ARE BUSINESS CYCLES ALL ALIKE IN EUROPE?

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Resumo: Investigamos nesse artigo se os ciclos econômicos têm um comportamento semelhante a partir do cálculo do custo de bem-estar do ciclo econômico para a União Européia como uma solução do problema proposto por Lucas (1987). Tais países foram escolhidos porque possuem uma longa tradição de integração e comércio, se comportando com um "experimento natural" para investigar o quão similar são os custos de bem-estar dos ciclos econômicos entre os países. Admitindo preferências do tipo CES e uma forma reduzida razoável para o consumo, estimamos os custos de bem-estar usando três métodos alternativas para decomposição tendência-ciclo, contudo focando o exercício sobre a decomposição de Beveridge-Nelson multivariada. Nossos resultados mostram que os custos de bem-estar são muito diferentes entre os países da União Européia e entre esses e os Estados Unidos, sendo uma forte evidência de que os ciclos econômicos não têm comportamento semelhante na Europa.

Classificação JEL: E32, C32, C53
Palavras-chave: Ciclos Econômicos, Custos de Bem-estar, Cointegração, Decomposição de Beveridge-Nelson.

Abstract: We investigate whether business cycles are all alike computing the welfare costs of business cycles for European-Union (EU) as the solution of the problem proposed by Lucas (1987). Because these countries have a long tradition of integration and trade, it is a "natural experiment" to investigate how similar their welfare costs of business cycles are. Using standard assumptions on preferences and a reasonable reduced form for consumption, we computed welfare costs using three alternative trend-cycle decomposition methods, but focusing on the multivariate Beveridge-Nelson decomposition. Our results show that welfare costs are very different across EU countries and between US and EU countries, and thus it is a strong evidence that business cycles are not alike in Europe.

JEL Codes: E32, C32, C53
Keywords: Business cycles, welfare costs, cointegration, Beveridge-Nelson decompositon.

ÁREA 3 - MACROECONOMIA, ECONOMIA MONETÁRIA E FINANÇAS

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Are Business Cycles All Alike in Europe?

1. Introduction

A discussion with a long tradition in macroeconomics is on what generates business cycles. According to one view, which we label the *institutional* view, business cycles are generated by large and infrequent shocks that hit macroeconomic variables, leading them to fluctuate about their trend. Because institutional settings vary from country to country, these shocks are different across countries and business cycles are not all alike. According to a different view, which we label the *dynamic-stochastic-general-equilibrium* – DSGE – view, business cycles are generated by small and frequent white-noise shocks that hit macroeconomic variables, which have a dynamic path qualitatively well approximated by a dynamic stochastic general equilibrium model. Because these shocks are small (low variance), and there is no reason to believe that propagation mechanisms vary from country to country, business cycles are all alike. Indeed, Lucas (1977), in his opening statement of this classic paper, asks: "*Why is it that, in capitalist economies, aggregate variables undergo repeated fluctuations about trend, all of essentially the same character?*"

Of course, it is not trivial to investigate if business cycles are all alike. First, one has to define in which sense they should be alike and different ways to measure similarities. A first approach, followed by Blanchard and Watson (1986), is to look directly into shocks themselves, investigating whether they are small or large, as well as their nature. Usually this is done using a structural econometric model. Since there is no consensus on how shock identification should be performed, and several shock-identification techniques have been criticized on different grounds, it is hard to come out with a satisfactory answer once this direct approach is followed.

The shortcomings of the direct approach can be overcome if instead of focusing directly on shocks, one uses an indirect approach, focusing on a fundamental difference in the nature of business cycles entailed by these two types of shocks. A concept that has received some attention recently, and that can be used to investigate whether business cycles are alike is the welfare cost of business cycles. The idea is straightforward: Lucas (1987) calculates the proportion of extra consumption, in all dates and states of nature, a rational consumer would require in order to be indifferent between an infinite sequence of consumption under uncertainty and a certain sequence which is cycle free. This proportion is labelled the *welfare cost of business cycles*, and can be directly computed using consumption data and a parametric version of the utility function; see the variants in Imrohoroglu (1989), Obstfeld (1994), Van Wincoop (1994), Atkeson and Phelan (1995), Pemberton (1996), Dolmas (1998), Tallarini (2000), Otrok (2001), and Franco, Guillen and Issler (2003).

If shocks are frequent and similar across countries, in which they have a low variance, and if the propagation mechanism is similar in nature to that in dynamic stochastic general equilibrium models, one should find that the welfare costs of business cycles across economies are all similar. However, if institutional factors are important, shocks will be different in nature and the welfare costs of business cycles will be different across economies. Of course, one can always find a set of countries that have similar institutional settings. For them, finding similar welfare costs of business cycles may just be a consequence of similar institutions. However, if the opposite is true for this set of countries, then it is hard to argue for the DSGE view.

In this paper, we investigate whether business cycles are all alike computing the welfare costs of business cycles for an important subset of European countries – European-Union (EU) countries: Austria, Belgium, Denmark, Finland, France, Germany, Great Britain, Greece, Ireland, Italy, Luxembourg, Portugal, Spain, Sweden and The Netherlands. As is well known, these countries have a long tradition of integration and trade dating well before the common-currency Euro area was implemented. Because of this feature, it is a “natural experiment” to investigate how similar their welfare costs of business cycles are, in the sense that we will be surprised to find major differences between them.

In computing the welfare costs of business cycles for EU countries we use the techniques in Beveridge and Nelson (1981) to decompose (the log of) consumption in a trend and a cyclical component. In this case, the trend will be stochastic and modeled as a random walk.[1] This choice relies on a sizable amount of econometric evidence available on consumption, or, alternatively, on the amount of authors that have used the unit-root specification, e.g., Hall (1978), Nelson and Plosser (1982), Campbell (1987), King et al. (1991), Cochrane (1994), *inter alia*. Moreover, to make our results comparable to previous work, we also modeled the trend as either a deterministic linear process (with and without a break) or following a slowly evolving secular process captured by the Hodrick and Prescott (1997) filter.

Our results show that the welfare costs of business cycles are very different across EU countries. Using the Beveridge and Nelson decomposition, and plausible values for the risk aversion coefficient and the discount rate of future utility, we find that the welfare cost of Spain (4.1% of consumption) is almost ten times that of the UK (0.45% of consumption) – median of 2.85%. Major differences in welfare costs are also found when alternative trend-cycle decomposition methods are employed, although they are not as pronounced as the ones obtained using the Beveridge and Nelson decomposition.

The paper is divided as follows. Section 2 provides a theoretical and statistical framework to evaluate the welfare costs of business cycles. Section 3 provides the estimates that are used in calculating them. Section 4 provides the calculations results, and Section 5 concludes. There is also an Appendix providing the econometric background necessary to implement the calculations carried out in the paper.

[1] Lucas (1987, pp. 22-23, footnote 1) explicitely considers the possibility that the trend in consumption is stochastic as in Nelson and Plosser (1982).

