Collateral Value and Forbearance Lending

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Abstract

We investigate the foreclosure policy of collateral-based loans in which the endogenous collateral value plays a crucial role. If creditors are able to commit, then the equilibrium arrangement is more likely to feature forebearance lending by specifying a lower level of liquidation (or roll over all of the loans) relative to the ex-post efficiency criterion for each realization of the interim signal. The key is that collateral value may drop too low when banks call in loans by auctioning off borrowers' collateral and this makes clearing up non-performing loans less attractive. We attribute the banks' leniency as we have observed in Japan during the 1990s to an equilibrium arrangement where banks can commit due to either relationship banking or an implicit lender-borrower contract, such as the arrangement under Japan's main-bank system.

Key words: Collateral value, forbearance lending, government guarantee.
JEL classification: E44, E51, G3

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1 Introduction

Regulatory forbearance has been considered as one of the main suspects for financial institutions to roll over non-performing loans and delay in clearing up bad loans, and thus eventually responsible for the weakness of financial institutions and prolonged contractions in real economic activity.[1] The measures may include relaxing regulatory supervision, outright government subsidies and bailout. For example, Cargill et al. (1997) argue that Japan's government has adopted "buy time" policies since the burst of its real estate market and stock market booms, such as relaxing bank capital requirements, allowing banks to hold non-performing loans without special write-offs, and also allowing insolvent financial institutions to operate, in the hope that the economy and the real-estate market would soon recover, so as to float these financial institutions back to health.

In this paper we argue that the "forbearance" phenomenon needs not necessarily be a consequence of anticipated government subsidy or bailout, rather, it can arise from an equilibrium arrangement between financial institutions and their borrowers. This "private forbearance" story is motivated Okina and Shiratsuka (2001) and Mori et al. (2001), who documented that even though Japanese banks' non-performing loans rose significantly after stock and real estate prices crashed in the early 1990s, financial institutions continued to lend to unprofitable firms in order to prevent loan losses from materializing, because collateral was of little value to cover their losses. The empirical testing by Kobayashi et al. (2002) also found evidence that Japanese banks exercised forbearance lending in construction and real estate industries during 1993-1999.

We investigate the foreclosure policy of collateral-based loans in which the endogenous collateral value plays a crucial role. Creditors may or may not be able to commit to the

[1] For example, Chang and Velasco (2001), McKinnon and Pill (1998) proclaim the potential catastrophic consequence of the "overborrowing" syndrome and the boom-bust cycle of asset prices due to the anticipation of government rescue. See also Krugman (1998) and Kim and Lee (2002) how government guarantee causes large boom-bust fluctuations in asset prices and economic activity. For example, Kim and Lee construct a dynamic model which stresses the relationship between collateralized asset and government subsidy to firms. They *assume* that banks are willing to supply funds to money-losing firms as long as the value of collateral still covers the amount of debt. A crisis erupts when the value of the asset falls below the amount of cumulated loans.

pre-specified foreclosure policy. With commitment the foreclosure policy may specify a lower level of liquidation (or roll over all of the loans) relative to the ex post efficiency criterion, which we dub as "forbearance lending." When the world interest rate is lower, the public is initially more optimistic about the prospects of project returns, firms hold a higher level of collateralizable asset initially, or when creditors can better enforce repayments, the equilibrium arrangement with commitment tends to feature forbearance lending. On the other hand, the model with no commitment always specifies a higher level of liquidation than ex post efficiency requires. We attribute the observed leniency of banks to their customers as the equilibrium arrangement when banks are able to pre-commit due to either relationship banking or implicit lender-borrower contract, such as the arrangement under Japan's main-bank system.[2]

Our model is based on the building blocks of recent works that emphasize the interaction between the credit constraints and the value of collateralized assets.[3] Structurally, our model is closer to Holmstrom and Tirole (1998). They consider a three-period model in which at the interim period a stochastic amount of liquidity is required to keep the project going. Lenders will reconsider whether to supply the extra working capital based on the realization of liquidity shock. If terminated, the liquidity value of the project is zero. Investment is subject to moral hazard in that an entrepreneur can gain some private benefit by privately choosing to shirk, which affect the project's probability of success. They find that the optimal cutoff is always between the ex post first-best cutoff and the other cutoff of which lenders cannot pre-commit,[4] there can be no leniency arising from the second-best contract.

This paper is also related to the literature of soft budget constraint (SBC), in particular to those works that conceive SBC as a dynamic commitment problem, due to, for

[2]See, for example, Aoki and Dore (1994), Hoshi (2000), and Hoshi and Kashyap (1999).

[3]This literature, such as Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Chen (2001), concentrates on the transmission mechanism of an exogenous shock which generates large fluctuations in asset prices and persistent effects on economic activity.

[4]The reason for the second-best cutoff falls below the first-best cutoff (i.e., more stringent liquidity supply policy) is that since firms are credit-constrained and thus the rate of return of their internal fund exceeds the market rate, they do not want to be full insured in order to exchange for more investment ex ante.

example, government subsidy and expected bailout, financial structure, and asymmetric information.[5] For example, a mainstream view of SBC is that failure to enforce bankruptcy rule due to expectations of government bailout creates creditor passivity.[6] Our paper challenges the general view of soft budget constraint literature that failure to commitment generally leads to leniency. We shows that creditors tend to liquidate more ex post when they cannot commit ex ante, while they may roll over more loans with commitment. Also the results do not rely on asymmetric information, government subsidy or expected bailout.

Morris and Shin (2001) also investigate how creditors decide to roll over or terminate loans, based on other creditors' moves. They show that the coordination problem leads to multiple equilibria when the state of the economy is common knowledge. On the other hand, when private information is introduced, there is a unique equilibrium provided that the private signal is sufficiently precise and multiplicity of equilibrium re-emerges if the private signal is not informative enough. In our paper there is no private information and the equilibrium is unique. The key is that the value of collateral depends on the fraction of loans to be liquidated and in their model the yields from foreclosure (liquidation value) does not depend on the number of creditors who foreclose.

The rest of the paper is organized as follows. Section 2 outlines the environment of the model. Section 3 analyzes financial contracting when creditors are able to commit. We discuss under what conditions forbearance lending arises. Section 4 contrasts the previous section by considering the equilibrium arrangement when creditors can not commit. Section 5 concludes.

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[5]See recent surveys by Mitchell (2000) and Maskin and Xu (2001). For applications of soft budget constraint to dynamic commitment problems and banking sector problems, see also Berglof and Roland (1997) and Huang and Xu (1999), and the references within.

[6]For example, Mitchell (1998) argues that banks choose to roll over bad loans if they expect government bailout ex post. In turn, the government fails to commit not to rescue insolvent financial institutions if there are too many of them.

2 The Environment

Consider a small open economy with three groups of agents: creditors, entrepreneurs, and landlords. There are three periods indexed by $t = 0$, 1, and 2. All agents are assumed to be risk neutral and consume only at date-2. In each period there is a single consumption good and also a durable asset. The durable asset is initially held by entrepreneurs and landlords and its aggregate supply is assumed to be fixed at $\overline{K}$. After production, the asset becomes valueless at the end of date-2.

Each entrepreneur has access to a two-period investment project in which the durable asset serves as the sole input:

$$y_2 = \widetilde{A}k_0,$$

where k_0 is the level of the durable asset invested at date-0 and $\widetilde{A}$ is random and represents the project's productivity. Assume that $\widetilde{A}$ equals A with probability π and equals 0 with probability $1 - \pi$, where $A > 0$. The realization of $\widetilde{A}$ is publicly observable. An aggregate shock π is drawn from the set $(0, 1)$ according to the cumulative distribution function $G(\bullet)$ with a density $g(\bullet)$. Conditioned on π, $\widetilde{A}$ is independently and identically distributed across borrowers. The distribution $G(\bullet)$ can be considered as the public's prior assessment of the prospects of the projects. We assume that projects are not reversible but can be divisible at date-1. Assume that

$$E(\pi) A > r^2, \tag{1}$$

where $E(\pi) = \int \pi dG(\pi)$ and r is the world gross interest rate. This condition says investing in projects is socially desirable given the initial assessment of the probability of success.

At date-1 a public signal is revealed which is perfectly correlated with the realization of the probability π at date-2. We will study how this interim information affects lending, investment, and date-1 loan foreclosure policy.

Each entrepreneur is endowed with a quantity of durable asset k_{-1}. The date-0 the budget constraint faced by each entrepreneur is

$$q_0k_0 = q_0k_{-1} + b_0, \tag{2}$$

where q_0 is the date-0 price of the durable asset in terms of consumption goods and b_0 is the amount of borrowing at date-0. The determination of b_0 will be explained below.

We assume that the investment technology is specific to each entrepreneur: if the entrepreneur who has invested at date-0 abandons his project before date-2, the project produces nothing and is left only with its liquidation value; and the entrepreneur is free to walk away from the project. Foreseeing the possibility that entrepreneurs can threaten to walk away from production at date-1, and also that the asset becomes valueless at the end of date-2, creditors do not lend more than the date-1 expected value of the collateral:

$$b_0 \leq \frac{q_1 k_0}{r}, \tag{3}$$

where q_1 is the expected date-1 asset price.[7]

The above borrowing constraint implicitly assumes that entrepreneurs can not divert fixed asset for their own benefit after the initial investment has been in place. On the other hand, we assume that an entrepreneur can divert a fraction $(1 - \theta)$ of the date-2 project return, $0 < \theta < 1$. This says the entrepreneur can at most pledge a fraction θ of total project returns to his creditor. Thus, when the banking sector is competitive and the entrepreneur has strong bargaining power, the entrepreneur can reduce his debt repayment down to a fraction θ of total project returns.[8] This assumption can be justified by the creditor's limited capability to track down run-away borrowers.

Given the above assumptions, the financial contract specifies whether and the extent to roll over loans based on the new information available at date-1. When foreclosure is taken place, creditors seize a fraction or all of the assets and auction them off to the landlords, causing the asset price to fall.

