Debt, Financial Fragility and Economic Growth: A Post-Keynesian Macromodel[♦]

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Abstract: It is developed a mathematical post-keynesian macromodel of capacity utilization and growth in which the supply of credit-money is endogenous and firms' debt position – and thus the financial fragility of the economy – is explicitly modeled. Both the influence of interest rate and indebtedness on capacity utilization and the rates of profit and growth, on the one hand, and the effect of the parameters of the saving and investment functions on financial fragility, on the other hand, are carefully analyzed.

Key words: indebtedness; financial fragility; growth

Resumo: O artigo desenvolve um macromodelo matemático pós-keynesiano de utilização e crescimento da capacidade, no qual a oferta de crédito é endógena e o endividamento das firmas – e, assim, a fragilidade financeira da economia – é explicitamente modelado. São analisadas cuidadosamente a influência da taxa de juros e do grau de endividamento na utilização da capacidade e nas taxas de lucro e de crescimento, de um lado, e a influência dos parâmetros da função investimento das firmas e da propensão a poupar dos capitalistas na fragilidade financeira da economia, de outro lado.

Palavras-chave: endividamento; fragilidade financeira; crescimento

JEL classification: E12; E22
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1. Introduction

The paper develops a post-keynesian macromodel of capacity utilization and growth in which the supply of credit-money is endogenous and firms' debt position – and thus the financial fragility of the economy à la Minsky – is explicitly modeled. Both the influence of interest rate and indebtedness on capacity utilization and the rates of profit and growth, on the one hand, and the effect of the parameters of the saving and investment functions on financial fragility, on the other hand, are carefully analyzed.

Following the post-keynesian monetary approach, the supply of credit-money at any point in time is endogenous, demand-driven at an exogenously given nominal interest rate. The underlying presumption is that at any given point in time banks are price makers and quantity takers in their markets for loans, and price takers and quantity makers in the markets where they raise funding. Saving, in turn, is generated by and concurrently with investment. The choice variable is therefore investment rather than saving, with investment being financed by credit-money generated by entrepreneurial borrowing from the banking system and not by prior saving.

More broadly, and in line with the financial instability hypothesis developed by Minsky (1975, 1982), the capital development of the economy is conceived of as being accompanied by exchanges of present money for future money. While the present money pays for resources that are used in the production of investment output, the future money is the amount of profits that will accrue to firms as their capital assets are used in production. As a result of the process through which investment by producing units is financed, the liabilities on their balance sheet determine a series of prior payment commitments, while their assets generate a series of conjectured cash inflows. With investment being financed by credit-money generated by entrepreneurial borrowing from the banking system, the flow of money to firms is a response to expectations of future profits, while the flow of money from firms is financed by profits that are actually realized.

As it turns out, even tough a rise in the real wage will lead to an increase in capacity utilization, it will nonetheless leave the growth rate unchanged. The impact of a change in the interest rate on the rates of capacity utilization and growth is negative (ambiguous) in case financial and productive capitalists have a common (different) saving propensity. In turn, a change in the ratio of debt to capital has an impact (nonetheless ambiguous) on capacity utilization and growth only in case those saving propensities are different.

Moreover, it is formally derived a version of the minskyan taxonomy of finance regimes or postures (hedge, speculative and ponzi). What then follows is a detailed – and actually innovative, to the best of our knowledge – analysis of how the propensity of each one of those regimes to prevail, given the rate of interest and the debt ratio, is affected by changes in structural parameters such as the saving ratio of capitalists and the sensitivity of investment to changes in the profit rate and in the interest rate. Indeed, while the propensity of the economy to a hedge finance regime rises with a higher sensitivity of investment to the interest rate, the propensity to a Ponzi regime falls with it. Similarly, the propensity to a hedge regime rises with a higher sensitivity of investment to the profit rate, while the propensity to a Ponzi regime falls with it. Lastly, while the propensity to a hedge regime falls with a higher saving rate by capitalists, the propensity to a Ponzi regime rises with it.

Given the interest rate and the debt ratio, therefore, a higher sensitivity of investment either to the interest rate or to the profit rate will make for a higher propensity of the economy to a hedge regime. However, the accompanying impact of a higher sensitivity of investment to the profit rate (interest rate) on capacity utilization and growth is positive (negative), so that a more financially robust economy do not (do) come at the cost of a lower level and growth of economic activity. Similarly, the higher propensity to a less financially fragile regime generated by a lower propensity to save by capitalists is accompanied by higher capacity utilization and growth.

The remainder of the paper is organized as follows. Section 2 describes the structure of the model, whereas Section 3 analyzes its behavior in terms of the determination of capacity utilization and growth. In turn, Section 4 develops a formalization of the minskyan taxonomy of finance regimes and analyzes the influence of structural parameters on the propensity of the economy to each one of these regimes, given the interest rate and the debt ratio. The last section summarizes the main conclusions derived along the way.