2. The Problem

Lucas (1987) proposed a way to evaluate the welfare gains of cycle smoothing. Suppose an agent that chooses a consumption sequence $\{c_t\}_{t=0}^{\infty}$ that maximizes intertemporal utility, U, subject to a budget constraint:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

where $E_t(\cdot) = E(\cdot|\Omega_t)$ is the conditional expectation operator of a random variable, using Ω_t as the information set, and $\beta \in (0,1)$ is a constant discount factor. He worked with a class of consumption streams with trend and cycle components such as:

$$c_t = \alpha_0 (1+\alpha_1)^t \exp\left(-\frac{1}{2}\sigma_z^2\right), \quad t = 0,1,\ldots \tag{2}$$

where $\{z_t\}$ is a stationary stochastic process with a stationary distribution given by $\ln(z_t) \sim N(0,\sigma_z^2)$. Cycle-free consumption will be the sequence $\{c_t^*\}_{t=0}^{\infty}$, where $c_t^* = \alpha_0(1+\alpha_1)^t$ since $E\left[\exp\left(-\frac{1}{2}\sigma_t^2\right)z_t\right] = 1$. Notice that $\{c_t^*\}_{t=0}^{\infty}$ is the resulting sequence when we replace the random variable c_t with its unconditional mean. Hence, for any particular time period, c_t represents a mean-preserving spread of c_t^*.

Risk averse consumers prefer $\{c_t^*\}_{t=0}^{\infty}$ to $\{c_t\}_{t=0}^{\infty}$, so the costs of the economic instability can be measured by calculating λ which solves the following equation:[2]

$$E\left(E_0 \sum_{t=0}^{\infty} \beta^t u((1+\lambda)c_t)\right) = \sum_{t=0}^{\infty} \beta^t u(c_t^*) \tag{3}$$

Then λ is the compensation required by consumers that makes them indifferent between the uncertain stream $\{c_t\}_{t=0}^{\infty}$ and the stream $\{c_t^*\}_{t=0}^{\infty}$. Notice that uncertainty here comes in the form of stochastic business cycles, since the trend in consumption is deterministic.

Lucas (1987) assumed that the utility function is in CES class:

$$u(c_t) = \frac{c_t^{1-\phi} - 1}{1-\phi}, \tag{4}$$

where $\phi > 0$ is the constant coefficient of relative risk-aversion and $u(c_t)$ converges to $\ln(c_t)$ as $\phi \to 1$. It calculated λ that satisfies (3) for some values of β and ϕ using US data for post-war period.

Obviously there are others forms of c_t besides (2). If we suppose c_t is difference stationary then it can be decomposed as the sum of a deterministic trend, a random walk trend and a stationary cycle (ARMA process), as shown in Beveridge and Nelson (1981),

[2] Notice that Lucas (1987) uses the unconditional mean operator instead of the conditional mean operator in λ. The same problem can be proposed using the conditional expectation instead. This is exactly how we proceed in this paper.

$$
\begin{aligned}
\ln(c_t) &= \ln(\alpha_0) + t.\ln(1+\alpha_1) - \frac{\omega_t^2}{2} + \sum_{i=1}^{t} \varepsilon_i + \sum_{j=0}^{t-1} \psi_j \mu_{t-j} \\
&= \ln(\alpha_0 (1+\alpha_1)^t) - \frac{\omega_t^2}{2} + \ln(X_t) + \ln(Y_t)
\end{aligned}
\qquad (5)
$$

where $\ln(\alpha_0 (1+\alpha_1)^t \exp(-\omega_t^2/2)$ is the deterministic term, $\ln(X_t) = \sum_{i=1}^{t} \varepsilon_i$ is the random walk component, $\ln(Y_t) = \sum_{j=0}^{t-1} \psi_j \mu_{t-j}$ is the $MA(\infty)$ representation of the stationary part (cycle), and $\omega_t^2 = \sigma_{11} t + 2\sigma_{12} \sum_{j=0}^{t-1} \psi_j + \sigma_{22} \sum_{j=0}^{t-1} \psi_j^2$ is the conditional variance of $\ln(c_t)$. The permanent shock ε_t and the transitory shock μ_t are assumed to have a bi-variate normal distribution as follows,

$$
\begin{pmatrix} \varepsilon_t \\ \mu_t \end{pmatrix} \sim iidN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \right) \qquad (6)
$$

i.e., shocks are independent, thus serially uncorrelated, but contemporaneously correlated if $\sigma_{12} \neq 0$.

Calculating the welfare cost of business cycles for the difference-stationary case requires first a discussion on how to deal with the fact that now uncertainty comes both in the trend and the cyclical component of $\ln(c_t)$. Moreover, since the trend component has a unit root, its unconditional mean and variance are not defined. Notice that, in the exercise proposed by Lucas, all the cyclical variation in $\ln(c_t)$ is eliminated, which is equivalent to eliminating *all* its variability, since the trend is deterministic. Here, this equivalence is lost, because the trend is stochastic as well.

To deal with this issue, we follow Obstfeld (1994) in considering the conditional expectation operator $E_o(\cdot)$ in (3), in spite of the unconditional expectation operator $E(\cdot)$. In this case, c_t^* is now redefined as $c_t^* = E_o(c_t)$. Therefore, we are assuming that it is possible to offer the consumer an certain consumption stream c_t^* (with no trend and cyclical variation) based on information available at the outset of the problem. Of course, the alternative for the consumer is to face c_t, which has a conditional variance that depends on ω_t^2. Consumption has now a unit root and so $\omega_t^2 \to \infty$, as $t \to \infty$ (although $\omega_t^2 < \infty$ for all t finite). Hence, uncertainty can get relatively large as the horizon increases, which may be balanced by the fact that there is discounting in the welfare function.

As in Obstfeld (1994), the problem we propose solving here is

$$
E_0 \sum_{t=0}^{\infty} \beta^t u((1+\lambda)c_t) = \sum_{t=0}^{\infty} \beta^t u(E_0(c_t)). \qquad (7)
$$

Under (4), (5) and (6), and using the properties of the moments of log-normal distributions, we can calculate (7). Apart from an irrelevant constant term, its left-hand side is given by

$$
E_0 \sum_{t=0}^{\infty} \beta^t u((1+\lambda)c_t) = \frac{[\alpha_0(1+\lambda)]^{1-\phi}}{1-\phi} \sum_{t=0}^{\infty} \left[\beta(1+\alpha_1)^{1-\phi}\right]^t \exp\left[-\frac{(1-\phi)\phi\omega_t^2}{2}\right]. \qquad (8)
$$

Notice that, (8) converges if $\beta(1+\alpha_1)^{1-\phi}\exp\left[-\frac{(1-\phi)\phi\sigma_{11}}{2}\right]<1$.

Calculating the conditional mean of c_t yields $c_t^* = E_0(c_t) = \alpha_0(1+\alpha_1)^t \exp\left(-\frac{\omega_t^2}{2}\right)E_0(X_t Y_t) = \alpha_0(1+\alpha_1)^t$. Hence, apart from an irrelevant constant term, the right-hand side of (7) is

$$\sum_{i=0}^{\infty}\beta^t u(c_t^*) = \frac{\alpha_0^{1-\phi}}{1-\phi}\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\phi}\right]^t \tag{9}$$

which converges if $\beta(1+\alpha_1)^{1-\phi}<1$.

