Party platforms in electoral competition with heterogeneous constituencies

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This paper shows how political parties differentiate to reduce electoral competition. Two parties choose platforms in a unidimensional policy space, and then candidates from these parties compete for votes in a continuum of constituencies with different median voters. Departing from their parties' platforms is costly enough that candidates do not take the median voter's preferred position in every constituency. Because the candidate whose party is located closer to the median voter gets a higher expected payoff, parties acting in their candidates' best interests differentiate—when one party locates right of center, the other prefers to locate strictly left of center to carve out a "home turf," constituencies that can be won with little to no deviation from the platform of the candidate's party. Hence, competition that pulls candidates together pushes parties apart. Decreasing "campaign costs" increases party differentiation as the leftist party must move further from the rightist party to carve out its home turf, as does increasing heterogeneity across constituencies.

KEYWORDS. Political parties, median voter, Hotelling competition.

JEL CLASSIFICATION. C72.

1. INTRODUCTION

Competition among political parties is surprisingly lackluster. A significant fraction of seats in the American House of Representatives go uncontested by one of the two major parties—Democrats or Republicans—each electoral cycle (seven percent of the seats in the 2006 elections). Moreover, elections that are contested by both major parties tend not to be close: in only twelve percent of the 2006 elections was the winning margin less than ten percent (New York Times 2006). Why is political competition so uncompetitive?

This paper shows how political competition drives parties apart. Perhaps the most famous result in formal political theory is that two candidates competing in a single first-past-the-post election for political office should jointly adopt the median voter's preferred position (Hotelling 1929, Downs 1957, Black 1958). We use the Hotelling–Downs model of electoral competition as a point of departure to explore party positioning. What types of platforms do political parties acting in their candidates' best interests espouse? We show that parties separate precisely because when party platforms are identical candidates compete so vigorously. They differentiate to reduce political competition.

We model a political party as a collection of candidates, each campaigning in a different constituency. In each constituency, candidates from two parties compete in a first-past-the-post election. Voters in every constituency have single-peaked preferences over the same unidimensional policy space; different constituencies have median voters with different ideal points. Voters in each constituency care only about their elected candidate's position and not the platform of that candidate's party. Knowing the distribution of the median voters' ideal points, the two political parties choose where to place their platforms in policy space. A party's platform serves as its candidates' default policy position. In any constituency, either candidate may depart from her party's position, but at a cost: the further her position from her party platform, the higher the candidate's cost. The most natural interpretation of these costs is campaign costs—informing voters of a position different from the party's requires costly advertising. (For expositional simplicity, we refer to these costs of departing from party platform as campaign costs throughout.) Candidates trade campaign costs off against the private benefit of winning elections.

In this setting, we ask where political parties seeking to maximize their candidates' payoffs choose to locate their platforms. Our main result is that if campaign costs are high enough that candidates do not adopt the median voter's preferred position in each constituency, then parties do not jointly adopt the median among the median voters' preferred positions; they differentiate from each other. This happens because the closer the two parties locate to each other, the more vigorously their candidates compete to win election in any given constituency. Consequently, each has an incentive to move away from the other—giving up heavily contested elections—in order to carve out politically sympathetic constituencies where it wins elections without much costly repositioning. While competition may drive candidates together, it drives parties apart.

Political parties provide their candidates with funds and organizational infrastructure as well as signaling their candidates' policy positions. Candidates deviate from party policy to cater to their constituents—Republicans in Maryland espouse more liberal positions than their colleagues in Virginia—yet clearly do not go so far as to adopt the median voter's preferred position. Parties systematically lose elections in politically unfavorable constituencies. Ansolabehere et al. (2001) present evidence that Republican House candidates in 1996 were without exception more conservative than their Democratic challengers and lost in more liberal constituencies. In particular, candidates do not espouse the preferred position of their median constituent. We model this

party effect in a simple way: candidates pay campaign costs to deviate from their party platform.

Candidates' costs of deviating from party policy play a crucial role in our analysis. We regard these costs as a reduced form of the many reasons why candidates may wish to mimic their parties. Most literally, advertising or publicizing a new policy may be costly (buying television spots, etc.), and the further is a candidate's position from her party's, the more expensive it might be to convince voters of the candidate's actual position. These costs also could represent unpalatable payments or promises to special interests necessary to finance the publicizing of a departure from party policy. Political action committees—pressure groups—provide forty percent of funding for U.S. Congressional elections (Herrnson 1997). Because the formal model does not depend on whether parties or candidates pay these costs, an alternative interpretation is that candidates have policy preferences and parties find it harder to enlist candidates with preferences further from their platforms. For the results, what matters most is that the overall game not be zero sum: given the number of elections they win, parties prefer that their candidates adhere to the party platform.

Strictly speaking, in our model candidates prefer not to belong to either party, so as to be able to adopt any position without cost. But for any number of reasons outside our formal model, candidates benefit from party membership. Parties may reduce the costs of elections by sharing fixed costs across candidates. They may also enjoy legal privileges benefiting their candidates: party candidates automatically appear on the ballot in many elections, whereas unattached candidates must submit petitions signed by enough voters. Likewise, we ignore candidates' party assignments. In our model, both candidates prefer to belong to the party closer to their constituency's median voter. Moreover, the candidate belonging to the party further away receives zero expected payoff. Yet if candidates enjoy other benefits from campaigning, and there are enough potential candidates, neither party will have trouble fielding a candidate. We believe that we sacrifice little in realism or applicability by assuming that for exogenous reasons each constituency has an election comprising one candidate from each of two parties.

Finally, our assumption that parties seek to maximize the sum of their candidates' payoffs means that they trade off the total number of seats they win against the cost of winning them. In particular, parties care about more than winning a majority. After seeing its Parliamentary majority slump from 165 in 2001 to 66 in the 2005 election, the Labour party recently lost its first vote under Tony Blair. The U.S. Republican party uses a strategy of "catch and release"; first, it whips ("catches") party members so as to guarantee a majority on controversial legislation; second, it "releases" members running in elections predicted to be close to vote against the party (Hacker and Pierson 2005). Both demonstrate the value of super-majorities. Likewise, parties benefit from having larger submajorities. For example, a submajority of 41 in the 100-member U.S. Senate can prevent the majority from closing debate. Naturally some seats have more value than others, e.g. the 51st in the U.S. Senate; for analytical tractability, we abstract from this issue by assuming that all seats are equally valuable. Since parties also like money (which, among other things, helps them to win future elections), they trade winning seats against the cost of winning them.

Section 2 presents our model. In Section 3, we analyze candidate competition in a given constituency, taking party platforms as given. In our model, in any constituency, the candidate whose party platform lies closer to the median voter has an advantage and is more likely to win. In some constituencies, both candidates toe the party line, and the advantaged candidate wins the election with probability one. In others, candidates mix over policy positions. In these constituencies, the advantaged candidate sometimes loses but always wins with higher likelihood than the disadvantaged candidate. Importantly, when a candidate's advantage increases—her party is located closer to her constituency's median voter—her expected payoff increases: she can win the election with any given probability at lower cost. This effect provides parties with an incentive to move away from the center.

Section 4 analyzes parties that simultaneously choose platforms in the best interests of their members. When moving away from the center (the median among median voters) parties trade off the expected number of seats their candidates win against the campaign costs their candidates pay. Moving right causes the rightist party to lose elections in the center but allows it to carve out a home turf on the right, where elections can be won at little or no cost. Because constituencies with centrist median voters are heavily contested, they can be won only at considerable cost.[1] Each party's incentive to win any particular constituency depends not upon the value of election alone but rather upon its value net of campaign costs.

When campaign costs are high—candidates only reluctantly deviate from their parties' platforms—parties position themselves near the center of the policy spectrum but sufficiently far apart that candidates from the leftist party do not contest constituencies whose median voter lies to the right of the rightist party, and vice versa. Candidates only ever depart from their parties' positions to locate at the center. Consequently, the leftist party's position lies (weakly) to the left of all leftist candidates' positions, and likewise for the rightist party. With high campaign costs, each party adopts a platform more extreme than its most extreme candidate.

For lower campaign costs, parties locate further apart. Knowing that candidates from the other party will compete more intensely in every constituency, each party must separate more from the other in order to carve out its home turf. But once costs become low enough that candidates adopt the median voter's position in every constituency, party platforms go back to the median among median voters, which minimizes candidates' campaign costs.

Section 5 discusses how our results are robust to changes in our assumption about the distribution of voter preferences across constituencies. Section 6 concludes.

Literature

Several authors explore the role of parties in electoral competition. Austen-Smith (1984) develops a model where in each of many constituencies two parties field candidates in

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[1]Indeed, Ansolabehere et al. (2001) find the gap in candidates' policy positions is smallest in Congressional districts whose voters split fifty-fifty in the vote for President. This corresponds to an equilibrium in our model where in contested constituencies candidates must depart from their party platforms.

first-past-the-post elections. Voters recognize that government policy depends upon which party wins and the location of its candidates. Candidates seek to maximize vote shares. Austen-Smith shows that in a coalition-proof Nash equilibrium of the game, candidates position to set their party's policy at the median voter's bliss point. A crucial difference between his paper and ours is that his voters care about the location of the national party—turning party competition into Downsian competition—whereas ours care about the location of the local candidate. In that sense, his model may better resemble the British system of parliamentary democracy, where MPs have only loose ties to their constituencies, and ours the U.S. congressional system, where congress members have much stronger ties to their constituencies. Yet we suspect that voters in both systems pay careful attention to their local candidates' positions.[2] Certainly our model better reflects U.S. gubernatorial elections, where voters care only about their own governors.

Snyder (1994) also models two parties competing across a finite number of heterogeneous constituencies. Party incumbents in the different constituencies set party platform through majoritarian voting; they have lexicographic preferences over winning and the number of seats won by their party. In each constituency voters elect the politician whose party locates closer to its median bliss point and in the event of a tie randomize with equal probability. Snyder shows that in equilibrium parties differentiate, dividing the political spectrum into left and right, where leftist candidates win in leftist constituencies and only in such constituencies. One way that our model differs from Snyder's is in the size of differentiation: in his model, ordering constituencies from left to right by their median bliss points, both parties locate between the medians in some constituencies n and $n+1$. Another difference is the number of equilibria, as the cutoff n in his model is arbitrary. At a more conceptual level, Snyder's differentiation hinges on a conflict of interest within the party; non-incumbents would like to compensate the incumbents to move the party platform to the center, but cannot. By contrast, in our model the party maximizes the total payoff of its candidates.