2. Structure of the model

We model an economy that is closed and has no government fiscal activities. A single good that can be used for both investment and consumption is produced with two factors of production, capital and labor, which are combined through a fixed-coefficient technology.

Production activity is carried out by oligopolistic firms. They produce – and hire labor – according to demand, it being considered only the case in which there is not enough demand to ensure full capacity utilization at the ongoing price. Hence, firms are assumed to hold idle physical capital.[1] Firms also make investment plans, which are described by the following desired investment function:

$$g^d = \alpha + \beta r - \gamma i \qquad (1)$$

where α , β e γ are positive parameters, and g^d is the desired investment as a ratio of the existing physical capital stock, K. In turn, r is the rate of profit defined as the flow of money profits generated by physical capital, R, divided by the value of capital stock at output price, while i is the interest rate. We follow Rowthorn (1981) and Dutt (1990), who in turn follow Kalecki (1971) and Robinson (1962), in assuming that desired investment

[1] For Steindl (1952), firms plan idle capacity so as to be ready for a sudden expansion of sales. First, the existence of fluctuations in demand means that the producer wants to be in a boom first, and not to leave the sales to new competitors who will press on her market when the boom is over. Second, it is not possible for the producer to expand her capacity step by step as her market grows due to the indivisibility and durability of the equipment. Finally, entry deterrence is an issue: if prices are high enough, entry of new competitors is feasible even when required capital is large; hence, the holding of idle capacity allows firms to confront new entrants by suddenly raising supply and driving prices down. Along similar lines, Spence (1977) and Cowling (1982) argue that the holding of excess capacity inhibits entry by making potential entrants unsure about post-entry profits.

depends positively on the rate of profit. Regarding the negative dependence of desired investment on the interest rate, we follow Dutt (1994).

The economy is inhabited by two classes, capitalists and workers. Following the tradition of Marx, Kalecki (1971), Kaldor (1956), Robinson (1962), and Pasinetti (1962), we assume that they have a different saving behavior. Workers, who are always in excess supply, provide labor and earn only wage income, which is all spent in consumption. This assumption that workers as a class do no saving does not, of course, rule out the possibility that individual workers might save. What this view amounts to is the assumption that for workers as a class the saving of some of them is matched by the dissaving of others (Foley & Michl, 1999). Capitalists, in turn, receive profit income, which is the entire surplus over wages, with productive and financial capitalists both saving constant fractions, s_p and s_f, respectively, of their share in profits. Hence, the functional division of income is given by:

$$X = (W/P)L + R \qquad (2)$$

where X is the output level, W is the nominal wage, P is the price level and L is the employment level. Hence, financial capitalists' profit income appear as a deduction of the general flow of money profits generated by the stock of physical capital:

$$rK = r_p K + r_f K \qquad (3)$$

where r_p is the portion of the general rate of profit which accrues to productive capitalists and r_f is the portion of the general rate of profit which accrues to financial capitalists.

From eq. (2), the general rate of profit can be expressed as:

$$r = (1 - Va)u \qquad (4)$$

where V is the real wage, a is the labor-output ratio and, therefore, $(1-Va)$ is the profit share in income. Since we assume that the ratio of capacity output to the capital stock is given, we can identify capacity utilization with the actual output-capital ratio, $u = X/K$. For simplicity, we also assume that the profit share in income is given, which implies that changes in the general rate of profit in the short run are driven solely by changes in capacity utilization. The division of profit income between productive and financial capitalists, in turn, depends on the stock of debt, D, of the former with the latter, with $\delta = D/K$ being the debt-capital ratio. Hence, the debt service, which is the portion of financial capitalists in profit income, is given by iD.

Therefore, the aggregate saving as a proportion of the capital stock is given by:

$$g^s = s_p r + (s_f - s_p) i\delta \qquad (5)$$

which follows from our assumptions that workers do not save and productive and financial capitalists save constant fractions, s_p and s_f, respectively, of their profit income.

3. The behavior of the model in the short run

The short run is here defined as a time span through which the capital stock, K, the nominal interest rate, i, the stock of debt, D, the price level, P, the nominal wage, W, and the labor-output ratio, a, can all be taken as given. The existence of excess productive capacity means that the macroeconomic equality between desired investment and saving is brought about by changes in output through changes in capacity utilization.[2] Using eqs. (1), (4) and (5), we can solve for the short-run equilibrium value of u, given V, i, δ and parameters of the model:

$$u^{*} = \frac{\alpha - [\gamma + (s_{f} - \mathrm{s_{p}})\delta]\, i}{(s_{p} - \beta)(1 - Va)} \tag{6}$$

Regarding stability, we employ a keynesian short-run adjustment mechanism stating that output will change in proportion to the excess demand in the goods market. Hence, u^{*} will be stable provided the denominator of eq. (6) is positive, which we assume to be the case. We also assume that the numerator of u^{*} is positive, which will ensure a positive value for u^{*} itself.

