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The Use of the AHP in Civil Engineering Projects

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Abstract: Most engineering, economic, social and institutional decisions are made with explicit notions of optimal behavior and implicit human motivations. In such a process, manipulation of both tangible and intangible data and satisfaction of multiple criteria are essential to the success of decision-making. In this paper an approach to multiple-criteria decision making known as the analytic hierarchy process (AHP) is presented. Some mathematical details of the procedure are briefly discussed. The application of the method to a real life civil engineering project for the selection of an appropriate bridge design is also presented.

Keywords: multi-criteria decision making, analytic hierarchy process, bridge design

1 Introduction

The analytic hierarchy process (AHP) is a basic approach to decision making. This multiple criteria scaling method was founded by Saaty [9]. It is designed to cope with both the rational and the intuitive to select the best from a number of alternatives evaluated with respect to several criteria. In this process, the decision maker carries out simple pairwise comparison judgments. These are used to develop overall priorities for ranking the alternatives. The AHP both allows for inconsistency in the judgements and provides a means to improve consistency. The procedure starts with development of alternative options, specification of values and criteria, then, it follows the evaluation and recommendation of an option.

As perhaps the most popular and widespread multi-criteria decision making (MCDM) method the AHP has extensively been used in the economics/management area in subjects including auditing, database selection, design, architecture, finance, macro-economic forecasting, marketing, consumer choice, product design and development, strategy, planning, portfolio selection, facility location, resource allocation, transportation, and performance analysis. In political problems the AHP is used in such areas as conflicts and negotiations, political candidacy, security assessments, and world influence. For social concerns, it is applied in education, environmental issues, health, law, medicine, population dynamics, and public sector. Some technological applications include innovation projects, portfolio selection and technology transfer.

In Section 2, an overview of the AHP methodology is presented building upon the work of Saaty and Vargas [11], while in Section 3, an application of this method to a civil engineering project (a bridge selection problem) is reported.

2 Overview of the AHP methodology

In this section we describe the major characteristics of the AHP.

2.1 Structure

The most effective form used to structure a decision problem is a hierarchy. Consisting usually of three levels: the goal of the decision at the top level, followed by a second level containing the criteria by which the alternatives, located in the third level, will be evaluated (see Figure 1.). Hierarchical decomposition of complex systems appears to be a basic device used by the human mind to cope with diversity. One organizes the factors affecting the decision in gradual steps from the general, in the upper levels of the hierarchy, to the particular, in the lower levels. The purpose of the structure is to make it possible to judge the importance of the elements in a given level with respect to some or all of the elements in the adjacent level above.

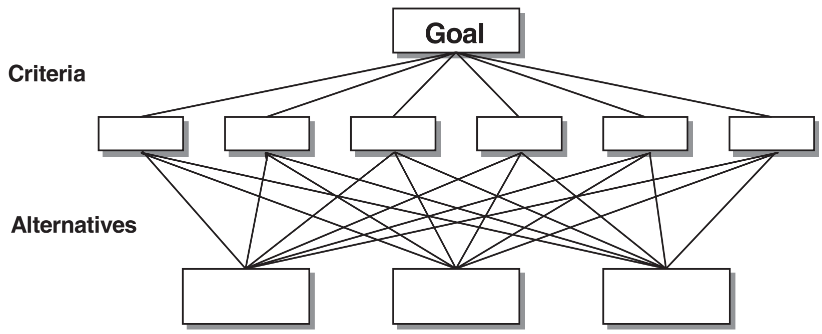


Figure 1

A three level decision hierarchy

Perhaps the most creative task in making a decision is deciding what factors to include in a hierarchic structure. At this phase one must include enough relevant detail to represent the problem as thoroughly as possible. Considering the environment surrounding the problem, identifying the issues, attributes or parameters etc. that the individual decision maker (or a group of participants associated with the problem) feels should contribute to the solution.