The role of landlords is to serve as a buffer absorbing the changes of entrepreneurs'

[7]See Hart and Moore (1994, 1998) and Kiyotaki and Moore (1997) for more details in the analysis of renegotiation and debt repayment. Empirically, this is consistent with the literature of corporate finance, stating that a weak contractual enforcement is closely associated with credit market constraint. See, for example, La Porta et al. (1997), whereby weak enforcement of shareholder rights explains a great deal of the variation in how firms are funded and owned across countries.

[8]See Holmstrom and Tirole (1998) for a somewhat different derivation of the "pledgeable returns" to creditors. It is the wedge between total project unit return and the entrepreneur's pledgeable unit return that results in a foreclosure rule deviating from the first-best cutoff.

demand for assets. They hold the rest of the asset k' which is not employed by the entrepreneurs. The per period rental rate of the asset for alternative uses is given by

$$u_t \equiv H'(k'_t), \tag{4}$$

where $H(\bullet)$ satisfies the usual neoclassical assumptions, $H'(x) > 0$, $H''(x) < 0$ for all x, and $H'(0) = \infty$ and $H'(\infty) = 0$.

To close the model, we require the asset market clears at each period,

$$\overline{K} = k'_t + k_t,\ \ t = 0, 1. \tag{5}$$

Also, condition (6) governs the behavior of asset prices over time so that no arbitrage opportunity is allowed:

$$q_0 = H'(k'_0) + q_1/r. \tag{6}$$

2.1 Ex post efficiency

Note that at date-1 ex post efficiency requires a project to be stopped if the signal indicates that the project's expected return is smaller than or equal to the liquidation value per unit of the investment,

$$\pi A/r \leq q_1(\pi),$$

and continued if otherwise, where $q_1(\pi)$ is date-1 equilibrium asset price given the observation of the signal. Note that $q_1(\pi) = H'(\overline{K} - k_1(\pi))$, where $k_1(\pi)$ is date-1 investment given π, then the condition can be restated as

$$\pi A/r \leq H'(\overline{K} - k_1(\pi)).$$

Suppose the signal indicates that $\pi A/r > H'(\overline{K} - k_1(\pi))$, then it is optimal to continue the project and thus $k_1(\pi) = k_0$. We denote the threshold value $\overline{\pi}$ such that there will be no liquidation at all when $\pi > \overline{\pi}$, where $\overline{\pi}$ satisfies $\overline{\pi}A/r = H'(\overline{K} - k_0)$. Then we have

$$\overline{\pi} = \frac{rH'(\overline{K} - k_0)}{A}.$$

On the other hand, we denote $\underline{\pi}$ to be such that when $\pi < \underline{\pi}$, the entire project should be liquidated, where the cutoff $\underline{\pi}$ satisfies $\underline{\pi}A/r = H'(\overline{K})$, or

$$\underline{\pi} = \frac{rH'(\overline{K})}{A} < \overline{\pi}.$$

When π is between $\underline{\pi}$ and $\overline{\pi}$, an equilibrium requires that $\pi A/r = H'(\overline{K} - k_1(\pi))$. The date-1 investment $k_1(\pi)$ can be solved accordingly, which is increasing in π. In Figure 1, the solid curve is the locus of date-1 asset price. The set of thresholds $(\underline{\pi}, \overline{\pi})$ can be referred as the ex post efficiency cutoffs.

3 Commitment

The financial contract consists of date-0 investment and date-1 investment given π, $\{k_0, k_1(\pi)\}$, and a foreclosure policy which specifies a set of threshold values of π. When creditors are able to commit, the entrepreneur's problem is to maximize his expected returns, $E\left[(1-\theta)\pi A k_1(\pi)\right]$, subject to the borrower's budget constraint and borrowing constraint, (2) and (3), and the creditor's participation constraint

$$E\left[\pi\theta A k_1(\pi) + r(k_0 - k_1(\pi))q_1(\pi)\right] \geq r^2 b_0,$$

and also the constraint $0 \leq k_1(\pi) \leq k_0$. On the left-hand of the creditor's participation constraint, the first term is the expected project returns accrued to the creditor and the other term is the revenue from liquidating the asset. The right-hand side is the opportunity cost of the funds.

Substitute the budget constraint (2) into creditor's participation constraint and the borrowing constraint (3):

$$\begin{aligned}
&Max. \quad E\left[(1-\theta)\pi A k_1(\pi)\right]/r^2, \\
&s.t. \quad E\left[\pi\theta A k_1(\pi) + r(k_0 - k_1(\pi))q_1(\pi)\right]/r^2 \geq q_0\left(k_0 - k_{-1}\right), && (7)\\
&\qquad q_0 k_{-1} \geq \left(q_0 - q_1/r\right)k_0, && (8)\\
&\qquad k_1(\pi) \leq k_0, \text{ for all } \pi. && (9)\\
&\qquad 0 \leq k_1(\pi), \text{ for all } \pi. && (10)
\end{aligned}$$

Let the Lagrangian multipliers of the four constraints be λ_1, λ_2, $\mu_1(\pi)$ and $\mu_2(\pi)$, respectively.

Proposition 1 *Suppose $E(\pi)\theta A > rq_1$ for any q_1. Then*

(i) the financial contract features $k_1(\pi) = k_0$ for all π;

(ii) the creditor's participation constraint is slack; and

(iii) the entrepreneurs borrow up to the maximum limit of the borrowing constraint.

The first proposition shows that if the creditors' ex-ante expected project return per unit of investment is larger than the expected date-1 collateral value, then the financial contract features no asset liquidation for any realization of π. This occurs when either creditors can better enforce repayments, the public is initially more optimistic about the economy's outlook, or the project's potential unit return is large.

In this case, entrepreneurs borrow up to the maximum limit of the borrowing constraint so that the equilibrium investment is given by

$$k_0 = \frac{q_0 k_{-1}}{q_0 - q_1/r},$$

which says that the quantity equals the entrepreneur's net worth times an investment multiplier which is the reciprocal of the amount of down payment per unit of investment.

We are, however, more interested in the case in which the data-1 investment $k_1(\pi)$ may be lower than the initial investment k_0 for some π. Suppose that $E(\pi)\,\theta A \leq rq_1$ and $\max_\pi \pi\theta A > rq_1$ for some q_1, then $k_1(\pi)$ cannot equal to k_0 for all π. We can show that there exist $\overline{\pi}^c$ such that $k_1(\pi) = k_0$ for $\pi \geq \overline{\pi}^c$, and $k_1(\pi) < k_0$ for $\pi < \overline{\pi}^c$. The superscript is referred to as commitment. By the first order conditions, we have

$$A\pi\left[(1-\theta)+\lambda_1\theta\right]/\lambda_1 r = q_1(\pi) + r\mu_1(\pi)/\lambda_1 \text{ for } \pi \geq \overline{\pi}^c, \quad (11)$$

$$A\pi\left[(1-\theta)+\lambda_1\theta\right]/\lambda_1 r = q_1(\pi) - r\mu_2(\pi)/\lambda_1 \text{ for } \pi < \overline{\pi}^c. \quad (12)$$

Note that for $\pi \geq \overline{\pi}^c$, $k_1(\pi)$ equals k_0 and the date-1 asset price remains at $q_1(\pi) = H'(\overline{K} - k_0)$. Using (11), the critical value $\overline{\pi}^c$ is given by

$$\overline{\pi}^c = \frac{r\lambda_1 H'(\overline{K} - k_0)}{A\left[(1-\theta)+\lambda_1\theta\right]}.$$

Next, when $k_1(\pi)$ is strictly lower than k_0, there are two sub-cases. The first case is when $k_1(\pi)$ drops to zero. When $k_1(\pi) = 0$, the date-1 asset price reaches its lowest level, $q_1(\pi) = H'(\overline{K})$, which no longer depends on π. Together with (12), the condition that $k_1(\pi)$ drops to zero is when π is low enough,

$$\pi < \frac{r\lambda_1 H'(\overline{K})}{A\left[(1-\theta)+\lambda_1\theta\right]} \equiv \underline{\pi}^c.$$

Comparing with $\overline{\pi}^c$, it is immediate that $\underline{\pi}^c < \overline{\pi}^c$. Note that $\overline{\pi}^c$ and $\underline{\pi}^c$ are both increasing in λ_1 and both drop to zero as λ_1 approaches zero. The second case is when $k_1(\pi)$ is strictly positive but smaller than k_0, which arises when $\underline{\pi}^c < \pi < \overline{\pi}^c$. By (12), the date-1 asset price is

$$q_1(\pi) = \frac{\pi A\left[(1-\theta)+\lambda_1\theta\right]}{r\lambda_1}, \text{ for } \underline{\pi}^c < \pi < \overline{\pi}^c. \tag{13}$$

Note that $q_1(\pi)$ is increasing in π and decreasing in λ_1. Since $q_1(\pi) = H'(\overline{K} - k_1(\pi))$, the date-1 investment $k_1(\pi)$ is implied by

$$H'(\overline{K} - k_1(\pi)) = \frac{\pi A}{r\lambda_1}\left[(1-\theta)+\lambda_1\theta\right]. \tag{14}$$

It is straightforward to show that $k_1(\pi)$ is also increasing in π and decreasing in λ_1.[9]

To solve for the endogenous variables, plugging $\overline{\pi}^c$, $\underline{\pi}^c$, (8), (13), and (14) into the creditor's participation constraint (7),

$$\begin{aligned} &E\left(\pi \mid \pi \geq \overline{\pi}^c\right)\theta A k_0 + r k_0 G(\underline{\pi}^c) H'(\overline{K}) + \\ &E\left[\pi\theta A k_1(\pi) + r(k_0 - k_1(\pi)) q_1(\pi) \mid \underline{\pi}^c < \pi < \overline{\pi}^c\right] = r^2 q_0\left(k_0 - k_{-1}\right), \end{aligned}$$

then we can solve for the multiplier λ_1 as a function of date-0 asset price and investment, $\lambda_1 = \lambda_1\left(q_0, k_0\right)$. Next, using (13), we can express the expected date-1 asset price as

$$q_1 = \left[1 - G(\overline{\pi}^c)\right] H'(\overline{K} - k_0) + E\left[\pi \mid \underline{\pi}^c < \pi < \overline{\pi}^c\right] \frac{A\left[(1-\theta)+\lambda_1\theta\right]}{\lambda_1 r} + G(\underline{\pi}^c) H'(\overline{K}).$$

Together with the cutoffs $\overline{\pi}^c$ and $\underline{\pi}^c$, we can solve for $q_1 = q_1\left(k_0, \lambda_1\left(q_0, k_0\right)\right)$. Finally, using (6) and (8), we can solve for q_0 and k_0 in terms of model parameters, θ, r, k_{-1}, $\overline{K}$, and H'. Substituting them back to λ_1, $\overline{\pi}^c$ and $\underline{\pi}^c$, all endogenous variables are solved. Proposition 2 summarizes the above results.

