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# Industrialization and Technological Progress with Many Countries under a Non-homothetic Preference

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## Abstract

This paper examines industrialization in each country by using a model with a continuum of countries. Our model is mainly based on Yanagawa's (1996) model. However, unlike Yanagawa's model, our model adopts the Stone-Geary utility function of a non-homothetic preference. The main results are as follows. First, we find that an increase in agricultural productivity leads to industrialization under the non-homothetic preference, whereas it leads to deindustrialization under the homothetic preference. Second, the widening disparity of manufacturing productivity among countries leads to an increase in the number of agricultural countries in the world, even if it is under the non-homothetic preference.

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# 1 Introduction

For a long time, many people have considered industrialization essential for economic development. In particular, an increase in agricultural productivity is considered an important step toward achieving industrialization. However, does an increase in agricultural productivity necessarily lead to industrialization? Moreover, does industrialization actually lead to economic development?

For these issues, Matsuyama (1992) shows that an increase in agricultural productivity does not lead an agricultural country to industrialize under the small open economy setting. In addition, he shows that if industrialization does not occur, then the country specializes in the agricultural sector, which consequently lowers the growth rate of the economy. Matsuyama uses the Stone-Geary utility function of a non-homothetic preference, which implies that the income elasticity of demand for the agricultural goods is less than unity. The engine of growth is learning-by-doing in the manufacturing sector. However, his result depends largely on the assumption of the small open economy. Hence, the model mainly analyses the home country without the other countries.

Yanagawa (1996) extends Dornbusch, Fischer, and Samuelson's (1977) Ricardian model with a continuum of goods and builds a two-good Ricardian model with a continuum of countries. Because his model is a multi-country model, we can observe the effects of technological improvement in each country. In this respect, his model is better than Matsuyama's (1992) model. Yanagawa (1996) shows that whether a home country becomes an agricultural or a manufacturing country depends on the performances of other countries. In addition, he shows that the structural change of the home country depends on the performances of other countries. However, unlike Matsuyama (1992), Yanagawa (1996) assumes that all countries share a common Cobb-Douglas utility function.

As stated above, Yanagawa's (1996) model is interesting, but it has the shortcoming of using a homothetic preference. Given the empirically indisputable Engel's law, it is desirable that we use a non-homothetic preference. In this paper, we attempt to overcome this shortcoming.

Our model is mainly based on Yanagawa's (1996) model. However, unlike Yanagawa (1996), we adopt the Stone-Geary utility function of a non-homothetic preference, following Matsuyama (1992), Spilimbergo (2000), and Kongsamut et al. (2001). Using this extended model, we show that the difference between the cases of homothetic preference and non-homothetic preference affects industrialization. Moreover, we analyze how exogenous technical progress promotes industrialization or deindustrialization.

The main results in this paper are summarized as follows. An increase in agricultural productivity leads to deindustrialization under the homothetic preference, whereas it leads to industrialization under the non-homothetic preference.

The remaining paper is organized as follows. Section 2 presents the model. Section 3 defines the equilibrium relative price and the boundary country that produces both agricultural goods and manufacturing goods under free trade. In section 4, we analyze how exogenous technical progress produces industrialization and conduct numerical analysis. Section 5 concludes the paper.

## 2 Model

In this section, we explain the basic framework of our model. We change the homothetic utility function in Yanagawa (1996) model to the non-homothetic Stone-Geary utility function in this paper.

