the growth rate. We also achieve insights into the impact of education policy on growth, by generalizing the main conclusion of Glomm and Ravikumar (1997). Improving education allows the economy to enjoy a higher growth and a better environment, provided the tax rate is initially below a critical boundary related to the share of human capital in output.

In the second case we assume that environmental awareness is positively influenced by education. We call this *education-led environmental awareness*. It turns out that multiple equilibria of different nature coexist. If awareness depends on the level of human capital then agents' tradeoffs change towards more environmental friendly decisions as they accumulate knowledge. the economy may then either reach a steady state or follow an asymptotic balanced growth path in the long run. To understand which of these options will occur, the stability properties of the model are scrutinized. When the sensitivity of environmental awareness to knowledge is low, the economy may be driven towards an asymptotic balanced growth path. However, if environmental awareness is very sensitive to human capital, then it substantially affects the dynamics of the economy and may drive it to a steady state. Focusing on the steady state, we finally analyze the effects of a change in the tax rate on long term capital, knowledge and the environment. Again, our conclusions differ from those in the existing literature. Higher taxation does not necessarily enhance wealth and the environmental quality, even if the tax rate is initially below the critical boundary identified in the first case. In other words, such a policy is less likely to produce a *win-win* situation than is generally stated in the literature.

Our contribution to the literature on growth, education and the environment is twofold. First, even if environmental awareness is not constant, there is still room for balanced growth. Indeed, under the weaker condition that awareness is bounded from above, we show the existence of an asymptotic balanced growth path. Second, whereas balanced growth is possible, the economy may also be caught in a steady state when the impact of education on awareness is taken into account. Of course, the economic and environmental implications of these two solutions are poles apart.

The paper is organized as follows. Section 2 presents the model. Section 3 shows the existence and uniqueness of the balanced growth path in the economy with *constant environmental awareness*. It also contains some comparative statics to highlight the impact of environmental awareness and taxation on growth. Section 4 performs the equilibrium analysis for the economy with *education-led environmental awareness* and shows how accounting for the impact of knowledge on environmental awareness modifies the results. Finally, we present our conclusions in Section 5.

2   The model

The present framework is an overlapping generations model that combines considerations for growth, knowledge and the environment. In a perfectly competitive world, the firms produce a single homogeneous good used both for consumption and in-

vestment. In addition, the use of physical capital in production generates polluting emissions. The government levies a tax on income in order to finance education.

2.1 Production

Firms produce the final good Y_t with a constant returns to scale technology using skilled labour L_t^s and physical capital K_t:

$$Y_t = F(K_t, L_t^s), \tag{1}$$

where L_t^s is equal to the stock of human capital H_t time the amount of unskilled labor L_t. Since the production function is homogeneous of degree one, it can be expressed by its intensive form: $f(k_t)$ with $k_t = K_t/L_t^s$ the capital-skilled labour ratio.

Assumption 1. $f(k) : R^+ \to R^+$ is C^2. $f(0) = 0$, $\forall k > 0$, $f(k) > 0$, $f'(k) > 0$, $f''(k) < 0$. There exists an upper bound to the attainable capital $\tilde{k} < \infty$ such that: $f(\tilde{k}) = \tilde{k}$. In addition, $\lim_{k \to +\infty} f(k) = +\infty$ and $\lim_{k \to +\infty} f'(k) = 0$.

Assuming capital depreciates at the rate $\delta \in [0, 1]$, profit maximization yields:

$$w_t = f(k_t) - k_t f'(k_t) \tag{2}$$

$$r_t = f'(k_t) - \delta \tag{3}$$

with w_t the wage rate and r_t the real rental rate of capital.

2.2 The households

We consider an infinite horizon economy composed of finite-lived agents. A new generation is born in each period $t = 1, 2, ...,$ and lives for two periods: youth and old age. There is no population growth and the size of a generation is normalized to one. The young agent born in period t is endowed with H_t units of human capital. As in Glomm and Ravikumar (1997), the stock of knowledge accumulates according to the following constant returns process:

$$H_{t+1} = G(H_t, E_t) \tag{4}$$

with E_t the amount of public expenditures on education at period t. Defining e_t as the ratio from public expenditure to knowledge, this technology rewrites: $H_{t+1} = H_t g(e_t)$.

Assumption 2. $g(e) : R^+ \to R^+$ is C^2. $g(0) = 0$, $\forall e > 0$, $g(e) > 0$, $g'(e) > 0$, $g''(e) < 0$. Moreover, $\lim_{e \to +\infty} g(e) = +\infty$.

Education is thus entirely financed by public expenditures. In that purpose the government taxes both labor income and the interests on savings at an uniform tax rate τ.

In her youth, the agent supplies knowledge H_t to firms for a real wage $w_t H_t$. She allocates this wage net of taxes to savings s_t and maintenance m_t,[5] taking as given the tax rate τ. When retired, the agent supplies her savings to firms and earns the return of savings $R_{t+1} s_t$ (with $R_{t+1} = 1 + (1-\tau) r_{t+1}$ the interest factor). Her income is entirely devoted to the consumption c_{t+1}. The two budget constraints respectively write:

$$(1 - \tau) w_t H_t = s_t + m_t \tag{5}$$

$$c_{t+1} = (1 + (1 - \tau) r_{t+1}) s_t. \tag{6}$$

We further assume the government's budget, at period t, is balanced:

$$E_t = \tau (w_t H_t + r_t s_{t-1}) \tag{7}$$

Polluting emissions are imputed to the use of physical capital and degrade environmental quality Q. It is possible to control the level of emissions and to improve environmental quality through the maintenance m_t. The dynamics of Q are then given by:

$$Q_{t+1} = (1 - \mu) Q_t - \rho K_t + \gamma m_t, \tag{8}$$

The variable Q is a broadly defined index of the quality of the environment whose autonomous level (the level in the absence of human activity) is zero. For example, Q may be understood as the inverse of the atmospheric concentration of greenhouse gases. Then $\mu \in (0, 1)$ is the speed at which Q goes back to its autonomous level and $\rho > 0$ (respectively, $\gamma > 0$) is the parameter that represents the effect of emissions on environmental quality (respectively, of the efficiency of maintenance).

In accordance with John and Pecchenino (1994), the preferences of the agent born at date t are defined on old age consumption c_{t+1} and environmental quality Q_{t+1}. They are described by the utility function $U(c_{t+1}, Q_{t+1})$.

Assumption 3. $U(c, Q) : R^+ \times R^+ \to R$ is C^2 with: $U_1 \geq 0$, $U_2 \geq 0$, $U_{11}, U_{22} \leq 0$ and $\lim_{c \to 0} U_1(c, Q) = +\infty$. The cross derivative is positive $U_{12} \geq 0$.

