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There Will be Money
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#### Abstract

A common belief among monetary theorists is that monetary equilibria are tenuous due to the intrinsic uselessness of fiat money (Wallace (1978)). In this article we argue that the tenuousness of monetary equilibria vanishes as soon as one introduces a small perturbation in an otherwise standard random matching model of money. Precisely, we show that the sheer belief that fiat money may become intrinsically useful, even if only in an almost unreachable state, might be enough to rule out nonmonetary equilibria. In a large region of parameters, agents' beliefs and behavior are completely determined by fundamentals.


Keywords: fiat money, autarky, equilibrium selection
JEL Codes: E40, D83

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# There will be money 

Luis Araujo and Bernardo Guimaraes

## 1 Introduction

The defining property of fiat money is its intrinsic uselessness. As pointed out by Wallace (1978), a direct implication of this property is the tenuousness of monetary equilibria in the sense that there always exists the possibility that fiat money has no current value because no one expects it to have value in the future. In fact, any model where the frictions that explain the need for fiat money are explicitly laid out and monetary equilibria exists (e.g., overlapping generations, turnpike, search) also exhibits equilibria where money has no value at any date. These nonmonetary equilibria are
usually viewed as natural. They are the confirmation of the idea that valued fiat money is simply a belief driven phenomenon.

In this paper, we argue that the tenuousness of monetary equilibria and the existence of nonmonetary equilibria is not as natural as one may think. In particular, we demonstrate that nonmonetary equilibria are tenuous in the sense that a small perturbation of the intrinsic uselessness of fiat money may eliminate such equilibria. Our analysis is cast in a random matching model of money along the lines of Kiyotaki and Wright (1993) modified by the assumption that there exists an arbitrarily small probability that the economy reaches a state where fiat money acquires intrinsic value. Under this assumption, we show that there exists a large region of parameters where a monetary equilibrium exists but there are no nonmonetary equilibria. The idea is that merely the belief that fiat money may became intrinsically useful, even if only in an almost unreachable state, is enough to rule out nonmonetary equilibria.

Our analysis unveils a natural mapping between agents' behavior and the fundamentals of the economy. In a large region of parameters, agents' beliefs and behavior are solely determined by economic fundamentals, so arbitrary beliefs cannot play any role in selecting an equilibrium. For any level of patience, autarky is the unique equilibrium if the ratio between the utility of consumption and the cost of production is small and money is the unique equilibrium if the same ratio is large. There also exists an intermediate region where the monetary equilibrium coexists with autarky, but this region gets smaller as agents become more patient.

Our paper relates to two strands of literature. First, there are a number of papers that apply a commodity-money refinement to fiat money equilibria with the objective of reducing the set of equilibria (McCandless and Wallace (1991), Zhou (2003), Zhu (2003,2005), Wallace and Zhu (2004)). In these papers, a fiat money equilibrium is said to pass the refinement test if it is the limit of a sequence of commodity-money equilibria when the intrinsic value of the commodity money converges to zero. In contrast to our work, this literature is focused on eliminating monetary equilibria.

Second, this paper is related to the literature on equilibrium selection in coordination games. The literature on global games (Carlsson and Van Damme (1993), Frankel, Morris and Pauzner (2003)) shows that multiplicity of equilibria disappears once the information structure of the game
is slightly perturbed. A related argument applies to dynamic games with complete information where a state variable is subject to shocks (Frankel and Pauzner (2000)). Those papers assume the existence of dominant regions and of strategic complementarities. Both assumptions hold in this model: there exists some state where accepting money is a dominant strategy; and the use of money intrinsically relies on coordination. However, differently from those models, here if there are significant gains from trade, refusing to accept money is never a dominant strategy. Thus, the condition for a unique equilibrium is not that there exists a threshold where an agent is indifferent between two choices, but that an agent strictly prefers one choice (in our case, to accept money) when close to a threshold. ${ }^{1}$

The paper is organized as follows. In the next section we present the model and deliver our main result. In section 3 we present some examples and in section 4 we conclude.

