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Abstract

Imperfect information has played a prominent role in modern business cycle theory. We assess its importance by estimating the New Keynesian (NK) model under alternative informational assumptions. One version focuses on confusion between temporary and persistent disturbances. Another, on unobserved variation in the inflation target of the central bank. A third on persistent mis–perceptions of the state of the economy (measurement error). And a fourth assumes perfect information (the standard NK–DSGE version). We find that imperfect information contains considerable explanatory power for business fluctuations. Signal extraction seems to provide a conceptually satisfactory, empirically plausible and quantitatively important business cycle mechanism.

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Introduction

The role of imperfect information in business fluctuations has been a prominent theme in modern business cycle theory. In the monetary, rational expectations model of Lucas, 1972, misperceptions of nominal aggregates give rise to confusion between nominal and relative price movements and constitute the main source of economic fluctuations.[1] Confusion about the shocks afflicting the economy is also present in the original version of the RBC model (Kydland and Prescott, 1982). In this model, the confusion arises from the agents' inability to distinguish between temporary and permanent changes in multi-factor productivity.

In spite of its presence in two of the most influential modern macroeconomic models, imperfect information is not considered an important element for understanding business cycles. There is a presumption that imperfect knowledge of the true values of nominal aggregates is not important enough to generate large and persistent movements in real economic activity. And the success of the Kydland-Prescott RBC model in accounting for business cycle owes little, if anything, to informational problems.

Recent work, however, has questioned the unimportance of imperfect information/signal extraction problems in influencing macroeconomic outcomes. For instance, Orphanides (2002, 2003) has forcefully argued that mis–perceptions of potential output may have played the key role behind the excessively loose monetary policy and the great inflation of the 70s. Collard and Dellas, 2007, claim that the failure of the rational expectations literature to find significant effects of mis-perceived money may be due to its use of inappropriate measures of mis–perceptions. In particular, using the proper measure leads to the uncovering of significant effects on economic activity. Moreover, they demonstrate that under –empirically plausible levels of– imperfect information the NK model can deliver considerable inertia. Collard and Dellas, 2006, argue that the success of imperfect information in accounting for business cycles is not limited to models with sticky prices. In particular, confusion regarding the types of shocks afflicting the economy plays a crucial role in allowing also the RBC model to fit a number of conditional, dynamic, stylized facts. Finally, Woodford, 2003, and Lorenzoni, 2006, show that a Phelps-Lucas island type of economy with strategic complementarities, idiosyncratic shocks and noisy aggregate *money supply* (in the former) or *productivity* (in the latter), can exhibit realistic business cycle patterns if the amount of noise is sufficiently high. In a similar vein, Angeletos and La'o, 2009, argue that incomplete information can be useful for understanding business-cycles.

Why should one expect that imperfect information about the shocks and the true state of the economy could play an important role in business cycles? The reason is to be found in the response of the agents to perceived shocks. First, the agents may over–react to some shocks

[1] Such mis-perceptions may arise either from the complete unavailability of up-to-date information. Or, from the fact that the available information is contaminated by measurement error (King, 1982)

and this generates "excessive" volatility.[2] And second, they under-react to some other shocks. In particular, the reaction to a particular shock may be initially mooted as the agents attribute part of the shock to other disturbances. And it may pick up gradually as the agents learn and update their perceptions of the actual shocks that haver occurred. Imperfect information may thus provide an endogenous propagation mechanism that can generate inertia, persistence and reversals, and it can often lead to hump shaped dynamics (see, for instance, Dellas, 2006).

The objective of this paper is to undertake a comprehensive study of the role of imperfect information in macroeconomic fluctuations. Comprehensive in the sense that alternative informational setups are considered. And that the alternative versions of imperfect information are compared not only among themselves but also to versions that assume perfect information.

Of the latter class of models we consider the two most widely used ones. Namely, the purely forward looking version of the NK model (Woodford, 2002). And its popular "hybrid" version with both forward and backward looking elements as well as real rigidities (Christiano et al, 2005, Galí and Gertler, 1999, Smets and Wouters, 2003). Of the class of models with imperfect information we consider three versions, each one relying on common, imperfect information but emphasizing a different source of mis–perception. The first one is motivated by Kydland and Prescott, 1982, and Orphanides, 2002, and involves confusion between temporary and permanent shocks. The second is motivated by Cogley and Sbordone, 2006, and relies on –unobserved– variation in the inflation target of the central bank. These two versions share in common the assumption that the agents observe perfectly all endogenous variables (inflation, output, interest rate) but not all of the shocks. The third and final version draws on Collard and Dellas', 2007, argument that, as revealed by the process of data revisions, very few aggregate variables are observed accurately. Consequently, unlike the two versions presented above, it assumes that the shocks as well as some of the endogenous variables are observed with noise.[3] This version thus represents a more severe case of information imperfection than the other two. Nevertheless, as we will establish later, the degree of mis–perception required by the model in order to fit the data is well within the range observed in the real world.

The models are estimated on US quarterly data over the 1966–2002 period using Bayesian methods. They are then evaluated and compared in terms of various criteria: Overall fit (the log-likelihood), unconditional second moments, and IRFs.[4]

[2]For instance, they respond to changes in monetary aggregates while such changes would have been otherwise neutral.

[3]While our story emphasizes measurement error in aggregate variables, rational inattention à la Sims, 2003, might also generate similar mis–perceptions. The implications of the latter have not been investigated econometrically yet.

[4]These comparisons constitute one of the main differences between this paper and other work in the literature that tests the empirical validity of the *perfect information* NK model under alternative specifications. For instance, Eichenbaum and Fisher, 2004, find that an estimated version of the NK model with backward indexation is consistent with the data (as judged by the J–statistic in the context of GMM estimation). De Walque, Smets and Wouters, 2004, find that the Smets and Wouters model performs well even when the parameter of backward

The main finding is that imperfect information has considerable explanatory power for the US business cycle. In particular, it improves the fit of the baseline NK model (that is, the forward looking model without real rigidities) according to the log-likelihood criterion. Moreover, a model that exhibits a more "severe" informational problem (the Collard and Dellas version) fares better with regard to not only the NK model but also the "hybrid" model (the one with perfect information, backward indexation and real rigidities) in spite of the fact that it does not itself contain any real rigidities. This is an important result as backward indexation and real rigidities have been the subjects of controversy and there is ongoing search in the profession to develop models that share its success but rely less –at least quantitatively – on these features. Moreover, we find that the success of the imperfect information model does not require implausible amounts of noise in the signal extraction problem. Namely, the amount of estimated noise corresponds closely to the measurement errors observed in real time data.

The remaining of the paper is organized as follows. Sections 1 and 2 present the model with and without a signal extraction problem (what we call perfect and imperfect information, respectively) and discuss the econometric methodology. Section 3 discusses the econometric methodology. Section 4 presents the main results. The last section offers some concluding remarks.

1 The model without signal extraction

1.1 The Household

There exists an infinite number of households distributed over the unit interval and indexed by $j \in [0,1]$. The preferences of household j are given by[5]

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \chi_{t+\tau} \left[\log(c_{t+\tau} - g_{t+\tau}) - \nu^h \frac{h_{t+\tau}^2}{2} \right] \tag{1}$$

where $0 < \beta < 1$ is a constant discount factor, c_t denotes consumption in period t, and h_{jt} is the quantity of labor supplied by the representative household of type j. χ_t is a preference shock that is assumed to follow an AR(1) process of the form

$$\log(\chi_t) = \rho_\chi \log(\chi_{t-1}) + (1-\rho_\chi)\log(\overline{\chi}) + \varepsilon_{\chi,t}$$

where $|\rho_\chi| < 1$ and $\varepsilon_{\chi,t} \rightsquigarrow \mathscr{N}(0, \sigma_\chi^2))$.

g_t denotes an external habit stock which is assumed to be proportional to past aggregate consumption:

$$g_t = \vartheta \overline{c}_{t-1} \text{ with } \vartheta \in [0,1).$$

indexation is close to zero. But the fact that the model without indexation is not rejected by the data does not mean that it performs the best within a particular class of models.

[5]A more general utility function would be $\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \chi_{t+\tau} \left[\frac{(c_{t+\tau}-g_{t+\tau})^{1-\sigma_c}}{1-\sigma_c} - \nu^h \frac{h_{t+\tau}^{1+\sigma_h}}{1+\sigma_h} \right]$. However, it turns out that both σ_c and σ_h are not identified in this model. Our specification amounts to setting $\sigma_c = \sigma_h = 1$.

In each period, household j faces the budget constraint

$$B_t + P_t c_t = R_{t-1} B_{t-1} + W_t h_t + \Pi_t \tag{2}$$

where B_t is nominal bonds. P_t, the nominal price of goods. c_t denotes consumption expenditures. W_t is the nominal wage. Ω_t is a nominal lump-sum transfer received from the monetary authority and Π_t denotes the profits distributed to the household by the firms.

This yields the following set of first order conditions

$$\nu_h h_t = \frac{w_t}{c_t - \theta \overline{c}_{t-1}} \tag{3}$$

$$\frac{\chi_t}{c_t - \theta \overline{c}_{t-1}} = \beta R_t E_t \frac{\chi_{t+1}}{(c_{t+1} - \theta \overline{c}_t)\pi_{t+1}} \tag{4}$$

1.2 The firms

1.2.1 Final Good Producers

The final good, y is produced by combining intermediate goods, y_i, by perfectly competitive firms. We follow Kimball, 1995, in assuming a production function of the type

$$\int_0^1 G\left(\frac{y_t(i)}{y_t}\right) \mathrm{d}i = 1 \tag{5}$$

where $G(\cdot)$ is an increasing and concave function. Profit maximization gives rise to the following demand function for good i

$$y_t(i) = y_t G'^{-1}\left(\frac{P_t(i)}{P_t}\kappa_t\right) \tag{6}$$

where $\kappa_t \equiv \int_0^1 G'\left(\frac{y_t(j)}{y_t}\right)\frac{y_t(j)}{y_t}\mathrm{d}j$.

The general price index

$$P_t = \int_0^1 G'^{-1}\left(\frac{P_t(i)}{P_t}\kappa_t\right) \mathrm{d}i \tag{7}$$

1.2.2 Intermediate goods producers

Each firm i, $i \in (0,1)$, produces an intermediate good by means of labor according to production function

$$y_t(i) = a_t h_t(i) - \Phi \tag{8}$$

where $h_t(i)$ denotes the labor input used by firm i in the production process. $\Phi > 0$ is a fixed cost. a_t is an exogenous technology shock which is assumed to follow an AR(1) process of the form

$$\log(a_t) = \rho_a \log(a_{t-1}) + (1-\rho_a)\log(\overline{a}) + \varepsilon_{a,t}$$

where $|\rho_a| < 1$ and $\varepsilon_{a,t} \rightsquigarrow \mathscr{N}(0, \sigma_a^2)$.

Intermediate goods producers are monopolistically competitive, and therefore set prices for the good they produce. We follow Calvo in assuming that firms set their prices for a stochastic number of periods. In each and every period, a firm either gets the chance to adjust its price (an event occurring with probability ξ) or it does not. If it does not get the chance, then it is assumed, following Christiano et al.,2005, to set prices according to

$$P_t(i) = \pi_{t-1} P_{t-1}(i). \tag{9}$$

Hence there is perfect indexation to past inflation. In the sensitivity analysis we consider alternative specifications involving partial indexation to past inflation combined with partial indexation to either the –variable– inflation target of the central bank or steady state inflation.

If a firm i sets its price optimally in period t then it chooses a price, $P_t^\star$, in order to maximize:

$$\max_{P_t^\star(i)} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Phi_{t+\tau}(1-\xi)^\tau \left(P_t^\star(i)\Xi_{t,\tau} - P_{t+\tau}\Psi_{t+\tau}\right) y_{t+\tau}(i)$$

subject to the total demand (6) and

$$\Xi_{t,\tau} = \begin{cases} \pi_t \times \ldots \times \pi_{t+\tau-1} & \text{if } \tau \geqslant 1 \\ 1 & \text{otherwise} \end{cases}$$

Ψ_t denotes the real marginal cost of the firm.[6] $\Phi_{t+\tau}$ is an appropriate discount factor derived from the household's evaluation of future relative to current consumption. This leads to the first order condition

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \Phi_{t+\tau}(1-\xi)^\tau y_{t+\tau}(i) \left(P_t^\star(i)\Xi_{t,\tau} + (P_t^\star(i)\Xi_{t,\tau} - P_{t+\tau}\Psi_{t+\tau}) \frac{1}{\varsigma_{t+\tau}} \frac{G'(\varsigma_{t+\tau})}{G''(\varsigma_{t+\tau})} \right) = 0 \tag{10}$$

where $\varsigma_{t+\tau} = G'^{-1}\left(\frac{P_t^\star(i)\Xi_{t,\tau}}{P_{t+\tau}} \kappa_{t+\tau}\right)$. Since the price setting scheme is independent of any firm specific characteristic, all firms that reset their prices will choose the same price, such that $P_t^\star(i) = P_t^\star$.

In each period, a fraction ξ of contracts ends and $(1-\xi)$ survives. Hence, from (7) and the price mechanism, the aggregate intermediate price index writes

$$P_t = \xi P_t^\star G'^{-1}\left(\frac{P_t^\star}{P_t}\kappa_t\right) + \xi \pi_{t-1} G'^{-1}\left(\frac{\pi_{t-1}P_{t-1}}{P_t}\kappa_t\right) \tag{11}$$

[6] Since there is only one common technology shock and labor is homogenous, the real marginal cost is the same across firms.

1.3 Monetary Policy

Monetary policy is conducted according to

$$\log(R_t) = \rho_r \log(R_{t-1}) + (1-\rho_r)[\log(R^\star) + \log(\overline{\pi}_t) + \alpha_y(\log(y_t) - \log(y_t^{\mathrm{N}})) + \alpha_\pi(\log(\pi_t) - \log(\overline{\pi}_t)] + \epsilon_{r,t}$$

where $\overline{\pi}_t$ represents the inflation target of the central bank and y_t^{N} is the natural rate of output, that is the output level that would prevail in a flexible price economy. $\epsilon_{r,t}$ is a monetary policy shock. It follows a Gaussian iid process ($\varepsilon_{r,t} \rightsquigarrow \mathscr{N}(0, \sigma_r^2)$).

We will consider two alternative cases for the inflation target. In the first one, the central bank targets the constant steady state inflation, hence $\overline{\pi}_t = \overline{\pi}$. In the second one, following Cogley and Sbordone, 2006, we assume that the inflation target varies over time. In particular, it follows an AR(1) process of the form

$$\log(\overline{\pi}_t) = \rho_\pi \log(\overline{\pi}_{t-1}) + (1-\rho_\pi)\log(\overline{\pi}) + \varepsilon_{\pi,t}$$

where $|\rho_\pi| < 1$ and $\varepsilon_{\pi,t} \rightsquigarrow \mathscr{N}(0, \sigma_\pi^2)$.

1.4 Equilibrium

In equilibrium, we have $y_t = c_t$. Log-linearization of the model around the deterministic steady state leads to the standard IS-AS-MP representation of the NK model.

$$\begin{aligned}
\widehat{y}_t =& \frac{\vartheta}{1+\vartheta}\widehat{y}_{t-1} + \frac{1}{1+\vartheta}\mathbb{E}_t\widehat{y}_{t+1} - \frac{1-\vartheta}{1+\vartheta}(\widehat{R}_t - \mathbb{E}_t\widehat{\pi}_{t+1}) + \widehat{x}_t && (12)\\
\widehat{\pi}_t =& \frac{\gamma}{1+\beta\gamma}\widehat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma}\mathbb{E}_t\widehat{\pi}_{t+1} + \frac{\xi(1-\beta(1-\xi))}{(1-\xi)(1+\beta\gamma)(\varphi\zeta+1)}\frac{2+\varphi-\vartheta}{1-\vartheta}\widehat{y}_t\\
&- \frac{\xi(1-\beta(1-\xi))}{(1-\xi)(1+\beta\gamma)(\varphi\zeta+1)}\frac{\vartheta}{1-\vartheta}\widehat{y}_{t-1} - \widehat{z}_t + \widehat{\nu}_t && (13)\\
\widehat{R}_t =& \rho_r\widehat{R}_{t-1} + (1-\rho_r)(\alpha_y(\widehat{y}_t - \widehat{y}_t^{\mathrm{N}}) + \alpha_\pi\widehat{\pi}_t + (1-\alpha_\pi)\widehat{\overline{\pi}}_t) + \epsilon_{r,t} && (14)\\
\widehat{y}_t^{\mathrm{N}} =& \frac{\vartheta(1+\varphi)}{2+\varphi-\vartheta}\widehat{y}_{t-1}^{\mathrm{N}} + \frac{(1+\varphi)(1-\vartheta)(1-\xi)(1+\beta\gamma)(\varphi\zeta+1)}{(2+\varphi-\vartheta)\xi(1-\beta(1-\xi))}\widehat{z}_t && (15)
\end{aligned}$$

where φ is the share of fixed costs in total output, and ζ is the curvature of the Kimball goods market aggregator. $\widehat{x}_t = \frac{1-\vartheta}{1+\vartheta}(\widehat{\chi}_t - \mathbb{E}_t\widehat{\chi}_{t+1})$, $\widehat{z}_t = \frac{2\xi(1-\beta(1-\xi))}{(1-\xi)(1+\beta\gamma)(\varphi\zeta+1)}\widehat{a}_t$.[7] $\widehat{\nu}_t$ is a cost push shock which is assumed to be iid and normally distributed with mean 0 and standard deviation σ_ν.

We will study five versions of the model, two with perfect and three with imperfect information. In particular, in the first version we assume that: the agents have perfect information about all shocks; the Central Bank targets a constant inflation rate ($\sigma_\pi = 0$, $\rho_\pi = 0$); there is no indexation to lagged inflation in price setting ($\gamma = 0$); and there are no real rigidities present($\vartheta = 0$). This

[7]Note that both $\widehat{x}_t$ and $\widehat{z}_t$ follow the same AR(1) process as their underlying fundamentals, χ_t and a_t, but the volatility of the innovations is a simple re–scaling of the original innovations. In order to save on notation, we will keep σ_χ and σ_a to denote the volatilities of the transformed shocks.

version, which we will call model 1, corresponds to the standard, baseline New Keynesian (NK) model.[8] The second version differs from the first regarding the price setting scheme as well as the presence of real rigidities. In particular, here we assume that firms that cannot optimally reset their prices adopt a backward indexation scheme. And that ϑ is allowed to differ from zero. This version is denoted as model 2.

