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PORTFOLIO CHOICE IN RETIREMENT: HEALTH RISK AND THE DEMAND FOR ANNUITIES, HOUSING, AND RISKY ASSETS

Motohiro Yogo

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1050 Massachusetts Avenue
Cambridge, MA 02138
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Portfolio Choice in Retirement: Health Risk and the Demand for Annuities, Housing, and Risky Assets
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ABSTRACT

This paper develops a life-cycle model in which a household faces stochastic health depreciation and chooses consumption, health expenditure, and the allocation of its wealth between bonds, stocks, and housing. The model is calibrated to explain the cross-sectional variation and the joint dynamics of health expenditure, health, and wealth for females, aged 65 or older, in the Health and Retirement Study. The calibrated model implies that the welfare gain from relaxing borrowing constraints on home equity is 5 percent of wealth at age 65. Similarly, the welfare gain from private annuitization is 16 percent of wealth at age 65.

Motohiro Yogo
Federal Reserve Bank of Minneapolis
Research Department
90 Hennepin Avenue
Minneapolis, MN 55401-1804
and NBER
myogo@nber.org

As a large cohort of baby boomers approach retirement, the design of products that ensure lifetime financial security is at the forefront of the agenda in the retail financial and insurance industries. In public policy, there are various proposals for reforming Social Security and Medicare, and their impact on private saving and the demand for health care remain unclear. Despite enormous practical interest, there is relatively little academic work on health expenditure and saving decisions in retirement when households face health risk, compared to the large literature that studies consumption and saving decisions in the working phase when households face labor-income risk. This paper is an attempt to fill this gap in the life-cycle literature.

I develop a life-cycle model in which a household faces stochastic health depreciation, which affects its marginal utility of consumption and life expectancy. The household receives retirement income (including Social Security) and chooses consumption, health expenditure, and the allocation of its wealth between bonds, stocks, and housing to maximize its lifetime utility that includes a bequest motive. The key inputs into the life-cycle model are the dynamics of health and health insurance (including Medicare), which affects the price of health care relative to non-health consumption.

I estimate the dynamics of health using data on health care utilization, health status, and mortality for females, aged 65 or older, in the Health and Retirement Study. I also estimate health insurance coverage using data on out-of-pocket versus total health expenditure. Given these inputs, the life-cycle model produces cross-sectional variation in health expenditure and wealth that is consistent with a number of key empirical facts. First, out-of-pocket health expenditure, as a share of income, falls in health and rises in age. Second, financial and housing wealth, as a share of total wealth that includes the present value of retirement income, rises in both health and age. Third, the portfolio share in stocks rises in both health and age. Finally, the portfolio share in housing rises in health for older households and falls in age.

The life-cycle model also produces joint dynamics of health and wealth that is consistent with the empirical evidence. In the data, households in good or better health reduce financial and housing wealth by 21 percent on average when their health declines to poor. A decomposition of this reduction in wealth shows that financial wealth accounts for 10 percent, while housing wealth accounts for 11 percent. For households in very good or better health, relatively small health shocks have essentially no impact on financial and housing wealth.

I use the calibrated life-cycle model as a laboratory for two quantitative experiments. The first experiment relaxes borrowing constraints on home equity. For a household in good health at age 65, the welfare gain from relaxed borrowing constraints is 5 percent of financial and housing wealth. The second experiment allows households to privately annuitize wealth, beyond what is implicitly annuitized through Social Security and defined-benefit pension plans. For a household in good health at age 65, the welfare gain from private annuitization is 16 percent of financial and housing wealth. These findings imply the presence of significant frictions, whether technological or psychological, that may be overcome through financial education, the design of better financial products, or public policy reform.

The remainder of the paper proceeds as follows. Section I develops a life-cycle model of consumption and portfolio choice in retirement. Section II measures the key inputs and outputs of the life-cycle model using the Health and Retirement Study. Section III uses simulations from the life-cycle model to explain the cross-sectional variation and the joint dynamics of health expenditure, health, and wealth in the data. Section IV examines the welfare implications of relaxing portfolio constraints in the life-cycle model. Section V concludes with a discussion of potentially interesting extensions for future work.

I. A Life-Cycle Model of Consumption and Portfolio Choice in Retirement

This section develops a life-cycle model of consumption and portfolio choice in retirement. The basic structure of the life-cycle model can be summarized as follows. An individual enters retirement with an initial endowment of health, financial wealth, and housing wealth. In each period while alive, the household faces stochastic health depreciation, which affects its marginal utility of consumption and life expectancy. In response to the health shock, the household chooses consumption, housing expenditure, health expenditure, and the allocation of financial wealth between bonds and stocks. Upon death, the household leaves financial and housing wealth as a bequest.

The life-cycle model in this paper allows health expenditure, wealth, and its allocation between bonds, stocks, and housing to all respond endogenously to health shocks. Individual features of the model have appeared in the life-cycle literature. For example, Gabriel Picone, Martin Uribe and R. Mark Wilson (1998) allow health expenditure to respond endogenously to health shocks, but they do not model housing or portfolio choice. A number of authors study housing and portfolio choice during the working phase when households face labor-income risk, instead of retirement when they face health risk (João F. Cocco 2005, Xiaoqing Hu 2005, Rui Yao and Harold H. Zhang 2005). Finally, a number of authors study portfolio choice between bonds, stocks, and annuities in the context of a life-cycle model in which health expenditure and mortality are exogenous (Ryan D. Edwards 2008, Wolfram J. Horneff, Raimond H. Maurer, Olivia S. Mitchell and Michael Z. Stamos 2009, Gaobo Pang and Mark Warshawsky 2010).

A. Housing Expenditure

The household enters each period t with an initial housing stock D_{t-1}. The level of the housing stock incorporates both the size and the quality of the home. Housing depreciates at a constant rate $\delta \in [0, 1)$ in each period. After depreciation, the household chooses housing expenditure E_t, which can be negative in the case of downsizing. Whenever housing expenditure is different from zero, the household incurs a transaction cost of $\tau(1-\delta)P_t D_{t-1}$, where $\tau \in [0, 1)$ and P_t is the home price. The presence of a transaction cost that is proportional to home value makes housing expenditure lumpy. The accumulation equation for housing is

$$D_t = (1 - \delta)D_{t-1} + E_t. \tag{1}$$

Housing is a unique asset that serves a dual purpose. On the one hand, the household enjoys a utility flow from living in a home. On the other hand, housing is a form of savings, which the household can use for consumption or health expenditure while alive and bequeath upon death. For example, a household that develops a physical disability can sell its home and use the proceeds to pay for nursing home care.

B. Health Expenditure

Analogous to housing, I model the household's health as an accumulation process (Michael Grossman 1972). The household enters each period t with an initial health stock H_{t-1}. Health depreciates at a stochastic rate $\omega_t \leq 1$ in each period t. As discussed in further detail below, the distribution of ω_t depends on the household's state variables in period t, including previous health. For example, whether you get a heart attack today is purely chance, but the likelihood of getting a heart attack depends on whether you have a history of heart disease. The household dies if $\omega_t = 1$, that is, if its health depreciates entirely. The household's maximum possible lifetime is T so that $\omega_{T+1} = 1$ with certainty.

After the realization of health depreciation in period t, the household chooses health expenditure $I_t \geq 0$ if still alive. Health expenditure is an investment in the sense that its impact on health can persist for more than one period. Health investment is irreversible in the sense that the household cannot reduce its health through negative expenditure. Irreversibility of investment is a key economic feature that makes health fundamentally different from financial assets or housing.

The accumulation equation for health is

$$H_t = (1 - \omega_t)H_{t-1} + \psi[(1 - \omega_t)H_{t-1}]^{1-\psi} I_t^{\psi}. \tag{2}$$

This specification for health production has two key features that are well suited for empirical analysis. First, health production is homogeneous in the health stock. Second, there are decreasing returns to health investment, captured by the parameter $\psi \in (0, 1]$ (Isaac Ehrlich and Hiroyuki Chuma 1990). Decreasing returns is a simple way to model the fact that treatment when unhealthy has a much larger impact on health than preventive care when healthy.

C. Budget and Portfolio Constraints

The household enters each period t with wealth W_t, which is the sum of financial wealth and the present value of retirement income (i.e., Social Security and defined-benefit pension plans). As discussed in further detail below, I model retirement income as a distribution from a non-traded real annuity. The household uses wealth for consumption C_t, housing expenditure E_t at the relative price P_t, and health expenditure I_t at the relative price Q_t. As discussed in further detail below, the relative price of health care accounts for the household's health insurance coverage.

Wealth remaining after consumption and expenditures can be saved in either bonds or stocks. Let $A_{1,t}$ denote savings in bonds in period t, and let $A_{2,t}$ denote savings in stocks. Similarly, let $A_{3,t}$ denote the present value of retirement income. Let $1_{\{E_t \neq 0\}}$ denote an indicator function that is equal to one if housing expenditure is different from zero in period t. The household's savings in period t is

$$\sum_{n=1}^{3} A_{n,t} = W_t - C_t - P_t E_t - \tau 1_{\{E_t \neq 0\}} (1-\delta) P_t D_{t-1} - Q_t I_t. \tag{3}$$

Let $R_{n,t+1}$ denote the gross rate of return on asset n from period t to $t+1$. The intertemporal budget constraint is

$$W_{t+1} = \sum_{n=1}^{3} R_{n,t+1} A_{n,t}. \tag{4}$$

Define total wealth as the sum of financial wealth, the present value of retirement income, and housing wealth:

$$\widehat{W}_t = W_t + \left(1 - \tau 1_{\{E_t \neq 0\}}\right)(1-\delta) P_t D_{t-1}. \tag{5}$$

Define savings in housing wealth as $A_{D,t} = P_t D_t$. Combined with the accumulation equation for housing (1), the household's total savings in period t is

$$\sum_{n=1}^{3} A_{n,t} + A_{D,t} = \widehat{W}_t - C_t - Q_t I_t. \tag{6}$$

Define the gross rate of return on housing from period t to $t+1$ as

$$R_{D,t+1} = \frac{(1-\delta)P_{t+1}}{P_t}. \tag{7}$$

I can rewrite the intertemporal budget constraint as

$$\widehat{W}_{t+1} = \sum_{n=1}^{3} R_{n,t+1} A_{n,t} + \left(1 - \tau 1_{\{E_{t+1} \neq 0\}}\right) R_{D,t+1} A_{D,t}. \tag{8}$$

Bonds

Bonds have a constant gross rate of return $R_{1,t+1} = \overline{R}_1$. The average real return on the one-year Treasury bond, deflated by the consumer price index for all items less medical care, was 2.5 percent for the period 1958–2008. I therefore set $\overline{R}_1 = 1.025$ annually.

For tractability, I model a mortgage or a home equity loan as a short position in bonds. The household can borrow up to $A_{1,t} \geq -\lambda A_{D,t}$ in each period t, where $\lambda = 0.2$ in the benchmark model. This low borrowing limit is based on evidence that older retired households have difficulty borrowing from their home equity, compared to younger working households (Todd Sinai and Nicholas S. Souleles 2008). Section IV examines the welfare implications of relaxing this borrowing constraint.

Stocks

Stocks have a stochastic gross rate of return

$$R_{2,t+1} = \overline{R}_2 \nu_{2,t+1}, \tag{9}$$

where $\log \nu_{2,t+1} \sim \mathbf{N}(-\sigma_2^2/2, \sigma_2^2)$ is independently and identically distributed. The real return on the Center for Research in Securities Prices value-weighted stock index, deflated by the consumer price index for all items less medical care, had a mean of 7 percent and a standard deviation of 18 percent for the period 1958–2008. Based on these estimates, I set $\overline{R}_2 = 1.065$ and $\sigma_2 = 0.18$ annually. An equity premium of 4 percent, which is slightly lower than its historical estimate of 4.5 percent, is a common assumption in life-cycle models of portfolio choice (João F. Cocco, Francisco J. Gomes and Pascal J. Maenhout 2005). The household cannot short stocks, so that it faces the portfolio constraint $A_{2,t} \geq 0$ in each period t.

Real Annuity

For convenience, I model retirement income as a distribution from a non-traded real annuity, under the assumption that Social Security and defined-benefit pension plans are indexed to the consumer price index. Let a unit of the annuity be a claim that pays off one unit of consumption in every period prior to death. Let p_t denote the actuarially fair survival probability in period t, which is a deterministic function of gender, birth cohort, and age. The annuity price in period t is

$$P_{3,t} = \sum_{s=1}^{T-t-1} \frac{\prod_{u=1}^{s} p_{t+u}}{\overline{R}_3^s}, \tag{10}$$

where $\overline{R}_3$ is the discount rate on the annuity. The annuity has a gross rate of return that is contingent on survival:

$$R_{3,t+1} = \begin{cases} \overline{R}_3/p_{t+1} & \text{if } \omega_{t+1} \neq 1 \\ 0 & \text{if } \omega_{t+1} = 1 \end{cases}. \tag{11}$$

To calibrate the annuity price, I use the survival probabilities for females born in the 1940 cohort from the Social Security life tables (Felicitie C. Bell and Michael L. Miller 2005, Table 7). I set the maximum possible age in the life-cycle model to 119 to match the maximum age in the life tables. I set $\overline{R}_3 = 1.025$ annually to match the riskless interest rate in the life-cycle model.