The elements being compared should be homogeneous. The hierarchy does not need to be complete; that is an element in a given level does not have to function as a criterion for *all* elements in the level below. Further, a decision maker can insert or eliminate levels and elements as necessary to clarify the task of setting priorities or to sharpen the focus on one or more parts of the system. E.g., elements that are of less immediate interest can be represented in general terms at the higher level of the hierarchy. The task of setting priorities requires that the criteria, the sub-criteria, the properties or features of the alternatives be compared among themselves in relation to the elements of the next higher level.

2.2 Philosophy, procedure and practice of AHP

The AHP is a general theory of measurement. It is used to derive the most advanced scales of measurement (called ratio scales) from both discrete and continuous paired comparisons in multilevel hierarchic structures. These comparisons may be taken from actual physical measurements or from subjective estimates that reflect the relative strength of preferences of the experts. By physical we mean the realm of what is fashionably known as the tangibles in so far as they constitute some kind of objective reality outside the individual conducting the measurement. By contrast, the psychological is the realm of the intangibles, comprising the subjective ideas, feelings and beliefs of the individual. The question is whether there is a coherent theory that can deal with both of these worlds of reality without compromising either. The AHP is a method that can be used to establish measures in both the physical and human domains. The AHP has special concern with departure from consistency and the measurement of this departure, and dependence within and between the groups of elements of its structure. This is made possible by taking several factors into consideration simultaneously, allowing for dependence and for feedback, and making numerical tradeoffs to arrive at a synthesis or conclusion.

In using the AHP to model a problem, one needs a hierarchic structure to represent that problem, as well as pairwise comparisons to establish relations within the structure. In the discrete case, comparisons lead to dominance matrices and in the continuous case to kernels of Fredholm operators, from which ratio scales are derived in the form of principal eigenvectors, or eigenfunctions, as the case may be. These matrices, or kernels, are positive and reciprocal. In a real world application of the AHP the required number of such matrices is equal to the number of the weighting factors. In addition, regarding that the number of the group members is 5–15, there is a need for aggregation what is called the process of synthesizing group judgments. By synthesizing the particular priorities with the average weighting factors of the attributes the ultimate output is yielded in the form of a weighted priority ranking indicating the overall preference scores for each of the alternatives under study.

2.3 Types of human measurements

There are two types of comparisons that humans make: absolute and relative. In absolute comparisons, alternatives are compared with a standard or a baseline which exists in one's memory and has been developed through experience. In relative comparisons, alternatives are compared in pairs according to a common attribute. The AHP has been used with both types of comparisons to derive ratio scales of measurement. Relative measurement, w_i, i=1,…,n, of each n elements is a ratio scale of values assigned to that element and derived by comparing it in pairs with the others. In paired comparisons two elements i and j are compared with respect to a property they have in common. The smaller i is used as the unit and the larger j is estimated as a multiple of that unit in the form $(w_i / w_j) /1$ where the ratio w_i / w_j is taken from a fundamental scale of absolute values. Thus, such a dominance matrix of these ratio comparisons, denoted by **A**, may be given in the form:

$$\mathbf{A} = \begin{bmatrix} 1 & \frac{w_1}{w_2} & . & . & . & \frac{w_1}{w_n} \\ \frac{w_2}{w_1} & 1 & . & . & . & \frac{w_2}{w_n} \\ . & & . & & & . \\ . & & & . & & . \\ . & & & & . & . \\ \frac{w_n}{w_1} & \frac{w_n}{w_2} & . & . & . & 1 \end{bmatrix}.$$

Absolute measurement is applied to rank the alternatives in terms of either the criteria or the ratings (intensities) of the criteria; for example: excellent, very good, good, average, below average, poor, and very poor. After setting priorities for the criteria, pairwise comparisons are also made between the ratings themselves to set priorities for them under each criterion and dividing each of their priorities by the largest rated intensity to get the ideal intensity. This process produces a ratio scale score for each alternative. The scores thus obtained of the alternatives can finally be normalized by dividing each of them by their sum.

2.4 The fundamental scale

Paired comparison judgements in the AHP are applied to pairs of homogeneous elements. The fundamental scale of values to represent the intensities of judgments is shown in Table 1. This scale has been validated for effectiveness by numerous applications in a variety of professional fields of interest.