## 2 Model

Our environment is a version of Kiyotaki and Wright (1993). Time is discrete and indexed by $t$. There are $k$ indivisible and perishable goods. The economy is populated by a unit continuum of agents uniformly distributed across $k$ types. A type $i$ agent derives utility $u$ per unit of consumption of good $i$ and is able to produce good $i+1$ (modulo $k$ ) at a unit cost of $c$, with $u>c$. Agents maximize expected discounted utility with a discount factor $\beta \in(0,1)$. There is also a storable and indivisible object, which we denote as money. An agent can hold at most one unit of money at a time, and money is initially distributed to a measure $m$ of agents.

Trade is decentralized and agents face frictions in the exchange process. We formalize this idea by assuming that there are $k$ distinct sectors, each one specialized in the exchange of one good. Agents observe the sectors but inside each sector they are anonymously and pairwise matched under a uniform random matching technology. Each agent faces one meeting per period, and meetings are independent across agents and independent over time. Thus, if an agent wants money he goes to the sector which trades the good he produces and searches for an agent with money. If he has money he goes to the sector that trades the good he likes and searches for an agent with the good.

[^1]Due to the unit upper bound on money holdings, a transaction can happen only when an agent with money (buyer) meets an agent without money (seller).

We assume that, in any given period, the economy is in some state $z \in \mathbb{Z}$. States evolve according to the random process $z_{t}=z_{t-1}+\Delta z_{t}$, where $\Delta z_{t}$ follows a probability distribution that is independent of $t$, with expected value $E(\Delta z)$. We further assume that there exists a state $\bar{z}$ such that money has no intrinsic value for all $z<\bar{z}$, and it generates a flow value $\gamma$ for all $z \geq \bar{z}$.

### 2.1 Benchmark

We initially consider the problem of an agent when $\Delta z_{t}=0$ for all $t$, and the economy starts in some state $z<\bar{z}$. In this case, the economy never reaches a state where money has intrinsic value. First, there always exists an equilibrium where agents do not accept money, and the economy is in permanent autarky. Now, assume that an agent believes that all other agents always accept money. Let $V_{0, z}$ be his value function if he does not have money and the state is $z$, and let $V_{1, z}$ be the corresponding value function if he has money and the state is $z$. We have

$$
V_{1, z}=m \beta V_{1, z}+(1-m)\left(u+\beta V_{0, z}\right),
$$

and

$$
V_{0, z}=m\left[\sigma\left(-c+\beta V_{1, z}\right)+(1-\sigma) \beta V_{0, z}\right]+(1-m) \beta V_{0, z},
$$

where $\sigma \in[0,1]$ is the probability that the agent accepts money. Assume that $\sigma=1$. This implies that

$$
V_{1, z}-V_{0, z}=(1-m) u+m c
$$

It is indeed optimal to always accept money as long as $-c+\beta V_{1, z} \geq \beta V_{0, z}$, i.e.,

$$
\begin{equation*}
\beta[(1-m) u+m c] \geq c . \tag{1}
\end{equation*}
$$

In summary, as long as (1) holds, the economy exhibits multiple equilibria. ${ }^{2}$

[^2]
### 2.2 General Case

Throughout, we assume that

$$
\begin{equation*}
\gamma /(1-\beta)>c . \tag{2}
\end{equation*}
$$

This ensures that for some large enough $z$, an agent will always accept money, regardless of his belief about the behavior of other agents. For simplicity, it is assumed that $\gamma /(1-\beta)<u$, which ensures that an agent always accepts to exchange the good he values for money.

First, note that the use of money exhibits strategic complementarities. Precisely, for any pair of states $\left(z, z^{\prime}\right) \in \mathbb{Z}^{2}$, the agent's payoff from holding money in state $z$ is increasing in the measure of agents that accept money in state $z^{\prime}$. In fact, while the flow value of money is independent of the number of agents that accept money, the exchange value of money is an increasing function of the number of agents that also accept money.

Fix some state $z \in \mathbb{Z}$ and let $\phi(t)$ denote the probability that any state larger than $z$ will be reached for the first time in period $t$. We are ready to present our main result.

