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A PROGRAM FOR RESEARCH ON

SOCIAL AND ECONOMIC DIMENSIONS OF AN AGING POPULATION

Macroeconomic Implications of Population Aging and Public Pensions

Malick Souare

SEDAP Research Paper No. 100

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Macroeconomic Implications of Population Aging and Public Pensions

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Abstract

We develop a calibrated general equilibrium model of a small open economy to examine some macroeconomic and distributional effects of an aging population. The model features overlapping generations with a public pension system, asymmetric information in the labor market, and includes some households that are liquidity constrained. Our main results are as follows. First, by analyzing the consequences of population aging in one country without taking into account the extent of aging throughout the world, one may systematically misestimate the effect which aging may have on that country's living standards and its net foreign asset position. Second, the magnitude of the effect of an aging population on people's average living standards, both in the short run and in the long run, significantly depends on whether or not they are liquidity constrained. Third, whether increases in contribution rates to finance the public pension system (as the elderly dependency ratio rises) are imposed on workers or firms has little effect on the impact of aging on living standards; however, it does matter for the unemployment rate.

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1. Introduction

Over the coming decades the populations of many countries, particularly the OECD countries, will experience aging. This demographic shock will be caused mainly by the post-war "baby boom" generation heading for retirement, although the lower fertility and the greater longevity are also contributing factors. For Canada, the so-called old-age dependency ratio, normally defined as the ratio of the number of elderly (aged 65 and over) to the population of working age (15-64), is expected to nearly double (from about 0.2 now) by 2035, and then remain at about that level (see Hviding and Mérette, 1998, p.7; Denton and Spencer, 2000, p.4). Among the economic implications of aging populations, the issue which is perhaps ultimately of most concern to policy makers is the effect which such aging may have on per capita living standards. The empirical magnitude of this effect is the object of much debate. For a single economy such as Canada, this question has been examined using two general approaches: closed and open-economy specifications. In the former case, real wages and real interest rates are sensitive to domestic demographic conditions. However, applying the closed-economy model can only be justified if the analysis refers at least to the entire North American economy. For the analysis to be exclusive to Canada, we have to consider a small open-economy model. In this case, the assumption of perfect capital mobility pegs the interest rate. When combined with a standard constant-returns-to-scale production technology, perfect capital mobility also pegs the wage rate. Thus, in a small open-economy model, factor prices are not affected by domestic demographic conditions.

Furthermore, examining the macroeconomic consequences of aging populations for a single economy – using a small open-economy framework – might also be problematic, because assuming that the rate of interest is constant implies implicitly that no similar changes take place abroad. However, in the real world, population aging appears to be a world-wide phenomenon – although not perfectly synchronized. With a general aging process one would instead expect a fall in the world interest rates. Therefore one objective of this present paper, is to use an overlapping generations small open-economy model – calibrated to Canada – to highlight the concern that by ignoring the influence of population aging on the rate of interest, we may systematically misestimate the effect which aging may have on a country's living standards and its net foreign asset position. As a consequence, the scope of policy responses – those designed to ameliorate or offset these effects – may be very inaccurate.

Moreover, following Mankiw's (2000) advice that all applied public finance analyses involving household saving must allow for the fact that a significant number of households live "hand-to-mouth" (and so do not save), we evaluate how aging populations may affect people differently depending on whether or not they save for the future.

Another aspect of the aging is its implications for public pension funding. The aging population may undermine the sustainability of pay-as-you-go (PAYG) social insurance arrangements like the Canada/Quebec Pension Plan (C/QPP). However, the financial viability of the PAYG system can be maintained – despite an aging population – if payroll taxes are increased (as is the ongoing plan in Canada). But some of these increases in payroll taxes will raise the natural unemployment rate, with the result that the transfers to retirees will be at least partly funded by reducing economic outcomes for younger individuals at the lower end of the income scale. This outcome is often overlooked. Thus, another purpose of this paper is to estimate the magnitude of this effect. We also examine whether it is better to have the contributions of workers or firms adjust to finance pension benefits as the elderly dependency ratio increases.

The remainder of the paper is organized as follows. The next section briefly describes our methodology. Section 3 reviews the literature to motivate our modeling approach. In Section 4 we fully describe our model and in Section 5 we discuss its theoretical results regarding the effects of payroll taxes on unemployment, and compare them to those derived in some other studies. Section 6 explains how we derived analytical expressions used in the computer simulation experiments to examine the broader consequences of an aging population. Section 7 presents and discusses the baseline calibration of the model. In Section 8 we report and analyze illustrative quantitative results (with sensitivity tests) from the computer simulations. Section 9 concludes the paper.

2. Research Methodology

The core of the analysis is a model of a small open economy that involves overlapping generations and life-cycle features. It builds on Blanchard's (1985) and Nielsen's (1994) extensions of the representative-agent model of household behavior.

In this paper, the overlapping generations model is extended in two important ways. First, following Mankiw's (2000) suggestion, there is a subset of households that are liquidity constrained and cannot afford to save. Second, a model of asymmetric information in the labor market (Summers, 1988) is added, so that involuntary unemployment emerges for a subset of the liquidity constrained households. This model of efficiency wages (which is well-suited for numerical calibration) is inserted within the overlapping generations structure without exposing the analysis to Ambler's (1994) criticism of earlier work (e.g. Phelps, 1994) that shared these objectives. As in Nielsen, there is a PAYG public pension system. The forward-looking households who plan for the future have both private savings and the public pension available during their retirement years; the liquidity-constrained ones have only the public pension and transfer payments in old age. Some in the latter group rely on income transfers before retirement as well – since there is an employment insurance system. In the model, as in Canada, both the public pension and employment insurance programs are financed by earmarked proportional wage taxes levied on firms and their employees.

In the literature, pension systems and unemployment have traditionally been studied in separate classes of models. Pension systems are typically analyzed in intertemporal models that focus on savings incentives provided by the various systems. The usual assumption is that the labor market is perfectly competitive, so that there is no unemployment. Models of equilibrium unemployment usually neglect the role of pension systems. In a recent paper, Corneo and Marquardt (2000) provide a step toward a more integrated analysis of the PAYG public pension and employment insurance programs. They develop a two-period overlapping generations model with endogenous growth that highlights the interaction between the two social security systems (SSS) in the presence of a labor market with union wage-setting. Although they do not examine the aging issues, they illustrate that valuable insights can be gained – particularly in terms of growth policy – by studying branches of the social security system in a unitary framework. For example, the contribution rates to the SSS are shown to exert growth effects, which would not arise if only one of these programs were present. As a result, some economic implications that arise from the externalities between programs may be overlooked if one program is analyzed in isolation. In line with these authors, this paper provides a more integrated approach and analyzes the spillover effects between the public pension system and unemployment within the same macro model in the context of an aging population. Moreover, the behavior of other important macro variables, such as consumption and foreign debt accumulation, are addressed.

3. Literature Review

In general there is no simple aggregate consumption function in an economy composed of finitely lived agents. This is because agents differ in two respects. Being of different ages, they have different levels and compositions of wealth. Having different horizons, they have different propensities to consume out of wealth. This problem makes exact or approximate aggregation impossible (Modigliani, 1966).

The solution adopted by Diamond (1965) was to choose a very simple population age structure, avoiding altogether the need for aggregation. The solution chosen by Blanchard (1985) is, instead, to make assumptions that allow aggregation. The central assumption is that agents face, throughout their life, a constant instantaneous probability of death p. Thus their expected life is $(1/p)$. Agents are of different ages and have different levels of wealth, but they have the same planning horizon and the same propensity to consume. This allows one to solve the aggregation problem.

The main advantage of this approach is its flexibility. If we think of $(1/p)$ as the horizon index, we can choose it anywhere between zero and infinity. In particular, by letting p go to zero, agents have infinite horizons (Ramsey, 1928; Barro, 1974). The aggregate consumption function in this overlapping generations model with no bequests has a particularly simple and tractable form: it is proportional to the sum of aggregate human and nonhuman wealth. The marginal propensity to consume out of total wealth is $(p+\delta)$, where δ is the rate of time preference. Introducing the government budget constraint, Blanchard demonstrates that government debt is a net contributor to wealth because future taxes are discounted by the probability of death plus the interest rate, implying that the present value of future primary balances is less than the stock of debt.

While this approach captures the finite horizon aspect of life, its main drawback is that ignores the change in behavior over life, the "life-cycle" aspect. In that respect, Nielsen (1994) extended Blanchard's model by allowing retirement. He investigates the consequences of changes in a small open economy's unfunded social security system (PAYG public pension system) for its net foreign asset position. Assuming that the social security system is self-financing (in the sense that at any time *t* the taxes collected from the young correspond to the benefits paid to the old), Nielsen shows how an increase in the social security system's benefit rate is likely to lead to higher foreign indebtedness, even when "induced" retirement is allowed for. Feldstein (1974) identified two effects on private saving from the introduction of a social security (PAYG pension) system. On one hand, the value of promised social security benefits would reduce people's need to save: social security wealth would replace private wealth. On the other hand, expected social security benefits might alter the timing of retirement: individuals might retire earlier and accordingly wish to save more during their years of employment to prepare for a longer period of retirement. The question was whether the former "replacement" effect would dominate the latter "induced retirement" effect. If it did, social security would lead to a reduction in private savings and thereby national wealth.

In Blanchard's model, there is no population growth. Weil (1989) considers a variant of Blanchard's model where new households continuously enter the economy over time but existing households do not leave. He illustrates that the arrival of new households is sufficient to generate most of the main results of the Diamond and Blanchard models. Buiter (1988) demonstrates that the combination of a nonzero birth rate and the absence of operative intergenerational bequest motives yields debt nonneutrality in the Blanchard-Weil framework.

James (1994) seeks to determine the real effects of government deficits and debt in Canada. To perform the analysis, he simulates a dynamic, computable, general equilibrium model of an open economy, calibrated to Canadian data. His model uses the Blanchard-Weil-Buiter overlapping generations framework. The key result is that debt reduction, by increasing total saving today and by allowing future distorting taxes to be reduced, leads to increased output and consumption in the future. If, however, the initial debt reduction is achieved by raising highly distorting taxes, then the short-term costs thus induced exceed the long-term benefits.

Burbidge and Scarth (1995) use Blanchard's (1985) formulation of the overlapping generations model and the social welfare function advocated by Calvo and Obstfeld (1988) to analyze interest taxation and tariffs in a small open economy. They find that when the revenue lost from the elimination of these taxes is replaced by raising the tax on labor, as in the Canadian experience, a "low" social discount rate is required for the model to support lower interest-income taxes, while a "high" discount rate is needed to support the elimination of tariffs.

Furthermore, since consumption smoothing is far from perfect, many people have net worth near zero, and bequests are an important factor in wealth accumulation, Mankiw (2000) suggests an alternative model for analyzing fiscal policy. According to Mankiw, a better model to analyze the macroeconomic effects of fiscal policy should acknowledge the great heterogeneity in consumer behavior that is apparent in the data. It should include both low-wealth households who fail to smooth consumption over time and high-wealth households who smooth consumption not only from year to year, but also from generation to generation. That is, there is a need for a model in which some consumers plan ahead for themselves and their descendants, while others are liquidity constrained and live paycheck-to-paycheck. All these features are shared by our model, except one: there is no bequest motive for the forward-looking households. There is a very good case against this assumption since the seminal work of Kotlikoff and Summers (1981). However, as noted by Rios-Rull (2001, p.2), two reasons can explain this assumption. First, assuming that all assets are held for life cycle motives is in part to be in line with the literature of population aging that has always used overlapping generations models with little role for bequests. Second, we do not have yet models that integrate a suitable theory of wealth inequality and a sophisticated demographic structure. In fact, this assumption simplifies the analysis (actually it makes it feasible). See Quadrini and Rios-Rull (1997) and Castaneda *et al.* (2000) for further arguments of what is a good theory of the wealth distribution.

In earlier work James (1994) considered an extension similar to that of Mankiw (2000) – with one group of households holding all the capital and receiving no labor income, the other holding all the government debt and earning all the labor income, and with neither being liquidity constrained.

Finally, although our model is much smaller, it has many broad characteristics in common with the Canadian Policy Analysis Model (CPAM) and the OECD's world model (Minilink).