Table 1. The fundamental scale [11]

Intensity of importance, Strength of preference	**Definition**	**Explanation**
1	**Equal importance**	Two activities contribute equally to the objective
2	Weak	
3	**Moderate importance**	Judgment slightly favor one Activity over another
4	Moderate plus	
5	**Strong importance**	Judgement strongly favor one activity over another
6	Strong plus	
7	**Very strong importance**	An activity is favored very strongly over another
8	Very, very strong	
9	**Extreme importance**	Favoring one activity over another is of the highest affirmation
Reciprocals of above	If activity *i* has one of the above nonzero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	

In real life problems, as a matter of fact, for these ratios arbitrary positive numbers can also be used, e.g. 4.1 or 6.87, or even beyond the lower and upper boundaries of the proposed scale, e.g. 23.6 or 0.05.

2.5 The eigenvector solution and the consistency of matrix A

The major objective of using a scaling method is to derive the vector of weights (called decision priorities) from the input data elicited from experts' judgements and/or from measurements. In the AHP, this task is accomplished by an eigenvalue-eigenvector formulation which is well-known in linear algebra. The components of the weights of the alternatives are given by the (normalized) components of the right hand side eigenvector associated to the maximal eigenvalue of matrix **A**. There are a great number of other methods to generate these priorities, e.g., extremum value procedures like the least squares optimization method [3,4], or using the singular value decomposition of the comparison matrix [6]. An excellent review about the benefits and the drawbacks of these procedures can be found in [7]. Another approach is to apply the multi-attribute utility theory. One well-known class of this approach is termed outranking methods, like e.g. the widely used PROMETHEE method [2]. Of course, there is no perfect scaling method which would outperform all the others with respect to the relevant properties. One of the most important features is the consistency, commonly interpreted in practice as the degree of inconsistency of a matrix of comparisons. This measure is directly related to the variance of the error incurred in estimating the entries of the matrix by the respondents. The AHP includes a consistency index for both the single matrices and also for the entire hierarchy.

2.6 Mathematical background

Let the finite set of alternatives (systems, objects) be denoted by A_i, i=1,2, ...,n. Let C_k, k=1,2, ...,m, denote a criterion (attribute) with respect to which every alternative is being evaluated. Let an $n\times n$ matrix $\mathbf{A}=[a_{ij}]$ with all entries positive numbers ($n\geq 3$) be introduced. Matrix **A** is called a *symmetrically reciprocal* (SR) matrix if the entries satisfy $a_{ij}a_{ji}=1$ for $i\neq j$, i,j=1,2, ...,n, and $a_{ii}=1$, i=1,2, ...,n. The use of these matrices was first proposed by Saaty [9]. Here an entry a_{ij} from R^n represents a *ratio*, i.e., a_{ij} indicates the strength with which alternative A_i dominates alternative A_j with respect to a given criterion C_k. Such a matrix is called a *pairwise comparison* matrix (PCM) and is usually being constructed by eliciting experts' judgements. The basic objective is to derive implicit *weights* (priority scores), $w_1,w_2, ...,w_m$, with respect to each criterion C_k. A vector of the *weights*, $\mathbf{w}=[w_i]$, $w_i>0$, i=1,...,n, may be determined by using the eigenvalue formulation $\mathbf{Aw}=\lambda\mathbf{w}$. Since the single criteria are usually not equally important, therefore, a vector of the *weighting factors* of each criterion, $\mathbf{s}=[s_k]$, where s_k, k=1,2, ...,m is often normalized so that $0< s_k <1$, should also be determined.