We suppose that the economy is composed of an agricultural sector and a manufacturing sector. The countries in the world continuously exist within the interval  $z \in [0, 1]$ . In this economy, all goods are produced by using only labor with a constant returns to scale technology.  $x_1(z) = a_1(z)L_1(z)$   $z \in [0, 1]$  and  $x_2(z) = a_2(z)L_2(z)$   $z \in [0, 1]$ . Labor employed in sector  $i$  of country  $z$  is denoted by  $L_i(z)$ . The labor productivity in sector  $i$  of country  $z$  is  $a_i(z)$ , and the output of good  $i$  of country  $z$  is  $x_i(z)$ . Both goods markets are competitive. We specify production technology as follows:

$$a_1(z) = \beta_1 + \gamma_1(1 - z), \quad (1)$$

$$a_2(z) = \beta_2 + \gamma_2 z, \quad (2)$$

where  $\beta_i (> 0)$  denotes *the common global productivity level* in sector  $i$  and  $\gamma_i (> 0)$ , *the biased technological productivity* in sector  $i$ .  $\gamma_i$  affects  $a_i$  by the form of  $\gamma_1(1 - z)$  or  $\gamma_2 z$  and these effects produce productivity difference among countries. For example, in the case of  $\gamma_1$ , the larger  $z$  is, the lower agricultural productivity  $a_1$  is.

Note that the production function of the agricultural sector is a decreasing function of  $z$  and that the production function of the manufacturing sector is an increasing function of  $z$ .

We assume that labor is fully employed. Suppose that the total labor endowment is given by  $\bar{L} = 1$  and is identical across countries. Then, in each country, the labor market clearing condition leads to  $L_1 + L_2 = 1$ .

We assume that labor is perfectly mobile between the two sectors. However, labor is immobile across the countries. Accordingly, nominal wage  $w$  is identical in the two sectors. Using profit functions and zero-profit conditions, we derive following result.

$$\frac{p_2}{p_1} = \frac{a_1}{a_2}, \quad (3)$$

where  $p_i$  denotes the price of good  $i$ . Equation (3) indicates that the relative price is equal to the relative productivity.

We formulate consumer behavior. Suppose that consumers obtain utility from the manufacturing and agricultural goods. All consumers in this economy share identical preferences. Then, the utility maximization problem is given by

$$\max_{c_1, c_2} u = (c_1 - \bar{c})^\alpha (c_2)^{1-\alpha}, \quad 0 < \alpha < 1, \quad \bar{c} > 0, \quad (4)$$

$$\text{s.t. } p_1 c_1 + p_2 c_2 = w, \quad c_1 > \bar{c}, \quad (5)$$

where  $c_i$  denotes the consumption of good  $i$ ;  $\bar{c}$ , the subsistence level of agricultural consumption; and  $\alpha$ , the degree of consumer preference for the agricultural good. The condition for utility maximization is given by

$$\left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{c_1 - \bar{c}}{c_2} \right) = \frac{p_2}{p_1}. \quad (6)$$

Using equations (5) and (6), we obtain the demand function for each good as follows:

$$c_1 = \frac{\alpha w}{p_1} + (1 - \alpha)\bar{c}, \quad c_2 = \frac{(1 - \alpha)w}{p_2} - (1 - \alpha)\frac{p_1\bar{c}}{p_2}. \quad (7)$$

Equation (7) is substituted into the utility function. Then, the indirect utility function is given by

$$u = \alpha^\alpha (1 - \alpha)^{1 - \alpha} \left( \frac{w}{p_1} - \bar{c} \right)^\alpha \left( \frac{w}{p_2} - \frac{p_1}{p_2} \bar{c} \right)^{1 - \alpha}. \quad (8)$$

Next, we consider the situation where the countries in the world are engaged in free trade. We derive the equilibrium relative price in the world and in the boundary country, whose relative price under autarky is equal to the equilibrium relative price in the world. In addition, we define structural change.

Suppose that under free trade, the relative productivity between the agricultural and manufacturing sectors satisfies the following condition.

$$\frac{a_2(z)}{a_1(z)} \leq \frac{a_2(\bar{z})}{a_1(\bar{z})} \quad \text{if } z < \bar{z}. \quad (9)$$

The larger  $z$  is, the higher the relative productivity between the two sectors becomes. Therefore, this assumption means that the order of comparative advantage in manufacturing is increasing in  $z$ .

The specified production functions in the previous section satisfy condition (9). Note that in the specified production functions, a change of  $\beta_i$  ( $\gamma_i$ ) does not affect the order of comparative advantage.