CPAM is designed to provide a reasonably complete representation of the Canadian macroeconomy.[1] It represents a small open economy that produces a single domestic good. There are four groups of domestic agents in the model: firms, consumers, and the fiscal and monetary authorities. Profit-maximizing firms combine labor and capital in a Cobb-Douglas technology to produce the single good. Trend population growth and trend productivity growth are exogenous. Consumers come in two types. There are forward-looking consumers who make decisions with a view to picking the best path for current and future consumption, and "rule-of-thumb" consumers who spend all their income in each period. Thus, all assets are held by the forward-looking agents. The behavior of the forward-looking consumers is characterized using the Blanchard (1985)-Weil (1989) model of overlapping generations, but in a discrete-time format. CPAM differs from our model in that our model has no monetary authority, no population growth, and our extension of Blanchard's model is in continuous time.

As for Minilink, it is a multi-region dynamic general equilibrium macroeconomic model of the world economy designed to examine policy issues over a time horizon of many decades.[2] It comprises five regional blocs – the United States, Japan, the European Union and the rest of the world, divided into fast and slow-aging regions – linked through international financial and goods market relationships. Each regional model involves three types of agents – households, firms and government. Some consumers are liquidity constrained and the behavior of the forward-looking

[1] For more details about CPAM see Black and Rose (1997).

[2] See the technical annex in Turner *et al.* (1998) for a complete and detailed description of Minilink and its properties.

consumers is based on an extended version of the Blanchard (1985) model – one in which labor income varies with age and the probability of death is a function of the old-age dependency ratio. The production technology used by the firms is a two-factor (capital and labor) constant-returns Cobb-Douglas function with exogenous labor-augmenting technical progress. Factors of production are assumed to be fully utilized. Each region is assumed to produce a single good, which is an imperfect substitute for the goods produced by other regions. Finally, the population growth rate is exogenous and the model operates in discrete time.

The OECD's economics department has used Minilink to analyze the macroeconomic implications of aging in global context. A "business-as-usual" case is examined in which, without improvements in labor market performance or specific policy adjustments to allow for the pressures of aging, economic growth is projected to slow significantly over the next 50 years in nearly all OECD countries; world real interest rates remain stable at current levels or even rise, because of the negative impact of aging on private savings, initially concentrated in the OECD where population aging occurs most rapidly. In the absence of specific policy adjustments, aging populations will also tend to reduce the growth of living standards (living standards measured as the level of output per capita) in the OECD as the output from any given number of workers is divided by a greater total population. Thus by 2050, the direct cumulative effects of the rise in dependency ratios is to reduce per capita living standards by about 10, 18 and 23 percent for the United States, the European Union and Japan, respectively, relative to a situation where (other things being equal) dependency ratios remain at current levels. Further analysis using model simulations, illustrate that only a combination of policies (such as measures which promote improvement in total factor productivity and increase public saving), carried out on a timely and coordinated international basis is likely to be successful in avoiding the slowdown in living standards that will be caused by aging.[3]

[3] Nonetheless, Lee (2001) and Lee and Edwards (2001) examine the fiscal impact of population aging in a probabilistic setting. They find that even in the experimental projections in which the rate of productivity growth is fixed at a higher level, the fiscal problems arising from aging are highly unlikely to go away. Moreover, they indicate that the fiscal consequences of population aging will indeed be severe for the US, but in many other industrial nations they will be simply staggering.

However, empirical evidence on the impact of population aging on private savings is very mixed. Meredith (1995), reviewing the empirical literature, shows that estimates of the sensitivity of the savings ratio to the dependency ratio vary considerably depending on whether they are based on microeconomic or macroeconomic, time series or cross-section data. For example, on the basis of pooled time-series estimation across the major industrial countries, Masson and Tryon (1990) obtain an estimate suggesting that an increase of 1 percentage point in the dependency ratio causes a corresponding fall of 1 percentage point in the savings ratio. At the other extreme, household survey evidence typically suggests only a weak (or even positive) effect of the dependency ratio on private savings rates (see for example Auerbach *et al*., 1991, Canari, 1994, and Borsch-Supan, 1996). A recent empirical study using pooled time-series evidence for 21 industrial countries by Masson *et al*. (1995) provides further empirical support for a weak effect of dependency ratios on private savings rates (according to which a 1 percentage point rise in the dependency ratio reduces the private savings rate by 0.14 percent).

In the version of the Minilink model used to generate the long-term reference scenario, it is assumed that a 1 percentage point rise in the dependency ratio reduces the private savings rate by 0.3 percent. This effect operates through a "probability of death" parameter which has a direct effect on the propensity to consume out of wealth.[4] In this case, world real interest rates rise by up to 0.7 of a percentage point by 2035. However, in an alternative simulation where there is no demographic effect on private savings behavior, world interest rates fall by 1.5 percentage points by 2035.[5] This effect stems from the world-wide slowdown in output growth, caused by a decline in the labor force, which reduces the marginal productivity of capital and so reduces interest rates. At the slower growth rate, less investment is needed to maintain the required capital stock.

Thus, from the perspective of the world economy, there is likely to be an *ex ante* reduction in both investment and savings, and the uncertainties follow from which is likely to fall by more.

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[4] Note that in Minilink, the death probability is endogenised as a function of the old-age dependency ratio.
[5] See Figure 7 in Turner *et al.* (1998, p. 69).

One of the main contributions in this paper is to insert *appropriately* a model of asymmetric information in the labor market within the overlapping generations framework, so that involuntary unemployment emerges. Phelps' (1994) earlier attempt has been criticized by Ambler (1994). As mentioned, the assumption of a constant probability of death facilitates the aggregation of individuals' demands. Since there is labor market rationing in Phelps' model, Ambler stresses that individual's demand and asset accumulation equations depend both on previously accumulated wealth and on employment status. This introduces an additional degree of heterogeneity compared to the original Blanchard model, and the associated aggregation problem is ignored by Phelps.

Involuntary unemployment appears to be a persistent feature of many modern labor markets (Shapiro and Stiglitz, 1984, 1985). The presence of such unemployment raises the question of why wages do not fall to clear labor markets. One explanation is that, in an asymmetric information and moral hazard situation, firms may not lower their wages, even in the face of unemployment, if net productivity depends on wages. Profits may fall when wages are reduced, if reducing wages influences productivity by affecting workers' effort. As a result, wage rigidities and involuntary unemployment arise.[6]

Theories in which there is a cost as well as a benefit to the firm of paying lower wages are known as *efficiency-wage* theories. (The name comes from the idea that higher wages may raise the productivity, or efficiency, of labor.) The central idea is that if firms cannot monitor their workers' effort perfectly, they may pay more than market-clearing wages to induce workers not to shirk. If it pays one firm to raise its wage, it will pay all firms to raise their wages. When they all raise their wages, the demand for labor decreases, and unemployment results.

However, different kinds of models come under the heading of "efficiency-wage theory". Three of the most important are: moral hazard or shirking, adverse selection, and sociological models (summaries of efficiency wage models appear in Johnson and Layard, 1986; Weiss, 1991; and Phelps, 1994). Here, we focus on shirking models.

[6] By involuntary unemployment we mean a situation where an unemployed worker is willing to work for less than the wage received by an equally skilled employed worker, yet no job offers are forthcoming.

Nonetheless, the ideas behind the latter two are the following: paying a higher wage can improve workers' ability along dimensions the firm cannot observe. Specifically, if higher-ability workers have higher reservation wages, offering a higher wage raises the average quality of the applicant pool, and thus raises the average ability of the workers the firm hires. In the last case, a high wage can build loyalty among workers, and hence induce high effort; conversely, a low wage can cause anger and desire for revenge, and thereby lead to shirking or sabotage. Thus, all efficiency wage models have one common feature, namely that the wage enters the production function in a labor-augmenting way.

The efficiency-wage theory of structural unemployment, of which Summers' (1988) model is an example, combines the rigor demanded by classical economists (given its grounding in formal optimization) with a well-defined source of market failure (which is at the core of Keynesian economics). As a result, many influential economists regard it as the most promising framework for understanding unemployment. (See, for example, Blanchard and Fischer, 1989, p.463).

The final prerequisite for model selection is that it can be readily calibrated, so that illustrative and empirically relevant *quantitative* results can be derived. We have chosen Summers' (1988) model of efficiency wages for this reason.[7] Romer (1996) has drawn attention to this model as a particularly compact version of efficiency wage theory, and we have extended it to allow for various taxes and transfer payments, a variable capital stock, and two types of labor, high and low-skilled workers or high and low-paid workers. It is worth mentioning that both the Summers model and our extension of it have a property which is consistent with economic history. It relates to the vast increase in productivity over the past century, associated with no long-term trend in the unemployment rate. In our model, productivity has no effect on the unemployment rate; some other formulations (for example, Shapiro and Stiglitz, 1984) do not involve this appealing property.

Finally, regarding the integrated approach, our model on the whole is very close in spirit to a recent paper by Corneo and Marquardt (2000). They develop a two-period

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[7] Pissarides (1998) notes that Shapiro and Stiglitz's (1984) model has a serious limitation for numerical simulations, since it cannot be readily calibrated.

overlapping generations model with endogenous growth that highlights the interaction between PAYG public pension and employment insurance programs in the presence of a labor market with union wage-setting. Although they do not examine the aging issues, they illustrate that valuable insights can be gained – particularly in terms of growth policy – by studying branches of the social security system in a unitary framework. As a result, some economic implications that arise from the externalities between programs may be overlooked if one program is analyzed in isolation.

However, in the two-period overlapping generations model, the length of a "period" is one generation – often taken to be 30 years (see Barro and Sala-i-Martin, 1995, p.131). One drawback of this framework – as noted by Barro and Sala-i-Martin – is that it involves the assumption that there is a 30-year lag between the act of abstaining from consumption and the actual use of the newly produced output as capital. Therefore, we opt to extend the Blanchard (1985)-Nielsen (1994) specification of overlapping generations. Moreover, while the presence of unions in the labor market is an important institutional feature of European labor markets, the rate of unionization is very small in North America. In Canada, the rate is about 30 percent and mostly concentrated in the public sector. As a result, we instead assume a model of asymmetric information in the labor market where firms set wages. This model of efficiency wages as well as the union model generates involuntary unemployment.

4. The Model

The core of the analysis is a model of a small open economy that involves overlapping generations and life-cycle features. There are three groups of domestic agents: firms, consumers, and the government. Profit-maximizing firms combine labor and capital in a Cobb-Douglas technology to produce a single good. Trend productivity growth is exogenous and there is no population growth. At any time, the economy comprises two age groups: the working age people (workers and unemployed) and the old (pensioners). Consumers come in two types. There are forward-looking consumers (high-paid workers) who make decisions with a view to picking the best path for current and future consumption, and "rule-of-thumb" consumers (low-paid workers) who spend all their income in each period. Thus, all assets are held by the forward-looking agents.

These two types of consumers are in the same proportion in both working age people and pensioners, π and $(1-\pi)$. The behavior of the forward-looking consumers is characterized using the Blanchard (1985)-Nielsen (1994) model of disjoint overlapping generations with retirement. In the model, as in Canada, both the PAYG pension and employment insurance programs are financed by earmarked proportional wage taxes levied on firms and their employees.

We now give the details of each of the "building blocks" of the model.

4.1 Firms

There is a large number of identical competitive firms. The production function for the representative firm is of the form:

$$Y = K^{\alpha}(QN_h)^{\beta}(Q\mu N_l)^{1-\alpha-\beta} \tag{1}$$

where N_h and N_l denote, respectively, the employment of high and low-paid workers, K is the capital stock, Q the labor-augmenting technical progress that grows exogenously at rate n, and μ is an index of effort per low-paid worker. Notice that there is no work effort index for high-wage workers because we make the assumption that these workers do not shirk, since they have "good" jobs which they enjoy. Assuming that firms cannot perfectly monitor the effort supplied by low-paid workers, they may pay them more than market-clearing wages to induce these workers not to shirk. As a result, only the low-wage workers can be unemployed in the economy. Though this assumption may be a little restrictive, in practice this group of workers represents the largest proportion of the unemployed (see Layard *et al*, 1991, p.22; OECD, 1994, p.243). In addition, as Shapiro and Stiglitz (1984, p.443) state in their conclusion, involuntary unemployment is a significant factor in the observed level of unemployment, especially in low-paid, lower-skilled, blue-collar occupations.

Compared to the Cobb-Douglas production technology with only one type of labor input, an appealing feature of having two types of labor is that the assumption of perfect capital mobility – in the small open economy model – does not peg the wage rates.