Given the parameters defining the processes $\{c_t^*\}_{t=0}^{\infty}$ and $\{c_t\}_{t=0}^{\infty}$, $\lambda(\phi,\beta)$ is

$$\lambda(\phi,\beta) = \left\{\frac{\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\phi}\right]^t}{\sum_{t=0}^{\infty}\left[\beta(1+\alpha_1)^{1-\phi}\right]^t \exp\left[-\frac{(1-\phi)\phi\omega_t^2}{2}\right]}\right\}^{1/(1-\phi)} - 1. \tag{10}$$

In the definition of ω_t^2 in (10), we replace $\sigma_{12}\sum_{j=0}^{t-1}\psi_j$ and $\sigma_{22}\sum_{j=0}^{t-1}\psi_j^2$ by their respective unconditional counterparts, $\tilde{\sigma}_{12} = \sigma_{12}\sum_{j=0}^{\infty}\psi_j$ and $\tilde{\sigma}_{22} = \sigma_{12}\sum_{j=0}^{\infty}\psi_j^2$ (which may be a reasonable approximation even for relatively small t, and a very good approximation for intermediate and large t), making $\omega_t^2 = \sigma_{11}t + 2\tilde{\sigma}_{12} + \tilde{\sigma}_{22}$. Assuming that the conditions for (8) and (9) holds, (10) converges to[3]

$$\lambda(\phi,\beta) = \begin{cases} \left\{\exp\left[\frac{\phi\left(2\tilde{\sigma}_{12}+\tilde{\sigma}_{22}\right)}{2}\right]\left\{\frac{1-\beta(1+\alpha_1)^{1-\phi}\exp\left[-\frac{(1-\phi)\phi\sigma_{11}}{2}\right]}{1-\beta(1+\alpha_1)^{1-\phi}}\right\}\right\}^{1/(1-\phi)} - 1, & \text{if } \phi \neq 1 \\ \exp\left[\frac{1}{2}\left(\frac{\beta\sigma_{11}}{1-\beta} + 2\tilde{\sigma}_{12} + \tilde{\sigma}_{22}\right)\right] - 1, & \text{if } \phi = 1 \end{cases} \tag{11}$$

which shows the way we chose to estimate $\lambda(\phi,\beta)$ in this paper.[4] In subsection 2.1 we discuss a methodology for calculating $\lambda(\phi,\beta)$ estimates standard errors. It's straightfoward to see that $\lambda(\phi,\beta)$ is increasing in β,[5] thus welfare cost of fluctuations is as large as agents are patient.

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[3] Equation (11) for $\phi = 1$ is derived on appendix A.

[4] In our results we have observed that, for all values of (ϕ,β) we considered here, $\beta(1+\alpha_1)^{1-\phi}<1$. However, it was not always the case that $\beta(1+\alpha_1)^{1-\phi}\exp\left[-\frac{(1-\phi)\phi\sigma_{11}}{2}\right]<1$, since the term $\exp\left[-\frac{(1-\phi)\phi\sigma_{11}}{2}\right]$ was always greater than unity, and sometimes large enough as to prevent the convergence condition to hold.

We now turn to other possible ways of modelling the trend component. If the trend is modeled as a deterministic function of time, as in (2), then the analysis is done as originally proposed by Lucas (1987). In spite of the fact that Lucas has proposed the analysis as in (3) above, he actually implemented it in a different way (see Lucas, 1987, footnote 2, p. 23)), removing the trend in consumption using the filtering procedure proposed in Hodrick and Prescott (1997). The filter is two sided, i.e., uses past and future consumption values to get the slowly-moving trend. In principle, the trend removed using such a procedure should be treated as a random variable. However, for simplicity, Lucas treated the trend as deterministic, which we also do here. Hence, when using the Hodrick and Prescott trend, our results should be viewed as a lower-bound for the welfare cost of business cycles. To implement the calculations in this case, we computed the deterministic growth rate present in the Hodrick and Prescott trend, treating the cyclical component as in (7) above. Hence, $c_t = \alpha_0'(1+\alpha_1')^t \exp(-\sigma_{z'}^2/2) z_t'$, $\ln(z_t') \sim N(0, \sigma_{z'}^2)$ and $c_t^* = \alpha_0'(1+\alpha_1')^t$, where α_0' and α_1' are now the deterministic components associated with the Hodrick-Prescott trend, and z_t' is the residual cyclical component associated with it. We may observe that for linear and Hodrick-Prescott trend, $\sigma_{11} = \tilde{\sigma}_{12} = 0$, and so λ in equation (11) does not depend of β and α_1 and is monotonicaly increasing in ϕ.

2.1 Standard Erros of $\lambda(\phi, \beta)$ Estimates

Let Ω the variance-covariance matrix of the permanent shock ε_t and the transitory shock μ_t of the log of consumption, as presented in equation (6), and $\hat{\Omega}$ \$ the maximum likelihood estimator of Ω,[6] thus,

$$\sqrt{T}\begin{bmatrix} \hat{\sigma}_{11,T} - \sigma_{11} \\ \hat{\sigma}_{21,T} - \sigma_{21} \\ \hat{\sigma}_{22,T} - \sigma_{22} \end{bmatrix} \xrightarrow{d} N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2\sigma_{11}^2 & 2\sigma_{11}\sigma_{21} & 2\sigma_{21}^2 \\ 2\sigma_{11}\sigma_{21} & \sigma_{11}\sigma_{22} + \sigma_{21}^2 & 2\sigma_{21}\sigma_{22} \\ 2\sigma_{21}^2 & 2\sigma_{21}\sigma_{22} & 2\sigma_{22}^2 \end{bmatrix}\right) \tag{12}$$

Let $\hat{\alpha}_1$ a consistent and assymptoticaly normally-distributed estimator of α_1, i.e.,

$$\sqrt{T}\left(\hat{\alpha}_1 - \alpha_1\right) \xrightarrow{d} N\left(0, \sigma_\alpha^2\right) \tag{13}$$

Let $\theta_0 = (\alpha_1, \sigma_{11}, \sigma_{21}, \sigma_{22})'$ and $\hat{\theta}_T = (\hat{\alpha}_{1,T}, \hat{\sigma}_{11,T}, \hat{\sigma}_{21,T}, \hat{\sigma}_{22,T})'$. From (12) and (13) and applying Delta Method, we have,

$$\sqrt{T}\left[\lambda(\hat{\theta}_T) - \lambda(\theta_0)\right] \xrightarrow{d} N\left(0, \mathrm{C}(\theta_0)\begin{bmatrix} \sigma_\alpha^2 & 0 \\ 0 & \Omega \end{bmatrix}\mathrm{C}(\theta_0)'\right) \tag{14}$$

where $\mathrm{C}(\theta_0)$ is the vector of partial derivatives of λ with respect to θ_0'.

3. Reduced Form and Long-Run Constraints

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[5] The term $\exp\left[-\frac{(1-\phi)\phi\sigma_{11}}{2}\right]$ is always greater than unity.

[6] See Hamilton (1994), pages 300-301.

Let $y_t = (\ln(c_t), \ln(I_t))'$ is a 2×1 vector containing the logarithms of consumption and disposable income.[7] Assume that both series individually contain a unit-root, and are generated by a *p*-th order vector autoregression (VAR),

$$y_t = \pi_1 y_{t-1} + \pi_2 y_{t-2} + \cdots + \pi_p y_{t-p} + \varepsilon_t, \quad \text{or,}$$

$$\Pi(\mathrm{L})\mathrm{y_t} = \varepsilon_t$$

where $\Pi(\mathrm{L}) = I_n - \pi_1 L - \pi_2 L^2 - \cdots - \pi_p L^p$. Decomposing $\Pi(\mathrm{L})$ as

$$\Pi(\mathrm{L}) = -\Pi(1)L^p + (1-L)\Gamma(L)$$

leading to the vector error-correction model (VECM)

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \cdots + \Gamma_{p-1}\Delta y_{t-p+1} - \Pi(1) y_{t-p} + \varepsilon_t, \qquad (14)$$

where $\Pi(1) = \gamma\alpha'$, $\Gamma_j = -I_n + \sum_{i=1}^{j}\pi_i$, $j = 1,2,\ldots,p-1$, α is the cointegration vector and γ is a 2×1 constant vector.