By analogy with the two-country Ricardian model, countries whose relative productivity is lower than  $P (= p_2/p_1)$ , where  $P$  is the world relative price, will specialize in producing agricultural goods. Countries whose relative productivity is higher than  $P$  will specialize in manufacturing goods. Let us define  $\bar{z}$  as the country whose relative productivity is exactly equal to  $P^*$  in equilibrium. We call the index  $\bar{z}$  the boundary country. While countries within the range of  $z \in [0, \bar{z})$  specialize in agricultural production, countries within the range of  $z \in (\bar{z}, 1]$  specialize in manufacturing production.

We define industrialization and deindustrialization. We term an increase in the number of manufacturing countries industrialization in the world. Then, these newly industrialized countries completely shift from agricultural production to manufacturing production. Moreover, we term an increase in the number of agricultural countries deindustrialization in the world. Then, these newly deindustrialized countries completely shift from manufacturing production to agricultural production.

Under free trade, all countries must specialize in agricultural goods or manufacturing goods, except the boundary country. Then, we obtain  $L_1 = 1$  for the countries specializing in A and  $L_2 = 1$  for the countries specializing in M, and the production functions  $x_1$  and  $x_2$  under free trade lead to  $x_1(z) = a_1(z)$ ,  $\forall z \in [0, \bar{z})$  and  $x_2(z) = a_2(z)$ ,  $\forall z \in (\bar{z}, 1]$ .

The relative productivity between the agricultural and manufacturing sectors becomes

$$\frac{a_1(z)}{a_2(z)} = \frac{\beta_1 + \gamma_1(1 - z)}{\beta_2 + \gamma_2 z}. \quad (10)$$

Then, we obtain the following equation relating the boundary country  $z^*$  to the world relative price:

$$P = \frac{a_1(\bar{z})}{a_2(\bar{z})} = \frac{\beta_1 + \gamma_1(1 - \bar{z})}{\beta_2 + \gamma_2\bar{z}}. \quad (11)$$

Using the aggregate supply functions under free trade, we obtain the total output in the world in each sector.

$$X_1(\bar{z}) = \int_0^{\bar{z}} x_1(z) dz = \bar{z} \left( \beta_1 + \gamma_1 - \frac{\gamma_1}{2} \bar{z} \right), \quad (12)$$

$$X_2(\bar{z}) = \int_{\bar{z}}^1 x_2(z) dz = (1 - \bar{z}) \cdot \left( \beta_2 + (1 + \bar{z}) \frac{\gamma_2}{2} \right). \quad (13)$$

We derive the world demand for good  $i$ . All countries are distributed within the range of  $[0, 1]$ , and we aggregate the quantity of demand in each country to derive the total output in the world. Aggregate demand  $C_1, C_2$  is given by  $C_1 = \int_0^1 c_1(z) dz$  and  $C_2 = \int_0^1 c_2(z) dz$ . Using equation (13), we aggregate the condition for utility maximization as follows:

$$P = \frac{1 - \alpha}{\alpha} \cdot \frac{C_1 - \bar{C}}{C_2}. \quad (14)$$

### 3 Equilibrium

The aggregate markets clearing conditions lead to  $X_1 = C_1$  and  $X_2 = C_2$ .

Substituting equations (12) and (13) into equation (14), we obtain the following equation.

$$P = \frac{1 - \alpha [X_1(\bar{z}) - \bar{C}]}{\alpha X_2(\bar{z})}. \quad (15)$$

We derive  $X_1$  and  $X_2$  from the market cleaning conditions and substitute them into equation (15). If the result is equal to equation (11), then  $\bar{z}$  determines. The resultant expression is as follows:

$$\frac{1 - \alpha}{\alpha} \frac{\bar{z} \left( \beta_1 + \gamma_1 - \frac{\gamma_1}{2} \bar{z} \right) - \bar{C}}{(1 - \bar{z}) \left( \beta_2 + (1 + \bar{z}) \frac{\gamma_2}{2} \right)} = \frac{\beta_1 + \gamma_1 - \gamma_1 \bar{z}}{\beta_2 + \gamma_2 \bar{z}}. \quad (16)$$