Define $y = Y/Q$, $k = K/Q$ and $w = W/Q$, where W is the wage level. The problem facing the representative firm is to maximize its profits, which are given by

$$\underset{k,\, N_h,\, N_l,\, w_l}{Max} \quad \Pr = k^{\alpha} N_h^{\beta} (\mu N_l)^{1-\alpha-\beta} - (r+\phi)k - w_h(1+\tau_f+\theta_f)N_h - w_l(1+\tau_f+\theta_f)N_l$$

subject to

$$\mu = \{w_l(1-t_w-\tau_w-\theta_w) - x\}^{\varepsilon}, \qquad (2)$$ [8]

where w_h and w_l denote, respectively, the wage paid to high and low-skilled workers (without including the effect of exogenous ongoing productivity growth on wages); τ_f and θ_f (τ_w and θ_w) are the firm's (employees') contribution rates to the pension and employment insurance programs, respectively; r is the interest rate (which is assumed to be fixed by perfect capital mobility internationally); ϕ the capital depreciation rate; t_w the personal tax rate applied to labor income; and x represents the alternative option available to low-wage workers, which is considered as given to the firm. Since ε is a positive fraction, equation (2) shows that a higher wage in the current job raises the low-paid worker's return relative to her alternative and thereby induces higher productivity. That gives rise to the interpretation of ε as the propensity to shirk.

Unlike financial and physical capital, both kinds of labor are assumed to be completely immobile across borders.

Setting the derivatives of profits with respect to k, N_h, N_l, and w_l[9] to zero, we have the following first-order conditions:

$$k: \qquad \alpha \frac{y}{k} = r + \phi \qquad (3)$$

$$N_h: \qquad \beta \frac{y}{N_h} = w_h(1+\tau_f+\theta_f) \qquad (4)$$

$$N_l: \qquad (1-\alpha-\beta)\frac{y}{N_l} = w_l(1+\tau_f+\theta_f) \qquad (5)$$

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[8] Although Summers (1988) and Romer (1996) did not provide micro foundations for this work effort function, Scarth and Moutos (2000) showed that it derives support from household optimization behavior.
[9] If there are unemployed workers, the firm can choose the wage freely. If unemployment is zero, on the other hand, the firm must pay at least the wage paid by other firms.

$$w_l: \qquad (1-\alpha-\beta)\frac{y}{N_l} = \frac{[w_l(1-t_w-\tau_w-\theta_w)-x]}{\varepsilon(1-t_w-\tau_w-\theta_w)}(1+\tau_f+\theta_f) \qquad (6)$$

Combining equations (5) and (6) yields the wage setting rule for the low-paid workers

$$w_l = \frac{x}{(1-\varepsilon)(1-t_w-\tau_w-\theta_w)} \qquad (7)$$

Without the asymmetric information that leads to shirking (that is, with $\varepsilon = 0$), firms simply set the wage such that the after-tax wage is equal to the workers' outside option. With shirking, however, the wage is higher, and with all firms behaving in this fashion, employment is lower when the threat of shirking exists. The resulting unemployment is involuntary since firms do not accept offers from the unemployed to work for less than the profit-maximizing wage.

The low-paid workers' alternative is a weighted average of the after-tax wage received at the other firms and what is received if the individual cannot find work (which Summers assumes is a fraction *f* times *w*). The weights in the average are the employment rate, (1-*u*), and the unemployment rate, *u*, respectively. Hence, the alternative income is $x = (1-u)w_l^o(1-t_w-\tau_w-\theta_w)+ufw_l$, where w_l^o is the pre-tax wage offered at other firms.[10] When this definition is substituted into the wage-setting rule (equation (7)) with $w_l^o = w_l$ in equilibrium, we get the following unemployment rate

$$u = \frac{\varepsilon(1-t_w-\tau_w-\theta_w)}{(1-t_w-\tau_w-\theta_w)-f} \qquad (8)$$

Thus, unemployment rises with the propensity to shirk (ε), the personal tax rate applied to labor income (t_w), employees' contribution rates to the social security systems (τ_w and θ_w), and with the generosity of employment insurance (*f*). Again, note that a rise in labor productivity has no effect on unemployment; instead, it simply raises real wages. This property of the model is consistent with economic history. There was a vast

[10] Notice that employment insurance benefit, fw_l – which is paid to those unemployed among low-income individuals – is tax free. Two arguments could be mentioned to justify this assumption. First, there exists a basic exemption for personal income tax. And second, only the employed individuals contribute to the public pension and employment insurance programs.

increase in productivity in the course of the twentieth century, with a corresponding increase in wages, but with no long-term trend in the unemployment rate. As noted earlier, many formulations of the efficiency-wage hypothesis do not have this appealing property.

4.2 The public pension system

The population of the economy is set equal to unity (at any time t). As in Blanchard's model, every individual has, regardless of the time of her birth, the same instantaneous probability of death, p, and the birth rate at any time is also equal to p. Thus, a cohort born at time zero has a size, as of time t, of pe^{-pt}. Since the population is split into those of working age and pensioners, the social security (pension) scheme determines – by way of a pension age, λ – the relative size of these two groups. Given the death (birth) rate p and the pension age λ, the former and latter groups constitute the shares $(1-e^{-p\lambda})$ and $e^{-p\lambda}$ of the population, respectively.[11] Because the pension system is constrained to be self-financing in the sense that all pension benefits in any given time are financed by the total amount of contributions paid in that time by the firms and their employees, we must have

$$(\tau_f + \tau_w)(N_h w_h + N_l w_l) = e^{-p\lambda} m \tag{9}$$

where m is the pension benefit adjusted for the productivity growth.

Furthermore, since high-paid workers (forward-looking consumers) and low-paid workers (liquidity-constrained consumers) are in the same proportion in both working age people and pensioners, π and $(1-\pi)$, and since only the low-paid workers can be unemployed, we have

$$N_h = \pi(1-e^{-p\lambda}) \tag{10}$$

and

$$N_l = (1-u)(1-\pi)(1-e^{-p\lambda}) \tag{11}$$

[11] $\int_{t-\lambda}^{t} pe^{-p(t-s)}ds = (1-e^{-p\lambda})$ and $\int_{-\infty}^{t-\lambda} pe^{-p(t-s)}ds = e^{-p\lambda}$, where s is the birth date.

Inserting equations (10) and (11) into the pension plan budget constraint (equation (9)) yields

$$(\tau_f + \tau_w)[\pi w_h + (1-u)(1-\pi)w_l] = \frac{e^{-p\lambda}}{1-e^{-p\lambda}} m \qquad (12)$$

where $\frac{e^{-p\lambda}}{1-e^{-p\lambda}}$ is the old-age dependency ratio – the ratio of retirees to those of working age.

Given that equation (12) is balanced in different demographic environments, it is important for the analysis that follows to know which variable – whether τ_w, τ_f (or both) or m – is set residually to adjust in response to unforeseen demographic changes (here, an increase in the elderly dependency ratio). This suggests a potential trade-off between efficiency and equity in the adjustment of the pension system to demographic shocks. On one hand, by holding the benefit payment fixed a higher dependency ratio would constitute a squeeze on the pension budget and the required rise in contribution rates could well have distortionary effects in e.g. the labor market. One the other hand, by targeting the contribution rates – and letting pension benefits fall endogenously – the associated intergenerational redistribution of income against pensioners would hardly be acceptable from a viewpoint of social equity. Since in the Canadian experience the contribution rates have been increased and since we are interested in estimating the magnitude of the distortionary effects on unemployment, we will fix (or target) the benefit payment at its previous steady state level and let the contribution rates adjust endogenously.

Note also that the projected increase in the old-age dependency ratio can be simulated by lowering either the retirement age, λ, or the probability of death, p (which is the birth rate as well; and $1/p$ is the life expectancy). In this paper, we will proceed by lowering the retirement age. While the share of elderly people will continue to increase with both lower fertility and greater longevity, demographers predict that the increase in the next three decades will be caused mainly by the baby-boom generation heading for retirement, and the only way that can be simulated in the current framework is through an assumed reduction in λ.

4.3 Consumers

4.3.1 Forward-looking consumers

In this subsection, we derive the aggregate consumption function for the forward-looking consumers. These individuals do not face any liquidity constraints. Their behavior is characterized using the Blanchard/Nielsen framework of overlapping generations with retirement. Since Blanchard’s generalization of the representative agent formulation is familiar, we focus more on Nielsen’s extension of this specification. The main drawback of Blanchard’s approach is that while it captures the finite horizon aspect of life it ignores change in behavior over life, the "life-cycle" aspect. In that respect, Nielsen extended Blanchard's model by allowing for retirement. Here, we extend Nielsen's model by introducing exogenous productivity growth and various taxes and transfer payments, and apply it only to the subset of households that are forward looking.

To simplify the exposition, we ignore for the moment the tax on interest income and the fact that only a portion of the population has a rate of time preference low enough to cause them to operate with a forward-looking plan. As in Blanchard (1985), each decision maker has an instantaneous utility function equal to the log of consumption, and she discounts the future at rate $(p+\delta)$, where δ and p are the rate of time preference and (constant) probability of death, respectively.

It is well-known under Blanchard's formulation (1985, p. 229, equation (5)) that aggregate consumption is a linear function of aggregate human wealth, $H(t)$, and financial wealth, $V(t)$,

$$C(t) = (p+\delta)[H(t)+V(t)]$$

Taking the time derivative of this equation gives:

$$\dot{C}(t) = (p+\delta)[\dot{H}(t)+\dot{V}(t)]$$

Now what are $\dot{V}(t)$ and $\dot{H}(t)$? The accumulation identity for aggregate financial wealth, $V(t)$, is standard:

$$\dot{V}(t) = rV(t) + (1-e^{-p\lambda})[1-t_w(t)-\tau_w(t)-\theta_w(t)]W(t) + e^{-p\lambda}M(t) - C(t)$$

where M is the pension benefit. Three assets are available to citizens in the economy: physical capital stock, $K(t)$, public debt, $B(t)$, and net foreign debt, $A(t)$. In symbols,

$$V(t) = K(t) + B(t) - A(t)$$

As for human wealth we have

$$\begin{aligned} H(t) = &\int_{-\infty}^{t-\lambda}\int_{t}^{\infty} pe^{-p(t-s)} M(i)R(t,i)dids \\ &+ \int_{t-\lambda}^{t}\int_{t}^{s+\lambda} pe^{-p(t-s)}[1 - t_w(i) - \tau_w(i) - \theta_w(i)]W(i)R(t,i)dids \\ &+ \int_{t-\lambda}^{t}\int_{s+\lambda}^{\infty} pe^{-p(t-s)} M(i)R(t,i)dids \end{aligned}$$

where $R(t,i)$ denotes the death-rate-inclusive discount factor, $R(t,i) = e^{-(r+p)(i-t)}$.

This expression consists of three integrals. The first is the discounted pension benefits to be received by that part of the population that is already old (pensioners) at time t - i.e. those who were born prior to time $t - \lambda$. The second integral corresponds to the after-tax wages to be received in the future by the young (workers) from time t to their retirement at $s + \lambda$, after which they will receive pension benefits. The latter benefits are captured by the third integral.

Reversing the order of integration and differentiating by making use of Leibniz's formula, we have

$$\begin{aligned} \dot{H}(t) = &(r + p)H(t) - (1 - e^{-p\lambda})[1 - t_w(t) - \tau_w(t) - \theta_w(t)]W(t) \\ &+ pe^{-p\lambda}\int_{t}^{t+\lambda} [(1 - t_w(i) - \tau_w(i) - \theta_w(i))W(i) - M(i)]\, e^{-r(i-t)}di \end{aligned}$$

Substituting $\dot{V}(t)$ and $\dot{H}(t)$ in the $\dot{C}(t)$ equation, we obtain

$$\begin{aligned} \dot{C}(t) = &(r - \delta)C(t) - p(p + \delta)V(t) + (p + \delta)e^{-p\lambda} M(t) + p(p + \delta)e^{-p\lambda}. \\ &\int_{t}^{t+\lambda} [(1 - t_w(i) - \tau_w(i) - \theta_w(i))W(i) - M(i)]\, e^{-r(i-t)}di \end{aligned}$$

To achieve a model which is stationary in equilibrium, this identity and the ones that follow are expressed in terms of variables that are ratios of each aggregate to labor-augmenting technical progress, Q, which grows at rate n.[12] Moreover, since tax rates are constant – except once-and-for-all changes – we have

$$\begin{aligned}\dot{c} &= (r-\delta-n)c - p(p+\delta)v + (p+\delta)e^{-p\lambda}m \\ &\quad + \frac{p(p+\delta)}{r-n}e^{-p\lambda}(1-e^{-(r-n)\lambda})[(1-t_w-\tau_w-\theta_w)w-m]\end{aligned} \tag{13}$$

This equation reduces to Blanchard's specification when $\lambda = \infty$, and it further reduces to the standard infinitely lived representative agent analysis if $p = 0$. Given this nesting, the model provides an appealing overlapping generations structure. Note that this equation differs from Nielsen's in three respects. First, since we split labor into two groups, equation (13) is derived without inserting the self-financing pension plan condition (equation (12)), and this explains the third element on the right-hand side of the equation (13). Second, tax payments are proportional to wages. And third, we have productivity growth.