Levy (2004) models parties in the citizen-candidate model of Osborne and Slivinski (1996) and Besley and Coate (1997) as being able to credibly commit to positions, whereas candidates cannot do so individually. In her model, parties are effective only in a multidimensional policy space, as they work by allowing groups to exploit gains from trade from different preferences across issues. Rather than view parties as alliances among autonomous candidates, we model them as the primary drivers of political competition.

Snyder and Ting (2002) model parties as brand names. Voters have no information about candidates' positions other than their party membership (or lack thereof) and have preferences that depend upon the mean and variance of their beliefs about the candidates' locations. They like candidates whose positions they expect to be near their

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[2]A growing psychology and economics literature describes how people use "narrow brackets," focusing on each decision in isolation without considering its global implications (see, inter alia, Kahneman and Tversky 1979, and Read et al. 1999). Voters using such "narrow brackets" pay more attention to their local candidate's position than fully rational voters and may be more likely to fall prey to the "catch and release" strategy described in the previous section.

own but dislike variance in their beliefs. Snyder and Ting show that when parties can reduce the variance of their members' positions by choosing extreme positions, then in equilibrium parties may prefer to locate at the extremes.[3]

A number of authors have modified the Hotelling–Downs model of electoral competition in ways that produce differentiated candidates. Wittman (1983) and Calvert (1985) show that candidates may not converge when they care both about winning the election and about the position of the winner. Banks (1990) models competition between two candidates who find departing from their ideal positions costly. He characterizes a semi-separating equilibrium where candidates with ideal points near the median voter adopt the median position, while those with more extreme positions separate. Hence centrist candidates pay costs to depart from their ideal points; in our model, candidates who likewise pay costs to differentiate from their parties do not always adopt the median voter's position. Palfrey (1984) demonstrates that when two established candidates choose positions before a third candidate enters the race, the established candidates differentiate to eliminate profitable entry opportunities. Callander (2005) extends Palfrey's framework to model parties competing across heterogeneous constituencies, where established national parties choose positions before local candidates compete in each constituency. Callander finds equilibria where parties differentiate even when challengers may choose not to enter, unlike in Palfrey's model.[4] Chan (2001), Heidhues and Lagerlöf (2003), and Bernhardt et al. (2007) construct models where candidates separate due to asymmetric information about voters' preferences. Bernhardt and Ingberman (1985) model an incumbent with a reputation facing a challenger who cannot reveal his position with certainty. When voters dislike risk, the incumbent need not move to the median voter to defeat the challenger; hence, candidates differentiate. With the exception of Palfrey (1984) and Callander (2005), all of these models bear more resemblance to our model of candidate competition than our model of party competition. In our model, *candidates* differentiate because they start from different party platforms, which produces an effect similar to the asymmetric information or heterogeneous preferences in these other papers. (Our candidates differentiate by using different (possibly mixed) strategies. In the context of price competition, Bester et al. (1996) have noted that common mixed strategies produce *ex post* differentiation.) By contrast, in our model *parties* differentiate despite symmetric starting points.

The intuition underlying our main result more closely resembles a literature in industrial organization on price competition between duopolists. Hotelling (1929) analyzes a model of two firms' choosing spatial locations knowing that consumers face transport costs. When prices are fixed, he shows that firms locate at the same position. D'Aspremont et al. (1979) show that when spatial location precedes price competition, and transport costs are quadratic, firms exhibit maximal differentiation. While firms

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[3]Of course, if moving away from the center increased variance—if extremist parties were more heterogeneous rather than more homogeneous as in Snyder and Ting (2002)—then parties in equilibrium would locate at the center.

[4]In Callander's model, parties may separate by more than the distance between median voters in the leftmost and right-most constituencies, something they never do in our model (even if we relax an assumption that prevents it in our formal model).

wish to move together for given prices, they wish to separate to gain market power in order to put up prices; this latter effect dominates. Costly relocation in our model plays a role similar to price competition in their model. Konrad (2000) makes a related point in a model where firms choose locations before competing in an all-pay auction for the right to sell a good to a customer whose location is initially unknown; the winning firm pays the cost of transportation to the consumer. Such all-pay auctions with heterogeneous, public valuations have been analyzed by Baye et al. (1996). Konrad (2000) shows that firms differentiate so as to minimize industry transport costs. One key difference from our paper is that in political competition, no candidate moves further than the median voter. This resembles a bid cap in an all-pay auction, which generates equilibria in the second stage of our game qualitatively different from those of Baye et al. (1996).

2. THE MODEL

Two parties A and B compete in elections across a continuum of heterogeneous constituencies. Each constituency is identified by its median voter's bliss point δ—which we assume to be unique—and we assume that the distribution of δ across constituencies is uniform on $[0,1]$.[5] The election game comprises two stages. In the first stage (Stage 1), the parties simultaneously choose platforms in the policy space $[0,1]$. After Stage 1, but before Stage 2, everyone observes the two parties' platform choices. In Stage 2, in each constituency $\delta \in [0,1]$, candidates from the two parties, A_δ and B_δ, respectively, compete in a first-past-the-post election. In each constituency δ, knowing the median voter's location δ, the two candidates simultaneously choose positions in $[0,1]$. A candidate may take any position she wishes in $[0,1]$, but advocating a position different from her party's platform has a cost: if Party A chooses platform a in Stage 1, then in any constituency δ, candidate A_δ pays $|\alpha - a|$ to take the position α, and likewise for B_δ. The further the candidate locates from her party's platform, the higher the cost; for simplicity the marginal cost is constant and normalized to one for all candidates. In δ the candidate who locates closer to the median voter's bliss point δ wins office; in particular, voters care only about their winning candidate's position and not about the platform of that candidate's party. In case of a tie both candidates win office with probability $\frac{1}{2}$. In every constituency δ, both candidates know δ before choosing their positions.[6] The winner of each election receives the private benefit $2V > 0$ and the loser no benefit. (As long as costs are linear, candidates' behavior depends only on the ratio of V to the marginal cost of positioning; hence, an increase in V can be interpreted also as a decrease in that marginal cost.) Candidates' payoffs as a function of parties' positions, their constituency, and their own policies α and β, respectively, are given by

$$U_{A_\delta}(a,b,\alpha,\beta) = \begin{cases} 2V - |\alpha - a| & \text{if } |\alpha - \delta| < |\beta - \delta| \\ V - |\alpha - a| & \text{if } |\alpha - \delta| = |\beta - \delta| \\ -|\alpha - a| & \text{if } |\alpha - \delta| > |\beta - \delta| \end{cases}$$

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[5]Section 5 relaxes this assumption.

[6]The formal model can equally well be interpreted as one with a single constituency, where parties share the common prior that the median voter is uniformly distributed on $[0,1]$.

$$U_{B_\delta}(a,b,\alpha,\beta) = \begin{cases} 2V - |\beta - b| & \text{if } |\beta - \delta| < |\alpha - \delta| \\ V - |\beta - b| & \text{if } |\beta - \delta| = |\alpha - \delta| \\ -|\beta - b| & \text{if } |\beta - \delta| > |\alpha - \delta|. \end{cases}$$

A candidate's (mixed) strategy maps any given combination of party platforms (a,b) (chosen by the parties in Stage 1) into a distribution over the policy space $[0,1]$. Thus, a strategy for candidate A_δ is given by $F_{A_\delta} : [0,1]^2 \mapsto \Delta[0,1]$, where for any $(a,b) \in [0,1]^2$, $F_{A_\delta}(a,b;\cdot) \in \Delta[0,1]$ is a (cumulative) distribution function on $[0,1]$, and likewise for B_δ. Note that these distributions need not be absolutely continuous with respect to Lebesgue measure; candidates' strategies may contain mass points.

Each party's payoff is the average of its candidates' payoffs across all constituencies, i.e., the payoff of party $i \in \{A, B\}$ as a function of the platforms (a,b) and the candidates' strategies $(F_{A_\delta})_{\delta \in [0,1]}$ and $(F_{B_\delta})_{\delta \in [0,1]}$ is

$$U_i(a,b,(F_{A_\delta})_{\delta \in [0,1]},(F_{B_\delta})_{\delta \in [0,1]})$$
$$= \int_{[0,1]} \int_{[0,1]} \int_{[0,1]} U_{i_\delta}(a,b,\alpha,\beta) \, dF_{A_\delta}(a,b;\alpha) \, dF_{B_\delta}(a,b;\beta) \, d\delta.$$

We analyze subgame-perfect equilibria in which parties play pure strategies (while candidates may play mixed strategies).[7] We adopt the convention that $a \le b$.

DEFINITION 1. The strategy profile $(a^*, b^*, (F^*_{A_\delta})_{\delta \in [0,1]}, (F^*_{B_\delta})_{\delta \in [0,1]})$ is a subgame-perfect equilibrium of the election game if

1. for each $\delta \in [0,1]$ and each pair of platforms (a,b), for each α in the support of the distribution described by $F^*_{A_\delta}(a,b;\cdot)$,

$$\alpha \in \underset{\alpha' \in [0,1]}{\arg\max} \int U_{A_\delta}(a,b,\alpha',\beta) \, dF^*_{B_\delta}(a,b;\beta),$$

   and for each β in the support of the distribution described by $F^*_{B_\delta}(a,b;\cdot)$,

$$\beta \in \underset{\beta' \in [0,1]}{\arg\max} \int U_{B_\delta}(a,b,\alpha,\beta') \, dF^*_{A_\delta}(a,b;\alpha)$$

2.
$$a^* \in \underset{a}{\arg\max} \, U_A(a,b^*,(F^*_{A_\delta})_{\delta \in [0,1]},(F^*_{B_\delta})_{\delta \in [0,1]})$$
$$b^* \in \underset{b}{\arg\max} \, U_B(a^*,b,(F^*_{A_\delta})_{\delta \in [0,1]},(F^*_{B_\delta})_{\delta \in [0,1]}).$$

In the next section we completely characterize the candidates' equilibrium strategies. Section 4 solves for the parties' subgame-perfect equilibrium strategies by backward induction.

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[7]The existence of such equilibria follows from Theorem 1 below. Furthermore, Proposition 1 shows that there do not exist equilibria in which the candidates' strategies are pure.

3. ELECTORAL COMPETITION

This section analyzes electoral competition between candidates, taking the parties' platforms as given. As before denote the platforms of Parties A and B by a and b, respectively. When $|\delta - a| < |\delta - b|$ ($|\delta - a| > |\delta - b|$), Party A is closer to (further from) the median voter, and we refer to Candidate A_δ as being advantaged (disadvantaged). In most of this section we assume that B_δ is not disadvantaged and further that $a \leq b \leq \delta$: the median voter's bliss point lies to the right of both parties. That is, for every $\delta \in [0,1]$ we determine $F_{A_\delta}^*(a,b;\cdot)$ and $F_{B_\delta}^*(a,b;\cdot)$ only for $a \leq b \leq \delta$.[8] Because the winner of the election depends only upon the distances between the two candidates and the voter's bliss point and not upon the distance between the candidates, this also determines $F_{A_\delta}^*$ and $F_{B_\delta}^*$ for the case where the median voter's bliss point lies to the left of one party. Equilibria in the other cases can be easily derived from symmetry properties, and a candidate's payoff given general party platforms can be deduced from the case $a \leq b \leq \delta$ by a transformation of the platforms.[9] We focus on this one case solely to shorten the exposition.