Let us denote by η the elasticity between consumption and the environmental quality,

$$\eta = \frac{Q U_Q}{c U_c}. \tag{9}$$

This parameter represents the agents' environmental awareness, as in Ono (2003). The larger η, the stronger the value attached to the environment. Thereafter, by convention, the utility function will be parameterized by η, $U^{\eta_{t+1}}(c_{t+1}, Q_{t+1})$, and we will focus on the specific class of utility functions with a constant elasticity: [6] In the case where environmental awareness is given beforehand, we define the following:

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[5]It is possible to reinterpret m_t as a tax levied by a one period lived government in order to finance the abatement activity, for the benefit of agents living during its period of office (John *et al.* 1995).

[6]This property holds for the cases of logarithmic and Cobb-Douglas utility functions (Zhang 1999).

Definition 1 *Constant Environmental Awareness (CEA)*
Preferences with Constant Environmental Awareness are such that: $\eta_t = \eta$, $\forall t$

We will use CEA preferences to elaborate a benchmark model. Typically, CEA preferences are also used in the papers that tackle with sustainable growth (see for instance Bovenberg and Smulders 1995, 1996 and Bovenberg and de Mooij 1997). Having η constant is presented in the literature as a necessary condition for a balanced growth path to exist.

Then, our purpose is to depart from this restriction on preferences by introducing a relationship between the education level and agents' concern for the environment. For the same class of utility functions, this leads us to consider another case where the environmental awareness is shaped by education:

Definition 2 *Education-Led Environmental Awareness (ELEA)*
Preferences with Education-Led Environmental Awareness are such that: $\eta_t = \eta(H_t)$

An interesting and original approach to modeling environmental awareness consists in recognizing that agents' tradeoffs and decisions are influenced by their level of knowledge, particularly when these decisions encompass an environmental dimension. Indeed, one may expect that, with knowledge accumulation, private agents' tradeoffs evolve toward environmental friendly decisions. In other words, the more educated the agents, the greener their actions. This is consistent with the empirical evidence reported in the introduction. These features are summarized in the following assumption.

Assumption 4. $\eta(H)$ *is such that:* $\eta'(H) > 0$, $\eta''(H) \leq 0$ *and* $\lim_{H \to \infty} \eta(H) = \bar{\eta} < \infty.$[7]

Environmental awareness grows at a decreasing rate with knowledge. There exists an upper bound for the weight of the environment in preferences. Even if knowledge can potentially grow indefinitely, environmental awareness asymptotically converges to a level of full awareness of environmental issues. It must be kept in mind that, in this model, the agent cannot influence her own environmental awareness.

Depending on the case considered, CEA or ELEA, the representative agent born at date t divides her resources between maintenance and savings in order to maximize her lifetime utility. Taking prices and pollution at the beginning of period t as given, the problem is written as:

$$\max_{m_t, s_t, c_{t+1}} U^{\eta_{t+1}}(c_{t+1}, Q_{t+1})$$

subject to,

$$\begin{cases} w_t H_t = s_t + m_t \\ c_{t+1} = (1 + (1 - \tau)r_{t+1})s_t \\ Q_{t+1} = (1 - \mu)Q_t - \rho K_t + \gamma m_t \\ m_t \geq 0 \end{cases}$$

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[7]As an illustration of ELEA preferences, one may consider a log-additive utility function: $U(c_{t+1}, Q_{t+1}, H_{t+1}) = \log c_{t+1} + \eta(H_{t+1}) \log Q_{t+1}$.

The first order condition (FOC) for the representative agent's problem reads

$$-R_{t+1}U_1(c_{t+1}, Q_{t+1}) + \gamma U_2(c_{t+1}, Q_{t+1}) = 0, \tag{10}$$

Let us now define the competitive equilibrium.

Definition 3 *Intertemporal competitive equilibrium*
Given the public policy in education, an intertemporal competitive equilibrium is a sequence of per capita variables $\{c_t, m_t, s_t\}$, aggregate variables $\{K_t, H_t, Q_t, E_t\}$ and prices $\{w_t, r_t\}$ such that:

  i/ households and firms are at their optimum: the FOC (10) and the two conditions (2) and (3), for profit maximization, are satisfied,

 ii/ all markets clear: $L_t = 1$ implying $L_t^s = H_t$ and $K_{t+1} = s_t$,

iii/ individual budget constraints (5) and (6) are satisfied,

iv/ the government budget (7) is balanced,

 v/ the dynamics of human capital are given by (4),

vi/ the dynamics of environmental quality are given by (8).

3 Competitive equilibrium in the CEA economy

In this section, the equilibrium analysis will be conducted for the case where preferences are of the CEA type.[8]

3.1 Balanced growth path

In the CEA economy, households preferences are given by Assumption 3. The FOC (10) becomes

$$c_{t+1} = \frac{(1 + (1 - \tau)r_{t+1})Q_{t+1}}{\gamma\eta} \tag{11}$$

and, according to the definition of the competitive equilibrium, it defines a relationship between Q_{t+1} and K_{t+1}

$$Q_{t+1} = \gamma\eta K_{t+1},$$

or equivalently

$$Q_{t+1} = \gamma\eta H_{t+1}k_{t+1}, \tag{12}$$

that governs the dynamics in the positive maintenance region.

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[8]For the sake of simplicity, we will not consider the non-negativity constraint on m_t, thereby concentrating on the interior solution.

Thereafter, we consider that capital does not depreciate: $\delta = 0$. Then, under conditions (2) and (3), the government's revenue expresses as a share of production:

$$\tau(w_t H_t + r_t s_{t-1}) = \tau H_t f(k_t),$$

which implies that the dynamics of H simplify to:

$$H_{t+1} = H_t g(\tau f(k_t)). \tag{13}$$

Finally, the maintenance decision is:

$$m_t = (1 - \tau)H_t(f(k_t) - k_t f'(k_t)) - H_{t+1}k_{t+1}. \tag{14}$$

Putting eqs. (12)-(14) together, equilibrium dynamics are given by the following system:

$$\begin{cases} Q_{t+1} = \gamma\eta H_{t+1}k_{t+1} \\ H_{t+1} = H_t g(\tau f(k_t)) \\ Q_{t+1} = (1 - \mu)Q_t - \rho H_t k_t + \gamma\left((1 - \tau)H_t(f(k_t) - k_t f'(k_t)) - H_{t+1}k_{t+1}\right) \end{cases} \tag{15}$$

Thereafter, we investigate whether a balanced growth path (BGP) exists. A BGP is defined as follows.

Definition 4 _Balanced growth path (BGP)_
A balanced growth path is a 4-tuple $\{\bar{Q}, \bar{H}, \bar{K}, \theta\}$ such that

$$\begin{cases} Q_t = \bar{Q}(1 + \theta)^t \\ H_t = \bar{H}(1 + \theta)^t \\ K_t = \bar{K}(1 + \theta)^t \end{cases} \tag{16}$$

solves exactly (15).

A BGP is a path where all state variables grow at the same constant rate θ.

