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**EQUILIBRIUM INCENTIVE CONTRACTS**

**by**

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# Equilibrium Incentive Contracts

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## Abstract

We study a labour market in which firms can observe workers' output but not their effort, and in which a worker's productivity in a given firm depends on a worker-firm specific component, unobservable for the firm. Firms offer wage contracts that optimally trade off effort and wage costs. As a result, employed workers enjoy rents, which in turn create unemployment. We show that the incentive power of the equilibrium wage contract is constrained socially efficient in the absence of unemployment benefits. We then apply the model to explain the recent increase in performance-pay contracts. Within our model, this can be explained by three different factors: (i) increased importance of non-observable effort, (ii) a fall in the marginal tax rate, (iii) a reduction in the heterogeneity of workers performing the same task. The likely effect of all three factors is an increase in the equilibrium unemployment rate.

**Key words:** Incentives, Contracts, Unemployment, efficiency

**JEL codes:** E24, J30, J41

## 1 Introduction

Within the economics profession, a large body of knowledge has been produced on how to design optimal contracts. Furthermore, a new subfield within economics, personnel economics, is primarily concerned with how this body of knowledge can be applied to construct optimal wage contracts within firms. It is puzzling therefore, that so little research effort in the last decade has been directed towards the question of how different kinds of wage contracts within firms may influence the overall performance of the economy.

Particularly because there is evidence that the use of wage contracts is changing in the direction of more performance-related pay.

It has not always been like this. In the eighties, the effects of different kinds of wage contracts on the macroeconomic behaviour of the economy was much in focus. Weitzman's (1985) idea that profit sharing between workers and firms may increase firms' incentives to hire new employees and vacuum clean the market for unemployed workers was intensely debated before it was rejected by most of the profession. The shirking model by Shapiro and Stiglitz (1984), where wages are increased as a response to costly monitoring of worker effort, is still on most Ph.D. students' reading list. In addition we have the literature on implicit contracts (Hart and Holmström 1987) and on wage bargaining (see Farber 1986 for an overview). However, the modelling of wage contracts in many of these papers is, to some extent, premature.

Our starting point is a standard contracting problem as described in Laffont and Tirole (1993). When deciding on a contract, the principal trades off incentives and rent extraction. As a result, workers obtain (in expected terms) some rents, in the sense that it is better to be employed than to be unemployed. Consequently, there is unemployment in equilibrium.

Within our model, workers' productivity depends on their general productivity, the match specific productivity, and their effort. We show that with *ex ante* identical workers and no unemployment benefits, the unemployment rate is constraint efficient in the sense that a planner would not like to alter the incentive contracts provided by the firms.

A recent study (Towers and Perrin 1998) indicates that performance-pay contracts have recently become more widely used. We interpret this as an increased incentive power of the contracts provided to these workers. Within our model framework we identify three possible sources for why the incentive power of labour contracts may increase: First, increased importance of unmonitored effort provided by the workers. Second, lower labour taxes. Third, less heterogeneity among workers within a given job category (given observable characteristics), due to a more segregated labour market and due to improved selection methods. We find that the likely effect of all these changes is an increase in the equilibrium unemployment rate.

As mentioned above surprisingly little work has been done on the relationship between optimal output based contracts and the performance of the labour market. An exception is Foster and Wang (1984) who show that rents associated with optimal contracts may lead to unemployment. Their paper differs in several respects from the present paper. First, Foster and Wang

assume a given number of firms. With free entry of firms, as we assume, unemployment is not an equilibrium outcome in their model. Second, Foster and Wang do not examine whether the incentives provided by the market are socially optimal. Finally their paper does not explicitly derive the determinants of the unemployment rate, thereby making it unsuitable for analysing the effects of changes in the incentive structures on unemployment.

Our paper is also related to the large literature on rents associated with employment, including the seminal contribution by Shapiro and Stiglitz (1984) mentioned above. In their model, individual output is unobservable, thereby ruling out incentive schemes of the type considered here.

The paper is organised as follows: In the next section, we first discuss on a broad level different trade-offs that an optimal contract may balance, and argue that the relationship between worker rents and incentives may be important. We characterise the optimal contract in this case, and derive the resulting labour market equilibrium. Section 3 concerns efficiency of this equilibrium. We show that the unemployment rate in this economy is constrained efficient, in the sense that a planner would not like to alter the incentive contracts provided by the firms. In section 4 we derive comparative statics result, while the last section concludes.

## **2 Modelling performance pay**

As mentioned in the introduction, there exist several different models of optimal contracts in which the costs and benefits of stronger incentives are balanced at the margin. In most of these models, the gain from providing stronger incentives to the agent is that this gives rise to higher effort. The costs (or agency costs) associated with stronger incentives may vary: Firstly, stronger incentives may give rise to a misallocation of risk, as the agent will carry a larger share of the risks than an optimal risk-sharing agreement would imply. Secondly, in a multi-tasking framework, stronger incentives may imply that the agent will allocate his effort on the different tasks inefficiently if the output from some of the tasks cannot be measured adequately. Thirdly, if the agent has private information about his ability, then providing him with stronger incentives implies that he captures more economic rents, implying that the principal faces a trade-off between incentives and rent extraction.

In this paper we focus on the last type of models, often referred to as adverse selection models. A seminal paper on adverse selection models is

Mirlees (1971), which characterises an optimal tax regime under asymmetric information. Maskin and Riley (1984) analyse the optimal price discrimination strategy of a monopolist, and Baron and Myerson (1982) the optimal regulation of a monopolist. The trade-off between incentives and rent extraction caused by asymmetric information is studied in detail in Laffont and Tirole (1993), and our model of optimal labour contracts will be closely related to the standard model in their book.

Two driving assumptions in our model of optimal contracts (as in models with asymmetric information more generally) are as follows:

1. There exists some *ex post* heterogeneities between the agents. This means that the productivity of an employee is unknown to the employer at the point when the wage contract is signed.
2. It is costly for a firm to replace an employee. These costs may be search and hiring costs or training costs and are sunk when the firm (eventually) learns the productivity of the employee in question.

We will argue that these assumptions have empirical support. The strongest empirical evidence is found in a unique study by Lazear (2000). Lazear analysed the effects of a shift from a flat wage rate to performance pay in a large corporation (Safelite). Lazear had access to data concerning individual worker productivity before and after the shift in pay structure. Many of his observations are striking:

1. Productivity differs between workers. The variance in monthly productivity as a percentage of the mean was 53 percent before and 49 percent after the switch to piece rate payments. However, this number includes both within-worker and between-worker components. When estimating the between-worker components (ability differences, or what Lazear refers to as fixed effects), controlling for time and tenure effects, the variance in percentage of the means are still 24 percent before and 20 percent after the switch to piece rate payments. The difference in productivity between the 90th and the 10th percentile of the workers in percent of the means are 47 percent before and 35 percent after the switch to piece rate payments
2. When introducing piece rate payments, the company included a wage floor approximately equal to the fixed wage introduced earlier. According to Lazear, the firm did this "*in order to avoid massive turnover*".