Next, when we add the tax rate on interest income and use the fact that this theory applies only to the proportion π of the population (forward-looking consumers or high-paid workers) who hold all assets $(k+b-a)$ in the economy, we get

$$\begin{aligned}\dot{c}_h &= (r(1-t_i)-\delta-n)c_h - p(p+\delta)(k+b-a) + \pi(p+\delta)e^{-p\lambda}m \\ &\quad + \pi\frac{p(p+\delta)}{(r(1-t_i)-n)}e^{-p\lambda}[1-e^{-(r(1-t_i)-n)\lambda}][(1-t_w-\tau_w-\theta_w)w_h-m]\end{aligned} \tag{14}$$

where t_i is the tax rate applied to interest income.

4.3.2 Liquidity-constrained consumers

These individuals live hand-to-mouth and therefore fail to smooth consumption over time. One can associate hand-to-mouth behavior with individuals who have high discount rates and often face binding borrowing constraints. Thus, the workers, unemployed, and pensioners of this group spend all of their incomes (which are after-

[12] As before, we will use lower case letters to denote variables adjusted for the productivity growth. Moreover, if we choose unity as the initial value for y, some lower case letters can be interpreted as the ratio of those variables to GDP.

tax wages, employment insurance benefits, and pension benefits, respectively, plus their transfer payments). By use of equation (11), we have the following aggregate spending equation

$$\begin{aligned} c_l &= (1-u)(1-\pi)(1-e^{-p\lambda})w_l(1-t_w-\tau_w-\theta_w) \\ &\quad + fw_l u(1-\pi)(1-e^{-p\lambda}) + (1-\pi)e^{-p\lambda}m + tp \end{aligned} \tag{15}$$

where c_l is consumption of the low income group, f and tp denote, respectively, the generosity of employment insurance and transfer payments. On the right-hand side of equation (15), the first term represents the after-tax labor income of workers, the second term the employment insurance paid to those unemployed, the third element the pension payments, and the fourth – transfer payments that go to low income individuals whether or not they are working, and whether or not they are retired.[13]

4.4 Government and foreign debts

For simplicity we assume that the public debt is owned entirely by domestic residents and that the domestic physical capital stock and net foreign assets are perfectly substitutable. Then, one might think of foreign debt as the share of the physical capital stock employed domestically that is owned by foreigners. Foreign debt, a, increases whenever net interest payments on pre-existing foreign debt exceed the country's net exports[14],

$$\dot{a} = (r(1-t_i)-n)a - [y - ((n+\phi)k + \dot{k}) - g - c_h - c_l] \tag{16}$$

where g is government direct spending. Since there are no adjustment costs involved with installing physical capital, investment is given by $(n+\phi)k+\dot{k}$. Thus, only the

[13] Assuming that transfer payments go only to the "poor" can be justified by the fact that income taxes are proportional in our model, whereas they are actually progressive.

[14] Although there is only one good, foreigners can buy domestic output, and domestic residents can buy foreign output. Thus, the only function of international trade in this model is to allow domestic production to diverge from domestic aggregate demand. In other words, we consider the intertemporal aspects of international trade but neglect the implications for patterns of specialization in production.

amount $z = k - a$ of the physical capital that is employed within the country is domestically owned.[15] By use of this definition, equation (16) can be written as:

$$\dot{z} = (r(1-t_i) - n)z + y - (r(1-t_i) + \phi)k - g - c_h - c_l \tag{17}$$

As for the evolution of government debt , *b*, we have

$$\dot{b} = d - nb \tag{18}$$

which states that public debt increases whenever the deficit, *d*, exceeds the economy's growth rate times the pre-existing debt. The government budget deficit is the excess of government spending over tax revenue. Using equations (10) and (11), we have

$$\begin{aligned} d = {} & g + r(1-t_i)b + fw_l u(1-\pi)(1-e^{-p\lambda}) + tp \\ & - t_i rk - (t_w + \theta_w + \theta_f)[\pi w_h + (1-u)(1-\pi)w_l](1-e^{-p\lambda}) \end{aligned} \tag{19}$$

The government spends on direct purchases, net interest payments on the outstanding debt, employment insurance benefits, and transfer payments. Revenue is derived from taxes on interest income and wages, and form the contributions to the employment insurance program made by firms and their employees.

We have assumed that the government collects tax on all the interest income that is generated within the country. In effect, the government levies a withholding tax on foreigners that is equal to the tax that is levied on domestic residents. Also, as in Canada, we considered employment insurance program surpluses (or deficits) as an element of the overall government budget constraint.

5. Theoretical results and comparisons with the existing literature

Some relevant theoretical results stem from the present model. In contrast to the employees', the firms' contribution rates to the pension and employment insurance programs are shown to have no influence on the unemployment rate; this follows from:

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[15] Indeed, if $a = k - z$, is positive, it corresponds to net claims by foreigners on the domestic economy. Conversely, if a is negative, it represents net claims by domestic residents on foreign economies.

$$u = \frac{\varepsilon(1-t_w-\tau_w-\theta_w)}{(1-t_w-\tau_w-\theta_w)-f} \qquad \text{or} \qquad u = \frac{\varepsilon}{1-\dfrac{f}{1-t_w-\tau_w-\theta_w}}$$

Hence, attempts to fight unemployment by means of reductions in the payroll tax on firms are useless in this model. Nevertheless, a decrease in tax rates on workers results in a lower unemployment rate. To understand the intuition behind these results, consider each category in turn.

Tax rates on firms

An increase in firms' contribution rates decreases their willingness to pay for labor. Workers recognize this fact and so generally reduce their wage claims in line with the firms' decreased ability to pay. The fact that all firms have a similarly decreased willingness to pay is what leaves workers no option but to accept the wage cut. Thus, an increase in the employer payroll tax will reduce the net return to working; and since employment insurance benefit is a fixed proportion of the gross wage, it will also reduce (proportionately) the net returns from being unemployed. As a result, raising employer taxes will not influence the comparative attractions of working and being jobless with benefits, so unemployment will be unaffected by firms' tax rates changes. In fact, most empirical studies find that these payroll taxes are, for all intents and purposes, fully passed on to employees in the form of lower wages[16], so firms' contribution rates are actually a "wage killer", not a "job killer". To put the point differently, in our model and in reality, a cut in these payroll taxes does not raise employment, it only raises the wages of those who already have jobs.

Tax rates on employees

An increase in taxes levied on employees makes the work less attractive for them and therefore raises their propensity to shirk. More specifically, if an employee payroll tax rate (or income tax rate) is increased, this will reduce the return to working but not reduce the prospective benefit if jobless, since that benefit is computed as a fixed proportion of the gross wage and assumed to be tax free. Hence, a hike in the

[16] There is an important empirical literature that reports that in the long-run, employer payroll taxes are fully shifted to labor in the form of lower wages. See, for example, Dahlby (1992, 1993); Kesselman (1997) for a survey.

employee tax rates will make work relatively less attractive and induce workers to shirk. Firms react by paying higher wages. As a result, the demand for labor decreases, and higher unemployment results. Thus, these taxes are a job killer. Clearly, cutting them decreases the propensity of employees to shirk. As the wage employers must pay decreases, firms hire more workers.

Regarding the effects of labor taxation in efficiency wage models, the existing literature leads to surprisingly different conclusions. Pisauro (1991) illustrates that the features of the effort function, μ , are the source of the difference in the outcomes. In particular, three elements turn out to be important:

(i) whether $\mu(w,.)$ [17] is a continuous and concave function;

(ii) whether $\mu(w,.)$ depends on the expected alternative income earned outside the firm, say x; and

(iii) whether $\mu(w,x)$ is homogeneous of degree zero in w and x.

The effort function in our model is consistent with (i) and (ii).

We can now compare our results with what other efficiency wage models predict. In the "rudimentary" efficiency wage model proposed for expositive purposes by Yellen (1984)[18], μ is continuous but does not depend on x; that is $\mu = \mu(w)$. In this case, an ad valorem tax on labor (a wage tax) has no effect on the after-tax wage (i.e. the tax is borne entirely by firms) and there is a negative effect on employment. On the other hand, a specific tax (an employment tax) leads to an increase in the after-tax wage.

In Johnson and Layard (1986), μ is continuous and homogeneous of degree zero in w and x, that is $\mu = \mu(w/x)$. In this framework, an ad valorem tax will be borne entirely by workers, leading to a reduction in the after-tax wage by the full amount of the tax and leaving employment unaffected (the same results as in our model, but the prediction is completely reversed when compared to the "rudimentary" model). And a specific tax may either lower or raise the after-tax wage but it will definitely cause a decrease in employment.

[17] The notation indicates that the effort or efficiency function depends on the real wage and other variables.

[18] Note that in Yellen's model, and in the subsequent (moral hazard) ones, tax refers to the employers' tax.

In Shapiro and Stiglitz (1984), μ depends also on *x*, but is not homogeneous of degree zero, and can assume only two values: 0 and $\mu^* > 0$. In this setting, the effect of an increase in the two taxes is exactly the same: a decrease in the after-tax wage, but by less than the full amount of the tax, and a decrease in employment.

Finally, in Pisauro (1991), μ is continuous between 0 and 1, concave, and is a positive function of the wage, unemployment rate, and detection rate, and a negative function of unemployment benefits and the (constant) marginal disutility of effort. (Recall that in our model, *x* also depends on *w*, *u, f*). In Pisauro's model, ad valorem and specific taxes on labor do not have the same impact on the real wage. The imposition of a specific tax causes an increase in the net real wage. However, both taxes have a negative effect on employment. This difference with respect to what our model predicts comes from the fact that in Pisauro's model, the effort function involves the restriction $\mu_{wu} = 0$.

Regarding these various assumptions about the effort function, Pisauro states that the alternative income, *x*, seems an essential ingredient of a moral hazard model with some micro foundation. But there are no reasons, apart from simplifying the problem (with some costs), to suppose either that there is only one possible positive level of effort (as should be obvious on the ground of realism), or that $\mu(w, x)$ should be homogenous of degree zero in *w* and *x*. On a priori grounds, a desirable effort function should therefore be continuous and have the general form $\mu(w, x)$. In that respect, readers can notice that the effort function in our model is consistent with these recommendations.

6. Population aging effects

In order to examine the broader effects of an aging population (as represented by an increase in the old-age dependency ratio) we have constructed a calibrated version of the model presented above. Collecting some equations together the model reduces to the following system:

$$y = \psi k^{\alpha}(1-e^{-p\lambda})^{1-\alpha}(1-u)^{1-\alpha-\beta}[w_l(1-t_w-\tau_w-\theta_w)]^{\varepsilon(1-\alpha-\beta)} \qquad (1')$$

where ψ embodies all constant terms.

$$\alpha\frac{y}{k} = r+\phi \qquad (3)$$

$$\beta\frac{y}{\pi(1-e^{-p\lambda})} = w_h(1+\tau_f+\theta_f) \qquad (4')$$

$$(1-\alpha-\beta)\frac{y}{(1-u)(1-\pi)(1-e^{-p\lambda})} = w_l(1+\tau_f+\theta_f) \qquad (5')$$

$$u = \frac{\varepsilon(1-t_w-\tau_w-\theta_w)}{(1-t_w-\tau_w-\theta_w)-f} \qquad (8)$$

$$(\tau_f+\tau_w)[\pi w_h+(1-u)(1-\pi)w_l] = \frac{e^{-p\lambda}}{1-e^{-p\lambda}}m \qquad (12)$$

$$\begin{aligned}\dot{c}_h = {} & (r(1-t_i)-\delta-n)c_h - p(p+\delta)(z+b)+\pi(p+\delta)e^{-p\lambda}m \\ & +\pi\frac{p(p+\delta)}{(r(1-t_i)-n)}e^{-p\lambda}(1-e^{-(r(1-t_i)-n)\lambda})[(1-t_w-\tau_w-\theta_w)w_h-m]\end{aligned} \qquad (14')$$

$$\begin{aligned}c_l = {} & (1-u)(1-\pi)(1-e^{-p\lambda})(1-\tau_w+\theta_f)w_l+(1-\pi)e^{-p\lambda}m \\ & +d-g-r(1-t_i)b+t_i rk+(t_w+\theta_w+\theta_f)\pi(1-e^{-p\lambda})w_h\end{aligned} \qquad (15')$$

$$\dot{z} = (r(1-t_i)-n)z+y-(r(1-t_i)+\phi)k-g-c_h-c_l \qquad (17)$$

These equations determine y, k, w_h, w_l, u, τ [19], c_h, c_l, z.