When both parties' platforms are close enough to the median voter's bliss point in constituency δ, then in any equilibrium of the second stage both candidates locate at this bliss point. When $\delta < V + a$, the unique equilibrium is for both candidates to locate at δ. If $\delta > V + a$, then A_δ is unwilling to locate at the median voter's bliss point to win the election with probability one-half; in equilibrium, both candidates cannot choose the median voter's bliss point with probability one. In this case, unless each candidate chooses to locate at her party's platform, the equilibrium must be in mixed strategies. Otherwise, some candidate would pay costs higher than necessary to win with probability zero or one, or each candidate would win with probability one-half yet could win for sure by moving infinitesimally closer to the median voter.

We develop $F_{A_\delta}^*(a,b;\cdot)$ and $F_{B_\delta}^*(a,b;\cdot)$ through a series of lemmata. To state the first lemma, we say that a distribution function F has a gap between x and $y > x$ if $F(x) = \sup_{t<y} F(t)$. All proofs are in the [Appendix](#).

LEMMA 1. *Let $a \leq b \leq \delta$. For each pair of candidates' strategies $((F_{A_\delta}^*)_{\delta \in [0,1]}, (F_{B_\delta}^*)_{\delta \in [0,1]})$ we have the following two properties.*

(i) *If $F_{A_\delta}^*(a,b;\cdot)$ ($F_{B_\delta}^*(a,b;\cdot)$) has a gap between $x \geq b$ and $y < \delta$ then it has a gap between x and δ and $F_{B_\delta}^*(a,b;\cdot)$ ($F_{A_\delta}^*(a,b;\cdot)$) also has a gap between x and δ.*

(ii) *$F_{A_\delta}^*(a,b;\cdot)$ ($F_{B_\delta}^*(a,b;\cdot)$) can have an atom only at A's (B's) party platform or at δ. If*

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[8]One can interpret a Stage-2 subgame as a complete-information, common-value, all-pay auction with a bid cap (the median voter's position) and handicap (Candidate A_δ's disadvantage). We know of no paper that addresses these auctions.

[9]Details can be found in the proof of [Proposition 2](#) in the [Appendix](#). For example, suppose $a \leq \frac{1}{2}(a+b) \leq \delta \leq b$. Because A_δ is disadvantaged and located to the left of δ, her equilibrium strategy is exactly the same as in the case where $a \leq \widetilde{b} \leq \delta$ for $\widetilde{b} = 2\delta - b$ (note that $|\delta - \widetilde{b}| = |\delta - b|$). Candidate B_δ is advantaged and puts the same mass on b and δ as she would put on δ and $\widetilde{b} = 2\delta - b$. She mixes on some (r,b) (with density $1/2V$) if and only if she would mix on $(\widetilde{b}, 2\delta - r)$ if her position were $\widetilde{b}$.

$F^*_{A_\delta}(a, b; \cdot)$ $(F^*_{B_\delta}(a, b; \cdot))$ *has an atom at* δ, *then* $F^*_{B_\delta}(a, b; \cdot)$ $(F^*_{A_\delta}(a, b; \cdot))$ *has an atom at* δ.

This result implies that the candidates' equilibrium strategies $F^*_{A_\delta}(a, b; \cdot)$ and $F^*_{B_\delta}(a, b; \cdot)$ have common support except at the parties' platforms.

LEMMA 2. *Let* $a \leq b \leq \delta$. *In any constituency* δ *with* $\delta > a + V$, *in equilibrium at least one candidate puts mass on her party's platform. Consequently, at least one candidate gets an equilibrium payoff of zero.*

Since $F^*_{A_\delta}(a, b; \cdot)$ and $F^*_{A_\delta}(a, b; \cdot)$ must have common support except at a or b, if neither candidate adopts her party's position, then equilibrium strategies must have common support. But this cannot be unless both adopt the median voter's bliss point with probability one; otherwise, given the equilibrium strategies have no atoms except at the median voter's bliss point, the disadvantaged candidate sometimes pays a positive cost to lose with probability one. In fact, Candidate A_δ must always put mass on a, otherwise B_δ would be unwilling to put mass on b, and we have a contradiction. Now note that if Candidate A_δ randomizes over some interval (u, v), with $b \leq u < v \leq \delta$, then she must be indifferent between locating at $t \in (u, v)$ and $t + \varepsilon \in (u, v)$, meaning that

$$F^*_{B_\delta}(a, b; t) 2V - (t - a) = F^*_{B_\delta}(a, b; t + \varepsilon) 2V - (t + \varepsilon - a)$$

$$\Rightarrow \quad \frac{1}{2V} = \frac{F^*_{B_\delta}(a, b; t + \varepsilon) - F^*_{B_\delta}(a, b; t)}{\varepsilon}.$$

Letting $\varepsilon \to 0$ (and repeating the argument for $t - \varepsilon$) gives a formula for the probability density function, $f^*_{B_\delta}(a, b; t) = 1/2V$ for any $t \in (u, v)$. Since the same argument holds for B_δ, both candidates randomize (over intervals) with the same density function.

We now know the structure of the candidates' equilibrium strategies. If $\delta > a + V$ one or both candidates put mass on their party's position. Candidates then may randomize over (b, r), for some r, with density $1/2V$. Finally candidates may put mass on the median voter's bliss point. If candidates put no mass on the median voter's bliss point, then $r = a + 2V$, the furthest away from a that A_δ is willing to locate to win the election with probability one. If candidates put mass on the median voter's bliss point, then $r < a + 2V$. Finding $F^*_{A_\delta}(a, b; \cdot)$ and $F^*_{B_\delta}(a, b; \cdot)$ consists merely of checking which of these strategy profiles is a mutual best response for each configuration of (a, b, δ).

PROPOSITION 1. *Let* $a \leq b \leq \delta$. *The candidates' equilibrium strategies* $(F^*_{A_\delta}, F^*_{B_\delta})$, *which are unique for* $\delta \neq a + V$, *are characterized as follows.*

(P1) *If* $b - a \geq 2V$, *then*

$$F^*_{A_\delta}(a, b; t) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t \geq a \end{cases}, \quad F^*_{B_\delta}(a, b; t) = \begin{cases} 0 & \text{if } t < b \\ 1 & \text{if } t \geq b \end{cases}$$

i.e. each candidate locates at her party's platform.

Suppose $b - a < 2V$.

(M1) *If $a + 2V \leq \delta$, then*

$$F^*_{A_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < a \\ \frac{b-a}{2V} & \text{if } b \geq t \geq a \\ \min\{\frac{t-a}{2V}, 1\} & \text{if } t > b \end{cases}, \quad F^*_{B_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < b \\ \min\{\frac{t-a}{2V}, 1\} & \text{if } t \geq b \end{cases}$$

i.e. each candidate randomizes over $(b, a + 2V)$ with density $1/2V$ and puts mass on her party's platform.

(M2) *If $\frac{1}{2}(a+b) + V \leq \delta \leq a + 2V$, then*

$$F^*_{A_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < a \\ \frac{b-a}{2V} & \text{if } b \geq t \geq a \\ \min\{\frac{t-a}{2V}, \frac{\delta-a-V}{V}\} & \text{if } \delta > t > b \\ 1 & \text{if } t \geq \delta \end{cases},$$

$$F^*_{B_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < b \\ \min\{\frac{t-a}{2V}, \frac{\delta-a-V}{V}\} & \text{if } \delta > t \geq b \\ 1 & \text{if } t \geq \delta \end{cases}$$

i.e. each candidates randomizes over $(b, 2\delta - a - 2V)$ with density $1/2V$ and puts mass on her party's platform and δ.

(M3) *If $a + V < \delta \leq \frac{1}{2}(a+b) + V$, then*

$$F^*_{A_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < a \\ \frac{V-\delta+b}{V} & \text{if } \delta > t \geq a \\ 1 & \text{if } t \geq \delta \end{cases}, \quad F^*_{B_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < b \\ \frac{\delta-V-a}{V} & \text{if } \delta > t \geq b \\ 1 & \text{if } t \geq \delta \end{cases}$$

i.e. each candidate randomizes between her party's platform and δ.

(M4) *If $\delta = a + V$, then the set of equilibria is the set of strategy pairs for which*

$$F^*_{A_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < a \\ q & \text{if } \delta > t \geq a \\ 1 & \text{if } t \geq \delta \end{cases}, \quad F^*_{B_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < \delta \\ 1 & \text{if } t \geq \delta \end{cases}$$

for some $q \leq (V - \delta + b)/V$; i.e., B_δ locates at δ, and A_δ locates at a with probability q and at δ with probability $1 - q$.

(P2) *If $\delta < a + V$, then*

$$F^*_{A_\delta}(a,b;t) = F^*_{B_\delta}(a,b;t) = \begin{cases} 0 & \text{if } t < \delta \\ 1 & \text{if } t \geq \delta \end{cases}$$

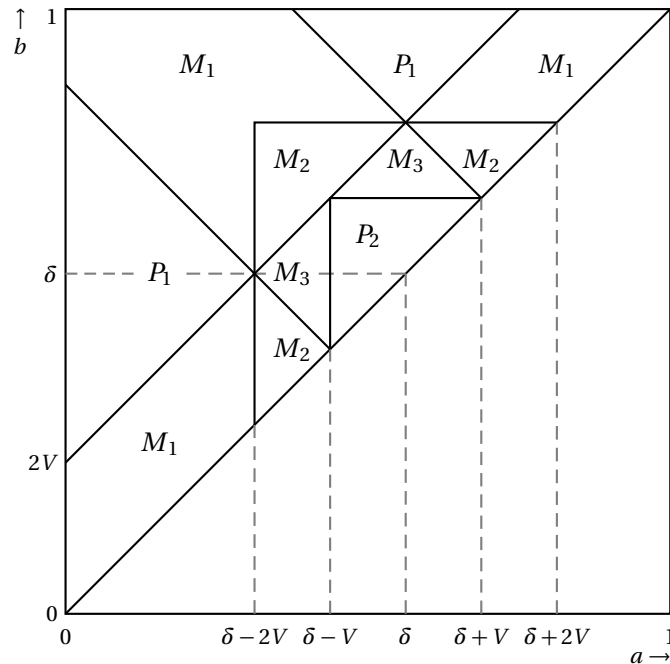*i.e. both candidates locate at δ.*

FIGURE 1. The form of the candidates' strategies across constituencies.