Thus, the firm regarded it as being in its interest to keep the workers at the lower end of the productivity scale. Furthermore, many workers ended up in the guarantee range.

3. Still, 92 percent of the workers experienced a wage increase, and a quarter of the workers received a wage increase exceeding 28 percent of the previous wage. Since workers within the guarantee wage range were no worse off than before the shift, all workers experiencing a wage increase must be better off after the shift. Furthermore, the numbers indicate that many workers were substantially better off after the shift to piece rate payment.

Lazear's findings thus clearly indicate that the assumptions listed above (that workers are *ex post* heterogenous and that it is in the firms' interest to retain not only the very best workers) are valid. It follows that an asymmetric information model, where firms trade off incentives and rent extraction, seems appropriate when modeling a firm's choice of wage contract.

## 2.1 The model

There are two types of agents in the economy, workers and firms. The measure of workers in the economy is constant and normalised to one. Workers leave the market for exogenous reasons at a rate  $s$ , and are replaced by new workers that enter the market as unemployed. Workers and firms have symmetric information about the worker's productivity at the hiring stage, and we capture this by assuming that workers are identical *ex ante*, i.e., before they are hired by a firm. We are thus studying a segment of the market in which workers have the same observable characteristics.<sup>1</sup> However, once employed, the productivity of a given worker also depends on a worker-firm

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<sup>1</sup>Observable differences in productivity will not change our results, as the optimal wage contract will be contingent on all observable characteristics. The important aspect of the assumption is that workers and firms are symmetrically informed about the worker's productivity. This is admittedly a strong assumption, as self-selection mechanisms may be empirically important (see for instance Lazear 2000). On the other hand, the mechanisms created by self-selection on contracts are very different from the mechanisms studied in this paper. We therefore find it rational to separate the effects of self-selection and of worker rents into two different studies, and refer the interested reader to Moen and Rosen (2001) for the effects of self-selection on contracts.

specific productivity term  $\epsilon$ , reflecting that a worker may fit better into some jobs than others.

The timing of the hiring process goes as follows:

1. The firm incurs a search cost  $K$
2. The firm advertises a wage contract
3. The firm receives job applications from unemployed workers
4. One of the applicants is hired
5. Production starts.

The time delay associated with the hiring process is assumed to be small relative to the duration of the employment relationship, and is therefore ignored.<sup>2</sup> As shown below, the firm will always attract applicants as long as the expected value of the contract to the worker exceeds his outside option  $U^0$ . As we will see, this constraint will not be binding, as firms will offer contracts that leave rents to the workers in expected terms.

The worker-firm specific term is revealed to the worker after he is hired. We assume that the time it takes before the worker learns his worker-firm specific productivity term is sufficiently long so that other applicants for the job are not available at that point in time. Thus, if the worker leaves at this point, the firm has to incur the search cost  $K$  over again to hire a new worker. Still, we assume that this time lag is relatively short compared to the expected duration of the employment relationship. As the focus of this paper is not on the behaviour of the worker during the learning process, we assume that  $\epsilon$  is revealed to the worker immediately after he is hired. We assume that  $\epsilon$  is unobservable for the firm.

The search cost  $K$  may be given various interpretations. The most direct interpretation is that  $K$  denotes the cost of advertising a vacancy, for instance in a newspaper.  $K$  may also include costs associated with evaluating and testing workers. More generally,  $K$  may consist of any costs incurred by the firm (not the worker) before the worker's productivity is revealed, which is wasted if the worker quits. Thus, if the firm pays for firm-specific training

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<sup>2</sup>Note, though, that  $K$  may partly reflect costs associated with time delays caused by a time-consuming hiring process.

costs in the initial phase of the employment relationship, this may also be included in  $K$ .

We assume that the value of  $\epsilon$  for any worker-firm pair is continuously distributed on an interval  $[\epsilon_{\min}, \epsilon^{\max}]$ , and we denote the cumulative distribution function  $F$ . Furthermore, we assume that for a given worker, the worker-firm specific productivity terms in any two firms are independent. Thus, the realisation of  $\epsilon$  does not convey any information regarding the firm-specific term in another firm. It follows that a worker's outside option  $U^0$  when employed is independent of his worker-firm specific term in that firm. This is a simplifying assumption. Our main results will hold if a worker's productivity term  $\epsilon$  in one firm is correlated with the value in other firms as long as the correlation is less than perfect.

We do not allow for up-front payments (bonding). Thus, the firm can not charge an applicant with an entrance fee at the moment he is hired. Entrance fees will eliminate unemployment in our model. We think the absence of entrance fees is the strongest assumption we make in the paper. There has been a debate within the profession regarding the plausibility of entrance fees and bonding, see for instance Carmichael and Lorne (1985) and MacLeod and Malcomson (1993). In our setting, the absence of bonding may be rationalised in several ways.<sup>3</sup>

First, an entrance fee must be paid before the worker learns his worker-firm productivity term. After this term is observed by the worker, it is optimal to leave rents to "high-type" workers. The contract we apply below optimally trades off worker incentives and worker rents, and at this stage bonds are superfluous as they will not increase firm profit. Thus, as long as the worker learns  $\epsilon$  relatively quickly, implicit bonding like deferred wage compensation or seniority wages as in Lazear (1981) does not work. A bond must be interpreted literally as an up-front payment from the worker to the firm (or at least as a payment that proceeds the revelation of  $\epsilon$ ).

There may be several reasons as to why a worker may be reluctant to pay its employer an up-front fee sufficiently high to eliminate all his expected rents. Ritter and Taylor (1994) shows that if firms have private information regarding its probability of bankruptcy, then requiring a bond can be interpreted as a sign that its probability of bankruptcy is high. As a result,

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<sup>3</sup>At least the Norwegian legislation does not allow for entrance fees paid to firms. The contracts act of 31th of May 1918 no 4, §36, in effect deems up-front payments as illegal. We believe similar legal restrictions on up-front payments exist in many European countries,

firms with a low probability of bankruptcy leaves rents to the employees. More generally, with up-front fees the firm may have an incentive to foul the workers in various ways, by hiring and collecting bonds from too many employees, by prematurely replacing the worker (to collect a new bond) etc. By requiring a low bond or no bond at all, a firm may signal that it has no such intentions.<sup>4</sup>

### 2.1.1 Optimal contracts

Following a standard approach to contract theory, as laid out in Laffont and Tirole (1993), we assume that the productivity of a worker  $i$  in a firm  $j$  can be written as  $y_{ij} = \bar{y} + \alpha\epsilon_{ij} + \gamma e_j$  where  $e_j$  is worker effort (unobservable to the firm) and  $\bar{y}$  is a constant. The parameter  $\alpha$  reflects the importance of the worker-firm specific term, and  $\gamma$  the importance of worker effort, for the output level. Output is observable, and wage contracts may therefore be made contingent on  $y$ . A worker's utility flow is given by  $u = w - c(e)$ , where  $w$  denotes the wage and  $c(e)$  the effort costs.<sup>5</sup> We assume that  $c(e)$  is increasing and that  $c'(e)$  is convex, with  $c'(0) = 0$ .