Cointegration between the logarithms of consumption and income may be explained using the theory of permanent-income. In this theory, consumption can be viewed as proportional to the expected present discounted value of all income stream. Hence, the expected present value of consumption and income are equal, and both series are proportional in the long run,[8] moreover, the cointegrating vector will be α =(-1,1)′.

We turn now to the discussion of how to extract trends and cycles from (14). First, put the system (14) in state-space form, as discussed in Proietti (1997),

$$\begin{aligned} \Delta y_t &= Zf_t \\ f_t &= Tf_{t-1} + Z'\varepsilon_t \end{aligned} \qquad (15)$$

where $f_t = \begin{bmatrix} \Delta y_t \\ \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \alpha' y_{t-p} \end{bmatrix}$, $T = \begin{bmatrix} \Gamma_1 \cdots \Gamma_{p-1} & -\gamma\alpha' & -\gamma \\ I_{2(p-1)} & 0 & 0 \\ 0_{1\times 2(p-1)} & \alpha' & 1 \end{bmatrix}$, $Z = \begin{bmatrix} I_2 & 0_{2\times(2p-)} \end{bmatrix}$.

From the work of Beveridge and Nelson (1981), and Stock and Watson (1988), ignoring initial conditions and deterministic components, the series in y_t can be decomposed into a trend (τ_t) and a cyclical component (ψ_t), as $y_t = \tau_t + \psi_t$ where,

$$\tau_t = y_t + \lim_{k\to\infty}\sum_{i=0}^{k} E_t\left[\Delta y_{t+i}\right] \text{ and } \psi_t = -\lim_{k\to\infty}\sum_{i=0}^{k} E_t\left[\Delta y_{t+i}\right] \qquad (16)$$

It is straightforward to show that τ_t is a multivariate random-walk. Using the state-space representation (15), we can compute the limits above. The cyclical and trend components will be, respectively,[9]

$$\begin{aligned} \psi_t &= -Z\left[I_m - T\right]^{-1} Tf_t \\ \tau_t &= y_t - \psi_t \end{aligned} \qquad (17)$$

[7] A full discussion of the econometric models employed here can be found in Beveridge and Nelson (1981), Stock and Watson (1988), Engle and Granger (1987), Campbell (1987), Campbell and Deaton (1989), and Proietti (1997).
[8] See Campbell (1987) and Campbell and Deaton (1989).
[9] See appendix B for cycle and trend equations derivation.

where m=2p+1, or, using formulas (6) and (7) in Proietti (1997),

$$\psi_t = -K\Gamma^*(L)\Delta y_t + Py_t, \quad \text{and} \tag{18}$$

$$\tau_t = K\sum_{i=1}^{t}\varepsilon_i \tag{19}$$

where $K = (I_N - P)(\Gamma(1)+\gamma\alpha')^{-1}$ and $\mathrm{P} = (\Gamma(1)+\gamma\alpha')^{-1}\gamma\left[\alpha'(\Gamma(1)+\gamma\alpha')^{-1}\gamma\right]\alpha'$ are projection matrices.

We can also use (15) to forecast trend and cyclical components at any horizon into the future. The forecast of ψ_{t+s}, given information up to *t*, is $\hat{\psi}_{t+s|t} = E_t\left[\psi_{t+s}\right] = -K\Gamma^*(L)ZTf_{t+s-1} + Py_t + PZ\left(\sum_{i=1}^{s}T^i\right)f_t$ and the forecast of τ_{t+s}, given information up to *t*, is $\hat{\tau}_{t+s|t} = \tau_t$ since the best forecast of a random walk *t+s* periods ahead is simply its value today.

To fully characterize the elements in (11), we need to compute the variance and the covariance of forecasts of trend and cyclical components. Recall that the conditional expectation of a log-normal random variable is just a function of the mean and variance of the normal distribution associated with it. Hence, to compute the variances of these forecasts, we have just to apply standard results of state-space representations. It is straightforward to show that:

$$E_t\left[\left(\tau_{t+s} - \hat{\tau}_{t+s|t}\right)\left(\tau_{t+s} - \hat{\tau}_{t+s|t}\right)'\right] = s.KQK'$$

where $E_t\left[\varepsilon_{t+i}\varepsilon'_{t+i}\right] = Q$, and that,

$$E_t\left[\left(\psi_{t+s} - \hat{\psi}_{t+s|t}\right)\left(\psi_{t+s} - \hat{\psi}_{t+s|t}\right)'\right] = VQV' + P\left(\sum_{i=1}^{s-1}W(i)QW(i)'\right)P'$$

and $E_t\left[\left(\tau_{t+s} - \hat{\tau}_{t+s|t}\right)\left(\psi_{t+s} - \hat{\psi}_{t+s|t}\right)'\right] = KQV' + K\left(\sum_{i=1}^{s-1}QW(i)'\right)P'$

where $V = \left[P - K\Gamma^*(1)\right]$, as computed in the appendix C.

Based on these last three covariance matrices, the correlations between trend and cyclical components of the data can be fully characterized. Hence, to get the corresponding element of means, variances, and covariances associated with $\ln(c_t)$, one has simply to choose the appropriate elements of these vectors and matrices.

4. Data

European Union (EU-15) countries[10] annual data for real income and population were obtained from Penn World Table (Summers and Heston) from 1950 to 2000. Annual data for household consumption were extracted from EUROSTAT, Statistics Sweden and Penn World Table from 1950 to 2000.[11]

[10] At present European Union is composed by 15 countries: Austria, Belgium, Denmark, Finland, France, Germany, Great Britain, Greece, Ireland, Italy, Luxembourg, Portugal, Spain, Sweden and The Netherlands.
[11] Data for Greece was available from 1951 to 2000 and for Germany from 1970 to 2000.

We tested cointegration between series of logs of per capita consumption and income of each country and EU-15. Table 1 presents results of the Johansen (1988, 1991) cointegration test.

Table 1 - Johansen Cointegration Test

Country	VAR Lag	Ho: nº of Coint Eq = None		Ho: nº of Coint Eq = At most 1	
		Trace Statistic	Max-Eigen Statistic	Trace Statistic	Max-Eigen Statistic
EU-15	6	49.71 **	44.01 **	5.70	5.70
Austria	2	27.98 **	23.47 **	4.51	4.51
Belgium	2	35.21 **	26.34 **	8.86	8.86
Denmark	1	4.78	4.21	0.57	0.57
Finland	3	29.61 **	23.31 **	6.29	6.29
France	3	13.10	12.00	1.10	1.10
Germany	1	6.46	5.20	1.26	1.26
Greece	1	9.81	8.12	1.69	1.69
Ireland	2	32.30 **	27.27 **	5.04	5.04
Italy	3	11.43	11.18	0.25	0.25
Luxembourg	1	29.03 **	23.87 **	5.17	5.17
Portugal	6	28.40 **	20.05 *	8.35	8.35
Spain	2	32.14 **	24.83 **	7.31	7.31
Sweden	2	22.03 *	16.37 *	5.66	5.66
The Netherlands	1	14.77	11.99	2.78	2.78
United Kingdom	3	27.35 **	23.02 **	4.32	4.32
Critical Values:	5%	15.41	14.07	9.24	9.24
	1%	24.60	20.20	12.97	12.97

*(**) indicates null hypothesis was rejected at 5%(1%) of significance.
VAR Lag indicates VAR order used in tests

The hypothesis of no cointegration equation was reject and the hypothesis of at most one cointegration equation was not rejected at 5% significance, except to Denmark, France, Germany, Greece, Italy and The Netherlands. Conditioning on the existence of one cointegrating vector, we tested the restriction that it was equal to (-1, 1)′ using the likelihood-ratio test in Johansen (1991). This hypothesis was not rejected for Austria, Ireland, Luxembourg, Sweden and United Kingdom (UK). Results are reported on table 2.