Rearranging equation (16), we obtain the following equation for  $\bar{z}$ .

$$\bar{z}^3 + \theta \bar{z}^2 + \mu \bar{z} + \lambda = 0, \quad (17)$$

where  $\theta = 3(\gamma_1\beta_2 - \beta_1\gamma_2 - \gamma_1\gamma_2)/(\gamma_1\gamma_2)$ ,  $\mu = 2(\gamma_2\bar{C} - \beta_2\gamma_1 - (\gamma_1\gamma_2)/2)/(\gamma_1\gamma_2)$ , and  $\lambda = 2[\beta_2(\beta_1 + \gamma_1 - \bar{C}) + \gamma_2(\beta_1 + \gamma_1)/2]/\gamma_1\gamma_2$ .

Does there exist an equilibrium value  $\bar{z}$ ? If  $\bar{z} = 0$ , then equation (17) is equal to  $\lambda > 0$ . Moreover, if  $\bar{z} = 1$ , then equation (17) is negative<sup>1)</sup>. Therefore, there is  $\bar{z}$  in  $[0, 1]$ .

From equation (17), we can derive  $\bar{z}$ . In addition, substituting  $\bar{z}$  into (15), we can derive the equilibrium relative price  $P^*$ .

In what follows, we will derive the indirect utility function under free trade. To do so, we must derive the indirect utility function under two cases: where countries specialize in manufacturing production and where countries specialize in agricultural production.

First, consider the indirect utility function of countries specializing in agricultural production. Using  $a_1 = w/p_1$  and the equilibrium relative price  $P^*$ , we derive the following utility function  $u_A^{\text{FT}}$ .

$$u_A^{\text{FT}}(z, \bar{z}) = \alpha^\alpha (1 - \alpha)^{1-\alpha} \left( \frac{\beta_2 + \gamma_2 \bar{z}}{\beta_1 + \gamma_1 - \gamma_1 \bar{z}} \right)^{1-\alpha} (\beta_1 + \gamma_1 + \gamma_1 z - \bar{c}). \quad (18)$$

Second, consider the indirect utility function of countries specializing in manufacturing production. Using  $a_2 = w/p_2$  and the equilibrium relative price  $P^*$ , we derive the following utility function  $u_M^{\text{FT}}$ .

$$u_M^{\text{FT}}(z, \bar{z}) = \alpha^\alpha (1 - \alpha)^{1-\alpha} \left( \frac{\beta_1 + \gamma_1 - \gamma_1 \bar{z}}{\beta_2 + \gamma_2 \bar{z}} \right)^\alpha \left( \beta_2 + \gamma_2 z - \frac{\beta_2 + \gamma_2 \bar{z}}{\beta_1 + \gamma_1 - \gamma_1 \bar{z}} \bar{c} \right). \quad (19)$$

In equations (18) and (19), the equilibrium value  $\bar{z}$  is determined by equation (17).<sup>2)</sup>

In the following section, we discuss how the boundary country  $\bar{z}$  shifts when the productivity parameters  $\beta_i$  and  $\gamma_i$  increase.

## 4 Transformation of industry caused by technological improvement

In this section, we analyze how exogenous technical progress (an increase in  $\beta_1$ ,  $\gamma_1$ ,  $\beta_2$ , or  $\gamma_2$ ) promotes industrialization or deindustrialization. First, we analytically examine the change of the boundary country. Second, using numerical examples, we concretely show this change.<sup>3)</sup>

First, we consider the effects of an increase in  $\beta_1$  on the effects of a change in  $z^*$  and suppose that  $\gamma_1$ ,  $\beta_2$  and  $\gamma_2$  are fixed parameters. We totally differentiate equation (16) with  $\beta_1$  and organize the result as follows.