The firm faces a trade-off between providing incentives to and extracting rents from the worker, and the optimal contract reflects this trade-off. In order to derive the optimal contract, we employ the revelation principle.<sup>6</sup> An optimal wage contract  $w(\epsilon), e(\epsilon)$  maximises firm profits  $\pi$  given 1) the worker's incentive compatibility (IC) constraint and 2) his individual rationality (IR) constraint.<sup>7</sup>

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<sup>4</sup>Suppose for instance that firms may choose to open a "fake" vacancy at cost  $\tilde{K} < K$ . A firm with a fake vacancy collects an entrance fee, and then fires the worker. If the workers cannot distinguish between a firm with a fake vacancy and a firm with an ordinary vacancy, the equilibrium entrance fee cannot exceed  $\tilde{K}$ , as the market then would be overflowed with fake vacancies. If  $\tilde{K}$  is not too high, there would still be (an endogenous amount of) rents in the economy.

<sup>5</sup>Strictly speaking, the relevant pay-offs are the expected discounted values of the income flows, or the asset values, not the flows themselves. Note also that the asset values can be obtained simply by dividing the associated flows with the discount rate  $r + s$ .

<sup>6</sup>We have assumed that the contract is advertised, and thus is constructed before the worker is hired. The revelation principle can therefore not be interpreted literally.

<sup>7</sup>When the firm learns the worker type, it has an incentive to renegotiate the contract. However, from an *ex ante* perspective it is optimal for the firm to commit to the contract.

A worker of "type"  $\epsilon$  can pretend that he is of type  $\tilde{\epsilon}$ . If he does so, he obtains a utility

$$\tilde{u}(\epsilon, \tilde{\epsilon}) = w(\tilde{\epsilon}) - c(e(\tilde{\epsilon})) + \frac{\alpha}{\gamma}(\tilde{\epsilon} - \epsilon)$$

The indirect utility flow can be written  $u(\epsilon) = \max_{\tilde{\epsilon}} u(\epsilon, \tilde{\epsilon})$ . The incentive compatibility constraint (truth-telling constraint) requires that  $\epsilon = \arg \max_{\tilde{\epsilon}} u(\epsilon, \tilde{\epsilon})$ , and necessary conditions for truth-telling is thus that

$$u'(\epsilon) = \frac{\partial \tilde{u}(\epsilon, \epsilon)}{\partial \epsilon}$$

i.e., that  $u'(\epsilon) = c'(e(\epsilon))\alpha/\gamma$ . Individual rationality requires that for any worker that stays with the firm,  $u(\epsilon) \geq u^0 = (r+s)U^0$ , otherwise the worker will do better by leaving the firm.

If a worker's worker-firm specific term is sufficiently low, it may not be in the firm's interest to keep him. Let  $\epsilon_c$  denote the associated cut-off level of  $\epsilon$ . Workers with worker-firm specific productivity term below  $\epsilon_c$  are not retained.

The profit flow of a firm with a worker of type  $\epsilon$  is given by  $\pi(\epsilon) = \bar{y} + \alpha\epsilon + \gamma e(\epsilon) - w(\epsilon)$ . Inserting  $u(\epsilon) = w - c(e)$  gives  $\pi(\epsilon) = \bar{y} + \alpha\epsilon + \gamma e(\epsilon) - c(e(\epsilon)) - u(\epsilon)$ . The optimal contract thus solves the problem

$$\begin{aligned} \max_{e(\epsilon), w(\epsilon), \epsilon_c} \int_{\epsilon_c}^{\epsilon^{\max}} [\bar{y} + \alpha\epsilon + \gamma e(\epsilon) - c(e(\epsilon)) - u(\epsilon)] dF \quad \text{S.T.} \quad (1) \\ u'(\epsilon) = c'(e(\epsilon))\alpha/\gamma \\ u(\epsilon_c) \geq u^0 \end{aligned}$$

This is an optimal control problem, with  $u$  as the state variable and  $e$  as control variable. The associated Hamiltonian is given by

$$H = [\bar{y} + \alpha\epsilon + \gamma e - c(e) - u]f(\epsilon) + \lambda(c'(e)\alpha/\gamma)$$

where  $\lambda$  is the adjungated function. First order conditions for maximum are given by (for a given cut-off  $\epsilon_c$ )

$$\begin{aligned}(\gamma - c'(e))f(\epsilon) + \lambda c''(e)\alpha/\gamma &= 0 \\ \dot{\lambda}(\epsilon) &= f(\epsilon)\end{aligned}$$

Since there are no terminal conditions at  $\epsilon^{\max}$  it follows that  $\lambda(\epsilon^{\max}) = 0$ , and thus that  $\lambda(\epsilon) = -(1 - F)$ . The first order condition for  $e$  can thus be written as

$$c'(e) = \gamma - \frac{1 - F(\epsilon)}{f(\epsilon)}c''(e)\alpha/\gamma \quad (2)$$

We assume that  $\frac{1-F(\epsilon)}{f(\epsilon)}$  is decreasing in  $\epsilon$ , i.e.  $f$  has an increasing hazard rate. If we assume that  $c'''(e) \geq 0$ , it follows that  $e(\epsilon)$  is increasing in  $\epsilon$ . Note that there are no distortions at the top. That is,  $c'(e(\epsilon^{\max})) = \gamma$ , which is the full information effort.

The optimal cut-off value solves the equation  $H(\epsilon_c) = 0$ , or

$$\bar{y} + \alpha\epsilon + \gamma e - u^0 - c(e) = \frac{1 - F(\epsilon_c)}{f(\epsilon_c)}c'(e)\alpha/\gamma \quad (3)$$

This equation uniquely determines  $e^c$  (see Appendix 1). Note that the optimal contract for workers that are hired is independent of the cut-off level. Note also that the expected profit of the firm can be written as a function of  $u^0$ , we write  $\pi = \pi(u^0)$ , or equivalently  $\Pi = \Pi(U^0)$  (where capital values indicate asset values rather than flow values), where  $\Pi$  is strictly decreasing in  $U^0$ .

Let  $(a, b)$  denote a linear contract of the form  $w = a + by$ . It is well known that the optimal non-linear contract can be represented by a menu  $(a(\epsilon), b(\epsilon))$  of linear contracts. For any  $b$ , the effort level chosen by a worker is such that  $c'(e) = b\gamma$ . We will refer to  $b$  as the incentive power of the associated linear contract. From (2) it follows that  $b(\epsilon)$  is given by

$$b(\epsilon) = 1 - \frac{1 - F(\epsilon)}{f(\epsilon)}c''(e)\alpha/\gamma^2 \quad (4)$$

Thus,  $b(\epsilon^{\max}) = 1$  reflecting that there are no distortions at the top. Assuming that  $c'''(e) \geq 0$  it follows from (11) that  $b$  is strictly increasing in  $\epsilon$ . Hence, for the lower types, the incentive power of the contract is strictly

less than one. The intuition is as follows: For any given worker type  $\epsilon$ , the firm faces a trade-off between giving this worker type stronger incentives and rent extraction from workers of higher types. The likelihood of obtaining a worker of type  $\epsilon$  is reflected in  $f(\epsilon)$ , while the measure of workers with a higher type is  $1 - F(\epsilon)$ . The extra rents obtained by higher worker types by increasing  $e$  with one unit is  $c''(e)/\gamma$ . The optimal contract thus scales down  $b$  with an amount equal to  $\frac{1-F(\epsilon)}{f(\epsilon)}c''(e)\alpha/\gamma^2$  (the denominator includes  $\gamma$  squared because  $b = c'(e)/\gamma$ ).<sup>8</sup>

In what follows, we are interested in comparing different wage contracts. We say that wage contract  $A$  is *more incentive powered* than wage contract  $B$  if,  $b^A(\epsilon) \geq b^B(\epsilon)$  for all  $\epsilon$ , with strict inequality for some  $\epsilon$  (with strictly positive measure).