[19] We have removed the subscript, since following an increase in the elderly dependency ratio the government might choose τ_w or τ_f (or both) to balance the pension plan budget constraint. We derive quantitative results for all three cases.

To get equation (1') we divided (1) by Q, then substituted in equations (2) (using the x definition), (10) and (11). Equations (4') and (5') result from using equations (10) and (11), respectively. The use of the definition, $z = k - a$, implies (14'). Equation (15') follows from inserting (19) (substituting out transfer payments, tp) into (15). Finally, note that equation (18) is ignored since the government debt is kept constant in all experiments reported below. In fact, the level of government transfer payments adjusts residually to keep the budget deficit ratio constant.

Next, we take the total differential of the system and eliminate the changes in y and k by substitution. Then, we focus on a set of seven equations that determine changes in z, c_h, c_l, w_h, w_l, u and τ. We denote absolute changes by d and percentage changes by Δ.

Thus, the model can be summarized as follows:

$$\Omega[\mathrm{d}\dot{z} \;\; \mathrm{d}\dot{c}_h \;\; \mathrm{d}c_l \;\; \Delta w_h \;\; \Delta w_l \;\; \mathrm{d}u \;\; \mathrm{d}\tau]' = \Theta[\mathrm{d}z \;\; \mathrm{d}c_h]' + \Gamma[\mathrm{d}\lambda \;\; \mathrm{d}r]' \qquad (20)$$

where Ω, Θ, and Γ are (7×7), (7×2) and (7×2) matrices, respectively.[20]

[20] The elements of these matrices are fully reported in Appendix A.

7. Model calibration

As a starting point for the analysis, the model is solved for an initial steady-state equilibrium in which all key ratios (most stock and variables being expressed as proportion of output)[21] are assumed to be constant. This numerical steady state is obtained by choosing values for the fundamental parameters of the model and all the stock and flow variables, to reflect as far as possible the economic situation in Canada in the years 1999-2000. This is achieved by selecting ratios on the basis of values observed in data and then using the model's equations to determine the remaining variables residually to insure internal consistency. The values for the key ratios and parameters are reported in Table 1.

Table 1

Variables and parameters		benchmark	perturbation
r	interest rate	0.05	0.046
n	productivity growth rate	0.02	
ϕ	capital depreciation rate	0.06	
λ	retirement age	40	34
p	probability of death	0.02	
π	proportion of population not liquidity constrained	0.6	
δ	rate of time preference, forward-looking consumers	0.0332	
α	capital's share of output	0.33	
β	high-skilled labor's share of output	0.536	
k	capital-output ratio	3	
a	initial foreign debt ratio	0.3	
b	government debt ratio	0.53	
g	government spending ratio	0.14	
t_i	tax rate on interest income	0.2	
t_w	tax rate on labor income	0.2	
u	proportion of unemployed among low-paid workers	0.175	
f	unemployment benefits as a proportion of wage	0.4	
θ_w	employee contribution rate to employment insurance	0.015	
θ_f	employer contribution rate to employment insurance	0.021	
τ_w	employee contribution rate to the PAYG pension plan	0.085	
τ_f	employer contribution rate to the PAYG pension plan	0.085	

[21] As noted above, we have chosen unity as the initial value for y, so that most of the variables in lower case letters can be interpreted as relative to GDP.

Since we exclude childhood and the period of life when people receive their education (people aged 1-20, say), our model does not distinguish between start of life and start of *working* life. Hence the period from age zero to λ covers the working life period which we set to 40. These assumptions imply that people retire on average at about age 60. Also, the assumed probability of death, 0.02, implies a life expectancy of 50 years[22] (or 70 years if we think of life "beginning" at age 20). These values appear to be a fair approximation for Canada and most OECD countries.

However, as just indicated, given the assumptions regarding the steady-state of certain stock variables, the values of many flow variables are automatically determined by equilibrium conditions and, hence, some differ significantly from recently-observed values. For example, since the government debt and economy's growth rate are positive, one of the government budget identities (equation (18)) implies a budget deficit-to-GDP ratio of 1.06 percent in the baseline "steady state".[23] The initial ratios of total private consumption, investment and net exports to GDP (0.614, 0.24 and 0.006 respectively) are implied by the variable and parameter values in Table 1.

As noted, the selected parameter values must satisfy all steady-state restrictions of the model. To accomplish this, some parameters have to be chosen residually, and the rate of time preference for forward-looking consumers and capital's share of output were selected in this regard. Since this implied rate of impatience (0.0332) is less than the interest rate (0.05), the consumption of these individuals rises as they age. (No specific rate of time preference is involved for the hand-to-mouth group; it is assumed that these individuals, in addition to earning lower labor income, are sufficiently impatient that they never save.)

Beach and Slotsve's (1996) examination of data indicates that the lowest two quintiles of the earnings distribution in Canada get about 20 percent of earnings. On the other hand, Wolff (1998) reports that the lowest two quintiles of the financial wealth

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[22] Recall that, in the model, expected life is given by $1/p$.

[23] It is worth noting that such a discrepancy also appears in the OECD's world model, Minilink. Besides, in Minilink, since the United States has net foreign liabilities, it is necessary to assume that it is running a trade surplus (which is not what we observe) in the baseline steady state to cover interest payments, so that net liabilities remain a stable share of GDP.

distribution hold only 0.2 percent of household wealth. Based on these facts, we have chosen appropriate values for β and π.

The reported values for the employment insurance (EI) contribution and payout rates are lower than the actual legislated values. The reason for this discrepancy is that the actual program involves no payroll taxes beyond certain income levels, a qualifying period involving no EI benefits, and a maximum number of weeks of benefits. Since we have followed Summers and modeled an EI system without such limits, we have had to reduce the replacement ratio and the payroll tax rates accordingly (as OECD analysts do when making international comparisons; see Martin, 1996). Nonetheless, the ratio of the employer and the employee contribution rates reflects the 1.4 factor in the Canadian system.[24]

Next, we justify the calibrated values for the employee and employer pension contribution rates (the last two parameters in Table 1) which are higher than the actual values. The reason for this discrepancy is that in the model, the old-age dependency ratio is given by $DR = \frac{e^{-p\lambda}}{1-e^{-p\lambda}}$ (the ratio of the share of retirees to the share of working-age people in the population), where p and λ are the death probability and retirement age, respectively. Hence, it is obvious that we cannot choose independent values for p, λ and DR; one parameter has to be determined by the equation. As a consequence, although our selected values for the probability of death and retirement age are individually reasonable, together they imply a dependency ratio that corresponds more closely to the "overall" dependency ratio[25] than to the elderly dependency ratio in Canada. With this implied (higher) dependency ratio, the actual contribution rates to the public pensions (CPP/QPP) lead to a relatively small individual pension payment (as a proportion of the average wage) of 9 percent. Therefore, in the baseline steady state, we have raised the contribution rates such that the pension payment as a proportion of the average wage is about 21 percent – which is a sensible number.

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[24] Since 1971 the employer contribution rate for EI is 1.4 times the rate paid by employees. Pre-1971, employers and employees paid matching contributions which varied with earnings.

[25] The overall dependency ratio refers to the ratio of the population who are not of working age (both young and old) to those who are.

Finally, the third column in Table 1 shows our "estimate" of the shock to the retirement age that leads to the projected increase in the dependency ratio as the baby-boom generation retires, and the change in the world real interest rates that might be expected to accompany this demographic event. Over the next 35 years, the ratio of the Canadian labor force to the Canadian population is expected to fall from 0.525 to 0.475 – a 10 percent reduction.[26] The main reason for this development is that the baby-boom generation is growing older (see Mankiw and Scarth (2001, p.104)). The perturbation to the retirement age in Table 1 is consistent with this fact.[27] Furthermore, according to the estimates reported by the OECD's economics department, world-wide population aging would lead to changes in world real interest rates that vary between (-1.5 to + 0.7) percentage points by 2035, depending on the assumed effect of population aging on private savings (see Figure 7 in Turner *et al.* (1998, p. 69)).[28] We have chosen a number that lies in the middle of this range – a drop of 40 basis points.

Furthermore, although we focus on the small open economy, the model could be solved in steady state as a closed economy to generate an endogenous interest rate change following an increase in the dependency ratio. Obviously, this procedure may be appealing if we assume that the calibration to the Canadian economy is representative of the entire Western world, and that the aging population phenomenon will proceed roughly the same way throughout all these countries. Thus, applying this procedure generates a change in the interest rate that is just below 0.4 percentage point.

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[26] If we normalize the population at 1, these numbers imply that the Canadian overall dependency ratio will increase by 22.16 percent in three decades.

[27] Since our baseline calibration generates an overall dependency ratio, we found it consistent to use the projection of this ratio in estimating the shock to the retirement age – despite the fact that the projection of the old-age dependency ratio may give a more accurate description of the economic burden arising from population aging.

[28] The first number follows by assuming that demographic change has no effect on private savings behavior; whereas, for the last one, it is assumed that a 1 percentage point rise in the dependency ratio reduces the private savings rate by 0.3 percent. However, as mentioned earlier, the empirical evidence on the impact of population aging on savings is very mixed.

8. Quantitative results

8.1 Long and short-run effects of increased elderly dependency ratio

We consider first the cases in which domestic population aging (or an increase in the domestic old-age dependency ratio) is not accompanied by aging in the rest of the world, and so does not involve any change in the rate of interest.[29] At the start, we assume that both the employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases. Then, using Gauss to solve the model yields the following two differential equations

$$\begin{bmatrix} \mathrm{d}\dot{z} \\ \mathrm{d}\dot{c}_h \end{bmatrix} = \begin{bmatrix} 0.020000 & -1.000000 \\ -0.001064 & -0.013209 \end{bmatrix} \begin{bmatrix} \mathrm{d}z \\ \mathrm{d}c_h \end{bmatrix} + \begin{bmatrix} 0.007102 \\ -0.000022 \end{bmatrix} \mathrm{d}\lambda \qquad (21)$$

Denote the 2x2 matrix above by Ψ, henceforth. Since c_h is a "jump" variable, while z is predetermined at each point in time, a unique convergent solution requires that there be just one negative eigenvalue for matrix Ψ. Given that the determinant of Ψ is the product of the two eigenvalues, saddlepath stability requires det (Ψ) < 0; that condition is met.

Following an unexpected but permanent reduction in the retirement age, which raises the dependency ratio, the dynamics of evolution in (domestic) forward-looking aggregate consumption and financial wealth (composed of ownership of domestic capital stock and net financial claims on foreigners) is illustrated in the phase diagram in Figure 1.[30] From matrix Ψ it is clear that the $\mathrm{d}\dot{c}_h = 0$ locus is negatively sloped and the $\mathrm{d}\dot{z} = 0$ locus is positively sloped. Further, since equation system (21) is linear, so is the saddlepath, SS, which equation is given by

$$c_h^o = c_h^* + \varphi(z^o - z^*) \qquad (22)$$

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[29] Hereafter, we refer to these cases as the reference scenario cases.

[30] Although the public debt belongs in this group's financial wealth, it is irrelevant in the dynamic analyses, since it is kept constant through time.

where c_h^o and z^o are the initial values (after the jump onto the saddlepath in the case of consumption), c_h^* and z^* are the steady-state values, and φ is the slope of the saddlepath. Both this slope and the negative eigenvalue – which defines the system's speed of adjustment once on the saddlepath – can be readily calculated from the matrix Ψ. (See appendix A for a brief sketch of that procedure.)