Table 2 - Testing $H_0 : \alpha' = (1, -1)$

Country	LR test for binding restrictions (rank = 1)		
	Cointegration Vector	Statistic Test [$\chi(1)$]	Probability
EU-15	(1 -1.3036)	22.992	0.0000
Austria	(1 -1.1301)	0.011	0.9179
Belgium	(1 -0.9169)	9.034	0.0027
Finland	(1 -0.9216)	7.097	0.0077
Ireland	(1 -0.566)	2.832	0.0924
Luxembourg	(1 -0.9523)	0.307	0.5794
Portugal	(1 -0.8471)	11.699	0.0006
Spain	(1 -0.9142)	9.461	0.0021
Sweden	(1 -0.605)	0.551	0.4578
United Kingdom	(1 -1.0709)	1.109	0.2923

The presence of unit root was investigated in consumption and income series for those countries which series do not cointegrate. At 5%, the unit root hypothesis was not rejected in all cases using the ADF test; see the same results obtained using the KPSS test.

Table 3 - Unit Root Tests

Country	Level / 1st difference	Log per capita Consumption		Log per capita GDP	
		KPSS Statistic	ADF Statistic	KPSS Statistic	ADF Statistic
Denmark	level	0.09667	-0.11656	0.19865 **	-1.20567
	1st difference	0.07542	-6.24076 ***	0.20865	-3.78836 ***
France	level	0.24246 ***	-0.33044	0.23844 ***	-0.46426
	1st difference	0.06699	-4.53792 ***	0.07984	-4.21962 ***
Germany	level	0.70563 **	-0.52329	0.72877 **	-0.98216
	1st difference	0.07968	-3.28340 ***	0.11754	-2.54650 **
Greece	level	0.21436 ***	-1.85539	0.86037 ***	-2.86528 *
	1st difference	0.33663	-1.88616 *	0.11701	-1.34423 *
Italy	level	0.93510 ***	-2.30782	0.23836 ***	-0.48263
	1st difference	0.07038	-2.03137 **	0.04587	-3.89228 **
The Netherlands	level	0.15361 **	-0.82406	0.20894 **	-1.99065
	1st difference	0.09047	-2.85621 ***	0.30862	-2.12404 **

*(**)[***] indicates null hypothesis was rejected at 10%(5%)[1%] of significance.
Ho ADF test: serie has unit root; Ho KPSS test: serie is stationary

5. Empirical Results

A *pth*-order vector error-correction model (VECM) with an unrestricted constant term for the logs of consumption and income was fitted using data for each country where we found cointegration. Otherwise, a vector autoregression model for the first differences of those series was estimated. We selected lag length by the use of information criteria, coupled with diagnostic test results. Based on VECM estimates we implemented the multivariate Beveridge and Nelson decomposition as suggested in Proietti (1997). We compute trend and cycle components of consumption using either equations (18) and (19) or equation (16).

Welfare costs of business cycles (λ) for EU-15 and EU countries was computed using equation (11) considering Beveridge-Nelson decomposition, linear time trend and Hodrick-Prescott trend. As a benchmark, we also computed the welfare cost of business cycles for the USA using aggregated consumption data from 1950 to 2000.

Results for reasonable preference parameter and discount values (β = 0.971, ϕ = 2) are presented in Table 4. Standard errors were calculated using Delta Method as discussed above and, as we may observe, they are negligible if compared to λ. Thus, welfare cost estimates are statistically different from zero at 1% of significance. Results for β={0.950;0.971;0.985} and ϕ={1;5;10;20} are presented in Appendix D.

Table 4 - Welfare Cost of Business Cycles

	Country	$\lambda(\%)$ for $\beta = 0.971$ and $\phi=2$		
		Beveridge-Nelson Decomposition	Hodrick-Prescott for Trend	Linear Time Trend
USA [Franco, Guillen and Issler (2003)]		0.25	0.04	0.40
USA		0.75 (0.0227)	0.04 (0.0011)	0.10 (0.0029)
European Union Countries	EU-15	0.18 (0.0041)	0.02 (0.0007)	0.21 (0.0059)
	The Netherlands	2.33 (0.0720)	0.13 (0.0038)	0.43 (0.0121)
	Italy	2.85 (0.0876)	0.06 (0.0016)	0.80 (0.0228)
	United Kingdom	0.45 (0.0134)	0.04 (0.0011)	0.09 (0.0024)
	France	2.57 (0.0787)	0.03 (0.0290)	0.89 (0.0254)
	Austria	1.33 (0.0375)	0.09 (0.0026)	0.23 (0.0066)
	Spain	4.10 (0.1171)	0.13 (0.0035)	1.49 (0.0426)
	Portugal	2.82 (0.0631)	0.31 (0.0089)	1.28 (0.0365)
	Belgium	2.91 (0.0698)	0.05 (0.0015)	0.35 (0.0099)
	Finland	3.72 (0.1051)	0.16 (0.0045)	0.67 (0.0190)
	Denmark	2.96 (0.0919)	0.16 (0.0046)	0.57 (0.0162)
	Ireland	2.42 (0.0737)	0.12 (0.0033)	0.57 (0.0161)
	Greece	3.26 (0.1005)	0.08 (0.0023)	0.92 (0.0262)
	Sweden	0.80 (0.0243)	0.18 (0.0051)	0.43 (0.0122)
	Luxembourg	1.23 (0.0297)	0.27 (0.0075)	0.62 (0.0175)
	Germany	3.91 (0.1233)	0.16 (0.0045)	0.36 (0.0102)

Note: Standard error in parentesis

On the one hand, for Beveridge-Nelson decomposition welfare costs for most EU countries is much greater than that for EU-15 as a whole and for the USA. Numbers for UK (0.45%) and Sweden (0.80%) are of the same order magnitude as for USA (0.75%). However, the result for the EU-15 as a whole is even smaller. On the other hand, there is a group of countries whose welfare costs are more than 2.5%: Spain (4.10%), Finland (3.72%), Germany (3.91%), Greece (3.26%), Belgium (2.91%), Italy (2.85%) and Portugal (2.82%). Comparing with Franco, Guillen and Issler (2003) results for USA for post-WWII period,[12] our result is three times greater.

Using Hodrick-Prescott Filtering we were able to reproduce Lucas (1987) and Franco, Guillen and Issler (2003) results for USA, i.e. λ_{USA} =0.04%. Welfare cost for EU-15 as a whole (0.02%) is lower than that for USA. Results for France (0.03%), UK (0.04%), Belgium (0.05%) and Italy (0.06%) are similar to that of the USA. For the remaining EU countries, particularly Portugal (0.31%) and Luxembourg (0.27%), λ is between 4 and 8 times that of the USA.

[12] They use non-durables and services annualy data from 1947-2000.

Summarily, welfare costs are very different across EU countries and between US and EU countries, and thus it is a strong evidence that business cycles are not alike in Europe. Differences in institutional settings from country to country, and consequentely the effects of shocks in the economies, are good explanation for variations in business cycles. Thus, our result is a contrary evidence of the *dynamic-stochastic-general-equilibrium* view.

6. Conclusions

In this paper, we investigate whether business cycles are all alike computing the welfare costs of business cycles for an important subset of European countries -- European-Union (EU) countries: Austria, Belgium, Denmark, Finland, France, Germany, Great Britain, Greece, Ireland, Italy, Luxembourg, Portugal, Spain, Sweden and The Netherlands. As is well known, these countries have a long tradition of integration and trade dating well before the common-currency Euro area was implemented. Because of this feature, it is a "natural experiment" to investigate how similar their welfare costs of business cycles are, in the sense that we will be surprised to find major differences between them.