The remaining versions introduce imperfect information.[9] The third version of the model is similar to model 1 ($\gamma = \sigma_\pi = \rho_\pi = \vartheta = 0$) with the exception that the agents do not observe the shocks directly and can only infer them through their —perfect— observation of output, inflation and the nominal interest rate. While the observation of the nominal interest rate and the level of output enables them to infer the preference and monetary policy shock, the observation of inflation is not sufficient to help them perfectly distinguish between the persistent technology and transient cost push shocks. This version, which we call model 3, is —loosely— motivated by Kydland and Prescott, 1982, and Orphanides, 2002, in the sense that it involves confusion between temporary and persistent shocks. Under these circumstance, the Central Bank is assumed to use the best possible estimates of the output gap and the inflation gap it can produce. This leads us to consider a Taylor rule of the type[10]

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1 - \rho_r)[\alpha_y(\widehat{y}_t - \widehat{y}^{\mathrm{N}}_{t|t}) + \alpha_\pi \widehat{\pi}_t)] + \epsilon_{r,t} \tag{16}$$

where $x_{t|t}$ denotes the expected value of variable x conditional on the information set of the Central Bank, which now consists of output, inflation and the nominal interest rate.

The fourth version of the model is similar to the previous one but emphasizes a different source of mis–perception. The agents still observe the same variables, but the imperfect information problem does not regard distinguishing between persistent technology shocks and transient cost push shock, but rather distinguishing between persistent inflation target shocks and transient monetary policy shocks. This version –which abstracts from the cost push shocks ($\sigma_\nu = 0$) in order to be differentiated from version three above– is motivated by the work of Cogley and Sbordone, 2006, who emphasize the role of –unobserved– variation in the inflation target of the central bank as a driving force of fluctuations in output and inflation. The central bank is now

[8] One could also attempt to estimate the baseline NK model augmented with real rigidities. We run into identification problems when we tried this. But more importantly, real rigidities are a rather controversial modeling feature, so it makes sense not to impose them on models that have shunned away from them, such as the baseline NK model or the imperfect information models. Their exclusion from the latter class of models is particularly useful as a means of assessing the ability of imperfect information to replace –or ameliorate– all the main inertial mechanisms of the hybrid NK model.

[9] We make the assumption that all agents in the model, including the central bank, have the same information set. Hence, we abstract from issues of private information and informational asymmetries.

[10] Note that since output and inflation are perfectly observed, their perceived value is identical to their actual value.

assumed to follow an interest rate rule of the form:[11]

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1-\rho_r)[\alpha_y(\widehat{y}_t - \widehat{y}^{\mathrm{N}}_{t|t}) + \alpha_\pi(\widehat{\pi}_t - \widehat{\pi}^\star_t)] + \epsilon_{r,t} \tag{17}$$

This version is denoted as model 4. Admittedly, the assumption of symmetric information between the private agents and the monetary authorities is rather tenuous under these circumstances. But the technical complications arising from dropping this assumption are rather prohibitive in this context.

The last —fifth— version of the model is like the third version, that is, it re–introduces the cost–push shocks and drops the inflation target shocks. But it adopts a different information structure that makes the informational problem more severe. In particular, here we assume that some of the variables of the model (shocks and endogenous variables) are measured imprecisely. We assume that the shocks, output and the inflation rate are measured with error, that is, for variable ω we have that

$$\omega^\star_t = \omega^{\mathrm{T}}_t + \upsilon_{\omega,t}$$

where ω^{T}_t denotes the true value of the variable and $\upsilon_{\omega,t}$ is a noisy process that satisfies $E(\upsilon_{\omega,t}) = 0$ for all t; $E(\upsilon_{\omega,t}\varepsilon_{z,t}) = E(\upsilon_{\omega,t}\varepsilon_{x,t}) = E(\upsilon_{\omega,t}\varepsilon_{\pi,t}) = E(\upsilon_{\omega,t}\varepsilon_{\nu,t}) = 0$; and

$$E(\upsilon_{\omega,t}\upsilon_{\omega,k}) = \begin{cases} \eta^2_\omega & \text{if } t = k \\ 0 & \text{Otherwise} \end{cases}$$

The nominal interest rate is still observed perfectly. In this version, the Central Bank follows the rule

$$\widehat{R}_t = \rho_r \widehat{R}_{t-1} + (1-\rho_r)[\alpha_y(\widehat{y}_{t|t} - \widehat{y}^{\mathrm{N}}_{t|t}) + \alpha_\pi \widehat{\pi}_{t|t}] + \epsilon_{r,t} \tag{18}$$

Consequently, the agents face a more severe —and hence, potentially more consequential— signal extraction problem in the fifth version in comparison to the other imperfect information versions. The motivation for this specification[12] comes from Orphanides, 2002, who argues that a large fraction of the output gap mis–perception during the great inflation of the 70s can be attributed to the mis-measurement of *actual* output. Using the real time data constructed by the Philadelphia FED, Collard and Dellas, 2007, have argued that such mis–measurement is present in all macroeconomic series (including GDP inflation) and that it is quantitatively substantial. The existence of real time data can help assess the plausibility of the estimated amount of noise in the model by comparing it to that present, say, in data revisions. Or, by comparing perceived

[11] Note that since output and inflation are perfectly observed, their perceived values coincide with their actual values.

[12] It should be noted that we are not the first one to estimate a NK model with signal extraction due to measurement errors. Lippi and Neri, 2006, estimate such a model but, having a different objective, they do not examine the stochastic properties of their model. Moreover, they do not compare the performance of their model to that of alternative NK specifications, so one cannot judge its *relative* success.

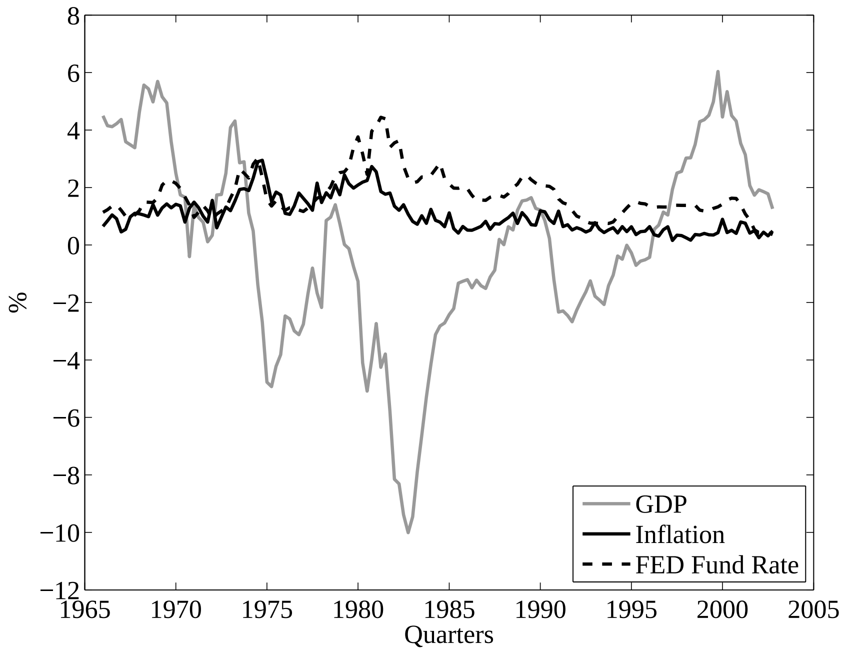
values to the corresponding real time data.[13] Nonetheless, it should be noted that imperfect observation of the true values of some of the variables of the system does not have to derive exclusively or even predominantly from measurement error in aggregate variables for the story to work. An alternative source could be a Sims', 2003, type of inattention on the part of the agents.

2 Econometric Methodology

2.1 Data

The model is estimated on US quarterly data for the period 1966:I–2002:IV. The data description and sources can be found in the appendix. Output is measured by real GDP, inflation is the annualized quarterly change in the GDP deflator, while the nominal interest rate is the Federal Funds Rate. The output series is detrended using a *linear* trend.

Figure 1: US Data



2.2 Estimation method

We do not estimate all the parameters of the model as some of them cannot be identified in the steady state and do not enter the log–linear representation of the economy. This is the case

[13]In addition to the measurement error specification of the signal extraction problem presented above there exists an alternative specification that relies on the distinction between idiosyncratic and aggregate shocks. We have opted for the former approach because it is much easier to implement and can also be evaluated using available extraneous information on real time data. The latter approach involves asymmetric information and higher order expectations and it much harder to solve and test.

for the demand elasticity, ζ.[14] All the other structural parameters are estimated, including the steady state level of annualized inflation, $\pi^\star$, and real interest rate, $r^\star$. The discount factor, β, is then computed using the estimated level of the real interest rate.[15] We therefore estimate the vector of parameters $\Theta = \{\vartheta, \xi, r^\star, \pi^\star, \rho_r, \alpha_\pi, \alpha_y, \rho_a, \rho_\chi, \rho_\pi, \sigma_a, \sigma_\chi, \sigma_r, \sigma_\nu, \sigma_\pi, \eta_y, \eta_\pi, \varphi\}$. Θ is estimated relying on a Bayesian maximum likelihood procedure. As a first step of the procedure, the log–linear system (12)–(15) is solved using the Blanchard-Khan method. In the specification with the signal extraction, the model is solved according to the method outlined in the companion technical Appendix.[16] The one sided Kalman filter is then used on the solution of the model to form the log–likelihood, $\mathcal{L}_m(\{\mathscr{Y}_t\}_{t=1}^T, \Theta)$, of each model, m. Once the posterior mode is obtained by maximizing the likelihood function, we obtain the posterior density function using the Metropolis–Hastings algorithm (see Lubik and Schorfheide, 2004).

Table 1 presents the –quite diffuse– prior distribution of the parameters.[17] The habit persistence parameter, ϑ, is beta distributed as it is restricted to belong to the (0,1) interval. The average of the distribution is set to 0.50, which is in line with the prior distribution used by Smets and Wouters, 2004. The steady state inflation rate and real interest rate have a Γ-distribution with means 1% and 0.5% per quarter respectively.

The parameter pertaining to the nominal rigidity is distributed according to a beta distribution as it belongs in the (0,1) interval. The average probability of price resetting, ξ, is set to 0.50, implying that a firm expects to reset prices on average every two quarters. The confidence interval, though, encompasses average price resetting from 1.1 to 10 quarters.

The persistence parameter of the Taylor rule, ρ_r, has a beta prior over (0,1) so as to guarantee the stationarity of the rule. The prior distribution is centered on 0.5. The reaction to inflation, α_π, and output, α_y, is assumed to be positive, and with a normal distribution centered at 1.5 and 0.125 respectively. These values correspond to values commonly used in the literature.

We have little knowledge of the processes that describe the forcing variables. We assume a beta distribution for the persistence parameters in order to guarantee the stationarity of the processes. Each distribution is centered on 0.5. Volatility is assumed to follow an inverse gamma distribution (to guarantee positiveness). However, in order to take into account the limited knowledge we have regarding these process we impose non informative priors. The same strategy is also applied to the process of noise in the signal extraction model.

[14] Following Smets and Wouters, 2007, we set $\zeta = 10$.

[15] More precisely, we have $\beta = (1 + r^\star/100)^{-1/4}$.

[16] The technical appendix is available from `http://fabcol.free.fr/index.php?page=research`.

[17] In the sensitivity analysis we report results with even more diffuse —essentially uninformative— priors as a means of establishing that the tightness of the priors used does not matter for the obtained ranking of the alternative models.

Table 1: Priors

Param.	Type	Param 1	Param 2	95% HPDI
θ	Beta	0.50	0.25	[0.096,0.903]
φ	Beta	0.25	0.125	[0.067,0.457]
ξ	Beta	0.50	0.25	[0.096,0.903]
$r^\star$	Gamma	0.50	0.50	[0.026,1.501]
$\pi^\star$	Gamma	1.00	1.00	[0.051,2.996]
ρ_r	Beta	0.50	0.25	[0.096,0.903]
α_π	Normal	1.50	0.25	[1.088,1.909]
α_y	Normal	0.125	0.05	[0.045,0.207]
ρ_a	Beta	0.50	0.25	[0.096,0.903]
ρ_χ	Beta	0.50	0.25	[0.096,0.903]
ρ_π	Beta	0.50	0.25	[0.096,0.903]
σ_a	Inv. Gamma	0.20	4.00	[0.17,0.83]
σ_χ	Inv. Gamma	0.20	4.00	[0.17,0.83]
σ_r	Inv. Gamma	0.20	4.00	[0.17,0.83]
σ_π	Inv. Gamma	0.20	4.00	[0.17,0.83]
σ_ν	Inv. Gamma	0.20	4.00	[0.17,0.83]
η_y	Inv. Gamma	0.20	4.00	[0.17,0.83]
η_π	Inv. Gamma	0.20	4.00	[0.17,0.83]

Note: The parameters are distributed independently from each other. [a] 95-percent highest probability density (HPD) credible intervals. The Param 1 and Param 2 report the mean and the standard deviation of the prior distribution for Beta, Gamma and Normal distributions. They report the s and ν parameters of the inverse gamma distribution, where $f(\sigma|s,\nu) \propto \sigma^{-(1+\nu)}\exp(-\nu s^2/2\sigma^2)$.

3 Estimation Results

Table 2 reports the posterior estimates for the five model versions.[18] Figures 2–6 record IRFs and Table 8 provides information on unconditional moments. The main findings are summarized below.

First, the estimated parameters are within the range typically found in the literature. The main exception to this regards the "persistence" parameters in the hybrid model. In particular, the persistence parameters of the technology and preference shocks are very low relative to the values typically reported in the literature (0.04 and 0.26, respectively). At the same time, habit persistence and the average duration of prices are too high (0.94 and 12 quarters respectively). Inspection of the model indicates how difficult it is to solve this weak identification problem.[19] Moreover, there is information in the sample that helps identify the parameters of the models as the comparison of the prior to posterior densities reveals. That is, the data are informative.

Second, the models can be ranked in terms of fit as measured by the log–likelihood. Within the models with perfect information, the hybrid version does better than the baseline NK model. Hence, the inertial dynamics induced by real rigidities and backward indexation are vital for empirical success. Within the models of imperfect information, the model with the most severe signal extraction problem (version 5) does best. Comparing across the two classes of models one can see that the while the hybrid model does better than models 3 and 4 (which in turn perform better than the baseline version 1), it falls short of model 5. This indicates that while imperfect information can be a substitute for real rigidities and backward inflation indexation, it must be present in a sufficiently large amount in order to be a perfect substitute. To see this point notice that models 4 and –in particular– 3 lack sufficient persistence in the signal extraction problem. For instance, consider model 3. The complete absence of serial correlation in the cost–push shock allows the agents to quickly discriminate between this and the persistent technology shock, leaving little room for inertial dynamics. Inspection of the corresponding IRFs confirms this. A more direct confirmation can be derived when one compares the paths of actual and perceived shocks (see Figure 7 in the Appendix). As can be seen, mis–perceptions are resolved fairly fast. Hence, introducing a limited signal extraction problem to the baseline NK model and expecting it to outperform the hybrid model may amount to asking for too much. Nevertheless, making the signal extraction problem somewhat more severe/persistent does the

[18]The interested reader may consult the technical appendix that accompanies the paper for more details regarding the distribution of posterior estimates.

[19]Note that setting the indexation parameter, γ equal to one ameliorates the identification problem significantly as it allows the Metropolis–Hastings algorithm to converge. But it does not manage to solve the weak identification problem. Imposing fixed values for some of the persistence parameters would take care of the problem but might prove unduly restrictive and also undermine the performance of the hybrid model. For instance, when fixing the persistence in the technology and preference shocks to 0.95, the parameter of habit persistence decreases –to 0.52– and the Calvo parameter increases to 0.38. But marginal likelihood also drops to -277.

Table 2: Estimation Results

Parameter	(1)	(2)	(3)	(4)	(5)
θ	–	0.94	–	–	–
		[0.90,0.97]			
ξ	0.49	0.08	0.49	0.68	0.22
	[0.33,0.64]	[0.03,0.14]	[0.34,0.64]	[0.51,0.83]	[0.14,0.31]
φ	0.26	0.29	0.27	0.24	0.28
	[0.04,0.48]	[0.07,0.51]	[0.05,0.49]	[0.02,0.46]	[0.06,0.50]
$r^\star$	0.61	0.66	0.62	0.56	0.62
	[0.31,0.86]	[0.36,0.97]	[0.34,0.87]	[0.11,0.92]	[0.22,1.02]
$\pi^\star$	0.90	1.01	0.91	0.86	0.97
	[0.53,1.24]	[0.69,1.33]	[0.57,1.26]	[0.34,1.30]	[0.57,1.34]
ρ_r	0.67	0.84	0.67	0.21	0.27
	[0.60,0.75]	[0.77,0.91]	[0.60,0.74]	[0.02,0.39]	[0.14,0.42]
α_π	1.61	1.40	1.58	2.37	1.64
	[1.29,1.90]	[1.06,1.75]	[1.31,1.90]	[2.05,2.70]	[1.19,2.10]
α_y	0.13	0.04	0.15	0.13	0.20
	[0.04,0.23]	[0.00,0.12]	[0.05,0.24]	[0.03,0.22]	[0.11,0.28]
ρ_a	0.98	0.07	0.98	0.99	0.95
	[0.96,1.00]	[0.00,0.18]	[0.96,1.00]	[0.97,1.00]	[0.92,0.97]
ρ_χ	0.91	0.26	0.91	0.92	0.86
	[0.85,0.97]	[0.09,0.42]	[0.85,0.97]	[0.85,0.98]	[0.80,0.92]
ρ_π	–	–	–	0.92	–
				[0.86,0.98]	
σ_a	0.27	0.11	0.25	0.81	0.12
	[0.14,0.44]	[0.08,0.14]	[0.12,0.40]	[0.35,1.38]	[0.09,0.16]
σ_χ	0.16	0.62	0.17	0.20	0.29
	[0.11,0.22]	[0.48,0.77]	[0.12,0.23]	[0.16,0.24]	[0.21,0.37]
σ_r	0.36	0.26	0.35	0.49	0.13
	[0.30,0.41]	[0.23,0.29]	[0.30,0.41]	[0.37,0.63]	[0.09,0.16]
σ_ν	0.14	0.15	0.16	–	0.21
	[0.10,0.19]	[0.11,0.18]	[0.11,0.22]		[0.18,0.25]
σ_π	–	–	–	0.11	–
				[0.08,0.15]	
η_y	–	–	–	–	0.28
					[0.10,0.52]
η_π	–	–	–	–	6.72
					[2.18,13.15]
$\mathcal{L}$	-323.472	-267.388	-321.101	-296.148	-264.729

Note: This table reports the mean of the posterior distribution of each parameter and the associated 95% HPDI (in brackets). (1): Baseline NK, (2): Hybrid NK (Backward Indexation, Real Rigidities), (3): Imperfect Info. Temporary vs Permanent Shocks, (4): Imperfect Info., Cogley–Sbordone, (5): Imperfect Info., Noisy Signals. $\mathcal{L}$ denotes the average log marginal density of the model. 95% HPDI in brackets.