Figure 1 illustrates the form that candidates' equilibrium strategies take in constituency δ as a function of a and b, where $a \leq b$; the case where $a \leq b \leq \delta$ covered in Proposition 1 corresponds to the triangle with vertices $\{(0,0),(0,\delta),(\delta,\delta)\}$. When the distance between A and B is large ($b \geq a + 2V$) both candidates choose their party's platform (P1). For the remaining cases, assume that this condition does not hold. If the distance between A and the median voter's bliss point is large ($\frac{1}{2}(a+b)+V \leq \delta$), then both candidates mix over positions to the right of b: A_δ tries to steal B_δ's election without locating at δ, forcing B_δ to take a position between her party's platform and the median voter's bliss point to fend off A_δ (M1 and M2).[10] When the distance between A and the median voter's bliss point is small, then A_δ finds it more profitable to directly adopt the median voter's position than to attempt to outmaneuver B_δ; in this case, each candidate locates at her party's position with positive probability and locates at δ with the complementary probability (M3). Finally, when A is close enough to the median voter's bliss point ($a + V > \delta$), then both candidates adopt the median voter's position δ (P2). The remainder of Figure 1 shows how these different forms of equilibria apply when $a \leq \delta \leq b$ (and cases where $b \leq a$ can be found by mirroring the regions about the 45-degree line). An essential feature of candidates' equilibrium strategies is that they mix over positions: in many constituencies, neither candidate knows her rival's

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[10]Candidates in this case put mass on the median voter (M2 but not M1) when A is sufficiently close to δ, namely $a + 2V > \delta$.
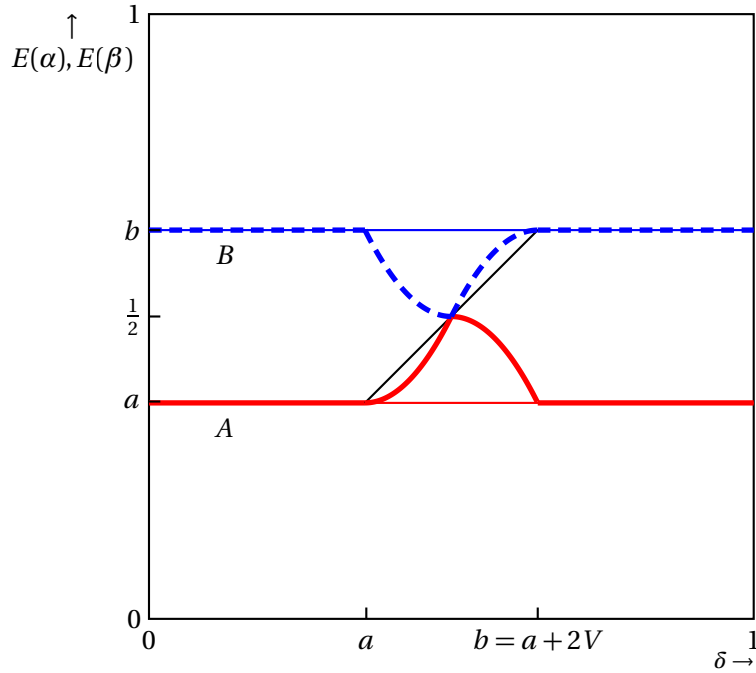
FIGURE 2. The candidates' expected positions across constituencies for $V = \frac{1}{7}$ and $(a,b) = (\frac{5}{14}, \frac{9}{14})$.

equilibrium position when selecting her own position. However, mixing *per se* does not drive our party differentiation result, as discussed in Section 6.

Figure 2 depicts a different facet of the candidates' equilibrium strategies by showing each candidate's expected position in each constituency, using a bold dashed line for B candidates and a bold solid one for A's. It uses the parameter values $V = \frac{1}{7}$ and $(a,b) = (\frac{1}{2} - V, \frac{1}{2} + V) = (\frac{5}{14}, \frac{9}{14})$.[11] For $\delta \notin (a,b)$, each candidate chooses to locate at her party platform (P1). In centrist constituencies, each candidate puts mass on her party platform and the median voter—the lighter 45-degree, a, and b lines in this region depict the support of the candidates' mixed strategies—adopting the median voter's position with probability one for $\delta = \frac{1}{2}$ (M3).

Figure 3 shows the candidates' expected positions in each constituency, again using a bold dashed line for B and a bold solid one for A, when $V = \frac{1}{3}$ and $(a,b) = (\frac{7}{24}, \frac{17}{24})$. When $\delta < b - V$, B_δ is unwilling to move to δ to win with probability one-half. In these constituencies, each candidate sometimes adopts her party's platform and otherwise mixes over policies to the left of a, putting mass on the median voter in more centrist constituencies (M1 and M2). In constituencies $\delta \in (b - V, a + V)$, both candidates adopt the median voter's positions (P2). As δ moves above $a + V$, A_δ's expected position jumps

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[11]In Figures 2 and 3, we choose party platforms that turn out to be equilibrium platforms for the chosen values of V.
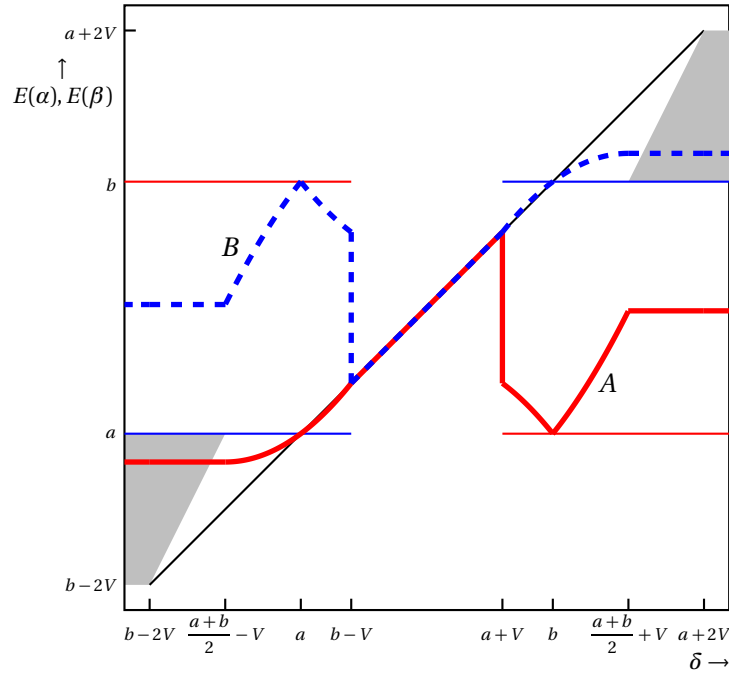
FIGURE 3. The candidates' expected positions across constituencies for $V = \frac{1}{3}$ and $(a,b) = (\frac{7}{24}, \frac{17}{24})$.

down as she adopts position δ with probability significantly less than one, a discontinuity discussed below (M3). For constituencies further to the right, A_δ mixes between a, δ, and positions to the right of b (M1 and M2, where the shaded region depicts positions to the right of b in the support of the candidates' mixed strategies).

Together, the figures illustrate some central predictions of our model. Constituencies at the center of the political spectrum are heavily contested as candidates take similar positions close to that of the median voter. More extremist constituencies are less heavily contested as candidates adopt positions closer to their party platforms, even when that means that one candidate seldom wins. As a result, candidates in more rightist constituencies do not always take more rightist positions on average than those in more leftist constituencies.

Characterizing the parties' equilibrium platform choices requires only the candidates' equilibrium payoffs; as shown in Proposition 1 these are unique for $a \leq b \leq \delta$ as long as $\delta \neq a + V$, i.e. for given (a,b) they are unique for almost all constituencies δ. We denote candidate i's equilibrium payoffs (for the subgame characterized by (a,b,δ)) by

$$\overline{U}_{i_\delta}(a,b,\delta) = \int_{[0,1]} \int_{[0,1]} U_{i_\delta}(a,b,\delta,\alpha,\beta)\, d\, F^*_{A_\delta}(a,b;\alpha)\, d\, F^*_{B_\delta}(a,b;\beta)$$

for $i \in \{A, B\}$.

COROLLARY 1. *Assume $a \leq b \leq \delta$. The disadvantaged Candidate A_δ's equilibrium expected payoff in any subgame is*

$$\overline{U}_{A_\delta}(a,b,\delta) = \begin{cases} V - \delta + a & \text{if } \delta - a < V \\ 0 & \text{otherwise.} \end{cases}$$

The advantaged Candidate B_δ's equilibrium expected payoff in any subgame is

$$\overline{U}_{B_\delta}(a,b,\delta) = \begin{cases} \min\{2V, b - a\} & \text{if } V \leq \delta - \frac{1}{2}(a+b) \\ 2(V - \delta + b) & \text{if } \delta - \frac{1}{2}(a+b) < V < \delta - a \\ V - \delta + b & \text{if } \delta - a < V \\ t \in [V - \delta + b, 2(V - \delta + b)] & \text{if } \delta - a = V. \end{cases}$$

When $\delta - a < V$, both candidates locate at the median voter's position, in which case each gets one-half the value of winning the election minus the costs of location. When $\delta - a > V$, the disadvantaged candidate's expected payoff is zero because she selects her party's platform with positive probability, and receives a payoff of zero when she does so.

The advantaged candidate's payoff depends upon her distance to the disadvantaged candidate as well as each candidate's distance to the median voter. To organize constituencies, we divide them into two classes depending upon the location of their median voter's bliss point relative to the party platforms.

DEFINITION 2. *If parties A and B locate at a and b, respectively, then constituency δ is* extremal *if $\delta \notin [\frac{1}{2}(a+b) - V, \frac{1}{2}(a+b) + V]$ or $|b - a| > 2V$. A constituency is* central *if it is not extremal.*

In particular, when $a \leq b \leq \delta$, the constituency δ is extremal if $\delta > V + \frac{1}{2}(a+b)$. When $b - a > 2V$, $a < b \leq \delta$ implies that $\delta > V + \frac{1}{2}(a+b)$; in this case, all constituencies $\delta \geq b > a$ are extremal. A constituency is extremal if its median voter lies sufficiently far from the party platforms. A constituency where each candidate chooses its party's platform is extremal. The only other extremal constituencies are those where A_δ mixes to the right of B_δ. Corollary 1 states that in extremal constituencies the advantaged candidate's expected payoff does not depend upon the distance between her party's platform and the median voter: either she wins for sure without departing from her party's position and gets $2V$, or A_δ competes with her, and B_δ's expected payoff equals the distance between the parties' platforms.