Further, let an $n\times n$ matrix $\mathbf{B}=[b_{ij}]$ denote an element-wise, positive matrix whose entries are all nonzero numbers. Matrix **B** is called a *transitive* matrix if $b_{ij}b_{jk}=b_{ik}$, for i,j,k=1,2, ...,n. In Farkas and Rózsa [5] it is proven that any transitive matrix is a one-rank SR matrix. In the AHP, a transitive matrix **B** is termed *consistent* matrix. If the PCM is not transitive, then it is termed *inconsistent*. Saaty [9] proved that the priority score of an alternative, what he called the *relative dominance* of the *i*th alternative A_i, is the *i*th component of the principal right eigenvector of **B**, u_i, i.e., even if the PCM is *not* transitive. The *principal* right eigenvector belongs to the eigenvalue of largest modulus. The eigenvalue of largest modulus will be called *maximal* eigenvalue. By Perron's theorem, for matrices with positive elements, the maximal eigenvalue is always positive, simple and the components of its associated eigenvector are positive [12]. Since any transitive matrix can be expressed as the product of a column vector **u** and a row vector $\mathbf{v}^T$, **B** can be written in the form of an the outer product: $\mathbf{B}=\mathbf{u}\mathbf{v}^T$ (the superscript indicates the transpose). This way it can be shown that the characteristic polynomial of **B**, $p_n(\lambda)$, can be obtained in the form: $\lambda^{n-1}(\lambda-1)$. From this expression it is apparent that **B** has a zero eigenvalue with multiplicity $n-1$ and one simple positive eigenvalue: $\lambda=n$, with its corresponding right and left eigenvectors, **u** and $\mathbf{v}^T$, respectively. The weights w_i, $i=1,...,n$, of the alternatives are given by the components of **u**. This solution for the weights is *unique* up to a multiplicative constant. Conventionally, it is normalized so that its components sum to unity.

In the transitive case the eigenvector method provides the true relative dominance of the alternatives. In reality, however, an individual cannot give his/her estimates such that they would conform to perfect consistency. Recognizing this fact, Saaty [10] proposed a *measure* for the *inconsistency* of a PCM: $\mu=(\lambda_{max}-n)/(n-1)$. Results might be accepted if $\mu\leq 0.08$. Otherwise the problem should be reconsidered and the associated PCM must be revised [10]. Obviously, for a consistent PCM: $\mu=0.00$, since this follows apparently from the above considerations (i.e. in that case: $\lambda_{max}=n$).

To compute the components of the *overall priority scores*, π_1, π_2, ..., π_n, (overall weights) for the set of the alternatives (i.e. when taking into account the weighting factors of each of the criteria) the AHP utilizes an *additive* type aggregation function: $\pi_i=\sum^{m}_{k=1}s_k w_i$, $i=1,2, ...,n$. We note that there are other ways of computing the overall priorities, e.g. a *multiplicative* weighted-geometric-mean aggregation is proposed in [1].

3 The selection of a bridge design: A case-study

This section presents an application of the use of the AHP for selecting the most appropriate bridge design. Here, we show that the AHP is able to link hard measurement to human values in the physical and the engineering sciences. The following study concerns an actual construction project to provide an alternative route across the Monongahela River at the city of Pittsburgh, USA. The author of this article participated in one of the seven decision making groups of this project. A detailed report of this study has appeared in [8]. The three types of bridges considered by The Port Authority of Allegheny County were (n=3):

A = A **Cable-stayed bridge** (Figure 2); it belongs to the group of the longest bridges called suspension bridges. The deck is hung from suspenders of wire rope, eyebars or other materials. Materials for the other parts also vary: piers may be steel or masonry; the deck may be made of girders or trussed. This type of bridge is usually applied with very high tensile strength, which minimizes beam deflection as the span is increased significantly. Moreover, adding several stay cables allows the use of more slender deck beams, which require less flexural stiffness. By decreasing the cable spacing supports, local bending moments in the girders are also reduced. Simple double-edge girders supporting transverse floor beams and top slabs provide a synergistic reinforcing action. The economic viability and aesthetic appeal make this type of bridge to be very popular.

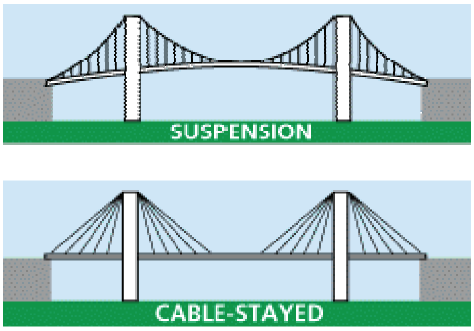


Figure 2

Suspension bridges including their cousin the cable-stayed bridge [13]

B = A **Truss bridge** (Figure 3); which allows applied loads to be resisted primarily by axial forces in its straight truss members. Its open web system permits the use of a greater overall depth than for an equivalent solid web girder. These factors lead to an economy in material and a reduced dead weight. Deflection is reduced and the structure is more rigid. However, fabrication and maintenance costs are increased. In addition, a truss bridge rarely possesses aesthetic beauty.