Hence, a rise in the real wage will lead to an increase in capacity utilization. Like in the models developed by Rowthorn (1981) and Dutt (1984, 1990), an increase in the real wage, by redistributing income from capitalists who save to workers who do not, raises consumption demand, and therefore increases capacity utilization. However, this rise in the real wage will leave the general rate of profit unchanged, since it will lower the profit share in income in the same extent that it will raise capacity utilization. Indeed, the equilibrium value of the general profit rate, r^{*}, which can be obtained by substituting the expression for u^{*} into eq. (4), is given by:

$$r^{*} = \frac{\alpha - [\gamma + (s_{f} - \mathrm{s_{p}})\delta]\, i}{(s_{p} - \beta)} \tag{7}$$

In turn, the impact of a change in the interest rate or in the debt-capital ratio on the short-run equilibrium values of capacity utilization and general profit rate is ambiguous:

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[2] A post-keynesian macrodynamic model of income distribution, capital accumulation and capacity utilization that considers both cases regarding capacity utilization, namely full and less-than-full, is developed in Lima & Meirelles (2003). The supply of credit-money is endogenous and interest rate is set by banks as a markup over the base rate, which is exogenously determined by the monetary authority. Over time, banking markup falls with firms' profit rate on physical capital and rises with the inflation rate. The dynamic stability properties of the system are carefully analyzed for both cases regarding capacity utilization, which makes for the possibility of multiple equilibria for the state variables real wage and interest rate. Unlike the present (static) model, however, firms' finance regime – and thus the financial fragility of the economy – is not modeled. In turn, a post-keynesian dynamic macromodel of utilization and growth of productive capacity in which firms' debt position is explicitly modeled is developed in Lima & Meirelles (2004). Regarding dynamics, it is shown, for instance, the possibility of relating the stability properties of a system having the interest rate and the debt position by firms as state variables to the type of minskyan finance regime that prevails across firms.

$$du^* / di = u_i^* = \frac{-[\gamma + (s_f - \mathrm{s_p})\delta]}{(s_p - \beta)(1 - Va)} \tag{8}$$

$$dr^* / di = r_i^* = \frac{-[\gamma + (s_f - \mathrm{s_p})\delta]}{(s_p - \beta)} \tag{9}$$

$$du^* / d\delta = u_\delta^* = \frac{-(s_f - s_p)i}{(s_p - \beta)(1 - Va)} \tag{10}$$

$$dr^* / d\delta = r_\delta^* = \frac{-(s_f - s_p)i}{(s_p - \beta)} \tag{11}$$

Eqs. (8)-(11) show that either a higher interest rate or a higher debt ratio will unambiguously lower the rates of capacity utilization and profit in case financial capitalists save a higher proportion of their profit income than productive capitalists do. A higher interest rate, for instance, will reduce demand not only by lowering desired investment, but by lowering consumption as well, as it will imply a intra-capitalist income redistribution towards those capitalists who save a higher proportion of their income. A higher debt ratio, in turn, despite not reducing desired investment directly, but only through the accelerator effect embodied in the r-argument, will reduce demand by redistributing profit income towards financial capitalists who consume relatively less than the productive capitalists do.

Now, only the impact of changes in the interest rate remains ambiguous when productive capitalists have a higher saving propensity than financial capitalists do. Indeed, eqs. (10)-(11) show that a higher debt ratio will unambiguously raise the rates of capacity utilization and profit in this case. The reason is that it will raise consumption demand by redistributing profit income towards those capitalists whose saving propensity is lower – and, despite not raising desired investment directly, it will end up doing so indirectly through the accelerator effect. Regarding the impact of changes in the interest rate, eqs. (8)-(9) show that it will depend on the relative impact of two opposite effects. On the one hand, a higher interest rate will unambiguously raise consumption demand by redistributing profit income towards those capitalists whose saving rate is lower. On the other hand, this higher interest rate will have a negative immediate impact on desired investment. Hence, the rates of capacity utilization and profit will end up raising (falling) in case the relative impact of these two effects is such that aggregate demand ends up raising (falling).[3]

The corresponding growth rate of the capital stock, which is actually the growth rate of this one-good economy, can be obtained by substituting the equilibrium value of the general rate of profit, eq. (7), into either eq. (1) or eq. (5):

[3] In case we had assumed that financial and productive capitalists share a common propensity to save, all of these ambiguities regarding the impact of a change in the interest rate would vanish. It can be checked that both capacity utilization and the general profit rate would unambiguously fall in response to a higher interest rate, while a change in the real wage would have the same qualitative impact as before. Besides, the equilibrium values of capacity utilization and the general profit rate would cease to depend on the debt ratio.

$$g^* = \frac{s_p(\alpha - \gamma i) - \beta(s_f - \mathrm{s_p})\delta\, i}{(s_p - \beta)} \tag{12}$$

which implies that the impact of changes in the interest rate or the debt ratio on the rate of growth is given by:

$$dg^* / di = g_i^* = \frac{-[s_p\gamma + \beta(s_f - \mathrm{s_p})\delta]}{(s_p - \beta)} \tag{13}$$

$$dg^* / d\delta = g_\delta^* = \frac{-\beta(s_f - \mathrm{s_p})i}{(s_p - \beta)} \tag{14}$$