In central constituencies, B_δ's payoff increases the closer her party's platform is to the median voter. However, the rate at which her payoff changes as a function of the distance between her party's platform and the median voter is not constant. When $\delta - a = V$, B_δ locates at the median voter's bliss point for sure, and A_δ, who is indifferent between locating at the median voter's bliss point and choosing his party's platform, locates at the median voter's bliss point with sufficiently high probability.[12] As

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[12]The probability that A_δ stays at his party's platform affects B_δ's equilibrium payoff, which explains why

$\delta - a$ increases—holding everything else constant—A_δ prefers to remain at his party platform. To keep A_δ indifferent over choosing the median voter's bliss point and her party platform, B_δ cannot locate at the median voter's bliss point with probability one. To make B_δ indifferent over the median voter's bliss point and her party's platform, A_δ must adhere to his party platform with sufficiently high probability. Thus, the probability that A_δ chooses the median voter's bliss point jumps down as $\delta - a$ moves through V, and so B_δ's expected payoff jumps up. For $\delta - a < V$—competition is tough—B_δ benefits much less from being close to the median voter than when $\delta - a > V$—competition is weak.

Definition 3 organizes constituencies into those where candidates locate closer than their parties to the median voter and those where they do not, allowing for the possibility of mixed strategies.

DEFINITION 3. The constituency δ is *contested in the equilibrium* $(a^*, b^*, (F^*_{A_\delta})_{\delta \in [0,1]}$, $(F^*_{B_\delta})_{\delta \in [0,1]})$ if $F^*_{A_\delta}(a^*, b^*, a^*) < 1$ or $F^*_{B_\delta}(a^*, b^*, b^*) < 1$. A constituency is *uncontested in the equilibrium* $(a^*, b^*, (F^*_{A_\delta})_{\delta \in [0,1]}, (F^*_{B_\delta})_{\delta \in [0,1]})$ if it is not contested in that equilibrium.

A constituency is uncontested if both candidates adhere to their parties' platforms with probability one. While our terminology differs from the more standard notion of an uncontested constituency as one where one of the two parties fails to field a candidate, there is little substantive difference: in an uncontested constituency as we define it, the disadvantaged candidate locates at her party's platform and loses with probability one; neither the winner's location nor anyone's payoff would change if the party did not field the candidate at all. If Candidate A_δ must incur costs larger than the value of winning the election to locate at Party B's platform ($b - a \geq 2V$), then in equilibrium each candidate locates at her party's platform; such constituencies are uncontested. On the other hand, if A_δ can choose Party B's platform at a cost less than the value of winning the election ($b - a < 2V$), then the constituency δ is contested.

4. PLATFORM LOCATION

This section analyzes the parties' platform choices. Parties maximize their candidates' payoffs taking into account how their platform choices affect subsequent campaigning. When the private benefit of winning election, $2V$, is not too large, parties adopt distinct platforms, one to the right of $\frac{1}{2}$ and the other to the left. In central constituencies, the advantaged candidate's payoff decreases in her distance from the median voter; this gives parties an incentive to minimize the expected distance between their platform and the median voter. In extremal constituencies, the advantaged candidate's payoff increases the further her party's platform lies from the opponent's; this provides parties with an incentive to differentiate from each other. However, the fact that in not all central constituencies are the candidates' payoffs affected by platforms in the same way—candidates have more incentive to be near the median voter when equilibria are

Proposition 2 excludes the case $\delta - a = V$. For any a and b, the event that $\delta - a = V$ occurs with zero probability and therefore does not affect the parties' expected continuation payoffs.
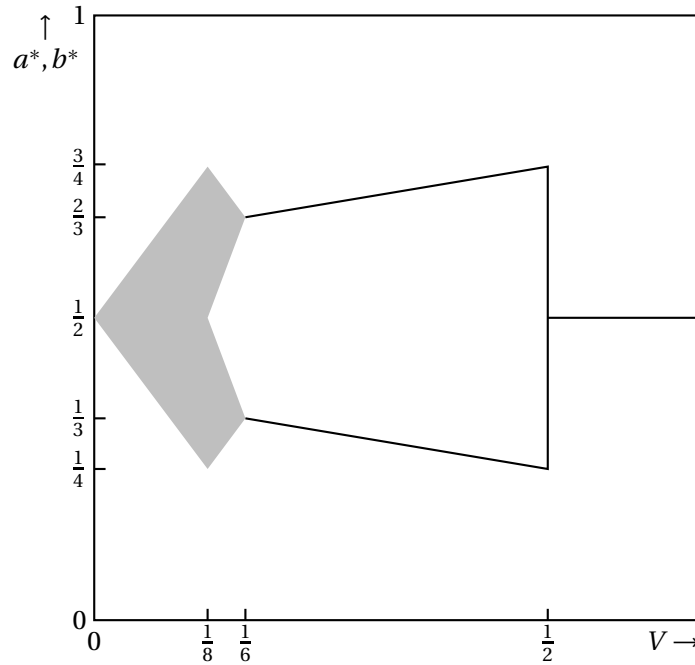
FIGURE 4. Party Platforms in Equilibrium. For $V \leq \frac{1}{6}$, any pair (a, b) of positions where b is in the shaded region and $a = 2V - b$ is an equilibrium. For $V = \frac{1}{2}$, any pair (a, b) where $a = 1 - b$ and $\frac{1}{4} \leq b \leq \frac{3}{4}$ is an equilibrium.

in mixed strategies—creates another motive for party differentiation. When choosing platforms, parties trade off these effects.

Theorem 1 completely characterizes parties' (pure) equilibrium strategies, denoted by a^* and b^*.[13] Figure 4 illustrates the parties' equilibrium strategies.

THEOREM 1. *The parties' equilibrium strategies are given by*

$$(a^*, b^*) = \begin{cases} (\frac{1}{2}, \frac{1}{2}) & \textit{if } V > \frac{1}{2} \\ (1 - x, x) \textit{ for some } x \in [\frac{1}{2}, \frac{3}{4}] & \textit{if } V = \frac{1}{2} \\ (\frac{3}{8} - \frac{1}{4}V, \frac{5}{8} + \frac{1}{4}V) & \textit{if } \frac{1}{6} \leq V < \frac{1}{2} \\ (x - 2V, x) \textit{ for some } x \in I(V) & \textit{if } V < \frac{1}{6}, \end{cases}$$

where $I(V) = [\max\{\frac{1}{2}, 4V\}, \min\{1 - 2V, \frac{1}{2} + 2V\}]$.

If $V > \frac{1}{2}$, parties do not differentiate and locate at one-half. For high V winning the election with probability one-half is sufficiently valuable that candidates always adopt

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[13]Since the first stage is symmetric with expected payoffs that are continuous in party platform, the game has a symmetric equilibrium in which parties play mixed strategies. Naturally this equilibrium also generates candidate differentiation.

the median voter's position; as a result, neither party can soften competition by separating from the other.

If $V < \frac{1}{6}$, parties maintain an equilibrium distance of $2V$, the private value of winning the election. The shaded region in Figure 4 shows the multiplicity of such equilibria. In equilibrium, constituencies with a median voter's bliss point between the two party platforms are contested, while all others are not. Parties have no incentive to take more extreme positions, which would not affect their payoffs from extremal constituencies ($2V$) but would lower their payoffs from central constituencies. Nor do they have incentive to go to the center. The loss from moving ε closer to the center is proportional to ε for extremal constituencies and the gain proportional to ε for central constituencies; when V is small and parties separate by no more than $2V$, there are many more extremal constituencies, so that the first effect dominates.

As V increases, each party wishes to move away from the center to maintain a distance of $2V$ from the other in order to secure its "home turf"; by locating too close to each other, parties would eliminate uncontested constituencies where their payoffs are highest. On the other hand, as V increases, this set of extremal constituencies shrinks. Consequently, the parties' incentive to move to the center to decrease their costs of winning contested, central constituencies grows larger by comparison. For $V < \frac{1}{6}$, the first effect dominates, and parties keep a distance of $2V$: parties differentiate as V increases. For $V > \frac{1}{6}$, parties separate by less than $2V$, so that all constituencies are contested.

When the private benefit $2V$ of winning is low (or, alternatively, when campaign costs are high), the two parties maintain enough distance between themselves that the only contested constituencies are those with median voters lying between the parties. In other words, parties are always (weakly) more extreme than their members. When the private benefit of winning is high (or campaign costs are low), all constituencies are contested: parties separate by less than $2V$. Nevertheless, as V increases, electoral competition increases, which shifts the set of constituencies that a given party wins with probability greater than one-half away from one-half. Since it is in these constituencies that parties have the most incentive to locate close to the median voter, they move away from one-half as V increases.

5. General distribution functions

Until now, we have assumed the distribution of median voters across constituencies to be uniform on $[0, 1]$. In this section we explore the robustness of our results to less heterogeneity among constituencies.

To do this, we restrict attention to distributions of median voters that are single-peaked, symmetric, and non-atomic in the sense that their cdf and pdf, G and g, respectively, satisfy the following two conditions: $G(x) = 1 - G(1 - x)$, and g is strictly increasing on $[0, \frac{1}{2}]$. Proposition 2 shows the robustness of our main result that parties offer differentiated platforms.

PROPOSITION 2. *For any single-peaked and symmetric distribution, there exists a threshold $\overline{V} > 0$ such that for each $V < \overline{V}$, $(b - 2V, b)$ is a pair of equilibrium strategies for the parties for any $\frac{1}{2} < b < \frac{1}{2} + 2V$.*
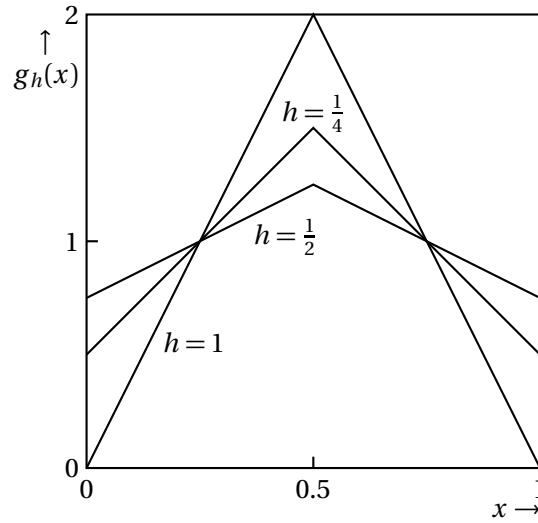
FIGURE 5. Distributions of constituencies.

Proposition 2 establishes that equilibria where parties differentiate by $2V$ are robust to small deviations from uniformity in the distribution of median voters across constituencies.