Proposition 2 *Suppose $E\left(\pi\right)\theta A \leq r q_1$, $\max_\pi \pi\theta A > r q_1$ for some q_1, then*

(i) the creditor's participation constraint binds;

(ii) the financial contract features $k_1(\pi) < k_0$ for some π. There exist a set thresholds $(\underline{\pi}^c, \overline{\pi}^c)$ such that $k_1(\pi) = k_0$ for $\pi \geq \overline{\pi}^c$, $0 < k_1(\pi) < k_0$ for $\underline{\pi}^c < \pi < \overline{\pi}^c$, and $k_1(\pi) = 0$ for $\pi < \underline{\pi}^c$.

[9] The sign of the second derviative w.r.t. π is less obvious, $\frac{d^2 k_1(\pi)}{d\pi^2} = \frac{A[(1-\theta)+\lambda_1\theta]}{r\lambda_1 H'''(\overline{K}-k_1(\pi))}\frac{dk_1(\pi)}{d\pi}$, which depends on the curvature of the function $H\left(\cdot\right)$.

Figure 2a illustrates the equilibrium foreclosure policy when $\lambda_1 \in (\underline{\lambda}, 1)$. The line $A\pi\left[(1-\theta)+\lambda_1\theta\right]/r\lambda_1$ is strictly increasing in π, while the dotted curve is the path of date-1 asset price $q_1(\pi)$ which is non-decreasing in π and bounded by $H'(\overline{K})$ and $H'(\overline{K}-k_0)$. By (11) and (12), we have $A\pi\left[(1-\theta)+\lambda_1\theta\right]/r\lambda_1 > q_1(\pi)$ if $\pi > \overline{\pi}^c$, and $A\pi\left[(1-\theta)+\lambda_1\theta\right]/r\lambda_1 < q_1(\pi)$ if $\pi < \underline{\pi}^c$, where $A\pi\left[(1-\theta)+\lambda_1\theta\right]/r\lambda_1$ is the expected return of the project, weighted by the multiplier λ_1. In the region $\underline{\pi}^c < \pi < \overline{\pi}^c$, the weighted expected return of the project must be equal to the equilibrium liquidation value of collateral, and thus any realization of π in this region yields a partial liquidation, $0 < k_1(\pi) < k_0$.

The dotted line in Figure 2b depicts $k_1(\pi)$ for each realization of π. The date-1 investment remains at k_0 as long as $\pi > \overline{\pi}^c$, starts to decline monotonically when π falls below $\overline{\pi}^c$, and drops to zero when $\pi \leq \underline{\pi}^c$.

3.1 Defining forbearance lending

We say that forbearance lending arises when the equilibrium contract specifies a lower level of liquidation for a given realization of the signal, compared with the foreclosure policy under ex post efficiency.

Proposition 3 *(i) If $\lambda_1 > \overline{\lambda}$, then $\underline{\pi} < \overline{\pi} < \underline{\pi}^c < \overline{\pi}^c$, and*

(ii) if $0 < \lambda_1 < \underline{\lambda}$, then $\underline{\pi}^c < \overline{\pi}^c < \underline{\pi} < \overline{\pi}$.,

where $\underline{\lambda} \equiv \frac{(1-\theta)H'(\overline{K})}{H'(\overline{K}-k_0)-\theta H'(\overline{K})} < 1 < \frac{H'(\overline{K}-k_0)}{H'(\overline{K})} \equiv \overline{\lambda}$.

To understand Proposition 3, first starts with the case $\lambda_1 = 1$. Then the weighted expected return of the project, $A\pi\left[(1-\theta)+\lambda_1\theta\right]/r\lambda_1$, reduces to $A\pi/r$, which coincides with expected return of the project under ex post efficiency criterion. Thus, $\underline{\pi}^c = \underline{\pi}$ and $\overline{\pi}^c = \overline{\pi}$. When λ_1 decreases, say, $\lambda_1 \in (\underline{\lambda}, 1)$, then we have $\underline{\pi}^c < \underline{\pi} < \overline{\pi}^c < \overline{\pi}$. This is depicted in Figure 2a. The dotted line is the locus of date-1 asset price with commitment, while the solid line is that of asset price under ex post efficiency criterion. There is an overlapping range between the two sets of cutoffs, but the former curve is everywhere higher than the latter, except on the upper and lower boundaries. This corresponds to the date-1 investments in Figure 2b: the quantity of liquidation under commitment is everywhere smaller than that under ex post efficiency except on the boundaries, suggesting a tendency of forbearance lending.

The intuition behind this is that λ_1 corresponds to the shadow price of ex ante expected net rate of return of creditors' net worth. When λ_1 drops to zero, the creditor's break-even condition is slack and thus the equilibrium contract favors entirely the entrepreneur's objective, as we have seen in Proposition 1. When λ_1 is strictly positive, a lower λ_1 means that the creditor's break-even condition is less binding, thus the weighted expected return of the project gives more weight to the entrepreneur's expected return. This in turn lowers the cutoffs that trigger liquidation. By (13) and (14), it is clear that $dq_1(\pi)/d\lambda_1 < 0$ and $dk_1(\pi)/d\lambda_1 < 0$ for $\underline{\pi}^c < \pi < \overline{\pi}^c$. This says when creditors's break-even condition is less binding, date-1 investment and asset price will be higher, echoing the fact that the cutoffs $(\underline{\pi}^c, \overline{\pi}^c)$ are lower.

The result also can be understood from Figure 2b directly. To see this, suppose we consider the case that the realized signal happens to be $\pi = \underline{\pi}$. Then, the equilibrium arrangement with commitment requires liquidation down to the point C, while the ex post efficiency requires to liquidate all assets (see point D). Suppose now creditors continue to liquidate more, the date-1 asset price will be pushed further downward from point A to B in Figure 2a. Then the date-1 asset price falls below the weighted expected rate of return of the project, $q_1(\pi) < A\pi\left[(1-\theta) + \lambda_1\theta\right]/r\lambda_1$. But this cannot be an equilibrium. In other words, since liquidation depresses the collateral value, the equilibrium requires that liquidation stops at the point when it is not worth selling an additional unit of the asset.

When λ_1 is even smaller, that is $\lambda_1 < \underline{\lambda}$, as depicted in Figure 3a, forbearance lending is particularly acute. For example, when the signal indicates that $\overline{\pi}^c < \pi < \underline{\pi}$, the project should be completely terminated from the viewpoint of ex post efficiency, but will be entirely rolled over when lenders can commit. In contrast, when λ_1 is large enough, say, $\lambda_1 > \overline{\lambda}$, as shown in Figure 3b, the foreclosure policy is completely opposite to the previous case. A moderately poor signal may trigger liquidation under commitment, even when ex post efficiency suggests continuation. This is because now the weighted expected return of the project gives more weight to the creditor's expected return, and thus the foreclosure rule will lean towards the creditor's interest.

Despite the simplicity of the model, it is not possible to assess the impacts of exogenous variables implicitly, because both thresholds $(\underline{\pi}^c, \overline{\pi}^c)$ contain endogenous variables. We therefore use a numerical example to investigate the model's properties of comparative statics. Given a uniform distribution of π and a reasonable concave function for $H(\cdot)$, we demonstrate that λ_1 becomes monotonically is increasing in r, and decreasing in k_{-1}, θ, A, and a first-order stochastic dominance (FSD) shift in the distribution in π. Furthermore, date-0 investment k_0 and asset price q_0 are both increasing in r, k_{-1}, θ, A, and a first-order stochastic dominance (FSD) shift in the distribution in π.

Therefore, when the world interest rate is lower, the entrepreneurs hold more collateral, creditors can better enforce repayments, or the public is initially more optimistic about the economy's outlook, the expected external finance premium λ_1 becomes lower and entrepreneurs are able to borrow more, and at the same time this also bid up the asset price. The pair of thresholds $(\underline{\pi}^c, \overline{\pi}^c)$ with commitment tend to become lower relative to the pair $(\underline{\pi}, \overline{\pi})$, so that forbearance lending emerges as an equilibrium arrangement. Therefore, the realization of a low π may result in only partial liquidation or even rollover of entire loans.

The result that forbearance lending emerges as an equilibrium arrangement corresponds to the observed leniency of creditors to their customers as we have observed in Japan during the 1990s. Recall that in the second half of 1980s, rapidly soaring land price raised the value corporations' collateral and rendered them to acquire more collateral and borrow more for investment. Growth rates were higher and optimism abounded. Together with Japan's main-bank system in which relationship banking or implicit lender-borrower contract is an essential feature, this creates a fertile ground for forbearance lending. Note also that we do not have to resort to the expected government subsidy or rescue to generate this seemingly "soft budget constraint" (SBC) phenomenon.