Let $B_{3,t}$ denote the retirement income in period t, so that savings in the annuity is $A_{3,t} = P_{3,t}B_{3,t}$. In the benchmark model, the household receives a constant stream of retirement income through Social Security and defined-benefit pension plans, which is equivalent to the portfolio constraint $B_{3,t} = B_{3,0}$ in each period t. Section IV examines the welfare implications of allowing the household to privately annuitize wealth, which is equivalent to relaxing the portfolio constraint to be $B_{3,t} \geq B_{3,t-1}$.

Housing

Housing has a stochastic gross rate of return

$$R_{D,t+1} = \overline{R}_D \nu_{D,t+1}, \tag{12}$$

where $\log \nu_{D,t+1} \sim \mathbf{N}(-\sigma_D^2/2, \sigma_D^2)$ is independently and identically distributed. The dynamics of the relative price of housing is then governed by equation (7), where the initial price level is normalized to $P_1 = 1$. Based on equation (7), I compute housing return using the Office of Federal Housing Enterprise Oversight price index and a depreciation rate of 1.14 percent for private residential fixed assets. The real housing return, deflated by the consumer price index for all items less medical care, had a mean of 0.4 percent and a standard deviation of 3.5 percent for the period 1976–2008. I therefore set $\overline{R}_D = 1.004$ and $\sigma_D = 0.035$ annually.

D. Objective Function

If the household is alive in period t, it has utility flow from consumption, housing, and health. Its utility flow over consumption and housing is given by the Cobb-Douglas function. Its utility flow over non-health consumption and health is given by the constant elasticity of substitution function:

$$U(C_t, D_t, H_t) = \left[(1-\alpha)\left(C_t^{1-\phi} D_t^{\phi}\right)^{1-1/\rho} + \alpha H_t^{1-1/\rho}\right]^{1/(1-1/\rho)}. \tag{13}$$

The parameter $\phi \in (0,1)$ is the utility weight on housing, and $\alpha \in (0,1)$ is the utility weight on health. The parameter $\rho \in (0,1]$ is the elasticity of substitution between non-health consumption and health.

If the household dies in period t, it bequeathes financial and housing wealth. Its utility flow over the bequest is

$$G\left(\widehat{W}_t, P_t\right) = \overline{u}\widehat{W}_t \left(\frac{\phi}{(1-\phi)P_t}\right)^{\phi}. \tag{14}$$

The parameter $\overline{u} > 0$ determines the strength of the bequest motive. This specification is the indirect utility function that corresponds to a Cobb-Douglas function over financial wealth and the housing (i.e., $\overline{u}W_t^{1-\phi}D_t^{\phi}$). It captures the notion that financial wealth and housing are not perfectly substitutable forms of bequest (see Yao and Zhang (2005) for a similar approach).

Let $1_{\{\omega_{t+1}=1\}}$ denote an indicator function that is equal to one if the household dies in period $t+1$, and let $1_{\{\omega_{t+1}\neq 1\}} = 1 - 1_{\{\omega_{t+1}=1\}}$. I define the household's objective function recursively as

$$\begin{aligned} J_t = \Big\{&(1-\beta)U(C_t, D_t, H_t)^{1-1/\sigma} \\ &+\beta \mathbf{E}_t \left[1_{\{\omega_{t+1}\neq 1\}} J_{t+1}^{1-\gamma} + 1_{\{\omega_{t+1}=1\}} G\left(\widehat{W}_{t+1}, P_{t+1}\right)^{1-\gamma}\right]^{(1-1/\sigma)/(1-\gamma)}\Big\}^{1/(1-1/\sigma)}, \end{aligned} \tag{15}$$

where the terminal value is $J_{T+1}^{1-\gamma} = 0$ (Larry G. Epstein and Stanley E. Zin 1991). The parameter $\beta \in (0,1)$ is the subjective discount factor. The parameter $\sigma > 0$ is the elasticity of intertemporal substitution, and $\gamma > 1$ is relative risk aversion.

If $\rho < \sigma$, non-health consumption and health are complements in the sense that the marginal utility of non-health consumption rises in health. For example, the marginal utility of a fine meal may be low if you have diabetes. If $\rho > \sigma$, non-health consumption and health are substitutes. For example, the marginal utility of cable television may be high if you have a physical disability. The complementarity between non-health consumption and health also captures the fact that the composition of consumption may change with respect to health.

E. Homogeneity in Total Wealth

In addition to age, the state variables of the life-cycle model are health, retirement income, the housing stock, the home price, and total wealth. However, homogeneity of the objective function allows me to eliminate total wealth as a state variable. I redefine the state variables of the life-cycle model as their values relative to total wealth:

$$\widehat{H}_t = \frac{(1-\omega_t)Q_t H_{t-1}}{\widehat{W}_t}, \tag{16}$$

$$\widehat{D}_t = \frac{(1-\delta)P_t D_{t-1}}{\widehat{W}_t}, \tag{17}$$

$$\widehat{B}_{3,t} = \frac{P_{3,t} B_{3,t-1}}{\widehat{W}_t}. \tag{18}$$

Homogeneity is a common assumption in life-cycle models of consumption and portfolio choice, which simplifies the model to make it suitable for empirical analysis. In order to preserve homogeneity, I make two additional parametric assumptions. First, the distribution of health depreciation ω_{t+1} depends on present health only through $\widehat{H}_t$. Second, the relative price of health care, or health insurance coverage, depends on present health only through $\widehat{H}_t$. In Section II, I estimate the distribution of health depreciation and the relative price of health care in the Health and Retirement Study.

Let $\widehat{C}_t = C_t/\widehat{W}_t$, $\widehat{I}_t = Q_t I_t/\widehat{W}_t$, and $\widehat{A}_{n,t} = A_{n,t}/\widehat{W}_t$ for each asset $n = 1, 2, 3, D$. I rescale the intratemporal budget constraint (6) as

$$\sum_{n=1}^{3} \widehat{A}_{n,t} + \widehat{A}_{D,t} = 1 - \widehat{C}_t - \widehat{I}_t. \tag{19}$$

I rescale the intertemporal budget constraint (8) as

$$\Delta W_{t+1} = \frac{\widehat{W}_{t+1}}{\widehat{W}_t} = \sum_{n=1}^{3} R_{n,t+1}\widehat{A}_{n,t} + \left(1 - \tau 1_{\{\widehat{A}_{D,t} \neq \widehat{D}_t\}}\right) R_{D,t+1}\widehat{A}_{D,t}. \tag{20}$$

Combining these two budget constraints, I eliminate $\widehat{A}_{1,t}$ as a policy variable:

$$\begin{aligned}\Delta W_{t+1} =& R_{1,t+1}\left(1-\widehat{C}_t-\widehat{I}_t\right)+\sum_{n=2}^{3}(R_{n,t+1}-R_{1,t+1})\widehat{A}_{n,t} \\ &+\left(1-\tau 1_{\left\{\widehat{A}_{D,t}\neq\widehat{D}_t\right\}}\right)(R_{D,t+1}-R_{1,t+1})\widehat{A}_{D,t}.\end{aligned} \tag{21}$$

Similarly, I rescale the utility function (13) as

$$\widehat{U}_t=\frac{U(C_t,D_t,H_t)}{\widehat{W}_t}=\widehat{C}_tV_t, \tag{22}$$

where

$$V_t=\left[(1-\alpha)\left(\frac{\widehat{A}_{D,t}}{P_t\widehat{C}_t}\right)^{\phi(1-1/\rho)}+\alpha\left(\frac{\widehat{H}_t\left[1+\psi\left(\widehat{I}_t/\widehat{H}_t\right)^{\psi}\right]}{Q_t\widehat{C}_t}\right)^{1-1/\rho}\right]^{1/(1-1/\rho)}. \tag{23}$$

I also rescale the bequest function (14) as

$$\widehat{G}_t=\frac{G\left(\widehat{W}_t,P_t\right)}{\widehat{W}_t}=\overline{u}\left(\frac{\phi}{(1-\phi)P_t}\right)^{\phi}. \tag{24}$$

I am now ready to restate the household's problem as follows. In each period t, the household chooses $\widehat{C}_t$, $\widehat{I}_t$, and $\widehat{A}_{n,t}$ $(n=2,3,D)$ to maximize its objective function:

$$\begin{aligned}\widehat{J}_t=\frac{J_t}{\widehat{W}_t}=&\Big\{(1-\beta)\widehat{U}_t^{1-1/\sigma} \\ &+\beta\mathbf{E}_t\left[\Delta W_{t+1}^{1-\gamma}\left(1_{\{\omega_{t+1}\neq 1\}}\widehat{J}_{t+1}^{1-\gamma}+1_{\{\omega_{t+1}=1\}}\widehat{G}_{t+1}^{1-\gamma}\right)\right]^{(1-1/\sigma)/(1-\gamma)}\Big\}^{1/(1-1/\sigma)}.\end{aligned} \tag{25}$$

The budget and portfolio constraints are

$$\widehat{C}_t + \widehat{I}_t + \widehat{A}_{2,t} + \widehat{A}_{3,t} + (1-\lambda)\widehat{A}_{D,t} \leq 1, \tag{26}$$

$$\widehat{A}_{3,t} = \widehat{B}_{3,t}. \tag{27}$$

The law of motion for the state variables are

$$\widehat{H}_{t+1} = \frac{(1-\omega_{t+1})Q_{t+1}\widehat{H}_t}{Q_t \Delta W_{t+1}} \left[1 + \psi \left(\frac{\widehat{I}_t}{\widehat{H}_t}\right)^{\psi}\right], \tag{28}$$

$$\widehat{D}_{t+1} = \frac{R_{D,t+1}\widehat{A}_{D,t}}{\Delta W_{t+1}}, \tag{29}$$

$$\widehat{B}_{3,t+1} = \frac{P_{3,t+1}\widehat{A}_{3,t}}{P_{3,t}\Delta W_{t+1}}, \tag{30}$$

and equation (7).

II. Calibrating the Life-Cycle Model Using the Health and Retirement Study

The Health and Retirement Study is a panel survey designed to study the health and wealth dynamics of the elderly in the United States. The data consist of eight waves, covering every two years between 1992 and 2006. This section explains how I use the data to measure the key inputs (i.e., health transition probabilities and health insurance coverage) and outputs (i.e., health expenditure, wealth, and asset allocation) of the life-cycle model. Appendix A contains details on the construction of the relevant variables for my analysis.

A. *Sample of Households*

My sample consists of households whose female respondent is born 1891–1940, aged 65 or older, and retired (including disabled or out of labor force) at the time of interview. The choice of females is motivated by the fact that they live longer, which leads to a larger sample of older households. The sample includes the Study of Assets and Health Dynamics Among the Oldest Old (born before 1924), the Children of Depression (born 1924–1930), and the initial HRS cohort (born 1931–1941). Households must have both positive income and net worth to be included in the sample.

The life-cycle model in this paper applies to single households, or to previously married households once they are widowed or divorced. Because the female respondent is typically married at the time of first interview, I must make some measurement assumptions in calibrating the life-cycle model. I measure health outcomes for the female respondent only, while I measure health expenses, income, and wealth at the household level. All of my empirical specifications control for marital status to account for potential differences between single and married households. I use the predicted values for single households to calibrate the life-cycle model.[1]

The Health and Retirement Study continues to interview respondents that enter nursing homes. However, any respondent that enters a nursing home receives a zero sampling weight because these weights are based on the non-institutionalized population of the Current Population Survey. Therefore, the use of sampling weights would lead me to underestimate the cost of nursing home care, which accounts for a significant share of out-of-pocket health expenditure in old age. Therefore, I do not use sampling weights in my analysis.

B. Health Problems and Health Care Utilization

The primary measure of health for my study is self-reported general health status. At each interview, the respondent reports whether her health is poor, fair, good, very good, or excellent. Insofar as health enters the household's utility function, self-reported health status is a relevant measure of health for calibrating the life-cycle model. As shown below, self-reported health status is highly correlated with doctor-diagnosed health problems, difficulty with activities of daily living, health care utilization, and future mortality.

Panel A of Table 1 reports the share of respondents who have ever had doctor-diagnosed health problems, separately by health status. The table shows that health status is highly

[1] An alternative approach, employed in an previous version of this paper, is to focus on the sub-sample of single households. The drawback of this approach is smaller sample size because households would only enter this sub-sample as they are widowed or divorced.

correlated with all measures of physical and mental health (Robert B. Wallace and A. Regula Herzog 1995). For example, 54 percent of respondents in poor health have had heart problems, which is higher than 39 percent of those in fair health, 26 percent of those in good health, 15 percent of those in very good health, and 9 percent of those in excellent health.

