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Abstract

We develop a moral hazard model with auditing where both the principal and the agent can influence the probability that the true state of nature is verified. This setting is widely applicable for situations where fraudulent reporting with costly state verification takes place. However, we use the framework to investigate tax evasion. We model tax evasion as a concealment-detection contest between the taxpayer and the authority. We show that higher tax rates cause more evasion and increase the resources wasted in the contest. Additionally, we find conditions under which a government should enforce incentive compatible auditing in order to reduce wasted resources.

JEL-Classification: H26, D82, K42

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1 Introduction

The purpose of this paper is to develop a moral hazard model with auditing where both the principal and the agent can influence the probability that the true state of nature is verified. We do not allow the principal to commit to an audit strategy before observing the signal from the agent. Such a setting is widely applicable to situations of fraud. Fraudulent claims for benefits, insurance payments, or loans are examples. It even could be applied to the broad range of situations where bilateral trade of goods takes place. Whenever it is hard and expensive to verify the value of a good for a potential buyer (antiques, paintings), while the seller has private information about this value, such a moral-hazard situation may arise. However, the application we choose is the case of tax evasion. This will enable us to draw conclusions about the impact of tax rates on tax evasion and the resources wasted by the agents' attempts to influence the detection probability.

The early neoclassical approach to income tax evasion (e.g. Allingham and Sandmo, 1972; Yitzhaki, 1974) treats the detection probability as an exogenous parameter.[1] In later contributions the audit probability was endogenized in two different ways. Reinganum and Wilde (1985) derive an optimal audit rule under the assumption that the authority has to invest in the audit probability.[2] In a neoclassical optimal taxation framework Cremer and Gahvari (1994) allow for the taxpayer to influence the audit probability by spending some resources on covering actions. In this paper, we explicitly model both the tax authority investing in detection and the taxpayer spending some income to cover evasion activity. The detection probability is determined by the effort exerted by both parties. We believe that for many countries the relationship between taxpayer and tax authority is quite competitive, and accordingly is accurately described by such a contest.

Furthermore, in the real world we observe that different sources of income lead to different evasion and concealment opportunities.[3] We include this fact in our model by just focusing on single components of income with different marginal coverage and fixed evasion costs. So we end up with separate evasion, coverage and detection decisions for different possible income components. The sum of all these decisions determines the over all income after tax - including possible fines.[4] We think that this approach, that allows for income structures with distinct income parts, is more realistic than the widely used framework where the aggregate income is considered to be homogenous and evasion decisions are modelled as continuous choices. This approach is related to Macho-Stadler and Perez-Castrillo (1997). There taxpayers are heterogeneous in income and income sources are heterogeneous

[1] For a detailed survey and many extensions to the basic neoclassical model see Cowell (1990).

[2] For a more general characterization of optimal enforcement schemes see Chander and Wilde (1998).

[3] The Taxpayer Compliance Measurement Program of the U.S. (IRS, 1983) e.g. estimates for 1981 that tax compliance for wages and salaries was 93.9%, 59.4% for capital gains, and only 37.2% for rents.

[4] By restricting our analysis to uncorrelated earnings probabilities, a linear tax system, and a penalty that does not depend on the over-all income, we can treat these decisions as independent.

in the (exogenous) probability of verification if an audit takes place. We endogenize the verification probability by introducing a contest. Furthermore, we do not assume that the tax authority can commit to an audit strategy. This reflects our aim to analyze the interaction between tax authority and taxpayer positively instead of characterizing an optimal, committable audit, penalty, and tax structure. We think that the normative approaches in the latter tradition suffer the problem that maximizing social welfare only with respect to tax evasion does not take into account that tax rates and fines may have more influence on welfare through other channels. The results of those models may be misleading for this reason.

We examine the equilibrium predictions of the model and find conditions the parameters have to satisfy in order that certain equilibria are obtained (such as e.g. "contest" or "honest taxpayer"). Our main finding is that in the tax evasion setting with incomplete and imperfect information no credible strategy for the tax authority exists that prevents tax evasion with certainty if the taxpayer has the opportunity of evading.[5] This finding differs from the standard literature (see e.g. Reinganum and Wilde (1985), Border and Sobel (1987), Mookherjee and Png (1989), Mookherjee and Png (1990) or Chander and Wilde (1998)), where optimal incentive-compatible enforcement schemes are derived. There the possibility of committing to a certain strategy is the key for the nonexistence of evasion. We follow the non-commitment assumption, which was introduced into the tax evasion literature by Reinganum and Wilde (1986) and Graetz et al. (1986). Our model shares some characteristics with Khalil (1997), who uses the price-regulation setup of Baron and Myerson (1982) and combines it with production-cost auditing. We think that the result in our model - i.e. the taxpayer always evades at least with a very small probability if he has the opportunity to do so - is empirically more realistic. In addition, our model predicts more tax evasion if tax rates are raised. This empirically established fact is hardly explainable with the traditional neoclassical models.[6] In the normative part of this paper we analyze the impact of tax rates on resources wastefully invested in detection and concealment. Additionally, we examine the conditions under which a government directive that commits the authority to an evasion preventing audit rule, reduces the resources wasted in the enforcement process.

The remainder of the paper is organized as follows: In the next section we discuss the timing of the game and our main assumptions. Then we develop the basic setup and analyze the impact of tax rates on evasion and wasted resources. In the following section we examine the conditions an external commitment device such as a law or government directive that commits the authority to an effort leading to truthful revelation has to satisfy in order to be waste reducing. In section 5 we analyze the robustness of our results in a richer framework where tax and fine systems can be non-linear. We conclude with some remarks on the policy implications of the presented model.

[5] We define a positive evasion opportunity as a situation where the fixed evasion costs are not prohibitive.

[6] A concise overview over the logic of different generations of tax-evasion models can be found in Franzoni (1999).

2 Timing and basic assumptions

In this section we develop the structure of the model and briefly discuss the underlying assumptions. We begin with the timing.

2.1 Timing

Before we comment on the reasons for choosing the present structure, we introduce the timing of our model and some notation. The sequence of events is as follows:

1. Nature determines the actual income y_i^a for every possible income source i.

2. The taxpayer observes y_i^a.

3. The taxpayer declares his income $d_i \in \{0, y_i^a\}$, and chooses the effort $e_i \in [0, \infty)$ to cover a possible evasion for every possible income source i.

4. The authority observes the declared income d_i for every source i. It does not observe the true income y_i^a and the concealment effort e_i exerted by the taxpayer.

5. The authority chooses a certain verification effort $a_i \in [0, \infty)$ for every possible income source.

6. Nature decides whether evasion is verifiable or not. The probability of verifiability is given by $p_i(a_{i,} e_i)$ for the different possible income components.

7. Taxpayer and authority receive their payoffs U_i and R_i, respectively.

Since we are not primarily interested in the effects of taxes on the income generation decision, we treat income as endogenously determined by nature. Furthermore, the induction of the income-generating mechanism does not lead to additional strategic effects. For reasons of clarity and simplicity we prefer not to model them.

2.2 The basic assumptions

Here we will explain the basic assumptions to be used in the main part of this chapter.

A1 Declaration is a binary decision for the different income sources, i.e. $d_i \in \{0, y_i^a\}$.

This assumption closely corresponds with our usage of the term "income" sources. Income sources in our sense are specific components of possible income, which are not divisible in terms of certification. Usually, the tax authorities - at least in systems with developed tax collection - ask for documents proving the value of declared income components. These are e.g. payment certificates issued by the employer, bank certificates for interest payments or copies of bills for deductions of expenses. So we assume that it is only possible to declare and certify a certain income component or not to declare it at all. However, this assumption is not crucial at all. In the linear framework we use, an interior declaration level is never optimal. To exclude the possibility of interior declaration levels from the beginning makes the notation easier and proves convenient for expositional reasons.

A2 Assume that the distribution of the realizations for different income sources y_i^a is dichotomous:

$$y_i^a = \begin{cases} y_i & \text{with probability} \quad \lambda_i \\ 0 & \text{with probability} \quad 1-\lambda_i \end{cases}.$$

The income sources are assume to be uncorrrelated.[7]

A2 considerably simplifies the analysis and could indeed be regarded as an oversimplification. However, the main results still hold if we relax this assumption.

A3 Both taxpayer and tax authority are risk neutral. They maximize expected net income and net revenue, respectively.

To assume a risk-neutral agents is a standard assumption in tax evasion games (e.g. Reinganum and Wilde, 1985). Risk-aversion complicates the model without adding any new qualitative insights. However it is less obvious what the objective function for the tax authority should be. There are alternative formulations that seem reasonable. Assuming that the authority maximizes net penalties instead of net revenue does not have any qualitative influence on our results. We would obtain qualitatively equivalent results if the tax authority were assumed to maximize net recovered revenue - an assumption indicating that bureaucrats care about their perceived performance.

A4 The tax system is linear (i.e. $T(d) = t\sum_{i=1}^{n} d_i$) and the penalty is proportional to the amount of taxes evaded or avoided (i.e. $F(d, y^a) = f \cdot t\sum_{i=1}^{n}(y_i^a - d_i)$ with $f > 1$).

This assumption serves two purposes. Firstly, it makes the results of this paper comparable to most of the existing work on tax evasion, since such tax and penalty systems are widely used in the literature. Secondly, this assumption makes sure that we can treat the overall tax liability and possible penalties as a simple sum of outcomes for the single income components. In this setting the choices of declared income, concealment and detection effort are independent for the different income sources. In section 5 we explore the robustness of our results under non-linear tax and fine systems.

A5 The verification probability p_i increases with detection effort a_i and decreases with concealment effort e_i. The marginal cost of influencing the verification probability in the favorable direction increases with the effort.

[7] In section (5) we allow for correlation.

To achieve this we use a formulation for the verification probability that is commonly used in the contest literature:

$$p_i(a_i, e_i) = \begin{cases} 0 & \text{if} \quad a_i, e_i = 0 \\ \frac{a_i}{a_i+e_i} & \text{else} \end{cases} \tag{1}$$

A6 The concealment costs C_i and the detection costs A_i are linear in effort.