Eqs. (13)-(14) show that a rise in the real wage, despite raising capacity utilization, will leave the growth rate unchanged. The reason, as seen above, is that this rise will leave the general rate of profit unchanged. In turn, either a higher interest rate or a higher debt ratio, by lowering the general rate of profit, will also lower the growth rate in case financial capitalists save a higher proportion of their profit income than productive capitalists do. Now, only the impact of changes in the interest rate on the growth rate remains ambiguous when productive capitalists have a higher saving propensity than financial capitalists do. Indeed, eq. (14) shows that a higher debt ratio, by raising the rate of profit, will raise the growth rate. Eq. (13), in turn, when compared to eq. (9), reveals that for a higher interest rate to end up raising the rate of growth, it is not sufficient that it raises the rate of profit along the way – recall that stability of all these short-run equilibrium values requires $s_p > \beta$.[4]

4. Indebtedness and finance regimes

Minsky's (1982) insightful taxonomy of firm finance regimes in terms of a firm's cash flow accounting categories has been neatly formalized by Foley (2003) as follows.[5] In a highly aggregated form, the cash flow identity equates the firm's source of funds from net operating revenues, R, and new borrowing, B, to its uses of funds for investment, I, and debt service, F:

$$R + B \equiv I + F \tag{15}$$

Hence, the change in debt, $\dot{D} = dD / dt$, is given by new borrowing:

[4] In line with the preceding footnote, financial and productive capitalists sharing a common saving propensity would make for an unambiguously inverse relationship between the rates of interest and growth. While a change in the real wage would keep having no impact on the equilibrium growth rate, the latter would now become independent from the debt ratio.

[5] According to Minsky´s broad characterization, hedge financing units are those that can fulfill all of their contractual payment obligations by their cash flows, while speculative units are those that can meet their interest payment commitments on outstanding debts, even as they are unable to repay the principle out of income cash flows. For Ponzi units, in turn, the cash flow from operations are not enough to fulfill either the rapayment of principle or the interest due on outstanding debts.

$$\dot{D} = B = I + F - R \tag{16}$$

Now, recalling that the growth rate is given by the investment as a proportion of the capital stock, $g = I/K$, that the profit rate is given by the flow of profits as a ratio of the capital stock, $r = R/K$, and that the debt service is given by iD, we can express the change in debt as follows:

$$\dot{D} = (g - r)K + iD \tag{17}$$

The minskyan taxonomy, in turn, can be derived as follows:

Hedge: $R \geq I + F$ ou $B \leq 0$ (18)
Speculative: $R < I + F$ ou $I > B > 0$ (19)
Ponzi: $R \leq F$ ou $B \geq I$ (20)

Now, using the definitions that led to eq. (17), and recalling that eq. (3) implies $r = r_p + i\delta$, our version of the minskyan taxonomy can then be expressed as follows:

Financing regime			
Hedge	$r - i\delta \geq g$	$r_p \geq g$	(21)
Speculative	$r - i\delta < g$	$r_p < g$	(22)
Ponzi	$r - i\delta \leq 0$	$r_p \leq 0$	(23)

Actually, by combining these inequalities with the respective expressions for the equilibrium values for the rates of profit and growth, eqs. (7) and (12), and by assuming for simplicity that financial and productive capitalists share a common saving propensity, s, we can derive the corresponding demarcation lines in the (i, δ)-space:

$$\delta_{h-s} = \frac{(1-s)\,\alpha}{(s-\beta)}\,\frac{1}{i} - \frac{(1-s)\gamma}{(s-\beta)} \tag{24}$$

$$\delta_{s-p} = \frac{\alpha}{(s\;-\beta)}\,\frac{1}{i} - \frac{\gamma}{(s-\beta)} \tag{25}$$

where δ_{h-s} and δ_{s-p} are the debt levels corresponding to the regime transition from hedge to speculative, and from speculative to Ponzi, respectively, as shown in Figure 1.[6]

[6] Even though we built on Foley's (2003) interesting formalization of the minskyan regimes, and actually drew a lot of inspiration from the whole paper, our version of the corresponding taxonomy turned out a different one. Indeed, Foley claimed that a model – like Taylor & O'Connell (1985) or, we should add, the one developed here – that maintains the closed economy Kaleckian relation between the growth rate and the savings out of profits, $g = sr$, which is eq. (5) above when it is assumed that productive and financial capitalists share a common saving propensity, does impose a regime of hedge finance in which $g < r$ in case it is assumed $s < 1$. Foley's ingenious solution is