Panel B of Table 1 reports the share of respondents who have some difficulty with activities of daily living at the time of interview, separately by health status. The table shows that health status is highly correlated with all measures of functional limitation. For example, 43 percent of respondents in poor health have some difficulty dressing, which is higher than 19 percent of those in fair health, 9 percent of those in good health, 4 percent of those in very good health, and 3 percent of those in excellent health.

Panel C of Table 1 reports the share of respondents who have used health care in the two years prior to the interview, separately by health status. The table shows that health status is highly correlated with the use of home health care, nursing home stay, and prescription drugs. For example, 34 percent of respondents in poor health have used home health care in the two years prior to the interview, which is higher than 17 percent of those in fair health, 9 percent of those in good health, 5 percent of those in very good health, and 3 percent of those in excellent health. Similarly, 19 percent of respondents in poor health have stayed at a nursing home in the two years prior to the interview, which is higher than 8 percent of those in fair health, 5 percent of those in good health, 3 percent of those in very good health, and 2 percent of those in excellent health. The fact that respondents in poor health are more likely to use home health care and to stay at a nursing home is consistent with higher incidences of functional limitation among these respondents.

C. Measuring the Inputs of the Life-Cycle Model

Health Transition Probabilities

Let H_t^* denote the respondent's self-reported health status at each interview. I model health status as a function of unobserved health $\widehat{H}_t$ through the following response function (see Adam Wagstaff (1986) and Ahmed W. Khwaja (2002) for a similar approach):

$$H_t^* = \begin{cases} 0 \quad \text{Dead} & \text{if } \widehat{H}_t < h_1 \\ 1 \quad \text{Poor} & \text{if } h_1 \leq \widehat{H}_t < h_2 \\ 2 \quad \text{Fair} & \text{if } h_2 \leq \widehat{H}_t < h_3 \\ 3 \quad \text{Good} & \text{if } h_3 \leq \widehat{H}_t < h_4 \\ 4 \quad \text{Very good} & \text{if } h_4 \leq \widehat{H}_t < h_5 \\ 5 \quad \text{Excellent} & \text{if } h_5 \leq \widehat{H}_t \end{cases} . \tag{31}$$

I use an ordered probit model to estimate how future health status at two years from the present interview depends on present health status, age, total wealth, measures of health care utilization, vigorous physical activity, smoking, marital status, and birth cohort. I interact the measures of health care utilization with health status to allow for the possibility that the marginal product of health care varies with health. I control for total wealth since the relevant measure of health for the life-cycle model is the variation in health that is independent of total wealth (i.e., $\widehat{H}_t$).

Column (1) of Table 2 reports the estimated coefficients and t-statistics for the ordered probit model. The sign of the coefficients can be interpreted as the direction of the marginal effects for the likelihood of the extreme health outcomes, namely death and excellent health. Present health status is a significant predictor of future health status. The negative coefficients for poor and fair health imply that these respondents are more likely to die prior to the next interview, compared to those in good health. Conversely, the positive coefficients for very good and excellent health imply that these respondents are less likely to die. The negative coefficient for age implies that older respondents are more likely to die. The positive coefficient for total wealth implies that wealthier respondents are less likely to die, holding everything else constant.

Measures of health care utilization that are positive and significant predictors of future health status are a dentist visit and a cholesterol test. Measures of health care utilization that are positive but insignificant predictors of future health status are a doctor visit, outpatient surgery, and a mammogram. For both a doctor visit and outpatient surgery, the coefficient on their interaction with poor health is positive, which implies that health care has a larger impact on future health for respondents that are already in poor health. Home health care, nursing home stay, and prescription drugs predict future health status with a negative coefficient, which is likely due to an omitted variables bias as discussed below. In addition to health care utilization, I examine vigorous physical activity and smoking as non-monetary measures of health investment. Vigorous physical activity is a positive and significant predictor of future health status, while smoking is a negative and significant predictor. A joint Wald test for measures of health care utilization, vigorous physical activity, smoking, and their interaction with health status rejects strongly. Taken together, this evidence suggests that the choices that respondents make regarding health investment have a significant impact on their future health.

Respondents in poor health are more likely to use health care. Therefore, the coefficients for health care utilization are potentially downward biased, insofar as health care utilization is negatively correlated with unobserved heterogeneity in health. To investigate this possibility, column (2) of Table 2 introduces doctor-diagnosed health problems and difficulty with activities of daily living as additional measures of present health. These additional measures are significant predictors of future health status, implying that present health status does not fully capture heterogeneity in health. The coefficients for health care utilization in column (2) are generally higher than those in column (1). For example, prescription drugs have a statistically insignificant coefficient of -1.56 in column (2), which is higher than the statistically significant coefficient of -17.58 in column (1). Because this evidence implies the presence of unobserved heterogeneity in health, Table 7 below takes an alternative approach by examining the data in first differences.

I use the estimates from column (1) of Table 2 to compute the predicted probabilities for single females who were born 1931–1940, have the average total wealth for her cohort and age, have not used health care in the two years prior to the interview, and does not regularly participate in vigorous physical activity and smokes at the time of interview. In other words, I estimate the counterfactual of how health status transitions from the present interview to the next in the absence of health investment. Figure 1 reports the predicted transition probabilities by present health status and age. The figure shows that health status is persistent and that present health is a significant predictor of future mortality. Conditional on being in poor health at any given age, death is the most likely outcome at the next interview. Conditional on being in excellent health at any given age, death is the least likely outcome at the next interview.

Let $\Pr(H^*_{t+1} = j | H^*_t = i)$ denote the predicted transition probability, from health status i in period t to health status j in period $t+1$, in the absence of health investment. For a respondent whose present health is $h_i \leq \widehat{H}_t < h_{i+1}$, health depreciation in period $t+1$ is

$$(32) \qquad 1 - \omega_{t+1} = \begin{cases} 0 & \text{with probability } \Pr(H^*_{t+1} = 0 | H^*_t = i) \\ h_1/h_i & \text{with probability } \Pr(H^*_{t+1} = 1 | H^*_t = i) \\ h_2/h_i & \text{with probability } \Pr(H^*_{t+1} = 2 | H^*_t = i) \\ h_3/h_i & \text{with probability } \Pr(H^*_{t+1} = 3 | H^*_t = i) \\ h_4/h_i & \text{with probability } \Pr(H^*_{t+1} = 4 | H^*_t = i) \\ h_5/h_i & \text{with probability } \Pr(H^*_{t+1} = 5 | H^*_t = i) \end{cases}.$$

Relative Price of Health Care

Virtually all households in the sample report health insurance coverage through Medicare, Medicaid, or an employer-provided health plan. Nevertheless, households report significant out-of-pocket health expenditure, especially in old age, which can arise for a number of reasons. Medicare does not cover nursing home care, and Medicaid only covers a limited and capped amount of nursing home care for those that qualify. In addition, households may choose out-of-network or higher quality care that is not covered by their health insurance.

For each household at each interview, I compute the out-of-pocket expenditure share as the ratio of out-of-pocket to total health expenditure. I use a censored regression model to estimate how the out-of-pocket expenditure share depends on health status, age and its interaction with health status, total wealth and its interaction with health status, marital status, and birth cohort. I then compute the predicted values for single females who were born 1931–1940 and have the average total wealth for her cohort and age. Let $q_t(H_t^*)$ denote the predicted out-of-pocket expenditure share for health status H_t^* in period t. I model the relative price of health care as

$$Q_t = e^{q(t-1)} q_t(H_t^*). \tag{33}$$

The first part accounts for the secular growth in the relative price of health care. The average log growth rate of the consumer price index for medical care, relative to that for all items less medical care, was 1.9 percent for the period 1958–2008. I therefore set $q = 0.019$ annually. The second part accounts for health insurance coverage at the household level.

Figure 2 reports the relative price of health care by health status and age. The relative price of health care rises in health, especially for younger households. For example, the relative price of health care is 0.35 for households in poor health at age 65, which is lower than 0.47 for those in excellent health. The fact that health insurance has better coverage for the unhealthy is consistent with the existence of copays and deductibles. The relative price of health care grows rapidly in old age. Part of this growth is explained by an out-of-pocket expenditure share that rises in age. For households in good health, the out-of-pocket expenditure share is 0.54 at age 77, which is lower than 0.60 at age 89. The remainder of the growth is explained by the secular growth in the relative price of health care.

D. Measuring the Outputs of the Life-Cycle Model

Out-of-Pocket Health Expenditure

In Table 3, I use a linear regression model to estimate how the logarithm of out-of-pocket health expenditure, as a share of income, depends on health status, age and its interaction with health status, total wealth and its interaction with health status, marital status, and birth cohort. Out-of-pocket health expenditure is 21 percent higher for households in poor health at age 65, compared to those in good health. For households in good health, out-of-pocket health expenditure rises by 65 percent for every ten years in age. The negative coefficient for the interaction of age with excellent health implies that out-of-pocket health expenditure rises less rapidly in age for households in excellent health.

To facilitate the comparison of the data with simulations from the life-cycle model, Panel A of Table 4 reports the predicted out-of-pocket health expenditure, as a share of income, by health status and age. The reported estimates are for single females who were born 1931–1940 and have the average total wealth for her cohort and age. The table does not extend beyond age 89 because sample attrition through death makes such extrapolation potentially unreliable. The key targets for the life-cycle model are that out-of-pocket health expenditure falls in health and rises in age. Households in poor health at age 65 spend 9 percent of their income on health care, which is higher than 7 percent for those in excellent health. Similarly, households in poor health at age 89 spend 48 percent of their income on health care, which is higher than 25 percent for those in excellent health. Households in good health spend 7 percent of their income on health care at age 65, which is lower than 35 percent at age 89.

Panel B of Table 4 reports the predicted health distribution by age for the same group of households. The health distribution at age 65 is 7 percent in poor health, 20 percent in fair health, 33 percent in good health, 30 percent in very good health, and 10 percent in excellent health. While households in poorest health tend to die and drop out of the sample, health declines even among healthier households that remain alive. The health distribution at age 89 is 16 percent in poor health, 28 percent in fair health, 32 percent in good health, 19 percent in very good health, and 4 percent in excellent health.

Financial and Housing Wealth

In Table 5, I use a censored regression model to estimate how housing and financial wealth, as a share of total wealth that includes the present value of retirement income, depends on health status, age and its interaction with health status, total wealth and its interaction with health status, marital status, and birth cohort. Financial and housing wealth, as a share of total wealth, is 4 percentage points higher for households in excellent health at age 65, compared to those in good health. For households in good health, financial and housing wealth, as a share of total wealth, rises by 12 percentage points for every ten years in age. The negative coefficient for the interaction of age with poor health implies that financial and housing wealth, as a share of total wealth, rises less rapidly in age for households in poor health.

Panel C of Table 4 reports the predicted financial and housing wealth, as a share of total wealth, by health status and age. The reported estimates are for single females who were born 1931–1940 and have the average total wealth for her cohort and age. The key targets for the life-cycle model are that financial and housing wealth, as a share of total wealth, rises in both health and age. Financial and housing wealth is 19 percent of total wealth for households in poor health at age 65, which is lower than 23 percent for those in excellent health. Similarly, financial and housing wealth is 43 percent of total wealth for households in poor health at age 89, which is lower than 50 percent for those in excellent health. As households age, the value of their liquid wealth rises relative the declining present value of retirement income. For households in good health, financial and housing wealth is 20 percent of total wealth at age 65, which is lower than 47 percent at age 89.

In Table 6, I use a censored regression model to estimate how the allocation to stocks, as a share of financial and housing wealth, depends on health status, age and its interaction with health status, total wealth and its interaction with health status, marital status, and birth cohort. The portfolio share in stocks is positively related to health, even after controlling for total wealth (Harvey S. Rosen and Stephen Wu 2004). The portfolio share in stocks is 4 percentage points lower for households in poor health at age 65, compared to those in good health. The portfolio share in stocks is 2 percentage points higher for households in excellent health at age 65, compared to those in good health. For households in good health, the portfolio share in stocks rises by 2 percentage points for every ten years in age.

Panel E of Table 4 reports the predicted portfolio share in stocks by health status and age. The key targets for the life-cycle model are that the portfolio share in stocks rises in both health and age. The portfolio share in stocks is 3 percent for households in poor health at age 65, which is lower than 8 percent for those in excellent health. Similarly, the portfolio share in stocks is 10 percent for households in poor health at age 89, which is lower than 12 percent for those in excellent health. For households in good health, the portfolio share in stocks is 6 percent at age 65, which is lower than 12 percent at age 89.