Let us define capital's share of output and the elasticity of substitution between physical and human capital as follows:

$$s(k) = \frac{kf'(k)}{f(k)}. \tag{17}$$

$$\sigma(k_t) = -\frac{(1 - s(k))f'(k)}{kf''(k)}. \tag{18}$$

The first proposition establishes the existence of a BGP.

Proposition 1 *If*

$$\lim_{k \to 0} \frac{f(k) - kf'(k)}{k} > \frac{1}{1 - \tau}, \tag{19}$$

$$\sigma(k) \geq s(k) \ \forall k > 0 \tag{20}$$

and

$$\eta(1 - \mu) \geq \frac{\rho}{\gamma}, \tag{21}$$

then there exists a unique BGP in the CEA economy. Admissibility of the BGP requires:

$$\frac{(1 - \tau)(f(\bar{k}) - \bar{k}f'(\bar{k}))}{\bar{k}} \geq g(\tau f(\bar{k})) \tag{22}$$

with $\bar{k}$, the long run physical to human capital ratio.

Proof. See Appendix A. ■

Condition (19) is common in the literature that studies the equilibrium properties of the OLG-one-sector model (without environmental issue). It is also used in Prieur (2008) who deals with this kind of issue. This condition generalizes *the strengthened Inada condition*, introduced by Galor and Ryder (1989), with a public policy. It ensures that the first unit of capital is sufficiently efficient, in terms of labor productivity (recall that the numerator in (19) corresponds to the wage $w(k)$), thereby avoiding the trivial equilibrium with zero capital. It is also a necessary condition for the existence of a non trivial equilibrium (see de la Croix and Michel (2000)). The second condition states that the elasticity of substitution between capital and labor is higher than the capital share of output.

These conditions are notably satisfied by the CES technologies, $F(K, H) = (\alpha K^{-\phi} + (1 - \alpha)H^{-\phi})^{-\frac{1}{\phi}}$, when $\phi \in (-1, 0]$. In addition, the second one seems quite reasonable since most of the estimations of the capital share of output and the elasticity of substitution give values respectively comprised in the range $[0.3, 0.4]$ and close to 1.

The sufficient condition (21) involves many parameters and can be interpreted as follows. Let us consider the present generation with high environmental concern η. She is willing to devote a lot of resources in order to maintain the environmental quality. But, this decision is done at the expense of savings and negatively affects capital accumulation. The next generation will, in turn, inherits from a lower stock of capital. The negative income effect will translate again into a reduction of the investment in physical capital. In order to balance this negative effect, the current generation must, at the same time, bequeath an important amount of environmental quality. Indeed, in this situation, the next generation will be able to substitute maintenance for savings. It requires ρ and/or μ (respectively γ) to be low (respectively high). This substitution allows the economy to achieve, in the long run, a sustainable growth path. Condition (22), for admissibility, ensures that long term maintenance is non-negative.

Let us denote by $\chi(k)$ the growth factor, $\chi(k) = g(\tau f(k))$,[9] and let us define $\bar{e}^x$, for $x = w, \chi$, as the long run elasticity with respect to k: $\bar{e}^x = \frac{\bar{k}x'(\bar{k})}{x(\bar{k})}$. Then, a sufficient condition for local stability is: $\bar{e}^\chi \leq \bar{e}^w$ (see Appendix B). Actually, we have $\bar{e}^w = s(\bar{k})s(\bar{e})$, where $s(\bar{e})$ is the share of public expenditures in the education technology. Thus, we also have that $\bar{e}^\chi \leq \bar{e}^w \leftrightarrow s(\bar{e}) \leq \frac{1}{\sigma(\bar{k})}$. This condition requires the share of public expenditures in education not to be too high.[10]

3.2 Comparative statics

This section assesses the impact of a change in the tax rate and of a change in environmental awareness on the long run growth rate. Along the BGP, the dynamics (15) reduce to

$$\begin{cases} \chi = g(\tau f(\bar{k})) \\ (1+\eta)g(\tau f(\bar{k})) = \eta(1-\mu) - \frac{\rho}{\gamma} + (1-\tau)\left(\frac{f(\bar{k})}{\bar{k}} - f'(\bar{k})\right) \end{cases} \tag{23}$$

where $\chi = \chi(\bar{k})$ is the growth factor common to all the state variables: $\chi = \frac{K_{t+1}}{K_t} = \frac{H_{t+1}}{H_t} = \frac{Q_{t+1}}{Q_t}$.

Proposition 2 *Along the balanced growth path,*
  i/ when the CEA economy grows at a non negative constant rate ($\chi \geq 1$), the stronger the concern for the environment, the lower the growth rate.
  ii/ increasing the tax enhances economic growth if and only if $\tau \leq \bar{\tau}$, with

$$\bar{\tau} = \frac{1 - s(\bar{k}) + e^{f'}s(\bar{k})}{s(\bar{k})(1 - s(\bar{k}) + e^{f'}s(\bar{k})) + 1 - s(\bar{k})} \tag{24}$$

where $e^{f'}$ is the elasticity of the marginal productivity with respect to k.

  Proof. Total differentiation of equations in (23) yields:

$$\begin{cases} d\chi = g'(\tau f(\bar{k}))(f(\bar{k})d\tau + \tau f'(\bar{k})dk) \\ (1+\eta)d\chi + \chi d\eta = (1-\mu)d\eta - \frac{\rho}{\gamma} + (1-\tau)\left(\frac{f(\bar{k})}{\bar{k}} - f'(\bar{k})\right)' dk - \left(\frac{f(\bar{k})}{\bar{k}} - f'(\bar{k})\right)d\tau \end{cases} \tag{25}$$

Rearranging and assuming $d\eta = 0$, we get

$$\frac{d\chi}{d\tau} \geq 0 \leftrightarrow \tau \leq \bar{\tau} \tag{26}$$

Now assuming $d\tau = 0$,

$$\frac{d\chi}{d\eta} \leq 0 \leftrightarrow \chi \geq 1 - \mu$$

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[9]In the long run, the growth rate is thus equal to $g(\tau f(\bar{k})) - 1$.
[10]Since $s(\bar{e}) < 1$, it holds for the Cobb-Douglas and the CES with $\phi \geq 0$ and for some $\phi < 0$ not too close to -1.

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The impact of environmental awareness on growth differs from the one generally detected in the literature. Gradus and Smulders (1993), Vellinga (1999) and Pautrel (2008) have shown that the long run growth rate is independent of the degree of environmental concern. In their setting, the negative impact of greener preferences on physical capital accumulation (the polluting input) is exactly offset by the greater incentive to accumulate human capital (the clean input). Here, because education does not rely on agents' decisions (from their point of view, it is exogenous), this compensation does not occur. Stronger awareness implies that agents allocate more resources to environmental maintenance. Consequently, the accumulation of both K and H is slowed down.