The rent to a worker of type  $\epsilon'$  that is hired by a firm is given by (since  $u'(\epsilon) = \alpha c'(e(\epsilon))/\gamma = \alpha b$ )

$$\begin{aligned}\rho &= \int_{\epsilon_c}^{\epsilon'} u'(\epsilon) d\epsilon \\ &= \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon\end{aligned}$$

Let  $\tilde{F} = F/(1 - F(\epsilon_c))$  denote the distribution of  $\epsilon$  conditional on being above  $\epsilon_c$ . The expected rent to a worker with a worker-firm specific productivity term is (see Appendix 2 for details)

$$\begin{aligned}E\rho &= \int_{\epsilon_c}^{\epsilon^{\max}} \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon d\tilde{F}(\epsilon') \\ &= \int_{\epsilon_c}^{\epsilon^{\max}} \alpha b(\epsilon) (1 - \tilde{F}) d\epsilon \\ &= \frac{\alpha}{1 - F(\epsilon_c)} \int_{\epsilon_c}^{\epsilon^{\max}} b(\epsilon) (1 - F(\epsilon_c) - F(\epsilon)) d\epsilon\end{aligned}\tag{5}$$

The expected income flow for a hired worker can thus be written as  $E[u(\epsilon)] = u^0 + E[\rho]$ . The expected discounted income for an employed worker is thus  $W = U^0 + R$ , where  $R = E[\rho/(r + s)]$ . The next lemma shows that  $E[\rho]$  is always strictly positive

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<sup>8</sup>We have not imposed any restrictions on  $b$ . A natural restriction would be that  $b$  (or  $e$ ) are nonnegative. This will always be the case if  $c'(0) = c''(0) = 0$ .

**Lemma 1** *Suppose  $\epsilon_c < \epsilon^{\max}$ . Then the expected rent  $\rho$  to the worker is strictly positive.*

*Proof:* No worker type can obtain negative rent, as in this case they would quit. From (5) it thus follows that  $\rho$  is zero if and only if  $b$  is zero almost everywhere. However, from (4) it follows that  $b(\epsilon)$  is strictly positive for all  $\epsilon$  sufficiently close to  $\epsilon^{\max}$ . QED

### 2.1.2 Matching

A natural starting point when modelling matching in the labour market is the urn-ball process (Hall 1979, Montgomery 1991). However, in order to simplify the analysis, we let the search frictions (but not the search costs) converge to zero. In order to do this, we first divide time into periods, and then let the time periods converge to zero. In each period, the matching process goes as follows:

1. Firms advertise vacancies and wage contracts attached to them
2. Unemployed workers observe the advertisements and send an application to one of the firms
3. All firms that obtain vacancies choose one applicant at random and start production. The rest of the workers remain unemployed

Let  $\Delta$  denote the time length of each period. We study the frictionless limit in which  $\Delta \rightarrow 0$ . We can then show the following result:

**Lemma 2** *Consider the limit equilibrium when  $\Delta \rightarrow 0$ . Suppose there is unemployment in this limit equilibrium. Consider a firm that advertises a wage contract that gives its employees an expected income that exceeds an unemployed worker's expected discounted income  $U^0$ . Then this firm will receive an application with probability 1.*

*Proof:* First note that as  $\Delta \rightarrow 0$ , the probability that a worker obtains a job offer in any given period converges to zero. If not, there would be no unemployment in the limit equilibrium. Consider a firm that advertises a wage contract that gives the employee an expected income  $E > U^0$ . The probability of obtaining a job in this firm must converge to zero when  $\Delta \rightarrow 0$

as well. If not, unemployed workers would be strictly better off applying for this job than for any other job with a finite wage. This again implies that the probability that the firm fills its vacancy converges to one as  $\Delta \rightarrow 0$ . QED

Thus, if there is unemployment in the limit equilibrium (hereafter just equilibrium), all vacancies that are advertised are filled. There is a continuum of unemployed workers on one side of the market and a flow of new vacancies that are filled immediately (so that the stock of vacancies has measure zero) on the other.

## 2.2 Equilibrium

It is now time to derive the equilibrium of the labour market. The first thing to note is that free entry of firms ensures that the expected income  $\Pi$  of a firm equals the search cost  $K$ . Our first equilibrium condition can thus be written as

$$\Pi(U^0) = K \tag{6}$$

Since  $\Pi$  is strictly decreasing in  $U^0$  this equation determines the equilibrium value of  $U^0$  uniquely. We denote this equilibrium value by  $U^{0*}$ .

The labour market is supposed to be in steady state. Let  $p$  denote the transition rate from unemployment to employment and  $z$  the utility flow of unemployed workers. The relationship between  $U^0$  and  $p$  is then given by

$$(r + s)U^0 = z + p(W - U^0) \tag{7}$$

where  $W$  is the expected discounted income when employed. By definition, the expected rent  $R$  is equal to  $W - U$ , hence we can rewrite the equation as

$$(r + s)U^0 = z + pR \tag{8}$$

The equilibrium in the labour market can be defined as a pair  $(p, U^0)$  satisfying equation (6) and (8).

With no unemployment, unemployed workers by definition find a job immediately, which implies that  $p$  is infinite. However, this leads to a contradiction, as  $U^0$  defined by (8) then goes to infinity and thus exceeds  $U^{0*}$  as defined by (6). This motivates our first proposition.

**Proposition 1** *In equilibrium, the unemployment rate is strictly positive*

Proof: Given  $U^{0*}$ , equation (3) determines  $\epsilon_c^*$ . Furthermore,  $\epsilon_c^* < \epsilon^{\max}$ , otherwise the firm would not capitalise  $K$ . It then follows from lemma 1 that the rent  $\rho$  is strictly positive. But then it follows from equation (8) that  $U^0$  goes to infinity if  $p$  does. Thus,  $p$  is finite. QED

Thus, rent associated with employment translates into unemployment. As being unemployed is the outside option for a worker, rent implies that it is strictly better (in expected terms) to be employed than to be unemployed. But this is inconsistent with full employment.

The transition rate to employment is determined so that the rents are dissipated. Inserting  $U^0 = U^{0*}$  into (8) and re-arranging gives

$$p = \frac{(r + s)U^{0*} - z}{R} \quad (9)$$

Let  $x$  denote the unemployment rate in the economy. Using the fact that  $px = s$  in steady state, yields

$$x = \frac{s}{r + s} \frac{R}{U^{0*} - Z} \quad (10)$$

where  $Z = z/(r + s)$  is the asset value of staying unemployed forever. Note that if we disregard discounting and set the unemployment benefit equal to zero, the unemployment rate is  $x = R/U^*$ . Thus, the expected fraction of the time the worker is unemployed equals the fraction of rents to total expected income when unemployed.