In computing the welfare costs of business cycles for EU countries we use the techniques in Beveridge and Nelson (1981) to decompose (the log of) consumption in a trend and a cyclical component. In this case, the trend will be stochastic and modeled as a random walk. Moreover, to make our results comparable to previous work, we also modeled the trend as either a deterministic linear process (with and without a break) or following a slowly evolving secular process captured by the Hodrick and Prescott (1997) filter.

Our results show that the welfare costs of business cycles are very different across EU countries. Using the Beveridge and Nelson decomposition, and plausible values for the risk aversion coefficient and the discount rate of future utility, we find that the welfare cost of Spain (4.1% of consumption) is almost ten times that of the UK (0.45% of consumption) – median of 2.85%. Major differences in welfare costs are also found when alternative trend-cycle decomposition methods are employed, although they are not as pronounced as the ones obtained using the Beveridge and Nelson decomposition.

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Appendix

Appendix A - Convergence of $\lambda(\phi,\beta)$ for $\phi \to 1$

Let $A(\phi) \equiv \exp\left[\frac{\phi(2\tilde{\sigma}_{12}+\tilde{\sigma}_{22})}{2}\right]$ and $B(\phi,\beta)=\left\{\frac{1-\beta(1+\alpha_1)^{1-\phi}\exp\left[-\frac{(1-\phi)\phi\sigma_{11}}{2}\right]}{1-\beta(1+\alpha_1)^{1-\phi}}\right\}^{1/(1-\phi)}$, then $\lambda(\phi,\beta)=A(\phi).B(\phi,\beta)-1$.

Rewriting

$$B(\phi,\beta)=\left\{\left[1+B_1(\phi,\beta)\right]^{\frac{1}{B_1(\phi,\beta)}}\right\}^{B_{21}(\phi,\beta).B_{22}(\phi,\beta)}$$

where $B_1(\phi,\beta)=\frac{\beta(1+\alpha_1)^{1-\phi}\left(1-\exp\left[-\frac{(1-\phi)\phi\sigma_{11}}{2}\right]\right)}{1-\beta(1+\alpha_1)^{1-\phi}}$, $B_{21}(\phi,\beta)=\frac{\beta(1+\alpha_1)^{1-\phi}}{1-\beta(1+\alpha_1)^{1-\phi}}$ and $B_{22}(\phi)=\frac{1-\exp\left[-\frac{(1-\phi)\phi\sigma_{11}}{2}\right]}{1-\phi}$

Thus $\lim_{\phi\to 1} B_{21}(\phi,\beta)=\frac{\beta}{1-\beta}$. Applying L'Hospital's Rule in $B_{22}(\phi)$ we have $\lim_{\phi\to 1} B_{22}(\phi)=\frac{\sigma_{11}}{2}$.

So, it's possible to resume $\lim_{\phi\to 1} B_2(\phi,\beta)=\frac{\beta\sigma_{11}}{2(1-\beta)}$.

Since that $\lim_{\phi\to 1} B_1(\phi,\beta)=0$, applying the definition of *e* number (base of the natural logarithm) we have,

$$\lim_{\phi\to 1} B(\phi,\beta)=\exp\left[\frac{\beta\sigma_{11}}{2(1-\beta)}\right] \text{ and}$$

$$\lim_{\phi\to 1} \lambda(\phi,\beta)=\exp\left[\frac{1}{2}\left(\frac{\beta\sigma_{11}}{1-\beta}+2\tilde{\sigma}_{12}+\tilde{\sigma}_{22}\right)\right]-1$$

Appendix B - Deriving Trend and Cycle Formulae from Space-state Form

Space-state form: $$\Delta y_t = Zf_t$$ $$f_t = Tf_{t-1} + Z'\varepsilon_t$$

Trend-cycle representation from Beveridge and Nelson (1981): $y_t = \tau_t + \psi_t$

where $\tau_t = y_t + \lim_{k\to\infty}\sum_{i=0}^{k} E_t\left[\Delta y_{t+i}\right]$ and $\psi_t = -\lim_{k\to\infty}\sum_{i=0}^{k} E_t\left[\Delta y_{t+i}\right]$

Solving the space-state form recursively we get,

$$\Delta y_{t+i} = ZT^{i} f_t + \sum_{j=1}^{i} ZT^{i-j} Z'\varepsilon_{t+j}$$

Applying E_t on both sides and summing up from $i = 1$ to ∞ we have,

$$\sum_{i=1}^{\infty} E_t \Delta y_{t+i} = Z\left(\sum_{i=1}^{\infty} T^{i}\right) f_t = Z\left(\sum_{i=0}^{\infty} T^{i}\right) Tf_t = Z\left[I_m - T\right]^{-1} Tf_t$$

where m = 2p + 1.

The cyclical and trend components will be, respectively,

$$\psi_t = -Z\left[I_m - T\right]^{-1} Tf_t$$

$$\tau_t = y_t - \psi_t$$

It is straightforward to see that $\Delta\tau_t = Z\left[I_m - T\right]^{-1} Z'\varepsilon_t$, i.e., τ_t is a multivariate random-walk.

Appendix C - Computing Conditional Covariances

From Proposition 2 in Proietti (1997),

$$\psi_{t+1} = -(I_N - P)(\Gamma(1) + \gamma\alpha')^{-1}\Gamma^*(L)\Delta y_{t+1} + Py_{t+1},$$

$$\tau_{t+1} = (I_N - P)(\Gamma(1) + \gamma\alpha')^{-1}\Gamma(L) y_{t+1}, \text{ or}$$

$$\Delta\tau_{t+1} = (I_N - P)(\Gamma(1) + \gamma\alpha')^{-1}\varepsilon_{t+1}$$

where, $P = (\Gamma(1) + \gamma\alpha')^{-1}\gamma\left[\alpha'(\Gamma(1) + \gamma\alpha')^{-1}\gamma\right]\alpha'$ and $\Gamma(L) = I - \Gamma_1 L$, which is decomposed as $\Gamma(L) = \Gamma_1(1) + (1-L)\Gamma^*(L)$, where $\Gamma^*(L) = \Gamma_1$, in the present context.

So, we have, $\tau_{t+s} = \tau_t + (I_N - P)(\Gamma(1) + \gamma\alpha')^{-1}\sum_{i=1}^{s}\varepsilon_{t+i}$ which implies that $\hat{\tau}_{t+s|t} = E_t(\hat{\tau}_{t+s}) = \tau_t$.

Denoting $K = (I_N - P)(\Gamma(1) + \gamma\alpha')^{-1}$, and $(\tau_{t+s} - \hat{\tau}_{t+s|t}) = K\sum_{i=1}^{s}\varepsilon_{t+i}$ we have,

$$E\left[\left(\tau_{t+s} - \hat{\tau}_{t+s|t}\right)\left(\tau_{t+s} - \hat{\tau}_{t+s|t}\right)'\right] = K\sum_{i=1}^{s} Q_i K' = s.KQK'$$

where $E_t\left[\varepsilon_{t+i}\varepsilon'_{t+i}\right] = Q_i = Q$. On the other hand, we can write,

$$\psi_{t+s} = -K\Gamma^*(L)\Delta y_{t+s} + Py_{t+s}$$

but, $\Delta y_{t+1} = ZTf_t + \varepsilon_{t+1}$, which implies that $\Delta y_{t+s} = ZTf_{t+s-1} + \varepsilon_{t+s}$. However, $y_{t+1} = y_t + ZTf_t + \varepsilon_{t+1}$, which implies that $y_{t+s} = y_t + Z\left(\sum_{i=1}^{s} T^i\right) f_t + \sum_{i=1}^{s}\left(I_N + Z\left\{\sum_{j=i}^{s-1} T^j\right\} Z'\right)\varepsilon_{t+i} + \varepsilon_{t+s}$.