The marginal concealment cost may depend on some parameters that describe the specific environment for the concealment of that income component. Banking system, laws to prevent money laundering, and the degree of transparency in capital markets are examples. Realistically, the marginal detection cost could depend on the amount of income concealed, since it is harder to conceal large amounts of money.[8]

$$C_i(e_i, \cdot) = c_i(\cdot) \cdot e_i. \tag{2}$$

Without loss of generality we can normalize the marginal detection cost to unity.

$$A_i(a_i) = a_i \tag{3}$$

This assumption reflects the observation that it is more costly for the tax evader to hide his evasion more effectively. He will take the cheaper measures to conceal before using the more expensive ones. On the other hand, it seems reasonable to assume that it is getting more and more expensive for the authority to achieve an extra percent of detection or verification probability, because tax inspectors should begin seeking where it is easiest to find evidence. We are aware that audit technologies may locally exhibit economies of scale or may not be continuous. However, the assumption that the authority uses the most effective means of detection first guarantees that all relevant audit efforts are on a concave function. Then the proposed continuous formulation can be seen as a convenient simplification that allows the application of calculus. The only property we need for our main results, is that the marginal costs of influencing the probability in its favoured direction are increasing. We do not allow the concealment cost to depend directly on the tax rate. We think, this is a realistic restriction that considerably simplifies the algebra.

A7 Tax evasion causes fixed evasion cost K_i to the evader.

Finally, we allow for some evasion costs K_i, which are incurred whenever the taxpayer tries to evade an income component. This reflects the observation that not only concealment, but also evasion may be costly. Sometimes an evasion opportunity has to be created in order to have the possibility to evade. There are expenses that do not vary with the level of concealment. Another part of K_i are the often cited moral costs of evasion. We use these moral cost as - admittedly, a somewhat crude - black

[8] To simplify the notation in what follows we will drop the possible arguments of the marginal cost function.

box variable that describes psychological differences of taxpayers (like ethics, attitudes etc.) leading to different evasion behaviour in identical situations.[9]

2.3 The payoffs and some notation

Let us now specify the payoff functions for the two players. It follows from our assumptions that the expected interim payoff - after declaration and efforts are determined - for the taxpayer can be written as

$$\begin{aligned} EU(\mathbf{d},\mathbf{e},\mathbf{a}) &= (1-t)\sum_{i\in H} y_i^a + \sum_{i\notin H} y_i^a\left[1 - f\cdot t\cdot p_i(e_i,a_i)\right] \\ &\quad -\sum_{i\notin H}(c_i\cdot e_i + K_i) \end{aligned} \tag{4}$$

where H is the set of all i with $d_i = y_i^a$, which is the set of truthfully declared income sources. The first sum gives the certain after-tax income for all income components that are declared. The second represents the income for the undeclared income parts - expected penalties included. The final, negative sum contains the concealment and evasion cost.

Following our assumption about the objective function of the tax authority (A3), we can write the expected payoff of the authority as:

$$ER(\mathbf{d},\mathbf{e},\mathbf{a}) = (1-t)\sum_{i\in H} y_i^a + \sum_{i\notin H}\left[y_i^a\cdot f\cdot t\cdot p_i(e_i,a_i)\right] - \sum_{i=1}^{n} a_i \tag{5}$$

Because of the linear system and the risk neutrality assumption we immediately see that the maximization for both objective functions is piecewise if income sources are uncorrelated. This leads to the following lemma:

Lemma 1 *The decisions (declaration d_i, concealment effort e_i and detection effort a_i) for income source i are independent of the decisions for all other income sources d_j, e_j, and a_j with $j \neq i$.*

Proof. Obvious. ■

Lemma 1 tells us that we can restrict ourselves to examining the decisions for a single income component. To simplify our notation we drop the indices for the potential income sources.

In order to be able to interpret the results we will derive, it is helpful to define the ratio of marginal detection cost to marginal concealment cost as the relative *concealment opportunity* η for the income component:[10]

$$\eta = \frac{1}{c} \tag{6}$$

[9] For some experimental evidence on psychological differences as predictors for evasion behaviour see Bayer and Reichl (1997) or Anderhub et al. (2001).

[10] Note, that η depends on the same arguments as c.

A further definition that will help the intuition is to use the ratio of fixed evasion cost to the possible tax-bill reduction to define the *evasion opportunity* ω as follows:

$$\omega = 1 - \frac{K}{ty} \tag{7}$$

An evasion opportunity ω of 0 means that a taxpayer has to invest the same amount of money (or time, nerves, and moral tension, respectively) to be able to evade as the possible tax bill reduction would be. More precisely, the evasion opportunity here is a percentage measure of the possible tax bill reduction net of evasion cost.

It remains to define the amount of resources invested wastefully in the process of declaration and auditing. The waste is defined as the sum of the costs for evasion, covering action, and detection activity:

$$W = ce + a + \phi K, \tag{8}$$

where ϕ is an indicator variable, which is equal to 1 if the income component is earned and evaded and 0 otherwise.

3 Signaling with hidden action

We concentrate on the genuine tax evasion situation, where the authority can neither observe the true income, nor the concealment effort. This might be the most frequent situation the tax authority faces when receiving tax returns. The only information the tax authority has is the probability distribution over the distinct income parts. As a simplifying assumption (see A2) we assumed a dichotomous distribution. Consequentially, the tax inspector knows that the taxpayer has the income component (worth an amount of y) with probability λ. With probability $1-\lambda$ the income from this income source is 0. This is common knowledge.

The solution concept we use is that of a Perfect Bayesian Equilibrium (PBE). The game the actors face can be classified as a signaling game with hidden action. In our case an equilibrium consists of three elements: a strategy for the taxpayer, a strategy for the authority, and the authority's beliefs about the true income of the taxpayer. The strategy for the taxpayer specifies a declaration (signal) and a concealment effort (hidden action) conditioned on whether he earned the income or not. The strategy for the authority is a detection effort depending on the observed declaration. The authority's beliefs assign probabilities to the income of the taxpayer. They depend on the observed declaration and are updated by using Bayes' Rule. In equilibrium the strategies maximize the actors' payoffs given the beliefs. The beliefs have to be consistent with the equilibrium strategies.

Unlike other models (e.g. Chander and Wilde, 1998) in our case the revelation principle does not hold. The reason is twofold. Firstly, we do not allow the authority to commit beforehand to a certain action. But even if we allowed for that, the revelation principle would fail, since secondly the nature of the contest restricts the set of feasible contracts. We will see the difference of outcomes when we compare an externally enforced incentive compatible effort scheme to the equilibrium in our original game.

3.1 Equilibria for different parameter settings

Let us begin with an obvious statement about the taxpayer's behaviour. Declaring any non-existent income never pays. So reporting zero if no income is earned is part of any equilibrium. The corresponding concealment effort is also zero.

$$d^*(y^a|y^a = 0) = 0 \tag{9}$$

$$e^*(y^a|y^a = 0) = 0 \tag{10}$$

When the tax authority has to decide how much to invest in detection, its only information is the declaration of the taxpayer. It may face a declaration of $d = 0$ or $d = y$. If an income declaration of y is observed it is optimal for the authority to do nothing. This is also part of any equilibrium:

$$a^*(d|d = y) = 0 \tag{11}$$

If the inspector representing the authority finds that the taxpayer declared no income, he might not be sure whether he faces a tax evader or just a person who really received no income from the source in question. He has to form some beliefs. Denote the belief that he faces a tax evader, which is the subjective probability that the true type of the taxpayer is y if he reports 0, as $\mu(y^a = y|d = 0)$. Applying Bayes' Rule this belief should be

$$\mu(y^a = y|d = 0) = \frac{\alpha \cdot \lambda}{\alpha \cdot \lambda + 1 - \lambda}, \tag{12}$$

where α is the probability that a taxpayer with positive income does not declare it.[11] We allow the taxpayer to play a mixed strategy. Now we can express the objective function of the authority if it faces a declaration of 0 as the expected fine collected net of detection costs:

$$ER(a, e, \mu, d|d = 0) = \mu \cdot f \cdot t \cdot y \cdot p(a, e) - a,$$

where μ is the abbreviated form of (12).

Investing valuable resources in concealment if there is nothing to conceal is a strictly dominated strategy for the taxpayer. So we have

$$e^*(y^a|y^a = y \ , \ d|d = y) = 0.$$

Taking this into account and allowing for mixing we can state the relevant ex ante objective function for the taxpayer under the condition that he earned the income component:

$$EU(e, a, y^a|y^a = y) = \alpha(y - p(e, a) \cdot f \cdot t \cdot y - c \cdot e) + (1 - \alpha)(1 - t)y$$

[11] Implicitly we already apply the consistency requirement that the authority puts a zero probability on the taxpayer declaring some income if he has not got it (see equation 9).

3.2 Pure strategy equilibrium

Let us now look for pure strategy equilibria. Note that equations (9) to (11) are part of any equilibrium. To find a pure strategy equilibrium we let $\alpha = 1$ (the taxpayer always evades) or $\alpha = 0$ (the taxpayer never evades). For the case of a pure evasion equilibrium we plug $\alpha = 1$ into both of the objective functions and find the optimal values for a and e. This is to make sure that the beliefs of the authority are consistent with the strategy of the tax evader. Later on, we have to check whether - given the outcome in the simultaneous effort stage - it is really optimal for the taxpayer to declare no income if he earned it. For $\alpha = 1$ - and $\mu = \lambda$ consequently - the two first-order conditions are:[12]

$$\frac{\partial}{\partial a} ER(a, e, \alpha, 0) = \frac{\lambda \cdot e \cdot f \cdot t \cdot y}{(a+e)^2} - 1 \leq 0 \tag{13}$$

$$\frac{\partial}{\partial e} EU(e, a, \alpha) = \frac{a \cdot f \cdot t \cdot y}{(a+e)^2} - c \leq 0 \tag{14}$$

The resulting efforts are:

$$a_p^* = \frac{\lambda^2 \cdot f \cdot t \cdot \eta \cdot y}{(\eta+\lambda)^2} \tag{15}$$

$$e_p^* = \frac{\lambda \cdot f \cdot t \cdot \eta^2 \cdot y}{(\eta+\lambda)^2} \tag{16}$$

This is an equilibrium whenever:

$$\begin{aligned} ER(e_p^*, a_p^*, d | d &= 0) \geq 0 \\ EU(e_p^*, a_p^*, \phi) &\geq (1-t)y \end{aligned}$$

The first condition always holds.[13] The second only holds for certain parameter configurations.[14] The condition on the parameters for a pure strategy evasion equilibrium to exist is:

$$K < t \cdot y \left[1 - f + f\left(\frac{\eta}{\eta+\lambda}\right)^2 \right] \text{, which simplifies to}$$

$$\omega \geq \frac{\lambda \cdot f(2\eta+\lambda)}{(\eta+\lambda)^2} \tag{17}$$

The question is now what the equilibrium looks like if the evasion opportunity ω is not high enough to ensure an evasion equilibrium. A natural candidate seems to be a pure non-evasion equilibrium. But in fact, pure strategy non evasion is not necessarily an equilibrium in this case. The argument goes as follows. Being honest dominates evasion if the authority exerts the best-response level of effort. It is a best response for the authority to exert no effort if it believes the taxpayer to be honest with

[12] The second order conditions are obviously satisfied.