Table 3: Moments (HP–filtered series)

	Data	(1)	(2)	(3)	(4)	(5)
σ_y	1.56	1.47	1.56	1.45	1.21	1.15
		[1.27,1.67]	[1.26,1.89]	[1.25,1.64]	[1.07,1.36]	[1.02,1.28]
σ_π	0.30	0.38	0.45	0.37	0.35	0.30
		[0.33,0.43]	[0.40,0.51]	[0.33,0.42]	[0.31,0.39]	[0.26,0.33]
σ_R	0.44	0.37	0.49	0.37	0.43	0.35
		[0.33,0.41]	[0.40,0.59]	[0.33,0.41]	[0.38,0.48]	[0.31,0.40]
$\rho(\pi, y)$	0.12	0.24	0.19	0.26	0.09	0.21
		[0.12,0.36]	[0.05,0.34]	[0.13,0.38]	[0.02,0.16]	[0.13,0.29]
$\rho(R, y)$	0.34	-0.25	-0.21	-0.25	-0.05	0.26
		[-0.37,-0.13]	[-0.40,-0.01]	[-0.36,-0.13]	[-0.12,0.02]	[0.11,0.41]
$\rho_y(1)$	0.87	0.65	0.85	0.63	0.71	0.68
		[0.63,0.68]	[0.81,0.89]	[0.60,0.66]	[0.69,0.72]	[0.66,0.71]
$\rho_\pi(1)$	0.48	0.37	0.72	0.40	0.42	0.33
		[0.30,0.45]	[0.69,0.75]	[0.33,0.46]	[0.33,0.52]	[0.22,0.43]
$\rho_R(1)$	0.82	0.65	0.81	0.65	0.73	0.71
		[0.59,0.70]	[0.77,0.86]	[0.60,0.71]	[0.69,0.76]	[0.66,0.77]
$\rho_y(2)$	0.69	0.41	0.64	0.39	0.47	0.43
		[0.38,0.44]	[0.58,0.71]	[0.34,0.42]	[0.46,0.48]	[0.40,0.47]
$\rho_\pi(2)$	0.31	0.13	0.47	0.14	0.25	0.20
		[0.07,0.20]	[0.42,0.50]	[0.08,0.21]	[0.18,0.31]	[0.12,0.27]
$\rho_R(2)$	0.58	0.40	0.61	0.40	0.47	0.44
		[0.33,0.46]	[0.55,0.67]	[0.34,0.46]	[0.43,0.51]	[0.38,0.51]
$\rho_y(4)$	0.27	0.08	0.25	0.07	0.12	0.08
		[0.06,0.10]	[0.16,0.34]	[0.04,0.10]	[0.11,0.12]	[0.05,0.11]
$\rho_\pi(4)$	0.24	-0.06	0.08	-0.06	0.03	0.01
		[-0.08,-0.03]	[0.03,0.13]	[-0.09,-0.03]	[-0.00,0.05]	[-0.01,0.04]
$\rho_R(4)$	0.25	0.06	0.22	0.06	0.09	0.06
		[0.01,0.10]	[0.16,0.29]	[0.01,0.10]	[0.05,0.13]	[0.01,0.11]

Note: (1): Baseline NK, (2): Hybrid NK (Backward Indexation, Real Rigidities), (3): Imperfect Info. Temporary vs Permanent Shocks, (4): Imperfect Info., Cogley–Sbordone, (5): Imperfect Info., Noisy Signals. 95% HPDI in brackets.

job. Namely, it gives the baseline model the feature it needs in order to match the data well.

The comparisons of the IRFs and moments across models offers some clues regarding not only the reasons for the differences in their relative performance but also about the driving forces in each model. The most successful models (2,4 and 5) tend to exhibit inertia in inflation dynamics while the less successful ones (1 and 3) do not. Good performance at this front seems to play an important role, as the differences in performance across models in the realm of unconditional moments tend to be quite small (see Table 3, which follows standard practice and used HP filtered data, or Table 8 in the appendix which used linearly detrended data.).

Note that the main contributor to inflation inertia differs across the three specifications. In the perfect information model with indexation, inflation inertia comes from the two demand disturbances (the preference and the monetary policy shock). In the model with the variable inflation target, it arises exclusively from the inflation target shock while in the model with measurement errors, from the preference shock. Hence the three models differ regarding their identification of the cause of inertia between —direct— policy and non–policy factors.

The mechanism for inflation inertia in the model of indexation is well known, so let us examine the mechanisms in the models with imperfect information. Figures 9–10 plot the actual and perceived paths of the shocks in the fourth and fifth models. As can be seen, the agents under–react to the shocks. It also takes them considerable time to recognize the true nature of the shock that has afflicted the economy. For instance, consider a positive inflation target shock (a higher inflation target). If the shock were perfectly observable, everybody's expectation and hence actual inflation would jump up and then, given the AR(1) assumption on the shock, it would monotonically decline. But if the shock is not observed then the agents cannot tell whether the lower nominal interest rate and higher inflation should be expected to persist in the future or not (as it would not if the shock represented a purely transient monetary disturbance ($\varepsilon_{e,t}$)). As time goes one and the agents continue observing a lower nominal interest rate they become more convinced that an inflation target shock has taken place and inflation increase relative to the initial period. Due to the AR(1) nature of the shock, it reaches a peak after a few periods and then starts declining towards the initial steady state.

Judging the mis–perceptions story

Models with measurement errors and signal extraction are often criticized as requiring implausibly large amounts of imperfect information in order to deliver good results. In model 5 we have attributed the mis–perception problem to measurement errors associated with preliminary data releases. In order to judge the contribution of this mechanism, one could compare the properties of the noise in the model to that present in real time data. A natural way of doing so is by computing the volatility of the data revisions in the real world (for instance, initial minus final release) and the volatility between the actual and the perceived values in the model. The underlying assumption here is that the actual value corresponds to the final revision while the perceived value to the preliminary release. Hence, the revision in, say, GDP_t is, $\log(Y_{t|t}) - \log(Y_t)$ where the former represents the initial and the latter the final release.

Table 4 presents information on the properties of these revisions.[20]

Table 4: The properties –moments– of data revisions

	Whole Sample		Post 1982	
	Data	Model	Data	Model
σ_{ε_π}	0.22,1.51	0.94	0.19,1.97	0.78
		[0.83,1.05]		[0.66,0.91]
$\sigma_{\varepsilon_{\Delta y}}$	0.54,0.68	0.51	0.40,0.58	0.37
		[0.41,0.61]		[0.29,0.46]
$\rho(\varepsilon_\pi, \pi)$	0.14,-0.09	0.24	0.08,-0.12	0.23
		[0.14,0.33]		[0.15,0.32]
$\rho(\varepsilon_{\Delta y}, \Delta y)$	0.38,0.28	0.86	0.53,0.28	0.74
		[0.78,0.93]		[0.60,0.88]
$\rho_{\varepsilon_\pi}(1)$	0.12,0.00	-0.03	0.04,-0.02	-0.04
		[-0.04,-0.02]		[-0.06,-0.03]
$\rho_{\varepsilon_{\Delta y}}(1)$	-0.09,-0.12	-0.03	-0.16,-0.24	-0.03
		[-0.04,-0.02]		[-0.04,-0.02]

Note: ε_π is the revision in inflation and $\varepsilon_{\Delta y}$ in output growth. σ is standard deviation, ρ is cross-correlation and $\rho(1)$ is 1st order autocorrelation. The first number in column 1 and 3 corresponds to the computation from a single vintage, and the second number to the computation from two successive vintages (for details see the footnote). 95% HPDI in brackets.

Several observations are in order. First, the amount of noise the model needs in fitting the data is not excessive relative to that present in data revisions. Second, the assumption made in the model that the correlation between the measurement error and the initial release is zero is satisfied in the data for inflation but not for output. In other words, the initial release seems to contain information about future releases, or equivalently, the measurement error is correlated with the business cycle. This is an interesting observation that has implications for the ability of the "noisy aggregate variables" theory of the business cycle to fit the data. In particular, it seems quite conceivable that incorporating a more realistic model of measurement error in model 5 could further improve its performance. While pursuing this track is outside the scope of the present paper it certainly represents a promising line of future research. And third, there

[20] Some caveats are in order. First, there is a difference in timing between the real world and the model. In particular, in the model we assume that the observation $x_{t|t}$ is available in period t. In the real world, data are released with a one period lag, that is, $x_{t|t+1}$ rather than $x_{t|t}$ is the data available. And second, in computing growth rates one may either use data from the same vintage (which then already includes a revision) or from two successive vintages. That is, either $\Delta x_{t|t} = \log(x_{t|t}) - \log(x_{t-1|t})$ or $\Delta x_{t|t} = \log(x_{t|t}) - \log(x_{t-1|t-1})$. It is not obvious which measure is more relevant for testing mis–perceptions in a model where agents' decisions involve levels of variables and there is no lag in the release of information. Consequently, we have chosen to report both measures.

is no serial correlation in the revisions.

4 Sensitivity analysis

In this section we discuss the implications of alternative specifications in order to judge the robustness of the main results and rankings reported above. The likelihood results from these exercises are reported in Tables 5 and 6.[21]

A natural question regards the importance of the distribution of the priors for the results obtained. It is should be clear from inspection of Table 1 that the priors we employ in the estimation above are quite diffuse. For instance, the 95% confidence interval for the Calvo parameter encompasses average price resetting from 1.1 to 10 quarters. Nonetheless, it may be worth repeating the analysis with even –essentially uninformative– flatter priors. Table 7 reports the new priors. Table 5 reports the posterior likelihood attained by each model both in the benchmark case and when diffuse priors are used. Inspection of the table shows an increase in the log–likelihood for all models, which indicates that the choice of the degree of tightness of the priors does matter. Nonetheless, the ranking of the alternative models is identical to that reported before. This is encouraging and should help dispel any concerns that our results may have been driven by the choice of the distribution of the priors.

Table 5: Likelihood: Sensitivity Analysis (Tightness of Priors, Period of Estimation)

Model	Benchmark	Diffuse Priors	Post–1982 period
(1)	-323.472	-313.883	-111.093
(2)	-267.388	-258.469	-111.131
(3)	-321.101	-317.726	-108.155
(4)	-296.148	-276.183	-109.042
(5)	-264.729	-255.363	-107.385

Monetary policy in the models considered here is perfectly credible. Consequently, and to the extent that the monetary authorities have a known, long term, inflation target, it is sensible to assume that the non-optimizing firms in model 2 would select to index their prices not only to past inflation but also to the inflation target itself. We have thus repeated the analysis using the price setting scheme $P_t(i) = \pi_{t-1}^{\gamma}\overline{\pi}_t^{1-\gamma}P_{t-1}(i)$, where $\overline{\pi}_t$, the inflation target of the central bank, follows an AR(1) process of the form:

$$\log(\overline{\pi}_t) = \rho_\pi \log(\overline{\pi}_{t-1}) + (1-\rho_\pi)\log(\overline{\pi}) + \varepsilon_{\pi,t}.$$

The second row in Table 6 presents the outcome of the estimation of this variant of model 2.

[21] More detailed results are reported in a companion technical appendix which is available on the authors' web pages.

There is no improvement relative the benchmark case reported above.

Allowing for partial indexation to past inflation also allows us to address a possible concern one might have from our having imposed perfect indexation ($\gamma = 1$) in the estimation in section 3. If the backward price indexation component is less than unity, then imposing a value of unity could disadvantage the hybrid model vis a vis the other models. We have therefore carried out the exercise for alternative values of the backward price indexation parameter[22] ($\gamma = 0.25, 0.50, 0.75$). Again there is no improvement in the performance of the model.

Table 6: Likelihood: Sensitivity Analysis (Hybrid NK Model)

Model	Likelihood
Benchmark	-267.388
Past Inflation (γ=0.25)	-281.865
Past Inflation (γ=0.50)	-279.587
Past Inflation (γ=0.75)	-275.926
C.B. Inflation Target (γ=0.58)	-278.186

For model 5, we have assumed that the agents observe both output and inflation with error. With only three endogenous variables this is necessary in order to make the signal extraction problem severe enough and give imperfect information a fighting chance to play a role in shaping the properties of the model. Nonetheless, it is of interest to see how the properties of the model would be affected if we minimized the role of mis–perceptions by allowing agents to also perfectly observe the inflation rate alongside the nominal interest rate.[23] In this case, the performance of this version of the model deteriorates significantly as the signal extraction problem becomes quite trivial. Our view is that the important issue here is not actually whether the inflation rate is accurately, contemporaneously observed or not but rather whether there is enough noise present in the system to make signal extraction a difficult task. A model with more variables could possess this property even if inflation were made observed with little noise as long as other endogenous variables were noisy and the structure of the model were such that the observation of inflation did not undermine the signal extraction problem.

A final sensitivity check carried out was to estimate the models for the post 1982 period. As is well known, this period has been associated with a significant decline in the volatility of inflation and output and, as revealed by IRFs of inflation, the reduction of inertia in inflation. This implies that a model, such as the hybrid, whose additional features relative to the baseline NK model aim to better capture inertia and volatility may not perform as well as in the full sample period. The last column of Table 5 shows that this conjecture is correct. The hybrid model does not improve upon the baseline model during that period. Table 5 also shows that the

[22]Price indexation now takes the form: $P_t(i) = \pi_{t-1}^{\gamma}\overline{\pi}^{1-\gamma}P_{t-1}(i)$, where $\overline{\pi}$ denotes steady state inflation.

[23]More detailed results are reported in the accompanying appendix.

imperfect information models do better than the two perfect information models. It is tempting to claim that these findings suggest that imperfect information may represent a more structural business cycle mechanism than other, commonly employed features (such as real rigidities or backward indexation). Nonetheless, the differences in the marginal likelihood across models are rather small, so this claim remains tenuous and deserves further investigation.

5 Conclusions

The concepts of imperfect information and mis–perceptions have a distinguished presence in macroeconomic theory going back to Lucas', 1972, and Kydland and Prescott's 1982, seminal work. Notwithstanding this history and also in spite of Orphanides' influential work on the role of mis–perceptions in the great inflation of the 70s, macroeconomic theory has more or less stayed clear of them. In this paper we have undertaken a comprehensive analysis of its role in US business cycles during the last 40 years using the New Keynesian model as the vehicle for the analysis. In particular, we have examined the performance of the model under different structures of information which are motivated by influential suggestions in the literature, such as unobserved variation in inflation target (Cogley and Sbordone, 2006) or, confusion between transitory and persistent real shocks (Kydland and Prescott, 1982, Orphanides, 2002). We found that imperfect information plays an important role in accounting for the US business cycle and it can help the NK model match key dynamic properties of the data (inertia, persistence) without any need for other, popular inertial features (such as real rigidities and backward price indexation schemes. Moreover, its success is not confined to a sample that also includes "unusual" macroeconomic behavior (the 70s) but it also extends to a period that only contains the great moderation. This makes us speculate that imperfect information may represent a structural business cycle mechanism than applies to a variety of macroeconomic environments.

Importantly, this good performance does not hinge on implausibly large amount of informational frictions. Even the version with the most severe information problems (the one with measurement errors) requires a degree of mis–perception that is well within the range observed in real time data. Nonetheless, given the fact that measurement errors in the real world do not fully conform to the standard assumption of independence made in the model, a fruitful line of future research may involve building more involved models of measurement error and incorporating them into the model in order to judge whether such errors truly constitute an important source of mis–perceptions.