In Table 6, I use a censored regression model to explain how the allocation to housing, as a share of financial and housing wealth, depends on health status, age and its interaction with health status, total wealth and its interaction with health status, marital status, and birth cohort. The coefficients for health status imply the absence of a robust relation between the portfolio share in housing and health at age 65. For households in good health, the portfolio share in housing falls by 13 percentage points for every ten years in age. The negative coefficient for the interaction of age with poor health implies that the portfolio share in housing falls more rapidly for households in poor health. Conversely, the positive coefficient for the interaction of age with excellent health implies that the portfolio share in housing falls less rapidly for households in excellent health.

Panel F of Table 4 reports the predicted portfolio share in housing by health status and age. The key targets for the life-cycle model are that the portfolio share in housing rises in health for older households and falls in age. The portfolio share in housing is 38 percent for households in poor health at age 89, which is lower than 53 percent for those in excellent health. For households in good health, the portfolio share in housing is 77 percent at age 65, which is higher than 45 percent at age 89.

Since stocks account for a small share of financial and housing wealth, Panel D of Table 4 shows that the portfolio share in bonds is essentially the mirror image of the portfolio share in housing. In other words, the portfolio share in bonds falls in health for older households and rises in age.

How Financial and Housing Wealth Responds to Declining Health

By documenting the cross-sectional variation in health and age, Table 5 reveals how financial and housing wealth responds to changes in health at the life-cycle frequency. Similarly, Table 6 reveals how the allocation of financial and housing wealth between bonds, stocks, and housing responds to changes in health at the life-cycle frequency. This section documents how wealth and its allocation respond to changes in health at the two-year frequency, between survey interviews.

For each household, I compute the growth rate of financial and housing wealth from the present interview to the next. In Table 7, I use a linear regression model to estimate how this change in financial and housing wealth depends on a set of dummy variables that measures the decline in health from the present interview to the next. Households in fair or better health reduce financial and housing wealth by 13 percent on average when their health declines to poor. Similarly, households in good or better health reduce financial and housing wealth by 8 percent when their health declines to fair or worse. Hence, the cumulative effect of declining health from good or better to poor is the sum of these two coefficients, which is a 21 percent reduction in financial and housing wealth. For households in very good or better health, relatively small health shocks have essentially no impact on financial and housing wealth.

To understand the source of the reduction in financial and housing wealth, I decompose the growth rate of financial and housing wealth into the sum of three parts due to bonds, stocks, and housing:

$$\frac{A_{1,t+1} - A_{1,t}}{A_{1,t} + A_{2,t} + A_{D,t}} + \frac{A_{2,t+1} - A_{2,t}}{A_{1,t} + A_{2,t} + A_{D,t}} + \frac{A_{D,t+1} - A_{D,t}}{A_{1,t} + A_{2,t} + A_{D,t}}. \tag{34}$$

Bonds account for 4 percent, stocks account for essentially none, and housing accounts for 9 percent of the 13 percent reduction in wealth when health declines from fair or better. Thus, households dissave primarily from housing wealth in response to relatively severe health shocks. Bonds account for 5 percent, stocks account for 3 percent, and housing accounts for essentially none of the 8 percent reduction in wealth when health declines from good or better. Thus, households dissave from financial wealth in response to more moderate health shocks.

III. Health Expenditure and Wealth in the Life-Cycle Model

Table 8 summarizes the preference and health parameters that I use to calibrate the life-cycle model. Following a common practice in the life-cycle literature, I set the subjective discount factor to $\beta = 0.96$ annually. The bequest motive is not well identified, separately from the elasticity of intertemporal substitution, based on the life-cycle wealth profile for typical households (ase discussed in Mariacristina De Nardi, Eric French and John Bailey Jones (2010) and John Ameriks, Andrew Caplin, Steven Laufer and Stijn Van Nieuwerburgh (2011)). I therefore set the bequest parameter to $\overline{u} = 1$, which is a fairly weak bequest motive that corresponds to one period (two years) of consumption. As discussed below, I calibrate the remaining preference and health parameters to match targeted moments in the data.

A. Optimal Consumption and Portfolio Policies

I solve the life-cycle model by numerical dynamic programming as described in Appendix B. Figure 3 reports the optimal consumption and portfolio policies at age 65, as functions of the health state $\widehat{H}_t$. The optimal policies are in units of total wealth that includes the present value of retirement income. My discussion will focus on the baseline policy evaluated at $\widehat{B}_{3,t} = 0.5$, $\widehat{D}_t = 0.5$, and $P_t = 1.0$.

Optimal consumption is increasing in health. The household consumes a lower share of its total wealth in poor health because non-health consumption and health are complements at the calibrated parameters (i.e., $\sigma > \rho$). Optimal out-of-pocket health expenditure is decreasing in health. The household spends a higher share of its total wealth on health care in poor health because of decreasing returns to health investment (i.e., $\psi < 1$).

The optimal portfolio share in bonds is decreasing in health, while the optimal portfolio share in stocks is increasing in health. Because the household has shorter life expectancy in poor health, this is analogous to the standard effect that an investor with shorter horizon should invest a lower share of its financial wealth in stocks (Zvi Bodie, Robert C. Merton and William F. Samuelson 1992). The optimal portfolio share in housing is increasing in health.

The household consumes less housing in poor health because non-health consumption and health are complements.

The dashed line represents the optimal policy for a higher housing stock, evaluated at $\widehat{D}_t = 0.9$. Any differences between this policy and the baseline policy can be interpreted as the effect of transaction costs because the housing stock would drop out as a state variable in the absence of such costs. The optimal housing policies coincide at good or worse health, which implies that a household in poor health is willing to pay the transaction cost to adjust the housing stock. The optimal housing policies differ at very good or better health, which implies that a household in excellent health is unwilling to pay the transaction cost to adjust the housing stock.

The dotted line represents the optimal policy for a higher home price, evaluated at $P_t = 1.5$. The primary effect of a higher home price is that the household substitutes from housing to consumption. The substitution effect dominates the income effect because consumption and housing are substitutes at the calibrated parameters (i.e., $\sigma < 1$).

I use the optimal consumption and portfolio policies to simulate a population of 100,000 households every two years from age 65 until death. I draw initial health from a lognormal distribution (i.e., $\log \widehat{H}_1 \sim \mathbf{N}(\mu_H, \sigma_H)$) to match the health distribution at age 65. I calibrate the initial level of retirement income, conditional on health, to match the present value of retirement income at age 65. Finally, I calibrate the initial housing stock, conditional on health, to match housing wealth at age 65.

B. Out-of-Pocket Health Expenditure

Panel A of Table 9 reports out-of-pocket health expenditure, as a share of income, by health status and age for simulated households. The utility weight on health is an important determinant of the level of out-of-pocket health expenditure. The returns to health investment is an important determinant of the relation between out-of-pocket health expenditure and health. Finally, the elasticity of intertemporal substitution is an important determinant of the relation between out-of-pocket health expenditure and age.

Consistent with Panel A of Table 4, the life-cycle model generates out-of-pocket health expenditure that falls in health and rises in age. Households in poor health at age 65 spend 91 percent of their income on health care, which is higher than 1 percent for those in excellent health. Similarly, households in poor health at age 89 spend 123 percent of their income on health care, which is higher than 4 percent for those in excellent health. This relation between out-of-pocket health expenditure and health is qualitatively consistent with the empirical evidence, despite being more pronounced.[2] Households in good health spend 12 percent of their income on health care at age 65, which is lower than 35 percent at age 89. This relation between out-of-pocket health expenditure and age matches the empirical evidence.

Panel B of Table 9 reports the health distribution by age for simulated households. The health distribution is jointly determined by health depreciation, whose magnitude is determined by the calibrated parameters μ_H and σ_H, and health expenditure. Consistent with Panel B of Table 4, health declines even among healthier households that remain alive. The health distribution at age 89 is 8 percent in poor health, 24 percent in fair health, 39 percent in good health, 26 percent in very good health, and 4 percent in excellent health.

C. Financial and Housing Wealth

Panel C of Table 9 reports financial and housing wealth, as a share of total wealth, by health status and age for simulated households. The elasticity of intertemporal substitution is an important determinant of the relation between wealth and age.

Consistent with Panel C of Table 4, the life-cycle model generates financial and housing wealth, as a share of total wealth, that rises in both health and age. Financial and housing wealth is 15 percent of total wealth for households in poor health at age 65, which is lower than 22 percent for those in excellent health. Similarly, financial and housing wealth is 43 percent of total wealth for households in poor health at age 89, which is lower than 50 percent for those in excellent health. This relation between wealth and health matches the empirical

[2]Lower returns to health investment (i.e., lower ψ) would better match the relation between out-of-pocket health expenditure and health, which was the calibration in a previous version of this paper. However, lower returns to health investment would reduce the response of financial and housing wealth to declining health, making it difficult for the life-cycle model to explain the empirical evidence in Table 7.

evidence. For households in good health, financial and housing wealth is 18 percent of total wealth at age 65, which is lower than 49 percent at age 89. This relation between wealth and age matches the empirical evidence.

Panel E of Table 9 reports the portfolio share in stocks by health status and age for simulated households. Relative risk aversion is an important determinant of the level of the portfolio share in stocks. The life-cycle model has difficulty explaining the low portfolio share in stocks because of the high equity premium, even with relative risk aversion of five. Fixed costs of participating in the stock market, which I have ignored in this paper for tractability, are likely to explain the low portfolio share in stocks (Francisco Gomes and Alexander Michaelides 2005).

Consistent with Panel E of Table 4, the portfolio share in stocks rises in both health and age. The portfolio share in stocks is 20 percent for households in poor health at age 65, which is lower than 34 percent for those in excellent health. Similarly, the portfolio share in stocks is 10 percent for households in poor health at age 89, which is lower than 23 percent for those in excellent health. This relation between the portfolio share in stocks and health is qualitatively consistent with the empirical evidence. For households in good health, the portfolio share in stocks is 20 percent at age 65, which is lower than 37 percent at age 89. This relation between the portfolio share in stocks and age is qualitatively consistent with the empirical evidence.

Panel F of Table 9 reports the portfolio share in housing by health status and age for simulated households. The utility weight on housing is an important determinant of the level of the portfolio share in housing. The elasticity of substitution between non-health consumption and health is an important determinant of the relation between the portfolio share in housing and health. Finally, the elasticity of intertemporal substitution is an important determinant of the relation between the portfolio share in housing and age.

Consistent with Panel F of Table 4, the portfolio share in housing rises in health for older households and falls in age. The portfolio share in housing is 57 percent for households in poor health at age 89, which is lower than 67 percent for those in excellent health. This relation between the portfolio share in housing and health for older households is qualitatively consistent with the empirical evidence. For households in good health, the portfolio share in housing is 82 percent at age 65, which is higher than 56 percent at age 89. This relation between the portfolio share in housing and age is qualitatively consistent with the empirical evidence.

D. How Financial and Housing Wealth Responds to Declining Health

For each simulated household, I compute the growth rate of financial and housing wealth from the present period to the next. In Table 10, I use a linear regression model to estimate how this change in financial and housing wealth depends on a set of dummy variables that measures the decline in health from the present period to the next. Households in fair or better health reduce financial and housing wealth by 12 percent on average when their health declines to poor. Similarly, households in good or better health reduce financial and housing wealth by 3 percent when their health declines to fair or worse. Hence, the cumulative effect of declining health from good or better to poor is the sum of these two coefficients, which is a 15 percent reduction in financial and housing wealth. For households in very good or better health, relatively small health shocks have essentially no impact on financial and housing wealth. Overall, how financial and housing wealth responds to declining health in the life-cycle model matches the empirical evidence in Table 7.

To understand the source of the decline in financial and housing wealth, I decompose the growth rate of financial and housing wealth into the sum of three parts due to bonds, stocks, and housing. Bonds and stocks account for 5 percent, and housing accounts for 7 percent of the 12 percent reduction in wealth when health declines from fair or better. The fact that households primarily dissave from housing wealth in response to relatively severe health shocks matches the empirical evidence in Table 7. However, the fact that households reallocate from stocks to bonds in response to declining health is inconsistent with the empirical evidence. Additional frictions, which I have ignored in this paper for tractability, are likely to explain the absence of reallocation from stocks to bonds at the two-year frequency.

IV. Welfare Analysis of Relaxing Portfolio Constraints in the Life-Cycle Model

A. Welfare Analysis of Relaxing Borrowing Constraints on Home Equity

In the benchmark model, the household can only borrow up to 20 percent of home value. This section examines the welfare implications of relaxing the borrowing constraint on home equity to be 60 percent of home value. This exercise can be interpreted as the impact of financial products such as reverse mortgages and home equity loans that relax borrowing constraints for older households.