When $V \in (\frac{1}{6}, \frac{1}{2}]$ and median voters are uniformly distributed, we saw that parties differentiate by less than $2V$. This can occur also under other distribution functions, but a precise characterization of the equilibria of this kind would be cumbersome, mainly because the equilibria depend on local properties of G and lack closed-form expressions. Nevertheless, it is interesting to analyze how a change in the polarization of median voters affects the parties' positions. We illustrate some comparative statics using a parametrized family of triangular distributions about one-half.

In what follows we consider the following family of single-peaked and symmetric distribution functions:

$$g_h(x) = \begin{cases} 1 - h + 4hx & \text{if } x \leq \frac{1}{2} \\ 1 + 3h - 4hx & \text{if } x > \frac{1}{2} \end{cases}$$

for $h \in (0, 1]$. As h decreases, the electorate (median voters' bliss points) becomes more polarized; Figure 5 illustrates $h \in \{\frac{1}{4}, \frac{1}{2}, 1\}$. We are interested in how party platforms change with h. It can be shown that (at least) for "small" values of h there exists an equilibrium (a^*, b^*) with the property that $2V \geq b^* - a^* \geq V$. This follows from the fact that the candidates' strategies do not depend on the distribution of median voters and that the proof of Theorem 1 extends in a continuous way: for "small" h the distribution of constituencies is close to our uniform baseline. In particular, for each $V \in (\frac{1}{6}, \frac{1}{2})$ the parties' equilibrium platforms are unique and have the property that all constituencies are contested (parties separate by less than $2V$). As shown in the proof of Theorem 1, Party A's best-response correspondence is single-peaked for $h = 0$ and thus also for "small"

h. Consequently, the platform a is a best response to the platform $b > \frac{1}{2}$ if and only if $\partial \overline{U}^h_A(a,b)/\partial a = 0$, namely Party A's payoff $\overline{U}^h_A(a,b)$ achieves a maximum. Where this occurs,

$$\frac{\partial \overline{U}^h_A(a,b)}{\partial a} = G_h\left(\tfrac{1}{2}(a+b) - V\right) - 4G_h(a) + G_h(b-V) + G_h(a+V).\text{[14]}$$

Using the symmetry of g_h, which implies that the equilibrium positions are symmetric as well, we can solve the quadratic equation $\partial \overline{U}^h_A(a,b)/\partial a|_{b=1-a} = 0$ and get as the unique (feasible) solution

$$a^* = \frac{1}{4} - \frac{1}{4h} + \frac{1}{4h}\sqrt{h^2 + h + 1 + 4h^2 V^2 - 2h^2 V - 2hV}. \tag{1}$$

Hence (a^*, b^*), with $b^* = 1 - a^*$, describes the parties' equilibrium strategies. As a^* increases in h, an immediate consequence of (1) is that the distance between the parties' platforms decreases in h. When constituencies are more homogeneous, parties choose more central platforms. The intuition is straightforward: as the number of central constituencies increases in h, parties care more about centrality and thus choose more central positions.

6. CONCLUSION

This paper provides a rationale for differentiation between *ex ante* identical political parties in a Hotelling–Downs-style model of electoral competition. In our model, parties are benevolent to their many purely opportunistic candidates running in heterogeneous constituencies with different voter preferences. Parties choose policy platforms that serve as default positions for their candidates. Candidates may deviate from their parties' platforms at a cost: the more they deviate, the higher these "campaign costs." In equilibrium, parties do not locate at the center of the political spectrum, which would maximize the number of elections that their candidates win. Instead, they locate their platforms less centrally; by separating, they avoid costly campaigns. In this way, each party carves out a "home turf" of constituencies that can be won with little or no campaigning. Decreasing campaign costs causes parties to move further apart: because candidates campaign more vigorously, each party moves away from the other to carve out its home turf. More heterogeneity across constituencies also increases party differentiation for the straightforward reason that there are more extremist constituencies to be won.

Our intention in this paper has been to re-explore the effects of political competition on electoral positioning. Many commentators have observed that political candidates seldom espouse common policies. This paper provides an explanation for this finding through political parties: candidates differ because their parties differ; parties differ to reduce political competition. Other authors have offered other compelling reasons for political differentiation, and we do not suggest that party competition constitutes the

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[14]This is the equivalent of subcase (iii) in the proof of Theorem 1.

sole impetus for differentiation. However, we think it is an important exercise to under-stand how competition between parties affects their candidates' positioning.[15]

A crucial assumption underlying party differentiation in our model is that candidates incur costs by deviating from their party platforms. In our model, candidates pay these campaign costs regardless of whether they win election. Yet some reasons why candidates may find deviating from party policy costly may loom larger when they win than when they lose. If campaign costs were paid only by winning candidates, our qualitative results would be unaffected. The closer the parties' platforms, the more in-tense would be electoral competition (the higher candidates' campaign costs in equilib-rium). One difference from our model is that the disadvantaged candidate could move $2V$ from her platform before incurring a negative payoff. When $V > \frac{1}{4}$, both parties in equilibrium choose platforms at $\frac{1}{2}$, and in every constituency both candidates adopt the median voter's position with certainty. When $V \leq \frac{1}{4}$, parties in equilibrium choose platforms at $\frac{1}{2} - V$ and $\frac{1}{2} + V$, and constituencies with extreme median voters go uncon-tested. Here too the degree to which candidates differentiate increases in V (or, equiva-lently, decreases in campaign costs). The fact that in this model candidates' equilibrium strategies are pure underscores the fact that differentiation in our model is not an ar-tifact of the candidates' playing mixed strategies. The absence of mixing in this model also allows it the following, alternative interpretation: candidates have policy prefer-ences; parties have none; and the winning party must pay a cost to attract a candidate whose preferred policy differs from its platform.

The assumption that both parties value winning a constituency by $2V$ can be re-laxed. In a more general model in which the valuations of parties A and B are $2V_A$ and $2V_B$ respectively, party differentiation can also occur in equilibrium. If, for example, $\frac{1}{2} \geq V_A \geq V_B$ and $V_A \leq \min\{7V_B - 1, \frac{3}{5}V_B + \frac{1}{5}\}$ (in particular, valuations are not too asym-metric), then in equilibrium parties locate at[16]

$$a^* = \tfrac{11}{8}V_A - \tfrac{13}{8}V_B + \tfrac{3}{8}$$
$$b^* = \tfrac{13}{8}V_A - \tfrac{11}{8}V_B + \tfrac{5}{8}.$$

For different valuations, the "stronger" Party A locates more centrally than the weaker B. As V_A increases, both parties move to the right (B faster than A), and the weaker party B is pushed to her end of the policy spectrum. When B becomes less competitive, the benefits from avoiding competition become larger and B moves to the right; A exploits this by capturing more of the policy space. Parties separate by $\frac{1}{4} + \frac{1}{4}(V_A + V_B)$ so that, as in our symmetric baseline model, as valuations increase parties move apart.

In our model, the distribution of median voters across constituencies is uniform. However, the way party differentiation depends upon V holds for any continuous, single-peaked distribution of the median voter's location. As V increases from zero, the

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[15]An open empirical question is how the presence or absence of parties in different elections affects candidate differentiation.

[16]The cumbersome derivation of this result is available in a supplementary file on the journal website, http://econtheory.org/supp/176/supplement.pdf.

parties' equilibrium platforms separate more and more until V reaches a critical size, at which point parties locate at one-half. Hence, the intuition that underlies equilibrium when $V \leq \frac{1}{6}$ does not rely on uniformity. For V sufficiently small, parties separate their platforms by $2V$.[17] Moving further apart only diminishes payoffs for central median voters, whereas moving further together costs more in the many extremal constituencies than it benefits in the few central constituencies. However, unlike in our model, when $V > \frac{1}{6}$ comparative statics may no longer be monotone, as the intuition behind our arguments depends upon a comparison of the measure of constituencies where candidates always adopt the median voter's preferred position to those where candidates mix between that and their parties' platforms.

Parties in our model choose their platforms to maximize their candidates' average payoffs. If instead each party chooses its party platform via a majoritarian election among its candidates, the result would be very different. Each candidate wishes her party would locate its platform at her constituency's median voter regardless of the other party's location. It is easy to verify that candidates' preferences over their party's location satisfy Gans and Smart's 1996 single-crossing condition, implying that their median bliss point—the median among median voters—is a Condorcet winner. This equilibrium differs so dramatically from ours because a majoritarian party does not trade gains in one constituency off against losses in another. Yet parties should facilitate mutually advantageous trades, for instance by having more extremist candidates pay centrists' campaign costs in return for moving party platform away from the center.

While we have interpreted our formal model in terms of parties and candidates, the results carry over to a single election without parties. Consider two candidates campaigning for election, where over the course of the campaign they learn information about voters' preferences. One formulation of this strategic setting coincides with our two-stage model: first candidates take initial policy positions with prior beliefs that the median voter's position is uniformly distributed on $[0, 1]$; then they learn the median voter's preferences; and finally they take new positions, where departing from their initial positions is costly. Notice that someone who maximizes total payoff while facing a distribution of median voters across constituencies that is uniform on $[0, 1]$ acts in the same way as someone who maximizes expected payoff while facing a single constituency whose median voter's position is drawn randomly from a uniform distribution on $[0, 1]$. In equilibrium, candidates at the outset of the campaign split the political spectrum, each one essentially betting on a median voter near her position that allows her to win the election at minimal cost. As the campaign progresses, either one candidate "concedes"—makes no movement to the median voter knowing that she will lose—or both move closer to the median voter.

APPENDIX

PROOF OF LEMMA 1.   1. Suppose Candidate i_δ's strategy has a gap between x and y but not between x and δ. Let $z = \sup\{w \mid F_{i_\delta}(w) = F_{i_\delta}(x)\}$ be the point where

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[17]The maximum value of V for which parties maintain a distance of $2V$ does depend on the distribution and can be no larger than $\frac{1}{6}$, for the uniform distribution has more variance than any single-peaked distribution.

the gap ends, so $x < z < \delta$. Because locating in a gap of the other candidate's strategy can never be optimal—decreasing the position a small amount decreases costs without changing the winning probability—both candidates' strategies must have gaps between x and z. But since both cannot have atoms at $z \neq \delta$ (for then either candidate could discretely increase her winning probability with neglible increase in cost), at least one candidate can benefit by locating in (x, z) (which decreases costs without affecting the winning probability).