### 3 Efficiency

Let us now analyse the efficiency of the equilibrium outcome. Obviously, the equilibrium outcome is not first best, as first best requires that  $c'(e) = \gamma$  and no unemployment. One can obtain (almost) full employment by having a arbitrarily high negative unemployment benefit, and the efficient level of effort can be approximated by a correctly designed negative income tax schedule on labour income (see below for the effect of taxes on the wage contract). These policy recommendations will probably not be taken seriously by any government, for reasons not captured by our model.

In our view, a more interesting question is as follows: given the behaviour of the workers and the entry decisions of firms, are the wage contracts chosen by firms socially optimal? Put differently, if a social planner were able to overrun the firms' design of wage contracts, while all other decisions were left to the agents in the market, would the planner like to do so?

We will say that the equilibrium wage contracts chosen by firms are *constrained efficient* if they maximise welfare given 1) the workers' incentive compatibility and individual rationality constraint, and 2) entry of firms satisfying the incentive compatibility constraint. We want to analyse whether the equilibrium wage contracts derived above are constrained efficient.

The planner maximises overall production less the costs of creating jobs. Let  $Y(\Phi)$  denote the expected discounted production value (net of effort costs) of a worker-firm pair as a function of the wage contract  $\Phi$ , and let  $R(\Phi)$  denote the associated expected rent the contract allocates to the worker. For each worker-firm pair that is formed, the search cost  $K$  is incurred  $1/[1 - F(\epsilon^*(\Phi))]$  times (a full specification of the contract includes a specification of the cut-off level, and we capture this by writing the cut-off level as a function of the contract). Finally, assume that also the social value of the utility flow of an unemployed worker is equal to  $z$ . The arrival rate of job offers to workers is determined by (9), and can thus be written as a function of  $\Phi$ . The planner's objective function is thus

$$S(\Phi) = \int_0^\infty [zx + xp(\Phi)[Y(\Phi) - \frac{K}{1 - F(\epsilon_c(\Phi))}]]dt.$$

(Recall that  $x$  denotes unemployment). The planner maximises  $S$  given the constraint that  $\dot{x} = s - (p + s)x$ .

It is shown in several contexts that the efficient solution maximises the pay-off to the unemployed workers, see for instance Acemoglu and Shimer (1999) and Moen and Rosen (2001), and Pissarides (2000). This is also the case in this context:

**Lemma 3** *The planner's problem is equivalent to the problem of maximising the expected discounted income,  $U^0$ , for an unemployed worker. Formally, the planner maximises  $U^0$  given by (7), given (9), workers' I.C. and I.R. constraints, and the constraint that  $\Pi = K$ .*

*Proof:* See Appendix

We are now in the position to show that the equilibrium wage contract is constrained efficient. The equilibrium wage  $w^e(y)$  solves the problem

$$\begin{aligned} \max_{\Phi} \Pi(\Phi) \quad \text{S.T. } U(\epsilon^*) &\geq U^0 \quad (\text{and the I.C. and I.R. constraints}) \\ \Pi(\Phi) &= K \end{aligned}$$

The constrained efficient contract solves the "dual" maximisation problem

$$\max_{\Phi} U^0(\Phi) \quad \text{S.T. } \Pi(\Phi) \geq K \quad (\text{and the I.C. and I.R. constraints})$$

**Proposition 2** *Suppose the equilibrium income  $z$  reflects the social value of staying unemployed. Then the equilibrium wage contract is constrained efficient.*

*Proof:* Let  $\Phi'$  denote the socially optimal contract and  $\Phi^e$  the equilibrium contract. Suppose the proposition does not hold. Then  $U^0(\Phi') > U^0(\Phi^e)$ . However, since  $\Pi(\Phi') = K$  when  $U = U^0(\Phi')$ , it follows that  $\Pi(\Phi') > K$  when  $U = U^0(\Phi^e)$ . But then  $\Phi^e$  cannot be a profit-maximising contract. We have thus derived a contradiction. QED.

When the firms choose wage contracts, they do not give weight to worker rents, but set the contract so as to balance rent extraction and worker effort. Increasing the incentive power of the contract  $b$  for some types thus gives rise to a positive externality on the employee, as this will tend to increase the rent (from equation (5)). At first glance one may therefore expect the incentive contracts to be too low powered (too low values of  $b$ ). However, this is not correct. The point is that higher worker rents feed directly back to the unemployment rate. The unemployment rate is determined so as to dissipate all rents, and increasing worker rents only leads to a corresponding increase in the unemployment rate so that the asset value of an unemployed worker stays constant.

We want to follow this argument a bit further. Suppose firms are free to choose which technology,  $\tau$ , to apply, which may influence overall production as well as the amount of rents allocated to the worker. To be more specific, suppose we could write  $Y = Y(\tau, \Phi)$  and  $R = R(\tau, \Phi)$ . The firm chooses the contract that maximises  $\Pi = (1 - F(\epsilon_c(\tau, \Phi)))[Y(\tau, \Phi) - R(\tau, \Phi) - U^0]$ . The

firms will choose the value of  $\tau$ ,  $\tau^*$ , that maximises  $\Pi(\tau)$ , and in equilibrium,  $\Pi(\tau^*) = K$ .

It is straight forward to show that the planner's objective still is to maximise the asset value of an unemployed worker. Proposition 1 implies that the planner, for any give  $\tau$ , chooses the same contract as the profit-maximising firms. Furthermore, since worker rents have no value for the planner (as it is dissipated away through a higher unemployment rate anyway), the planner chooses the same technology as the profit-maximising firms in the market.

**Lemma 4** *The firms' choice of production technology is constrained efficient.*

*Proof:* Let  $\tau'$  denote the constrained efficient value of  $\tau$ , and let  $\Phi' = \Phi(\tau')$  denote the associated optimal contract (which is equal to the equilibrium contract with this technology). Free entry implies that

$$U^{0'} = Y(\tau', \Phi') - R(\tau', \Phi') - \frac{K}{(1 - F(\tau', \epsilon_c^*(\Phi'))}$$

It follows that  $\tau^*$  is constrained efficient. If not,  $U^{0'} > U^{0*}$ . But then the firm in the market could do better by choosing  $\tau'$  and contract  $\Phi'$  minus an arbitrarily small constant. QED

Note, however, that overall production in the economy can be increased by production subsidies, financed for instance by a lump-sum tax. Such a subsidy leaves the equilibrium incentives (and thus the expected rents) unaltered, but increases  $U^{0*}$ , and from (10) it follows that unemployment falls and hence that employment increases.