Table 11 reports out-of-pocket health expenditure, financial and housing wealth, and its allocation between bonds, stocks, and housing for simulated households. Relaxed borrowing constraints do not have a significant impact on health expenditure or the level of financial and housing wealth, compared to the benchmark model in Table 9. However, relaxed borrowing constraints have a significant impact on the allocation of financial and housing wealth. Households borrow from home equity, especially when young, to take advantage of the equity premium. For example, households in good health at age 65 allocate −60 percent of their financial and housing wealth to bonds, 60 percent to stocks, and 100 percent to housing.

I calculate the welfare gain as the ratio of the value function achieved with relaxed borrowing constraints to that in the benchmark model, normalized to units of financial and housing wealth. As reported in Table 12, the welfare gain from relaxed borrowing constraints is 5 percent of financial and housing wealth for a household in good health at age 65. Healthier households with longer life expectancy achieve higher welfare gains. The welfare gain is 4 percent of financial and housing wealth for households in poor health at age 65, and 7 percent for those in good health.

B. Welfare Analysis of Private Annuitization

In the benchmark model, the household has only an implicit claim on a real annuity through Social Security and defined-benefit pension plans. This section examines the welfare implications of allowing households to privately annuitize wealth. In this exercise, I adopt two important institutional features of the annuity market in the United States. First, the pricing of annuities depends on age but not on health. As a frictionless benchmark, I assume that annuities are actuarially fair with respect to the Social Security life tables, having an average return of 2.5 percent that matches the riskless interest rate. Second, annuitization is irreversible, which amounts to a portfolio constraint $B_{3,t} \geq B_{3,t-1}$ in each period t.

Table 13 reports out-of-pocket health expenditure, financial and housing wealth, and its allocation between bonds, stocks, housing, and annuities for simulated households. Households reduce health expenditure and slightly increase financial and housing wealth, relative to the benchmark model in Table 9. For example, households in good health at age 77 spend 14 percent of income on health care, which is lower than 24 percent in the benchmark model. Although markets are incomplete in the model, the endogenous response of health expenditure to health shocks reduces the degree of market incompleteness. Consequently, households annuitize most of their financial and housing wealth, up to the borrowing constraint on home equity (Menahem E. Yaari 1965, Thomas Davidoff, Jeffrey R. Brown and Peter A. Diamond 2005). For example, households in good health at age 77 allocate −4 percent of their financial and housing wealth to bonds, 27 percent to housing, and 77 percent to annuities. Healthier households have a higher portfolio share in housing, due to the complementarity between non-health consumption and health, which leads to a lower portfolio share in annuities.

I calculate the welfare gain as the ratio of the value function achieved with private annuitization to that in the benchmark model, normalized to units of financial and housing wealth. As reported in Table 12, the welfare gain from private annuitization is 5 percent of financial and housing wealth for a household in good health at age 65. Healthier households with longer life expectancy achieve higher welfare gains, partly because the pricing of annuities is more advantageous for them. The welfare gain is 11 percent of financial and housing wealth for households in poor health at age 65, and 17 percent for those in good health.

The life-cycle model has two elements that make the welfare gain from private annuitization smaller than what it would otherwise be. First, housing wealth is more efficient than financial wealth for self-insuring longevity risk because of the utility flow from living in a home (Thomas Davidoff 2010). Second, the household can endogenously reduce health expenditure, which reduces the utility cost of a wealth shortfall in old age (see Samuel Marshall, Kathleen M. McGarry and Jonathan S. Skinner (2010) for empirical evidence). There are two additional elements, turned off in my exercise for clarity, that can further reduce the welfare gain from private annuitization: a stronger bequest motive (Benjamin M. Friedman and Mark J. Warshawsky 1990, Joachim Inkmann, Paula Lopes and Alexander Michaelides 2011) and annuities that are not actuarially fair (Olivia S. Mitchell, James M. Poterba, Mark J. Warshawsky and Jeffrey R. Brown 1999).

V. Conclusion

This paper has shown that a life-cycle model, in which health expenditure and saving decisions respond endogenously to health shocks, can explain the cross-sectional variation and the joint dynamics of health expenditure, health, and wealth in the Health and Retirement Study. There are a number of potentially interesting extensions for future work. First, the life-cycle problem can be generalized to encompass married households, so that health expenditure and saving decisions depend on the health and survival of both partners (Lee A. Lillard and Yoram Weiss 1997, Lena Jacobson 2000, David A. Love 2010). Second, the life-cycle problem can be extended to include the working phase prior to retirement. A number of interesting issues then arise such as the correlation between health and labor income (Julien Hugonnier, Florian Pelgrin and Pascal St-Amour 2009) and the endogenous response of labor supply to both public and employer-provided health insurance (David M. Blau and Donna B. Gilleskie 2008, Eric French and John Bailey Jones 2011). Finally, the life-cycle model can be used for welfare analysis of insurance products other than annuities such as life insurance, Medigap insurance, long-term care insurance (Ralph S. J. Koijen, Stijn Van Nieuwerburgh and Motohiro Yogo 2011).

Table 1—Health Problems and Health Care Utilization by Health Status

	Health status				
	Poor	Fair	Good	Very good	Excellent
Panel A: Doctor-diagnosed health problems (percent of respondents)					
High blood pressure	69	65	59	49	33
Diabetes	26	22	14	7	3
Cancer	22	18	15	13	10
Lung disease	23	14	8	5	3
Heart problems	54	39	26	15	9
Stroke	28	15	9	5	3
Psychiatric problems	30	20	13	8	6
Arthritis	78	73	65	55	40
Panel B: Some difficulty with activities of daily living (percent of respondents)					
Bathing	45	18	8	4	2
Dressing	43	19	9	4	3
Eating	22	7	3	2	1
Panel C: Health care utilization (percent of respondents)					
Doctor visit	98	97	96	94	91
Dentist visit	41	51	61	69	71
Home health care	34	17	9	5	3
Nursing home stay	19	8	5	3	2
Outpatient surgery	21	21	21	19	16
Prescription drugs	96	94	89	82	68
Cholesterol test	80	80	81	81	75
Mammogram	56	64	71	73	71

Note: Panel A reports the share of respondents who have ever had doctor-diagnosed health problems, separately by health status. Panel B reports the share of respondents who have some difficulty with activities of daily living at the time of interview, separately by health status. Panel C reports the share of respondents who have used health care in the two years prior to the interview, separately by health status. The sample consists of retired females who were born 1891–1940, aged 65 or older, and interviewed by the Health and Retirement Study during 1992–2006.

Table 2—Estimating How Future Health Depends on Present Health and Health Investment

Explanatory variable	(1)		(2)	
Health status:				
Poor	-154.02	(-5.94)	-118.08	(-4.45)
Fair	-73.87	(-5.43)	-62.82	(-4.42)
Very good	69.85	(6.25)	68.58	(6.04)
Excellent	135.36	(7.70)	131.04	(7.40)
(Age − 65)/10	-13.56	(-4.88)	-7.87	(-2.79)
× Poor	14.21	(3.86)	10.50	(2.81)
× Fair	7.71	(2.60)	6.84	(2.28)
× Very good	-3.77	(-1.18)	-2.87	(-0.89)
× Excellent	-16.13	(-2.78)	-15.07	(-2.57)
Total wealth	8.85	(4.48)	7.81	(3.92)
× Poor	-17.92	(-4.98)	-17.42	(-4.82)
× Fair	-6.38	(-2.29)	-7.49	(-2.68)
× Very good	6.16	(2.03)	6.36	(2.09)
× Excellent	8.67	(1.67)	9.53	(1.83)
Doctor visit	0.04	(0.01)	2.74	(0.34)
× Poor	13.25	(0.58)	10.06	(0.43)
× Fair	14.59	(1.09)	13.30	(0.95)
× Very good	-5.12	(-0.47)	-6.28	(-0.57)
× Excellent	9.98	(0.61)	10.51	(0.64)
Dentist visit	8.95	(3.28)	7.24	(2.64)
× Poor	6.67	(1.22)	3.62	(0.66)
× Fair	1.70	(0.41)	1.03	(0.25)
× Very good	8.12	(1.81)	7.85	(1.75)
× Excellent	16.97	(1.93)	16.50	(1.87)
Home health care	-25.22	(-4.96)	-15.74	(-3.06)
× Poor	-5.99	(-0.85)	0.86	(0.12)
× Fair	0.36	(0.05)	2.15	(0.32)
× Very good	-13.89	(-1.44)	-16.30	(-1.67)
× Excellent	-24.89	(-1.07)	-22.69	(-1.00)
Nursing home stay	-14.56	(-1.61)	-7.59	(-0.83)
× Poor	-9.88	(-0.83)	-6.92	(-0.57)
× Fair	-14.14	(-1.18)	-4.93	(-0.41)
× Very good	-25.24	(-1.48)	-22.48	(-1.33)
× Excellent	-97.01	(-2.19)	-96.88	(-2.22)
Outpatient surgery	2.58	(0.86)	4.45	(1.47)
× Poor	2.16	(0.37)	-0.69	(-0.11)
× Fair	-5.31	(-1.16)	-5.05	(-1.09)
× Very good	-0.93	(-0.19)	-1.42	(-0.30)
× Excellent	-4.33	(-0.45)	-4.20	(-0.44)
Prescription drugs	-17.58	(-4.21)	-1.56	(-0.36)
× Poor	11.08	(0.83)	10.15	(0.77)
× Fair	-10.87	(-1.29)	-9.84	(-1.17)
× Very good	-1.36	(-0.23)	-6.36	(-1.07)
× Excellent	-8.59	(-0.93)	-16.81	(-1.82)

Explanatory variable	(1)		(2)	
Cholesterol test	8.89	(2.46)	11.74	(3.23)
× Poor	-12.76	(-1.84)	-12.44	(-1.78)
× Fair	-5.23	(-0.95)	-5.56	(-1.00)
× Very good	0.67	(0.12)	-0.46	(-0.08)
× Excellent	-6.17	(-0.61)	-6.15	(-0.61)
Mammogram	4.86	(1.52)	4.81	(1.49)
× Poor	-0.39	(-0.07)	-4.77	(-0.84)
× Fair	-1.21	(-0.25)	-1.63	(-0.34)
× Very good	0.66	(0.13)	1.58	(0.31)
× Excellent	5.35	(0.55)	4.70	(0.48)
Vigorous physical activity	18.89	(7.27)	15.14	(5.79)
× Poor	-1.65	(-0.23)	-8.85	(-1.20)
× Fair	3.52	(0.79)	2.05	(0.46)
× Very good	0.08	(0.02)	2.22	(0.56)
× Excellent	4.82	(0.66)	6.64	(0.91)
Smoking	-16.94	(-3.78)	-17.40	(-3.87)
× Poor	11.25	(1.44)	7.48	(0.94)
× Fair	4.63	(0.68)	2.85	(0.42)
× Very good	-4.53	(-0.64)	-4.64	(-0.66)
× Excellent	-18.32	(-1.25)	-18.97	(-1.30)
Doctor-diagnosed health problems:				
High blood pressure			-10.41	(-6.30)
Diabetes			-19.79	(-9.24)
Cancer			-15.98	(-7.37)
Lung disease			-24.13	(-9.00)
Heart problems			-16.30	(-9.00)
Stroke			-7.72	(-2.85)
Psychiatric problems			-10.24	(-4.67)
Arthritis			-12.78	(-7.46)
Some difficulty with activities of daily living:				
Bathing			-21.45	(-6.93)
Dressing			-12.66	(-4.53)
Eating			-27.72	(-6.12)
Married	-2.24	(-1.25)	-3.38	(-1.87)
Birth cohort:				
1891–1900	-98.19	(-4.96)	-112.28	(-5.87)
1901–1910	-33.38	(-4.89)	-43.93	(-6.34)
1911–1920	-7.57	(-2.02)	-16.82	(-4.42)
1921–1930	1.67	(0.74)	-4.13	(-1.80)
Wald test for health investment	505.50	(0.00)	300.62	(0.00)
Observations	20,557		20,434	

Note: An ordered probit model is used to explain future health status at two years from the present interview. The table reports the estimated coefficients in percentage points and heteroskedasticity-robust t-statistics in parentheses. The Wald test for the dependence of future health status on health investment includes measures of health care utilization (i.e., doctor visit, dentist visit, home health care, nursing home stay, outpatient surgery, prescription drugs, cholesterol test, and mammogram), vigorous physical activity, smoking, and the interaction of these explanatory variables with present health status. The p-value for the Wald test is reported in parentheses. The sample consists of retired females who were born 1891–1940, aged 65 or older, and interviewed by the Health and Retirement Study during 1992–2006.