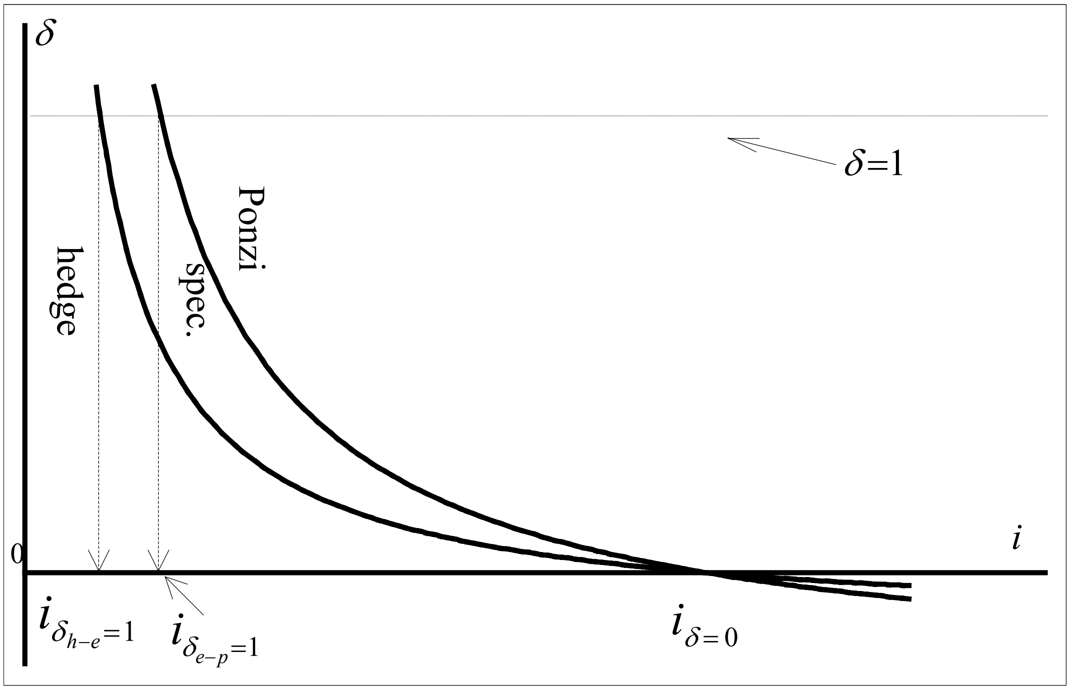

Figure 2: Minskyan financing regimes in the (i, δ)-space.

Note that for very low levels of debt these demarcation lines approach each other, with the lower limit of the debt ratio meaning an interest rate given by $i_{\delta=0} = \alpha/\gamma$. In turn, for a debt ratio given by $\delta = 1$ the interest rate levels corresponding to the respective transition lines – from hedge to speculative and from speculative to Ponzi – are given by:

$$i_{\delta_{h-s}=1} = \frac{\alpha(1-s)}{[(s-\beta)+\gamma(1-s)]} \tag{26}$$

$$i_{\delta_{s-p}=1} = \frac{\alpha}{[(s-\beta)+\gamma]} \tag{27}$$

We can analyze the influence of the structural parameters of the saving and investment functions on the propensity of this model economy to be located in each one of the financing regimes, given the interest rate and the debt ratio, by calculating the size of the corresponding areas in the (i, δ)-space. First of all, we have to delimitate the sub-space of validity of the present model in that space. In addition to being greater than or equal to zero, i and δ should also be constrained with respect to their maximum values, so as to

the opening of the economy to capital inflows from abroad, which would allow the domestic growth rates to exceed the domestic profit rate. Hence, the model developed here, by distinguishing between productive and financial capitalists, to each one accruing a portion of the general rate of profit, provides a different solution to the issue raised by Foley – by allowing the growth rate to exceed the portion of the general profit rate that accrues to indebted productive capitalists. Indeed, Minsky himself did not rely on the opening of the economy to develop a taxonomy of finance regimes, having nonetheless built on Kaleckian profit equations.

ensure positive equilibrium values for the rates of capacity utilization, profit and growth – which requires $i \le \alpha / \gamma$.

We are then in position to calculate not only the total area of validity of the model, but also the sub-areas corresponding to each one of the minskyan financing regimes. The total area, A_T, is given by:

$$A_T = \int_0^1 \frac{\alpha}{\gamma} \mathrm{d}\delta = \frac{\alpha}{\gamma} \tag{28}$$

An analogous procedure for the calculation of the hedge sub-area, A_H, will yield:

$$A_H = \int_0^1 \frac{\alpha(1-s)}{[(s-\beta)+\gamma(1-s)]} \mathrm{d}\delta = \frac{\alpha(1-s)}{(s-\beta)} \ln\left[1+\frac{(s-\beta)}{\gamma(1-s)}\right] \tag{29}$$

Regarding the speculative and Ponzi sub-areas, respectively A_S and A_P, we have:

$$A_S = \frac{\alpha}{(s-\beta)} \ln\left[1+\frac{(s-\beta)}{\gamma}\right] - \frac{\alpha(1-s)}{(s-\beta)} \ln\left[1+\frac{(s-\beta)}{\gamma(1-s)}\right] \tag{30}$$

$$A_P = \frac{\alpha}{\gamma} - \frac{\alpha}{(s-\beta)} \ln\left[1+\frac{(s-\beta)}{\gamma}\right] \tag{31}$$

Therefore, eqs. (29)-(31) can be interpreted as showing that the propensity of each one of the minskyan financing regimes to prevail, given the rate of interest and the debt ratio, is dependent on the saving ratio of capitalists, s, on autonomous investment, α, and on the sensitivity of investment to changes in the profit rate, β, and in the rate of interest, γ.