In Figure 1, the initial steady state is represented by the point 1. After a once-and-for-all reduction in the retirement age, consumption drops immediately to a lower level corresponding to point 2 on the saddlepath SS, and then the system moves over time (along SS) toward point 3 (the new steady state, where consumption is still below its initial steady-state level).[31]

On impact, consumption falls by 12.88 percent of its initial full-equilibrium value (see Table 2, Bm.1, Col.1)[32]. This effect is calculated according to equation (22), and the outcome is simply a weighted average of the steady-state effects. (Note that since z is a predetermined variable, $\mathrm{d}z^o / \mathrm{d}\lambda$ is zero.) During the transition period, consumption increases gradually until the new full equilibrium is reached; at that stage the drop is 6.91 percent of the initial steady-state value (see Table 2, Bm.1, Col.1).[33] What are the mechanisms that lead to these results?

The initial fall in consumption is the net effect of forces working in opposite directions. On the one hand, a lowering of the retirement age means that the labor force shrinks. This (other things being equal) reduces the marginal productivity of capital – which leads to a downward pressure on the interest rate – and increases the marginal product of labor – which pushes pre-tax wages up. On the other, since perfect capital mobility pegs the rate of interest (and since we assume initially that the increased dependency ratio has no effect on the interest rate), the physical capital stock has to decrease. This reduction in the capital stock (that ensures the constant interest rate) lowers the marginal product of labor – and that reduces pre-tax wages. In addition, firms are now willing to pay less for labor – since their contribution rate to the public

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[31] Since the share of forward-looking individuals in the population is constant through time, the average consumption of this group also displays a similar pattern.
[32] Bm. and Col. denote Benchmark and Column, respectively.
[33] For the same reason as in the footnote 31, these numbers also correspond to the variations in average consumption among forward-looking agents.

pension system increases with demographic changes. According to our results (see Table 3, Bm.1, Col.1), these downward pressures on pre-tax wages dominate. Moreover, after-tax wages also decrease – since the employees' pension contribution rate has increased as well. Thus, these negative effects on pre and after-tax wages combined with the fact that the number of workers has shrunk explain why the average consumption of this group has declined on impact.

Furthermore, since z is a predetermined variable, $z = k - a$ implies that on impact, the variations in domestic physical capital stock correspond to the variations in foreign indebtedness. Therefore, as indicated in Table 3 (Bm.1, Col.1), the physical capital stock ratio falls on impact by an amount that is just equal to the initial foreign debt ratio. Hence, following the "aging" shock, the remaining physical capital is entirely owned by domestic (forward-looking) residents. After these initial adjustments, the dynamics of z is determined by that of a – since the change in the physical capital stock is once-and-for-all. Thus, as shown in Figure 1, during the transition to the new full equilibrium, the forward-looking group's consumption rises gradually as their claims on foreign agents increase. However, in the new steady state their living standards remain below the initial full-equilibrium level – despite the resulting higher level of non-human wealth.

As for the hand-to-mouth group, the decline in their average consumption is *greater* (18.54 percent of the initial steady-state value – both initially and in new full equilibrium) (see Table 2, Bm.1, Col.1). The causes of that large fall are the following. First, for the same reasons as before, the labor force has shrunk and also the low-paid workers in this group suffer a reduction in their pre- and after-tax wages. However, since firms induce these workers not to shirk, the fall in their pre-tax wages is lower than that of the high-wage workers (see Table 3, Bm.1, Col.1). As a result, the unemployment rate among lower-skilled workers increases by 0.7 of one percentage point (see Table 2, Bm.1, Col.1). Second, the government transfer payments – which go only to this group of low-income individuals – have decreased. The reduction in pre-tax wages, physical capital stock employed within the country, and the lower number of workers result in a loss of revenue for the government. Therefore, transfer payments have to adjust accordingly to maintain the government budget deficit. Although, the domestic (forward-looking) residents have built up a positive stock of net foreign assets (see Table 3, Bm.1, Col.1), that does not have any revenue implications for the domestic government – since by assumption it collects tax on the interest income that is generated within the country irrespective of who owns that capital.

The first column of benchmark 2 in Tables 2 and 3 show how the results change when *only* the employees' pension contribution rate adjusts residually as the dependency ratio rises. Relative to the previous case, there are fewer downward pressures on the pre-tax wages – since the firms' contribution rate is unaffected. Besides, the wage paid to lower-skilled workers increases (see Table 3, Bm.2, Col.1). The intuition runs as follows. A relatively higher increase in taxes levied on employees makes the work less attractive for low-paid workers and therefore raises their propensity to shirk. Knowing that, firms react by paying higher wages. As a result, the labor demand for these workers decreases more, and a consequent higher unemployment emerges. In this case, the rate of unemployment among these workers increases by 1.3 percentage points (see Table 2, Bm.2, Col.1) – compared to 0.7 percentage point above.

Finally, the first column of benchmark 3 in Tables 2 and 3 report how the results are affected by the assumption that *only* the firms' contribution rate varies endogenously following an increase in the dependency ratio. In this case, the effort supplied by lower-skilled workers (and hence their productivity) is unaffected. This is because an increase in the firms' pension contribution rate – which reduces market wage rates – lowers the net return both to working and to drawing employment insurance benefits, since raising this tax affects the net returns to work and non-work proportionately. Therefore, the low-paid workers' comparative attractions of working and being jobless with benefits are unaffected. Then, since there is no threat of shirking, both groups' pre-tax wages decline by an equivalent amount (see Table 3, Bm.3, Col.1). As a result, the unemployment rate is unchanged (see Table 2, Bm.3, Col.1).

In summary, comparisons of the first column of the three benchmarks in Table 2 suggest that the choice of which contribution rate becomes the dependent variable (as the dependency ratio increases) has little effect on the impact of aging on average living standards – this is true whether or not agents are forward-looking. However, the choice matters for labor market distortions, and therefore, for the unemployment rate. More specifically, in contrast to the employees', the firms' contribution rates to both the public pension and employment insurance programs are shown to have no influence on the unemployment rate. It also appears that the hand-to-mouth individuals are those who take a bigger hit in their average living standards – both in the short run and in the long run. Nevertheless, on the whole, we should regard population aging as an upcoming threat to living standards, and therefore there is a need for policy actions that may ameliorate or reverse the effect of aging on per-capita living standards.

Furthermore, notice the substantial swings in net foreign asset positions (see the last row in Table 3, Col.1 for each benchmark). Although, the magnitude of these swings is considerably larger than has been experienced, it is reassuring to note that it is similar to ones reported by the OECD's world model, Minilink (see Figures 6, 9 on pages 68 and 71, respectively, in Turner *et al.* (1998)).

So far, we have assumed that there are no demographic changes in the rest of the world and more explicitly that the rate of interest is constant. However, in the real world, the aging population seems to be a world-wide phenomenon – although not perfectly synchronized. With a general aging process one would instead expect a fall in the world interest rates. Therefore, we now consider the possibility that an increase in the elderly dependency ratio will have a negative effect on the world rate of interest – which we assume decreases by four tenths of one percentage point. To examine how this eventuality modifies the results, we initially revert to the assumption that both the employee and employer pension contribution rates adjust (in equal proportion) as the dependency ratio increases. Then using Gauss to solve the model yields the following two differential equations

$$\begin{bmatrix} \mathrm{d}\dot{z} \\ \mathrm{d}\dot{c}_h \end{bmatrix} = \begin{bmatrix} 0.020000 & -1.000000 \\ -0.001064 & -0.013209 \end{bmatrix} \begin{bmatrix} \mathrm{d}z \\ \mathrm{d}c_h \end{bmatrix} + \begin{bmatrix} 0.007102 & 0.691234 \\ -0.000022 & 0.239747 \end{bmatrix} \begin{bmatrix} \mathrm{d}\lambda \\ \mathrm{d}r \end{bmatrix} \quad (23)$$

Following an increase in the dependency ratio – which is accompanied by a reduction in the rate of interest, the full dynamic response of (domestic) forward-looking aggregate consumption and financial wealth is shown in Figure 2; and the numerical results are reported on the second column of benchmark 1 in Tables 2 and 3.

As illustrated in Figure 2, after the "aging" shock (and the accompanying interest rate effect), the system moves from its initial full equilibrium, point 1, to point 2 immediately; then as time passes, the system moves to point 3. The initial fall in consumption is less, 8.52 percent – compared to 12.88 percent in the reference scenario case (see the two columns of benchmark 1 in Table 2). This result stems from the fact that there are fewer downward pressures on both pre- and after-tax wages. First, since the rate of interest decreases, the required reduction in the physical capital stock is smaller (see Table 3, Bm.1, Col.2). As a result, the negative effect on pre-tax wages of a decline in capital stock is mitigated. Second, with the lower reduction in pre-tax wages, the rise in the two contribution rates that is needed to balance the pension plan is lower – for the same increase in the elderly dependency ratio. This implies a smaller decrease in the after-tax wage. Consequently, by ignoring the world-wide aging – and hence a reduction in world real interest rates, we overestimate the initial impact of population aging on domestic savers' living standards.

Furthermore, during the transition to the new full equilibrium, the forward-looking group's living standards decrease gradually as their level of foreign indebtedness increases; that is seen in Figure 2. Indeed, the lower interest rate means a reduced incentive for private saving and therefore a lower level of non-human wealth. At the new steady state, the average consumption of this group is reduced by 10.48 percent – compared to 6.91 percent in the reference scenario case (see the two columns of benchmark 1 in Table 2). Therefore, by ignoring demographic changes in the rest of the world, we underestimate the final impact of population aging on domestic forward-looking individuals' living standards.

As for the hand-to-mouth group, things are worsened less than in the reference scenario (see the two columns of benchmark 1 in Table 2). The smaller reduction in physical capital stock and its effect on pre-tax wages have another implication for this group; the required reduction in government transfer payments is lower. Here also, the reference scenario case overestimates the effect of aging on this group's living standard.

Finally, we also derive the results for the cases in which either the employee or employer pension contribution rate is the only rate that adjusts when the demographic structure changes. These results are reported on the second column of benchmarks 2 and 3, respectively, in both Tables 2 and 3.

To sum up, comparisons of the two columns of each benchmark in Table 2 suggest that by ignoring the impact of the general aging process on the world rate of interest, we may systematically overestimate or underestimate the effect which aging populations may have on per-capita living standards. As a consequence, policy responses – that are designed to offset this effect – may be inappropriate. Therefore, any conclusions we make about the implications of population aging on living standards should be based on what we believe regarding the effect of aging throughout the world on interest rates. Moreover, when domestic population aging is accompanied by aging in the rest of the world, and hence a reduction in world real interest rates, changes in the country's net external liabilities are very small (see the last row in Table 3, Col.2 for each benchmark). The intuition runs as follows. Broadly speaking, to the extent that imbalances in savings and investment – which arise from aging – occur at the global level they are likely to be reflected in movements in real interest rates, whereas to the extent that they occur in particular regions they will be reflected in changes in net foreign asset positions (and exchange rates).[34] In addition, when we compare the second column of the three benchmarks in Table 2, it appears again that the results are not much affected by whose contribution rate changes, expect for the unemployment rate. Also, the hand-to-mouth individuals are those who are most affected by aging.

Furthermore, it is conceivable that the pressure on the public pension system due to population aging may induce people to postpone their retirement age. But since the retirement age is exogenous in this model, the only way we can examine this eventuality is to assume a lower increase in the elderly dependency ratio. For example, one may assume the limit case where the dependency ratio increases by half of what is expected. The results concerning this possibility are presented in Tables 4 and 5.

[34] Regional imbalances need not lead to global imbalances if the weight of the region in the world economy is small and/or if there are offsetting imbalances in other regions.

Although the aging consequences are mitigated in this case, it appears that our basic results are not much affected.

8.2 Sensitivity analysis

To evaluate the relative influence of selected parameters for model results, we have conducted a number of sensitivity tests in the case where both the employee and employer pension contribution rates adjust in equal measure as the dependency ratio increases.

As explained earlier, one unappealing aspect of this model is that we cannot simultaneously choose appropriate values for the death probability (p), retirement age (λ), and the old-age dependency ratio (*DR*). In our baseline calibration, we have chosen sensible values for the first two parameters, and the model generates a dependency ratio that corresponds more closely to the overall dependency ratio than to the elderly dependency ratio in Canada. Therefore, we have had to raise the pension contribution rates such that the individual pension payment (as a proportion of the average wage) equals about 21 percent. Since the old-age dependency ratio may give a more accurate description of the economic burden arising from population aging, we now consider two sets of values for p and λ that generate a sensible elderly dependency ratio of roughly 25 percent. As a result, we set both pension contribution rates at the actual values of 4.5 percent.