Hence, $\hat{\psi}_{t+s|t} = E_t(\psi_{t+s}) = -K\Gamma^*(1)ZTf_{t+s-1} + Py_t + PZ\left(\sum_{i=1}^{s} T^i\right) f_t$, which implies that

$$\psi_{t+s} - \hat{\psi}_{t+s|t} = \left[P - K\Gamma^*(1)\right]\varepsilon_{t+s} + \sum_{i=1}^{s}\left(I_N + Z\left\{\sum_{j=1}^{s-i} T^j\right\} Z'\right)\varepsilon_{t+i} \ .$$

Denoting $V = \left[P - K\Gamma^*(1)\right]$ and $W(i) = \left(I_N + Z\left\{\sum_{j=1}^{s-i} T^j\right\} Z'\right)$ we have,

$$E_t\left[\left(\psi_{t+s} - \hat{\psi}_{t+s|t}\right)\left(\psi_{t+s} - \hat{\psi}_{t+s|t}\right)'\right] = VQV' + P\left(\sum_{i=1}^{s-1} W(i)QW(i)'\right)P'$$

and $E_t\left[\left(\tau_{t+s} - \hat{\tau}_{t+s|t}\right)\left(\psi_{t+s} - \hat{\psi}_{t+s|t}\right)'\right] = KQV' + K\left(\sum_{i=1}^{s-1} QW(i)'\right)P'$

Appendix D - Tables

Tabela D.1 - Welfare Cost of Business Cycles ($\lambda\%$)

(continues)

Lucas(1987) Benchmark Values (USA Data)	β = 0.950, 0.971, 0.985	ϕ=1	ϕ=5	ϕ=10	ϕ=20
		0.008	0.042	0.08	0.17

Country	Model	β	ϕ=1	ϕ=5	ϕ=10	ϕ=20
USA (Franco *et alli*, 2003)	Beveridge-Nelson Decomposition	0.950	0.14	0.25	0.27	0.30
		0.971	0.24	0.29	0.30	0.31
		0.985	0.47	0.33	0.32	0.32
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.01	0.04	0.08	0.17
	Linear Time Trend	0.950, 0.971, 0.985	0.14	0.70	1.41	2.84
USA (Total Consumption)	Beveridge-Nelson Decomposition	0.950	0.38 (0.0115)	0.66 (0.0200)	0.70 (0.0217)	0.68 (0.0226)
		0.971	0.67 (0.0203)	0.79 (0.0240)	0.78 (0.0242)	0.73 (0.0242)
		0.985	1.33 (0.0402)	0.90 (0.0277)	0.83 (0.0262)	0.76 (0.0254)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.02 (0.0005)	0.09 (0.0027)	0.19 (0.0054)	0.38 (0.0107)
	Linear Time Trend	0.950, 0.971, 0.985	0.05 (0.0015)	0.26 (0.0073)	0.51 (0.0146)	1.03 (0.0294)
European Union (EU-15)	Beveridge-Nelson Decomposition	0.950	0.10 (0.0021)	0.27 (0.0062)	0.44 (0.0115)	0.77 (0.0225)
		0.971	0.14 (0.0034)	0.28 (0.0065)	0.45 (0.0116)	0.78 (0.0226)
		0.985	0.25 (0.0065)	0.30 (0.0067)	0.46 (0.0117)	0.78 (0.0226)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.01 (0.0003)	0.06 (0.0017)	0.12 (0.0035)	0.25 (0.0069)
	Linear Time Trend	0.950, 0.971, 0.985	0.10 (0.0029)	0.52 (0.0148)	1.05 (0.0298)	2.11 (0.0602)
The Netherlands	Beveridge-Nelson Decomposition	0.950	1.33 (0.0403)	1.98 (0.0630)	2.11 (0.0730)	2.32 (0.1012)
		0.971	2.36 (0.0717)	2.31 (0.0744)	2.31 (0.0810)	2.47 (0.1094)
		0.985	4.68 (0.1439)	2.60 (0.0842)	2.47 (0.0870)	2.57 (0.1153)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.07 (0.0019)	0.34 (0.0096)	0.68 (0.0192)	1.36 (0.0387)
	Linear Time Trend	0.950, 0.971, 0.985	0.21 (0.0060)	1.07 (0.0304)	2.15 (0.0613)	4.34 (0.1253)
Italy	Beveridge-Nelson Decomposition	0.950	1.74 (0.0522)	2.16 (0.0682)	2.00 (0.0711)	1.78 (0.1018)
		0.971	3.13 (0.0952)	2.54 (0.0817)	2.24 (0.0808)	1.97 (0.1141)
		0.985	6.29 (0.1947)	2.86 (0.0931)	2.41 (0.0882)	2.10 (0.1232)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.03 (0.0008)	0.14 (0.0039)	0.28 (0.0079)	0.56 (0.0158)
	Linear Time Trend	0.950, 0.971, 0.985	0.40 (0.0114)	2.02 (0.0577)	4.08 (0.1178)	8.33 (0.2453)

Tabela D.1 - Welfare Cost of Business Cycles ($\lambda\%$)

(continues)

Country	Model	β	$\phi=1$	$\phi=5$	$\phi=10$	$\phi=20$
United Kingdom	Beveridge-Nelson Decomposition	0.950	0.23 (0.0067)	0.42 (0.0122)	0.47 (0.0137)	0.52 (0.0154)
		0.971	0.40 (0.0119)	0.50 (0.0146)	0.52 (0.0151)	0.55 (0.0162)
		0.985	0.78 (0.0234)	0.57 (0.0167)	0.56 (0.0162)	0.57 (0.0168)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.02 (0.0006)	0.10 (0.0028)	0.20 (0.0057)	0.40 (0.0114)
	Linear Time Trend	0.950, 0.971, 0.985	0.04 (0.0012)	0.21 (0.0061)	0.43 (0.0122)	0.86 (0.0244)
France	Beveridge-Nelson Decomposition	0.950	1.35 (0.0402)	2.16 (0.0677)	2.26 (0.0788)	2.44 (0.1240)
		0.971	2.41 (0.0730)	2.61 (0.0837)	2.56 (0.0914)	2.69 (0.1411)
		0.985	4.83 (0.1482)	3.01 (0.0982)	2.79 (0.1014)	2.88 (0.1542)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.01 (0.0004)	0.07 (0.0020)	0.14 (0.0041)	0.29 (0.0082)
	Linear Time Trend	0.950, 0.971, 0.985	0.45 (0.0126)	2.25 (0.0642)	4.54 (0.1314)	9.29 (0.2747)
Austria	Beveridge-Nelson Decomposition	0.950	0.68 (0.0190)	1.29 (0.0333)	1.56 (0.0394)	1.97 (0.0537)
		0.971	1.17 (0.0339)	1.51 (0.0401)	1.70 (0.0434)	2.05 (0.0561)
		0.985	2.28 (0.0677)	1.70 (0.0463)	1.80 (0.0467)	2.11 (0.0579)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.05 (0.0013)	0.23 (0.0065)	0.46 (0.0130)	0.92 (0.0261)
	Linear Time Trend	0.950, 0.971, 0.985	0.12 (0.0033)	0.58 (0.0166)	1.17 (0.0333)	2.36 (0.0675)
Spain	Beveridge-Nelson Decomposition	0.950	2.29 (0.0637)	3.88 (0.1023)	4.86 (0.1358)	7.55 (0.3401)
		0.971	4.00 (0.1164)	4.48 (0.1243)	5.27 (0.1541)	8.07 (0.3962)
		0.985	7.90 (0.2401)	5.00 (0.1439)	5.59 (0.1690)	8.47 (0.4432)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.06 (0.0018)	0.31 (0.0089)	0.63 (0.0178)	1.26 (0.0358)
	Linear Time Trend	0.950, 0.971, 0.985	0.74 (0.0211)	3.78 (0.1089)	7.70 (0.2260)	15.99 (0.4867)
Portugal	Beveridge-Nelson Decomposition	0.950	1.56 (0.0346)	4.54 (0.1152)	7.97 (0.2278)	15.11 (0.4796)
		0.971	2.26 (0.0532)	4.74 (0.1178)	8.09 (0.2288)	15.18 (0.4803)
		0.985	3.83 (0.0997)	4.90 (0.1202)	8.17 (0.2297)	15.23 (0.4807)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.16 (0.0044)	0.79 (0.0224)	1.58 (0.0451)	3.19 (0.0916)
	Linear Time Trend	0.950, 0.971, 0.985	0.64 (0.0181)	3.24 (0.0931)	6.58 (0.1921)	13.59 (0.4096)