4 No Commitment

For the equilibrium characterized above to be viable, there must be some commitment technology by which creditors are obligated to the pre-specified foreclosure policy. Suppose creditors cannot commit to the above foreclosure policy, then creditors decide whether and to what extent loans are to be rolled over *after* observing the date-1 signal. We need to solve the equilibrium backwards: creditors set their date-1 foreclosure rule given the signal and entrepreneurs' date-0 investment decisions, and then the financial contract determines the amount of date-0 loans and investment.

4.1 Banks' date-1 decision rules

We first consider the date-1 ex post decision of creditors given the observation of π and entrepreneurs' date-0 borrowing and investment decisions. Given the signal, if the given amount of loan k_0 is rolled over, then the creditor receives an expected value of $\theta\pi A k_0$ at date-2. On the other hand, if a certain fraction or all of loan is recalled, then the creditor receives $q_1(\pi)$ for each unit of asset liquidated and an expected return $\theta\pi A$ for each unit of the remaining investment. This yields an expected return $rq_1(\pi)(k_0 - k_1(\pi)) + \theta\pi A k_1(\pi)$. Comparing these two alternatives, liquidation is better off than no-liquidation if

$$(\theta\pi A - rq_1(\pi))\,(k_0 - k_1(\pi)) \leq 0.$$

Since in this case $k_0 > k_1(\pi)$, we must have $\theta\pi A \leq rq_1(\pi)$. Thus, from the creditors' ex post point of view, the foreclosure policy is specified as follows: roll over ($k_1(\pi) = k_0$) if $q_1(\pi) < \theta\pi A/r$, and liquidate ($k_1(\pi) < k_0$) if $q_1(\pi) \geq \theta\pi A/r$. The latter condition says that if the date-1 equilibrium asset price is greater than or equal to the present value of per unit debt repayment to creditors, then creditors are better off to liquidate the assets. The liquidation will not stop until the condition holds with equality.Since $q_1(\pi) = H'(\overline{K} - k_1(\pi))$, we have $\theta\pi A/r \leq H'(\overline{K} - k_1(\pi))$.

We define the cutoff $\overline{\pi}^n$ to be such that when $\pi > \overline{\pi}^n$, the loan is rolled over and thus $k_1(\pi)$ equals to k_0, then

$$\overline{\pi}^n = \frac{rH'(\overline{K} - k_0)}{\theta A}.$$

When $\pi < \overline{\pi}^n$, the creditor will start to liquidate loans. The superscript is referred to as no-commitment. We also define $\underline{\pi}^n$ such that when $\pi < \underline{\pi}^n$, $k_1(\pi)$ is equal to zero,

$$\underline{\pi}^n = \frac{rH'(\overline{K})}{\theta A} < \overline{\pi}^n.$$

Thus, when the realized π locates inside the range $(\underline{\pi}^n, \overline{\pi}^n)$, the date-1 investment is implied by

$$H'(\overline{K} - k_1(\pi)) = \theta\pi A/r.$$

4.2 Date-0 borrowing and investment decision

At date-0, given the date-1 liquidation policy $(\underline{\pi}^n, \overline{\pi}^n)$, the entrepreneur maximizes the objective,

$$(1-\theta)A\left\{k_0 \int_{\overline{\pi}^n}^{1} \pi dG + \int_{\underline{\pi}^n}^{\overline{\pi}^n} \pi k_1(\pi)\, dG\right\}/r^2,$$

subjective to the creditor's participation constraint,

$$\begin{aligned} s.t. \quad & \theta A k_0 \int_{\overline{\pi}^n}^{1} \pi dG + r k_0 H'(\overline{K}) G(\underline{\pi}^n) + \\ & \int_{\underline{\pi}^n}^{\overline{\pi}^n} \left[\pi\theta A k_1(\pi) + r(k_0 - k_1(\pi)) q_1(\pi)\right] dG \geq r^2 q_0 (k_0 - k_{-1}), \end{aligned} \tag{15}$$

the budget constraint (2), and the borrowing constraint (3).

Note that in the objective function the effect of a change in k_0 on $\overline{\pi}^n$ is given by

$$(1-\theta)A\left[k_0(-\overline{\pi}^n) + \overline{\pi}^n k_1(\overline{\pi}^n)\right]/r^2 \frac{d\overline{\pi}^n}{dk_0}.$$

Since $k_1(\overline{\pi}^n) = k_0$, the effect of a change in k_0 on $\overline{\pi}^n$ is cancelled out. Furthermore, since the date-1 investment is implied by $H'(\overline{K} - k_1(\pi)) = \theta\pi A/r$ for $\pi < \overline{\pi}^n$, we can rewrite the objective function as

$$Max. \quad (1-\theta)A\left\{k_0 \int_{\overline{\pi}^n}^{1} \pi dG + \int_{\underline{\pi}^n}^{\overline{\pi}^n} \pi\left[\overline{K} - (H')^{-1}(\theta\pi A/r)\right] dG\right\}/r^2,$$

which is an increasing and linear function of k_0. Therefore, it is easy to see that in equilibrium the creditors' participation constraint and borrowing constraint are binding.

For $\underline{\pi}^n < \pi < \overline{\pi}^n$ the date-1 asset price is given by $q_1(\pi) = \pi\theta A/r$, substituting into the creditors' participation constraint yields

$$r^2 q_0 k_{-1} = \left[r^2 q_0 - \theta A \int_{\underline{\pi}^n}^{1} \pi dG - r H'(\overline{K}) G(\underline{\pi}^n)\right] k_0,$$

where $\underline{\pi}^n = rH'(\overline{K})/\theta A$. After some manipulation, we can solve for the date-0 investment k_0 by

$$\frac{k_0 - k_{-1}}{k_{-1}} r^2 H'(\overline{K} - k_0) = \theta A \int_{\underline{\pi}^n}^{1} \pi dG + r H'(\overline{K}) G(\underline{\pi}^n). \tag{16}$$

Together with borrowing constraint $q_0 k_{-1} = (q_0 - q_1/r)\, k_0$ and condition (6), these three equations solve for the three endogenous variables, denoted (k_0^n, q_0^n, q_1^n).

The dotted line in Figure 4a illustrates the locus of date-1 asset price under no commitment $q_1^n(\pi)$. The date-1 asset price under ex post efficiency always lie above $q_1^n(\pi)$. It can be immediately observed that

$$\underline{\pi} \equiv \frac{rH'(\overline{K})}{A} < \frac{rH'(\overline{K})}{\theta A} \equiv \underline{\pi}^n, \overline{\pi} \equiv \frac{rH'(\overline{K})}{A} < \frac{rH'(\overline{K} - k_0)}{\theta A} \equiv \overline{\pi}^n.$$

Therefore, the quantity of liquidation under no-commitment is always larger than that under ex post efficiency foreclosure policy. The intuition why the cutoffs with no commitment $(\underline{\pi}^n, \overline{\pi}^n)$ differs from the ex post efficiency cutoffs $(\underline{\pi}, \overline{\pi})$ is that there is a wedge between the project's expected return (πA) and the pledgeable expected return to lenders $(\theta \pi A)$. An entrepreneur can at most pledge to the lender $\theta \pi A$ per unit of investment, due to imperfect enforceability, thus lenders tends to liquidate more than the quantity required by ex post efficiency if they can review their foreclosure policy ex post.

From Figure 4, it can be immediately observed that the result of foreclosure policy with no commitment resembles the case with commitment when λ_1 is large, where there is no forbearance lending. Thus, the observation of no-forbearance-lending may be either due to the fact that creditors are not able to pre-commit and simply follows the wait-and-see policy, or that even with commitment the economy's fundamentals or prospects deteriorate and thus creditors abandon forbearance lending policy.

We conduct some comparative statics analysis. First, to see the effect of a change in θ, note that the direct effect of a better enforceability lowers the cutoff $\overline{\pi}^n$, while the indirect effect encourages lenders to lend more given the same level of collateral, raising both the date-0 investment and asset price ($dk_0^n/\theta > 0$, $dq_0^n/\theta > 0$), and $\overline{\pi}^n$. Which effect dominates depends on model parameters. A change in θ also affect $\overline{\pi}$. We then have $d\overline{\pi}/\theta > |d\overline{\pi}^n/\theta| > 0$, that is, $\overline{\pi}^n$ is less sensitive to a change in θ than $\overline{\pi}$ is. An increase in productivity A has a similar effect on $\overline{\pi}^n$ and $\overline{\pi}$, and again $\overline{\pi}^n$ is less sensitive to changes in A.

Second, a higher entrepreneur' initial holding of collateral k_{-1} increases investment and asset price ($dk_0^n/k_{-1} > 0$, $dq_0^n/k_{-1} > 0$), and at the same time raises $\overline{\pi}^n$, which says the initial booms in the credit market and asset market make it vulnerable to bad news.

Finally, a first-order stochastic dominance (FSD) shift in the distribution of π indicates that initially the public has a more optimistic assessment about the prospects of project returns, which raises both date-0 investment and asset price but also pushes $\overline{\pi}^n$ to a higher level, making it more likely to start liquidating assets. The effect of a FSD shift to $\overline{\pi}$ is similar and again less than that to $\overline{\pi}^n$.

Note that an shift in k_{-1} or FSD also raises $\overline{\pi}$. The effects of a shift in k_{-1} or FSD are presented in Figure 5. Either shift raises the initial investment k_0^n to a higher level. Since the line $\theta\pi A/r$ is flatter than the line $\pi A/r$, the threshold $\overline{\pi}^n$ under no commitment rises to a larger extent than the ex post efficiency threshold $\overline{\pi}$ does. Note that the results of comparative statics analysis regarding to changes in k_{-1}, θ, and FSD to date-0 investment and asset price are the same as the numerical analysis in section 3 where creditors are able to commit. Proposition 4 summarizes the above results.