[13] ER is equal to $fty\lambda^3/(\eta+\lambda)^2 > 0$.

[14] Note that ϕ again is the abbreviation for $y^a = y$, $d = 0$.

certainty ($\alpha = 0$). But if the authority is exerting no effort it is not a best response for the taxpayer to be honest if the fixed evasion costs are not prohibitive. Then the beliefs off the equilibrium path required for this equilibrium would not be consistent. The equilibrium would require $\mu > 0$, but since α should be equal to zero by using Bayes' rule, μ should be equal to zero as well. This is an obvious contradiction.

The condition for a pure strategy non-evasion equilibrium to exist is the trivial case where evasion is a dominated strategy. This is the case whenever the fixed evasion costs K are higher than the maximal gain from evasion ty. In terms of the evasion opportunity we have a pure strategy non-evasion equilibrium, whenever

$$\omega \leq 0. \tag{18}$$

3.3 Hybrid equilibrium

For all the cases where the evasion opportunity is too low for a pure strategy evasion equilibrium, but too high for pure strategy non evasion, we can find a hybrid equilibrium. This is an equilibrium where one type (in our case $y^a = 0$) plays a pure strategy, while the other type ($y^a = y$) randomizes. To find this equilibrium we have to find the evasion probability α that yields the same payoff in equilibrium as reporting truthfully. To do so we use the first-order conditions for optimal efforts as functions of α. Then we solve for the α that guarantees the taxpayer the honesty payoff $y(1-t)$. The first-order conditions for the efforts are:

$$\begin{aligned} \frac{\partial}{\partial a} ER(a, e, \alpha, 0) &= \mu(\alpha) \frac{e \cdot f \cdot t \cdot y}{(a+e)^2} - 1 \leq 0 \\ \frac{\partial}{\partial e} EU(e, a, \alpha, y) &= \alpha\left(\frac{f \cdot t \cdot y \cdot a}{(a+e)^2} - c\right) \leq 0 \end{aligned}$$

Solving simultaneously for the optimal effort depending on α leads to

$$a^*(\alpha) = \frac{\mu(\alpha)^2 \cdot f \cdot t \cdot y \cdot \eta}{(\eta + \mu(\alpha))^2} \tag{19}$$

$$e^*(\alpha) = \frac{\mu(\alpha) \cdot f \cdot t \cdot y \cdot \eta^2}{(\eta + \mu(\alpha))^2} \tag{20}$$

Equating the resulting expected payoff $EU\left(e^*(\alpha), a^*(\alpha), y\right)$ to the honesty payoff $(1-t)\,y$ gives the equilibrium belief of facing a tax evader the authority has to have, whenever it observes a zero income declaration:

$$\mu(\alpha^*) = \eta\left(\sqrt{\frac{f}{f-\omega}} - 1\right) \tag{21}$$

The requirement that this belief has to be consistent with behaviour leads us to the equilibrium probability of evasion that the taxpayer will use when mixing:

$$\alpha^* = \frac{\eta\,(1-\lambda)\left(\sqrt{f} - \sqrt{f-\omega}\right)}{\lambda\left[(1+\eta)\sqrt{f-\omega} - \eta\sqrt{f}\right]} \tag{22}$$

Substituting α^* back into the optimal effort function gives the equilibrium efforts for the hybrid equilibrium:

$$a_h^* = \begin{cases} 0 & \text{if} \quad d = y \\ t \cdot y \cdot \eta \left(\sqrt{f} - \sqrt{f-\omega}\right)^2 & \text{if} \quad d = 0 \end{cases} \tag{23}$$

$$e_h^* = \begin{cases} t \cdot y \cdot \eta \left(\sqrt{f(f-\omega)} - f + \omega\right) & \text{if} \quad y^a = y \wedge d(\alpha^*) = 0 \\ 0 & \text{else} \end{cases} \tag{24}$$

Insert figure 1 about here

Figure 1 shows the dependence of the equilibrium type on the income source parameters. We see that for a lower earning probability λ the concealment opportunity η for every fine level f has to be lower to deter the taxpayer from always evading. The intuition is the following: A lower earning probability reduces the expected recoverable income (including fines) for the authority. The tax authority reduces its detection effort. Knowing this the taxpayer realizes that evading with certainty pays. Note that the auditing office knows (according to its equilibrium beliefs) that the taxpayer will evade, whenever he got the income: but it just does not pay to step up the effort, because the probability of facing a honest taxpayer, who did not earn the income, is too high. Also intuitive is the result that a higher fixed evasion cost K (= lower evasion opportunity to evade ω) ceteris paribus requires a higher concealment opportunity for a taxpayer to cheat with certainty.

Less intuitive, however, is our result that the detection effort is deterministic. In models with commitment and perfect auditing it may be optimal for the authority to mix between auditing and doing nothing (e.g. Mookherjee and Png, 1989). On the first sight, this feature seems to be very appealing, because we observe in reality that similar tax declarations may trigger different auditing behaviour. However, the superiority of a random audit rule is driven by the restrictive assumptions that audits are perfect, that the authority can commit to an audit strategy, and that taxpayers are risk-averse. The authority commits beforehand to an audit probability that is just high enough to deter every single taxpayer from evasion. Perfect auditing without commitment, does not cause random audits to be optimal.

In our model, where the detection probability is determined by the efforts in a contest, not even allowing for commitment would cause the authority to mix over different detection efforts (see section 4). We do not believe that the observed randomness in auditing stems from a situation where the authority can commit to perfect audits, because that would imply that the authority knowingly audits honest taxpayers.[15] We believe that the tax authority instead conditions its audits on the belief of facing a tax evader after having received a tax declaration. If we enrich our model by assuming that the authority has limited auditing resources it might become optimal to concentrate resources randomly on single taxpayers. In our view this reason for random audits is the more plausible.

[15] In these models in equilibrium all taxpayers report truthfully.

Returning to our main purpose, to examine the effect of tax rates on tax evasion and waste, we can state the following propositions.

Proposition 1 *In the imperfect and incomplete information scenario a higher tax rate ceteris paribus weakly increases tax evasion for a specific income component if there are fixed evasion costs.*

Proof. See appendix. ■

Proposition 2 *In the imperfect and incomplete information scenario a higher tax rate ceteris paribus weakly increases the wastefully invested resources for a specific income component if there are fixed evasion costs.*

Proof. See appendix. ■

The intuition behind these results is straightforward. A higher tax rate provides stronger incentives for the taxpayer to evade by increasing the possible gain from tax evasion. This effect is strengthened by the fact that the evasion opportunity increases with the tax rate, since the ratio between possible gains and fixed evasion cost becomes more favourable. The tax authority, anticipating the stronger evasion incentives, has an incentive to exert more effort, because the potential revenue to recover rises with higher incentives for evasion. The nature of the contest forces the taxpayer to raise his concealment effort, as well, to keep track with the higher detection effort of the authority. These effects over all lead to more tax evasion, higher efforts, and consequentially to more wastefully invested resources.

Insert figure 2 about here

Figure 2 shows how the evasion probability for a certain income component depends on the tax rate (dashed line).[16] For low tax rates the income is reported because the fixed evasion cost are prohibitive ($\omega < 0$). As the tax rate rises, the taxpayer (in the hybrid equilibrium) evades with increasing probability, until it pays to evade with certainty if he earned the income component (pure evasion equilibrium). The solid line depicts the expected waste in percent of the expected earned income for the same parameter configuration.

It is also interesting to investigate the role of λ, which is the prior probability that a specific income component is earned. The influence of the earnings probability comes from its relevance for the beliefs the tax authority might have, whenever it observes a zero declaration. A very low earnings probability tells the authority that it is very unlikely to face a tax evader after a zero declaration - even if it believes that the taxpayer evades with certainty if he earns the income. Knowing this, the tax man will not exert a big detection effort. In return it is likely that for the taxpayer evasion will pay. So he evades with certainty. With an increasing earnings probability the expected recoverable income

[16]The parameter settings were $y = 1$, $\eta = 2/3$, $\lambda = .1$, $K = .2$, and $f = 3$.

for the tax authority increases. Consequentially, it increases the detection effort. This makes the taxpayer - still evading with certainty - try harder to conceal his evasion. The over-all waste increases with the earnings probability. At a certain level of the earnings probability the detection effort of the authority is becoming so massive that for the taxpayer evasion with certainty no longer pays. The equilibrium switches from pure evasion to the hybrid case. The higher the earnings probability becomes the larger is the expected revenue from detection effort for the authority. In order to reduce the tax inspector's belief that he is facing an evader the taxpayer reduces the evasion probability α. In return the authority reduces the effort. The expected waste now falls with an increasing probability that the income source generates the income.

Figure 3 illustrates the intuition above. It shows how the amount of waste depends on the earnings probability λ.[17] The two graphs correspond to two different tax rates (dashed line .4, solid line .25). On the left of the spike the taxpayer is evading with certainty (pure evasion equilibrium), while we have a hybrid equilibrium to the right where the taxpayer mixes between evasion and reporting truthfully.

Insert figure 3 about here

4 Externally enforced incentive compatibility

In this section we consider a situation, where the government externally forces the tax authority to exert as much effort as necessary to deter tax evasion with certainty. We examine the resources required under this regime and compare them to the expected waste under the discretionary audit rule without such an external commitment device.

The reason that in our model the revelation principle does not hold is - besides the restriction of the set of feasible contracts - the fact that we do not allow the authority to commit beforehand to a certain effort level if it observes a declaration of zero. Assume that the government externally enforces an incentive compatible effort level upon the authority. That means the government puts a law or a directive in place that forces the authority to exert an effort level for any zero declaration that makes sure that the taxpayer always truthfully reports his income. Here, it is necessary that this law is common knowledge. Our aim is to examine whether such an external commitment device is suitable for reducing the wastefully invested resources.

Let Ψ denote the set of all possible parameter configurations. Let $\psi \in \Psi$ be a specific parameter configuration. Such a configuration contains values for the earnings probability λ, the tax rate t, the evasion opportunity ω, the concealment opportunity η, and the potential income y.