An underlying assumption behind proposition 1 is that the unemployment income  $z$  reflects the social value of being unemployed. Thus,  $z$  may reflect the value of leisure, of home production, or alternatively wages in a secondary labour market (without unemployment). An interesting observation, following directly from equation (6) and that  $\Pi$  is independent of  $z$  is the following:

**Lemma 5** *Irrespective of whether  $z$  represents the social value of being unemployed or unemployment benefits, welfare is independent of  $z$*

Thus  $z$ , even if it reflects wages in a secondary sector, does not influence welfare. The reason is that a higher  $z$  makes it more time-consuming

to dissipate rents, and thus increases unemployment (or the time spent in the inferior secondary sector) exactly so much that the unemployed workers obtain the same utility level.

Suppose then that  $z$  (partly) consists of transfers from the government (unemployment benefit), and hence does not reflect the social value of being unemployed. The first thing to note is that the lemma above implies that the unemployment benefits are a total waste of resources, as they do not influence the well-being of unemployed workers (from equation 6). It follows that the unemployment rate does not influence the equilibrium wage contract or the equilibrium cut-off rate either. However, the unemployment rate increases with the unemployment benefit, as it takes more time to dissipate the rents associated with employment.

The constrained optimal wage contract in the presence of unemployment benefits is, however, of lower incentive power than the equilibrium wage contract.

**Lemma 6** *With strictly positive unemployment benefits, the equilibrium wage contract is more incentive powered than the constrained efficient wage contract*

Proof: With zero unemployment benefit, worker rents have zero social value. With positive unemployment benefit, worker rents have strictly negative social value. The planner's maximisation problem is thus identical with the maximisation problem (1), but with  $u(\epsilon)$  replaced with  $ku(\epsilon)$ , where  $k > 1$ . The first-order condition is thus given by (from 4)

$$b = 1 - k \frac{1 - F(\epsilon)}{f(\epsilon)} c''(e) \alpha / \gamma^2$$

As  $b$  decreases in  $k$ , this completes the proof. QED.

To gain intuition, note that a positive unemployment benefit makes the government bear part of the burden associated with being unemployed. This is not taken into account when the incentive contracts are determined.

By contrast, taxes on labour income will have a tendency to reduce the equilibrium incentive power of the contracts below its constrained optimal level as long as effort is not deductible. We return to this point shortly.

## 4 Determinants of the unemployment rate

As discussed in the introduction, there exists anecdotal evidence that wage contracts have tended to be more incentive powered lately, and in any case it may be interesting to analyse the effect of more incentive-powered wage contracts on the unemployment rate. As we will see, the effect on the unemployment rate depends on the reasons *why* the incentive power of the contracts has increased. That is, which of the structural parameters in the model has triggered the increase in  $b(\epsilon)$ .

We are not able to characterise the effects of a change in the cut-off level on the total amount of rents allocated to the worker in the general case (see below for special cases). We therefore make the assumption that the cut-off level is equal to zero. Thus, the firm accepts all matches. As a description of firm behaviour, this assumption may actually be a good approximation: given that a firm has selected a worker out of many, and possibly spent resources on training him (depending on the interpretation of  $K$ ), the likelihood that he will be dismissed when his worker-firm specific productivity component is revealed may be fairly low.

Let us first analyse the effects of changes in  $b(\epsilon)$  around the optimal schedule  $b^*(\epsilon)$ . As we have seen,  $U^o(b)$  is maximised at  $b^*(\epsilon)$ . From the envelope theorem it follows that  $U^o$  is approximately constant for wage contracts close to  $b^*(\epsilon)$ . From equation (5) and (10) it follows that *stronger incentives, cet. par., tend to yield more rents to the workers and thereby higher unemployment rate.*

Note also that it follows directly from (4) that  $b^*(\epsilon)$  is independent of  $\bar{y}$ .<sup>9</sup> Still, an increased  $\bar{y}$  means that rents are less important relative to overall productivity (and therefore to  $U^o$ ) and the unemployment rate  $x$  falls (from 10). The point is that the cost of being unemployed relative to the expected rent when becoming employed increases. We refer to this as the productivity effect.

### **Effort provision more important ( $\gamma$ increases)**

The point here is that the *unobservable* part of the workers' effort becomes more important. The part of the job tasks that can be observed by the firm can be monitored directly through contracts. Incentive schemes are

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<sup>9</sup>This actually stems from an artefactual assumption in the model, that the disutility of effort in terms of money is independent of the overall wealth of the agents in the economy. If the marginal disutility of effort increases with overall wealth, an increase in  $\bar{y}$  will reduce  $b$ .

important in order to promote unobservable effort.

Anecdotal evidence suggests that jobs have become more autonomous lately, as more authority is delegated to individual workers. This may indicate that worker effort is less observable, and therefore that effort provision through wage contracts is more important.

We analyse the effect of a change in technology which increases the impact of unobservable effort, keeping the relative importance of the worker-firm specific productivity term constant. An increase in  $\gamma$  implies that the deterministic part of worker productivity becomes more important relative to the stochastic productivity term. In order to neutralise this effect, we scale down the constant term  $\bar{y}$  in such a way that  $u^0$  is constant.<sup>10</sup> We refer to this as a balanced increase in  $\gamma$ .

An increase in  $\gamma$  implies that effort provision becomes more important. For a given  $e$ , an increase in  $\gamma$  tends to increase the right-hand side of equation (4), and thus  $b^*(\epsilon)$ . On the other hand, for a given  $b$ , an increase in  $\gamma$  tends to increase  $e$ . Given that  $c'''(e) \geq 0$  this tends to reduce incentives, as rent extraction becomes more important. The net effect is therefore in principle undetermined. However, very unrestrictive assumptions on the cost-function ensure that the first effect dominates so that  $b$  increases in  $\gamma$ . More specifically, it is sufficient to assume that  $c''(e)/c'(e)$  is nonincreasing.<sup>11</sup> This is satisfied for all polynomials on the form  $x^n$  as well as for the exponential  $e^x$ , for which it is constant.

**Proposition 3** *Suppose  $c''(e)/c'(e)$  is strictly decreasing in  $e$ . Then a balanced increase in  $\gamma$  increases the incentive power of the wage contract and increases the unemployment rate*

*Proof:* For any given  $b$ ,  $c'(e) = \gamma b$ , hence  $c''(e)/\gamma^2 = b^2 c''(e)/c'(e)^2$ . We can thus write equation (4) as  $b(\epsilon) = h(\epsilon, \gamma, b)$  where  $h(\epsilon, \gamma, b) = 1 - \alpha \frac{1-F(\epsilon)}{f(\epsilon)} b^2 c''(e)/c'(e)^2$ . Since  $c''(e)/c'(e)^2$  decreases in  $e$   $\frac{\partial h(\epsilon, \gamma, b)}{\partial \gamma} < 0$ . Since also  $\frac{\partial h(\epsilon, \gamma, b)}{\partial \gamma} < 0$  it follows directly that  $b$  increases in  $\gamma$ . From (5) it follows

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<sup>10</sup>Exactly the same results can be obtained if we assume that expected worker productivity stays constant. In this case, we will actually obtain a reinforcement through the effects of the change in  $\gamma$  on  $u^0$ . If expected worker productivity stays constant, increased worker rents imply a fall in  $u^0$ , which again will increase unemployment even further.