Proposition 1 If

$$
\begin{equation*}
\left(\sum_{t=1}^{\infty} \beta^{t} \phi(t)\right)[(1-m) u+m c] \geq c, \tag{3}
\end{equation*}
$$

then there exists a unique equilibrium. In this equilibrium, money is always accepted.

Proof. First, note that for some $z^{M} \gg \bar{z}$, an agent will find it optimal to accept money even if everyone else does not. Precisely, as $z \rightarrow \infty$, the payoff of holding money if no one else accepts it approaches $\gamma / 1-\beta$, which is larger than the cost of getting money, $c$, given the assumption in (2). Now, since the use of money exhibits strategic complementarities, it is a dominant strategy for all agents to accept money for $z>z^{M}$.

The proof is done by induction, where at each step strictly dominated strategies are eliminated. Starting from a threshold $z^{*}$, it is shown that an agent finds it optimal to accept money at state $z^{*}-1$, even if money has no intrinsic value. Then it is argued that incentives for accepting money are even larger if money has an intrinsic value.

Suppose that money has no intrinsic value and that all agents accept money if and only if $z \geq z^{*}$. We need to compare the payoff of such an agent with the one received by someone who
accepts money if $z \geq z^{*}-1$. An agent that accepts money in exchange for his good at state $z^{*}-1$ obtains

$$
-c+\sum_{t=1}^{\infty} \beta^{t} \phi(t)\left(\sum_{i=0}^{\infty} \pi\left(z^{*}+i \mid t_{\phi}=t\right) V_{1, z^{*}+i}\right) \equiv V_{z^{*}-1}^{a}
$$

where $t_{\phi}$ denotes the period a state larger than $z^{*}-1$ is reached and $\pi\left(z \mid t_{\phi}=t\right)$ denotes the probability that the state $z$ is reached conditional on $t_{\phi}$ equal to $t$. Since no agent is accepting money when $z \leq z^{*}-1$, the money received by the agent will not be useful until a state $z$ larger than $z^{*}-1$ is reached. When a state $z$ larger than $z^{*}-1$ is reached, the agent's value function is $V_{1, z}$. The term in brackets is the average of such value functions, weighted by their probabilities. The expected payoff of an agent that accepts money equals the discounted value of such averages, weighted by their own probabilities, minus $c$. Now, if an agent does not accept money at state $z^{*}-1$, he obtains

$$
\sum_{t=1}^{\infty} \beta^{t} \phi(t)\left(\sum_{i=0}^{\infty} \pi\left(z^{*}+i \mid t_{\phi}=t\right) V_{0, z^{*}+i}\right) \equiv V_{z^{*}-1}^{n} .
$$

This implies that the agent accepts money at state $z^{*}-1$ as long as

$$
\begin{equation*}
V_{z^{*}-1}^{a}-V_{z^{*}-1}^{n}=-c+\sum_{t=1}^{\infty} \beta^{t} \phi(t)\left(\sum_{i=0}^{\infty} \pi\left(z^{*}+i \mid t_{\phi}=t\right)\left[V_{1, z^{*}+i}-V_{0, z^{*}+i}\right]\right) \geq 0 . \tag{4}
\end{equation*}
$$

Now, since all other agents are accepting money at any state $z>z^{*}-1$, the value function of an agent with money at some state $z>z^{*}-1$ is

$$
V_{1, z}=m \beta E_{z} V_{1}+(1-m)\left(u+\beta E_{z} V_{0}\right),
$$

while the value function of an agent without money at some state $z>z^{*}-1$ is

$$
V_{0, z}=m\left(-c+\beta E_{z} V_{1}\right)+(1-m) \beta E_{z} V_{0},
$$

where $\beta E_{z} V_{1}$ and $\beta E_{z} V_{0}$ are the expected value of holding, respectively, one and zero unit of money at the end of the period, when the current state is $z$. Now, note that

$$
\begin{equation*}
V_{1, z}-V_{0, z}=(1-m) u+m c . \tag{5}
\end{equation*}
$$

Substituting 5 into 4 yields

$$
V_{z^{*}-1}^{a}-V_{z^{*}-1}^{n}=-c+\sum_{t=1}^{\infty} \beta^{t} \phi(t)[(1-m) u+m c] \geq 0
$$