Then the government wants the authority to exert an effort $a^*(\psi, d)$ such that the incentive con-

[17] The parameter settings are the same as for the previous figure, except K is reduced to .1.

straint (IC) for the taxpayer holds for every possible parameter configuration:

$$EU\left(d=y,y^{a}=y,a^{*}(\psi,y),e^{*}\right)\geq EU\left(d=0,y=y^{a},a^{*}(\psi,0),e\right)\quad\forall e\geq 0\ \ \forall\psi\in\Psi \tag{IC}$$

This just means that the expected payoff of the taxpayer if he earned an income component and declared it is at least as high as if he evaded it. We do not have to bother with the IC for the case the taxpayer did not earn the income component, since it is a dominant strategy to truthfully report zero, no matter what the effort of the tax authority will be. Since we are interested in the minimal a^* we already know the optimal effort, in the case that the taxpayer reports truthfully if he got the income, which has to be zero.

$$a^{*}(\psi,y)=0\ \ \forall\psi\in\Psi \tag{25}$$

The best the taxpayer can do if he is forced to report his earned income is to exert no effort ($e^*=0$). Therefore, the left hand side of (IC) reduces to $EU(y,y,0,0)=y(1-t)$. We also know from our previous analysis that the expected ex post income after evasion decreases with the authorities effort. In order to minimize a^* the authority will choose to make (IC) binding. Then the best a tax evader can do is to choose a concealment effort that maximizes his payoff, given the commitment effort of the authority. (IC) becomes:

$$y(1-t)=EU\left(0,y,a^{*}(\psi,0),e^{*}(a^{*})\right)\quad\forall\psi\in\Psi. \tag{26}$$

To solve this problem for $a^*(\psi,0)$ is straightforward. Using the envelope theorem we maximize $EU(0,y,\cdot)$ with respect to a given a, and choose a such that the equality holds. This leads to the optimal incentive-compatible detection effort for the authority, whenever it observes $d=0$:[18]

$$a^{*}(\psi,0)=\begin{cases} t\cdot y\cdot\eta(2f-2\sqrt{f(f-\omega)}-\omega & \text{if}\quad \omega>0\\ 0 & \text{else}\end{cases} \tag{27}$$

The question is whether this external commitment that deters the taxpayer from cheating is generally resource saving. That this is not the case is easily seen if we express the waste in terms of a percentage of the expected income. Then the expected waste W_c for positive evasion probability ω is given by

$$W_{c,\%}=\frac{1-\lambda}{\lambda\cdot y}[t\cdot y\cdot\eta(2f-2\sqrt{f(f-\omega)}-\omega],$$

which is the effort that is pointlessly exerted in the case of a true income declaration of 0, times the probability that the income is not earned, divided by the expected income. We see that W_c tends to infinity whenever λ approaches zero. Recall the expected waste in our non-commitment scenario, when for a small earnings probability a pure evasion equilibrium is played:

$$W_{nc,\%}=\frac{(2t\cdot f\cdot\eta\cdot\lambda)}{(\eta+\lambda)^{2}}+(1-\omega)\,t.$$

[18] Here the taxpayer is indifferent between evading or not evading. With an infinitesimal higher effort deterrence would be certain.

In this case the expected percentage waste tends to $t(1-\omega)$ when λ approaches zero. This suggests that the external commitment might be not a good solution for income sources where the earning probability is small. For high earning probabilities though this policy might reduce wasted resources. This is documented in the simulation in figure 4, where the solid (dashed) line represents the waste in the non-commitment (commitment) case.

Insert figure 4 about here

This result yields some important policy implications. The presence of a tax evasion contest causes an extra welfare loss. This loss consists of the resources that are unproductively spent on concealment and detection. The analysis above provides some guidelines how a government should organize tax enforcement activities in order to keep this loss as small as possible. The regime appropriate for income sources that are common to most citizens should be different from the regime for sources that generate income for only few people. For likely income sources such as income from dependent employment or interest payments on savings a resource saving enforcement policy has to guarantee that the costs for concealment are prohibitive. A policy that deters evasion of such income components may reduce wasted resources even if it is expensive to set up such a policy. The intuition is straightforward. The resources saved by deterring the many people that earn such income components from investing in concealment may outweigh the costs for conducting the policy. This fact may be a reason why in most countries taxes on income from dependent work and taxes on interest payments are deducted at source. This regime causes considerable costs for firms, banks, and authorities, but makes evasion almost impossible.

The enforcement of taxes paid on income from unlikely sources should consist of audits conducted by an authority with certain discretionary powers. In this case a regime that eliminates all evasion incentives may cost more than it saves concealment costs by deterring the few people that earn such income components from entering a contest.

5 Alternative tax and fine systems

In this section we explore the robustness of the finding that a higher tax rate leads to more evasion. First, we shortly comment on a non-linear fine scheme. Then we extend our model in a way that allows us to deal with non-linear tax systems.

5.1 Alternative fine schemes

So far we assumed that the fine payable for detected evasion is linearly dependent on the tax evaded. However, many real world fine schemes are not linear. A system where the evasion of large amounts leads to imprisonment may be an example where the fine increases more than linearly with the tax evaded. On the other hand, a fine that is calculated on the basis of concealed income instead of evaded taxes is totally independent of the tax rate. We are not aware of any empirically relevant system where the fine depends negatively on evaded taxes. We therefore exclude this case.

If we replace the specific fine (fty) by F and solve for the optimal pure-evasion efforts we get

$$a_p^* = \frac{\lambda^2 \cdot \eta \cdot F}{(\eta+\lambda)^2} \tag{28}$$

$$e_p^* = \frac{\lambda \cdot \eta^2 \cdot F}{(\eta+\lambda)^2} \tag{29}$$

instead of (23) and (24). So the pure evasion efforts only depend on the tax rate if the fines also depend on the tax rate. This is intuitively obvious as the prize of the detection-concealment contest is just the fine. We now investigate, how tax rate changes influence evasion behaviour if the tax scheme is not linear, by looking at the maximum moral cost an evading taxpayer can have.

The maximum moral cost $\hat{K}$ can be found by looking at a taxpayer which is indifferent between evading and reporting truthfully. Solving the indifference condition $EU(e_p^*, a_p^*, y^a = y, d = 0) = (1-t)y$ for $\hat{K}$ gives:

$$\hat{K} = ty - F\left[1 - \frac{1}{(1+\lambda/\eta)^2}\right]. \tag{30}$$

The cutoff for the moral cost $\hat{K}$ increases with the tax rate whenever

$$\frac{(\eta+\lambda)^2}{\lambda(\lambda+2\eta)} > \frac{dF}{dt}/y. \tag{31}$$

As the *lhs* is greater than one we can say that by increasing the tax rate a taxpayer that was previously indifferent between evasion and truthful reporting will only report truthfully if the tax hike increases the fine-income ratio by more than factor one. Note that for a fine that depends on the tax evaded a rising tax rate can only cause a former evader to (at least sometimes) report truthfully if the fine increases more than proportionally with the tax evaded. If the fine does not depend on the tax evaded the opposite is true. Then some formerly at least sometimes truthfully reporting taxpayers will evade with certainty after a tax rise. If we assume that taxpayers do not get a psychological gratification from evasion (negative moral cost) we have an upper bound for the fine $F_{\max}$, which allows for at least some evasion. This maximum fine can be found by solving equation (30) for F and setting $\hat{K}$ equal to zero. Rearranging gives

$$\frac{F_{\max}}{yt} = \frac{(\eta+\lambda)^2}{\lambda(\lambda+2\eta)}. \tag{32}$$

Note that in a fine system where the fine is linear in the tax evaded $F/(yt) = F'/y = f$ holds. This shows that in such a tax system the maximum fine condition implies the condition for more evasion (31). Put differently, in a system with linear fines higher taxes lead to more evasion if the fines are not prohibitive for all taxpayers

For a systems that does not compensate for a rising tax rate by also increasing the fine considerably higher tax rates lead to more taxpayers evading with certainty. A corollary is that if the fine does not rise considerably with the tax rate (inequality 31 holds) then more resources are wasted due to more evasion.[19]

We now turn to the taxpayers with moral cost which are too high for evading with certainty. The efforts in the hybrid equilibrium for a more general fine can be found by simply replacing (fty) by F in equations (19) and (20)

$$a_h^*(\alpha) = \frac{\mu(\alpha)^2 \cdot \eta \cdot F}{(\eta + \mu(\alpha))^2} \quad (33)$$

$$e_h^*(\alpha) = \frac{\mu(\alpha) \cdot \eta^2 \cdot F}{(\eta + \mu(\alpha))^2} \quad (34)$$

As in the section with specific fines we need to determine the equilibrium beliefs of the authority that are consistent with the taxpayer being indifferent between evasion and truthfully reporting. Using (19) and (20) with the replaced fine we can solve the indifference condition $EU\left(e^*(\alpha), a^*(\alpha), y\right) = (1-t)y$ for the equilibrium belief $\mu^*(\alpha)$.[20]:

$$\mu^*\left(\alpha\right) = \eta\left[\frac{F}{\sqrt{F(F+K-ty)}} - 1\right].$$

As $\mu^*(\alpha)$ is derived from α using Bayes' rule we basically could now solve for the evasion probability α^*. However, this is not really necessary as ceteris paribus an increase in μ implies that α must have increased.[21] So for a given taxpayer with moral cost such that partial evasion is optimal, the evasion probability increases with the tax rate if $d\mu^*/dt > 0$. Suppose F potentially depends on the tax rate. Then we get the following condition for more tax evasion

$$\frac{\eta F\left(yF - (ty - K)\frac{dF}{dt}\right)}{2\left(F(k - ty + F)\right)^{3/2}} > 0$$

which simplifies to

$$\frac{F}{ty - K} > \frac{dF}{dt}/y.$$

It is interesting that the rise in the fine necessary to prevent a taxpayer from evading more often as a response to a tax rise increases with the moral cost of the taxpayer. Morally constrained tax payers are reacting more strongly to higher taxes.

To find a sufficient condition for a higher tax rate to lead to more evasion for all taxpayers, we can use a lower bound for F. The lowest F for the hybrid equilibrium is the F that makes taxpayers indifferent between evading with certainty and playing a hybrid equilibrium. This is just the F that solves (30). Substituting into the condition above gives

$$\frac{(\eta+\lambda)^2}{\lambda(\lambda+2\eta)} > \frac{dF}{dt}/y,$$

[19] In the case where the fines do not depend on the tax rate the waste per evader stays the same. However, more people will evade with certainty.

[20] Recall that $\mu(\alpha)$ is the probability the tax authority assigns to the event that an observed declaration of zero comes from an evader which evades with probability α in case she earned the income.

[21] This has to be the case as $d\mu/d\alpha > 0$, which implies that $\mu(\alpha)$ is invertible and $d\alpha/d\mu > 0$.

which identical to the condition we get for pure evasion. We summarize this finding in the following proposition.