Table 3—Estimating How Out-of-Pocket Health Expenditure Relates to Health

Explanatory variable	Coefficient	t-statistic
Health status:		
Poor	21.14	(3.37)
Fair	4.64	(1.01)
Very good	-6.99	(-1.59)
Excellent	-8.26	(-1.25)
(Age − 65)/10	64.77	(23.78)
× Poor	3.35	(0.78)
× Fair	3.47	(1.06)
× Very good	-6.22	(-1.98)
× Excellent	-11.55	(-2.43)
Total wealth	-29.66	(-11.77)
× Poor	8.45	(1.79)
× Fair	8.56	(2.30)
× Very good	0.38	(0.11)
× Excellent	-7.16	(-1.40)
Married	48.96	(24.88)
Birth cohort:		
1891–1900	-116.24	(-7.21)
1901–1910	-85.55	(-15.07)
1911–1920	-65.70	(-18.73)
1921–1930	-39.16	(-16.42)
Observations	29,925	

Note: A linear regression model is used to explain the variation in the logarithm of out-of-pocket health expenditure, as a share of income. The table reports the estimated coefficients in percentage points and heteroskedasticity-robust t-statistics in parentheses. The sample consists of retired females who were born 1891–1940, aged 65 or older, and interviewed by the Health and Retirement Study during 1992–2006.

Table 4—Health Expenditure and Wealth in the Health and Retirement Study

Health status	Age				
	65	71	77	83	89
Panel A: Out-of-pocket health expenditure (percent of income)					
Poor	9	14	21	32	48
Fair	8	12	18	27	40
Good	7	11	16	24	35
Very good	7	10	14	20	28
Excellent	7	9	13	18	25
Panel B: Health distribution (percent of households)					
Poor	7	9	11	13	16
Fair	20	22	24	26	28
Good	33	34	34	33	32
Very good	30	27	25	22	19
Excellent	10	8	7	5	4
Panel C: Financial and housing wealth (percent of total wealth)					
Poor	19	25	31	37	43
Fair	19	25	32	38	45
Good	20	26	33	40	47
Very good	21	28	34	41	48
Excellent	23	29	36	43	50
Panel D: Bonds (percent of financial and housing wealth)					
Poor	19	29	38	45	52
Fair	12	23	32	40	47
Good	17	25	31	38	43
Very good	19	25	30	36	40
Excellent	14	20	25	30	34
Panel E: Stocks (percent of financial and housing wealth)					
Poor	3	5	6	8	10
Fair	4	5	7	9	11
Good	6	7	9	10	12
Very good	8	9	10	11	12
Excellent	8	9	10	11	12
Panel F: Housing (percent of financial and housing wealth)					
Poor	77	66	56	47	38
Fair	83	72	61	51	42
Good	77	68	60	52	45
Very good	73	66	60	53	47
Excellent	78	71	65	59	53

Note: Panel A reports the predicted values from the regression model in Table 3. Panel B reports the predicted values from an ordered probit model that explains health status as a function of age, total wealth, marital status, and birth cohort. Panel C reports the predicted values from the censored regression model in Table 5. Panels E and F report predicted values from the censored regression model in Table 6. All predicted values are for single females who were born 1931–1940 and have the average total wealth for her cohort and age.

Table 5—Estimating How Financial and Housing Wealth Relates to Health

Explanatory variable	Elasticity	t-statistic
Health status:		
Poor	-0.18	(-0.26)
Fair	-0.74	(-1.38)
Very good	1.89	(3.43)
Excellent	3.63	(4.30)
(Age – 65)/10	11.67	(33.54)
× Poor	-1.80	(-3.61)
× Fair	-0.47	(-1.17)
× Very good	-0.37	(-0.92)
× Excellent	-0.46	(-0.77)
Total wealth	15.72	(44.02)
× Poor	0.97	(1.46)
× Fair	0.18	(0.34)
× Very good	-0.73	(-1.48)
× Excellent	-1.63	(-2.22)
Married	-7.60	(-27.50)
Birth cohort:		
1891–1900	-12.87	(-8.26)
1901–1910	-6.60	(-9.64)
1911–1920	-4.34	(-9.82)
1921–1930	-1.43	(-4.62)
Observations	31,689	

Note: A censored regression model is used to explain the variation in housing and financial wealth, as a share of total wealth that includes the present value of retirement income. The table reports the estimated elasticities in percentage points, at the mean of the explanatory variables, and heteroskedasticity-robust t-statistics in parentheses. The sample consists of retired females who were born 1891–1940, aged 65 or older, and interviewed by the Health and Retirement Study during 1992–2006.

Table 6—Estimating How the Portfolio Shares in Stocks and Housing Relate to Health

Explanatory variable	Stocks		Housing	
Health status:				
Poor	-4.40	(-6.64)	0.54	(0.17)
Fair	-3.29	(-5.86)	6.13	(2.72)
Very good	2.48	(3.99)	-3.04	(-1.63)
Excellent	2.19	(2.45)	0.99	(0.39)
(Age – 65)/10	2.49	(7.09)	-13.19	(-12.25)
× Poor	1.39	(2.32)	-3.62	(-1.83)
× Fair	1.25	(2.82)	-4.21	(-2.95)
× Very good	-0.79	(-1.98)	2.30	(1.86)
× Excellent	-0.63	(-1.13)	3.40	(2.11)
Total wealth	12.39	(42.32)	3.64	(4.27)
× Poor	1.51	(2.33)	12.79	(7.54)
× Fair	0.40	(0.84)	5.24	(3.90)
× Very good	-0.82	(-2.01)	-4.02	(-3.26)
× Excellent	-0.97	(-1.79)	-8.00	(-4.89)
Married	-1.68	(-6.77)	5.73	(7.48)
Birth cohort:				
1891–1900	-6.45	(-5.64)	-27.86	(-5.00)
1901–1910	-4.82	(-9.70)	-12.02	(-5.91)
1911–1920	-2.59	(-6.32)	-6.56	(-4.93)
1921–1930	-0.56	(-1.83)	-2.85	(-2.88)
Observations	31,689		31,689	

Note: A censored regression model is used to explain the variation in the portfolio shares in stocks and housing, as a share of financial and housing wealth. The table reports the estimated elasticities in percentage points, at the mean of the explanatory variables, and heteroskedasticity-robust t-statistics in parentheses. The sample consists of retired females who were born 1891–1940, aged 65 or older, and interviewed by the Health and Retirement Study during 1992–2006.

Table 7—Estimating How Financial and Housing Wealth Responds to Declining Health

Explanatory variable	Percent change in			
	Financial and housing wealth	Bonds	Stocks	Housing
Health declined from				
Fair or better	-12.79	-3.52	0.03	-9.30
	(-2.59)	(-0.97)	(0.02)	(-2.53)
Good or better	-8.28	-4.67	-3.13	-0.48
	(-2.83)	(-2.28)	(-2.77)	(-0.22)
Very good or better	-2.90	-7.74	-0.60	5.43
	(-1.21)	(-3.89)	(-0.58)	(2.62)
Excellent	3.07	2.05	0.80	0.22
	(1.01)	(0.98)	(0.52)	(0.11)
(Age − 65)/10	-9.37	-4.41	0.38	-5.34
	(-6.63)	(-3.78)	(0.64)	(-4.57)
Observations	21,054	21,054	21,054	21,054

Note: A linear regression model is used to explain the change in financial and housing wealth over two years, from the present interview to the next, and to decompose that change into the sum of the parts due to bonds, stocks, and housing. The table reports the estimated coefficients in percentage points and heteroskedasticity-robust t-statistics in parentheses. The sample consists of retired females who were born 1891–1940, aged 65 or older, and interviewed by the Health and Retirement Study during 1992–2006.

Table 8—Parameters Used to Calibrate the Benchmark Model

Parameter	Symbol	Value
Preferences		
Subjective discount factor	β	0.96
Elasticity of intertemporal substitution	σ	0.6
Relative risk aversion	γ	5
Utility weight on housing	ϕ	0.6
Utility weight on health	α	0.01
Elasticity of substitution between non-health consumption and health	ρ	0.5
Strength of the bequest motive	$\overline{u}$	1.0
Financial assets		
Bond return	$\overline{R}_1 - 1$	2.5%
Average stock return	$\overline{R}_2 - 1$	6.5%
Standard deviation of stock returns	σ_2	18%
Housing		
Depreciation rate	δ	1.14%
Average housing return	$\overline{R}_D - 1$	0.4%
Standard deviation of housing returns	σ_D	3.5%
Borrowing limit	λ	20%
Transaction cost	τ	8%
Health		
Average of log health	μ_H	-10
Standard deviation of log health	σ_H	2
Returns to health investment	ψ	0.21

Note: The life-cycle model is solved and simulated at a two-year frequency to match the frequency of interviews in the Health and Retirement Study. The subjective discount factor, the average and standard deviation of asset returns, and the depreciation rate are reported annually.

Table 9—Health Expenditure and Wealth in the Benchmark Model

Health status	Age				
	65	71	77	83	89
Panel A: Out-of-pocket health expenditure (percent of income)					
Poor	91	91	105	115	123
Fair	33	45	54	62	68
Good	12	18	24	31	35
Very good	2	5	8	11	12
Excellent	1	1	4	4	4
Panel B: Health distribution (percent of households)					
Poor	7	6	7	7	8
Fair	20	18	20	22	24
Good	33	36	37	38	39
Very good	29	33	30	28	26
Excellent	10	7	6	5	4
Panel C: Financial and housing wealth (percent of total wealth)					
Poor	15	25	34	40	43
Fair	17	27	36	43	46
Good	18	28	37	45	49
Very good	20	29	38	45	50
Excellent	22	30	38	46	50
Panel D: Bonds (percent of financial and housing wealth)					
Poor	20	29	28	30	33
Fair	8	24	18	15	15
Good	-2	7	5	5	7
Very good	-9	-3	0	8	11
Excellent	-12	-6	-1	11	10
Panel E: Stocks (percent of financial and housing wealth)					
Poor	20	8	6	7	10
Fair	20	17	20	25	30
Good	20	24	26	32	37
Very good	27	26	24	26	28
Excellent	34	29	25	22	23
Panel F: Housing (percent of financial and housing wealth)					
Poor	59	63	66	63	57
Fair	71	59	62	60	54
Good	82	69	69	62	56
Very good	82	78	76	66	62
Excellent	79	77	76	66	67

Note: The solution of the life-cycle model is used to simulate a cross section of 100,000 households starting at age 65. The table reports the mean of the relevant variable across the population of households that remain alive at the given age. Table 8 reports the parameters of the life-cycle model.

Table 10—How Financial and Housing Wealth Responds to Declining Health in the Benchmark Model

Explanatory variable	Percent change in			
	Financial and housing wealth	Bonds	Stocks	Housing
Health declined from				
Fair or better	-11.98	12.48	-17.50	-6.97
Good or better	-3.48	14.01	-8.02	-9.47
Very good or better	-0.42	3.75	1.74	-5.91
Excellent	0.63	3.42	-2.44	-0.35
(Age − 65)/10	-2.14	-2.14	0.69	-0.69

Note: The solution of the life-cycle model is used to simulate a cross section of 100,000 households starting at age 65. A linear regression model is used to explain the change in financial and housing wealth over two years, from one period to the next, and to decompose that change into the sum of the parts due to bonds, stocks, and housing. The table reports the estimated coefficients across the population of households that remain alive at the given age. Table 8 reports the parameters of the life-cycle model.

Table 11—Health Expenditure and Wealth in a Life-Cycle Model with Relaxed Borrowing Constraints on Home Equity

Health status	Age				
	65	71	77	83	89
Panel A: Out-of-pocket health expenditure (percent of income)					
Poor	86	94	110	123	131
Fair	25	48	57	66	72
Good	5	19	26	30	34
Very good	1	6	9	10	10
Excellent	0	2	4	3	3
Panel B: Health distribution (percent of households)					
Poor	7	6	7	7	8
Fair	20	18	20	22	24
Good	33	36	37	38	39
Very good	29	33	30	28	26
Excellent	10	7	6	5	3
Panel C: Financial and housing wealth (percent of total wealth)					
Poor	15	26	35	43	46
Fair	17	28	37	45	49
Good	19	29	39	47	52
Very good	21	30	39	48	53
Excellent	22	30	39	48	53
Panel D: Bonds (percent of financial and housing wealth)					
Poor	-29	9	26	33	33
Fair	-24	3	10	13	12
Good	-60	-17	-7	-4	-4
Very good	-59	-28	-9	-4	-4
Excellent	-42	-30	-5	-2	-3
Panel E: Stocks (percent of financial and housing wealth)					
Poor	59	20	19	20	21
Fair	50	33	39	44	45
Good	60	51	56	58	60
Very good	59	57	53	53	55
Excellent	56	58	48	48	50
Panel F: Housing (percent of financial and housing wealth)					
Poor	70	70	55	47	47
Fair	74	64	50	44	42
Good	100	67	52	46	44
Very good	100	71	55	51	49
Excellent	86	72	57	53	53

Note: The solution of a life-cycle model, in which the household can borrow up to 60 percent of home value, is used to simulate a cross section of 100,000 households starting at age 65. The table reports the mean of the relevant variable across the population of households that remain alive at the given age. The parameters of the life-cycle model are the same as those for the benchmark model in Table 8, except for $\lambda = 0.6$.