The condition (24) which determines whether a change in the tax rate is beneficial for growth or not only depends on technological features, and in particular on the (long run) share of human capital in production, $1 - s(\bar{k})$. To illustrate the meaning of $\bar{\tau}$, let us consider the specification of Glomm and Ravikumar (1997), $f(k) = Ak^\alpha$, $\alpha \in (0, 1)$. Then, condition (24) reduces to $\tau \leq 1 - \alpha$: increasing the tax enhances the growth rate if and only if the tax is initially below the share of human capital in production. This proposition generalizes the standard result of the literature (see Glomm and Ravikumar, 1997) when the environment is taken into account. An increase in the tax rate has two opposite effects on the agent's income. There is first a direct negative income effect. On the one hand, in the first period, the agent has less resources to devote to maintenance and savings. On the other, in the second period, she consumes less since the returns on savings decrease. But, at the same time, a higher tax increases the government revenue and stimulates public expenditures on education. More human capital means more production and more income distributed to households. This corresponds to an indirect positive income effect. When $\tau \leq 1 - \alpha$, the benefits of a tax increase exceed the costs, and the growth rate of the economy is enhanced.

Having understood the properties of the CEA economy we can now turn to the case where environmental awareness is influenced by the education level, what we label the Education-Led Environmental Awareness (ELEA) economy.

4   Competitive equilibrium in the ELEA economy

The preferences in the ELEA economy are given by Assumption 4. The definition of the competitive equilibrium remains unchanged. The FOC for the representative agent's problem, given by (10), now reads

$$Q_{t+1} = \gamma \eta(H_{t+1}) H_{t+1} k_{t+1}, \tag{27}$$

and equilibrium dynamics become:

$$\begin{cases} Q_{t+1} = \gamma \eta(H_{t+1}) H_{t+1} k_{t+1} \\ H_{t+1} = H_t g(\tau f(k_t)) \\ Q_{t+1} = (1 - \mu) Q_t - \rho H_t k_t + \gamma \left((1 - \tau) H_t (f(k_t) - k_t f'(k_t)) - H_{t+1} k_{t+1} \right) \end{cases} \tag{28}$$

Looking at these dynamics, it turns out that the ratio between environmental quality and physical capital cannot be kept constant because of the endogeneity of environmental awareness. Let us investigate the kind of outcome that may emerge from (28).

4.1 Long run equilibria

Definition 5 *Asymptotic balanced growth path (ABGP)*
An economy follows an asymptotic balanced growth path if it approaches a balanced growth path asymptotically.

An asymptotic BGP occurs when a BGP cannot be reached but is gradually converged as time goes to infinity. The very characteristics of an asymptotic BGP is that, even if no proper BGP exists, the growth rates of state variables of the economy still approach a constant and common value. In the following proposition, we show that the ELEA economy may follow such an asymptotic BGP, or a steady state that solves:

$$\begin{cases} Q = \gamma\eta(H)Hk \\ \mu Q = \left(\gamma\left((1-\tau)(f(k)-kf'(k))-k\right)-\rho k\right)H \\ g(\tau f(k)) = 1 \end{cases} \tag{29}$$

Proposition 3 *In the ELEA economy:*
- *there exists a unique ABGP under conditions (19)-(21);*
- *there exists a unique steady state if and only if:*

$$(1-\tau)\frac{f(\tilde{k})-\tilde{k}f'(\tilde{k})}{\tilde{k}} - 1 \geq \frac{\rho}{\gamma} \tag{30}$$

with $\tilde{k}$, the long run physical to human capital ratio.

Proof. The proof of existence of an ABGP is provided in Appendix C.

As for the existence of a SS, the last equation in (29) gives the equilibrium ratio between K and H

$$\tilde{k}(\tau) = f^{-1}\left(\frac{g^{-1}(1)}{\tau}\right), \tag{31}$$

and combining the two first equations yields:

$$\mu\gamma\eta(\tilde{H}) = \gamma(1-\tau)\frac{f(\tilde{k})-\tilde{k}f'(\tilde{k})}{\tilde{k}} - \gamma - \rho \geq 0 \text{ if (30) holds.}$$

Thus the steady state level of human capital is:

$$\tilde{H}(\tau) = \eta^{-1}\left(\frac{1}{\mu\gamma}\left(\gamma(1-\tau)\frac{f(\tilde{k}(\tau))-\tilde{k}(\tau)f'(\tilde{k}(\tau))}{\tilde{k}(\tau)} - \gamma - \rho\right)\right),$$

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and is uniquely defined under the assumptions regarding $f(k)$ and $\eta(H)$. Finally, we obtain: $\tilde{K}(\tau) = \tilde{k}(\tau)\tilde{H}(\tau)$ and $\tilde{Q}(\tau) = \gamma\eta(\tilde{H}(\tau))\tilde{K}(\tau)$. ∎

In the previous section, we have shown that the accumulation of knowledge, together with environmental maintenance, was sufficient to promote sustainable growth. This result does not necessarily hold once the positive impact of education on environmental awareness is taken into account. To understand this property, simply refer to the trade-offs governing the agent's decisions. An increase in the effort m_t is a means to improve environmental quality and thus to enhance welfare. However, it also implies a fall in the non-environmental component of welfare since both savings and old age consumption decrease. Consequently, the agent chooses m_t to equate the marginal benefit of maintenance to its marginal cost. Now, other things equal, with the accumulation of knowledge the agent pays more attention to the environment. The marginal benefit of maintenance tends to be higher than its marginal cost, giving an incentive to the agent to allocate more resources to maintenance at the expense of savings. This substitution effect is accompanied by an income effect, since more knowledge means more production and more resources to be devoted to savings and maintenance. However, the latter effect does not always offset the former and economic growth may not be sustained.

The literature on endogenous growth and the environment has defined the necessary conditions on technology, preferences and environmental dynamics for a balanced growth to be possible (for a review of these conditions, refer to Bovenberg and Smulders 1995, 1996 and Bovenberg and de Mooij 1997). In particular, in optimal growth models, the elasticity or environmental awareness η is required to be constant. That is the reason why the papers in this field systematically consider utility functions with constant elasticity. In our setting, there exists an ABGP, which is economically similar to a BGP, even if this condition does not hold. Indeed, it turns out that this condition can be relaxed by simply requiring environmental awareness to be bounded from above (since $\eta'(H) > 0$), this boundary corresponding to a state of full awareness.

The result in proposition 3 clearly challenges the optimistic vision shared in the literature that the accumulation of knowledge fosters economic growth while being compatible with environmental improvement. If the economy may follow an ABGP, it may also be caught in a steady state, thereby loosing the opportunity to accumulate wealth and to improve environmental quality forever. An interesting issue is to determine under which conditions the economy may reach one trajectory or the other. A first answer is provided with the analysis of local stability. Let us define $e^\eta = \frac{H\eta'(H)}{\eta(H)} > 0$, the elasticity of environmental awareness with respect to H.