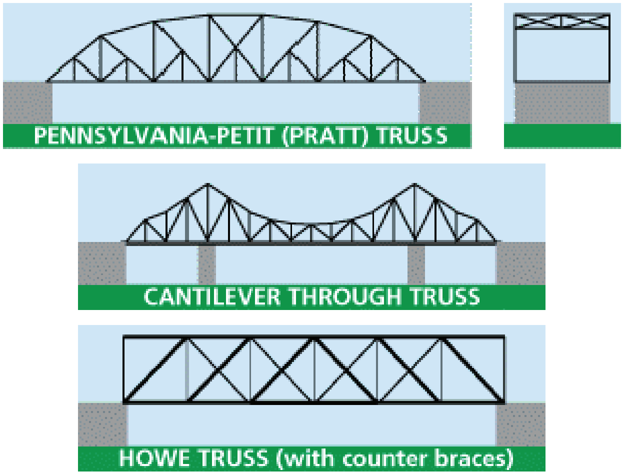
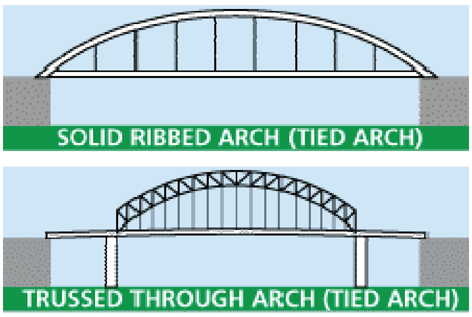


Figure 3

Bridges of Truss type [13]

C = A **Tied-Arch bridge** (Figure 4); which has been used for its architectural beauty and outstanding strength for centuries. With the aid of its inward-acting horizontal components, the arch is capable of distributing loads both above and below its structure. In a tied-arch design the horizontal reactions to the arch rib are supplied by a tie at deck level. It reduces bending moments in the superstructure and is fairly economical. Aesthetically, the arch has been perhaps the most appealing of all bridge types. It has, however, high relative fabrication and building costs.



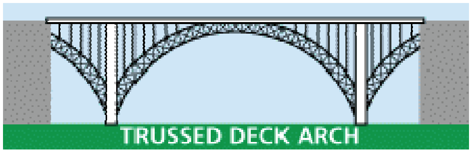


Figure 4

Arch bridges of different configurations including the tied-arch type bridges [13]

The most desirable bridge type would conceivably be the one that brings the most satisfaction to the greatest number of stakeholders. Keeping an eye on this goal, a hierarchy was developed with major stakeholders at the second level, the driving criteria at the third level and the three alternative bridge types at the fourth level. The major stakeholders were then arranged into seven groups each with a number of 8-15 people:

FWHA = A Federal Agency; which represents an array of federal departments. It is a key financier of the project and will have dictates with respect to the engineering integrity of any bridge type.

CBD = The Commercial Business District; which broadly represents the businesses in the downtown of Pittsburgh. Its interest implies to maintain the historical appearance of the building site as well.

PUB = The Public; which represents the population of the city that would use the new bridge.

DOT = The Pennsylvania Department of Transportation; which represents the complex interest of the state. These interests are financial (as the state provides part of the capital), political, technical and environmental.

DES = The Designers; who represent engineers, architects and planners and their professional organizations. They provide crucial technical input and so, they have a great influence.

SIG = Special Interest Groups; this means a very broad category with diverse and possibly conflicting interests. They are the concrete suppliers, the steel manufacturers and the environmentalists. Steel industry has declined in size and influence in this region, however, the concrete industry remained strong. Environmentalists are active and vocal.