<sup>11</sup>As will be clear from the proof, it is actually sufficient to assume that  $c''(e)/c'(e)^2$  is decreasing.

that  $E[\rho]$ , and thereby also  $R$ , increases in  $\gamma$ , but then it follows from (10) that  $x$  increases in  $\gamma$  as well. QED.

The intuition is straight-forward. If unobserved effort becomes more important, firms will provide their workers with stronger incentives, as incentive provision becomes more important relative to rent extraction. As a result, the expected rent associated with employment increases, and thus also unemployment.

### Reduced importance of unobserved productivity term (reduced $\alpha$ )

A reduction in  $\alpha$  may have different causes. Firstly, it may follow from better selection procedures and screening tests available for employers, for instance due to new batteries of personal and aptitude tests and the emergence of professional hiring agencies. In some countries, improved quality of schooling as well as more specialised education may lead to similar effects. Secondly, a reduction in  $\epsilon$  may also follow from a greater segregation in the labour market. This may tend to reduce job heterogeneity within each segment, implying that the set of jobs a given worker applies for becomes more homogeneous. Acemoglu (1999) gives some evidence that the degree of heterogeneity among workers has indeed declined over the past decades.

From (4) it follows that an increase in  $\alpha$  will lead to a reduction in  $b$  for all types  $\epsilon$ . Thus, as workers' *ex post* heterogeneity increases, the optimal wage contract becomes less incentive powered, as rent extraction becomes more important. A *reduction* in  $\alpha$  thus leads to more incentive-powered wage contracts. A reduction in  $\alpha$  will reduce overall worker productivity, and this will tend to increase the unemployment rate. However, as for changes in  $\gamma$ , we adjust  $\bar{y}$  correspondingly so that the equilibrium value of  $u^0$  remains constant, and refer to this as a balanced reduction in  $\alpha$ .

A reduction in  $\alpha$  has two opposing effects on worker rents. For a given wage contract, less *ex post* worker heterogeneities lead to lower expected rents. On the other hand, a more incentive-powered contract tends to increase expected worker rents. From (5) and (4) it follows that

$$E\rho = \int_{\epsilon^{\min}}^{\epsilon^{\max}} \alpha \left[ 1 - \frac{1 - F(\epsilon)}{f(\epsilon)} \frac{\alpha c''(e)}{\gamma^2} \right] d\epsilon$$

In order to get clear-cut results, we assume that  $c'''(e) = 0$  (i.e., that  $c(e)$  is

quadratic). The derivative with respect to  $\alpha$  is then

$$\begin{aligned}\frac{dE\rho}{d\alpha} &= \int_{\epsilon^{\min}}^{\epsilon^{\max}} \left[1 - \frac{1 - F(\epsilon)}{f(\epsilon)} \frac{\alpha c''(e)}{\gamma^2}\right] - \alpha \frac{1 - F(\epsilon)}{f(\epsilon)} \frac{c''(e)}{\gamma^2} d\epsilon \\ &= \int_{\epsilon^{\min}}^{\epsilon^{\max}} [2b(\epsilon) - 1] d\epsilon\end{aligned}$$

We define the average value of  $b$  as  $\bar{b} = \int_{\epsilon^{\min}}^{\epsilon^{\max}} b(\epsilon) / (\epsilon^{\max} - \epsilon^{\min})$ . It follows that if  $\bar{b} \leq 1/2$ , then a reduction in  $\alpha$  increases worker rents. We have thus shown the following proposition:

**Proposition 4** *Consider a balanced reduction in  $\alpha$  (the importance of ex post heterogeneities). This will lead to an increase in the unemployment rate if  $\bar{b} < 1/2$  and a decrease in the unemployment rate if  $\bar{b} > 1/2$ , where  $\bar{b}$  is the average incentive power of the contract as defined above.*

One should note that the average value  $\bar{b}$ , as we have defined it, only corresponds to the expected value of  $b(\epsilon)$  in the special case when  $\epsilon$  is uniformly distributed. Since  $b'(\epsilon) > 0$ , it follows that  $\bar{b}$  may be less than  $1/2$  even if  $E[b(\epsilon)]$  is greater than  $1/2$  if most of the probability mass is located at high values of  $\epsilon$  while the opposite is true if most of the probability mass is located at the lower part of the distribution. Finally, if  $c''(e)$  is strictly positive, this will reduce the responsiveness of the optimal contract to changes in  $\alpha$ , and will therefore make it more likely that reduced  $\alpha$  leads to reduced unemployment than what the proposition indicates.

## The effect of taxes

The wage contracts may also be influenced by the marginal tax rate. As will be clear shortly, a reduction in the marginal tax rate will (under reasonable assumptions) tend to increase the incentive power of the wage contract. This is particularly interesting because marginal tax rates have fallen in most western countries the last decade.

Consider first the effect of a proportional tax on the firms' net profits when all costs (including  $K$ ) are deductible. In this case, firms choose  $b$  to maximise  $(1 - t)\pi$  instead of just  $\pi$ . It follows that the equilibrium remains unchanged. A tax on firm profits is thus neutral.

Consider next a linear tax on wages. The income tax  $T$  is given by

$$T = tw + A$$

where  $A$  is a constant. We refer to  $t$  as the marginal tax rate. Below we analyse the effects of reductions in the marginal tax rate. In order to avoid the productivity effect, we simultaneously increase the fixed term  $A$  so as to keep  $u^0$  constant, and refer to this as a balanced reduction in the marginal tax rate.

The optimal contract can be derived in exactly the same manner as before. Worker utility is given by  $u(\epsilon) = w(1 - t) - c(e)$ . Firm profit can thus be written as

$$\pi(\epsilon) = \bar{y} + \alpha\epsilon + \gamma e - \frac{u(\epsilon) + c(e)}{1 - t}$$

The truth-telling condition is given by  $u'(\epsilon) = c'(e)\alpha/\gamma$ , as before. The first order conditions for the optimal contract are thus given by the following equations:

$$\begin{aligned} \frac{c'(e)}{1 - t} &= \gamma + \frac{1}{f(\epsilon)}\lambda c''(e)\alpha/\gamma \\ \dot{\lambda} &= \frac{f(\epsilon)}{1 - t} \end{aligned}$$

Integrating up  $\lambda$ , inserting and re-arranging thus gives

$$c'(e) = \gamma(1 - t) - \frac{1 - F(\epsilon)}{f(\epsilon)}c''(e)\alpha/\gamma$$

In a menu of linear contracts, the corresponding incentive parameter  $b$  is such that  $c'(e) = \gamma(1 - t)b$ , or that  $b = c'(e)/[\gamma(1 - t)]$ . It thus follows that

$$b(\epsilon) = 1 - \frac{1}{1 - t} \frac{1 - F(\epsilon)}{f(\epsilon)}c''(e)\alpha/\gamma^2 \quad (11)$$

It follows that for a given  $e$ , a reduced marginal tax rate  $t$  shifts the right-hand side of this equation up, thus tending to increase  $b$ . On the other hand, reduced marginal taxes tend to increase the effort level (for a given  $b$ ), which will increase  $c''(e)$ , and this pushes in the opposite direction. However, we can show that the latter effect dominates provided that  $c''(e)/c'(e)$  is decreasing in  $e$