Let us first consider the impact of changes in the sensitivity of investment to interest rate, whose corresponding partial derivatives are given by:

$$\frac{\partial A_T}{\partial \gamma} = \frac{-\alpha}{\gamma^2} < 0 \tag{32}$$

$$\frac{\partial A_H}{\partial \gamma} = \frac{-\alpha(1-s)}{\gamma[\gamma(1-s)+(s-\beta)]} < 0 \tag{33}$$

$$\frac{\partial A_S}{\partial \gamma} = \frac{-\alpha\, s\,(s-\beta)}{\gamma(\gamma+s-\beta)[\gamma(1-s)+(s-\beta)]} < 0 \tag{34}$$

$$\frac{\partial A_P}{\partial \gamma} = \frac{-\alpha(s-\beta)}{\gamma^2(\gamma+s-\beta)} < 0 \tag{35}$$

where the corresponding signs are given by all the restrictions imposed on the parameters along the way. Eqs. (32)-(34) show that a higher sensitivity of investment to interest rate will imply not only a smaller total area of validity of the model, but also smaller sub-areas for all financing regimes. Regarding the impact of changes in the sensitivity of investment to the rate of profit, we have the following partial derivatives:[7]

$$\frac{\partial A_H}{\partial \beta} = \frac{\alpha(1-s)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\;\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} > 0 \tag{36}$$

$$\frac{\partial A_S}{\partial \beta} = \frac{\alpha}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\;\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} - \frac{\alpha(1-s)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\;\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} \tag{37}$$

$$\frac{\partial A_P}{\partial \beta} = -\frac{\alpha}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\;\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} < 0 \tag{38}$$

Hence, a higher sensitivity of investment to the profit rate implies a larger hedge sub-area and a smaller Ponzi sub-area. Regarding the speculative sub-area, whether it gets larger or smaller depends on which one of these effects on the other sub-areas will prevail.[8]

Regarding the impact of the term given by $(s-\beta)$, we have the following expressions, after taking the corresponding series expansion:

$$\frac{\partial A_H}{\partial (s-\beta)} = -\frac{\alpha(1-s)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\;\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} < 0 \tag{39}$$

$$\frac{\partial A_S}{\partial (s-\beta)} = -\frac{\alpha}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\;\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} + \frac{\alpha(1-s)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\;\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} \tag{40}$$

$$\frac{\partial A_P}{\partial (s-\beta)} = \frac{\alpha}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\;\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} > 0 \tag{41}$$

Therefore, a higher $(s-\beta)$ implies a smaller hedge sub-area and a larger Ponzi sub-area. Regarding the speculative sub-area, the sign of the corresponding partial derivative is likewise ambiguous.[9] As for the impact of changes in the saving propensity of capitalists, in turn, we have the following partial derivatives:

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[7] Since these partial derivatives include a natural logarithm term, we replaced them for their respective series expansion, which will allow them to be unambiguously signed. Such an expansion is carried out with the expression $\ln x = \sum_{n=1}^{\infty}[(x-1)^n / nx^n]$, convergent to $x > 1/2$. Given all the parametric restrictions imposed along the way, the corresponding argument of these logarithmic functions, $[1+(s-\beta)/(1-s)\gamma]$ or $[1+(s-\beta)/\gamma]$, is greater than one, which makes it valid the series expansion of them throughout the range of interest.

[8] If the corresponding series can be truncated in the first term, it can be shown that $\partial A_S / \partial \beta \leq 0$ when $\gamma \geq (s-\beta)/\sqrt{1-s}$.

[9] Again, if the corresponding series can be truncated in the first term, it can be demonstrated that $\partial A_S / \partial (s-\beta) \leq 0$ when $\gamma \leq (s-\beta)/\sqrt{1-s}$, which is the opposite result of the previous one.

$$\frac{\partial A_H}{\partial s} = -\frac{\alpha(1-\beta)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} < 0 \tag{42}$$

$$\frac{\partial A_S}{\partial s} = -\frac{\alpha}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} + \frac{\alpha(1-\beta)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} \tag{43}$$

$$\frac{\partial A_P}{\partial s} = \frac{\alpha}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} > 0 \tag{44}$$

meaning that the a higher saving ratio of capitalists implies a smaller hedge sub-area and a larger Ponzi sub-area. Regarding the speculative sub-area, the sign of the corresponding partial derivative is again ambiguous.[10]