Proposition 4 *Given the no-commitment equilibrium,*

(1) The creditors always liquidate more under no-commitment than they do under ex post efficiency foreclosure policy, except on the boundaries;

(2) A better enforceability or productivity (higher θ or A) raises both the date-0 investment and asset price and lowers $\underline{\pi}^n$ and $\underline{\pi}$, while the effect to $\overline{\pi}^n$ and $\overline{\pi}$is ambiguous;

(3) A higher entrepreneur' initial holding of collateral k_{-1} or a shift in the distribution of π in the sense of FSD increases the date-0 investment and asset price, and also raises the cutoffs $\overline{\pi}^n$ and $\overline{\pi}$. Also the threshold $\overline{\pi}^n$ rises more than $\overline{\pi}$ does.

5 Discussions and Concluding Remarks

In this paper we show that, as an alternative view to the government's regulatory forbearance, private forbearance can emerge as an equilibrium result with commit when the world interest rate is lower, the entrepreneurs hold more collateral, creditors can better enforce repayments, or the public is initially more optimistic about the economy's outlook. The key to this result is that the endogenous collateral value acts as a safe valve to prevent creditors from liquidating too much.

This explains why creditors sometimes roll over loans even when the returns seem not so promising, without resorting to expected government subsidy or bailout. Our model thus provides an alternative aspect of problems of a bank-based financial system in which loan-making is determined by collateralizable asset's value.

Thus, we may empirically observe that the amount of loans supplied by creditors exhibits a negative correlation with firms' financial stance and collateral value, which may seem counter-intuitive if forbearance lending is not taken into account. Thus, we can test the hypothesis of existence of forbearance lending by controlling for the government's behavior.

References

Aoki, M., Dore, R. (eds.), 1994. The Japanese Firm: The Sources of Competitive Strength. Oxford, UK; Oxford University Press.

Berglof, E., and Roland, G., 1997. Soft Budget Constraints and Credit Crunches in Financial Transition. European Economic Review, 41(4), 807-817.

Bester, Helmut, 1994. The Role of Collateral in a Model of Debt Renegotiation, Journal of Money, Credit, and Banking, 26(1), 72-86.

Browne, L., Rosengren, E. S., 1992. Real Estate and Credit Crunch. Conference Proceedings No.36, Federal Reserve Bank of Boston.

Cargill, T. F., Hutchison, M. M., Ito, T., 1997. The Political Economy of Japanese Monetary Policy. Cambridge and London: MIT Press.

Carlstrom, C. T., Fuerst, T. S., 1997. Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. American Economic Review, 87, 893-910.

Chang, R., Velasco, A., 2001. Financial Crises in Emerging Markets: A Canonical Model. Quarterly Journal of Economics, 116 (2), 489-517.

Chen, N.-K., 2001. Bank Net Worth, Asset Prices and Economic Activity. Journal of Monetary Economics, 48(2), 415-436.

Hart, O., Moore, J., 1998. Default and Renegotiation: A Dynamic Model of Debt. Quarterly Journal of Economics, 113 (1), 1-41.

Hart, O., Moore, J., 1994. A Theory of Debt Based on the Inalienability of Human Capital. The Quarterly Journal of Economics, 109 (4), 841-79.

Holmstrom, B., Tirole, J., 1998. Public and Private Supply of Liquidity. Journal of Political Economy, 106, 1-40.

Hoshi, T., Kashyap, A., 1999. The Japanese Banking Crisis: Where Did It Come From and How Will It End?" NBER Macroeconomics Annual 1999, 14, 129-201.

Hoshi, T., Patrick, H. T., (Eds.), 2000. Crisis and Change in the Japanese Financial System. Boston, MA; Kluwer Academic Publishers.

Huang, H., Xu, C., 1999. Financial Institutions, Financial Contagion, and Financial Crises. Mimeo, London School of Economics.

Kim, Yong Jin and Jong-Wha Lee, 2002. Over-Investment, Collateral Lending, and Economic Crisis. Japan and the World Economy, 14(2), 181-201.

Kiyotaki, N., Moore, J., 1997. Credit Cycles. Journal of Political Economy, 105 (2), 211-48.

Kobayashi, K., Saita, Y., Sekine, T., 2002. Forbearance Lending: A Case for Japanese Firms. Working Paper 02-2, Bank of Japan.

Krugman, P., 1998. What Happened to Asia? Manuscript.

La Porta, R., Lopez-de-Silanes, F., Shleifer, A., Vishny, R., 1997. Legal Determinants of External Finance. Journal of Finance, 52, 1131-1150.

Leung, Charles Ka-Yui, and Tse, Chung Yi, 2002. Increasing Wealth and Increasing Instability: The Role of Collateral. Review of International Economics. 10 (1), 45-52.

Maskin, E., and Xu, C., 2001. Soft Budget Constraint Theories: From Centralization to the Market," Economics of Transition, 9(1), 1-28.

McKinnon, R., Pill, H., 1998. International Overborrowing: A Decomposition of Credit and Currency Risks. World Development, 26 (7), 1267-82.

Mitchell, J., 1998. Strategic Creditor Passivity, Regulation, and Bank Bailouts. CEPR Discussion Paper, No. 1780.

Mitchell, J., 2000. Theories of Soft Budget Constraints and the Analysis of Banking Crises. Economics of Transition, 8(1), 59-100.

Mori, N., Shiratsuka, S., Taguchi, H., 2001. Policy Responses to the Post-Bubble Adjustments in Japan: A Tentative Review. Monetary and Economic Studies, 19, IMES, Bank of Japan, 53-102.

Okina, K., Shiratsuka, S., 2001. Asset Prices Bubbles, Prices Stability, and Monetary Policy: Japan's Experience. IMES Discussion Paper Series 2001-E-16.

Radelet, S., Sachs, J., 1998. The East Asian Financial Crisis: Diagnosis, Remedies, Prospects. Brookings Papers on Economic Activity, 0 (1), 1-74.

Proof of Proposition 1 and 2

The second-best financial contract solves the following problem, denoted as (P1)

$$\begin{aligned}
Max.\ & E_\pi\left[(1-\theta)\pi A k_1(\pi)\right]/r^2,\\
s.t.\ & E_\pi\left[\pi\theta A k_1(\pi)+r(k_0-k_1(\pi))q_1(\pi)\right]/r^2 \geq b_0,\\
& q_0 k_0 = q_0 k_{-1}+b_0,\\
& b_0 \leq \frac{q_1 k_0}{r},\\
& 0 \leq k_1(\pi) \leq k_0.
\end{aligned}$$

Substitute the budget constraint (2) into creditor's participation constraint (7) and the borrowing constraint (3):

$$\begin{aligned}
Max.\ & E_\pi\left[(1-\theta)\pi A k_1(\pi)\right]/r^2,\\
s.t.\ & E_\pi\left[\pi\theta A k_1(\pi)+r(k_0-k_1(\pi))q_1(\pi)\right]/r^2 \geq q_0\left(k_0-k_{-1}\right),\\
& q_0 k_{-1} \geq \left(q_0-q_1/r\right)k_0,\\
& k_1(\pi) \leq k_0,\ \text{for all } \pi,\\
& 0 \leq k_1(\pi),\ \text{for all } \pi.
\end{aligned}$$

$$\begin{aligned}
\mathcal{L} \;=\; & E_\pi\left[(1-\theta)\pi A k_1(\pi)\right]/r^2+\lambda_1\left\{E_\pi\left[\pi\theta A k_1(\pi)+r(k_0-k_1(\pi))q_1(\pi)\right]/r^2-q_0\left(k_0-k_{-1}\right)\right\}+\\
& \lambda_2\left[q_0 k_{-1}-\left(q_0-q_1/r\right)k_0\right]+E_\pi\left[\mu_1(\pi)\left(k_0-k_1(\pi)\right)\right]+E_\pi\left[\mu_2(\pi)k_1(\pi)\right].
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
-\left(q_0-q_1/r\right)\lambda_1-\left(q_0-q_1/r\right)\lambda_2+E_\pi\left[\mu_1(\pi)\right] &= 0, && (17)\\
(1-\theta)\pi A/r^2+\lambda_1\left[\pi\theta A-rq_1(\pi)\right]/r^2-\mu_1(\pi)+\mu_2(\pi) &= 0, && (18)\\
\lambda_1\left\{E_\pi\left[\pi\theta A k_1(\pi)+r(k_0-k_1(\pi))q_1(\pi)\right]/r^2-q_0\left(k_0-k_{-1}\right)\right\} &= 0, && (19)\\
\lambda_2\left[q_0 k_{-1}-\left(q_0-q_1/r\right)k_0\right] &= 0, && (20)\\
\mu_1(\pi)\left(k_0-k_1(\pi)\right) &= 0, && (21)\\
\mu_2(\pi)k_1(\pi) &= 0. && (22)
\end{aligned}$$

A.1 Suppose $\lambda_1 = 0$.

By (18), $(1-\theta)\pi A/r^2-\mu_1(\pi)+\mu_2(\pi)=0$ for all π. This means $\mu_1(\pi)>0$ for all π. Thus, $k_1(\pi)=k_0$ and $\mu_2(\pi)=0$ for all π. Note that $\mu_1(\pi)=(1-\theta)\pi A/r^2$. The multiplier $\mu_1(\pi)$ is the ex post expected rate of return to entrepreneurs per unit of investment at date-1. By (17),

$$\lambda_2=\frac{E(\pi)(1-\theta)A}{\left(q_0-q_1/r\right)r^2}.$$

Thus, by (20), the investment equals

$$k_0 = \frac{q_0 k_{-1}}{q_0 - q_1/r}.$$

By the creditor's participation constraint (7), $k_1(\pi)$ equals k_0 for all π requires that $E(\pi)\,\theta A k_0 > r^2 q_0 (k_0 - k_{-1})$. Plugging in k_0, the condition requires

$$E(\pi)\,\theta A/r > q_1,$$

for any q_1, so that creditors are willing to participate.

A.2 Suppose $\lambda_1 > 0$.