2. Clearly, no candidate locates in (a, b). If i_δ's strategy has an atom at $x \in [b, \delta)$, then for some $\varepsilon > 0$ $j_\delta \neq i_\delta$ does not locate in $(x - \varepsilon, x)$, because moving slightly above x increases payoffs. But with a gap between $x - \varepsilon$ and x, i_δ could benefit by going into $(x - \varepsilon, x)$ unless $x = b_1$ and $i_\delta = B_\delta$. Suppose only A_δ has an atom at δ. Then B_δ must have a gap between some $x < \delta$ and δ—going to $\delta - \varepsilon$ for some $\varepsilon > 0$ cannot be as good for B_δ as going to δ—in which case A_δ can benefit by moving mass from δ into (x, δ).   □

PROOF OF LEMMA 2.   If not, then we know from the proof of Proposition 1 that $\inf\{x : F_{A_\delta}(x) > 0\} = \inf\{x : F_{A_\delta}(x) > 0\} = \delta$, a contradiction.   □

PROOF OF PROPOSITION 1.   When $\delta - a < V$, each candidate optimally locates at the median voter δ given that the other does the same. First note that because each candidate receives a strictly positive payoff by choosing δ regardless of the other's strategy, each has a strictly positive expected payoff in equilibrium. We claim that $\inf\{t : F_{A_\delta}^*(a, b; t) > 0\} = \inf\{t : F_{B_\delta}^*(a, b; t) > 0\} = \delta$. If the infima are unequal, the candidate with the lower infimum, i, does not get a positive expected payoff for all positions in the support of $F_{i_\delta}^*(a, b; \cdot)$. For each candidate to get a strictly positive payoff at this common infimum, each must put mass on the infimum. But this can happen only at δ, for otherwise each could discretely increase her winning probability by increasing her costs only infinitesimally.

When $\delta - a \geq V$, then the result follows from the considerations before the proposition.   □

PROOF OF THEOREM 1.   Assume Party A takes the platform a and Party B takes the platform b.

CLAIM 1.   *If $y > \frac{1}{2}$, a position $x > y$ yields strictly lower payoff for the party at x than the mirrored position $2y - x$. If $y < \frac{1}{2}$, a position $x < y$ yields strictly lower payoff for the party at x than the position $2y - x$. In particular there is no equilibrium where $a \neq b$ and both parties are strictly above $\frac{1}{2}$ or both are strictly below $\frac{1}{2}$.*

PROOF OF CLAIM 1.   By symmetry, the total payoff from constituencies with median voters in $[2y - 1, 1]$ is the same whether the party facing another party at y locates at $x > y$ or at $2y - x$. The first part of the claim follows from the fact that the total payoffs from constituencies with median voters in $[0, 2y - 1)$ are strictly higher at the latter position. The second part of the claim is proven similarly.   □

Together with symmetry, Claim 1 implies that it suffices to establish the best-response correspondence for A, as B's can be obtained by mirroring the parties' positions. Candidates' payoffs in any constituency δ as a function of the party platforms (a,b) can be derived from Proposition 1 by relabeling variables: A_δ's payoff $\overline{U}_{A_\delta}(a,b)$ is $U_{A_\delta}(x,y,\delta)$ as given by Corollary 1, where[18]

$$(x,y) = \begin{cases} (2\delta - b, 2\delta - a) & \text{if } \delta < a \\ (2\delta - b, a) & \text{if } a \leq \delta \leq \frac{1}{2}(a+b) \\ (a, 2\delta - b) & \text{if } \frac{1}{2}(a+b) < \delta \leq b \\ (a,b) & \text{if } b < \delta. \end{cases}$$

CLAIM 2. *Party A never chooses a such that $b - a > 2V$.*

PROOF OF CLAIM 2.   When $b - a > 2V$, candidates in constituencies with medians below $\frac{1}{2}(a+b) - V$ (or above $\frac{1}{2}(a+b) + V$) always choose their parties' platforms. In no constituencies do the candidate either both adopt the median voter's location or both mix between their parties' platforms and the median voter. Hence A's total payoff is

$$\overline{U}_A(a,b) = 2V\left(\tfrac{1}{2}(a+b) - V\right) + \int_{\frac{1}{2}(a+b)-V}^{\frac{1}{2}(a+b)} (b - 2\delta + a)\,d\delta$$

and $\partial \overline{U}_A(a,b)/\partial a = V > 0$. This proves Claim 2.   □

When $0 \leq b - a \leq 2V$, equilibrium can take any of the mixed forms described in Proposition 1 with payoffs described in Corollary 1. This gives

$$\overline{U}_A(a,b) = (b-a)\max\left\{\tfrac{1}{2}(a+b) - V, 0\right\}$$
$$+ 2\int_{\max\{\frac{1}{2}(a+b)-V,0\}}^{\max\{\min\{b-V,a\},0\}} (V + \delta - a)\,d\delta + \int_{\max\{\min\{b-V,a\},0\}}^{a} (V + \delta - a)\,d\delta$$
$$+ 2\int_{a}^{\max\{b-V,a\}} (V - \delta + a)\,d\delta + \int_{\max\{b-V,a\}}^{\min\{V+a,1\}} (V - \delta + a)\,d\delta.$$

We use this together with Claims 1 and 2 to establish A's best response to $b \geq \frac{1}{2}$. Claims 1 and 2 imply that we can restrict attention to platforms a with $0 \leq b - a \leq 2V$. In order to find the maxima of $\overline{U}_A(a,b)$, we calculate its derivative for the following cases: (i) $b \leq V$, (ii) $b > a + V$ and $\frac{1}{2}(a+b) < V$, (iii) $V < b < a + V$ and $\frac{1}{2}(a+b) < V$, (iv) $b > a + V$ and $\frac{1}{2}(a+b) > V$, and (v) $b < a + V$ and $\frac{1}{2}(a+b) > V$.

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[18]Note here that x and y may not lie in $[0,1]$. However, since none of the results in Section 3 depend on this assumption, we can continue to use them here.

(i) As $b \leq V$ and $a \leq b$ imply that $\frac{1}{2}(a+b) \leq V$,

$$\overline{U}_A(a,b) = \int_0^a (V + \delta - a)\, d\delta + \int_a^{\min\{a+V,1\}} (V - \delta + a)\, d\delta.$$

If $a + V < 1$, $\partial \overline{U}_A(a,b)/\partial a = V - a \geq 0$ (with strict inequality for $a < b$). If $1 \leq a + V$, $\partial \overline{U}_A(a,b)/\partial a = 1 - 2a$.

(ii) We have

$$\overline{U}_A(a,b) = 2\int_0^a (V + \delta - a)\, d\delta + 2\int_a^{b-V} (V - \delta + a)\, d\delta + \int_{b-V}^{a+V} (V - \delta + a)\, d\delta$$

and $\partial \overline{U}_A(a,b)/\partial a = b - 3a > 0$, where the inequality follows from $\frac{1}{2}(a+b) < V < b - a$.

(iii) We have

$$\overline{U}_A(a,b) = 2\int_0^{b-V} (V + \delta - a)\, d\delta + \int_{b-V}^a (V + \delta - a)\, d\delta + \int_a^{\min\{a+V,1\}} (V - \delta + a)\, d\delta$$

and $\partial \overline{U}_A(a,b)/\partial a = 2V - b - a > 0$ if $a + V < 1$ and $\partial \overline{U}_A(a,b)/\partial a = V - 2a - b + 1$ if $a + V \geq 1$.

(iv) We have

$$\overline{U}_A(a,b) = (b-a)\left(\tfrac{1}{2}(a+b) - V\right) + 2\int_{\frac{1}{2}(a+b)-V}^a (V + \delta - a)\, d\delta$$
$$+ 2\int_a^{b-V} (V - \delta + a)\, d\delta + \int_{b-V}^{a+V} (V - \delta + a)\, d\delta,$$

and $\partial \overline{U}_A(a,b)/\partial a = \frac{3}{2}b - \frac{5}{2}a - V$. From this, we conclude that $\overline{U}_A(a,b)$ has a unique maximizer in a on $(2V - b, b - V)$, since $\partial \overline{U}_A(a,b)/\partial a|_{a=2V-b} = 4b - 6V > 0$ and $\partial \overline{U}_A(a,b)/\partial a|_{a=b-V} = \frac{3}{2}V - b < 0$ (both since $2V - b < b - V \Rightarrow b > \frac{3}{2}V$).

(v) We have

$$\overline{U}_A(a,b) = (b-a)\left(\tfrac{1}{2}(a+b) - V\right) + 2\int_{\frac{1}{2}(a+b)-V}^{b-V} (V + \delta - a)\, d\delta$$
$$+ \int_{b-V}^a (V + \delta - a)\, d\delta + \int_a^{\min\{a+V,1\}} (V - \delta + a)\, d\delta.$$

If $a + V < 1$, $\partial \overline{U}_A(a,b)/\partial a = V - \frac{1}{2}(a+b) < 0$. If $a + V > 1$, $\partial \overline{U}_A(a,b)/\partial a = 1 - a - \frac{1}{2}(a+b) < 0$.

We can now find the optimal $a \in (b - 2V, b)$ by distinguishing four cases:

1. When $b \leq V$ we must have $V \geq \frac{1}{2}$, where Case i demonstrates that $a = \frac{1}{2}$ is the best reply to b.

2. When $0 < b - V \leq 2V - b$ and $b > 3V - 1$, $\overline{U}_A(a, b)$ has a unique maximizer at $a = 2V - b$. To see this note that Cases ii and v show that $\overline{U}_A$ is (strictly) increasing in a when $a < b - V$ and (strictly) decreasing when $a > 2V - b$. Because $1 - V > 2V - b$, $\overline{U}_A$ is (strictly) increasing for $a < 2V - b$ (Case iii) and therefore has a unique maximizer at $a = 2V - b$.

3. When $0 < b - V \leq 2V - b$ and $b \leq 3V - 1$, $\overline{U}_A(a, b)$ has a unique maximizer at $a = \frac{1}{2}(V - b + 1)$. To see this note that Cases ii and v show that $\overline{U}_A$ is (strictly) increasing for $a < b - V$ and (strictly) decreasing for $a > 2V - b$. As $\overline{U}_A$ is strictly increasing for $a < 1 - V$ and as $b \leq 3V - 1$ we have $\frac{1}{2}(V - b + 1) \geq 1 - V$ which shows that $\overline{U}_A$ is (strictly) increasing for $1 - V \leq a \leq \frac{1}{2}(V - b + 1)$. Furthermore, $\overline{U}_A$ is (strictly) decreasing for $a > \frac{1}{2}(V - b + 1) \geq 1 - V$ (see Case iii).

4. When $b - V > 2V - b$, Cases ii, iv, and v show that $\overline{U}_A(a, b)$ has a unique maximum at $a = \frac{3}{5}b - \frac{2}{5}V$.