Another alternative to analyze the effect of changes in the capitalists' saving ratio and in the parameters of the investment function involves the definition of a new variable, which will measure the propensity of the economy to a given financing regime. This variable is given by the ratio of the corresponding sub-area to the total area. Hence, the propensity of the economy to a hedge regime, P_H, is given by:

$$P_H = \frac{A_H}{A_T} = \frac{\gamma(1-s)}{(s-\beta)}\ln\left[1+\frac{(s-\beta)}{\gamma(1-s)}\right] \tag{45}$$

An analogous procedure will yield the propensity of the economy to a speculative regime and to a Ponzi regime:

$$P_S = \frac{\gamma}{(s-\beta)}\ln\left[1+\frac{(s-\beta)}{\gamma}\right] - \frac{\gamma(1-s)}{(s-\beta)}\ln\left[1+\frac{(s-\beta)}{\gamma(1-s)}\right] \tag{46}$$

$$P_P = 1-\frac{\gamma}{(s-\beta)}\ln\left[1+\frac{(s-\beta)}{\gamma}\right] \tag{47}$$

The partial derivatives of these propensities with respect to the sensitivity of the investment to interest rate are the following, after taking a series expansion of the natural logarithmic function:

$$\frac{\partial P_H}{\partial \gamma} = \frac{(1-s)}{(s-\beta)}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} > 0 \tag{48}$$

$$\frac{\partial P_S}{\partial \gamma} = \frac{1}{(s-\beta)}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} - \frac{(1-s)}{(s-\beta)}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} \tag{49}$$

$$\frac{\partial P_P}{\partial \gamma} = -\frac{1}{(s-\beta)}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} < 0 \tag{50}$$

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[10] Likewise, if the corresponding series can be truncated in the first term, it can be demonstrated that $\partial A_S / \partial s \geq 0$ when $\gamma \geq [(s-\beta)+s\sqrt{1-\beta}]/[1+s(1-s)/(s-\beta)]$.

Therefore, while the propensity of the economy to a hedge regime rises with a higher sensitivity of investment to interest rate, the propensity of the economy to a Ponzi regime falls with it. Recalling eqs. (32)-(33), it follows that the fall in the Hedge sub-area is less intense than the accompanying fall in the total area. Regarding the Ponzi sub-area, the opposite is the case: the fall in it is more intense than the accompanying fall in the total area, thus implying that the propensity to a Ponzi regime falls with a higher sensitivity of investment to interest rate. As for the propensity to a speculative regime, the resulting impact is ambiguous.[11] However, eqs. (6) and (12), recalling that in this section it is being assumed a common propensity to save by capitalists, show the accompanying impact of a higher sensitivity of investment to the interest rate on capacity utilization and growth is negative, so that a more financially robust regime do come at the cost of a lower level and growth of economic activity.

By analogous procedure, the partial derivatives for the effect on those propensities of changes in the sensitivity of the investment to the rate of profit are the following:

$$\frac{\partial P_H}{\partial \beta} = \frac{\gamma(1-s)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} > 0 \tag{51}$$

$$\frac{\partial P_S}{\partial \beta} = \frac{\gamma}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} - \frac{\gamma(1-s)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} \tag{52}$$

$$\frac{\partial P_P}{\partial \beta} = -\frac{\gamma}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} < 0 \tag{53}$$

Therefore, while the propensity of the economy to a hedge regime rises with a higher sensitivity of investment to the profit rate, the propensity of the economy to a Ponzi regime falls with it. As for the propensity to a speculative regime, the resulting impact is ambiguous. Now, eqs. (6) and (12), recalling that it is being assumed a common saving propensity by capitalists, show that the accompanying impact of a higher sensitivity of investment to the profit rate on the rates of capacity utilization and growth is positive. Unlike the instance of a higher sensitivity of investment to the interest rate, therefore, a more financially robust economy do not come at the cost of a lower level and growth of economic activity.

In turn, the partial derivatives for the effect on those propensities of changes in the saving rate of capitalists are the following:

$$\frac{\partial P_H}{\partial s} = -\frac{\gamma(1-\beta)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} < 0 \tag{54}$$

$$\frac{\partial P_S}{\partial s} = -\frac{\gamma}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} + \frac{\gamma(1-\beta)}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\frac{(s-\beta)^n}{[\gamma(1-s)+s-\beta]^n} \tag{55}$$

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[11] If the corresponding series can be truncated in the first term, it can be shown that $\partial P_S / \partial \gamma \leq 0$ when $\gamma \geq (s-\beta)/\sqrt{1-s}$.

$$\frac{\partial P_P}{\partial s} = \frac{\gamma}{(s-\beta)^2}\sum_{n=2}^{\infty}\frac{1}{n}\,\frac{(s-\beta)^n}{[\gamma+s-\beta]^n} > 0 \tag{56}$$

Hence, while the propensity of the economy to a hedge regime falls with a higher saving rate by capitalists, the propensity of the economy to a Ponzi regime rises with it. As for the propensity to a speculative regime, the resulting impact is again ambiguous. Since eqs. (6) and (12) show that the accompanying impact of a lower saving propensity by capitalists on the rates of capacity utilization and growth is positive, a more financially robust economy do not come at the cost of a lower level and growth of economic activity.