Proposition 3 *Condition (31) on the fine system and parameters is sufficient for a tax rise leading to more tax evasion.*

It now remains to check if the wasted resources are increasing with higher tax rates in the hybrid equilibrium. It is sufficient to check whether the wasted resources increase with the equilibrium belief μ^*. If this is the case then condition (31) is sufficient for higher tax rates increasing the waste of resources. The expected wasted resources in the contest for the hybrid equilibrium are given by

$$EW := \lambda\alpha(\mu^*)\left(e_h^* + K\right) + \left(\lambda\alpha(\mu^*) + 1 - \lambda\right)a_h^*.$$

Note that we can write the equilibrium evasion probability as a function of the beliefs. Using the equilibrium values for the efforts from equations (19) and (20) and differentiating leads to

$$\frac{dEW}{d\mu^*} = \frac{2F\mu\eta^2\left(1-\lambda\right) + \lambda\left(\eta+\mu\right)\left(F\mu\eta^2\alpha(\mu^*) + \left(\eta+\mu\right)\left(F\mu\eta + K\left(\eta+\mu\right)\right)\alpha'(\mu^*)\right)}{\left(\eta+\mu\right)^3} > 0.$$

So increasing equilibrium beliefs that an evasion has occurred whenever the tax authority observes a zero declaration are sufficient for more waste. As higher tax rate increase the evasion probability and therefore the believed evasion probability if condition (31) holds, we have the same sufficient condition for higher tax rates leading to more waste as we obtained for more tax evasion. Adding the insight we obtained for taxpayers, who always evade, leads to the following proposition.

Proposition 4 *Condition (31) on the fine system and parameters is sufficient for a tax rise leading to more wasted resources.*

We want to emphasize two points: Firstly, the condition we state is only a sufficient condition, which means that we may have more wasted resources due to higher tax rates even if the condition does not hold. Secondly, the condition tells us that waste increases with the tax rate for every taxpayer with $K < ty$. So aggregate waste could still increase if the condition is violated as taxpayers with high K may still evade more often if tax rates increase.

5.2 Non-linear tax systems

Suppose we have an income source which may yield the same amount of income more than once over the course of a financial year. Think of the rent from an investment property. Depending on the time occupied the income received will be a multiple of the weekly rent. The typical information the tax authority will have is that the taxpayer owns the property and the weekly rent paid for such a property in the market. However, what the authority does not know is the time the property is actually occupied.

The incentives to evade taxes on rent income are very similar to those in previous sections if we keep the assumption of a linear tax system, where fines are proportional to evaded taxes. However, if the tax system is non-linear then the average tax and fine rate changes with every additional week worth of rent declared to the tax authority, which influences the incentives to evade and to conceal for the taxpayer.

We use a simplified framework of an income source which can generate the same amount of income (normalized to 1) once, twice, or does not generate any income at all. This will give rise to a framework rich enough to explore the effects different tax systems have on evasion frequencies and resources wasted in the contest. The earnings probabilities, which are common knowledge, are given by:[22]

$$y = \begin{cases} 2 & & \lambda_2 \\ 1 & \text{with probability} & \lambda_1 \\ 0 & & 1 - \lambda_1 - \lambda_2. \end{cases} \tag{35}$$

For simplicity we exclude partial evasion. A taxpayer who earned the income twice cannot mimic a truthful taxpayer who earned the income once. This restriction greatly simplifies the analysis while the main incentives are still present.[23] Denote the taxpayer's mixed strategy probabilities of evasion as α_2 if the true income is 2 and as α_1 if the true income is 1. A strategy for the taxpayer consists of these probabilities and a concealment effort for every possible combination of earnings and declarations.[24] Denote the concealment effort the taxpayer exerts if he earned an income of θ and declared d as $e_d(\theta)$. So $e_0(1)$ denotes the effort a taxpayer with $y^a = 1$ exerts if he declares an income of 0. Then a strategy of a taxpayer - excluding the trivial case when no income is earned - is given by the vector $(\alpha_1, \alpha_2, e_0(1), e_0(2), e_1(1), e_2(2))$. A strategy for the authority is given by a detection effort for every possible declaration (a_0, a_1, a_2).

In order to be able to investigate the influence of progressive and regressive tax systems we allow the average tax rates to vary with income. The legal average tax rate for income levels of 1 and 2 are denoted by t_1 and t. We restrict our analysis to tax systems where the tax liability increases with income, i.e. $t_2 > t_1/2$.[25] We return to a fine which is proportional to the evaded income. The fine factor is given by f once again. Without loss of generality we normalize the marginal concealment and detection costs to 1. The moral costs a taxpayer has to bear if he evades (indicated by $\phi = 1$) are denoted by K. Then the interim expected payoff for the taxpayer is given by

[22] This earnings structure is very general and allows for correlation among income parts.

[23] If we allow for partial evasion we have to deal with multiple equilibria depending on off-equilibrium beliefs of the tax authority.

[24] The concavity of the taxpayer's objective function in efforts for every given income and declaration ensures that we don't have to take mixing among effort levels into account.

[25] Note that this does not rule out regressive tax systems (with decreasing average tax rate $t_2 < t_1$).

$$EU(y, d, e_d(y), a_d) = y - t_d d - p(a_d, e_d(y))\,[y - t_d d]\, f - e_d(y) - \phi K, \tag{36}$$

where $p(a_d, e_d(y))$ is the detection probably depending on concealment and detection effort.

The interim expected payoff of the authority for an observed declaration d can be written as

$$ER(d, e_d(\theta), a_d) = \begin{cases} f \sum_{\theta=1}^{2} \mu_\theta \cdot p(a_0, e_0(\theta))\theta - a_0 & if \quad d = 0 \\ t_d d - a_d & if \quad d > 0 \end{cases} \tag{37}$$

The expected payoff for the authority after observing a zero declaration is given by the fine it expects to collect net of detection costs. The expected fine depends on the beliefs μ_θ that the declaration of 0 comes from a taxpayer who actually earned an income of θ. If the tax authority observes a declaration $d > 0$ then there cannot be any evasion. So no fines will ever be collected.

We now determine the optimal concealment and detection efforts given a declaration and the authority's beliefs. It is obvious that a taxpayer who reports truthfully should never exert any effort:

$$e^*_\theta(\theta) = 0 \; \forall \theta \in \{0, 1, 2\}.$$

Equally trivial are the optimal detection efforts for positive declarations. As the authority cannot expect any fines to be collected it should not exert any effort in these cases:

$$a^*_d = 0 \; \forall d \in \{1, 2\}.$$

The non trivial case of efforts for zero declarations requires the taxpayer to maximize the expected payoff given the income, while the tax authority maximizes the expected revenue given the zero declaration.

Now we determine the efforts for the remaining cases. The taxpayer chooses an effort given his actual income and declaration decision maximizing expected after tax income (36), while the authority chooses its detection effort which maximizes expected revenue (37) depending on beliefs and the observed declaration. The first-order conditions for the authority depending on observing $d = 0$ is:

$$\frac{\partial ER}{\partial a_0} = 2f \cdot t_2 \frac{\mu_2 \cdot e_0(2)}{(a_0 + e_0(2))^2} + f \cdot t_1 \frac{\mu_1 \cdot e_0(1)}{(a_0 + e_0(1))^2} - 1 = 0$$

For the taxpayer the first-order conditions depending on actual income and under-reporting are:[26]

$$\begin{aligned} \frac{\partial EU}{e_0(1)} &= \frac{a_0 \cdot f \cdot t_1}{(a_0 + e_0(1))^2} - 1 = 0 \\ \frac{\partial EU}{e_0(2)} &= \frac{a_0 \cdot f \cdot 2t_2}{(a_0 + e_0(2))^2} - 1 = 0 \end{aligned}$$

[26] Global concavity of ER and EU guaranties that the second-order conditions are satisfies.

Solving the three dependent first-order conditions above gives the following optimal efforts:

$$e_0^*(2) = f\rho\left(\sqrt{2t_2} - \rho\right) \tag{38}$$

$$e_0^*(1) = f\rho\left(\sqrt{t_1} - \rho\right) \tag{39}$$

$$a_0^* = f\rho^2, \tag{40}$$

where

$$\rho = \frac{\mu_1\sqrt{t_1} + \mu_2\sqrt{2t_2}}{1 + \mu_1 + \mu_2}. \tag{41}$$

In equilibrium the beliefs μ_1 and μ_2 have to be consistent with behaviour. Denoting the evasion probabilities if an income of 2 or 1 is earned as α_2 and α_1, we have the following equilibrium beliefs

$$\mu_2^* = \frac{\lambda_2\alpha_2}{1 - \lambda_2(1-\alpha_2) - \lambda_1(1-\alpha_1)} \tag{42}$$

$$\mu_1^* = \frac{\lambda_1\alpha_1}{1 - \lambda_2(1-\alpha_2) - \lambda_1(1-\alpha_1)}. \tag{43}$$

5.3 Pure strategy evasion equilibrium

In this section we will explore the conditions under which pure strategy equilibria are feasible. There is an obvious pure strategy equilibrium, where neither evasion nor concealment and detection takes place. This is the case whenever the taxpayer has a moral cost that is so high that it outweighs the potential monetary benefit from evasion. For $K > 2t_2$ evasion never occurs as the maximum monetary benefit from evasion ($2t_2$) is always smaller than the moral cost. The more interesting case is a pure strategy evasion equilibrium, where the taxpayer evades with certainty if a positive income is earned. Evasion with certainty has to be better than compliance for both income levels. We can write the constraints to be satisfied as

$$EU^*(\theta = 1, d = 0) \geq EU(\theta = 1, d = 1) \tag{C1}$$

$$EU^*(\theta = 2, d = 0) \geq EU(\theta = 2, d = 2). \tag{C2}$$

In such an equilibrium the evasion probabilities conditioned on the income (α_1 and α_2) are both one. As the beliefs have to be consistent with the taxpayer's strategy, we can conclude that in a pure strategy evasion equilibrium $\mu_1 = \lambda_1$ and $\mu_2 = \lambda_2$ are true. We denote $\tilde{\rho}$ as ρ from (41) with the beliefs μ_1 and μ_2 replaced by the pure strategy evasion equilibrium beliefs λ_1 and λ_2. Then we can calculate the payoffs and rewrite the two constraints:

$$t_1 + f\tilde{\rho}\left(\tilde{\rho} - 2\sqrt{t_1}\right) - K \geq 0 \tag{C1'}$$

$$2t_2 + f\tilde{\rho}\left(\tilde{\rho} - 2\sqrt{2t_2}\right) - K \geq 0. \tag{C2'}$$

Examining the conditions above shows that under a non-linear tax system an increase of the tax rates t_1 or t_2 does not necessarily provide stronger incentives for full evasion to taxpayers. If we look at the condition for a taxpayer with $y = 2$ to declare 0 (C2'), for example, we see that increasing t_1 does not effect the gain from evasion ($2t_2$) but increases the expected losses. The reason for the latter is that the expected gain for the tax authority increases with t_1. An increased t_1 increases the potential fine from a pure evader with $y = 1$. So the authority increases the detection effort if a declaration of 0 is received as it could potentially come from a taxpayer with $y = 1$. This negatively effects a taxpayer with $y = 2$ who plans to declare 0. The same effect makes it less favourable for a taxpayer who earned $y = 1$ to evade if t_2 increases (see C1'). Moreover, we can show that only the condition for a low-income type (C1') is relevant if we restrict the analysis to tax systems where the tax liability increases with income ($2t_2 > t_1$).