Proposition 4 *i/ Local stability of the ABGP imposes: $\tilde{e}^\chi \leq \tilde{e}^w$;*
   ii/ For the SS, an additional condition is required: $\tilde{e}^\eta \in [\underline{\tilde{e}^\eta}, \overline{\tilde{e}^\eta}]$, with

$$\underline{\tilde{e}^\eta} = \frac{\Lambda}{(1-\mu)\eta(\tilde{H})\tilde{e}^\chi}, \quad \overline{\tilde{e}^\eta} = \frac{1+\eta(\tilde{H})}{\eta(\tilde{H})\tilde{e}^\chi}$$

$and \ \Lambda = (1 + \eta(\tilde{H}))(1 - \tilde{e}^\chi) - \left(\frac{\rho}{\gamma} + \mu\eta(\tilde{H}) + 1\right)(1 - \tilde{e}^w) > 0.$

In other words, the stability of the SS is more difficult to obtain than the one of the ABGP. Actually, the curvature of $\eta(H)$ is crucial for the stability analysis. When $\eta(H)$ is flat, environmental awareness is relatively insensitive to the level of knowledge. The dynamics behave as if we were in the CEA economy. Thus, there is no room for convergence toward the SS and the economy will be drawn along the ABGP. In contrast, if environmental awareness exhibits a strong reaction to a change in H, then the accumulation of knowledge substantially affects the dynamics, through this new channel, and may drive the ELEA economy to the SS. The influence of human capital on awareness must be significant, that is, $\eta'(H)$ must be high enough. This is the sense of $\tilde{e}^\eta \geq \underline{\tilde{e}}^\eta$. Now, at the same time, stability requires the magnitude of the impact of knowledge to be bounded from above ($\tilde{e}^\eta \leq \overline{\tilde{e}^\eta}$).

4.2 Comparative statics

Again, we are interested in the impacts of a policy promoting education but now, on the steady state. From the definition of $k^*(\tau)$ (see eq. (31)), we have $k^{*\prime}(\tau) < 0$: an increase in the tax rate lowers the equilibrium ratio from physical to human capital. This corresponds to a crowding-out effect according to which public expenditures on education are done at the expense of private savings.

In order to assess the impact of a change in τ on wealth and the environment, the same approach as in section 3.2 is followed. We use the specifications of Glomm and Ravikumar (1997) and, in addition, we define the expression of $\eta(H)$:

$$f(k) = Ak^\alpha, \ \alpha \in (0, 1)$$
$$g(e) = Be^{1-\beta}, \ \beta \in (0, 1)$$
$$\eta(H) = \bar{\eta}\left(1 - \frac{\varepsilon}{H}\right) \ \text{with} \ \bar{\eta} > 0, \varepsilon > 0$$

Here, the parameter $\bar{\eta}$ may be identified to the constant elasticity discussed in Section 3. It means that, starting with a stock of human capital $H_0 \geq \varepsilon$, the value of $\eta(H)$ is first lower than $\bar{\eta}$ and then monotonically converges to $\bar{\eta}$ as the economy accumulates knowledge. Considering these functional forms, the steady state values of human and physical capital and environmental quality are:

$$H^*(\tau) = \frac{\mu\bar{\eta}\varepsilon}{\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau)}, \tag{32}$$

$$K^*(\tau) = \frac{\mu\bar{\eta}\varepsilon}{\left(AB^{\frac{1}{1-\beta}}\right)^{\frac{1}{\alpha}} \tau^{\frac{1}{\alpha}} \left(\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau)\right)}, \tag{33}$$

$$Q^*(\tau) = \frac{\gamma\bar{\eta}\varepsilon\left(\pi(\tau) - \frac{\rho}{\gamma}\right)}{\left(AB^{\frac{1}{1-\beta}}\right)^{\frac{1}{\alpha}} \tau^{\frac{1}{\alpha}} \left(\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau)\right)}, \tag{34}$$

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with

$$\pi(\tau) = \left(AB^{\frac{1-\alpha}{1-\beta}}\right)^{\frac{1}{\alpha}}(1-\alpha)(1-\tau)\tau^{\frac{1-\alpha}{\alpha}} - 1.$$

In equilibrium, maintenance can be expressed as a share of physical capital, and this share is precisely given by $\pi(\tau)$: $m^*(\tau) = \pi(\tau)K^*(\tau)$. Condition (30) reads $\pi(\tau) \geq \frac{\rho}{\gamma}$, it implies equilibrium maintenance is positive. Moreover, the existence of the steady state requires the coefficient $\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau)$ to be strictly positive, which implies that $\pi(\tau) \in [\frac{\rho}{\gamma}, \mu\bar{\eta} + \frac{\rho}{\gamma})$.

Proposition 5 *In the steady state,*
    i/ if $\tau \geq 1 - \alpha$, then raising the tax decreases physical and human capital, and environmental quality;
    ii/ if $\tau < 1 - \alpha$, then raising the tax increases human capital, but the effect on physical capital and the environment is unclear.

    Proof. See Appendix E. ∎

Even if the usual argument developed in the literature (and discussed in Section 3.2) holds for the sensitivity of knowledge to the tax, it is no longer the case for physical capital and environmental quality. In other words, having initially $\tau < 1-\alpha$ is necessary but not sufficient to get a double dividend. Indeed, an additional effect plays with the increase in the tax rate. A higher tax tends to stimulate the accumulation of knowledge. But this, in turn, modifies the agent's preferences that become greener. Consequently, she allocates more resources to maintenance at the expense of savings and physical capital accumulation. This is a possible explanation of the fact that the critical level of the tax rate,[11] if it exists, is located somewhere below the labor share in output.

The additional effect may lead to a steady state with a lower level of physical capital. The impact on environmental quality is more complicated since, according to the first condition in (29), environmental quality is positively related to human and physical capital. If physical capital increases in response to the higher tax, then environmental quality increases too. There exists a double dividend. Otherwise, the overall impact is indeterminate since H increases whereas K decreases.

In sum, the policy might procure a double dividend (more growth and better environmental quality), but it requires the initial tax to be rather low, and lower than the usual bound $1 - \alpha$.

5   Conclusion

In this paper we have developed an OLG model with public expenditures on education and the environment. Our purpose was to assess a new channel through which education and the accumulation of knowledge may influence economic and environmental dynamics: the environmental awareness. For this purpose, we compared two

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[11]In the sense that it determines whether or not the reform has a positive impact on K and Q.

cases. First, assuming constant environmental awareness, we have generalized the conclusion of the literature on education and growth by showing the existence of a balanced growth path that is environmentally sustainable. Next, we have explicitly modelled the relationship between knowledge and environmental awareness. The competitive equilibrium then exhibits distinct features. In contrast with the previous case, constant growth is no longer the rule. Rather, the economy may reach either a steady state or an asymptotic balanced growth path in the long run. The convergence towards one or the other equilibrium is dictated by the environmental awareness sensitivity to knowledge. An analysis of the impact of the education policy on growth and the environment, still based on the comparison between the two cases, was also conducted. Again, the role played by the endogenous awareness is highlighted. The critical level of taxation that determines whether, by raising the tax, the economy may experience a higher growth rate and a better environmental quality is lowered when awareness is endogenous.