$$\frac{d\bar{z}}{d\beta_1} = \frac{A_4 - A_2 \bar{z}}{A_5}, \quad (20)$$

where  $A_1 = \beta_1 + \gamma_1(1 - \bar{z}/2)$ ,  $A_2 = \beta_2 + \gamma_2 \bar{z}$ ,  $A_3 = \beta_1 + \gamma_1 - \gamma_1 \bar{z}$ ,  $A_4 = (1 - \bar{z})(\beta_2 + (1 + \bar{z})(\gamma_2/2))$ , and  $A_5 = (A_1 \gamma_2 - \bar{C} \gamma_2) + A_2(\beta_1 + \gamma_1(1 - \bar{z}/2)) + (A_3 \gamma_1 + A_4(\beta_2 + \gamma_2 \bar{z})) > 0$ . From equation

1) See Appendix for a more detailed calculation

2) If  $\bar{C} = 0$ , the preference is homothetic, which corresponds to the case of Yanagawa (1996).

3) In this section, we suppose  $\alpha = 1/2$ . This assumption does not affect the following results from a qualitative standpoint.

(20),  $A_4 - A_2\bar{z} > 0$  because in this case,  $d\bar{z}/d\beta_1 > 0$ . Moreover,  $A_4 - A_2\bar{z} < 0$  because in this case,  $d\bar{z}/d\beta_1 < 0$ .

In the same way, we totally differentiate equation (16) with  $\gamma_1$  and derive  $d\bar{z}/d\gamma_1$  as follows.

$$\frac{d\bar{z}}{d\gamma_1} = \frac{A_4(1 - \bar{z}) - A_2\bar{z}(1 - \bar{z}/2)}{A_5}. \quad (21)$$

Then,  $A_4(1 - \bar{z}) - A_2\bar{z}(1 - \bar{z}/2) > 0$  because in this case,  $d\bar{z}/d\gamma_1 > 0$ . Moreover,  $A_4(1 - \bar{z}) - A_2\bar{z}(1 - \bar{z}/2) < 0$  because in this case,  $d\bar{z}/d\gamma_1 < 0$ .

In the same way, we totally differentiate equation (16) with  $\beta_2$  and derive  $d\bar{z}/d\beta_2$  as follows.

$$\frac{d\bar{z}}{d\beta_2} = \frac{(1 - (\bar{z} + (A_1 - \bar{C})))}{A_5}. \quad (22)$$

Then,  $(1 - (\bar{z} + (A_1 - \bar{C}))) > 0$  because in this case,  $d\bar{z}/d\beta_2 > 0$ . Moreover,  $(1 - (\bar{z} + (A_1 - \bar{C}))) < 0$  because in this case,  $d\bar{z}/d\beta_2 < 0$ .

Finally, in the same way, we totally differentiate equation (16) with  $\gamma_2$  and derive  $d\bar{z}/d\gamma_2$  as follows.

$$\frac{d\bar{z}}{d\gamma_2} = \frac{A_3 \left( \frac{1 - \bar{z}^2}{z} \right) - (\bar{z}(A_1 - \bar{C}))}{A_5}. \quad (23)$$

Then,  $A_3((1 - \bar{z}^2)/(\bar{z}) - (\bar{z}(A_1 - \bar{C}))) > 0$  because in this case,  $d\bar{z}/d\gamma_2 > 0$ . Moreover,  $A_3((1 - \bar{z}^2)/(\bar{z}) - (\bar{z}(A_1 - \bar{C}))) < 0$  because in this case,  $d\bar{z}/d\gamma_2 < 0$ .

We cannot completely understand the above results because we do not know the explicit solution of  $\bar{z}$  from (17). However, we comprehend, to some degree, these results as follows. The effects of a change of the boundary country by the increase in the agricultural productivity ( $\beta_1, \gamma_1$ ) depend on the parameters of manufacture ( $\beta_2, \gamma_2$ ) and vice versa. For example, even if the manufacturing productivity of a country is low, when its comparative advantage is high, this country will specialize the manufacturing production.