Proposition 5 *For tax systems where the liability does not decrease with income* ($2t_2 \geq t_1$) *a pure strategy evasion equilibrium exists if* $t_1 + f\tilde{\rho}\left(\tilde{\rho} - 2\sqrt{t_1}\right) - K \geq 0$.

Proof. We have to show that only C1' is binding if the additional condition $2t_2 \geq t_1$ is imposed. The proof is in two steps:

First we show that if C1' holds for a tax-rate combination (t_1, t_2) with $t_1 = 2t_2$ then C2' holds for all tax rate combinations $(\hat{t}_1, t_2)$ with $\hat{t}_1 < 2t_2$. This guarantees that C2' can be replaced by $2t_2 \geq t_1$ for all t_1 where C2' holds at (t_1, t_2) with $t_1 = 2t_2$. If C1' holds for a tax rate combination with $2t_2 \geq t_1$ then C2' obviously holds, as C2' and C1' are identical for $2t_2 \geq t_1$. So C2' is slack if C2' relaxes when t_1 decreases. Differentiating *lhs* of C2' with respect to t_1 gives

$$\frac{\lambda_1 f\left(\lambda_1\sqrt{t_1} - (1+\lambda_1)\sqrt{2t_2}\right)}{(1+\lambda_1+\lambda_2)^2\sqrt{t_1}} < 0 \text{ for } 2t_2 \geq t_1.$$

Secondly, we need to show that condition C1' is not contained in the condition $2t_2 \geq t_1$. C1' cannot be contained in $2t_2 \geq t_1$ if C1' gets stricter if t_2 increases. Differentiating *lhs* of C1' with respect to t_2 gives

$$2f(\tilde{\rho} - \sqrt{t_1})\frac{\partial\tilde{\rho}}{\partial t_2} < 0,$$

as $\partial\tilde{\rho}/\partial t_2 > 0$ and $\tilde{\rho} - \sqrt{t_1} < 0$ for an interior solution. This concludes the proof. ■

The fact that we can solely concentrate on C1' and the monotonicity of the tax system combined with the fact that C1' gets stricter with increasing t_2, but less strict with increasing t_1, provides us with unique minimum tax rates for a pure strategy equilibrium. The minimum tax rates are such that C1' and $2t_2 \geq t_1$ both hold with equality. Solving the two equations gives the following corollary.

Corollary 1 *In every pure strategy evasion equilibrium we must have* $t_1 \geq t_{\min}$ *and* $t_2 \geq t_{\min}/2$, *where*

$$t_{\min} = \frac{K\left(1+\lambda_1+\lambda_2\right)^2}{1+(\lambda_1+\lambda_2)(2+\lambda_1+\lambda_2)(1-f)}.$$

We can now check whether higher tax rates lead to more wasted resources if tax rates rise in a pure strategy evasion equilibrium. The expected pure-strategy evasion-equilibrium waste is given by

$$\begin{aligned} EW_p^* &= (\lambda_1 + \lambda_2) K + \lambda_1 e_0^*(1) + \lambda_2 e_0^*(2) + a_0^* \\ &= (\lambda_1 + \lambda_2) K + 2f\tilde{\rho}^2 \end{aligned}$$

We immediately see that the waste increases with the tax rates t_1 and t_2 as $\tilde{\rho}$ increases with the tax rates.

Proposition 6 *In a pure evasion equilibrium raising either t_1 or t_2 increases the wasted resources if the increased tax rates still lead to a pure evasion equilibrium.*

We have a pure non-evasion equilibrium if the moral evasion cost is prohibitive relative to possible evasion gains:

$$\alpha_1^* = \alpha_2^* = 0 \text{ iff } K > \max[t_1, t_2/2].$$

5.4 Partial evasion equilibria

In this section we shortly describe the outcome if moral constraints are too strong for a pure strategy evasion equilibrium, but not strong enough to totally prevent taxpayers from evading. First we establish the result that no equilibrium exists where both high and low income taxpayers evade with a positive probability smaller than 1.

Lemma 2 *There is no equilibrium with $\alpha_1^* \times \alpha_2^* \in (0,1) \times (0,1)$ and $2t_2 > t_1$.*

Proof. In an equilibrium where both types mix between evasion and truthful declaration the following indifference conditions must hold:

$$\begin{aligned} EU^*(\theta &= 1, d = 0, \alpha_1^*, \alpha_2^*) = EU(\theta = 1, d = 1) \\ EU^*(\theta &= 2, d = 0, \alpha_1^*, \alpha_2^*) = EU(\theta = 2, d = 2). \end{aligned}$$

This is equivalent to

$$\begin{aligned} t_1 + f\rho\left(\rho - 2\sqrt{t_1}\right) - K &= 0 \\ 2t_2 + f\rho\left(\rho - 2\sqrt{2t_2}\right) - K &= 0, \end{aligned}$$

where ρ depends on α_1^*, α_2^* through the equilibrium beliefs. Inspection shows that there is no ρ that jointly satisfies these two conditions. The only case where both equations can be satisfied is if $2t_2 = t_1$, where the two conditions are identical. ■

We now establish the result that the equilibrium evasion probability for a taxpayer who earned an income of i weakly increases with t_i. For this purpose it is sufficient to show that the evasion constraint for an i-income taxpayer relaxes, while the constraint for a j-income taxpayer becomes stricter with increasing t_i. If this is true we can also conclude that the evasion probability for an i-income taxpayer weakly decreases with t_j.

Proposition 7 *In equilibrium we have*

$$\begin{aligned} \frac{\Delta \alpha_i^*}{\Delta t_i} &\geq 0 \ \forall t_i, t_j, i = 1, 2 \\ \frac{\Delta \alpha_i^*}{\Delta t_j} &\leq 0 \ \forall t_i, t_j, i = 1, 2. \end{aligned}$$

Proof. See appendix. ■

The result above shows the main incentives for the taxpayers. In equilibrium a taxpayer will (weakly) increase evasion if the average tax rate for his particular income is raised. Evasion becomes more profitable. This is the same force driving the results as for an isolated one-off income source discussed in previous sections. However, there is a difference. In the richer model where we allow for a multiple income source the tax rate for a particular income creates a negative externality on the taxpayers with other income levels through the auditing efforts of the tax authority. The authority can not distinguish if a zero tax return comes from a truthfully reporting taxpayer with income 0 or from a tax cheat with income 1 or 2. Therefore its auditing effort will reflect something like an average of the fines it can potentially collect from the different types of taxpayers declaring zero. So raising the tax rate for a certain income level increases the average fine the authority expects to collect. Consequently, the tax authority steps up its effort, which makes evasion less attractive to the taxpayers with incomes for which the tax rate has not changed.

The observation that a raised individual tax rate increases the auditing effort of the authority and the concealment effort of the taxpayer affected, suggests that the resources wasted in the contest increase with the tax rate, too. This is not necessarily true as there is the countervailing effect of less evasion from taxpayers with different income. Below we plot the results of a typical simulation. We used the following parameter values: $\lambda_1 = .3$, $\lambda_2 = .4$, $K = .1$, and $f = 1.25$.

Insert figure 5 about here

In figure 5 we plot the equilibrium evasion probabilities depending on tax rates. The left panel shows the evasion probability for a high-income taxpayer. The evasion probability for a low-income earner is depicted in the right panel. The analytical results are confirmed. The evasion probability weakly increases with the tax rate for the own income level and weakly deceases with the tax rate for the other income level.[27] More interestingly, figure 6 plots the expected waste depending on the tax rates.

[27]Note that the empty area at the lower right of both graphs, relates to tax systems we have excluded ($2t_2 < t_1$).

Insert figure 6 about here

Both panels of figure 6 show the expected waste depending on the tax rates. The right panel gives a contourplot where lighter shading corresponds to higher waste. We can see that higher tax rates for low income earners in general weakly increase the expected waste from the contest. Increasing the high-income tax rate usually also increases the waste. The only occasion where this is not true is when the high income taxpayer evades with certainty already, while the low-income taxpayer evades with a probability between 0 and 1. There an increase of t_2 reduces the wasted resources as it reduces the evasion probability of low income earner. Over-all we can see that higher tax rates have the tendency to increase the waste of resources due to concealment and detection. However, there are situations where this is not necessarily true. On the other hand, a population of taxpayers, which are heterogeneous with respect to their moral cost K, makes it very likely that an increased tax rate will increase aggregate waste. Then the reduced evasion of a few taxpayers in the region where the waste decreases will be compensated for with all the other taxpayers who are in a region where a higher tax rate increases the waste. The actual change of waste, although likely to be positive, depends on the distribution of K in the population.

6 Conclusion

Our main interest in this paper was to examine the impact of tax rates on tax evasion and the resources spent on concealment and detection activities. Our finding is that higher tax rates lead to more tax evasion. This rather intuitive result does not lead per se to any policy implications based on welfare considerations. We do not want to enter the discussion about the relationship between welfare and tax evasion, since the widely used standard measures for welfare do not appear to be sufficient to make a sensible judgement if we look at tax evasion. The welfare effect of more or less tax evasion measured by some form of social welfare criterion is highly sensitive to the criterion used, to assumptions about the state the economy is in (distortions, public good provision etc.), and to assumptions about individual preferences.[28] Thus it seems to be reasonable to base judgements about the desirability of tax evasion on broader foundations than traditional welfare economics does. What we can say is that if one considers tax evasion as undesirable - which we implicitly do - then lower tax rates might be a good policy measure to reduce it.

More clear-cut are the consequences of our result that higher income tax rates usually imply more wasteful investment in income concealment and detection. Higher tax rates lead - beside a higher excess burden - to some extra cost. More scarce resources are unproductively absorbed by the contest between taxpayer and tax authority.

[28] A detailed discussion can be found in Cowell (1990), chapter 7.

Furthermore, our model provides an additional insight into the effectiveness and desirability of measures to prevent tax evasion. We saw that an external commitment device, such as law or governmental directives, which forces the tax authority to make sure that no tax evasion takes place, might not be desirable for income sources which rarely generate income. This is due to the fact that the detection resources needed to induce truthful revelation then become excessive compared to the small revenue collected.