Taking into account the positive influence of knowledge on environmental awareness challenges the very existence of a sustainable growth path, as it is generally advocated in the literature. Indeed, with the accumulation of knowledge awareness increases and agents progressively divert themselves from polluting activities. This evolution in tastes benefits to the environment, since agents get higher incentives to engage in green activities. This results in welfare improvement but, in the meantime, may preclude balanced growth.

Appendices

A   Existence of a BGP (prop. 1)

In the long run, the growth factor of K, H and Q is equal to: $\frac{H_{t+1}}{H_t} = g(\tau f(k))$. According to (12), the ratio from environmental quality to human capital is: $\frac{Q_{t+1}}{H_{t+1}} = \gamma\eta k$. Combining the first and third eqs. in (15) yields:

$$\gamma(1 + \eta)H_{t+1}k_{t+1} = (1 - \mu)Q_t - \rho H_t k_t + \gamma(1 - \tau)H_t(f(k_t) - k_t f'(k_t))$$

Dividing both sides of this eq. by H_t and evaluating this eq. along a BGP give:

$$\underbrace{\gamma(1 + \eta)g(\tau f(k))}_{I(k)} = \underbrace{\gamma(1 - \tau)\frac{f(k) - kf'(k)}{k} + \gamma\eta(1 - \mu) - \rho}_{J(k)} \tag{35}$$

Proving the existence of a BGP boils down to showing that there exists an intersection between $I(k)$ and $J(k)$.

From assumption 1. & 2., $I(k)$ is increasing and concave. Moreover, $I(0) = 0$ and $\lim_{k \to +\infty} I(k) = +\infty$.

The first derivative of $J(k)$ is

$$J'(k) = \gamma(1 - \tau)\frac{f''(k)}{s(k)}\left(\sigma(k) - s(k)\right),$$

therefore, $J'(k) \leq 0 \leftrightarrow \sigma(k) \geq s(k)$. Since there exists an upper bound to the attainable capital, we necessarily have $\lim_{k \to +\infty} \frac{f(k) - kf'(k)}{k} = 0 \leftrightarrow \lim_{k \to +\infty} J(k) = \gamma\eta(1 - \mu) - \rho \lessgtr 0$. Assume $\gamma\eta(1 - \mu) - \rho > 0$, then $\gamma(1 + \eta(1 - \mu)) \geq \rho$ and

$$\lim_{k \to 0} \frac{f(k) - kf'(k)}{k} > \frac{1}{1 - \tau}. \tag{36}$$

then $0 < \lim_{k \to 0} J(k) < \infty$.

There exists a unique intersection $\bar{k}$ between the two functions. Since $\frac{H_{t+1}}{H_t} = g(\tau f(\bar{k}))$, it means that the unique constant growth rate is $\theta = g(\tau f(\bar{k})) - 1$.

B   Local stability for BGP

Dynamics (15)

$$\begin{cases} q_{t+1} = \gamma\eta k_{t+1} \\ \chi(k_t)q_{t+1} = (1 - \mu)q_t - \rho k_t + \gamma(1 - \tau)w(k_t) - \gamma\chi(k_t)k_{t+1} \end{cases}$$

with $q_t = \frac{Q_t}{H_t}$ and $\chi(k_t) = g(\tau f(k_t))$. Linearize the dynamics around the steady state $(\bar{k}, \bar{q})$ and make use of (35), the system reduces to one dimensional dynamics:

$$dq_{t+1} = \psi dq_t \text{ with } \psi = \left(\frac{\left(\eta(1 - \mu) - \frac{\rho}{\gamma} \right)}{(1 + \eta)\chi(\bar{k})} (1 - \bar{e}^w) + \bar{e}^w - \bar{e}^\chi \right)$$

where $\bar{e}^x$, for $x = w, \chi$, is the long run elasticity with respect to k: $\bar{e}^x = \frac{\bar{k}x'(\bar{k})}{x(\bar{k})}$. Assume $\bar{e}^\chi < \bar{e}^w$. Since $\bar{e}^w = s(\bar{k})s(\bar{e})$, with $s(\bar{e})$ the share of public expenditures in the education technology, $\bar{e}^\chi < \bar{e}^w \leftrightarrow s(\bar{e}) < \frac{1}{\sigma(k)}$ (since $s(\bar{e}) < 1$, it holds for the Cobb-Douglas, the CES with $\phi \geq 0$ and for some $\phi < 0$ not to close to -1). $\bar{e}^\chi < \bar{e}^w$ together with (21) implies $\psi > 0$. Local stability then imposes: $\psi < 1$ that is equivalent to: $\left(\eta(1 - \mu) - \frac{\rho}{\gamma} - (1 + \eta)\chi(\bar{k}) \right)(1 - \bar{e}^w) < (1 + \eta)\chi(\bar{k})\bar{e}^\chi$. According to (35), this inequality is satisfied.

To summarize, if $\bar{e}^\chi < \bar{e}^w$, then the BGP is locally stable.

C   Existence of an ABGP (prop. 3)

We are interested in cases where agregate variables grow at non negative, non constant, rates. The growth factor satisfies: $\chi_i \geq 1$ for $i = K_t, H_t, Q_t$. Dynamics are:

$$\begin{cases} Q_{t+1} = \gamma\eta(H_{t+1})H_{t+1}k_{t+1} & (*) \\ H_{t+1} = H_t g(\tau f(k_t)) & (+) \\ Q_{t+1} = (1 - \mu)Q_t - \rho H_t k_t + \gamma \left((1 - \tau)H_t(f(k_t) - k_t f'(k_t)) - H_{t+1}k_{t+1} \right) & (\times) \end{cases}$$

Substitute $(*)$ into $(\times)$:

$$\left(1 + \frac{1}{\eta(H_{t+1})} \right) \frac{Q_{t+1}}{Q_t} = 1 - \mu - \frac{\rho}{\gamma\eta(H_t)} + \frac{(1 - \tau)}{\eta(H_t)} \left(\frac{f(k_t)}{k_t} - f'(k_t) \right) \qquad (37)$$

• Assume first $\lim_{t \to +\infty} k_t = 0 \leftrightarrow \lim_{t \to +\infty} K_t \ll \lim_{t \to +\infty} H_t$:

From $(+)$ and $g(0) = f(0) = 0$, $\lim_{t \to +\infty} \chi_{H_t} = 0$ which contradicts the condition $\chi_i \geq 1$ for all i.