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A Data

output = log(GDPC96/LNSindex)×100

inflation = log(GDPDEF/GDPDEF(-1))×100

interest rate = Federal Funds Rate/4

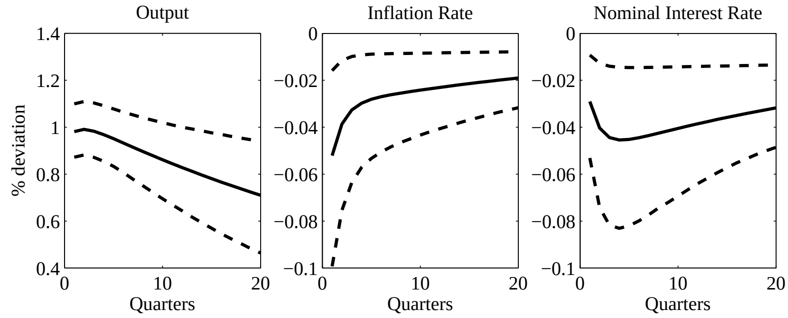
Source of the original data:

- GDPC96 : Real Gross Domestic Product - Billions of Chained 1996 Dollars, Seasonally Adjusted Annual Rate Source: U.S. Department of Commerce, Bureau of Economic Analysis.
- GDPDEF : Gross Domestic Product - Implicit Price Deflator - 1996=100, Seasonally Adjusted Source: U.S. Department of Commerce, Bureau of Economic Analysis
- Federal Funds Rate : Averages of Daily Figures - Percent Source: Board of Governors of the Federal Reserve System
- LNS10000000 : Labor Force Status : Civilian noninstitutional population - Age : 16 years and over - Seasonally Adjusted - Number in thousands Source: U.S. Bureau of Labor Statistics (Before 1976: LFU800000000 : Population level - 16 Years and Older)
- LNSindex : LNS10000000(1992:3)=1

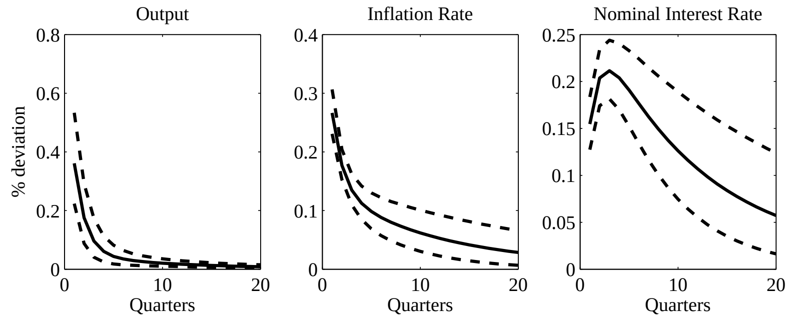
B Figures 1966–2002

Figure 2: Impulse Response Functions – Perfect info, forward NK

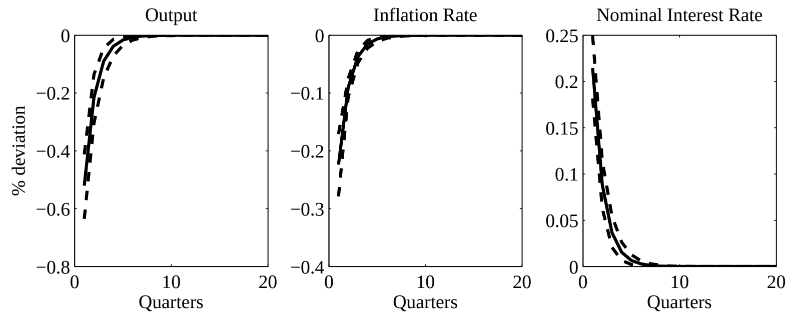
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Cost Push Shock

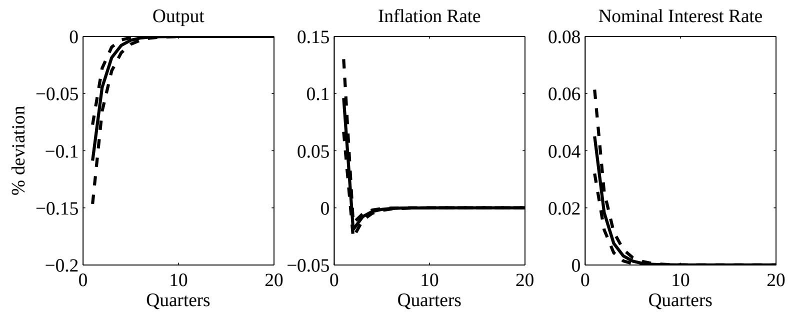
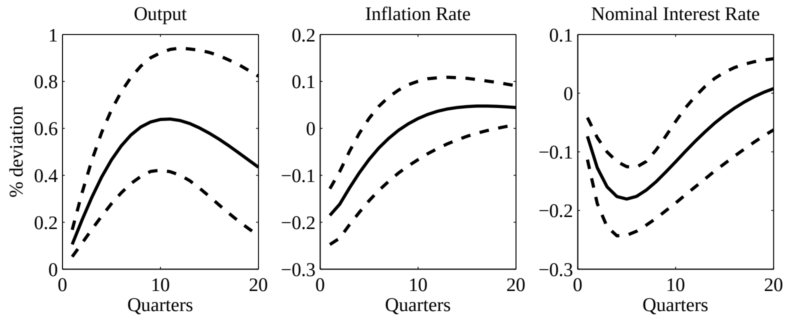
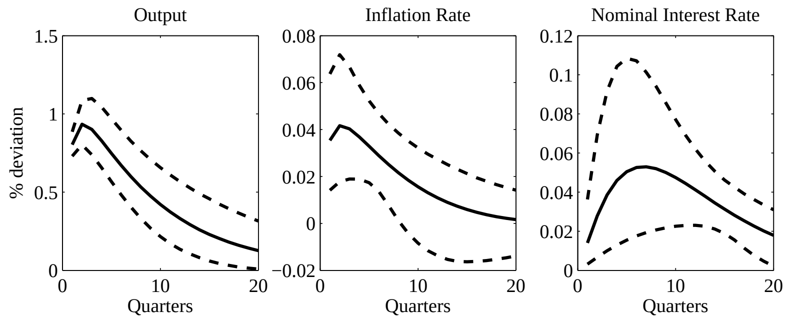


Figure 3: Impulse Response Functions – Perfect info, hybrid NK

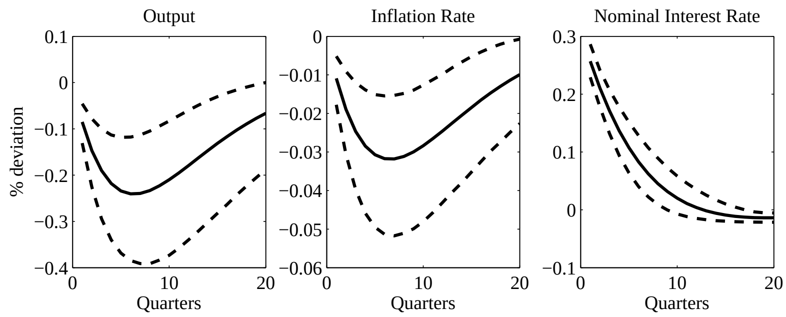
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Cost Push Shock

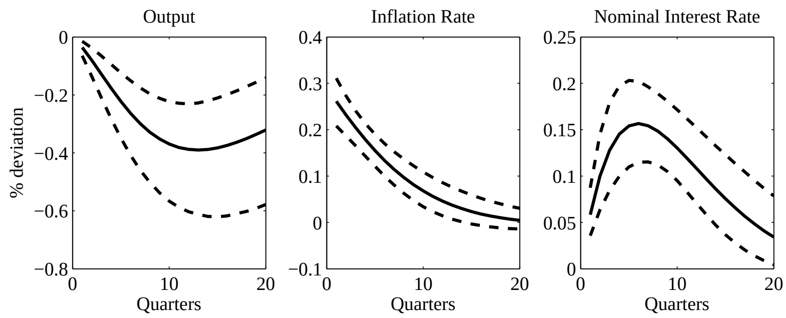
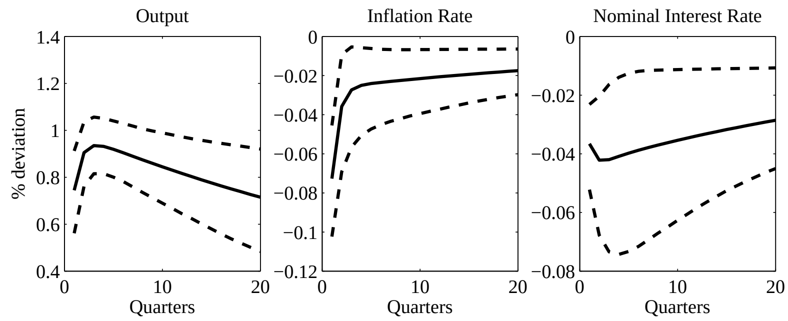
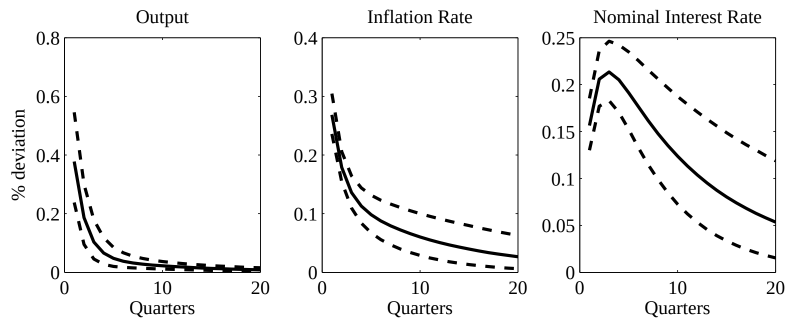


Figure 4: Impulse Response Functions – Imperfect info, Pers. vs Temp

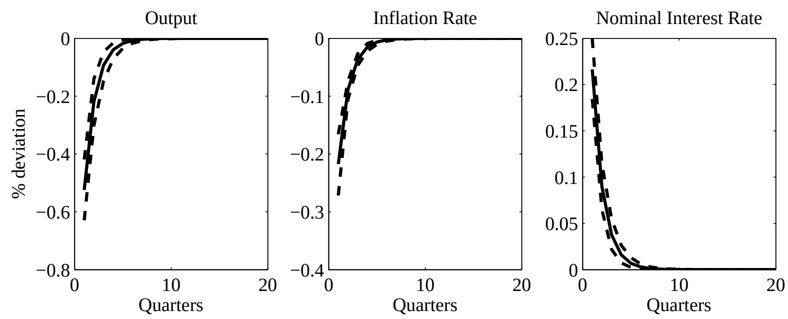
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Cost Push Shock

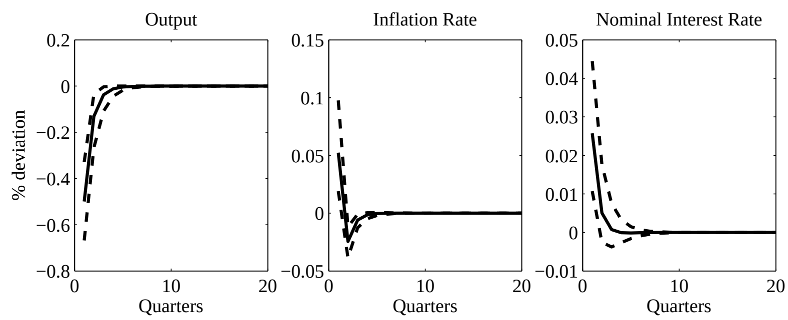
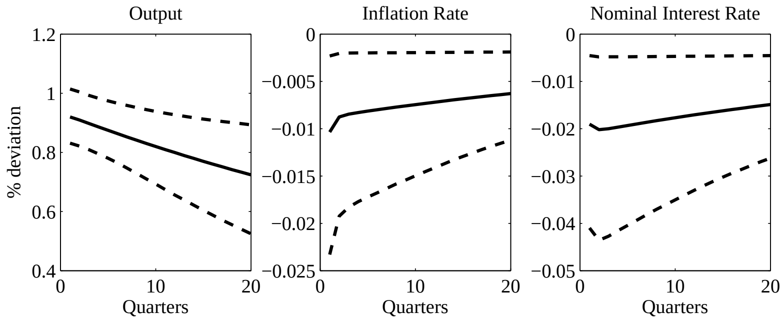
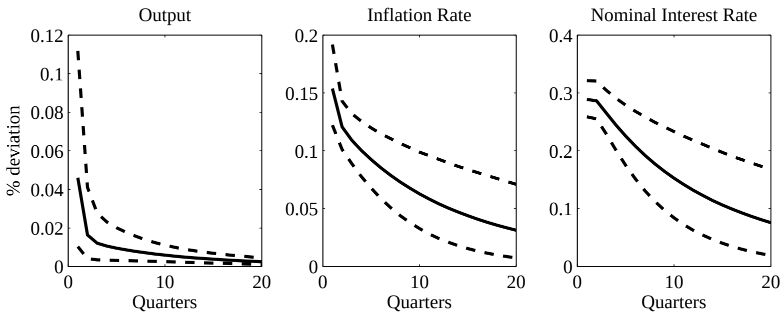


Figure 5: Impulse Response Functions– Imperfect info, Inflation target shock

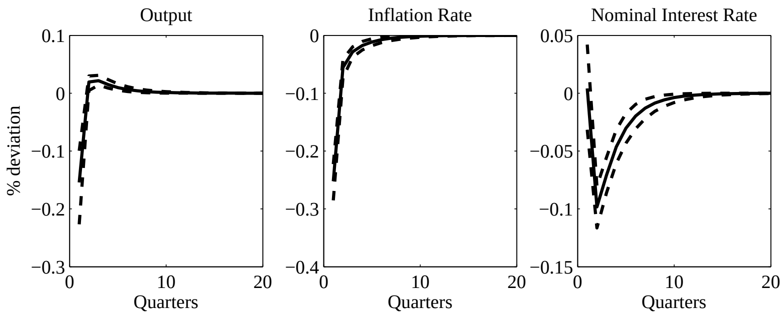
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Inflation Target Shock

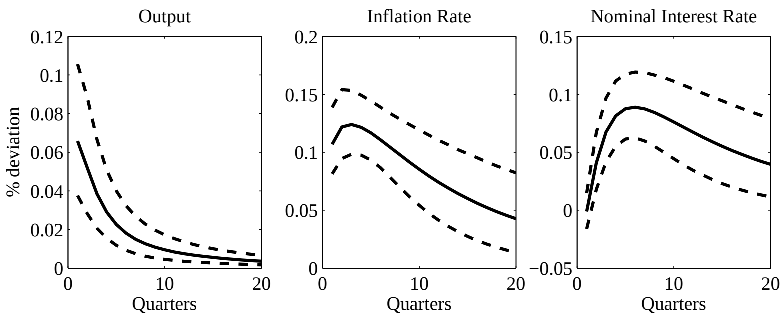
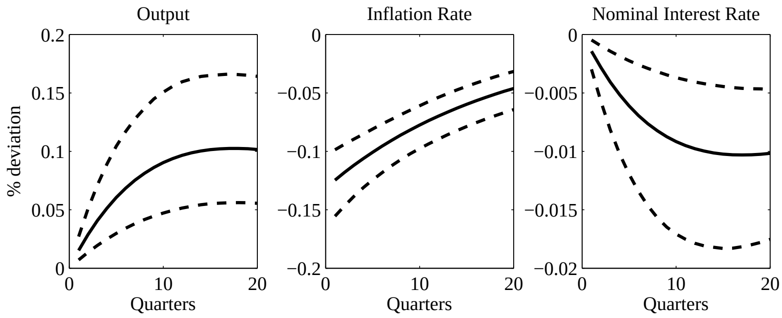
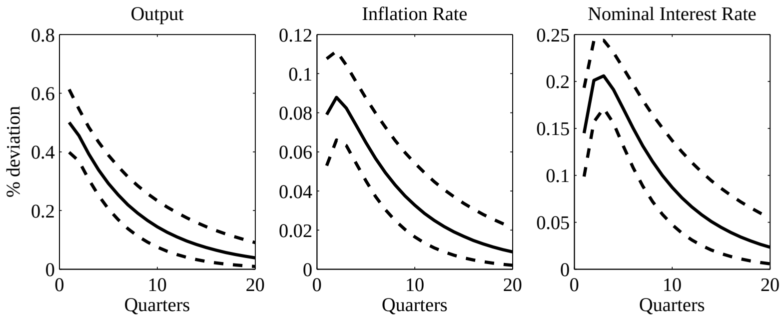


Figure 6: Impulse Response Functions – Imperfect info, measurement errors

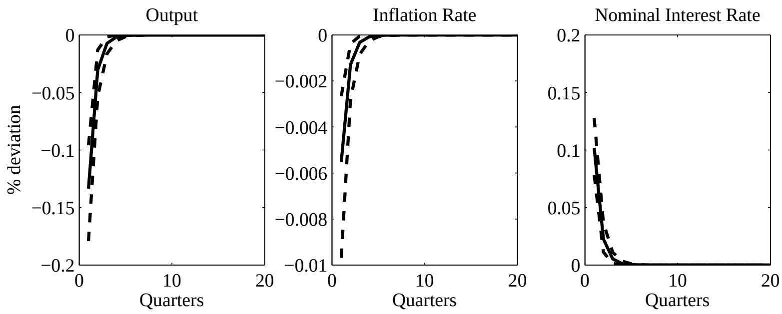
(a) Technology Shock



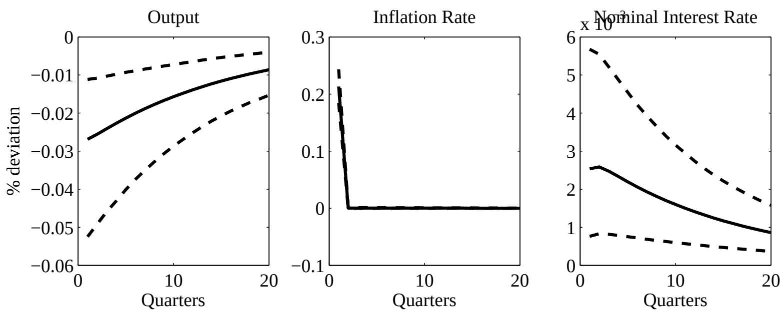
(b) Preference Shock



(c) Interest Rate Shock



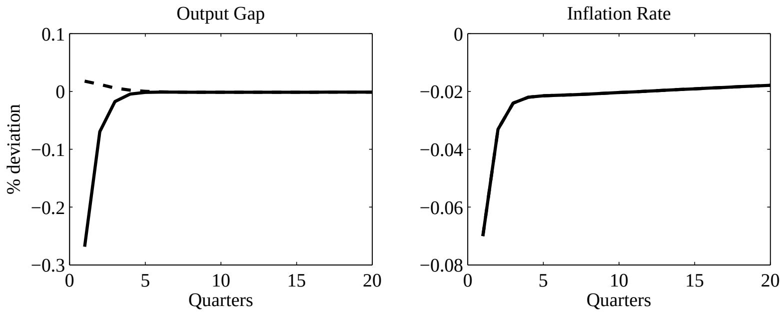
(d) Cost Push Shock



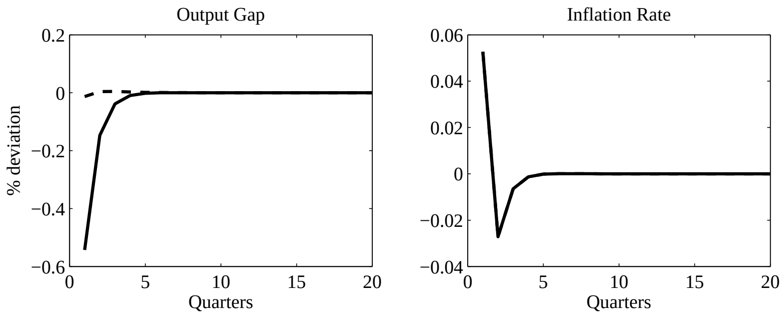
B.1 The degree of mis-perceptions of shocks

Figure 7: IRF: Actual vs Perceived — Imperfect info, Pers. vs Temp

(a) Technology Shock



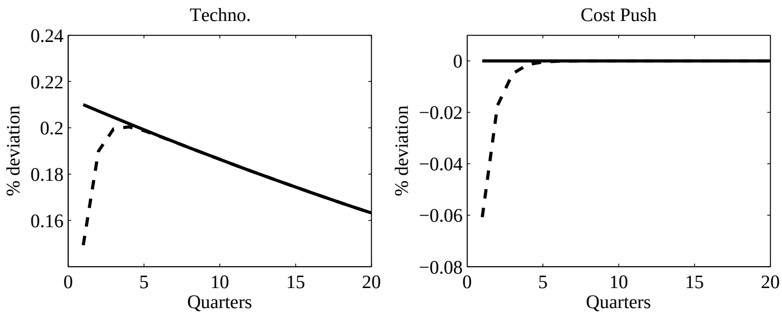
(b) Cost Push Shock



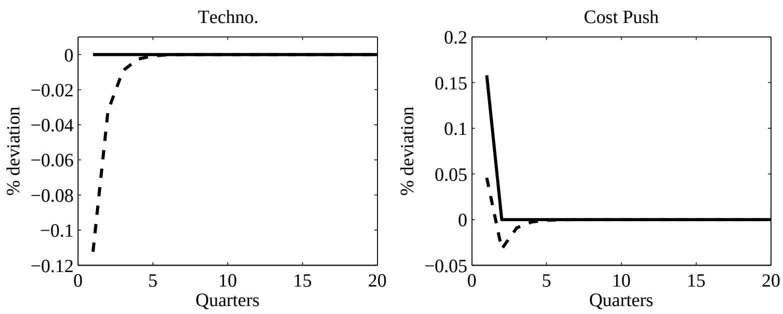
Note: Plain line: True, Dashed line: Perceived

Figure 8: IRF: Actual vs Perceived — Imperfect info, Pers. vs Temp.