Without loss of generality, we can assume that $b \geq \frac{1}{2}$ (otherwise we can transform the entire policy space by $x \mapsto 1 - x$ and relabel parties). Claims 1 and 2 imply that the best response for Party A to b is either $a = 2V - b$ or is defined by Cases 1–4 above. Furthermore, Claim 1 implies that in equilibrium we must have $b = \frac{1}{2}$ or $a = b$ or $a \leq \frac{1}{2}$. We first characterize all equilibria in which $b - a < 2V$. If $b \leq V$, then Case 1 applies: $V \geq \frac{1}{2}$, and $a = \frac{1}{2}$ is a best response to b. By symmetry, checking whether b is a best response for B to $a = \frac{1}{2}$ is equivalent to checking whether $\widetilde{a} = 1 - b \leq \frac{1}{2}$ is a best response for A to $b = \frac{1}{2}$. This requires (see Case 1 again) that $1 - b = \frac{1}{2}$ or $b = \frac{1}{2}$. In particular, these arguments demonstrate that the only equilibria with $\frac{1}{2} \leq b \leq V$ have $a = b = \frac{1}{2}$, which is an equilibrium for $V \geq \frac{1}{2}$. Assume henceforth that $b > V$.

When $b \leq \frac{3}{2}V$ and $b \leq 3V - 1$, Case 3 requires that $a = \frac{1}{2}(V - b + 1)$. We therefore have to find conditions under which b is a best response to a, or equivalently using symmetry those under which $\widetilde{a} = 1 - b$ is a best response to $\widetilde{b} = 1 - a = \frac{1}{2}(1 - V + b)$. As $\widetilde{b} \leq V$, Case 1 shows that $\widetilde{a}$ is a best response to $\widetilde{b}$ if and only if $a = b = V = \frac{1}{2}$, which cannot hold (as we assume $b > V$).

When $b \leq \frac{3}{2}V$ and $b > 3V - 1$, Case 2 implies that $a = 2V - b$. Once again, we must find conditions under which $\widetilde{a}$ is a best response to $\widetilde{b} = 1 - 2V + b$. As $\widetilde{b} > V$, Case 1 does not apply. If $5V - 2 < b \leq \frac{7}{2}V - 1$, we have $\widetilde{b} - V \leq 2V - \widetilde{b}$ and $\widetilde{b} > 3V - 1$ and thus Case 2 shows that we need $\widetilde{a} = 2V - \widetilde{b}$. This and $a = 2V - b$ are fulfilled if and only if $V = \frac{1}{2}$. Thus if $V = \frac{1}{2}$, $(1 - b, b)$ is an equilibrium if and only if $b \in (\frac{1}{2}, \frac{3}{4}]$. If $b \leq \min\{\frac{7}{2}V - 1, 5V - 2\}$ we have $\widetilde{b} \leq \min\{\frac{3}{2}V, 3V - 1\}$. Case 3 shows that we need to have $\widetilde{a} = \frac{1}{2}(V - \widetilde{b} + 1)$, or equivalently $1 - b = \frac{1}{2}(V - 1 + a + 1) = \frac{1}{2}(3V - b)$ which implies $b = 2 - 3V$. But this

implies that $a = 2V - b = 5V - 2$ such that $a = b = 5V - 2 = 2 - 3V$. However, since this requires $V = \frac{1}{2}$, it contradicts $b > V$. If $b > \frac{7}{2}V - 1$, then $\widetilde{b} > \frac{3}{2}V$; from Case 4, we need $\widetilde{a} = \frac{3}{5}\widetilde{b} - \frac{2}{5}V$, or equivalently $b = \frac{1}{4} + V$. Since we need $b \leq \frac{3}{2}V$, it must be that $V \geq \frac{1}{2}$. Because we also need $b > \frac{7}{2}V - 1$, $V < \frac{1}{2}$, which is a contradiction.

If $b > \frac{3}{2}V$, Case 4 implies $a = \frac{3}{5}b - \frac{2}{5}V$. Again we look for conditions under which $\widetilde{a}$ is a best response to $\widetilde{b} = 1 - \frac{3}{5}b + \frac{2}{5}V$. If $b \geq \frac{5}{3} - V$ (equivalently $\widetilde{b} \leq V$), Case 1 dictates that $\widetilde{a} = \frac{1}{2} = b$. This is not possible as $\frac{1}{2} = b > \frac{3}{2}V$ implies $V < \frac{1}{3}$, which contradicts $b \geq \frac{5}{3} - V$. Suppose now $b < \frac{5}{3} - V$. If $b \geq \frac{5}{3} - \frac{11}{6}V$ ($\Leftrightarrow \widetilde{b} - V \leq 2V - \widetilde{b}$) and $b < \frac{10}{3} - \frac{13}{3}V$ ($\Leftrightarrow \widetilde{b} > 3V - 1$), then we need $\widetilde{a} = 2V - \widetilde{b}$ (Case 2), which is equivalent to $b = \frac{5}{4} - V$. But this contradicts $\frac{5}{4} - V = b \geq \frac{5}{3} - \frac{11}{6}V \Rightarrow V \geq \frac{1}{2}$ and $\frac{5}{4} - V = b > \frac{3}{2}V \Rightarrow V < \frac{1}{2}$. If instead $b \geq \frac{5}{3} - \frac{11}{6}V$ and $b \geq \frac{10}{3} - \frac{13}{3}V$, then we need $\widetilde{a} = \frac{1}{2}(V - \widetilde{b} + 1) = \frac{1}{2}(V - (1 - \frac{3}{5}b + \frac{2}{5}V) + 1)$ (Case 3) or, equivalently, $b = \frac{10}{13} - \frac{3}{13}V$ (which implies that $a = \frac{6}{13} - \frac{7}{13}V$). As we require that $b > \frac{3}{2}V$ (which implies $V < \frac{4}{9}$) and $b \geq \frac{10}{3} - \frac{13}{3}V$ (which implies $V \geq \frac{5}{8}$) this cannot be an equilibrium. If $b < \frac{5}{3} - \frac{11}{6}V$ then we need $\widetilde{a} = \frac{3}{5}\widetilde{b} - \frac{2}{5}V$ (Case 4) or, equivalently, $b = \frac{1}{4}V + \frac{5}{8}$ (which implies that $a = \frac{3}{8} - \frac{1}{4}V$). Note that we have $a < \frac{1}{2}$ and $b > \frac{1}{2}$ so that this is an equilibrium if $b - a < 2V$ and $b < \frac{5}{3} - \frac{11}{6}V$ or, equivalently, if $\frac{1}{6} < V < \frac{1}{2}$.

We have characterized all equilibria for $b - a \neq 2V$. Assume now that $b - a = 2V$. Claim 2 establishes that A does not wish to lower a. Due to the single-peakedness of $\overline{U}_A(a,b)$, $a = b - 2V$ is a best response to b if and only if $\partial \overline{U}_A(a,b)/\partial a|_{a=b-2V} \leq 0$. This happens only in Cases iii, iv, and v. As $b - a = 2V$, only Case iv applies and requires $b - V \geq a$, $\frac{1}{2}(a + b) - V \geq 0$, and $a \geq \frac{3}{5}b - \frac{2}{5}V$. These conditions are equivalent to $b \geq 4V$. Similarly, b best responds to a if and only if $a \leq \frac{1}{2}$ (because of Claim 1) and $1 - a = 1 - b + 2V \geq 4V$, or equivalently if and only if $b \leq \min\{1 - 2V, \frac{1}{2} + 2V\}$. Thus, we have an equilibrium with $a = b - 2V$ if and only if $V \leq \frac{1}{6}$ and $b \in [\max\{\frac{1}{2}, 4V\}, \min\{1 - 2V, \frac{1}{2} + 2V\}]$. □

PROOF OF PROPOSITION 2. Assume Party A takes platform a, and B takes platform b, and denote the cumulative distribution function of median voters by G and its density by g. First note that A never chooses a such that $b - a > 2V$. This follows from a slight generalization of the proof of Claim 2 (see the proof of Theorem 1). As in Claim 1 in the proof of Theorem 1, when $b > \frac{1}{2}$ the position $a > b$ is strictly worse than the mirrored position $1 - a$, and thus the only candidates for best responses to $b > \frac{1}{2}$ lie in $[b - 2V, b]$. For $V \leq \frac{1}{2}(a + b)$ and $b - 2V \leq a < b - V$, A's payoff (as a function of b) is given by (similar to case (iv) in the proof of Theorem 1),

$$\overline{U}_A^G(a,b) = (b - a)G\left(\tfrac{1}{2}(a + b) - V\right) + 2\int_{\frac{1}{2}(a+b)-V}^{a}(V + \delta - a)g(\delta)\,d\delta$$

$$+ 2\int_{a}^{b-V}(V - \delta + a)g(\delta)\,d\delta + \int_{b-V}^{a+V}(V - \delta + a)g(\delta)\,d\delta,$$

from which we obtain

$$\frac{\partial \overline{U}_A^G(a,b)}{\partial a} = G\left(\tfrac{1}{2}(a+b)-V\right) - 4G(a) + G(b-V) + G(a+V) \leq -3G(a) + 2G(a+V).$$

If $V \leq \tfrac{1}{2}(a+b)$ and $a \geq b - V$, we have

$$\overline{U}_A^G(a,b) = (b-a)G\left(\tfrac{1}{2}(a+b)-V\right) + 2\int_{\tfrac{1}{2}(a+b)-V}^{b-V}(V+\delta-a)g(\delta)\,d\delta$$
$$+ \int_{b-V}^{a}(V+\delta-a)g(\delta)\,d\delta + \int_{a}^{a+V}(V-\delta+a)g(\delta)\,d\delta,$$

and hence

$$\frac{\partial \overline{U}_A^G(a,b)}{\partial a} = G\left(\tfrac{1}{2}(a+b)-V\right) - 2G(a) - G(b-V) + G(a+V) \leq -2G(a) + G(a+V).$$

Define $\overline{V}$ by $G(\tfrac{1}{2} - 2\overline{V}) = 0.4$ (which exists by the continuity of G). By symmetry, $G(\tfrac{1}{2} + 2\overline{V}) = 0.6$. Now take $V < \overline{V}$ and choose b with $\tfrac{1}{2} < b < \tfrac{1}{2} + 2V$. Then $G(a+V) < \tfrac{3}{2}G(a)$ for all a with $b - 2V \leq a < b - V$ and $G(a+V) < 2G(a)$ for all a with $b - V \leq a < b$. This establishes that for any $V < \overline{V}$, $\tfrac{1}{2} < b < \tfrac{1}{2} + 2V$, and $a \in (b - 2V, b]$ we have $\partial \overline{U}_A^G(a,b)/\partial a < 0$, and thus $a = b - 2V$ is a best response for Party A. Mirroring positions around $\tfrac{1}{2}$ and repeating the argument completes the proof. $\qquad\square$

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