To be more concrete, we present some numerical examples.

First, we compare the case of the homothetic preference ( $\bar{C} = 0$ ) with the case of nonhomothetic preference ( $\bar{C} > 0$ ). We set the baseline parameter values  $\beta_1, \gamma_1, \beta_2, \gamma_2$ , and  $\bar{C}$  as ① in Table 1. In each case, the results of increase in  $\beta_1$  are ② and ③ in Table 1. In the case of ②, if  $\beta_1$  increases, then  $\bar{z}$ , that is, deindustrialization in the world, decreases. However, in the case of ③,  $\bar{z}$  increases. That is, the case promotes deindustrialization in the world.<sup>4)</sup>

Second, we examine the numerical simulation of the change of the boundary country from an increase in the technological parameters. The numerical results are ④, ⑤, ⑥ and ⑦, in Table 2. In what follows, we give an intuitive interpretation of these results.

4) Whether or not homotheticity is assumed has no effect on the change in  $\bar{z}$  via the rise of  $\beta_2, \gamma_1$ , and  $\gamma_2$ .



First, we consider the effects of  $\gamma_i$ . The increase in  $\gamma_i$  affects the size of  $a_i$  in each country. In the case of ⑥, the larger the increase in  $z$  is, the larger the increase in  $a_2$  is from the increase in  $\gamma_2$ . After the increase in  $\gamma_2$ , the world manufacturing demand is satisfied with fewer countries than before. Hence, countries relatively low manufacturing productivity in the countries specializing in manufacturing are *crowded out* from the manufacturing production. Hence, the number of agricultural countries increases in the world. In the same way, in the case of an increase in  $\gamma_1$ , the number of manufacturing countries increases.

Second, we consider the effects of  $\beta_i$ . An increase in  $\beta_i$  leads to an equal increase in  $a_i$  of all countries. Hence, an increase in  $\beta_i$  does not produce the crowding-out effect as stated above. An increase in  $\beta_i$  affects the real income in all the countries via a decrease in the relative price. In this numerical example, an increase in  $\beta_2$  of ⑦ augments the demand of manufacturing goods via an decrease in the relative price. Moreover, in the case of an increase in  $\beta_1$ , a change in  $\bar{z}$  is differently affected under homothetic preference or under the non-homothetic preference, as noted above. In particular, an increase in  $\beta_1$  augments the demand of agricultural goods under the homothetic preference via an increase in the relative price, and therefore,  $\bar{z}$  increases. Meanwhile, under the non-homothetic preference, an increase in  $\beta_1$  augments the demand of manufacturing goods via income elasticity of agricultural goods which is lower than unity, and therefore,  $\bar{z}$  decreases.<sup>5)</sup>

Finally, we discuss the change in welfare of each country after the rise of the technological parameters. We substitute the results of Table 1 and Table 2 into equations (17) and (18) and derive the welfare of each country. As noted above, the increase of each technological parameter raises the welfare of all countries via the increase of the real income in each country.

## 5 Conclusions

This paper has examined industrialization and deindustrialization by using a model with a continuum of countries. Our model is mainly based on Yanagawa's (1996) model. However, unlike yanagawa's model, we adopt a non-homothetic Stone-Geary utility function unlike Yanagawa's model.

We conducted a numerical analysis to observe the effects of technical improvement. Then, an increase in agricultural productivity certainly leads to industrialization under the non-homothetic preference. However, it is not necessarily the case that an increase in agricultural technical improvement leads to industrialization. In particular, the case of an increase in the common global productivity in the agricultural sector under the homothetic preference promotes deindustrialization. The reason is that a increase in the real income of consumers via a reduction in the agricultural price promotes the consumption of agricultural goods. However, the case under the non-homothetic preference promotes industrialization. The reason is that the effects of promoting the consumption of the manufacturing goods via the income elasticity of agricultural goods, which is lower than unity, are greater than the effects of promoting the manufacturing consumption via an increase in the real income of consumers. Finally, we find that technological progress in each sector raises the welfare of all countries via the increase of the real income in all countries.<sup>6)</sup>

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5) In the case of  $\gamma_2$ , the effects of the crowding out are greater than this effect.