that is

$$
\sum_{t=1}^{\infty} \beta^{t} \phi(t)[(1-m) u+m c] \geq c
$$

This is so because $\sum_{i=0}^{\infty} \pi\left(z^{*}+i \mid t_{\phi}=t\right)=1$ and $(1-m) u+m c$ is a constant.
The above reasoning has been done assuming that agents will not accept money at states smaller than $z^{*}$, but if that were not the case, incentives for holding money would only increase, owing to the strategic complementarities in using money. Hence, if condition (3) holds, accepting money at state $z^{*}-1$ is a strictly dominant strategy given that all agents are accepting money at states larger or equal to $z^{*}$ and money has no intrinsic value.

Now, if money has intrinsic value $\gamma$ for some $z \leq z^{*}$, then the difference $V_{z^{*}-1}^{a}-V_{z^{*}-1}^{n}$ can only increase. In fact, while the expression for $V_{z^{*}-1}^{n}$ is the same as above, the expression for $V_{z^{*}-1}^{a}$ has to be modified to include the positive utility flow from holding money. So, condition (3) suffices to rule out not accepting money at $z^{*}-1$ if all agents are accepting money if $z \geq z^{*}$, regardless of whether we are in a region where money has intrinsic value.

As (i) it is a strictly dominant strategy to accept money if $z \geq z^{M}$, and (ii) for all $z^{*}$, if all agents accept money whenever $z \geq z^{*}$, accepting money at $z=z^{*}-1$ is a strictly dominant strategy, accepting money is the only strategy that survives iterative elimination of strictly dominated strategies.

Under the belief that everyone will always accept money, an agent find it optimal to accept money if the cost of producing the good $(c)$ is smaller than the benefit $(u(1-m)+m c)$ discounted by the discount rate $\beta$, because money can only be used in the next period (condition (1)). The condition for the monetary equilibrium being the unique equilibrium in the model (equation (3)) substitutes the discount factor $\beta$ by a weighted average of $\beta^{t}$ for all $t$. The weights come from the following exercise: assuming that at time 0 the economy is at state $z^{*}-1$, and everyone accepts money at states $z \geq z^{*}$, the weight of $\beta^{t}$ is the probability of reaching a state where money is accepted at time $t$ (and not before). If that condition holds, there cannot be a threshold $z^{*}$ such that money is accepted only if $z \geq z^{*}$, because agents would find it optimal to accept money at state $z^{*}-1$.

Owing to the intrinsic value of money in some states of the economy, there exists a range of
parameters such that holding money is a dominant strategy. This dominant region rules out beliefs that autarky is an equilibrium at all states: in the worst case (for money), there will be a threshold such that money is accepted only if the economy is above that threshold. That is the only part of the argument where the intrinsic value of money plays an important role. Then, if condition (3) holds, that allows us to rule out autarky at every state by iterative deletion of strictly dominated strategies. There is no mention of the intrinsic value of money in condition (3) because autarky is ruled out even at states where the expected discounted value of the intrinsic value of money is zero.

Remark 1 Proposition 1 holds even if the probability that $\bar{z}$ is ever reached is arbitrarily small.

The probability that $\bar{z}$ will ever be reached depends on the stochastic process of $\Delta z$. If the expected value of $\Delta z$ is non-negative, $\bar{z}$ will eventually be reached with probability 1 regardless of how far $z_{0}$ is from $\bar{z}$. In contrary, if $E(\Delta z)$ is negative, then the probability that $\bar{z}$ will ever be reached depends on how far $z_{0}$ is from $\bar{z}$. If this distance is large enough, the probability that $\bar{z}$ can ever be reached is arbitrarily small.