Table 12—Welfare Analysis of Relaxing Portfolio Constraints in the Life-Cycle Model

Health status	Relaxed borrowing constraints	Private annuitization
Poor	4	11
Fair	5	10
Good	5	16
Very good	7	20
Excellent	7	17

Note: Welfare gain is expressed as the share of financial and housing wealth at age 65 in the benchmark model. The borrowing limit is 60 percent of home value in the life-cycle model with relaxed borrowing constraints on home equity. The average annuity return is 2.5 percent in the life-cycle model with private annuitization.

Table 13—Health Expenditure and Wealth in a Life-Cycle Model with Private Annuitization

Health status	Age				
	65	71	77	83	89
Panel A: Out-of-pocket health expenditure (percent of income)					
Poor	76	69	77	81	96
Fair	19	30	35	36	41
Good	5	11	14	14	15
Very good	1	3	5	5	5
Excellent	0	1	3	3	3
Panel B: Health distribution (percent of households)					
Poor	7	7	7	8	10
Fair	20	18	21	23	26
Good	33	36	37	39	39
Very good	29	33	29	26	23
Excellent	10	7	5	4	2
Panel C: Financial and housing wealth (percent of total wealth)					
Poor	15	27	38	49	61
Fair	18	29	39	51	63
Good	19	30	40	52	64
Very good	20	30	40	52	64
Excellent	22	30	41	52	64
Panel D: Bonds (percent of financial and housing wealth)					
Poor	-3	-2	-2	-1	0
Fair	-9	-5	-3	-3	-1
Good	-16	-8	-4	-4	-3
Very good	-15	-10	-5	-4	-3
Excellent	-13	-12	-6	-4	-3
Panel E: Stocks (percent of financial and housing wealth)					
Poor	18	4	1	0	0
Fair	26	3	1	0	0
Good	34	4	0	0	0
Very good	30	6	0	0	0
Excellent	33	9	0	0	0
Panel F: Housing (percent of financial and housing wealth)					
Poor	53	39	23	16	10
Fair	63	43	24	20	13
Good	82	50	27	20	16
Very good	84	57	33	20	16
Excellent	79	62	37	20	16
Panel G: Annuities (percent of financial and housing wealth)					
Poor	32	59	78	84	89
Fair	20	59	78	83	88
Good	0	54	77	84	87
Very good	1	47	72	84	87
Excellent	1	41	69	84	87

Note: The solution of a life-cycle model, in which the household can save in annuities at an average return of 2.5 percent, is used to simulate a cross section of 100,000 households starting at age 65. The table reports the mean of the relevant variable across the population of households that remain alive at the given age. The parameters of the life-cycle model are the same as those for the benchmark model in Table 8.

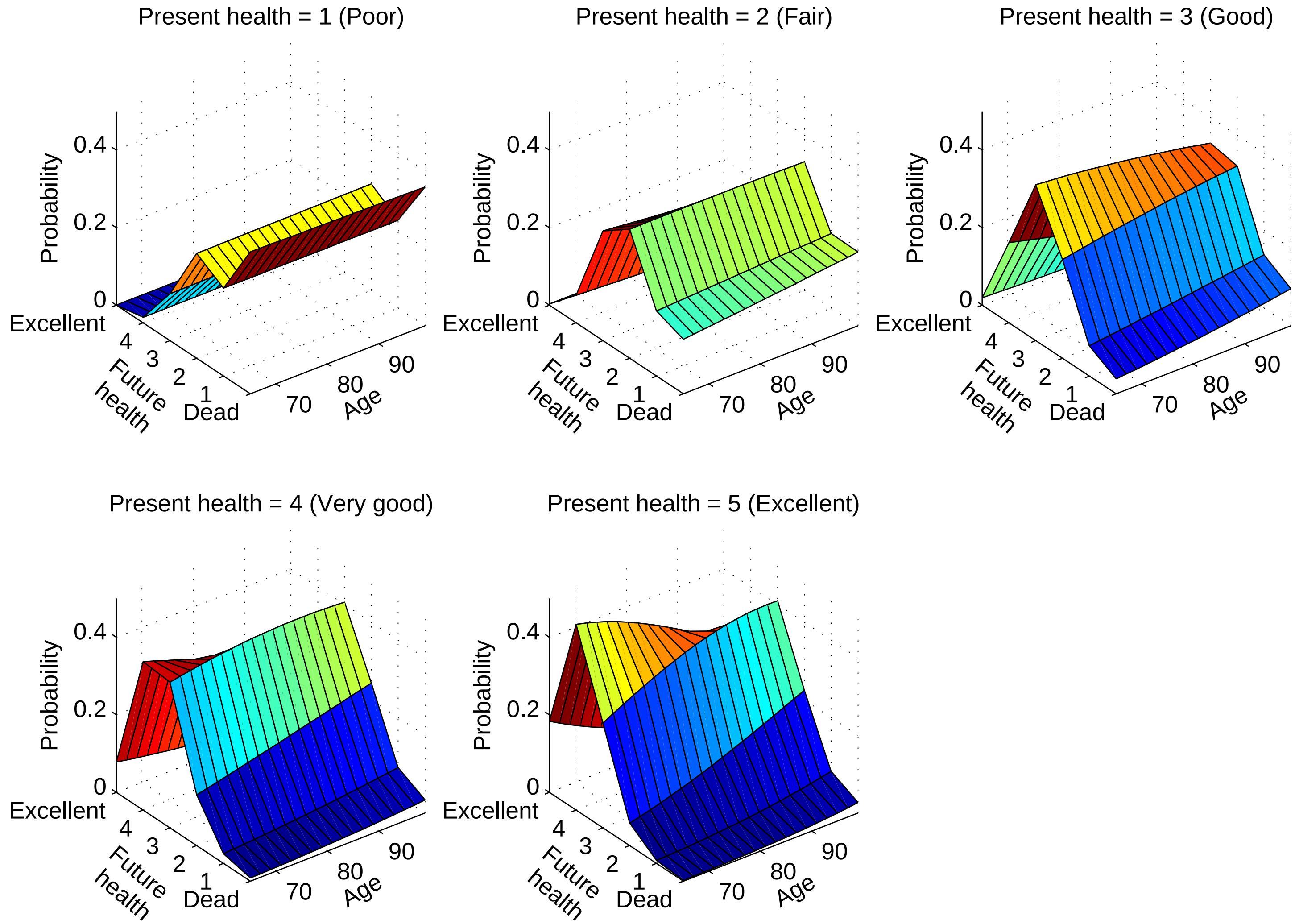

Figure 1. Health Transition Probabilities in the Absence of Health Investment

Note: This figure reports the predicted probabilities from the ordered probit model in column (1) of Table 2. The predicted probabilities are for single females who were born 1931–1940, have the average total wealth for her cohort and age, have not used health care in the two years prior to the interview, and does not regularly participate in vigorous physical activity and smokes at the time of interview.

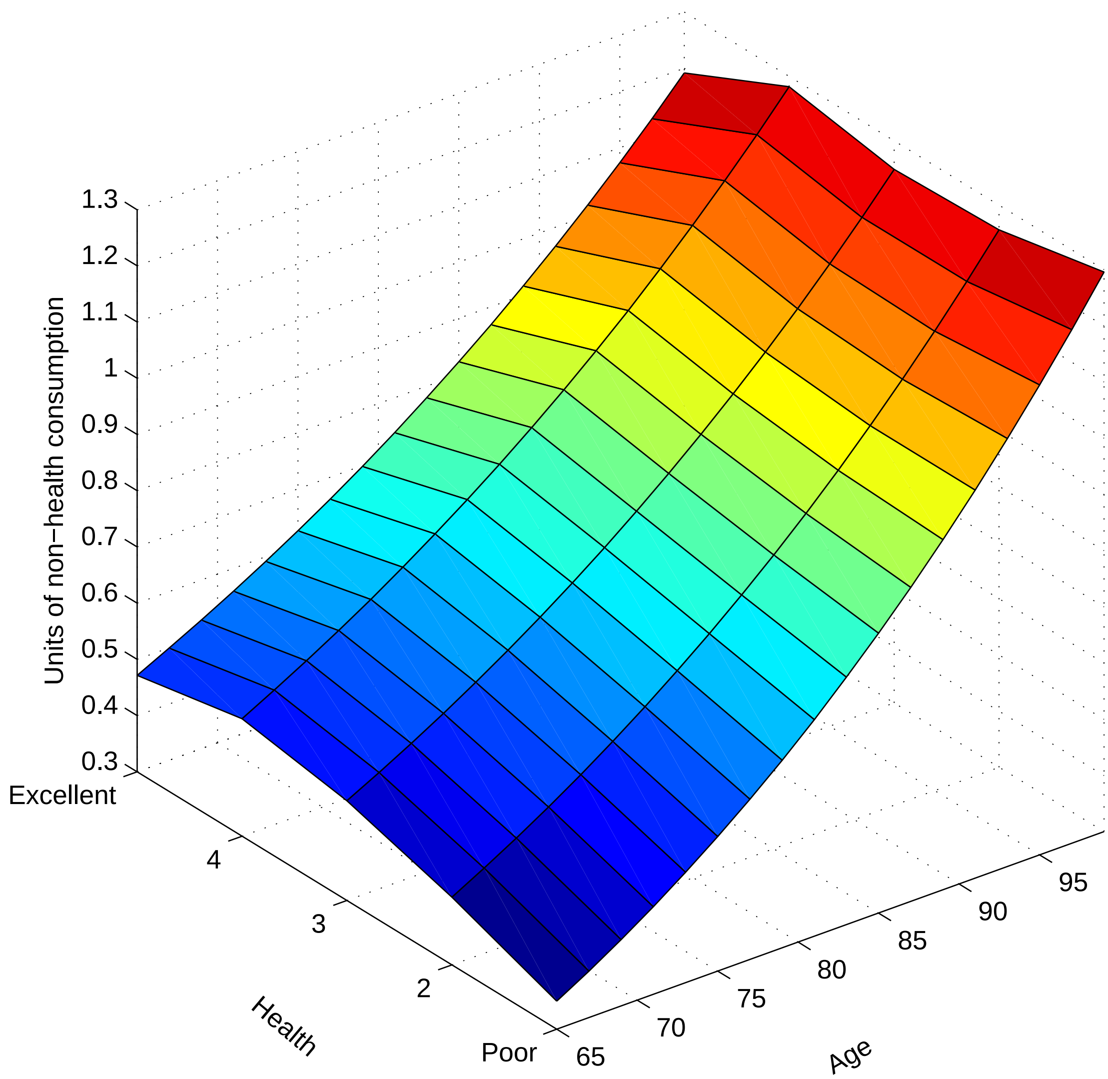

Figure 2. Relative Price of Health Care

Note: This figure reports the relative price of health care, calculated through equation (33) with $q = 0.019$. A censored regression model is used to estimate how the out-of-pocket expenditure share depends on health status, age and its interaction with health status, total wealth and its interaction with health status, marital status, and birth cohort. The predicted values for single females, who were born 1931–1940 and have the average total wealth for her cohort and age, are used to construct $q_t(H_t^*)$.