Then, there exist $\overline{\pi}^c$ such that $k_1(\pi) = k_0$ for $\pi \geq \overline{\pi}^c$, and $k_1(\pi) < k_0$ for $\pi < \overline{\pi}^c$, where by (18),

$$\begin{aligned}(1-\theta)\pi A/r^2 + \lambda_1 [\pi\theta A - rq_1(\pi)]/r^2 - \mu_1(\pi) &= 0 \\ \text{for } \pi &\geq \overline{\pi}^c, \text{ with } \mu_1(\pi) > 0, \mu_2(\pi) = 0 \quad (23) \\ (1-\theta)\pi A/r^2 + \lambda_1 [\pi\theta A - rq_1(\pi)]/r^2 + \mu_2(\pi) &= 0 \\ \text{for } \pi &< \overline{\pi}^c \text{ with } \mu_1(\pi) = 0, \mu_2(\pi) \geq 0. \quad (24)\end{aligned}$$

A.2.1 If $\pi \geq \overline{\pi}^c$, then $k_1(\pi) = k_0$.

By (23), the date-1 asset price is given by

$$q_1(\pi) = \frac{\pi\theta A}{r} + \frac{(1-\theta)\pi A - r^2\mu_1(\pi)}{r\lambda_1}.$$

In this case, since $q_1(\pi) = H'(\overline{K} - k_0)$ is a independent of π, the multiplier $\mu_1(\pi)$ can be written as

$$\mu_1(\pi) = \frac{\pi A [(1-\theta) + \lambda_1\theta]}{r^2} - \frac{\lambda_1 H'(\overline{K} - k_0)}{r}.$$

Note that $\mu_1(\pi)$ is continuous in π. For $\pi \geq \overline{\pi}^c$, $\mu_1(\pi) > 0$ and $d\mu_1(\pi)/d\pi > 0$; for $\pi < \overline{\pi}^c$, $\mu_1(\pi) = 0$. Thus, the cutoff $\overline{\pi}^c$ is defined such that $k_1(\pi) = k_0$ for $\pi \geq \overline{\pi}^c$, and $k_1(\pi) < k_0$ for $\pi < \overline{\pi}^c$, and is given by

$$\overline{\pi}^c = \frac{\lambda_1 r H'(\overline{K} - k_0)}{A[(1-\theta) + \lambda_1\theta]}.$$

A.2.2 If $\pi < \overline{\pi}^c$, then $k_1(\pi) < k_0$., and also that

$$\begin{aligned}(1-\theta)\pi A/r^2 + \lambda_1 [\pi\theta A - rq_1(\pi)]/r^2 + \mu_2(\pi) &= 0 \\ \text{if } k_1(\pi) &= 0 \ (\mu_2(\pi) > 0), \text{ and} \quad (25) \\ (1-\theta)\pi A + \lambda_1 [\pi\theta A - rq_1(\pi)] &= 0 \\ \text{if } k_1(\pi) &> 0 \ (\mu_2(\pi) = 0). \quad (26)\end{aligned}$$

(i) If $\mu_2(\pi) = 0$, then $k_1(\pi) > 0$. By (26), the date-1 asset price $q_1(\pi)$ is implied by

$$q_1(\pi) = \frac{\pi\theta A}{r} + \frac{\pi(1-\theta)A}{\lambda_1 r} > \frac{\pi\theta A}{r}, \tag{27}$$

which is increasing in π, and thus the quantity of $k_1(\pi)$ is implied by

$$\pi A\left[(1-\theta) + \lambda_1\theta\right] = \lambda_1 r H'(\overline{K} - k_1(\pi)). \tag{28}$$

Note that $k_1(\pi)$ is also increasing in π.

(ii) If $\mu_2(\pi) > 0$, then $k_1(\pi) = 0$. By (25), the date-1 asset price now becomes

$$q_1(\pi) = \frac{\pi\theta A}{r} + \frac{\pi(1-\theta)A + r^2\mu_2(\pi)}{\lambda_1 r}.$$

When $k_1(\pi) = 0$, the asset price drops to the lowest level, $q_1(\pi) = H'(\overline{K})$, which is a independent of π. The multiplier $\mu_2(\pi)$ can be written as

$$\mu_2(\pi) = \frac{\lambda_1 H'(\overline{K})}{r} - \frac{\pi A\left[(1-\theta)+\lambda_1\theta\right]}{r^2}.$$

Note that $\mu_2(\pi) > 0$ and $\frac{d\mu_2(\pi)}{d\pi} < 0$ for $k_1(\pi) = 0$, and $\mu_2(\pi) = 0$ for $k_1(\pi) > 0$. Thus, the cutoff $\underline{\pi}^c$ is defined such that $k_1(\pi) = 0$ for $\pi < \underline{\pi}^c$, and $k_1(\pi) > 0$ for $\pi \geq \underline{\pi}^c$, and is given by

$$\underline{\pi}^c = \frac{\lambda_1 r H'(\overline{K})}{A\left[(1-\theta)+\lambda_1\theta\right]}.$$

Proof of Proposition 3

$$\begin{aligned}
(1)\underline{\pi} - \underline{\pi}^c &= \frac{rH'(\overline{K})}{A} - \frac{r\lambda_1 H'(\overline{K})}{A\left[(1-\theta)+\lambda_1\theta\right]} \\
&= \frac{rH'(\overline{K})}{A}\left[\frac{(1-\theta)(1-\lambda_1)}{1-\theta+\lambda_1\theta}\right] \gtrless 0, \text{ if } \lambda_1 \lessgtr 1. \\
(2)\overline{\pi} - \overline{\pi}^c &= \frac{rH'(\overline{K}-k_0)}{A} - \frac{r\lambda_1 H'(\overline{K}-k_0)}{A\left[(1-\theta)+\lambda_1\theta\right]} \\
&= \frac{rH'(\overline{K}-k_0)}{A}\left[\frac{(1-\theta)(1-\lambda_1)}{1-\theta+\lambda_1\theta}\right] \gtrless 0, \text{ if } \lambda_1 \lessgtr 1.
\end{aligned}$$

$$\begin{aligned}
(3)\underline{\pi} - \overline{\pi}^c &= \frac{rH'(\overline{K})}{A} - \frac{r\lambda_1 H'(\overline{K}-k_0)}{A\left[(1-\theta)+\lambda_1\theta\right]} \gtrless 0, \text{ if } \lambda_1 \lessgtr \underline{\lambda}. \\
(4)\underline{\pi}^c - \overline{\pi} &= \frac{r\lambda_1 H'(\overline{K})}{A\left[(1-\theta)+\lambda_1\theta\right]} - \frac{rH'(\overline{K}-k_0)}{A} \gtrless 0, \text{ if } \lambda_1 \lessgtr \overline{\lambda}.
\end{aligned}$$

where $0 < \underline{\lambda} \equiv \frac{(1-\theta)H'(\overline{K})}{H'(\overline{K}-k_0)-\theta H'(\overline{K})} < 1$, and $\overline{\lambda} \equiv \frac{H'(\overline{K}-k_0)}{H'(\overline{K})} > 1$.

If λ_1 is large enough to be greater than $\overline{\lambda}$, according to the above conditions, then we have $\underline{\pi} < \overline{\pi} < \underline{\pi}^c < \overline{\pi}^c$. On the other hand, when λ_1 is small such that $0 < \lambda_1 < \underline{\lambda}$, then we have $\underline{\pi}^c < \overline{\pi}^c < \underline{\pi} < \overline{\pi}$.

Proof of Proposition 4

(1) θ and A: An increase in θ lowers $\underline{\pi}^n$, while the effect of a higher θ to $\overline{\pi}^n$ is ambiguous. To see this, note that a higher θ lowers $\underline{\pi}^n$, and thus decreases $G(\underline{\pi}^n)$ and raises $\int_{\underline{\pi}^n}^1 \pi dG$. Furthermore, in the right-hand-side of (16) the increase in the first term $\theta A \int_{\underline{\pi}^n}^1 \pi dG$ compensates more than the decrease in the second term $rH'(\overline{K})G(\underline{\pi}^n)$, because $\theta A\pi \geq rH'(\overline{K})$ for $\pi \geq \underline{\pi}^n$. Thus, the date-0 investment and asset price are both increasing in θ ($dk_0^n/\theta > 0$, $dq_0^n/\theta > 0$). Therefore, the net effect to $\overline{\pi}^n$ is

$$\frac{d\overline{\pi}^n}{d\theta} = -\frac{rH'(\overline{K}-k_0^n)}{\theta^2 A} - \frac{rH''(\overline{K}-k_0^n)}{\theta^2 A}\frac{dk_0^n}{d\theta},$$

where the first term lowers the cutoff because lenders now have better enforceability and the second term raises the cutoff due to the fact that higher initial investment and asset price are more sensitive to the realization of π. The net effect depends on the distribution of π and the functional form of $H'(\cdot)$, as well as parameters. An increase in productivity A has a similar effect.

(2) k_{-1}: It is easy to see from (16) that higher entrepreneur' initial holding of collateral k_{-1} increases investment and asset price ($dk_0^n/k_{-1} > 0$, $dq_0^n/k_{-1} > 0$), and at the same time raises $\overline{\pi}^n$, which says the initial booms in the credit market and asset market make it vulnerable to bad news.

(3) FSD: A shift in the distribution of π in the sense of first-order stochastic dominance (FSD) pushes $\overline{\pi}^n$ to a higher level. To see this, observe that if the distribution of π shifts upward in the sense of FSD, $G(\underline{\pi}^n)$ is lower and $\int_{\underline{\pi}^n}^1 \pi dG$ is larger, and thus the increase in $\theta A \int_{\underline{\pi}^n}^1 \pi dG$ is larger than the decline in $rH'(\overline{K})G(\underline{\pi}^n)$, and by the fact that $\theta A\pi \geq rH'(\overline{K})$ for $\pi \geq \underline{\pi}^n$. This raises RHS of (16) and thus increases date-0 investment and asset price. This, in turn, raises $\overline{\pi}^n$.