(a) Technology Shock



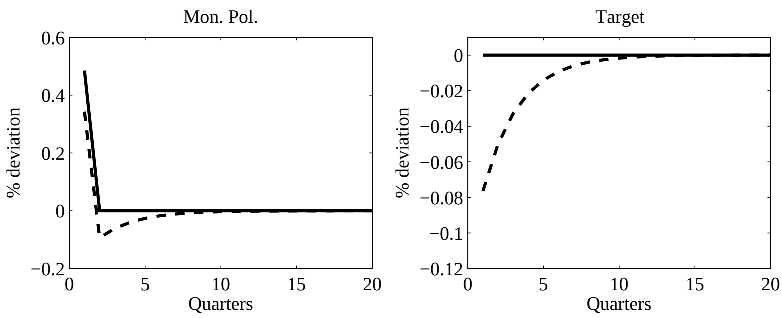
(b) Cost Push Shock



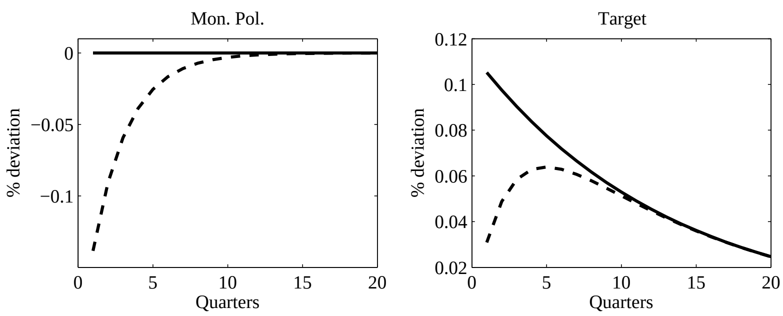
Note: Plain line: True, Dashed line: Perceived

Figure 9: IRF: Actual vs Perceived — Imperfect info, Inflation target shock

(a) Monetary Policy Shock



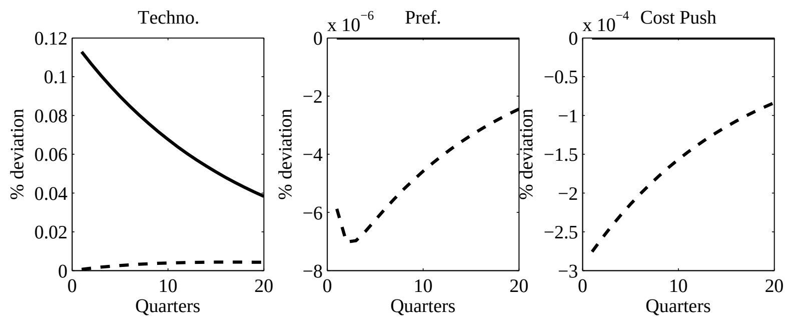
(b) Inflation Target Shock



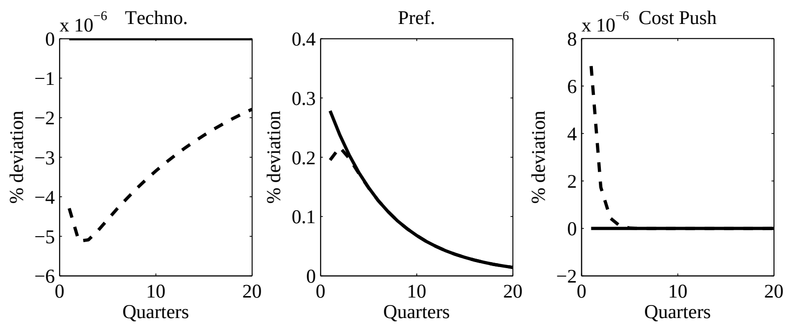
Note: Plain line: True, Dashed line: Perceived

Figure 10: IRF: Actual vs Perceived — Imperfect info, Measurement errors

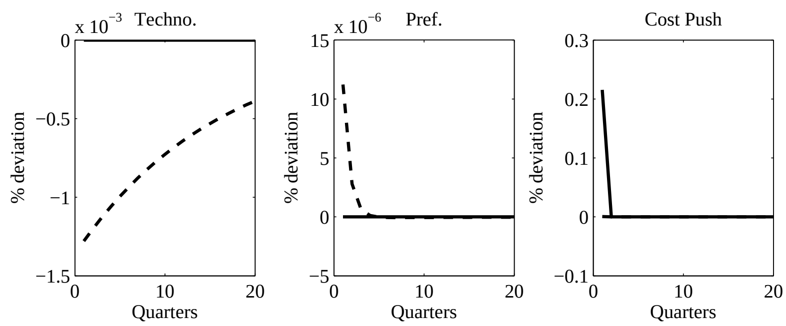
(a) Technology Shock



(b) Preference Shock



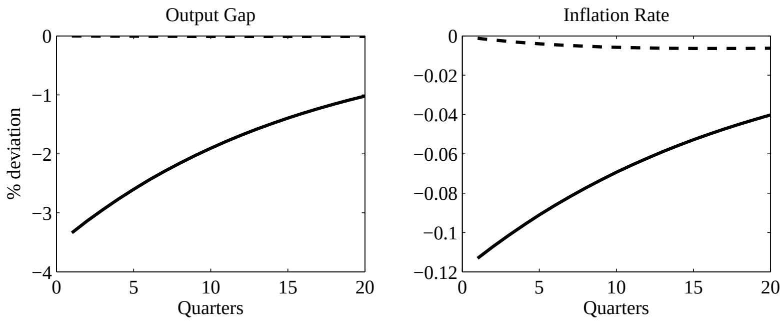
(c) Cost Push Shock



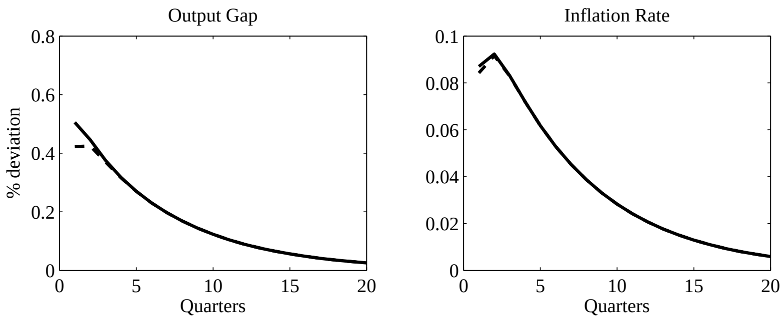
Note: Plain line: True, Dashed line: Perceived

Figure 11: IRF: Actual vs Perceived — Imperfect info, Measurement errors

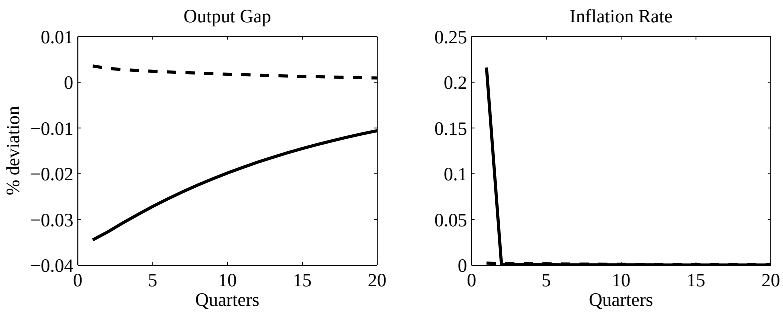
(a) Technology Shock



(b) Preference Shock



(c) Cost Push Shock



Note: Plain line: True, Dashed line: Perceived

Technical Appendix
Not Intended for Publication

A Robustness Analysis

Table 7: Diffuse Priors

Param.	Type	Param 1	Param 2	95% HPDI
ϑ	Uniform	0.00	1.00	[0.025;0.975]
ξ	Uniform	0.00	1.00	[0.025;0.975]
φ	Uniform	0.00	1.00	[0.025;0.975]
$r^{\star}$	Uniform	0.00	4.00	[0.10;3.90]
$\pi^{\star}$	Uniform	0.00	4.00	[0.10;3.90]
ρ_r	Uniform	0.00	1.00	[0.025;0.975]
α_π	Normal	1.50	0.50	[0.52;2.47]
α_y	Normal	0.125	0.05	[0.027;0.222]
ρ_a	Uniform	0.00	1.00	[0.025;0.975]
ρ_χ	Uniform	0.00	1.00	[0.025;0.975]
ρ_π	Uniform	0.50	0.50	[0.025;0.975]
σ_a	Invgamma	0.20	4.00	[0.10;0.38]
σ_χ	Invgamma	0.20	4.00	[0.10;0.38]
σ_r	Invgamma	0.20	4.00	[0.10;0.38]
σ_π	Invgamma	0.20	4.00	[0.10;0.38]
σ_ν	Invgamma	0.20	4.00	[0.10;0.38]
η_y	Invgamma	0.20	4.00	[0.10;0.38]
η_π	Invgamma	0.20	4.00	[0.10;0.38]

Note: The parameters are distributed independently from each other.
[a] 95-percent highest probability density (HPD) credible intervals (see **?**, p.57). The Param 1 and Param 2 report the lower and upper bounds for Uniform distributions, the mean and the standard deviation for the Normal distributions. They report the s and ν parameters of the inverse gamma distribution, where $f(\sigma|s,\nu) \propto \sigma^{-(1+\nu)}\exp(-\nu s^2/2\sigma^2)$.

Table 8: Moments: Data linearly detrended

	Data	(1)	(2)	(3)	(4)	(5)
σ_y	3.37	5.87	4.09	6.11	6.76	2.50
		[3.11,9.86]	[2.49,5.94]	[2.95,10.39]	[3.13,12.08]	[2.00,3.09]
σ_π	0.62	0.55	0.70	0.54	0.62	0.51
		[0.45,0.68]	[0.57,0.85]	[0.44,0.67]	[0.48,0.81]	[0.41,0.61]
σ_R	0.79	0.74	0.89	0.72	0.93	0.60
		[0.55,1.00]	[0.70,1.10]	[0.53,0.97]	[0.63,1.33]	[0.48,0.74]
$\rho(\pi, y)$	-0.27	-0.18	0.08	-0.17	-0.06	-0.01
		[-0.35,-0.04]	[-0.26,0.37]	[-0.32,-0.03]	[-0.10,-0.02]	[-0.15,0.15]
$\rho(R, y)$	-0.39	-0.32	-0.36	-0.31	-0.13	0.03
		[-0.52,-0.12]	[-0.65,-0.05]	[-0.50,-0.12]	[-0.23,-0.04]	[-0.17,0.27]
$\rho_y(1)$	0.97	0.97	0.97	0.97	0.99	0.93
		[0.94,1.00]	[0.95,0.99]	[0.94,1.00]	[0.97,1.00]	[0.90,0.96]
$\rho_\pi(1)$	0.87	0.69	0.88	0.70	0.81	0.76
		[0.56,0.82]	[0.83,0.92]	[0.57,0.83]	[0.71,0.90]	[0.66,0.85]
$\rho_R(1)$	0.93	0.90	0.94	0.90	0.93	0.89
		[0.85,0.96]	[0.91,0.96]	[0.84,0.96]	[0.89,0.98]	[0.85,0.94]
$\rho_y(2)$	0.92	0.95	0.93	0.95	0.97	0.86
		[0.89,0.99]	[0.87,0.98]	[0.89,1.00]	[0.94,1.00]	[0.81,0.91]
$\rho_\pi(2)$	0.83	0.56	0.75	0.57	0.74	0.71
		[0.39,0.74]	[0.67,0.84]	[0.39,0.73]	[0.61,0.87]	[0.60,0.80]
$\rho_R(2)$	0.84	0.83	0.86	0.82	0.86	0.79
		[0.74,0.92]	[0.80,0.91]	[0.73,0.91]	[0.77,0.95]	[0.71,0.87]
$\rho_y(4)$	0.79	0.91	0.82	0.91	0.95	0.76
		[0.81,0.99]	[0.70,0.93]	[0.82,0.99]	[0.89,1.00]	[0.67,0.84]
$\rho_\pi(4)$	0.79	0.44	0.54	0.44	0.64	0.62
		[0.25,0.64]	[0.40,0.69]	[0.26,0.65]	[0.46,0.80]	[0.51,0.73]
$\rho_R(4)$	0.69	0.70	0.68	0.69	0.74	0.61
		[0.58,0.86]	[0.55,0.80]	[0.55,0.84]	[0.59,0.90]	[0.50,0.73]

Note: (1): Baseline NK, (2): Hybrid NK (Backward Indexation, Real Rigidities), (3): Imperfect Info. Temporary vs Permanent Shocks, (4): Imperfect Info., Cogley–Sbordone, (5): Imperfect Info., Noisy Signals.

B Detailed Tables, 1966-2002

Table 9: Posteriors – Perfect Info, Forward NK

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.48	0.49	0.50	0.08	[0.33, 0.64]
φ	0.25	0.26	0.26	0.11	[0.04, 0.48]
$r^\star$	0.63	0.61	0.62	0.14	[0.31, 0.86]
$\pi^\star$	0.93	0.90	0.92	0.18	[0.53, 1.24]
ρ_r	0.68	0.67	0.68	0.04	[0.60, 0.75]
α_π	1.54	1.61	1.60	0.16	[1.29, 1.90]
α_y	0.13	0.13	0.13	0.05	[0.04, 0.23]
ρ_a	0.98	0.98	0.98	0.01	[0.96, 1.00]
ρ_χ	0.91	0.91	0.91	0.03	[0.85, 0.97]
σ_a	0.24	0.27	0.26	0.08	[0.14, 0.44]
σ_χ	0.15	0.16	0.16	0.03	[0.11, 0.22]
σ_r	0.34	0.36	0.35	0.03	[0.30, 0.41]
σ_ν	0.14	0.14	0.14	0.02	[0.10, 0.19]
Average log marginal density:					-323.472

Table 10: Posteriors – Perfect Info, Hybrid NK

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.94	0.94	0.94	0.02	[0.90, 0.97]
ξ	0.05	0.08	0.08	0.03	[0.03, 0.14]
φ	0.26	0.29	0.29	0.11	[0.07, 0.51]
$r^\star$	0.66	0.66	0.66	0.15	[0.36, 0.97]
$\pi^\star$	1.03	1.01	1.01	0.16	[0.69, 1.33]
ρ_r	0.86	0.84	0.84	0.04	[0.77, 0.91]
α_π	1.40	1.40	1.38	0.18	[1.06, 1.75]
α_y	0.00	0.04	0.03	0.04	[0.00, 0.12]
ρ_a	0.02	0.07	0.06	0.05	[0.00, 0.18]
ρ_χ	0.23	0.26	0.26	0.08	[0.09, 0.42]
σ_a	0.12	0.11	0.11	0.02	[0.08, 0.14]
σ_χ	0.61	0.62	0.62	0.07	[0.48, 0.77]
σ_r	0.26	0.26	0.26	0.02	[0.23, 0.29]
σ_ν	0.12	0.15	0.15	0.02	[0.11, 0.18]
Average log marginal density:					-267.388

Table 11: Posteriors – Imperfect Info, Pers. vs Temp.