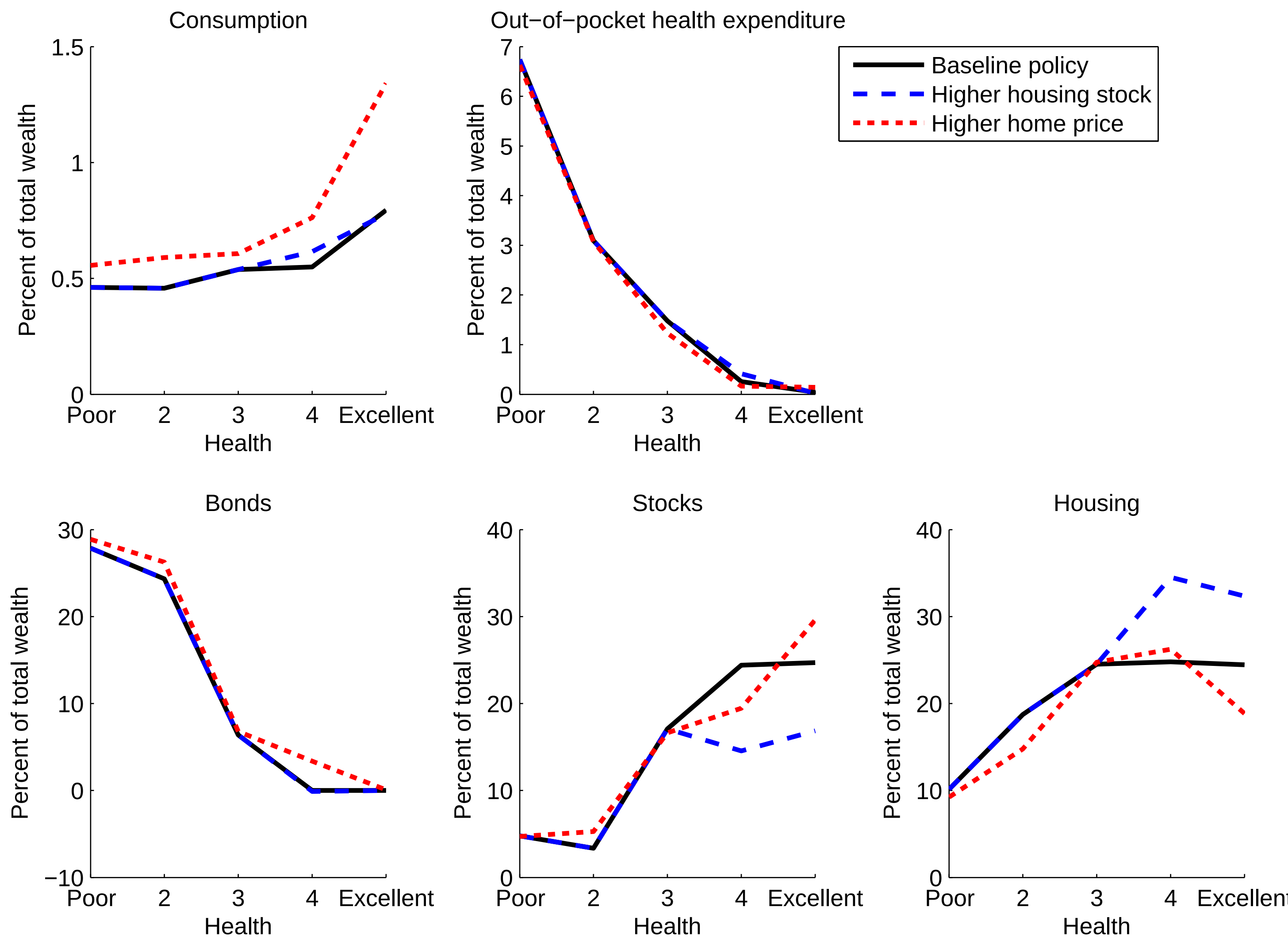

Figure 3. Optimal Consumption and Portfolio Policies in the Benchmark Model

Note: This figure reports the optimal consumption and portfolio policies (i.e., $\widehat{C}_t$, $\widehat{I}_t$, $\widehat{A}_{1,t}$, $\widehat{A}_{2,t}$, and $\widehat{A}_{D,t}$) at age 65, as functions of the health state $\widehat{H}_t$. The baseline policy is evaluated at $\widehat{B}_{3,t} = 0.5$, $\widehat{D}_t = 0.5$, and $P_t = 1.0$. The optimal policy for higher housing stock is evaluated at $\widehat{B}_{3,t} = 0.5$, $\widehat{D}_t = 0.9$, and $P_t = 1.0$. The optimal policy for higher home price is evaluated at $\widehat{B}_{3,t} = 0.5$, $\widehat{D}_t = 0.5$, and $P_t = 1.5$.

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A. Definition of Variables Using the Health and Retirement Study

Most of the variables are based on the RAND HRS (Version I), which is produced by the RAND Center for the Study of Aging with funding from the National Institute on Aging and the Social Security Administration. Out-of-pocket health expenditure is the sum of out-of-pocket health expenditure from the RAND HRS and payments of health insurance premiums. Out-of-pocket health expenditure from the RAND HRS is the total amount paid for hospitals, nursing homes, doctor visits, dentist visits, outpatient surgery, prescription drugs, home health care, and special facilities. Payments of health insurance premiums are the sum of premiums paid for Medicare/Medicaid HMO, private health insurance, long-term care insurance, and prescription drug coverage (i.e., Medicare Part D). I convert the premium reported at monthly, quarterly, semi-annual, or annual frequency to the total implied payment over two years.

Retirement income is the sum of labor income, employer pension and annuity income, Social Security disability and supplemental security income, Social Security retirement income, and unemployment or workers compensation. I calculate after-tax income by subtracting federal income tax liabilities, estimated through the NBER TAXSIM program (Version 9). I calculate the present value of retirement income as after-tax income times the price of a real annuity, matched to the female respondent by birth cohort and age. I calculate the price of a real annuity through equation (10), using the survival probabilities for females in the Social Security life tables (Bell and Miller 2005, Table 7) and a riskless interest rate of 2.5 percent.

Bonds consist of checking, savings, and money market accounts; CD, government savings bonds, and T-bills; bonds and bond funds; and an imputed value of bonds in IRA and Keogh accounts. Because the portfolio allocation in IRA and Keogh accounts is not known, I impute the portfolio share in bonds for each household to be the same as that in non-retirement accounts. I subtract the value of liabilities from the value of bonds. Liabilities consist all mortgages for primary and secondary residence, other home loans for primary residence, and other debt. Stocks consist of businesses; stocks, mutual funds, and investment trusts; and an imputed value of stocks in IRA and Keogh accounts. Housing consists of primary and secondary residence.

B. Numerical Solution of the Consumption and Portfolio-Choice Problem

I discretize health $\widehat{H}_t$ into five grid points, spaced to match the lognormal distribution for health at age 65. I discretize retirement income $\widehat{B}_{3,t}$ into nine grid points, equally spaced between 0.1 and 0.9. I discretize the housing stock $\widehat{D}_t$ into nine grid points, equally spaced between 0.1 and 0.9. I discretize the home price P_t into five grid points, equally spaced on a logarithmic scale between 1 and 1.5. I calculate the transition probabilities between these five grid points to match the moments for housing returns. Finally, I discretize the lognormal shock for stock returns $\nu_{2,t}$ into five grid points, spaced so that each grid point realizes with equal probability. I chose the fineness of the state space after some experimentation to minimize computation time without sacrificing accuracy.

Because the household dies with certainty in period $t = 28$ (i.e., age 119), its value function in that period is given by equation (24). For each period $t < 28$ and at each grid point in the state space, I solve the problem recursively through the following algorithm.

1) Suppose that paying the transaction cost to change the housing stock is optimal (i.e., $\widehat{A}_{D,t} \neq \widehat{D}_t$). Find the policies $\widehat{C}_t$, $\widehat{I}_t$, and $\widehat{A}_{n,t}$ $(n = 2, 3, D)$ that maximizes the objective function, using numerical interpolation to evaluate the value function in period $t + 1$.

2) If $\widehat{A}_{3,t} + (1 - \lambda)\widehat{A}_{D,t} \geq 1$, the policies from step 1 must be optimal because the household must reduce the housing stock to satisfy the budget constraint. Otherwise, proceed to step 3.

3) Suppose that avoiding the transaction cost by keeping the present housing stock is optimal (i.e., $\widehat{A}_{D,t} = \widehat{D}_t$). Find the policies $\widehat{C}_t$, $\widehat{I}_t$, and $\widehat{A}_{n,t}$ $(n = 2, 3)$ that maximizes the objective function, using numerical interpolation to evaluate the value function in period $t + 1$.

4) Compare the value of the objective function achieved in steps 1 and 3. The policy that

achieves the higher value is the optimal policy.

The use of analytical partial derivatives of the objective function makes the numerical optimization routine faster and more accurate than it would otherwise be. The partial derivative of the objective function with respect to consumption is

$$\text{(B1)}\quad \begin{aligned}\frac{\partial \widehat{J}_t}{\partial \widehat{C}_t} =& \widehat{J}_t^{1/\sigma}\Bigg\{(1-\beta)\widehat{U}_t^{-1/\sigma}\frac{\partial \widehat{U}_t}{\partial \widehat{C}_t}\\ &-\beta \mathbf{E}_t\left[\Delta W_{t+1}^{1-\gamma}\left(1_{\{\omega_{t+1}\neq 1\}}\widehat{J}_{t+1}^{1-\gamma}+1_{\{\omega_{t+1}=1\}}\widehat{G}_{t+1}^{1-\gamma}\right)\right]^{(\gamma-1/\sigma)/(1-\gamma)}\\ &\times \mathbf{E}_t\left[\Delta W_{t+1}^{-\gamma}R_{1,t+1}\left(1_{\{\omega_{t+1}\neq 1\}}\widehat{J}_{t+1}+1_{\{\omega_{t+1}=1\}}\widehat{G}_{t+1}^{1-\gamma}\right)\right]\Bigg\},\end{aligned}$$

where

$$\text{(B2)}\quad \frac{\partial \widehat{U}_t}{\partial \widehat{C}_t} = (1-\alpha)(1-\phi)V_t^{1/\rho}\left(\frac{\widehat{A}_{D,t}}{P_t\widehat{C}_t}\right)^{\phi(1-1/\rho)}.$$

The partial derivative of the objective function with respect to health expenditure is

$$\text{(B3)}\quad \begin{aligned}\frac{\partial \widehat{J}_t}{\partial \widehat{I}_t} =& \widehat{J}_t^{1/\sigma}\Bigg\{(1-\beta)\widehat{U}_t^{-1/\sigma}\frac{\partial \widehat{U}_t}{\partial \widehat{I}_t}\\ &-\beta \mathbf{E}_t\left[\Delta W_{t+1}^{1-\gamma}\left(1_{\{\omega_{t+1}\neq 1\}}\widehat{J}_{t+1}^{1-\gamma}+1_{\{\omega_{t+1}=1\}}\widehat{G}_{t+1}^{1-\gamma}\right)\right]^{(\gamma-1/\sigma)/(1-\gamma)}\\ &\times \mathbf{E}_t\left[\Delta W_{t+1}^{-\gamma}R_{1,t+1}\left(1_{\{\omega_{t+1}\neq 1\}}\widehat{J}_{t+1}+1_{\{\omega_{t+1}=1\}}\widehat{G}_{t+1}^{1-\gamma}\right)\right]\Bigg\},\end{aligned}$$

where

$$\text{(B4)}\quad \frac{\partial \widehat{U}_t}{\partial \widehat{I}_t} = \frac{\alpha\psi^2\widehat{C}_tV_t^{1/\rho}}{\widehat{H}_t^{\psi}\widehat{I}_t^{1-\psi}+\psi\widehat{I}_t}\left(\frac{\widehat{H}_t\left[1+\psi\left(\widehat{I}_t/\widehat{H}_t\right)^{\psi}\right]}{Q_t\widehat{C}_t}\right)^{1-1/\rho}.$$

The partial derivative of the objective function with respect to savings in asset $n = 2, 3$ is

$$\text{(B5)}\qquad \frac{\partial \widehat{J}_t}{\partial \widehat{A}_{n,t}} = \widehat{J}_t^{1/\sigma} \beta \mathbf{E}_t \left[\Delta W_{t+1}^{1-\gamma} \left(1_{\{\omega_{t+1} \neq 1\}} \widehat{J}_{t+1}^{1-\gamma} + 1_{\{\omega_{t+1} = 1\}} \widehat{G}_{t+1}^{1-\gamma} \right) \right]^{(\gamma - 1/\sigma)/(1-\gamma)} \times \mathbf{E}_t \left[\Delta W_{t+1}^{-\gamma} (R_{n,t+1} - R_{1,t+1}) \left(1_{\{\omega_{t+1} \neq 1\}} \widehat{J}_{t+1} + 1_{\{\omega_{t+1} = 1\}} \widehat{G}_{t+1}^{1-\gamma} \right) \right].$$

Finally, the partial derivative of the objective function with respect to savings in housing wealth is

$$\text{(B6)}\qquad \frac{\partial \widehat{J}_t}{\partial \widehat{A}_{D,t}} = \widehat{J}_t^{1/\sigma} \Bigg\{ (1-\beta) \widehat{U}_t^{-1/\sigma} \frac{\partial \widehat{U}_t}{\partial \widehat{A}_{D,t}} + \beta \mathbf{E}_t \left[\Delta W_{t+1}^{1-\gamma} \left(1_{\{\omega_{t+1} \neq 1\}} \widehat{J}_{t+1}^{1-\gamma} + 1_{\{\omega_{t+1} = 1\}} \widehat{G}_{t+1}^{1-\gamma} \right) \right]^{(\gamma - 1/\sigma)/(1-\gamma)} \times \mathbf{E}_t \left[\Delta W_{t+1}^{-\gamma} (1-\tau)(R_{D,t+1} - R_{1,t+1}) \left(1_{\{\omega_{t+1} \neq 1\}} \widehat{J}_{t+1} + 1_{\{\omega_{t+1} = 1\}} \widehat{G}_{t+1}^{1-\gamma} \right) \right] \Bigg\},$$

where

$$\text{(B7)}\qquad \frac{\partial \widehat{U}_t}{\partial \widehat{A}_{D,t}} = \frac{(1-\alpha)\phi \widehat{C}_t V_t^{1/\rho}}{\widehat{A}_{D,t}} \left(\frac{\widehat{A}_{D,t}}{P_t \widehat{C}_t} \right)^{\phi(1-1/\rho)}.$$