6) In this paper, we have used the linear productivity functions. If, instead, we use exponential produc-

## Appendix

We confirm the existence of  $\bar{z}$ .

The following equation determines  $\bar{z}$ .

$$\begin{aligned} \bar{z}^3 - \bar{z}^2 \left[ \frac{1}{\gamma_1 \gamma_2} \left[ \frac{3}{2}(\gamma_1 \gamma_2 + \gamma_2 \beta_2 - \beta_2 \gamma_1) \right] \right] \\ - \bar{z} \left( \frac{1}{\gamma_1 \gamma_2} \left( \frac{3}{2} \gamma_1 \beta_2 + \frac{3}{2} \gamma_1 \gamma_2 + \beta_1 \gamma_2 + \beta_1 \beta_2 \right) \right) \\ + \frac{1}{\gamma_1 \gamma_2} \left[ \bar{C} \gamma_2 + \beta_1 \beta_2 + \frac{\gamma_2 \beta_1}{2} + \gamma_1 \beta_2 + \frac{\gamma_1 \gamma_2}{2} \right] = 0. \end{aligned} \quad (24)$$

We consider the case of  $\bar{z} = 0$ . Then,

$$\bar{C} \gamma_2 + \beta_1 \beta_2 \frac{\gamma_2 \beta_1}{2} + \gamma_1 \beta_2 \frac{\gamma_1 \gamma_2}{2} > 0. \quad (25)$$

Meanwhile, we consider the case of  $\bar{z} = 1$ . We substitute it into equation (24) and derive the following results.

$$\beta_1 \gamma_1 + \frac{1}{2} \gamma_1 \beta_2 + \frac{3}{2} \gamma_1 \gamma_2 + \beta_1 \beta_2 - \bar{C}(\gamma_2 + \beta_2) = 0. \quad (26)$$

If the subsistence level of agricultural consumption  $\bar{C}$  becomes the maximum value  $\bar{C}(C_{max})$ , then  $\bar{z} = 1$  and

$$\bar{C}_{max} = \beta_1 + \frac{\gamma_1}{2}. \quad (27)$$

Substitute equation(26) into equation(27). The resultant expression is as follows.

$$-\gamma_1 \gamma_2 < 0. \quad (28)$$

Hence, there is equilibrium value  $\bar{z} \in [0, 1]$ .

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tivity functions, the effect of  $\beta_2$  disappears and only  $\gamma_2$  affects industrialization. Hence, an increase in agricultural productivity is a necessary condition for an increase in the number of industrialized countries. However, the condition is limited to the countries that are located near the boundary country. The detailed calculation for the result is available on request.

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	$\alpha$	$\beta_1$	$\gamma_1$	$\gamma_2$	$\beta_2$	$\bar{c}$	$z^*$	$P^*$
①	0.5	1	1	1	1	0.2	0.54960	0.93597
②	0.5	2	1	1	1	0.2	0.54420	1.59033
③	0.5	1	1	1	1	0	0.5	1
④	0.5	2	1	1	1	0	0.51279	1.64411

Table 1: The case of comparing under homothetic preference with under non-homothetic preference

	$\alpha$	$\beta_1$	$\gamma_1$	$\gamma_2$	$\beta_2$	$\bar{c}$	$z^*$	$P^*$
①	0.5	1	1	1	1	0.2	0.54960	0.93597
②	0.5	2	1	1	1	0.2	0.54420	1.59033
⑤	0.5	1	2	1	1	0.2	0.51995	1.28956
⑥	0.5	1	1	2	1	0.2	0.56046	0.67873
⑦	0.5	1	1	1	2	0.2	0.53951	0.57510

Table 2: The boundary country and the equilibrium relative price in the case of an increase in each technology parameter.