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.47	0.49	0.49	0.08	[0.34, 0.64]
φ	0.25	0.27	0.26	0.11	[0.05, 0.49]
$r^\star$	0.64	0.62	0.63	0.13	[0.34, 0.87]
$\pi^\star$	0.93	0.91	0.93	0.17	[0.57, 1.26]
ρ_r	0.68	0.67	0.68	0.04	[0.60, 0.74]
α_π	1.50	1.58	1.58	0.15	[1.31, 1.90]
α_y	0.15	0.15	0.15	0.05	[0.05, 0.24]
ρ_a	0.99	0.98	0.98	0.01	[0.96, 1.00]
ρ_χ	0.91	0.91	0.91	0.03	[0.85, 0.97]
σ_a	0.21	0.25	0.24	0.08	[0.12, 0.40]
σ_χ	0.16	0.17	0.17	0.03	[0.12, 0.23]
σ_r	0.34	0.35	0.35	0.03	[0.30, 0.41]
σ_ν	0.16	0.16	0.16	0.03	[0.11, 0.22]
Average log marginal density:					-321.101

Table 12: Posteriors – Imperfect info, Inflation target shock

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.70	0.68	0.69	0.08	[0.51, 0.83]
φ	0.24	0.24	0.24	0.12	[0.02, 0.46]
$r^{\star}$	0.59	0.56	0.58	0.21	[0.11, 0.92]
$\pi^{\star}$	0.89	0.86	0.88	0.24	[0.34, 1.30]
ρ_r	0.20	0.21	0.20	0.10	[0.02, 0.39]
α_π	2.33	2.37	2.37	0.17	[2.05, 2.70]
α_y	0.13	0.13	0.13	0.05	[0.03, 0.22]
ρ_a	0.99	0.99	0.99	0.01	[0.97, 1.00]
ρ_χ	0.92	0.92	0.92	0.03	[0.85, 0.98]
ρ_π	0.93	0.92	0.93	0.03	[0.86, 0.98]
σ_a	0.78	0.81	0.77	0.28	[0.35, 1.38]
σ_χ	0.19	0.20	0.20	0.02	[0.16, 0.24]
σ_r	0.49	0.49	0.49	0.07	[0.37, 0.63]
σ_π	0.11	0.11	0.11	0.02	[0.08, 0.15]
Average log marginal density:					-296.147

Table 13: Posteriors – Imperfect info, Measurement errors

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.23	0.22	0.22	0.04	[0.14, 0.31]
φ	0.26	0.28	0.28	0.11	[0.06, 0.50]
$r^{\star}$	0.64	0.62	0.62	0.20	[0.22, 1.02]
$\pi^{\star}$	0.98	0.97	0.97	0.20	[0.57, 1.34]
ρ_r	0.27	0.27	0.28	0.07	[0.14, 0.42]
α_π	1.62	1.64	1.64	0.23	[1.19, 2.10]
α_y	0.19	0.20	0.20	0.04	[0.11, 0.28]
ρ_a	0.94	0.95	0.95	0.01	[0.92, 0.97]
ρ_χ	0.85	0.86	0.86	0.03	[0.80, 0.92]
σ_a	0.11	0.12	0.12	0.02	[0.09, 0.16]
σ_χ	0.28	0.29	0.28	0.04	[0.21, 0.37]
σ_r	0.12	0.13	0.13	0.02	[0.09, 0.16]
σ_ν	0.22	0.21	0.21	0.02	[0.18, 0.25]
η_y	0.21	0.28	0.25	0.12	[0.10, 0.52]
η_π	4.34	6.72	6.10	3.02	[2.18, 13.15]
Average log marginal density:					-264.729

B.1 Detailed Tables, More diffuse priors

Table 14: Posteriors – Perfect Info, Forward NK

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.56	0.58	0.59	0.12	[0.35,0.77]
φ	0.40	0.48	0.46	0.28	[0.01,0.93]
$r^\star$	0.66	0.65	0.66	0.14	[0.36,0.94]
$\pi^\star$	0.96	0.96	0.96	0.18	[0.59,1.32]
ρ_r	0.68	0.68	0.68	0.04	[0.60,0.75]
α_π	1.55	1.64	1.64	0.18	[1.29,2.02]
α_y	0.14	0.15	0.14	0.07	[0.02,0.28]
ρ_a	0.99	0.98	0.98	0.01	[0.96,1.00]
ρ_χ	0.92	0.92	0.92	0.03	[0.86,0.98]
σ_a	0.26	0.29	0.28	0.09	[0.14,0.48]
σ_χ	0.15	0.16	0.16	0.03	[0.11,0.21]
σ_r	0.35	0.36	0.36	0.03	[0.30,0.42]
σ_ν	0.14	0.15	0.14	0.03	[0.10,0.19]
Average log marginal density:					-313.883

Table 15: Posteriors – Perfect Info, Hybrid NK

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.93	0.94	0.94	0.02	[0.90, 0.98]
ξ	0.08	0.09	0.09	0.04	[0.03, 0.17]
φ	1.00	0.56	0.58	0.26	[0.12, 1.00]
$r^\star$	0.66	0.70	0.70	0.15	[0.40, 0.99]
$\pi^\star$	1.15	1.03	1.03	0.16	[0.73, 1.34]
ρ_r	0.85	0.85	0.85	0.04	[0.77, 0.92]
α_π	1.26	1.40	1.37	0.22	[1.00, 1.83]
α_y	0.00	0.03	0.02	0.03	[0.00, 0.09]
ρ_a	0.00	0.05	0.04	0.05	[0.00, 0.15]
ρ_χ	0.42	0.24	0.24	0.08	[0.07, 0.40]
σ_a	0.08	0.11	0.11	0.02	[0.08, 0.14]
σ_χ	0.47	0.63	0.63	0.07	[0.49, 0.78]
σ_r	0.26	0.26	0.26	0.02	[0.23, 0.30]
σ_ν	0.15	0.14	0.14	0.02	[0.10, 0.18]
Average log marginal density:					-258.469

Table 16: Posteriors – Imperfect Info, Pers. vs Temp.

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.60	0.57	0.59	0.12	[0.34, 0.77]
φ	0.60	0.51	0.51	0.27	[0.06, 0.98]
$r^\star$	0.66	0.66	0.67	0.14	[0.37, 0.93]
$\pi^\star$	0.96	0.97	0.98	0.18	[0.60, 1.31]
ρ_r	0.68	0.68	0.68	0.04	[0.60, 0.76]
α_π	1.48	1.59	1.58	0.19	[1.20, 1.96]
α_y	0.17	0.17	0.17	0.07	[0.04, 0.31]
ρ_a	0.99	0.99	0.99	0.01	[0.97, 1.00]
ρ_χ	0.91	0.91	0.91	0.03	[0.85, 0.97]
σ_a	0.21	0.25	0.24	0.09	[0.11, 0.41]
σ_χ	0.15	0.17	0.16	0.03	[0.11, 0.23]
σ_r	0.34	0.36	0.35	0.03	[0.30, 0.42]
σ_ν	0.16	0.16	0.16	0.03	[0.11, 0.22]
Average log marginal density:					-317.726

Table 17: Posteriors – Imperfect info, Inflation target shock

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.70	0.76	0.78	0.10	[0.56, 0.91]
φ	0.14	0.38	0.36	0.23	[0.00, 0.74]
$r^\star$	0.65	0.68	0.67	0.25	[0.15, 1.16]
$\pi^\star$	0.95	0.94	0.94	0.27	[0.39, 1.47]
ρ_r	0.08	0.18	0.17	0.11	[0.00, 0.37]
α_π	2.90	3.03	3.01	0.29	[2.47, 3.61]
α_y	0.12	0.14	0.14	0.07	[0.02, 0.26]
ρ_a	1.00	0.99	0.99	0.01	[0.97, 1.00]
ρ_χ	0.92	0.92	0.93	0.03	[0.86, 0.98]
ρ_π	0.94	0.94	0.94	0.03	[0.89, 0.99]
σ_a	1.14	1.04	1.00	0.33	[0.44, 1.71]
σ_χ	0.20	0.21	0.21	0.02	[0.18, 0.25]
σ_r	0.64	0.61	0.60	0.09	[0.43, 0.78]
σ_π	0.11	0.11	0.11	0.02	[0.08, 0.14]
Average log marginal density:					-276.183

Table 18: Posteriors – Imperfect info, Measurement errors

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.29	0.30	0.30	0.07	[0.15, 0.42]
φ	0.50	0.60	0.64	0.27	[0.11, 1.00]
$r^\star$	0.69	0.68	0.68	0.21	[0.28, 1.11]
$\pi^\star$	0.98	0.97	0.97	0.19	[0.59, 1.34]
ρ_r	0.28	0.29	0.29	0.08	[0.14, 0.44]
α_π	1.61	1.76	1.74	0.39	[1.00, 2.45]
α_y	0.23	0.23	0.23	0.06	[0.11, 0.35]
ρ_a	0.95	0.95	0.95	0.01	[0.92, 0.97]
ρ_χ	0.85	0.86	0.86	0.03	[0.80, 0.93]
σ_a	0.11	0.12	0.12	0.02	[0.09, 0.16]
σ_χ	0.28	0.30	0.29	0.04	[0.22, 0.38]
σ_r	0.12	0.13	0.13	0.02	[0.09, 0.17]
σ_ν	0.22	0.21	0.21	0.02	[0.18, 0.25]
η_y	0.20	0.28	0.24	0.13	[0.11, 0.53]
η_π	4.33	6.39	5.81	2.85	[2.08, 12.47]
Average log marginal density:					-255.363

B.2 Detailed Tables, Alternative Specifications

Table 19: Posteriors – Perfect Info, Hybrid NK (**Partial indexation to the inflation target**)

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.94	0.92	0.93	0.02	[0.88,0.97]
ξ	0.02	0.07	0.07	0.02	[0.03,0.12]
γ	0.85	0.88	0.89	0.05	[0.79,0.97]
φ	0.26	0.28	0.28	0.11	[0.07,0.51]
$r^\star$	0.67	0.67	0.67	0.12	[0.42,0.92]
$\pi^\star$	1.02	1.02	1.02	0.13	[0.75,1.27]
ρ_r	0.84	0.76	0.76	0.06	[0.64,0.86]
α_π	1.21	1.21	1.19	0.13	[1.01,1.47]
α_y	0.00	0.02	0.02	0.01	[0.00,0.03]
ρ_a	0.03	0.08	0.07	0.06	[0.00,0.19]
ρ_χ	0.24	0.28	0.28	0.09	[0.12,0.45]
ρ_π	0.02	0.09	0.08	0.07	[0.00,0.22]
σ_a	0.12	0.12	0.12	0.02	[0.09,0.16]
σ_χ	0.61	0.60	0.59	0.07	[0.45,0.74]
σ_r	0.26	0.23	0.23	0.03	[0.17,0.29]
σ_π	0.12	0.12	0.12	0.02	[0.09,0.16]
Average log marginal density:					-278.186

Table 20: Posteriors – Imperfect info, Measurement errors (R **and** π **are perfectly observable**)

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.46	0.48	0.48	0.08	[0.34,0.64]
φ	0.25	0.27	0.27	0.12	[0.05,0.50]
$r^\star$	0.64	0.62	0.63	0.13	[0.37,0.89]
$\pi^\star$	0.93	0.92	0.94	0.17	[0.54,1.24]
ρ_r	0.68	0.67	0.68	0.04	[0.60,0.74]
α_π	1.49	1.57	1.56	0.16	[1.27,1.88]
α_y	0.15	0.15	0.14	0.05	[0.05,0.24]
ρ_a	0.99	0.98	0.99	0.01	[0.96,1.00]
ρ_χ	0.90	0.90	0.90	0.03	[0.84,0.96]
σ_a	0.21	0.24	0.23	0.07	[0.12,0.39]
σ_χ	0.17	0.19	0.18	0.03	[0.12,0.26]
σ_r	0.34	0.35	0.35	0.03	[0.30,0.41]
σ_ν	0.15	0.15	0.15	0.03	[0.10,0.20]
η_y	0.18	0.22	0.20	0.08	[0.10,0.38]
Average log marginal density:					-324.209

C Detailed Tables, Post–1982

Table 21: Posteriors – Perfect Info, Forward NK (Post 82 period)

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.62	0.75	0.76	0.09	[0.57,0.91]
φ	0.27	0.24	0.23	0.12	[0.02,0.46]
$r^\star$	0.15	0.75	0.73	0.38	[0.08,1.46]
$\pi^\star$	0.01	0.52	0.50	0.30	[0.00,1.04]
ρ_r	0.57	0.37	0.38	0.14	[0.10,0.61]
α_π	2.15	2.25	2.25	0.15	[1.98,2.55]
α_y	0.13	0.13	0.13	0.05	[0.04,0.23]
ρ_a	0.99	0.99	0.99	0.01	[0.97,1.00]
ρ_χ	0.99	0.98	0.98	0.01	[0.95,1.00]
σ_a	0.37	1.12	1.07	0.54	[0.28,2.22]
σ_χ	0.11	0.12	0.12	0.01	[0.10,0.15]
σ_r	0.27	0.36	0.35	0.06	[0.25,0.48]
σ_ν	0.13	0.15	0.14	0.03	[0.09,0.21]
Average log marginal density:					-111.093007

Table 22: Posteriors – Perfect Info, Hybrid NK (Post 82 period)

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
θ	0.58	0.89	0.91	0.09	[0.76,0.99]
ξ	0.47	0.14	0.12	0.09	[0.02,0.27]
φ	0.26	0.28	0.28	0.11	[0.06,0.51]
$r^{\star}$	0.93	0.80	0.80	0.22	[0.34,1.20]
$\pi^{\star}$	0.72	0.76	0.76	0.18	[0.39,1.14]
ρ_r	0.70	0.80	0.80	0.05	[0.69,0.90]
α_π	1.94	1.52	1.50	0.23	[1.08,1.99]
α_y	0.14	0.09	0.09	0.05	[0.01,0.18]
ρ_a	0.98	0.14	0.08	0.20	[0.00,0.74]
ρ_χ	0.87	0.63	0.64	0.13	[0.36,0.86]
σ_a	0.15	0.10	0.10	0.03	[0.07,0.14]
σ_χ	0.11	0.27	0.26	0.08	[0.12,0.41]
σ_r	0.23	0.20	0.20	0.02	[0.16,0.24]
σ_ν	0.15	0.13	0.13	0.02	[0.10,0.17]
Average log marginal density:					-111.131

Table 23: Posteriors – Imperfect Info, Pers. vs Temp. (Post 82 period)

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.65	0.66	0.66	0.10	[0.45,0.85]
φ	0.26	0.25	0.24	0.11	[0.03,0.46]
$r^{\star}$	0.15	0.78	0.77	0.37	[0.09,1.45]
$\pi^{\star}$	0.00	0.55	0.54	0.30	[0.01,1.08]
ρ_r	0.51	0.47	0.48	0.11	[0.24,0.65]
α_π	2.16	2.20	2.19	0.15	[1.90,2.48]
α_y	0.13	0.14	0.14	0.05	[0.05,0.24]
ρ_a	0.99	0.99	0.99	0.01	[0.97,1.00]
ρ_χ	0.99	0.98	0.98	0.01	[0.95,1.00]
σ_a	0.45	0.61	0.52	0.37	[0.15,1.30]
σ_χ	0.11	0.12	0.12	0.01	[0.10,0.15]
σ_r	0.29	0.32	0.31	0.05	[0.24,0.43]
σ_ν	0.17	0.18	0.18	0.04	[0.11,0.27]
Average log marginal density:					-108.155552

Table 24: Posteriors – Imperfect info, Inflation target shock (Post 82 period)

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.63	0.67	0.68	0.09	[0.50,0.83]
φ	0.24	0.25	0.24	0.12	[0.02,0.46]
$r^{\star}$	0.13	0.76	0.74	0.37	[0.07,1.44]
$\pi^{\star}$	0.01	0.52	0.51	0.30	[0.00,1.04]
ρ_r	0.40	0.30	0.31	0.12	[0.06,0.53]
α_π	2.20	2.25	2.24	0.15	[1.96,2.54]
α_y	0.13	0.13	0.13	0.05	[0.04,0.22]
ρ_a	0.99	0.99	0.99	0.01	[0.97,1.00]
ρ_χ	0.99	0.98	0.98	0.01	[0.95,1.00]
ρ_π	0.36	0.31	0.31	0.11	[0.10,0.55]
σ_a	0.44	0.62	0.57	0.22	[0.27,1.09]
σ_χ	0.12	0.13	0.13	0.01	[0.10,0.15]
σ_r	0.21	0.21	0.21	0.06	[0.11,0.32]
σ_π	0.19	0.20	0.20	0.03	[0.13,0.26]
Average log marginal density:					-109.042291

Table 25: Posteriors – Imperfect info, Measurement errors (Post 82 period)

Param.	Mode	Mean	Median	Std. Dev.	95% HPDI
ξ	0.17	0.17	0.17	0.04	[0.10, 0.26]
φ	0.26	0.29	0.28	0.11	[0.07, 0.51]
$r^{\star}$	0.97	0.95	0.96	0.19	[0.60, 1.33]
$\pi^{\star}$	0.75	0.75	0.75	0.15	[0.46, 1.04]
ρ_r	0.38	0.38	0.39	0.09	[0.20, 0.56]
α_π	1.70	1.73	1.73	0.22	[1.30, 2.17]
α_y	0.18	0.19	0.19	0.04	[0.10, 0.27]
ρ_a	0.89	0.89	0.90	0.03	[0.84, 0.94]
ρ_χ	0.92	0.92	0.92	0.03	[0.87, 0.96]
σ_a	0.09	0.10	0.10	0.01	[0.07, 0.13]
σ_χ	0.20	0.22	0.21	0.03	[0.16, 0.29]
σ_r	0.12	0.12	0.12	0.02	[0.09, 0.15]
σ_ν	0.16	0.16	0.16	0.02	[0.13, 0.20]
η_y	0.20	0.27	0.24	0.13	[0.11, 0.52]
η_π	2.97	4.43	4.04	2.22	[0.87, 8.78]
Average log marginal density:					-107.385365

Table 26: Moments (Post 82 period)

	Data	(1)	(2)	(3)	(4)	(5)
σ_y	2.08	5.38	3.64	5.53	5.52	1.76
		[2.20,9.81]	[1.82,6.14]	[2.34,10.06]	[2.30,9.92]	[1.36,2.19]
σ_π	0.28	0.62	0.64	0.60	0.64	0.37
		[0.32,1.10]	[0.42,0.97]	[0.31,1.05]	[0.32,1.12]	[0.29,0.46]
σ_R	0.65	1.32	0.90	1.25	1.34	0.64
		[0.58,2.43]	[0.65,1.22]	[0.55,2.25]	[0.58,2.51]	[0.46,0.86]
$\rho(\pi, y)$	-0.05	-0.07	0.35	-0.07	-0.06	0.31
		[-0.13,-0.01]	[-0.21,0.72]	[-0.14,-0.01]	[-0.12,-0.01]	[0.07,0.55]
$\rho(R, y)$	0.16	-0.09	-0.01	-0.10	-0.09	0.42
		[-0.17,-0.02]	[-0.42,0.42]	[-0.18,-0.02]	[-0.17,-0.02]	[0.14,0.69]
$\rho_y(1)$	0.94	0.99	0.97	0.98	0.99	0.90
		[0.97,1.00]	[0.94,1.00]	[0.96,1.00]	[0.97,1.00]	[0.85,0.94]
$\rho_\pi(1)$	0.62	0.85	0.87	0.87	0.88	0.72
		[0.69,0.99]	[0.79,0.96]	[0.73,0.99]	[0.74,0.99]	[0.61,0.84]
$\rho_R(1)$	0.90	0.98	0.95	0.97	0.98	0.94
		[0.94,1.00]	[0.91,0.98]	[0.94,1.00]	[0.95,1.00]	[0.90,0.97]
$\rho_y(2)$	0.86	0.97	0.90	0.97	0.97	0.81
		[0.94,1.00]	[0.82,0.99]	[0.93,1.00]	[0.94,1.00]	[0.73,0.89]
$\rho_\pi(2)$	0.56	0.82	0.73	0.83	0.84	0.66
		[0.63,0.99]	[0.58,0.91]	[0.65,0.98]	[0.65,0.99]	[0.53,0.78]
$\rho_R(2)$	0.76	0.96	0.86	0.95	0.96	0.86
		[0.90,1.00]	[0.79,0.94]	[0.89,1.00]	[0.90,1.00]	[0.79,0.94]
$\rho_y(4)$	0.59	0.95	0.72	0.95	0.95	0.67
		[0.88,1.00]	[0.52,0.95]	[0.88,1.00]	[0.88,1.00]	[0.55,0.79]
$\rho_\pi(4)$	0.55	0.79	0.49	0.79	0.79	0.55
		[0.56,0.98]	[0.25,0.81]	[0.58,0.98]	[0.57,0.98]	[0.41,0.69]
$\rho_R(4)$	0.57	0.92	0.66	0.91	0.92	0.73
		[0.81,1.00]	[0.50,0.82]	[0.80,0.99]	[0.81,1.00]	[0.59,0.86]

Note: (1): Baseline NK, (2): Hybrid NK (Backward Indexation, Real Rigidities), (3): Imperfect Info. Temporary vs Permanent Shocks, (4): Imperfect Info., Cogley–Sbordone, (5): Imperfect Info., Noisy Signals. 95% HPDI in brackets.