Figure 1

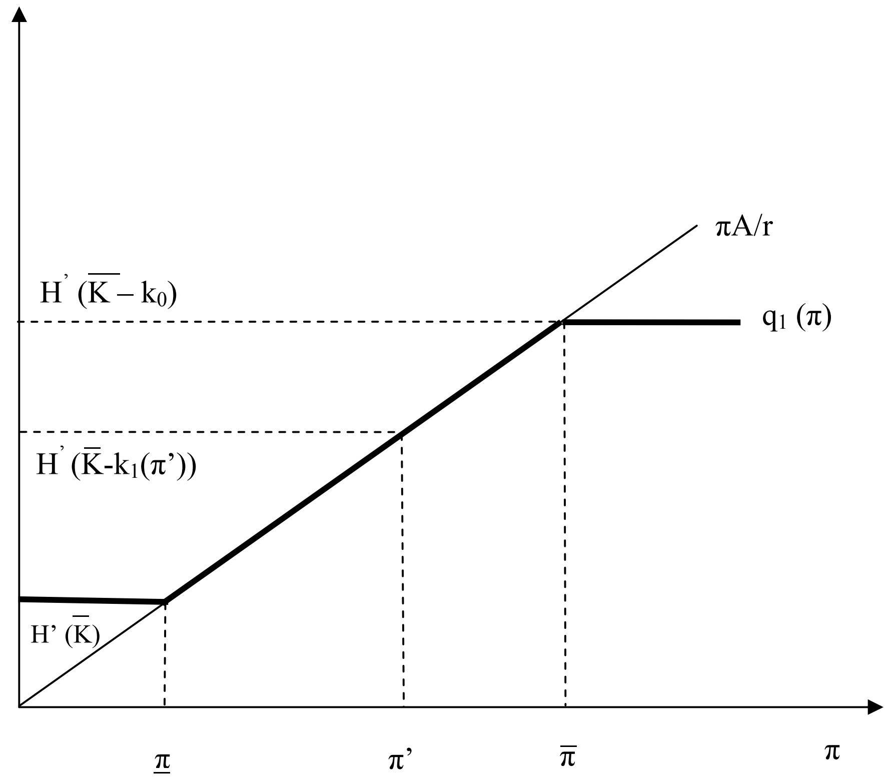


Figure 2a $\lambda_1 \in (\underline{\lambda}, 1)$

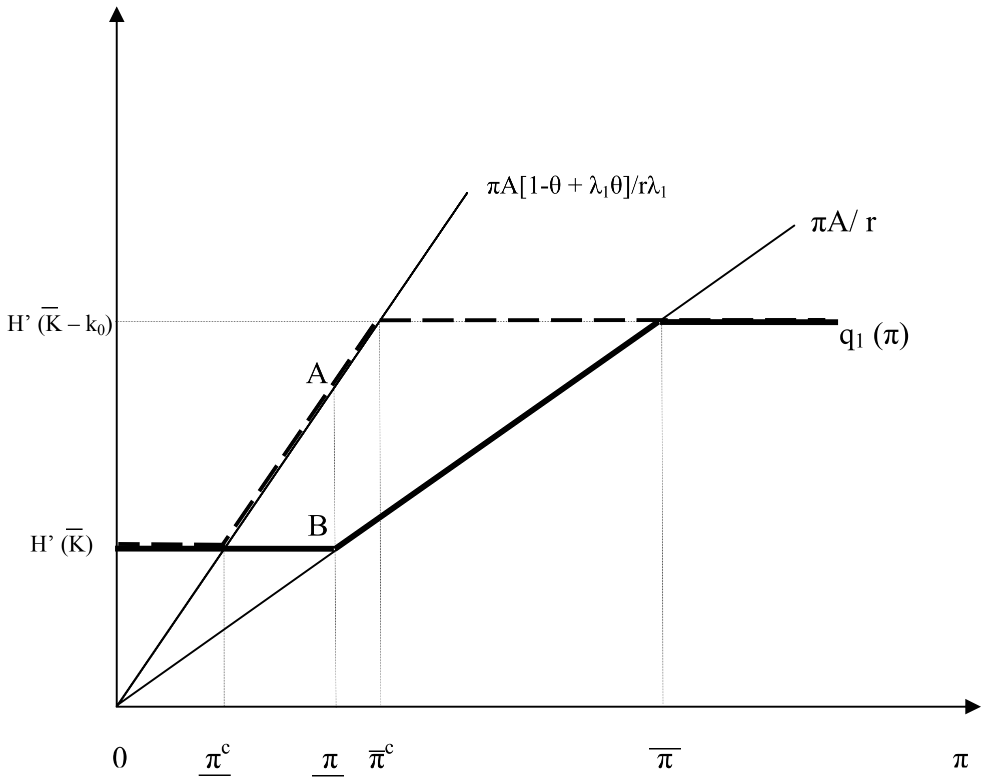


Figure 2b $\lambda_1 \in (\underline{\lambda}, 1)$

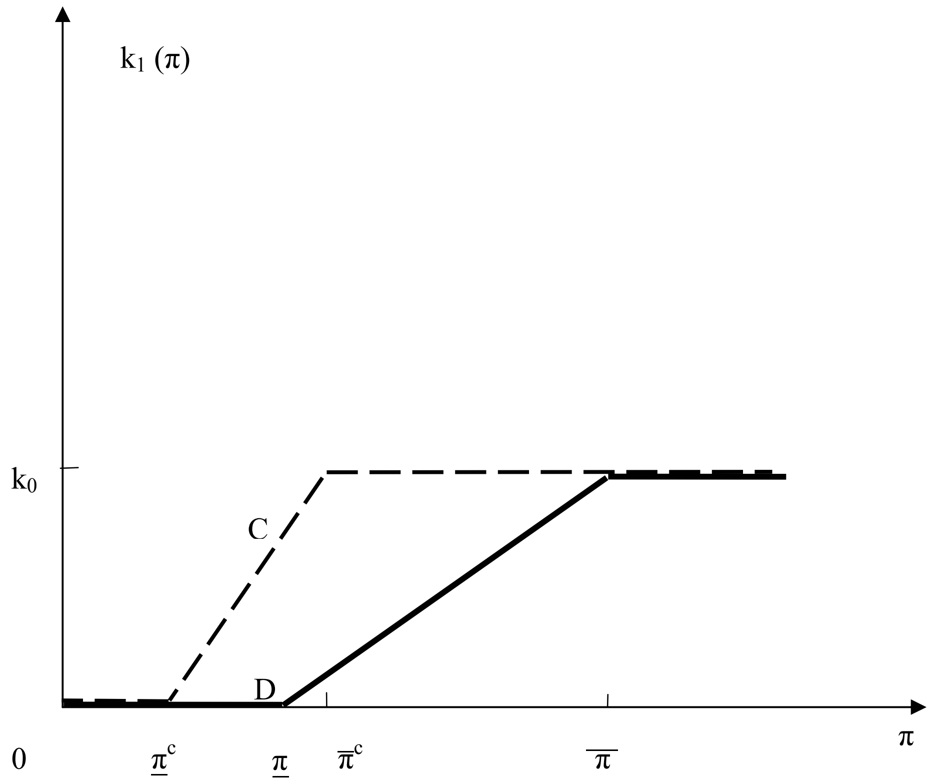


Figure 3a $\lambda_1 < \underline{\lambda}$

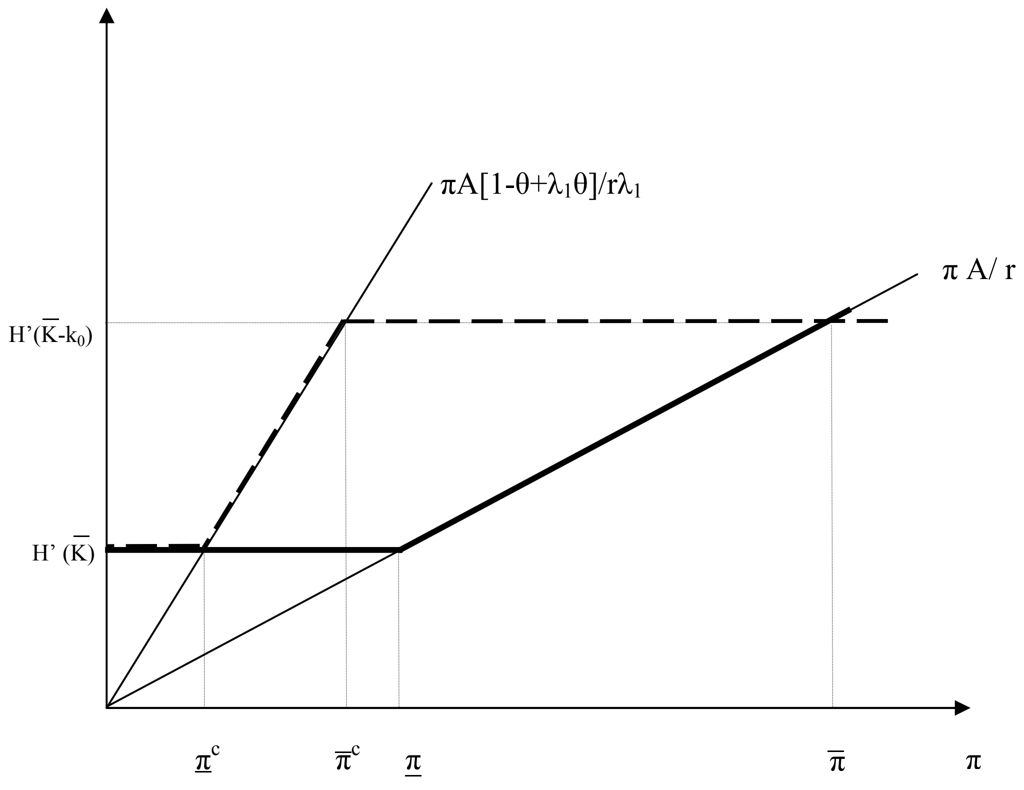