Such long term probabilities are not important in the computation for the condition of a unique equilibrium in (3). All that matters for that condition is the probability $\phi_{z}(t)$ of reaching a nearby state in the following periods, while the discount rate is still not too low. Hence, two very similar stochastic processes, one with $E(\Delta z)=0$ and another with a slightly negative $E(\Delta z)$ will yield a very similar condition for a unique equilibrium, although the difference between the probabilities of ever reaching $\bar{z}$ can be arbitrarily close to $1 .{ }^{3}$

## 3 Examples

In order to make easy the comparison between conditions (1) and (3), it is worth rewriting (3) as

$$
\begin{equation*}
\lambda \beta[(1-m) u+m c] \geq c \tag{6}
\end{equation*}
$$

[^3]so that the only difference between the condition for existence of a monetary equilibrium in (1) and the condition for the existence of no other equilibrium in (6) is the factor
$$
\lambda=\sum_{t=1}^{\infty} \beta^{t-1} \phi(t)
$$
$\lambda$ is a number between 0 and 1. $\lambda=0$ means that autarky is always an equilibrium, while $\lambda=1$ implies that autarky can only be an equilibrium if the monetary equilibrium does not exist.

### 3.1 Binary process

Consider a simple stochastic process where

$$
\operatorname{Pr}(\Delta z=1)=p \text { and } \operatorname{Pr}(\Delta z=-1)=1-p
$$

Departing from state $z^{*}-1$ in period 0 , the probability of reaching state $z^{*}$ in period 1 is $p$. Otherwise, the economy moves to state $z^{*}-2$. Then, state $z^{*}$ can only be reached in period 3 . The stochastic process until state $z^{*}$ is reached is illustrated in Figure 1. The probabilities that state $z^{*}$ will be reached at time $t$ are given by

$$
\phi(2 i+1)=\frac{(2 i)!}{i!(i+1)!} p^{i+1}(1-p)^{i}
$$

and $\phi(2 i)=0$ for all $i \geq 0$. The formula for $\phi(2 i+1)$ resembles a binomial distribution, but the usual combination is replaced with the Catalan Numbers. ${ }^{4}$ The value of $\lambda$ is given by

$$
\begin{equation*}
\lambda=\sum_{i=0}^{\infty} \beta^{(2 i)}\left(\frac{(2 n)!}{n!(n+1)!} p^{i+1}(1-p)^{i}\right) \tag{7}
\end{equation*}
$$

3.1.1 The case $p=0.5$

If $p=0.5, \lambda$ becomes:

$$
\begin{equation*}
\lambda=\sum_{i=0}^{\infty} \beta^{(2 i)}\left(\frac{(2 i)!}{i!(i+1)!}\left(\frac{1}{2}\right)^{2 i+1}\right) \tag{8}
\end{equation*}
$$

which is a function of $\beta$ only. Figure 2 shows $\lambda$ as a function of $\beta$. The factor $\lambda$ depends on the discounted sum of probabilities that state $z^{*}$ will be reached when departing at time 0 from $z^{*}-1$. As $\beta$ approaches $0, \lambda$ approaches 0.5 , which is the probability that $z^{*}$ will be reached in the next

[^4]

Figure 1: Binary process


Figure 2: Binary process, $p=0.5$
period. The probabilities that $z^{*}$ will be reached after 3,5 or more periods are not important if $\beta$ is small. But as $\beta$ approaches 1 , the discounted sum of probabilities that state $z^{*}$ will be reached approaches the sheer probability that the economy will at some point be at $z^{*}$, and if $p=0.5$, that probability is 1 . As $\beta$ approaches 1 , the region where the only equilibrium is the monetary one converges to the region where the monetary equilibrium exists. Thus agents' behavior is completely determined by fundamentals and sunspots play no role.


Figure 3: Binary process, $p=0.5$

The conditions for existence and uniqueness for a monetary equilibrium in (1) and (3), respectively, depend on $\beta, m, u$ and $c$. Normalizing $c=1$ and assuming $m=1 / 2$, which maximizes
the amount of exchanges, the possible equilibria are drawn in figure 3. The dotted curve depicts the condition for existence of a monetary equilibrium and the full curve shows the condition for the monetary equilibrium being unique. Below the dotted curve, autarky is the only equilibrium, between both curves both autarky and the monetary equilibrium exist, and above the full curve autarky cannot be an equilibrium. Autarky is not an equilibrium if the ratio $u / c$ is large enough. If $\beta$ is crucially determined by the frequency agents meet, then if agents meet often enough ( $\beta$ is high) and there are gains from trade ( $\frac{u}{c}$ is larger than one), money will arise in equilibrium.