Table 27: HP–filtered Moments (Post 82 period)

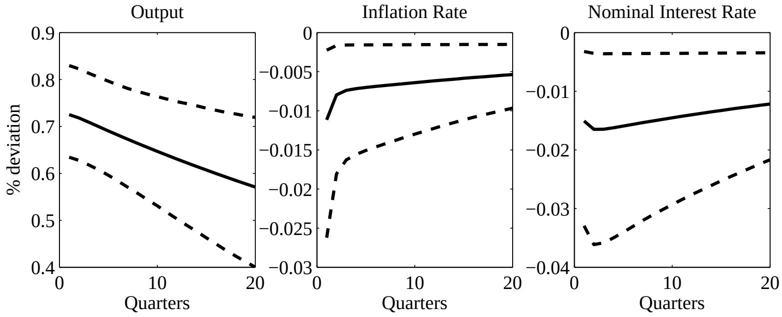
	Data	(1)	(2)	(3)	(4)	(5)
σ_y	1.00	0.96	1.72	0.99	1.00	0.95
		[0.81,1.13]	[1.14,2.32]	[0.83,1.17]	[0.84,1.17]	[0.81,1.10]
σ_π	0.18	0.23	0.42	0.22	0.23	0.24
		[0.20,0.27]	[0.33,0.53]	[0.19,0.26]	[0.20,0.27]	[0.20,0.27]
σ_R	0.29	0.29	0.52	0.29	0.29	0.31
		[0.25,0.33]	[0.39,0.65]	[0.25,0.34]	[0.25,0.34]	[0.26,0.36]
$\rho(\pi, y)$	0.13	0.05	0.38	0.07	0.10	0.24
		[-0.03,0.15]	[0.02,0.65]	[-0.03,0.16]	[0.02,0.20]	[0.14,0.35]
$\rho(R, y)$	0.58	-0.11	-0.04	-0.16	-0.10	0.26
		[-0.21,-0.03]	[-0.38,0.27]	[-0.27,-0.05]	[-0.20,-0.02]	[0.05,0.47]
$\rho_y(1)$	0.84	0.71	0.90	0.68	0.70	0.67
		[0.69,0.72]	[0.85,0.94]	[0.64,0.72]	[0.69,0.72]	[0.64,0.70]
$\rho_\pi(1)$	0.13	0.21	0.73	0.29	0.34	0.34
		[0.11,0.32]	[0.67,0.81]	[0.17,0.41]	[0.23,0.44]	[0.23,0.45]
$\rho_R(1)$	0.84	0.67	0.86	0.67	0.72	0.75
		[0.61,0.72]	[0.81,0.91]	[0.61,0.73]	[0.67,0.77]	[0.69,0.81]
$\rho_y(2)$	0.65	0.47	0.70	0.44	0.47	0.42
		[0.45,0.48]	[0.61,0.79]	[0.40,0.48]	[0.45,0.48]	[0.37,0.46]
$\rho_\pi(2)$	0.08	0.08	0.47	0.12	0.13	0.20
		[0.03,0.14]	[0.37,0.61]	[0.05,0.19]	[0.06,0.20]	[0.14,0.28]
$\rho_R(2)$	0.55	0.44	0.66	0.44	0.48	0.50
		[0.39,0.48]	[0.58,0.75]	[0.39,0.49]	[0.44,0.52]	[0.44,0.57]
$\rho_y(4)$	0.18	0.12	0.25	0.11	0.12	0.06
		[0.11,0.12]	[0.11,0.40]	[0.09,0.12]	[0.11,0.12]	[0.02,0.10]
$\rho_\pi(4)$	0.27	-0.02	0.06	-0.02	-0.03	0.01
		[-0.04,0.00]	[-0.12,0.19]	[-0.04,0.01]	[-0.06,-0.01]	[-0.01,0.04]
$\rho_R(4)$	0.12	0.11	0.24	0.11	0.12	0.11
		[0.09,0.12]	[0.09,0.38]	[0.08,0.13]	[0.10,0.14]	[0.06,0.15]

Note: (1): Baseline NK, (2): Hybrid NK (Backward Indexation, Real Rigidities), (3): Imperfect Info. Temporary vs Permanent Shocks, (4): Imperfect Info., Cogley–Sbordone, (5): Imperfect Info., Noisy Signals. 95% HPDI in brackets.

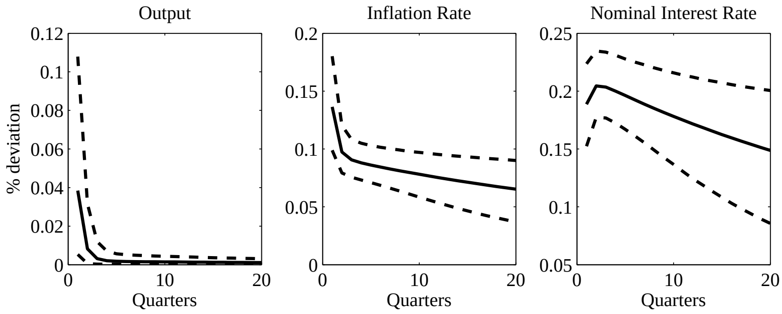
C.1 Figures, Post 1982 period

Figure 12: Impulse Response Functions – Perfect info, forward NK (Post 82 period)

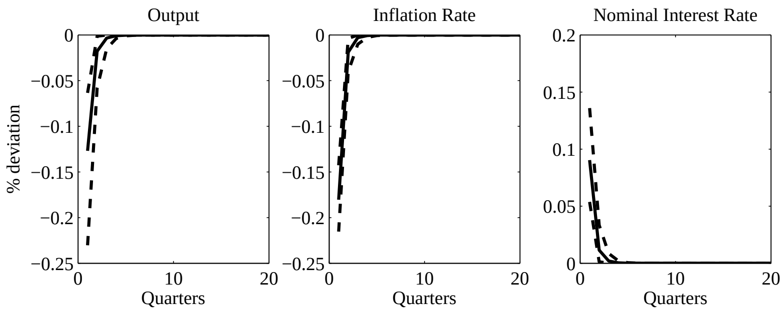
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Cost Push Shock

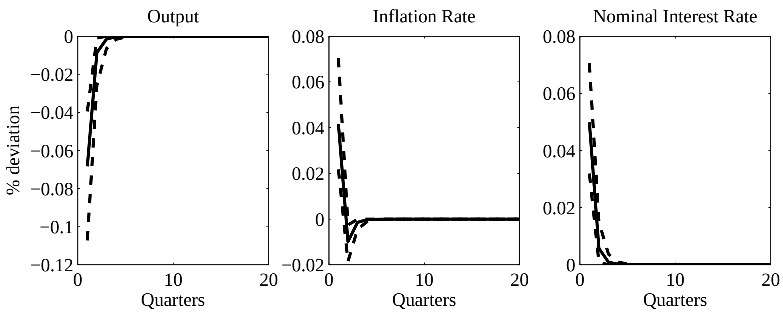
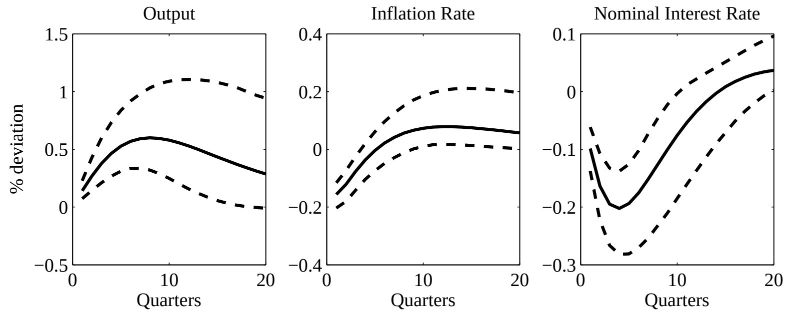
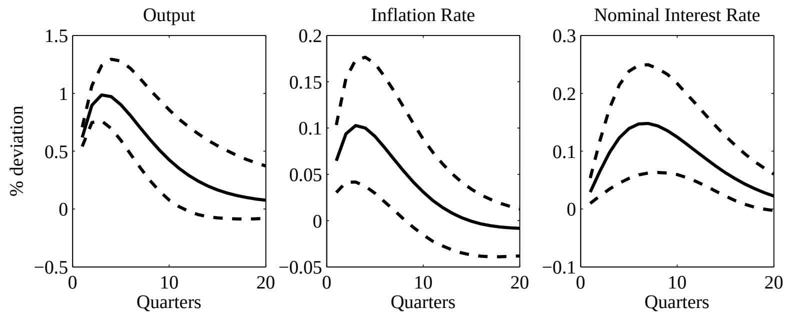


Figure 13: Impulse Response Functions – Perfect info, hybrid NK (Post 82 period)

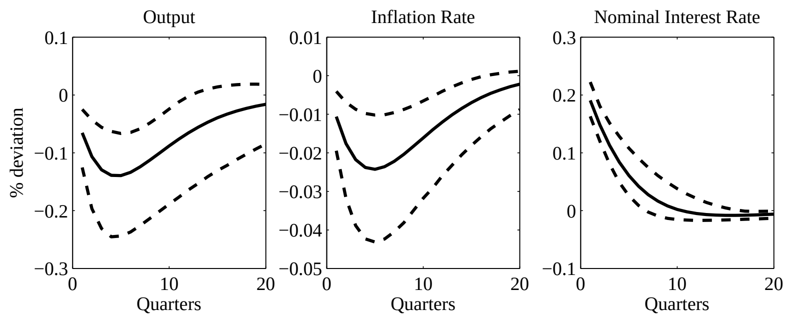
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Cost Push Shock

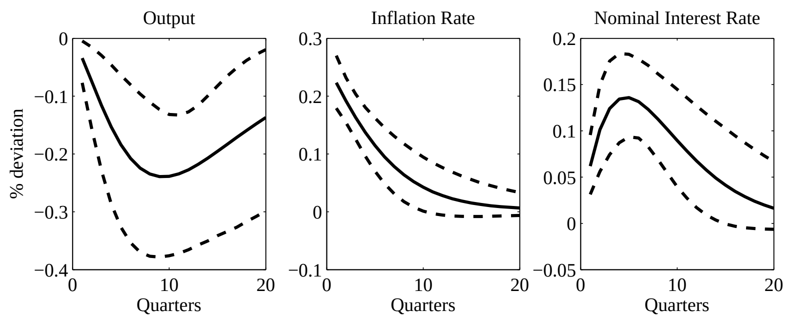
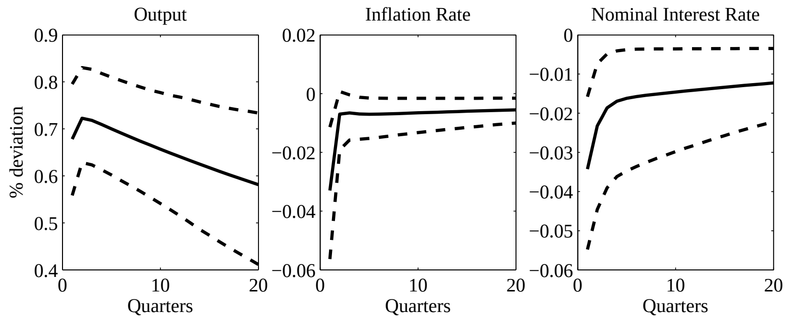
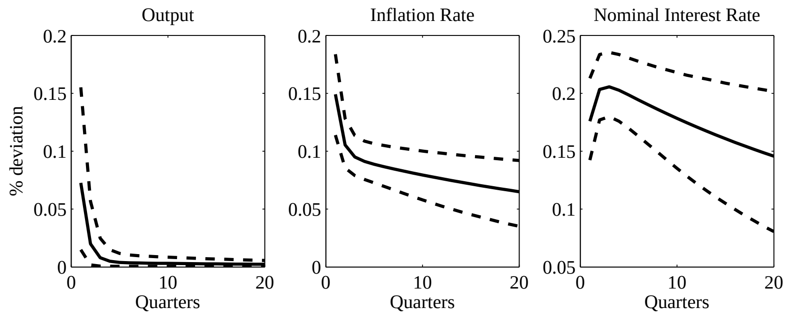


Figure 14: Impulse Response Functions – Imperfect info, Pers. vs Temp. (Post 82 period)

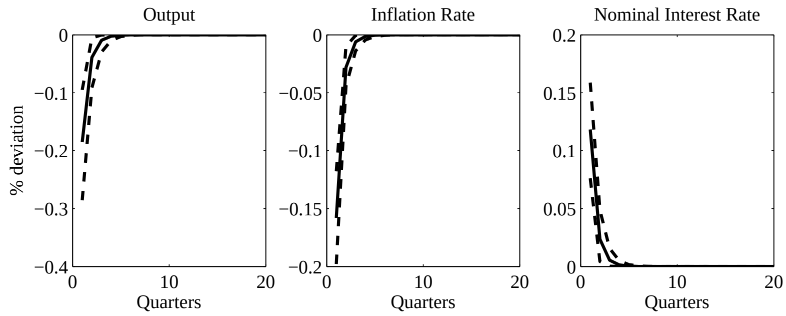
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Cost Push Shock

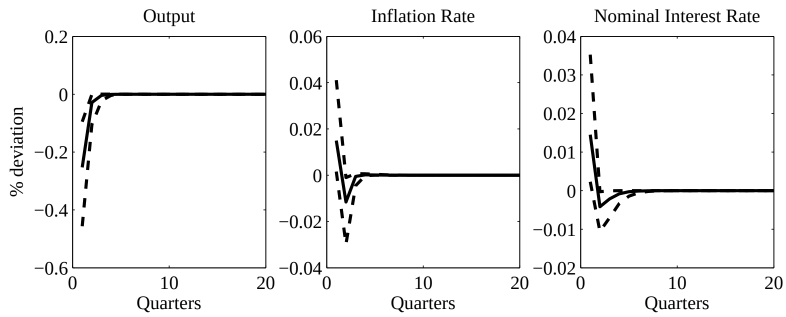
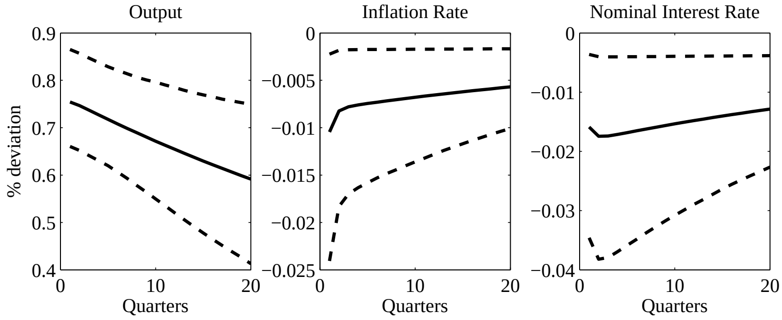
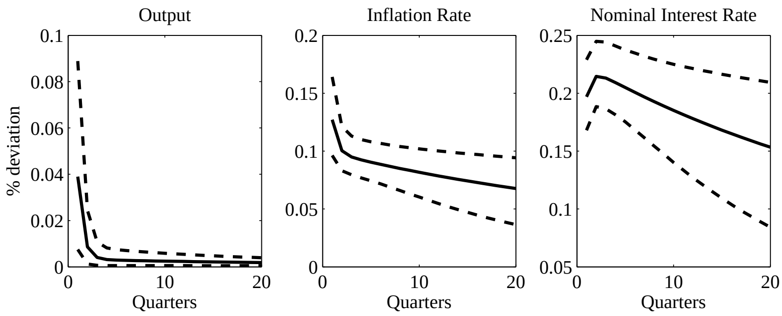


Figure 15: Impulse Response Functions– Imperfect info, Inflation target shock (Post 82 period)