**Proposition 5** *Suppose  $c''(e)/c'(e)$  is decreasing in  $e$ . Then a reduction of the marginal tax rate  $t$  increases the incentive power of the wage contract, and a balanced decrease reduces unemployment.*

*Proof:* For any given  $b$ ,  $c'(e) = \gamma b(1 - t)$ , and hence  $c''(e)/[(1 - t)\gamma^2] = bc''(e)/c'(e)\gamma$ . We can thus write equation (11) as  $b(\epsilon) = h(\epsilon, \gamma, b)$ , where  $h(\epsilon, \gamma, b) = 1 - \alpha \frac{1 - F(\epsilon)}{f(\epsilon)} bc''(e)/c'(e)\gamma$ . Since  $e$  decreases in  $t$  and since  $c''(e)/c'(e)$  falls in  $e$ , it follows that  $\frac{\partial h(\gamma, b)}{\partial t} < 0$ . Since also  $\frac{\partial h(\gamma, b)}{\partial b} < 0$  it follows that  $b$  decreases in  $t$ . The first part of the proposition thus follows. Since  $\rho(\epsilon) = \int_{\epsilon_c}^{\epsilon} u'(\epsilon) d\epsilon$  and  $u'(\epsilon) = c'(e)\alpha/\gamma = \alpha(1 - t)b$  it follows that a reduction in  $t$  increases  $\rho(\epsilon)$  for all  $\epsilon$ , and thereby also  $R$ . But then it follows from (10) that a balanced decrease in  $t$  increases the unemployment rate  $x$ . QED.

## 5 Conclusion

This paper studies the impact of performance pay contracts on the overall performance of the economy. We analyse whether the incentive power of the equilibrium wage contract is socially efficient. In the absence of unemployment benefits the incentive power is constrained efficient, while with positive unemployment benefits the incentive power is too high.

The model provides three possible explanations for the documented increase in the use of performance pay over the last decade: First, an increased importance of non-observable effort. Second, a fall in the marginal tax rate. Third, a reduction in the heterogeneity of workers performing the same task. All three changes are likely to increase the equilibrium unemployment rate.

## Appendix

### Appendix 1: Unique cut-off

Define

$$\Psi(\epsilon) = \bar{y} + \alpha\epsilon + \gamma e - u^0 - c(e) - \frac{1 - F(\epsilon)}{f(\epsilon)} c'(e)\alpha/\gamma$$

Equation (3) defines  $\epsilon$  uniquely iff  $\Psi(\epsilon) = 0$  is uniquely defined.

$$\frac{d\Psi(\epsilon)}{d\epsilon} = \alpha + \gamma \frac{de}{d\epsilon} - c'(e) \frac{de}{d\epsilon} - c'(e) \alpha / \gamma \frac{d}{d\epsilon} \frac{1 - F(\epsilon)}{f(\epsilon)} - \frac{1 - F(\epsilon)}{f(\epsilon)} c'(e) \alpha / \gamma \frac{de}{d\epsilon}$$

Inserting  $-\frac{1-F(\epsilon)}{f(\epsilon)} c''(e) \alpha / \gamma \frac{de}{d\epsilon} = (c'(e) - \gamma) \frac{de}{d\epsilon}$  (from equation (2)) yields

$$\frac{d\Psi(\epsilon)}{d\epsilon} = \alpha - c'(e) \alpha / \gamma \frac{d}{d\epsilon} \frac{1 - F(\epsilon)}{f(\epsilon)} > 0$$

Hence  $\Psi(\epsilon) = 0$  is uniquely defined.

## Appendix 2: Deriving equation (5)

The integral

$$E[\rho] = \int_{\epsilon_c}^{\epsilon^{\max}} \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon d\tilde{F}(\epsilon')$$

can be simplified using integration by parts. We use that  $\int_a^b u(x)v'(x)dx = \int_a^b u'(x)v(x)dx$ . Let  $v = 1 - \tilde{F}$ ,  $v' = -d\tilde{F}$ ,  $u = \int_{\epsilon_c}^{\epsilon'} \alpha b(\epsilon) d\epsilon$ ,  $u' = \alpha b(\epsilon)$ . This gives

$$\begin{aligned} E[\rho] &= -\int_{\epsilon_c}^{\epsilon^{\max}} (1 - \tilde{F}) \alpha b(\epsilon) d\epsilon + \int_{\epsilon_c}^{\epsilon^{\max}} \alpha b(\epsilon) (1 - \tilde{F}) d\epsilon \\ &= \int_{\epsilon_c}^{\epsilon^{\max}} \alpha b(\epsilon) (1 - \tilde{F}) d\epsilon \end{aligned}$$

## Appendix 3: Proof of Lemma 3

The current-value Hamiltonian associated with the planner's maximisation problem can be written as

$$H^c = zx + xp(\Phi) \left[ Y(\Phi) - \frac{K}{1 - F(\epsilon^*(\Phi))} \right] + \lambda [s - (p(\Phi) + s)x]$$

where the only state variable is  $x$ . The first order conditions are given by

$$\begin{aligned}
r\lambda &= \frac{\partial}{\partial x} H^c = z + p(\Phi) \left[ Y(\Phi) - \frac{K}{1 - F(\epsilon^*(\Phi))} \right] - \lambda(p + s) \\
\Phi &= \arg \max_{\Phi} H^c \\
&= \max_{\Phi} [zx + xp(\Phi) \left[ Y(\Phi) - \frac{K}{1 - F(\epsilon^*(\Phi))} \right] + \lambda[s - (p(\Phi) + s)x]]
\end{aligned}$$

The first of these conditions implies that

$$(r + s)\lambda = z + p \left[ Y(\Phi) - \frac{K}{1 - F(\epsilon^*(\Phi))} - \lambda \right] \quad (12)$$

In the maximisation problem in the second equation, the state variable  $x$  and the adjungated variable are regarded as constant, and the maximisation problem can equivalently be expressed as

$$\max_{\Phi} p(\Phi) \left[ Y(\Phi) - \frac{K}{1 - F(\epsilon^*(\Phi))} - \lambda \right]$$

which is equivalent with maximising  $\lambda$  defined by equation (12).

We want to show that this is equivalent with maximising  $U^0$  defined by (7), given (9) and the constraint that  $\Pi = K$ . The free entry assumption implies that the expected profit of a firm that hires a worker is equal to  $K/(1 - F(\epsilon^*))$ . The rest of the surplus is allocated to the worker. Thus,  $W(\Phi) = Y(\Phi) - K/(1 - F)$ . Inserted into (7), we find that

$$(r + s)U^0 = z + p \left[ Y(\Phi) - \frac{K}{1 - F(\epsilon^*(\Phi))} - U^0 \right] \quad (13)$$

The expression for  $U^0$  in (13) is formally identical to the expression for  $\lambda$  in equation (12). Maximising  $\lambda$  given that  $Y = Y(\Phi)$  and  $p = p(\Phi)$  must then be equivalent with maximising  $U^0$  given the same two constraints. QED

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