Figure 3b $\lambda_1 > \overline{\lambda}$

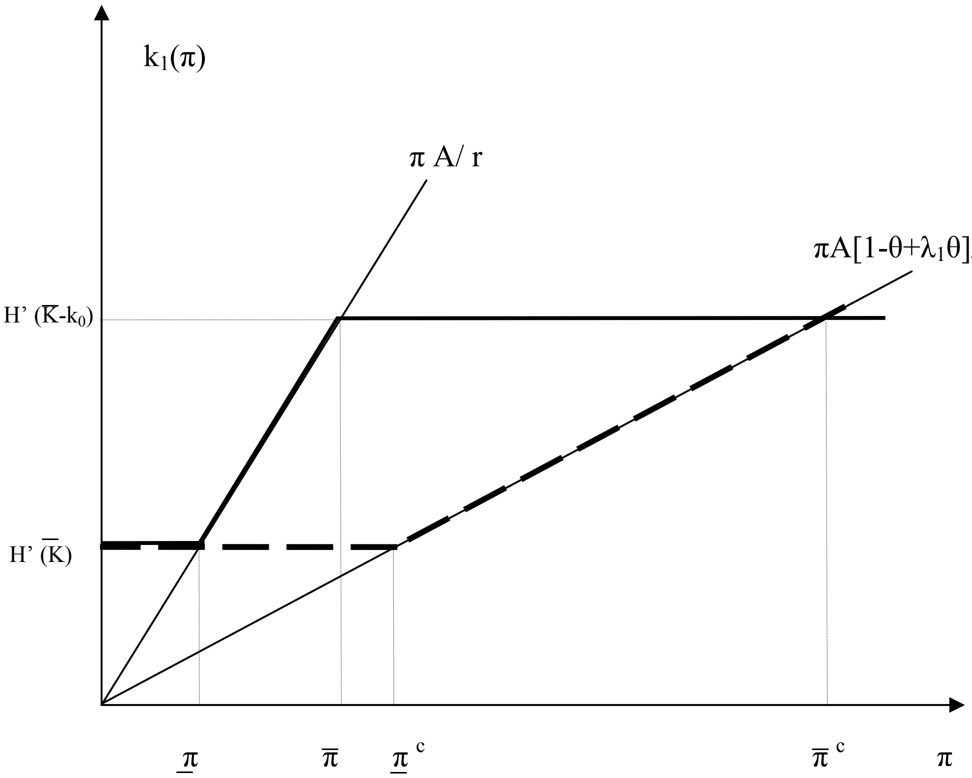


Figure 4a   No Commitment

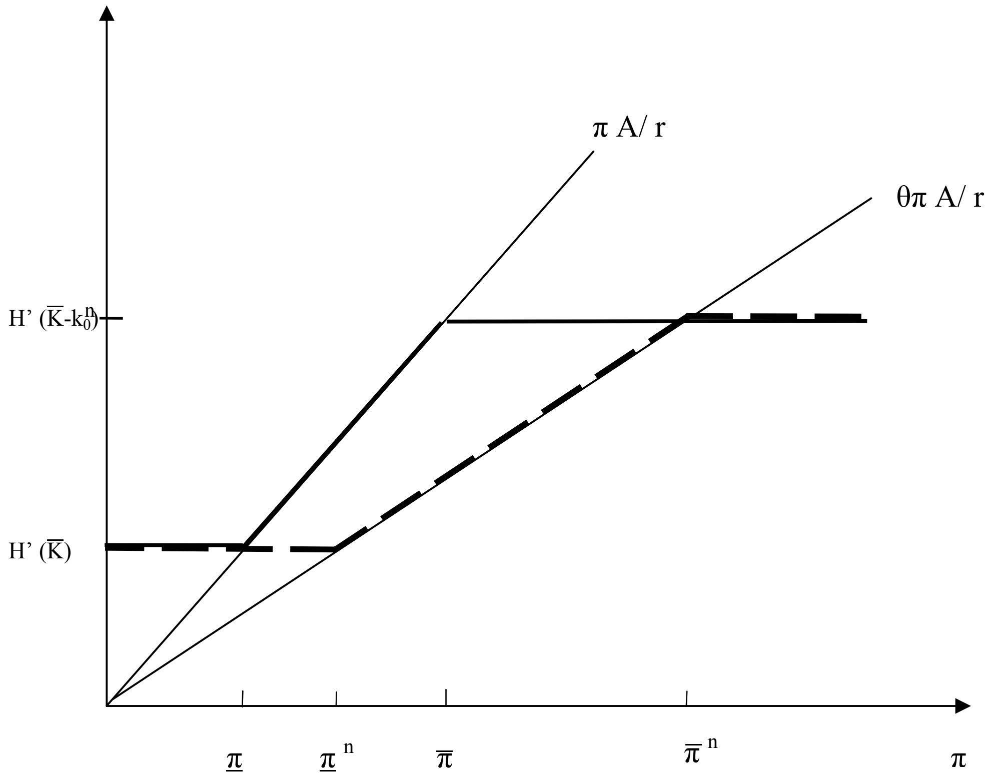


Figure 4b   No Commitment

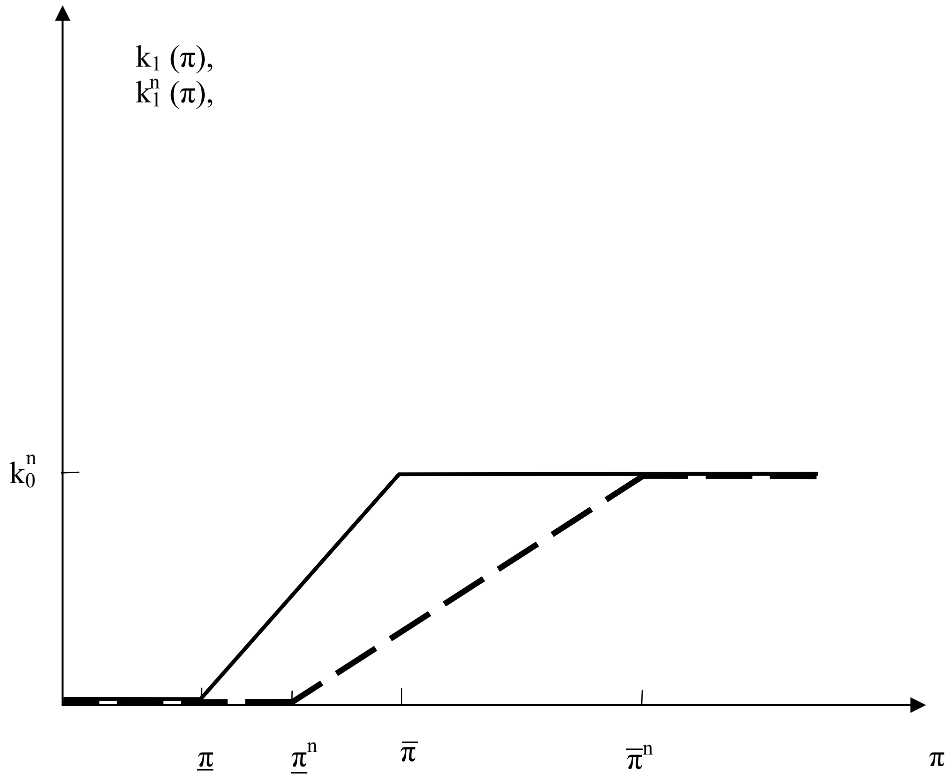
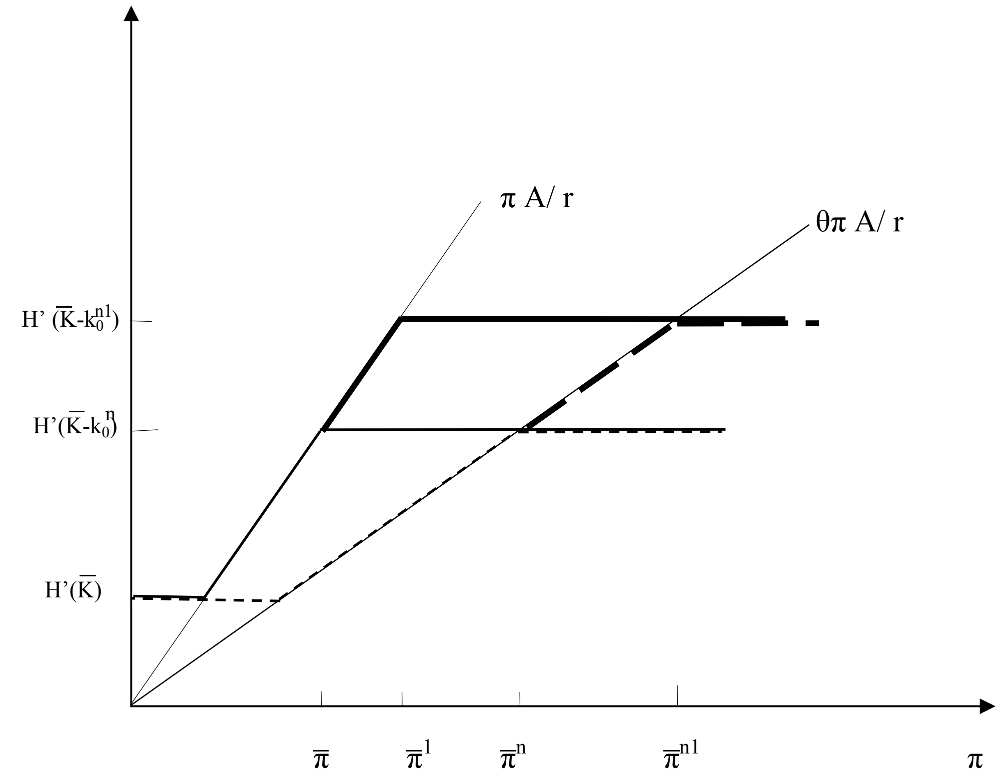


Figure 5   An increase in k_{-1} or a shift in FSD



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