Tabela D.1 - Welfare Cost of Business Cycles ($\lambda\%$)

(continues)

Country	Model	β	ϕ=1	ϕ=5	ϕ=10	ϕ=20
Belgium	Beveridge-Nelson Decomposition	0.950	1.52 (0.0360)	3.68 (0.0824)	5.65 (0.1397)	9.59 (0.2738)
		0.971	2.43 (0.0628)	4.06 (0.0916)	5.89 (0.1443)	9.78 (0.2774)
		0.985	4.47 (0.1257)	4.38 (0.1005)	6.08 (0.1482)	9.91 (0.2803)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.03 (0.0008)	0.13 (0.0038)	0.27 (0.0075)	0.53 (0.0151)
	Linear Time Trend	0.950, 0.971, 0.985	0.17 (0.0050)	0.88 (0.0249)	1.76 (0.0503)	3.56 (0.1024)
Finland	Beveridge-Nelson Decomposition	0.950	1.97 (0.0541)	3.63 (0.0942)	4.63 (0.1257)	7.11 (0.2881)
		0.971	3.41 (0.0986)	4.23 (0.1155)	5.05 (0.1435)	7.60 (0.3341)
		0.985	6.70 (0.2023)	4.76 (0.1350)	5.38 (0.1581)	7.98 (0.3727)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.08 (0.0022)	0.40 (0.0112)	0.79 (0.0225)	1.59 (0.0453)
	Linear Time Trend	0.950, 0.971, 0.985	0.33 (0.0095)	1.68 (0.0479)	3.39 (0.0975)	6.89 (0.2015)
Denmark	Beveridge-Nelson Decomposition	0.950	1.52 (0.0461)	2.73 (0.0887)	3.25 (0.1173)	4.35 (0.2174)
		0.971	2.68 (0.0817)	3.26 (0.1072)	3.60 (0.1327)	4.68 (0.2433)
		0.985	5.30 (0.1636)	3.72 (0.1240)	3.88 (0.1451)	4.94 (0.2637)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.08 (0.0023)	0.41 (0.0116)	0.82 (0.0233)	1.65 (0.0470)
	Linear Time Trend	0.950, 0.971, 0.985	0.29 (0.0081)	1.43 (0.0408)	2.89 (0.0828)	5.86 (0.1705)
Ireland	Beveridge-Nelson Decomposition	0.950	1.37 (0.0410)	2.03 (0.0629)	2.16 (0.0717)	2.35 (0.1002)
		0.971	2.43 (0.0735)	2.39 (0.0750)	2.37 (0.0802)	2.51 (0.1093)
		0.985	4.83 (0.1484)	2.69 (0.0855)	2.54 (0.0867)	2.63 (0.1159)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.06 (0.0017)	0.29 (0.0083)	0.59 (0.0167)	1.18 (0.0336)
	Linear Time Trend	0.950, 0.971, 0.985	0.28 (0.0080)	1.43 (0.0407)	2.88 (0.0825)	5.84 (0.1698)
Greece	Beveridge-Nelson Decomposition	0.950	2.00 (0.0602)	2.56 (0.0811)	2.54 (0.0909)	2.72 (0.1542)
		0.971	3.59 (0.1092)	2.99 (0.0965)	2.81 (0.1024)	2.96 (0.1719)
		0.985	7.20 (0.2237)	3.35 (0.1096)	3.01 (0.1112)	3.13 (0.1852)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.04 (0.0011)	0.20 (0.0057)	0.40 (0.0114)	0.80 (0.0228)
	Linear Time Trend	0.950, 0.971, 0.985	0.46 (0.0130)	2.32 (0.0663)	4.69 (0.1357)	9.59 (0.2840)

Tabela D.1 - Welfare Cost of Business Cycles ($\lambda\%$)

(conclusion)

Country	Model	β	ϕ=1	ϕ=5	ϕ=10	ϕ=20
Sweden	Beveridge-Nelson Decomposition	0.950	0.37 (0.0110)	0.73 (0.0225)	0.81 (0.0257)	0.81 (0.0281)
		0.971	0.65 (0.0196)	0.92 (0.0283)	0.93 (0.0297)	0.88 (0.0309)
		0.985	1.28 (0.0389)	1.09 (0.0337)	1.02 (0.0329)	0.94 (0.0330)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.02 (0.0006)	0.10 (0.0029)	0.21 (0.0058)	0.41 (0.0117)
	Linear Time Trend	0.950, 0.971, 0.985	0.09 (0.0025)	0.45 (0.0127)	0.90 (0.0254)	1.80 (0.0514)
Luxembourg	Beveridge-Nelson Decomposition	0.950	0.72 (0.0176)	1.49 (0.0340)	2.22 (0.0569)	3.65 (0.1093)
		0.971	1.16 (0.0306)	1.61 (0.0364)	2.29 (0.0577)	3.69 (0.1096)
		0.985	2.14 (0.0604)	1.71 (0.0386)	2.33 (0.0583)	3.71 (0.1097)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.13 (0.0038)	0.66 (0.0189)	1.33 (0.0380)	2.68 (0.0769)
	Linear Time Trend	0.950, 0.971, 0.985	0.31 (0.0087)	1.55 (0.0442)	3.13 (0.0898)	6.35 (0.1853)
Germany	Beveridge-Nelson Decomposition	0.950	2.18 (0.0661)	3.27 (0.1106)	3.58 (0.1454)	5.05 (0.3948)
		0.971	3.90 (0.1196)	3.90 (0.1346)	4.02 (0.1673)	5.63 (0.4689)
		0.985	7.83 (0.2448)	4.44 (0.1559)	4.35 (0.1848)	6.09 (0.5316)
	Hodrick-Prescott for Trend	0.950, 0.971, 0.985	0.08 (0.0023)	0.40 (0.0114)	0.81 (0.0229)	1.62 (0.0461)
	Linear Time Trend	0.950, 0.971, 0.985	0.18 (0.0051)	0.90 (0.0255)	1.81 (0.0515)	3.64 (0.1049)