PAT = The Port Authority Transit; it is the ultimate project owner. This premier stakeholder cares of all management issues from conception to construction, as well as maintenance.

In the level below the stakeholders are the six criteria with respect to which the bridge types were evaluated. They are interpreted as (m=6):

$C1$ = Engineering Feasibility (**EF**): The technical knowledge and experience of both the designers and contractors in regard to the bridge type.

$C2$ = Capital Cost (**CC**): Necessary funding. Because the costs were committed, low costs are included in the overall benefits hierarchy as one of the criteria.

$C3$ = Maintenance (**MA**): General cleaning, painting, repair and inspection vary dramatically with bridge type.

$C4$ = Aesthetics (**AE**): Architectural attractiveness.

$C5$ = Environmental Impact (**EI**): The ecological and historical adjustments that must be compromised.

$C6$ = Durability (**DU**): The lifetime of the bridge and the potential major repairs over and above the routine maintenance.

Tangible data supporting the engineering characteristics ($C1$, $C2$, $C3$, $C6$) have been derived from measurements, while the ratios for the intangible attributes ($C4$, $C5$) were judged by the groups of stakeholders. Numerical computations were done by the software package Expert Choice. First, the actors were compared to determine their relative importance (weighting factors). The 7×7 sized pairwise comparison matrix **A** is displayed on the next page. Note that matrix **A** is a slightly inconsistent matrix. Its inconsistency measure has been calculated that yielded: μ=0.03.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1/5 & 1 & 1/2 & 1/3 & 3 \\ 1/2 & 1 & 1/6 & 1/2 & 1/3 & 1/4 & 2 \\ 5 & 6 & 1 & 5 & 4 & 3 & 7 \\ 1 & 2 & 1/5 & 1 & 1/2 & 1/3 & 3 \\ 2 & 3 & 1/4 & 2 & 1 & 1/2 & 4 \\ 3 & 4 & 1/3 & 3 & 2 & 1 & 5 \\ 1/3 & 1/2 & 1/7 & 1/3 & 1/4 & 1/5 & 1 \end{bmatrix}.$$

The criteria were then compared according to each factor and the composite priorities calculated (see Table 2.).

Table 2. Weighting factors and weights (priorities) of the criteria

Weighting factor, s_k	0.135	0.221	0.029	0.136	0.085	0.056	0.337	

Stakeholder / Criterion C_k	FHWA	CBD	PUB	DOT	DES	SIG	PAT	Weight w_i
$C1$ = EF	0.117	0.048	0.037	0.216	0.313	0.033	0.260	0.173
$C2$ = CC	0.340	0.048	0.297	0.082	0.197	0.357	0.100	0.147
$C3$ = MA	0.069	0.116	0.297	0.052	0.118	0.097	0.260	0.154
$C4$ = AE	0.069	0.401	0.074	0.216	0.136	0.224	0.061	0.174
$C5$ = EI	0.202	0.270	0.114	0.352	0.117	0.224	0.061	0.181
$C6$ = DU	0.202	0.116	0.182	0.082	0.118	0.064	0.260	0.171
Inconsistency μ	0.02	0.07	0.05	0.04	0.08	0.06	0.05	

Finally, the alternatives were compared according to each criterion and the composite priority scores (weights) computed. This information was synthesized to yield the overall priority ranking and the overall priorities of the bridges:

Overall ranking and the overall priorities, π_i
B (0.371)
|
C (0.320)
|
A (0.309)

Thus, in this project, the most desirable bridge is of a **Truss** type. It is quite interesting to note that a couple of months later this result was reconsidered. The major difference in the duplicated decision making process was the addition of a new stakeholder, the US Coast Guard (USCG), the responsible authority for the river traffic, and the deletion of the Public (PUB). On the effect of the USCG concerning the reinforcement of the safety aspects of river transportation and the further ecological claims of the environmentalists the final ranking of the types of bridges has been changed in favor of a **Tied-arch** type bridge. Since then, the new bridge has been built to the Wabash Tunnel, consisting of three high occupancy vehicle lanes and a lane for pedestrian traffic.

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