Figure 4: Binary process

### 3.1.2 The case $p<0.5$

Assume now that $p=0.5-\varepsilon$. For a sufficiently small $\varepsilon$, a value of $\lambda$ very similar to that implied by (8) would be obtained, but the probability that $\bar{z}$ would ever be reached could be made arbitrarily close to 0 for some $z_{0}$. For lower values of $p$, the factor $\lambda$ is given by equation (7). Figure 4 shows the relation between $\beta$ and $\lambda$ for different values of $p$. As before, as $\beta$ approaches $0, \lambda$ approaches $p$. However, as $\beta$ approaches $1, \lambda$ does not approach 1 since there is a positive probability, bounded away from zero, that $z^{*}$ will never be reached by a process departing from $z^{*}-1$. The factor $\lambda$ is still increasing in $\beta$, owing to the fact that the late arrivals at $z^{*}$ are worth more for larger $\beta$.

### 3.2 Normal random walk

The analysis up to now has considered a discrete state space, but the results are easily extended to a continuous state space, when $z$ can be any real number. Suppose the economy in period 0 is at state $z^{*}$ and denote by $\varphi(t)$ the probability that a state $z \geq z^{*}$ will be reached at time $t$, and not before (after $t=0$ ). An argument identical to 1 yields the following condition for a unique equilibrium:


Figure 5: Normal process

$$
\left(\sum_{t=1}^{\infty} \beta^{t} \varphi(t)\right)[(1-m) u+m c]>c
$$

which is a version of $(3)$ with $\varphi(t)$ instead of $\phi(t)$. While in the discrete case we need to consider the probability that state $z^{*}$ will be reached if we depart from $z^{*}-1$, in the continuous case we consider the probabilities of reaching $z^{*}$ when the economy starts at a state that is arbitrarily close to $z^{*}$. If at that state not accepting money is not optimal, then the iterative process of elimination of strictly dominated strategies goes through, exactly as in 1 . The value of $\lambda$ is

$$
\lambda=\sum_{t=1}^{\infty} \beta^{t-1} \varphi(t) .
$$

Figure 5 compares the relation between $\lambda$ and $\beta$ for a binary process and a normal process assuming $E(\Delta z)=0$ in both cases. The probabilities $\varphi(t)$ for the normal case are obtained from Monte Carlo simulations. While the main characteristics of the curves are the same, the factor $\lambda$ is higher when $\Delta z$ follows a normal stochastic process. Intuitively, it is "easier" to arrive at a state larger than $z^{*}$
starting at (or close enough to) $z^{*}$ and following a normal random walk than starting at $z^{*}-1$ and following a binary process.

## 4 Conclusion

A widely held view among monetary theorists is that monetary equilibria are tenuous due to the intrinsic uselessness of fiat money (Wallace (1978)). In contrast, nonmonetary equilibria are resilient, and their existence underpins the fact that valued fiat money is purely a belief driven phenomenon. In this article we argue that tenuousness of monetary equilibria and the resilience of nonmonetary equilibria vanishes as soon as one introduces a small perturbation in an otherwise standard random matching model of money. We have chosen to present our analysis in a random matching model along the lines of Kiyotaki and Wright (1993) due to its tractability. However, we believe that our message is more general than that. More specifically, we conjecture that related results can be obtained in any environment with a region where accepting money is a dominant strategy, no matter how unlikely reaching this region is.

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[^1]:    ${ }^{1}$ In this sense, the argument resembles that in Rubinstein (1989), although this is a dynamic model with complete information and many players (as Frankel and Pauzner (2000)).

[^2]:    ${ }^{2}$ Kiyotaki and Wright (1993) prove that there exists an equilibrium where agents accept money with probability between zero and one. A similar equilibrium also exists here.

[^3]:    ${ }^{3}$ Under the usual assumption of common knowledge of rationality, the distance between the current state $z_{0}$ and $\bar{z}$ can be disregarded from the analysis. That distance could have some effect on the conditions if boundedly rational agents were not able to think too far ahead, for example.

[^4]:    ${ }^{4}$ See, e.g., http://mathworld.wolfram.com/CatalanNumber.html.