Parameter Set I involves: $p = 0.04$ (which implies a life expectancy of 45 years at age 20), $\lambda = 40$ (as in the baseline calibration), and $\delta = 0.0165$ (a corresponding adjustment in the forward-looking rate of time preference that keeps all steady state restrictions satisfied). Furthermore, according to Hviding and Mérette (1998, p.7) or Mérette (2001, p.56), the elderly dependency ratio in Canada is expected to nearly double by 2035. The "shock" to the retirement age that is consistent with this fact is $d\lambda = -10$. The simulation results corresponding to this set are reported in Tables 6 and 7 – lower life expectancy case.

Parameter Set II implies: $\lambda = 80$, $p = 0.02$ (as in the baseline calibration), $\delta =$ 0.0215, and $d\lambda = -20$. These results are reported in Tables 6 and 7 – higher retirement age case.

When we compare the lower life expectancy and higher retirement age results in Table 6, and in turn compare each to the benchmark 1 results in Table 2, the sensitivity analyses suggest that our initial findings are fairly robust.

Furthermore, calibrations for all aggregative models of this sort are always a little strained by the requirement that one interest rate must represent both the net marginal product of capital and the rate of return on risk-free government debt. For a risky investment, our baseline calibration of the interest rate is relatively small. Therefore, we now set the interest and capital depreciation rates (at 8 percent and 3 percent, respectively) such that the capital-output ratio and capital's share of output remain unaffected (see equation (3)). The results of these simulations are reported in Table 8. Again, it appears that our initial findings are truly robust.

9. Conclusion

This paper has examined some fiscal, macroeconomic and distributional consequences of population aging for a single economy. Our vehicle for addressing these issues was a general equilibrium model that involves overlapping generations with a public pension system, asymmetric information in the labor market, and a subset of households that are liquidity constrained. The analysis leads to three main conclusions.

First, it shows that by analyzing the aging problem in a small open-economy framework without taking into account its global dimensions, we may systematically misestimate the effect which aging may have on the country's living standards and its net foreign asset position. As a result, the scope of policy responses – that are designed to ameliorate or offset these effects – may be very inaccurate.

Second, the magnitude of the effect of an aging population on people's average living standards, in the short run as well as in the long run, significantly depends on whether or not they are liquidity constrained. Nevertheless, on the whole, we should regard population aging as an upcoming threat to living standards, and therefore there is a need for policy actions that may ameliorate or reverse its effect.

Third, the actual incidence of increased contributions to the public pension system – that is, whether the workers' or firms' contribution rate should increase to maintain the financial viability of the system as the elderly dependency ratio rises – has little effect on the impact of aging on living standards. This conclusion emerges whether or not agents save for the future. However, the incidence of contributions (to both the public pension and employment insurance programs) matters for labor market distortions, and hence for the unemployment rate. More specifically, in contrast to the employees', the firms' contribution rates to both programs are shown to have no influence on the unemployment rate.

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Figure 1: Dynamic adjustment of domestic (forward-looking group) consumption and financial wealth to an increase in the elderly dependency ratio – *with no* demographic change abroad.

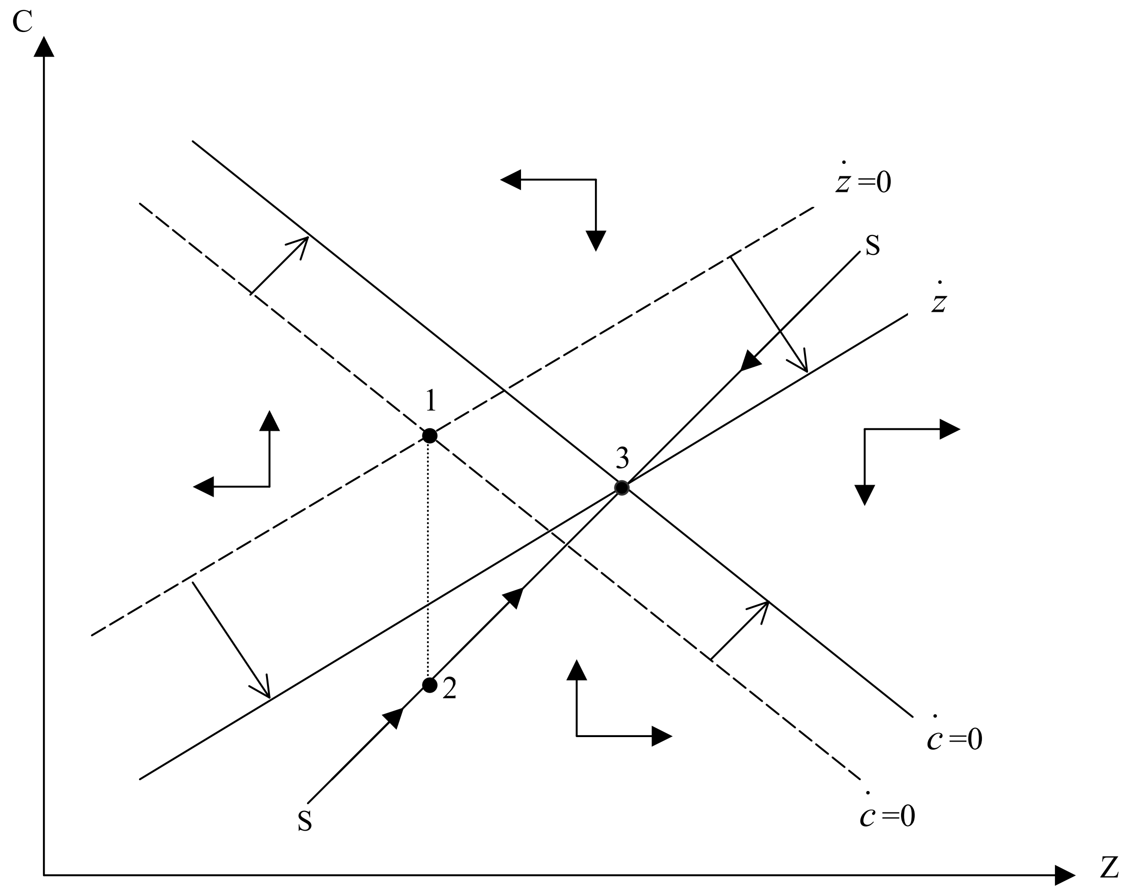
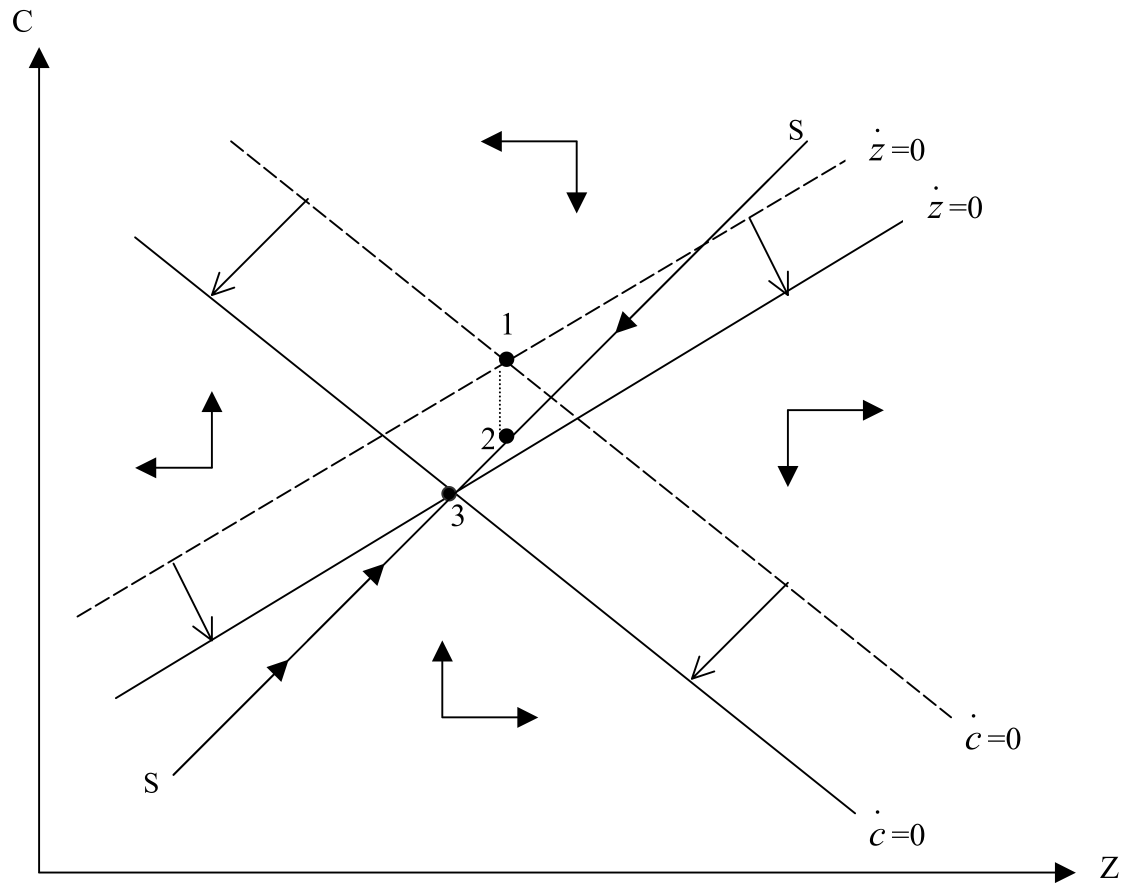


Figure 2: Dynamic adjustment of domestic (forward-looking group) consumption and financial wealth to an increase in the elderly dependency ratio – *with* similar demographic change abroad.



Long and Short-Run Effects of Increased Elderly Dependency Ratio

I – Benchmark Simulation Experiments

Table 2: Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers.

Variable	Benchmark 1		Benchmark 2		Benchmark 3	
	λ shock	λ and r shocks	λ shock	λ and r shocks	λ shock	λ and r shocks
c_h, impact effect	-12.88%	-8.52%	-13.31%	-8.91%	-12.39%	-8.07%
c_h, long-run effect	-6.91%	-10.48%	-7.58%	-11.10%	-6.12%	-9.76%
c_l, I and LR effects	-18.54%	-14.72%	-17.40%	-13.67%	-19.88%	-15.94%
u	+0.007	+0.006	+0.013	+0.011	0.000	0.000

Notes:

I: Denotes Impact
LR: Denotes Long Run

Benchmark 1: The employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases.
Benchmark 2: The employee contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.
Benchmark 3: The employer contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.

Table 3: Effects of increased elderly dependency ratio on wages, capital stock, output and foreign debt.

Variable	Benchmark 1		Benchmark 2		Benchmark 3	
	λ shock	λ and r shocks	λ shock	λ and r shocks	λ shock	λ and r shocks
Δw_h	-2.06%	-0.07%	-0.37%	+1.48%	-4.03%	-1.87%
Δw_l	-1.24%	+0.70%	+1.15%	+2.88%	-4.03%	-1.87%
$\mathrm{d}k$	-0.300	-0.136	-0.305	-0.140	-0.295	-0.132
Δy	-10.00%	-8.18%	-10.16%	-8.31%	-9.85%	-8.03%
$\mathrm{d}a^*$	-0.823	+0.035	-0.806	+0.051	-0.843	+0.017

Notes:

Δ : Denotes percentage changes
d : Denotes absolute changes

Benchmark 1: The employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases.
Benchmark 2: The employee contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.
Benchmark 3: The employer contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.

Table 4: Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers – DR increased by half of what is expected.

Variable	Benchmark 1		Benchmark 2		Benchmark 3	
	λ shock	λ and r shocks	λ shock	λ and r shocks	λ shock	λ and r shocks
c_h, impact effect	-6.63%	-2.27%	-6.85%	-2.46%	-6.38%	-2.06%
c_h, long-run effect	-3.56%	-7.13%	-3.90%	-7.42%	-3.15%	-6.80%
c_l, I and LR effects	-9.55%	-5.72%	-8.96%	-5.23%	-10.24%	-6.29%
u	+0.003	+0.003	+0.006	+0.005	0.000	0.000

Note: DR denotes Dependency Ratio

Table 5: Effects of increased elderly dependency ratio on wages, capital stock, output and foreign debt – DR increased by half of what is expected.