• Assume next $\lim_{t \to +\infty} k_t = +\infty \leftrightarrow \lim_{t \to +\infty} K_t \gg \lim_{t \to +\infty} H_t$:

At least asymptotically, we have $\chi_{K_t} > \chi_{H_t}$: $\lim_{t \to +\infty} \chi_{K_t} > \lim_{t \to +\infty} \chi_{H_t}$.

From $(+)$, $\lim_{k \to +\infty} f(k) = +\infty$ and $\lim_{e \to +\infty} g(e) = +\infty$, it turns out that $\lim_{t \to +\infty} \chi_{H_t} = g(\tau f(\lim_{t \to +\infty} k_t)) = +\infty$. Thus, $\lim_{t \to +\infty} H_t = +\infty$ and, from assumption 4, $\lim_{t \to +\infty} \eta(H_t) = \bar{\eta}$.

Together with $(*)$, $\lim_{t \to +\infty} \frac{Q_{t+1}}{K_{t+1}} = \gamma\bar{\eta}$. Therefore, $\lim_{t \to +\infty} \chi_{K_t} = \lim_{t \to +\infty} \chi_{Q_t}$.

But, according to (37),

$$\lim_{t \to +\infty} \chi_{Q_t} = \lim_{t \to +\infty} \frac{1 - \mu - \frac{\rho}{\gamma\eta(H_t)} + \frac{(1-\tau)}{\eta(H_t)} \left(\frac{f(k_t)}{k_t} - f'(k_t) \right)}{1 + \frac{1}{\eta(H_{t+1})}}$$

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under assumption 1, since $\lim_{t\to+\infty}\frac{f(k_t)}{k_t} = \lim_{k_t\to+\infty}\frac{f(k_t)}{k_t}$ when $\lim_{t\to+\infty} k_t = +\infty$, we obtain

$$\lim_{t\to+\infty}\chi_{Q_t} = \frac{1 - \mu - \frac{\rho}{\gamma\bar\eta}}{1 + \frac{1}{\bar\eta}} < +\infty$$

then $\lim_{t\to+\infty}\chi_{K_t} = \lim_{t\to+\infty}\chi_{Q_t} < +\infty = \lim_{t\to+\infty}\chi_{H_t}$. There is a contradiction.

$\Rightarrow$ The only possibility is $\lim_{t\to+\infty} k_t = \tilde k$, with $0 < \tilde k < \infty$, which implies: $\lim_{t\to+\infty}\chi_{K_t} = \lim_{t\to+\infty}\chi_{H_t}$. From (+), $\lim_{t\to+\infty}\chi_{H_t} = g(\tau f(\tilde k))$. If $g(\tau f(\tilde k)) \geq 1$, then $\lim_{t\to+\infty}\eta(H_t) = \bar\eta$. From (*), $\lim_{t\to+\infty}\frac{Q_{t+1}}{K_{t+1}} = \gamma\bar\eta$. Therefore, $\lim_{t\to+\infty}\chi_{K_t} = \lim_{t\to+\infty}\chi_{Q_t} = \lim_{t\to+\infty}\chi_{H_t}$. Now, from (37),

$$\gamma(1 + \bar\eta)g(\tau f(\tilde k)) = \gamma\bar\eta(1 - \mu) - \rho + \gamma(1 - \tau)\left(\frac{f(\tilde k)}{\tilde k} - f'(\tilde k)\right)$$

This equation is similar to (35), in appendix A. Under the conditions of proposition 1, $\tilde k$ exists (uniqueness) and defines the asymptotic constant growth rate $g(\tau f(\tilde k)) - 1$. There exists a asymptotic BGP for the dynamics above. The system will converge, in the long run, to a BGP defined in terms of $\tilde k$.

Remark: conditions for local stability of the ABGP are similar to the ones defined for the BGP except that $\tilde k$ now replaces $\bar k$.

D   Local stability for SS (prop. 4)

Linearizing (28) around the steady state $(\tilde k, \tilde H, \tilde Q)$, and making use of (30) yield:

$$\begin{cases} dk_{t+1} = \frac{\eta(\tilde H)}{1+\eta(\tilde H)}\left((1 - \mu)dQ_t + \gamma(1 + \mu\eta(\tilde H) + \tilde e^\eta)dH_t + \tilde H(\gamma((1 + \mu\eta(\tilde H))\tilde e^w + \tilde e^\chi\tilde e^\eta) - \rho(1 - \tilde e^w))dk_t\right) \\ dQ_{t+1} = \frac{\left((1-\mu)dQ_t - \gamma\tilde k\eta(\tilde H)(1-\mu+\tilde e^\eta)dH_t + \tilde H(\gamma((1+\mu\eta(\tilde H))\tilde e^w - (1+\eta(\tilde H)(1+\tilde e^\eta))\tilde e^\chi) - \rho(1-\tilde e^w))dk_t\right)}{\tilde H\gamma(1+\eta(\tilde H))} \\ dH_{t+1} = dH_t + \frac{\tilde H}{\tilde k}\tilde e^\chi dk_t \end{cases},$$

note that the same definitions as those of Appendix B. are used. In addition, we define: $\tilde e^\eta = \frac{\tilde H\eta'(\tilde H)}{\eta(\tilde H)}$. One root of the associated characteristic polynomial is nil, then it simplifies to:

$$P(\lambda) = -\lambda^2 + A\lambda + B$$

with,

$$A = \frac{1}{1+\eta(\tilde H)}\left((1 + \eta(\tilde H))(1 - \tilde e^\chi) - \left(\frac{\rho}{\gamma} + \mu\eta(\tilde H) + 1\right)(1 - \tilde e^w) + 1 + \eta(\tilde H) - \eta(\tilde H)\tilde e^\chi\tilde e^\eta\right)$$

$$B = -\frac{1}{1+\eta(\tilde H)}\left((1 + \eta(\tilde H))(1 - \tilde e^\chi) - \left(\frac{\rho}{\gamma} + \mu\eta(\tilde H) + 1\right)(1 - \tilde e^w) - (1 - \mu)\eta(\tilde H)\tilde e^\chi\tilde e^\eta\right)$$

Assume again $\tilde{e}^{\chi} < \tilde{e}^w$ and $(1 - \mu)\eta(\tilde{H}) \geq \frac{\rho}{\gamma}$ (rewriting of (21)), then $\Lambda = (1 + \eta(\tilde{H}))(1 - \tilde{e}^{\chi}) - \left(\frac{\rho}{\gamma} + \mu\eta(\tilde{H}) + 1\right)(1 - \tilde{e}^w) > 0$.

What is changing here, with regard to Appendix B, is the role played by education, through its influence on EA. Thus, a natural way to address the question of local stability is to look for conditions on $\tilde{e}^{\eta}$ that ensure it.