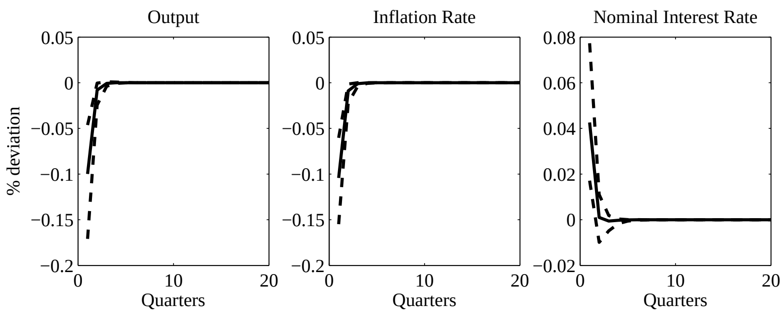
(a) Technology Shock



(b) Preference Shock



(c) Interest Rate Shock



(d) Inflation Target Shock

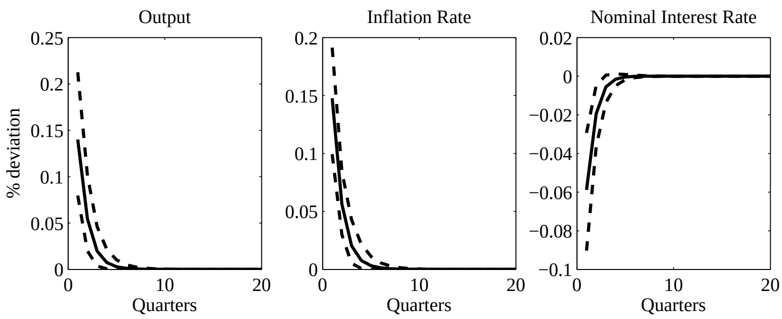
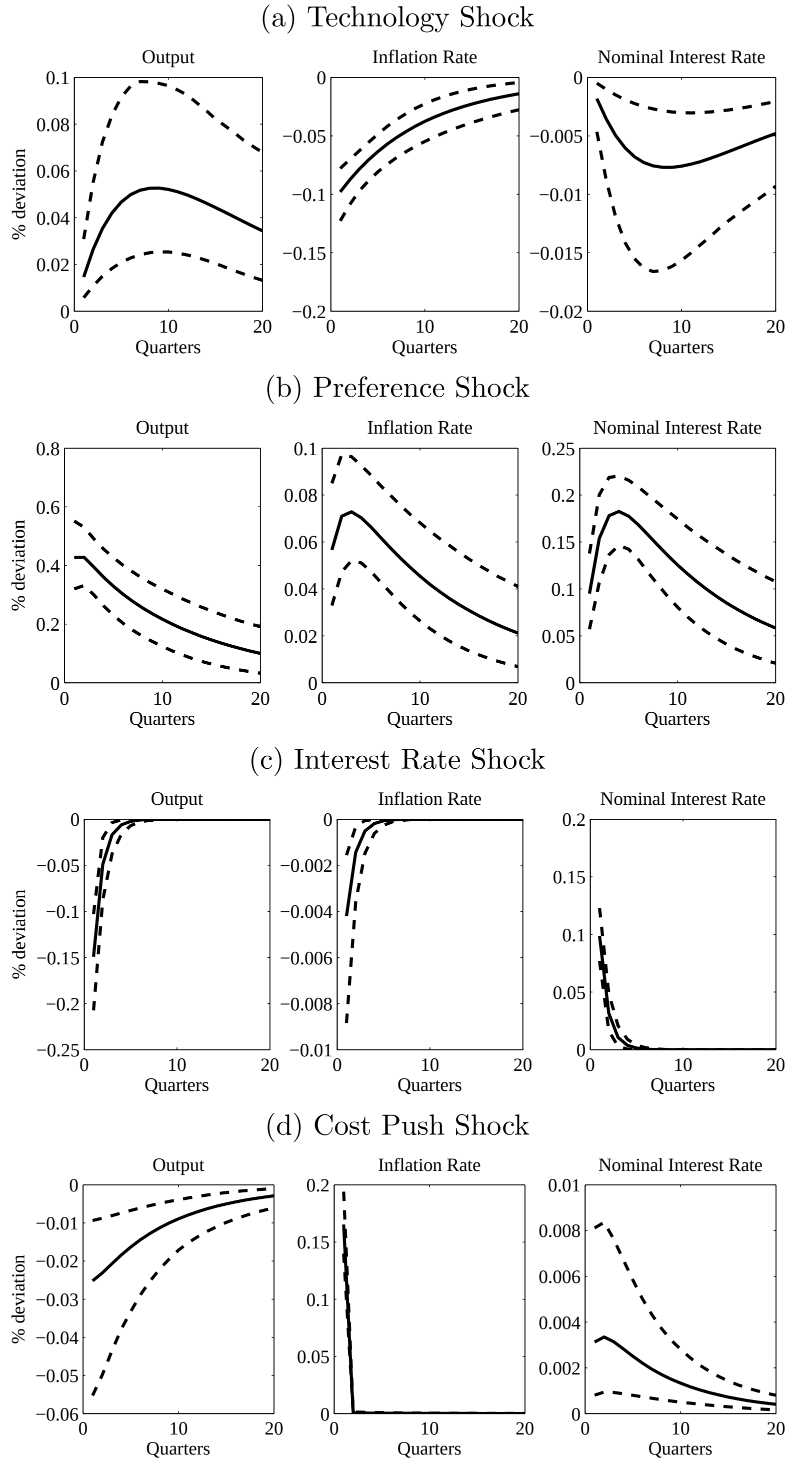


Figure 16: Impulse Response Functions – Imperfect info, measurement errors (Post 82 period)



D Solution Method

Let the state of the economy be represented by two vectors $\widetilde{X}_t^b$ and $\widetilde{X}_t^f$. The first one includes the predetermined (backward looking) state variables, i.e. $\widetilde{X}_t^b = (\widetilde{R}_{t-1}, \widetilde{z}_t, \widetilde{g}_t, \widetilde{\varepsilon}_t^R)'$, whereas the second one consists of the forward looking state variables, i.e. $\widetilde{X}_t^f = (\widetilde{y}_t, \widetilde{\pi}_t)'$. The model admits the following representation

$$M_0 \begin{pmatrix} \widetilde{X}_{t+1}^b \\ \mathbb{E}_t \widetilde{X}_{t+1}^f \end{pmatrix} + M_1 \begin{pmatrix} \widetilde{X}_t^b \\ \widetilde{X}_t^f \end{pmatrix} = M_2 \varepsilon_{t+1} \tag{19}$$

Let us denote the signal process by $\{S_t\}$. The measurement equation relates the state of the economy to the signal:

$$S_t = C \begin{pmatrix} \widetilde{X}_t^b \\ \widetilde{X}_t^f \end{pmatrix} + v_t. \tag{20}$$

Finally u and v are assumed to be normally distributed covariance matrices Σ_{uu} and Σ_{vv} respectively and $E(uv') = 0$.

$X_{t+i|t} = E(X_{t+i}|\mathscr{I}_t)$ for $i \geqslant 0$ and where $\mathscr{I}_t$ denotes the information set available to the agents at the beginning of period t. The information set available to the agents consists of *i)* the structure of the model and *ii)* the history of the observable signals they are given in each period:

$$\mathscr{I}_t = \{S_{t-j}, j \geqslant 0, M_0, M_1, M_2, C, \Sigma_{uu}, \Sigma_{vv}\}$$

The information structure of the agents is described fully by the specification of the signals.

D.1 Solving the system

Step 1: We first solve for the expected system:

$$M_0 \begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} + M_1) \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} = \tag{21}$$

which rewrites as

$$\begin{pmatrix} X_{t+1|t}^b \\ X_{t+1|t}^f \end{pmatrix} = W \begin{pmatrix} X_{t|t}^b \\ X_{t|t}^f \end{pmatrix} \tag{22}$$

where

$$W = -M_0^{-1} M_1$$

After getting the Jordan form associated to (22) and applying standard methods for eliminating bubbles, we get

$$X_{t|t}^f = G X_{t|t}^b$$

From which we get

$$X_{t+1|t}^b = (W_{bb} + W_{bf} G) X_{t|t}^b = W^b X_{t|t}^b \tag{23}$$

$$X_{t+1|t}^f = (W_{fb} + W_{ff} G) X_{t|t}^b = W^f X_{t|t}^b \tag{24}$$

Step 2: We have

$$M_0 \begin{pmatrix} X^b_{t+1} \\ X^f_{t+1|t} \end{pmatrix} + M_1 \begin{pmatrix} X^b_t \\ X^f_t \end{pmatrix} = M_2 u_{t+1}$$

Taking expectations, we have

$$M_0 \begin{pmatrix} X^b_{t+1|t} \\ X^f_{t+1|t} \end{pmatrix} + M_1 \begin{pmatrix} X^b_{t|t} \\ X^f_{t|t} \end{pmatrix} = 0$$

Subtracting, we get

$$M_0 \begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} + M_1 \begin{pmatrix} X^b_t - X^b_{t|t} \\ X^f_t - X^f_{t|t} \end{pmatrix} = M_2 u_{t+1} \tag{25}$$

which rewrites

$$\begin{pmatrix} X^b_{t+1} - X^b_{t+1|t} \\ 0 \end{pmatrix} = W^c \begin{pmatrix} X^b_t - X^b_{t|t} \\ X^f_t - X^f_{t|t} \end{pmatrix} + M_0^{-1} M_2 u_{t+1} \tag{26}$$

where, $W^c = -M_0^{-1} M_1$. Hence, considering the second block of the above matrix equation, we get

$$W^c_{fb}(X^b_t - X^b_{t|t}) + W^c_{ff}(X^f_t - X^f_{t|t}) = 0$$

which gives

$$X^f_t = F^0 X^b_t + F^1 X^b_{t|t}$$

with $F^0 = -{W^c_{ff}}^{-1} W^c_{fb}$ and $F^1 = G - F^0$.

Now considering the first block, we have

$$X^b_{t+1} = X^b_{t+1|t} + W^c_{bb}(X^b_t - X^b_{t|t}) + W^c_{bf}(X^f_t - X^f_{t|t}) + M^2 u_{t+1}$$

from which we get, using (23)

$$X^b_{t+1} = M^0 X^b_t + M^1 X^b_{t|t} + M^2 u_{t+1}$$

with $M^0 = W^c_{bb} + W^c_{bf} F^0$, $M^1 = W^b - M^0$ and $M^2 = M_0^{-1} M_2$.

We also have

$$S_t = C_b X^b_t + C_f X^f_t + v_t$$

from which we get

$$S_t = S^0 X^b_t + S^1 X^b_{t|t} + v_t$$

where $S^0 = C_b + C_f F^0$ and $S^1 = C_f F^1$

D.2 Filtering

Since our solution involves terms in $X^b_{t|t}$, we would like to compute this quantity. However, the only information we can exploit is a signal S_t that was described previously. We therefore use a Kalman filter approach to compute the optimal prediction of $X^b_{t|t}$.

In order to recover the Kalman filter, it is a good idea to think in terms of expectation errors. Therefore, let us define

$$\widetilde{X}^b_t = X^b_t - X^b_{t|t-1}$$

and

$$\widetilde{S}_t = S_t - S_{t|t-1}$$

Note that since S_t depends on $X^b_{t|t}$, only the signal relying on $\widetilde{S}_t = S_t - S^1 X^b_{t|t}$ can be used to infer anything on $X^b_{t|t}$. Therefore, the policy maker revises its expectations using a linear rule depending on $\widetilde{S}^e_t = S_t - S^1 X^b_{t|t}$. The filtering equation then writes

$$X^b_{t|t} = X^b_{t|t-1} + K(\widetilde{S}^e_t - \widetilde{S}^e_{t|t-1}) = X^b_{t|t-1} + K(S^0\widetilde{X}^b_t + v_t)$$

where K is the filter gain matrix, that we would like to compute.

The first thing we have to do is to rewrite the system in terms of state–space representation. Since $S_{t|t-1} = (S^0 + S^1)X^b_{t|t-1}$, we have

$$\begin{aligned}
\widetilde{S}_t &= S^0(X^b_t - X^b_{t|t}) + S^1(X^b_{t|t} - X^b_{t|t-1}) + v_t \\
&= S^0\widetilde{X}^b_t + S^1 K(S^0\widetilde{X}^b_t + v_t) + v_t \\
&= S^\star \widetilde{X}^b_t + \nu_t
\end{aligned}$$

where $S^\star = (I + S^1 K)S^0$ and $\nu_t = (I + S^1 K)v_t$.

Now, consider the law of motion of backward state variables, we get

$$\begin{aligned}
\widetilde{X}^b_{t+1} &= M^0(X^b_t - X^b_{t|t}) + M^2 u_{t+1} \\
&= M^0(X^b_t - X^b_{t|t-1} - X^b_{t|t} + X^b_{t|t-1}) + M^2 u_{t+1} \\
&= M^0\widetilde{X}^b_t - M^0(X^b_{t|t} + X^b_{t|t-1}) + M^2 u_{t+1} \\
&= M^0\widetilde{X}^b_t - M^0 K(S^0\widetilde{X}^b_t + v_t) + M^2 u_{t+1} \\
&= M^\star \widetilde{X}^b_t + \omega_{t+1}
\end{aligned}$$

where $M^\star = M^0(I - KS^0)$ and $\omega_{t+1} = M^2 u_{t+1} - M^0 K v_t$.

We therefore end–up with the following state–space representation

$$\widetilde{X}^b_{t+1} = M^\star \widetilde{X}^b_t + \omega_{t+1} \quad (27)$$

$$\widetilde{S}_t = S^\star \widetilde{X}^b_t + \nu_t \quad (28)$$

For which the Kalman filter is given by

$$\widetilde{X}^b_{t|t} = \widetilde{X}^b_{t|t-1} + PS^{\star\prime}(S^\star PS^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widetilde{X}^b_t + \nu_t)$$

But since $\widetilde{X}^b_{t|t}$ is an expectation error, it is not correlated with the information set in $t-1$, such that $\widetilde{X}^b_{t|t-1} = 0$. The prediction formula for $\widetilde{X}^b_{t|t}$ therefore reduces to

$$\widetilde{X}^b_{t|t} = PS^{\star\prime}(S^\star PS^{\star\prime} + \Sigma_{\nu\nu})^{-1}(S^\star \widetilde{X}^b_t + \nu_t) \tag{29}$$

where P solves

$$P = M^\star PM^{\star\prime} + \Sigma_{\omega\omega}$$

and $\Sigma_{\nu\nu} = (I + S^1K)\Sigma_{vv}(I + S^1K)'$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime}$

Note however that the above solution is obtained for a given K matrix that remains to be computed. We can do that by using the basic equation of the Kalman filter:

$$\begin{aligned}
X^b_{t|t} &= X^b_{t|t-1} + K(\widetilde{S}^e_t - \widetilde{S}^e_{t|t-1}) \\
&= X^b_{t|t-1} + K(S_t - S^1X^b_{t|t} - (S_{t|t-1} - S^1X^b_{t|t-1})) \\
&= X^b_{t|t-1} + K(S_t - S^1X^b_{t|t} - S^0X^b_{t|t-1})
\end{aligned}$$

Solving for $X^b_{t|t}$, we get

$$\begin{aligned}
X^b_{t|t} &= (I + KS^1)^{-1}(X^b_{t|t-1} + K(S_t - S^0X^b_{t|t-1})) \\
&= (I + KS^1)^{-1}(X^b_{t|t-1} + KS^1X^b_{t|t-1} - KS^1X^b_{t|t-1} + K(S_t - S^0X^b_{t|t-1})) \\
&= (I + KS^1)^{-1}(I + KS^1)X^b_{t|t-1} + (I + KS^1)^{-1}K(S_t - (S^0 + S^1)X^b_{t|t-1})) \\
&= X^b_{t|t-1} + (I + KS^1)^{-1}K\widetilde{S}_t \\
&= X^b_{t|t-1} + K(I + S^1K)^{-1}\widetilde{S}_t \\
&= X^b_{t|t-1} + K(I + S^1K)^{-1}(S^\star\widetilde{X}^b_t + \nu_t)
\end{aligned}$$

where we made use of the identity $(I + KS^1)^{-1}K \equiv K(I + S^1K)^{-1}$. Hence, identifying to (29), we have

$$K(I + S^1K)^{-1} = PS^{\star\prime}(S^\star PS^{\star\prime} + \Sigma_{\nu\nu})^{-1}$$

remembering that $S^\star = (I + S^1K)S^0$ and $\Sigma_{\nu\nu} = (I + S^1K)\Sigma_{vv}(I + S^1K)'$, we have

$$K(I{+}S^1K)^{-1} = PS^{0\prime}(I{+}S^1K)'((I{+}S^1K)S^0PS^{0\prime}(I{+}S^1K)'{+}(I{+}S^1K)\Sigma_{vv}(I{+}S^1K)')^{-1}(I{+}S^1K)S^0$$

which rewrites as

$$\begin{aligned}
K(I + S^1K)^{-1} &= PS^{0\prime}(I + S^1K)'\left[(I + S^1K)(S^0PS^{0\prime} + \Sigma_{vv})(I + S^1K)'\right]^{-1} \\
K(I + S^1K)^{-1} &= PS^{0\prime}(I + S^1K)'(I + S^1K)'^{-1}(S^0PS^{0\prime} + \Sigma_{vv})^{-1}(I + S^1K)^{-1}
\end{aligned}$$

Hence, we obtain

$$K = PS^{0\prime}(S^0PS^{0\prime} + \Sigma_{vv})^{-1} \tag{30}$$

Now, recall that

$$P = M^\star PM^{\star\prime} + \Sigma_{\omega\omega}$$

Remembering that $M^\star = M^0(I + KS^0)$ and $\Sigma_{\omega\omega} = M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime}$, we have

$$\begin{aligned} P &= M^0(I - KS^0)P\left[M^0(I - KS^0)\right]' + M^0K\Sigma_{vv}K'M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \\ &= M^0\left[(I - KS^0)P(I - S^{0\prime}K') + K\Sigma_{vv}K'\right]M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \end{aligned}$$

Plugging the definition of K in the latter equation, we obtain

$$P = M^0\left[P - PS^{0\prime}(S^0PS^{0\prime} + \Sigma_{vv})^{-1}S^0P\right]M^{0\prime} + M^2\Sigma_{uu}M^{2\prime} \tag{31}$$

D.3 Summary

We end–up with the system of equations:

$$\begin{aligned} X^b_{t+1} &= M^0X^b_t + M^1X^b_{t|t} + M^2u_{t+1} &(32)\\ S_t &= S^0_bX^b_t + S^1_bX^b_{t|t} + v_t &(33)\\ X^f_t &= F^0X^b_t + F^1X^b_{t|t} &(34)\\ X^b_{t|t} &= X^b_{t|t-1} + K(S^0(X^b_t - X^b_{t|t-1}) + v_t) &(35)\\ X^b_{t+1|t} &= (M^0 + M^1)X^b_{t|t} &(36) \end{aligned}$$

which fully describe the dynamics of our economy.

This may be recast as a standard state–space problem

$$\begin{aligned} X^b_{t+1|t+1} &= X^b_{t+1|t} + K(S^0(X^b_{t+1} - X^b_{t+1|t}) + v_{t+1}) \\ &= (M^0 + M^1)X^b_{t|t} + K(S^0(M^0X^b_t + M^1X^b_{t|t} + M^2u_{t+1} - (M^0 + M^1)X^b_{t|t}) + v_{t+1}) \\ &= KS^0M^0X^b_t + ((I - KS^0)M^0 + M^1)X^b_{t|t} + KS^0M^2u_{t+1} + Kv_{t+1} \end{aligned}$$

Then

$$\begin{pmatrix} X^b_{t+1} \\ X^b_{t+1|t+1} \end{pmatrix} = M_{\mathrm{X}}\begin{pmatrix} X^b_t \\ X^b_{t|t} \end{pmatrix} + M_{\mathrm{E}}\begin{pmatrix} u_{t+1} \\ v_{t+1} \end{pmatrix}$$

where

$$M_{\mathrm{X}} = \begin{pmatrix} M^0 & M^1 \\ KS^0M^0 & ((I - KS^0)M^0 + M^1) \end{pmatrix} \text{ and } M_{\mathrm{E}} = \begin{pmatrix} M^2 & 0 \\ KS^0M^2 & K \end{pmatrix}$$

and

$$X^f_t = M_{\mathrm{F}}\begin{pmatrix} X^b_t \\ X^b_{t|t} \end{pmatrix}$$

where

$$M_{\mathrm{F}} = \begin{pmatrix} F^0 & F^1 \end{pmatrix}$$