Although we have not formally modelled it, it is straightforward that reducing opportunities for evasion and concealment is a sensible strategy for reducing tax evasion and waste. There are numerous real world examples of governments trying to reduce these opportunities. Taxation at source reduces evasion opportunities, while the banks' duty to report high pay-ins in cash reduces concealment opportunities. We see in our model that lower concealment opportunities are more effective in reducing the waste, while small evasion opportunities control the extend of evasion more effectively.

Finally, note that all the influence factors on tax evasion known from economic psychology (subsumed under attitudes towards tax system, government and authority) play a role in our model. Such attitudes may be the main influence on the moral cost of evasion (contained in the fixed evasion cost). Dissatisfaction reduces the scruples (hence the moral cost) of evasion. So, beside the technical means, a tax system that is conceived as fair, efficient expenditure policy, and a good government performance may effectively deter tax evasion, as well as reduced opportunities or low tax rates.

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A Proofs of some propositions

This appendix contains proofs for propositions in the main text of the paper.

A.1 Proposition 1

Proof. It is sufficient to show that the probability of cheating α^* in the hybrid equilibrium increases with t and that some income sources that previously were reported with positive probability are evaded with a higher probability as t rises. Note: This also ensures that an income that was previously evaded with certainty will be evaded with certainty after the tax rise. Furthermore, an income source that has been evaded with positive probability will never be declared with certainty, since the condition for the pure non evasion equilibrium is $\omega(t) \leq 0$ and $\omega(t') > \omega(t)$ for $t' > t$.

To show that α^* is rising with t we use equation 22. Since $d\alpha^*/dt = d\alpha^*/d\omega \cdot dw/dt$ and $dw/dt = K/t^2y > 0 \;\forall\; K > 0$ it is sufficient to show that $d\alpha^*/d\omega > 0$ as well. Differentiation leads to:

$$d\alpha^*/d\omega = \frac{(1-\lambda)\eta\sqrt{f}}{2\lambda\sqrt{f-\omega}\left(\eta\sqrt{f}-(1+\eta)\sqrt{f-\omega}\right)^2} > 0$$

Since the fine parameter $f > 1$ and the evasion opportunity $\omega \leq 1$ the derivative is necessarily real. Since $0 < \lambda < 1$, the derivative is positive.

The condition for a taxpayer just to play a hybrid equilibrium was $\omega(t_0) = \lambda \cdot f(2\eta+\lambda)/(\eta+\lambda)^2 - \varepsilon$. A change of the tax rate does not effect the right hand side of this equation. The change on the left hand side is $d\omega/dt = K/ty^2$. It follows $\omega(t') > \lambda \cdot f(1+2\eta)/(1+\eta)^2 - \varepsilon$ if $t' > t_0$ and $K > 0$. This means that for some income sources taxpayers change from the hybrid equilibrium to the pure evasion equilibrium as the tax rate rises. ■

A.2 Proposition 2

Proof. The proof consists of three steps: Firstly (a), we show that an increase in the tax rate increases the waste for parameter configurations that lead to a hybrid equilibrium. Then we do the same for the pure strategy equilibrium (b). To conclude the proof it will be sufficient to show that the waste function is continuous at the point where the hybrid equilibrium becomes a pure evasion equilibrium (c). Note: For an income component where before and after the tax rise a pure non-evasion equilibrium was played ($\omega(t)$, $\omega(t') \leq 0$) the waste remains zero.

(a) We can write the expected waste in the hybrid equilibrium as $W = a^*(\lambda\alpha^* + 1 - \lambda) + (ce^* + K)\lambda\alpha^*$, where α^*, a^* and e^* depend on t.[29] Differentiation with respect to t leads to

$$\frac{dW}{dt} = \frac{da^*}{dt}(\lambda\alpha^* + 1 - \lambda) + \frac{d\alpha^*}{dt}a^* + \lambda\frac{d\alpha^*}{dt}(ce^* + K) + \lambda c\frac{de^*}{dt}\alpha^*.$$

If $d\alpha^*/dt$, da^*/dt, and de^*/dt are positive then dW/dt is positive, too. In the previous proof we showed that $d\alpha^*/dt > 0$ for $K > 0$. Taking the detection effort from equation 23 and differentiating with respect to t leads to:

$$\frac{da^*}{dt} = \eta y\left[(2f - 1 - \frac{2\sqrt{f(f-1+\frac{K}{ty})}(K + 2(f-1)ty)}{K + (f-1)ty}\right]$$

Since we cannot determine the sign globally, we have to look on the values for t that are relevant for the hybrid equilibrium. The lower bound condition is $t > K/y$. Since

$$\frac{da^*}{dt}\Big|_{t=K/y} = \eta y\left(2f - 1 - \frac{K + 2K(f-1)}{K}\right) = 0,$$

[29] e^* and a^* are to be understood as the equilibrium efforts in the case that the income is earned and evaded. To simplify the notation we drop the subscript h.

$da^*/dt > 0\ \forall t > K/y$ if a^* is convex for $t \geq K/y$. And indeed, a^* is globally convex in t - i.e.

$$\frac{d^2a^*}{dt^2} = \frac{K^2\eta\sqrt{f}}{2t^2\sqrt{(f-1+\frac{K}{ty})(K+(f-1)ty)}} > 0 \text{ for } K > 0,$$

since $f > 1$ by construction.

To finish part (a) we just have to make sure that $de^*/dt > 0$. Here, it is convenient to use equation 24, and again to express ω in terms of t. By differentiation we obtain:

$$\frac{de^*}{dt} = \frac{\eta y}{2}\left[(2-2f+\frac{\sqrt{f}(K+2(f-1)ty)}{ty\sqrt{f-1+\frac{K}{ty}}}\right]$$

some manipulation and replacing K/ty by the equivalent $1-\omega$ leads to:

$$\frac{de^*}{dt} = \frac{\eta y}{2}\left[-(2f-2)+(2f-2+1-\omega)\frac{\sqrt{f}}{\sqrt{(f-\omega)}}\right].$$

Since $2f-2+1-\omega > 2f-2$ and $\sqrt{f}/\sqrt{(f-\omega)} > 1$ for $\omega \in (0,1)$ the term in brackets, and also $de^*/dt > 0$ for the relevant range of the evasion opportunity ω.

(b) The expected waste in the pure evasion equilibrium is $W = a_p^* + \lambda e_p^*/\eta + \lambda K = (2tf\eta y\lambda^2)/(\eta+\lambda)^2 + \lambda K$ (from equation 15 and 16), which obviously increases with t.

(c) Suppose there exists (obtained from equation 17) a

$$\hat{t} = \frac{K}{y}\frac{(\eta+\lambda)^2}{\eta^2-2(f-1)\eta\lambda-(f-1)\lambda^2} > \frac{K}{y},$$

which is the maximum t for that a hybrid evasion equilibrium is obtained. If it does not exist then the parameters do not allow for a hybrid equilibrium, and we do not have to check for continuity. To show that the waste function is continuous at $\hat{t}$, where the hybrid equilibrium becomes a pure evasion equilibrium, we have to show that

$$W_p(\hat{t}) = \lim_{t->\hat{t}^+} W_h(t).$$

This is equivalent to

$$a_p^*(\hat{t}) + \frac{\lambda}{\eta}e_p^*(\hat{t}) = (\lambda \cdot \lim_{t->\hat{t}^+} \alpha^*(t) + 1 - \lambda) \cdot \lim_{t->\hat{t}^+} a_h^*(t) + \frac{\lambda}{\eta} \cdot \lim_{t->\hat{t}^+} \alpha^*(t) \cdot \lim_{t->\hat{t}^+} e_h^*(t).$$

The condition above is obviously fulfilled if

$$\lim_{t->\hat{t}^+} \alpha^*(t) = 1,\ \lim_{t->\hat{t}^+} a_h^*(t) = a_p^*(\hat{t}),\ \text{and} \lim_{t->\hat{t}^+} e_h^*(t) = e_p^*(\hat{t}).$$

Using the definition of α^* from equation 22, replacing ω by $1-K/ty$, and taking the right-hand limit at $\hat{t}$ leads to

$$\lim_{t->\hat{t}^+} \alpha(t) = \frac{\eta(1-\lambda)\left(\sqrt{f}-\frac{\sqrt{f}\eta}{\eta+\lambda}\right)}{\lambda\left[(1+\eta)\frac{\sqrt{f}\eta}{\eta+\lambda}-\eta\sqrt{f}\right]} = 1.$$

Using the definitions of the pure evasion equilibrium efforts from equations 15 and 16, expressing c as $1/\eta$, and substituting $\hat{t}$ gives:

$$\begin{aligned} a_p^*(\hat{t}) &= \frac{f \cdot \eta \cdot \lambda^2 \cdot K}{\eta^2 + 2 \cdot \eta \cdot \lambda(f-1) + \lambda^2(f-1)} \\ e_p^*(\hat{t}) &= \frac{f \cdot \eta^2 \cdot \lambda \cdot K}{\eta^2 + 2 \cdot \eta \cdot \lambda(f-1) + \lambda^2(f-1)}. \end{aligned}$$

Taking the limits of $a_h^*(t)$ and $e_h^*(t)$ at $\hat{t}$ in equations 23 and 24 (knowing that $\lim_{t->\hat{t}^+} \mu = \lambda$) yields

$$\begin{aligned} \lim_{t->\hat{t}^+} a_h^*(t) &= \frac{f \cdot \hat{t} \cdot y \cdot \eta^2 \cdot \lambda}{(\eta+\lambda)^2} = a_p^*(\hat{t}) \\ \lim_{t->\hat{t}^+} e_h^*(t) &= \frac{f \cdot \hat{t} \cdot y \cdot \eta \cdot \lambda^2}{(\eta+\lambda)^2} = e_p^*(\hat{t}), \end{aligned}$$

if $\hat{t}$ from equation 17 is plugged in. This concludes the proof. ■

A.3 Proposition 7

Proof. We have to show that the evasion conditions (C1, C2) for i-income taxpayers relax with increasing t_i and become stricter with increasing t_j. The following has to be true:

$$\frac{\partial}{\partial t_2} EU(\theta = 1, d = 0) - \frac{\partial}{\partial t_2} EU(\theta = 1, d = 1) \leq 0 \tag{1.t_2}$$

$$\frac{\partial}{\partial t_1} EU(\theta = 2, d = 0) - \frac{\partial}{\partial t_1} EU(\theta = 2, d = 2) \leq 0 \tag{2.t_1}$$

$$\frac{\partial}{\partial t_1} EU(\theta = 1, d = 0) - \frac{\partial}{\partial t_1} EU(\theta = 1, d = 1) \geq 0 \tag{1.t_1}$$

$$\frac{\partial}{\partial t_2} EU(\theta = 2, d = 0) - \frac{\partial}{\partial t_2} EU(\theta = 2, d = 2) \geq 0 \tag{2.t_2}$$

The first two inequalities (1.t_2 and 2.t_1) obviously hold. Substituting and differentiating gives

$$\begin{aligned} 2f(\rho - \sqrt{t_1}) \frac{\partial \rho}{\partial t_2} &< 0 \\ 2f(\rho - \sqrt{2t_1}) \frac{\partial \rho}{\partial t_1} &< 0, \end{aligned}$$

as $\partial\rho/t_2$, $\partial\rho/\partial t_1 > 0$, and $\rho < \sqrt{t_1} \leq \sqrt{2t_1}$.