Variable	Benchmark 1		Benchmark 2		Benchmark 3	
	λ shock	λ and r shocks	λ shock	λ and r shocks	λ shock	λ and r shocks
Δw_h	-1.06%	+0.93%	-0.19%	+1.66%	-2.08%	+0.08%
Δw_l	-0.64%	+1.28%	+0.59%	+2.32%	-2.08%	+0.08%
$\mathrm{d}k$	-0.155	+0.009	-0.157	+0.008	-0.152	+0.012
Δy	-5.16%	-3.32%	-5.23%	-3.38%	-5.07%	-3.25%
$\mathrm{d}a^*$	-0.424	+0.435	-0.415	+0.442	-0.435	+0.426

Note: DR denotes Dependency Ratio

II – Sensitivity Analysis Experiments

Unless otherwise indicated, the sensitivity analyses are conducted in the case where both the employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases.

Table 6: Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers – lower life expectancy and higher retirement age cases.

Variable	Lower life expectancy λ shock	Lower life expectancy λ and r shocks	Higher retirement age λ shock	Higher retirement age λ and r shocks
c_h, impact effect	-13.72%	-10.10%	-15.18%	-9.14%
c_h, long-run effect	-7.32%	-10.22%	-4.39%	-11.15%
c_l, I and LR effects	-19.10%	-15.16%	-19.10%	-15.16%
u	+0.007	+0.006	+0.007	+0.006

Table 7: Effects of increased elderly dependency ratio on wages, capital stock, output, and foreign debt – lower life expectancy and higher retirement age cases.

Variable	Lower life expectancy λ shock	Lower life expectancy λ and r shocks	Higher retirement age λ shock	Higher retirement age λ and r shocks
Δw_h	-2.46%	-0.55%	-2.46%	-0.55%
Δw_l	-1.66%	+0.22%	-1.66%	+0.22%
$\mathrm{d}k$	-0.311	-0.147	-0.311	-0.147
Δy	-10.36%	-8.53%	-10.36%	-8.53%
$\mathrm{d}a^*$	-0.843	-0.136	-1.533	+0.081

Table 8: Effects of increased elderly dependency ratio on living standards and unemployment rate among low-income workers – higher initial interest rate case.

Variable	S1		S2		S3	
	λ shock	λ and r shocks	λ shock	λ and r shocks	λ shock	λ and r shocks
c_h, impact effect	-10.78%	-7.68%	-11.18%	-8.05%	-10.31%	-7.25%
c_h, long-run effect	-3.64%	-9.32%	-4.28%	-9.90%	-3.00%	-8.65%
c_l, I and LR effects	-19.08%	-14.74%	-18.00%	-13.74%	-20.35%	-15.90%
u	+0.007	+0.006	+0.013	+0.011	0.000	0.000

Notes:

S1: The employee and employer contribution rates adjust (equally) to balance the pension plan budget constraint as the dependency ratio increases.

S2: The employee contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.

S3: The employer contribution rate alone adjusts to balance the pension plan budget constraint as the dependency ratio increases.

Appendix A:

Complete derivation of the simulation model

Taking the total differential of the equations (1'), (3) and (17)(with $z = k - a$) gives

$$\Delta y = \alpha \Delta k + \frac{(1-\alpha) p e^{-p\lambda}}{(1-e^{-p\lambda})} \mathrm{d}\lambda - \frac{(1-\alpha-\beta)}{(1-u)} \mathrm{d}u + \varepsilon(1-\alpha-\beta)\Delta w_l - \frac{\varepsilon(1-\alpha-\beta)}{(1-t_w-\tau_w-\theta_w)} \mathrm{d}\tau_w \qquad \text{(A1)}$$

$$\Delta k = \Delta y - \frac{1}{r+\phi} \mathrm{d}r \qquad \text{(A2)}$$

and

$$\mathrm{d}\dot{z} = (r(1-t_i)-n)\mathrm{d}z - \mathrm{d}c_h - \mathrm{d}c_l + y\Delta y - k(r(1-t_i)+\phi)\Delta k - a(1-t_i)\mathrm{d}r \qquad \text{(A3)}$$

respectively.

Using (A2) and $k = \frac{\alpha}{r+\phi} y$ (from equation (3)), (A3) can be written as:

$$\mathrm{d}\dot{z} = (r(1-t_i)-n)\mathrm{d}z - \mathrm{d}c_h - \mathrm{d}c_l + A_1 \Delta y + \left[\frac{(r(1-t_i)+\phi)k}{r+\phi} - (1-t_i)a \right] \mathrm{d}r \qquad \text{(A4)}$$

where $A_1 = \left((1-\alpha) + \frac{\alpha r t_i}{r+\phi} \right) y$.

Combining (A1) and (A2), then substituting the resulting equation into (A4) yields

$$\mathrm{d}\dot{z} + \mathrm{d}c_l - \frac{A_1 \varepsilon (1-\alpha-\beta)}{(1-\alpha)} \Delta w_l + \frac{A_1(1-\alpha-\beta)}{(1-\alpha)(1-u)} \mathrm{d}u + \frac{A_1 \varepsilon (1-\alpha-\beta)}{(1-\alpha)(1-t_w-\tau_w-\theta_w)} \mathrm{d}\tau_w =$$

$$(r(1-t_i)-n)\mathrm{d}z - \mathrm{d}c_h + \frac{A_1 p e^{-p\lambda}}{(1-e^{-p\lambda})} \mathrm{d}\lambda + \left[\frac{(r(1-t_i)+\phi)k}{r+\phi} - (1-t_i)a - \frac{\alpha A_1}{(1-\alpha)(r+\phi)} \right] \mathrm{d}r \qquad \text{(A5)}$$

This equation represents the first line in the matrix system.

After setting the total differential of the equations (14') and (15')(in this case, eliminating the change in k), we have respectively

$$\mathrm{d}\dot{c}_h - A_5(1-t_w-\tau_w-\theta_w)w_h\Delta w_h + A_5 w_h \mathrm{d}\tau_w = -p(p+\delta)\mathrm{d}z + (r(1-t_i)-\delta-n)\mathrm{d}c_h$$

$$+\left[A_6\left((r(1-t_i)-n)\frac{e^{-(r(1-t_i)-n)\lambda}}{(1-e^{-(r(1-t_i)-n)\lambda})}-p\right)-m\pi p(p+\delta)e^{-p\lambda}\right]\mathrm{d}\lambda \tag{A6}$$

$$+\left[A_6\left(\frac{(1-t_i)\lambda e^{-(r(1-t_i)-n)\lambda}}{(1-e^{-(r(1-t_i)-n)\lambda})}-\frac{(1-t_i)}{(r(1-t_i)-n)}\right)+(1-t_i)c_h\right]\mathrm{d}r$$

where

$$A_5 = \frac{\pi p(p+\delta)}{(r(1-t_i)-n)}e^{-p\lambda}(1-e^{-(r(1-t_i)-n)\lambda}) \quad \text{and} \quad A_6 = A_5((1-t_w-\tau_w-\theta_w)w_h - m)$$

and

$$\mathrm{d}c_l - A_3\Delta w_h - \left[\frac{A_2(1-\alpha)+A_4\varepsilon(1-\alpha-\beta)}{(1-\alpha)}\right]\Delta w_l + \left[\frac{A_2(1-\alpha)+A_4(1-\alpha-\beta)}{(1-\alpha)(1-u)}\right]\mathrm{d}u$$

$$+\left[\frac{A_2}{(1-\tau_w+\theta_f)}+\frac{A_4\varepsilon(1-\alpha-\beta)}{(1-\alpha)(1-t_w-\tau_w-\theta_w)}\right]\mathrm{d}\tau_w \tag{A7}$$

$$=\left[(A_2+A_3+A_4)\frac{pe^{-p\lambda}}{(1-e^{-p\lambda})}-m(1-\pi)pe^{-p\lambda}\right]\mathrm{d}\lambda + \left[t_i k-(1-t_i)b-\frac{A_4}{(1-\alpha)(r+\phi)}\right]\mathrm{d}r$$

where

$$A_2 = (1-u)(1-\pi)(1-e^{-p\lambda})w_l(1-\tau_w+\theta_f)$$

$$A_3 = (t_w+\theta_w+\theta_f)\pi(1-e^{-p\lambda})w_h \qquad \text{and} \qquad A_4 = \frac{rt_i\alpha y}{r+\phi}$$

These equations correspond respectively to the second and third rows in the matrix system.

By taking the total differential of the equations (4') and (5'), we get respectively

$$\Delta y = \Delta w_h + \frac{1}{(1+\tau_f+\theta_f)}\mathrm{d}\tau_f + \frac{pe^{-p\lambda}}{(1-e^{-p\lambda})}\mathrm{d}\lambda \tag{A8}$$

and

$$\Delta y = \Delta w_l + \frac{1}{(1+\tau_f+\theta_f)}\mathrm{d}\tau_f - \frac{1}{(1-u)}\mathrm{d}u + \frac{pe^{-p\lambda}}{(1-e^{-p\lambda})}\mathrm{d}\lambda \tag{A9}$$

Combining (A1) and (A2), then substituting the resulting equation into (A8) and (A9) gives

$$-\Delta w_h + \frac{\varepsilon(1-\alpha-\beta)}{(1-\alpha)}\Delta w_l - \frac{(1-\alpha-\beta)}{(1-\alpha)(1-u)}\mathrm{d}u - \frac{\varepsilon(1-\alpha-\beta)}{(1-\alpha)(1-t_w-\tau_w-\theta_w)}\mathrm{d}\tau_w - \frac{1}{(1+\tau_f+\theta_f)}\mathrm{d}\tau_f = \frac{\alpha}{(1-\alpha)(r+\phi)}\mathrm{d}r \tag{A8'}$$

and

$$\frac{\varepsilon(1-\alpha-\beta)-(1-\alpha)}{(1-\alpha)}\Delta w_l + \frac{\beta}{(1-\alpha)(1-u)}\mathrm{d}u - \frac{\varepsilon(1-\alpha-\beta)}{(1-\alpha)(1-t_w-\tau_w-\theta_w)}\mathrm{d}\tau_w - \frac{1}{(1+\tau_f+\theta_f)}\mathrm{d}\tau_f = \frac{\alpha}{(1-\alpha)(r+\phi)}\mathrm{d}r \tag{A9'}$$

These equations stand for the fourth and fifth rows in the matrix system, respectively.

The total differential of the equation (8) is

$$(1-t_w-\tau_w-\theta_w-f)\mathrm{d}u - (u-\varepsilon)\mathrm{d}\tau_w = 0 \tag{A10}$$

and it represents the sixth row in the matrix.

Finally, the seventh row in the system is the total differential of the equation (12), which is given by

$$(\tau_f + \tau_w)\pi w_h \Delta w_h + (\tau_f + \tau_w)(1-u)(1-\pi)w_l \Delta w_l - (\tau_f + \tau_w)(1-\pi)w_l \mathrm{d}u$$
$$+\left[\pi w_h + (1-u)(1-\pi)w_l\right](\mathrm{d}\tau_w + \mathrm{d}\tau_f) = -m\frac{pe^{-p\lambda}}{(1-e^{-p\lambda})^2}\mathrm{d}\lambda \qquad \text{(A11)}$$

Determination of the saddlepath slope and the negative eigenvalue

The solutions of the dynamic system defined by (21) are

$$z_t = z^* + \kappa_1 e^{\sigma_1 t} + \kappa_2 e^{\sigma_2 t}$$

$$c_t = c^* + \kappa_3 e^{\sigma_1 t} + \kappa_4 e^{\sigma_2 t}$$

But since $\sigma_1.\sigma_2 = \det(\Psi) < 0$, we can take $\sigma_1 < 0$ and $\sigma_2 > 0$. The initial jump to the saddlepath means that the unstable root, σ_2, does not operate, so $\kappa_2 = \kappa_4 = 0$. Further, since equation system (21) is linear, so is the saddlepath, equation (22). That equation and the two above are consistent if and only if $\kappa_3 = \varphi\kappa_1$. Thus, once on the saddlepath, the solutions simplify to

$$z_t = z^* + \kappa_1 e^{\sigma_1 t}$$

$$c_t = c^* + \varphi\kappa_1 e^{\sigma_1 t}$$

If these two equations and their time derivatives are substituted into system (21) and $\mathrm{d}\lambda$ is set to zero, the result simplifies to

$$\left[\sigma_1 \quad \varphi\sigma_1\right]' = \Psi\left[1 \quad \varphi\right]'$$

which can be used to solve for φ and σ_1.

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