Finally, the impact on financial fragility of these parametric changes can be as well analyzed through some limit cases. Let us consider, for instance, the case of an economy with a very high sensitivity of investment to interest rate. Taking the corresponding limit of the propensities given by eqs. (45)-(47), we have:

$$\lim_{\gamma\to\infty} P_H = 1,\ \ \lim_{\gamma\to\infty} P_S = \lim_{\gamma\to\infty} P_P = 0 \tag{57}$$

Therefore, the propensity of the economy to a hedge regime will be highest when the sensitivity of investment to interest rate is the highest. A similar result follows when the saving rate of capitalists converges to the sensitivity of investment to the profit rate:

$$\lim_{s\to\beta} P_H = 1,\ \ \lim_{s\to\beta} P_S = \lim_{s\to\beta} P_P = 0 \tag{58}$$

Another interesting limit case regards a very high propensity to save of capitalists:

$$\lim_{s\to 1}\left(P_S + P_P\right) = 1,\ \ \lim_{s\to 1} P_H = 0 \tag{59}$$

Hence, the propensity to a hedge regime will be the lowest when the saving propensity of capitalists is the highest. In turn, financial fragility will be the highest when the sensitivity of investment to interest rate is the lowest:

$$\lim_{\gamma\to 0} P_P = 1,\ \ \lim_{\gamma\to 0} P_H = \lim_{\gamma\to 0} P_S = 0 \tag{60}$$

5. Conclusion

This paper developed a post-keynesian macromodel of capacity utilization and growth, in which the supply of credit-money was made endogenous and firms' debt and the financial fragility of the economy were also explicitly modeled.

Production activity is carried out by oligopolistic firms, which produce according to demand, it being considered only the case in which there is not enough demand to ensure full capacity utilization at the ongoing price. Firms' desired investment depends positively on the rate of profit and negatively on interest rate.

In this context, a rise in the real wage will lead to an increase in capacity utilization. An increase in the real wage, by redistributing income from capitalists who save to workers who do not, raises consumption demand, and thus increases productive capacity utilization.

However, this rise in the real wage will leave the general rate of profit unchanged, since it will lower the profit share in income in the same extent that it will raise capacity utilization. In turn, the impact of a change in the interest rate or in the debt-capital ratio on the short-run equilibrium values of capacity utilization and general profit rate is ambiguous. A higher interest rate or a higher debt ratio will unambiguously lower the rates of capacity utilization and profit in case financial capitalists save a higher proportion of their profit income than productive capitalists do. Now, only the impact of changes in the interest rate remains ambiguous when productive capitalists have a higher saving propensity than financial capitalists do. Indeed, a higher debt ratio will unambiguously raise the rates of capacity utilization and profit in this case.

However, a rise in the real wage, despite raising capacity utilization, will leave the growth rate unchanged. The reason is that this rise will leave the general rate of profit unchanged. In turn, either a higher interest rate or a higher debt ratio, by lowering the general rate of profit, will also lower the growth rate in case financial capitalists save a higher proportion of their profit income than productive capitalists do. Now, only the impact of changes in the interest rate on the growth rate remains ambiguous when productive capitalists have a higher saving propensity than financial capitalists do. Indeed, a higher debt ratio, by raising the rate of profit, will raise the growth rate.

Having formally derived a version of the minskyan taxonomy of finance regimes, it was seen that the propensity of each one of them to prevail, given the rate of interest and the debt ratio, is dependent on structural parameters such as the saving ratio of capitalists and the sensitivity of investment to changes in the profit rate and in the rate of interest.

While the propensity of the economy to a hedge finance regime rises with a higher sensitivity of investment to interest rate, the propensity to a Ponzi regime falls with it. Indeed, financial fragility will be the highest when the sensitivity of investment to interest rate is the lowest. In turn, while the propensity of the economy to a hedge regime rises with a higher sensitivity of investment to the profit rate, the propensity of the economy to a Ponzi regime falls with it. In both case cases, the impact of the corresponding parametric change on the propensity to a speculative regime is ambiguous, insofar as it depends on the relative intensity of the accompanying opposite changes in the propensities to the two other regimes. However, the accompanying impact of a higher sensitivity of investment to the profit rate (interest rate) on capacity utilization and growth is positive (negative), so that a more financially robust economy do not (do) come at the cost of a lower level and growth of economic activity.

Finally, while the propensity of the economy to a hedge regime falls with a higher saving rate by capitalists, the propensity of the economy to a Ponzi regime rises with it. Indeed, the propensity to a hedge regime will be the lowest when the saving propensity of capitalists is the highest. As for the propensity to a speculative regime, the resulting impact of this parametric change is again – and for the same reason – ambiguous. Like in the instance of a higher sensitivity of investment to the profit rate, the higher propensity to a less financially fragile regime brought about by a lower propensity to save by capitalists is actually accompanied by higher capacity utilization and growth.

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