Note that (1.t_1) holds if $\partial\bar{\mu}_1/\partial t_1 > 0$, where $\bar{\mu}_1$ is the belief which ensures $EU(\theta = 1, d = 0) = EU(\theta = 1, d = 1)$. This is true as $EU(\theta = 1, d = 0)$ increases with μ_1, while $EU(\theta = 1, d = 1)$ is independent of μ_1.

$$\bar{\mu}_1 = \left((1+\mu_2)\sqrt{t_1} - \mu_2\sqrt{2t_2}\right)\sqrt{\frac{f}{K + (f-1)\,t_1}}$$

and

$$\frac{\partial \bar{\mu}_1}{\partial t_1} = \frac{\sqrt{f}\left[\left(K + \mu_2\left(K + (f-1)\sqrt{2t_1 t_2}\right)\right) + 2\sqrt{t_1}\left(K + (f-1)\,t_1\right)\left(\sqrt{t_1} - \sqrt{2t_2}\right)\mu_2'\right]}{2\sqrt{t_1}\left(K + (f-1)\,t_1\right)^{3/2}} > 0,$$

as $\sqrt{t_1} - \sqrt{2t_2}$, $\mu_2' \leq 0$, and $(f-1) > 0$. Note that $\mu_2' \leq 0$ directly follows from (2.t_1) above. The proof for (2.t_2) is analogous. ■

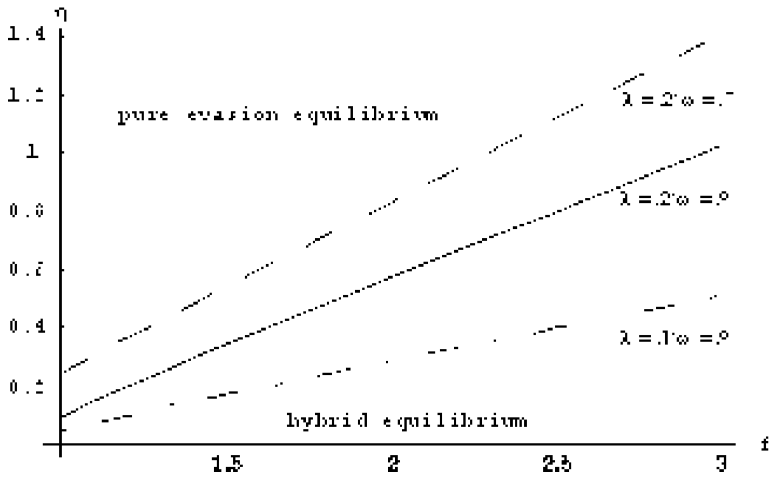

Figure 1: Parameter configurations for pure and hybrid evasion equilibria

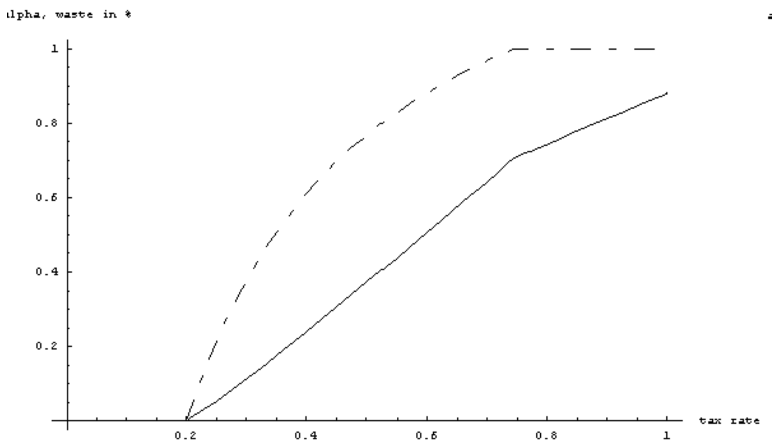

Figure 2: Evasion probability and waste pecentage for different tax rates

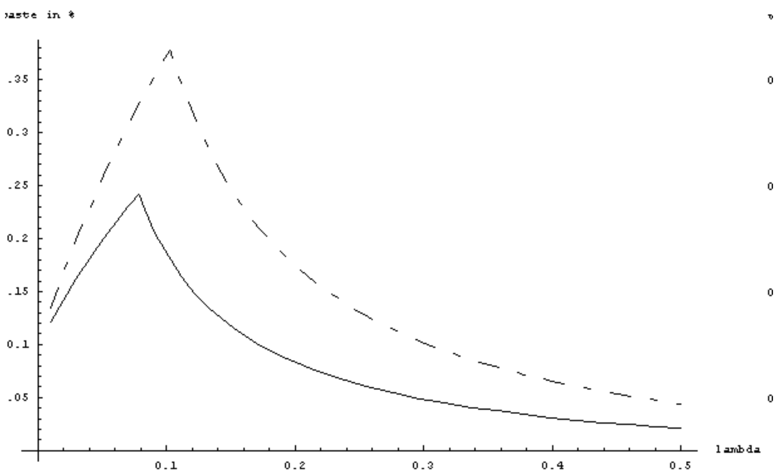

Figure 3: Waste percentage for different earnings probabilities

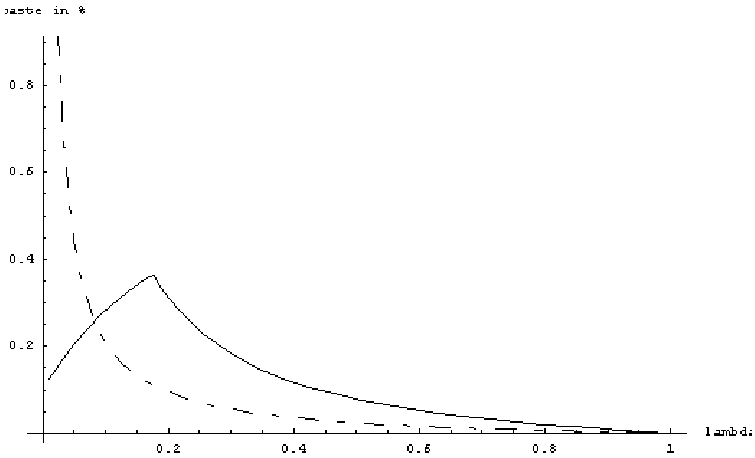

Figure 4: Waste percentages with and without commitment

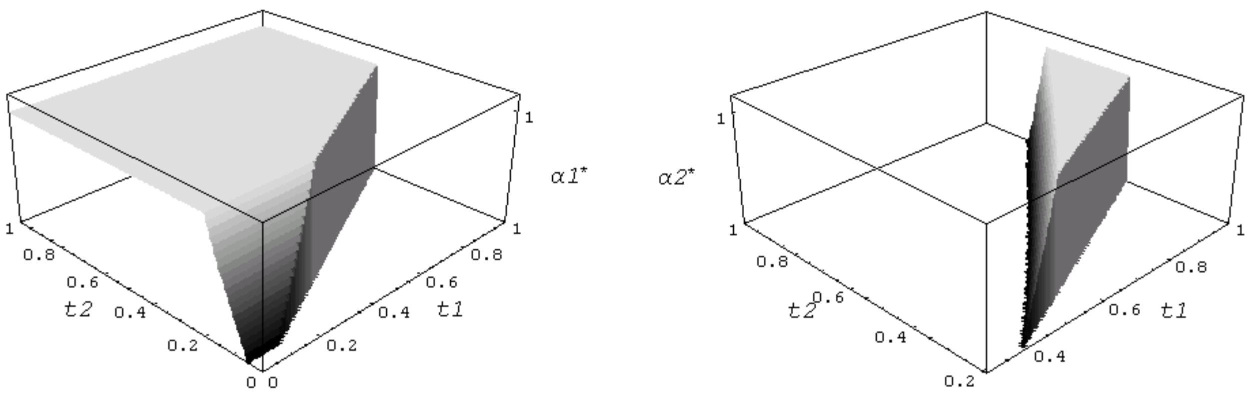

Figure 5: Equilibrium evasion probabilities for different tax systems

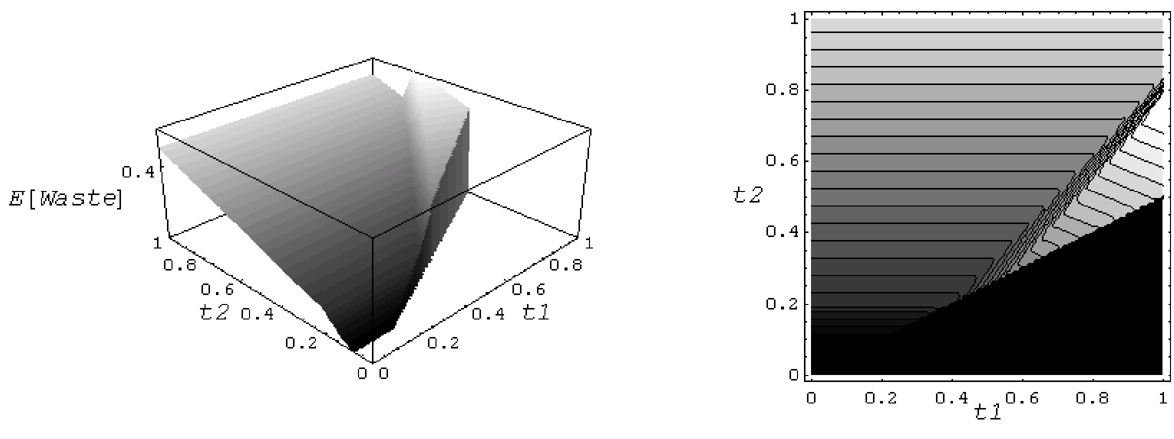

Figure 6: Expected equilibrium waste for different tax systems