Let us calculate the value of $P(\lambda)$ at the critical bounds 0, 1 and -1:

$$P(0) = B$$

$$P(1) = -\frac{\mu\eta(\tilde{H})}{1 + \eta(\tilde{H})}\tilde{e}^{\chi}\tilde{e}^{\eta} < 0$$

$$P(-1) = -2A + P(1)$$

We easily check there exists a non empty range $[\underline{\tilde{e}^{\eta}}, \overline{\tilde{e}^{\eta}}]$ (if $\tilde{e}^{\chi} > \mu$) for the EA's elasticity to education that is compatible with local stability with,

$$\underline{\tilde{e}^{\eta}} = \frac{\Lambda}{(1 - \mu)\eta(\tilde{H})\tilde{e}^{\chi}} \quad \text{and} \quad \overline{\tilde{e}^{\eta}} = \frac{1 + \eta(\tilde{H})}{\eta(\tilde{H})\tilde{e}^{\chi}}$$

since, in this case, we have $A, B > 0 \leftrightarrow P(-1) < 0$ and $P(0) > 0$. The two roots of $P(\lambda)$ belongs to $]-1, 1[$ and have opposite sign. Convergence is oscillatory.

To summarize, if $\tilde{e}^w > \tilde{e}^{\chi} > \mu$ and $\tilde{e}^{\eta} \in [\underline{\tilde{e}^{\eta}}, \overline{\tilde{e}^{\eta}}]$, then the SS is locally stable.

E   Impact of a change in τ on K and Q (prop. 5)

First note that
$$\pi'(\tau) = \frac{A^{\frac{1}{\alpha}}(1 - \alpha)B^{\frac{1-\alpha}{\alpha(1-\beta)}}\tau^{\frac{1-\alpha}{\alpha} - 1}(1 - \alpha - \tau)}{\alpha},$$

thus, $\pi'(\tau) \geq 0 \leftrightarrow \tau \leq 1 - \alpha$.

According to (32),
$$H^{*'}(\tau) = \frac{\mu\bar{\eta}\varepsilon\pi'(\tau)}{(\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau))^2},$$

therefore we have: $H^{*'}(\tau) \geq 0 \leftrightarrow \tau \leq 1 - \alpha$.

According to (33),

$$K^{*'}(\tau) = -\frac{\mu\bar{\eta}\varepsilon}{\alpha\left(AB^{\frac{1}{(1-\beta)}}\right)^{\frac{1}{\alpha}}\tau^{\frac{1}{\alpha}+1}} \frac{\left(\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau) - \alpha\tau\pi'(\tau)\right)}{\left(\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau)\right)^2},$$

it means that $\tau \geq 1 - \alpha \to K^{*'}(\tau) < 0$. Otherwise, the sign of $-\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau) - \alpha\tau\pi'(\tau)$ is *a priori* undeterminated.

According to (34),

$$Q^{*\prime}(\tau) = \frac{\gamma\bar{\eta}\varepsilon}{\left(AB^{\frac{1}{(1-\beta)}}\right)^{\frac{1}{\alpha}}} \frac{\left(\tau^{\frac{1}{\alpha}}\pi'(\tau)(\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau)) - \frac{\tau^{\frac{1}{\alpha}-1}}{\alpha}\left(\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau) - \alpha\tau\pi'(\tau)\right)\right)}{\tau^{\frac{2}{\alpha}}\left(\mu\bar{\eta} + \frac{\rho}{\gamma} - \pi(\tau)\right)^2},$$

and we also have $\tau \geq 1 - \alpha \rightarrow Q^{*\prime}(\tau) < 0$ whereas there is no simple condition allowing us to determine the impact when $\tau < 1 - \alpha$.

References

[1] Azariadis, C. and Drazen A.: Threshold externalities in economic development. The Quarterly Journal of Economics **105**, p. 501-526 (1990).

[2] Blanchard, O.: Debt, deficits and finite horizon. Journal of Political Economy **93**, p. 223-247 (1985).

[3] Bovenberg., A. and de Mooij, R.: Environmental tax reform and endogenous growth. Journal of Public Economics **63**, p. 207-237 (1997).

[4] Bovenberg, A. and Smulders, S.: Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. Journal of Public Economics **57**, p. 369-391 (1995).

[5] Bovenberg, A. and Smulders, S.: Transitional impacts of environmental policy in an endogenous growth model. International Economic Review **37**, p. 861-893 (1996).

[6] Brock, W. and Taylor, S.: Economic growth and the environment: A review of theory and empirics. In Handbook of Economic Growth, vol. I, part 2., p. 1749-1821. Elsevier North-Holland (2005).

[7] De la Croix, D. and Michel, Ph.: Myopic and Perfect Foresight in the OLG model. Economic Letters **67**, p. 53-60 (2000).

[8] European Commission : The attitudes of European Citizens towards the Environment. Eurobarometer 217, DG Environment (2005).

[9] European Commission : The attitudes of European Citizens towards the Environment. Eurobarometer 296, DG Environment (2008).

[10] Galor, O. and Ryder, H.: Existence, uniqueness and stability of equilibrium in an overlapping generations model with productive capital. Journal of Economic Theory **49**, p. 360-375 (1989).

[11] Glomm, G. and Ravikumar, B.: Productive Government Expenditures and Long Run Growth. Journal of Economic Dynamics and Control **21**, p. 183-204 (1997).

[12] Gradus, R. and Smulders, S.: The Trade-off between Environmental Care and Long-term Growth: Pollution in three Prototype Growth Models. Journal of Economics **58**, p. 25-51 (1993).

[13] John, A. and Pecchenino, R.: An overlapping generations model of growth and the environment. The Economic Journal **104**, p. 1393-1410 (1994).

[14] John, A., Pecchenino, R., Schimmelpfennig, D. and Schreft, S.: Short lived agents and long-lived environment. Journal of Public Economics **58**, p. 127-141 (1995).

[15] Krueger, A., B. and Lindhal, M.: Education for growth: why and for whom? Journal of Economic Literature **34**, p. 1101-1136 (2001).

[16] Lucas, R.: On the Mechanics of Economic Development. Journal of Monetary Economics **22**, p. 3-42 (1988).

[17] Ono, T.: Environmental tax policy in a model of growth cycles. Economic Theory **22**, p. 141-168 (2003).

[18] Pautrel, X.: Environmental policy, education and growth: a reappraisal when lifetime is finite. FEEM WP 57.08 (2008).

[19] Prieur, F.: The environmental kuznets curve in a world of irreversibility. Economic Theory, forthcoming (2009).

[20] UNFCCC: United Nations Framework Convention on Climate Change, United Nations, (1992).

[21] Vellinga, N.: Mutiplicative utility and the influence of environmental care on the short-term economic growth rate. Economic Modelling **16**, p. 307-330 (1999).

[22] Xepapadeas, A.: Economic growth and the environment. In Handbook of Environmental Economics, Vol. III, p. 1219-1271. Elsevier Science Publishers B.V (2005).

[23] Zhang, J.: Environmental sustainability, non linear dynamics and chaos. Economic Theory **14**, p. 489-500 (1999).

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