



School of Economics

Working Paper 2004-08

*Network Externalities, Demand Inertia and  
Dynamic Pricing in an Experimental  
Oligopoly Market*

Ralph-C Bayer and Mickey Chan

School of Economics  
University of Adelaide University, 5005 Australia

ISSN 1444 8866

# Network Externalities, Demand Inertia and Dynamic Pricing in an Experimental Oligopoly Market

Ralph-C. Bayer\* and Mickey Chan  
School of Economics, University of Adelaide,  
North Terrace, SA 5005, Australia

25th November 2004

## Abstract

This paper analyses dynamic pricing in markets with network externalities. Network externalities imply demand inertia, because the size of a network increases the usefulness of the product for consumers. Since past sales increase current demand, firms have an incentive to set low introductory prices to be able to increase prices as their networks grow. However, in reality we observe decreasing prices. This could be due to other factors dominating the network effects. We use an experimental duopoly market with demand inertia to isolate the effect of network externalities. We find that experimental price dynamics are rather consistent with real world observations than with theoretical predictions.

Keywords: Network Externalities, Demand Inertia, Experiments, Oligopoly

JEL-Codes: L13, C92

---

\*Corresponding Author. Tel.: ++61 (0)8 8303 5756; fax: ++ (0)8 8323 1460; email: ralph.bayer@adelaide.edu.au;

web site: <http://www.economics.adelaide.edu.au/staff/bayer.html>.

We want to thank Nick Robinson for great IT support. We are grateful for the many helpful suggestions we received from participants of various conferences and seminars. All remaining errors are ours.

# 1 Introduction

For many commodities, the individual utility of consumption depends on how many other people also consume the commodity. In their seminal paper, Katz and Shapiro (1985) refer to this phenomenon as network externalities. Such externalities may arise due to direct physical effects or indirect effects, which they refer to as consumption externalities. Examples for the direct physical effect are communication networks, like telephone or E-mail where the usefulness of having access obviously increases with the number of people that can be reached through the network. The classic example of indirect effects are computer operating systems, where the number of people using a particular system determines how many applications are written for it, which in turn determines how useful it is for the consumer.

The existence of network externalities has crucial effects on conduct in and the performance of markets. Issues such as compatibility, co-ordination to technical standards and effects on pricing and quality of services create challenges for economic theory (Economides, 1996). There is still some discussion about how significant network externalities are in producing market failures.<sup>1</sup> However, the literature that explores the adoption of technologies (Belleflamme, 1998; Kristiansen, 1996), standards and the lock-in of technologies (Witt, 1997; De Bijl and Goyal, 1995), compatibility issues (Baake and Boom, 2001), and product introduction (Katz and Shapiro, 1992) is extensive. We focus on another, less researched, aspect of network externalities: dynamic pricing under demand inertia.

In a market where at least two competing networks coexist, which are substitutes for consumers, current network size is correlated with future demand. The larger a network is today the higher is the demand tomorrow. Consequently, future demand is positively correlated with sales today. Thus, network externalities imply demand inertia. An

---

<sup>1</sup>Katz and Shapiro (1994) make the case that network externalities have a high significance, while in the same journal and volume Liebowitz and Margolis (1994) argue that network externalities are rarely a cause of market inefficiencies.

example of such coexisting networks is the market for game consoles. Currently there are three non-compatible competing systems: Sony Playstation2, Microsoft Xbox, and Nintendo GameCube (see Schilling 2003, for an analysis of the game console market). Coexistence of at least two standards has been the rule in the game console market since the late eighties (Sega Genesis and Nintendo SNES until 1994, Sony Playstation and Nintendo 64 from 1996 to 1999, and the three currently competing systems since 2001).

Demand inertia due to network externalities, *ceteris paribus*, puts pressure on competing firms to introduce their products with very low prices in order to increase the size of their network quickly. Cabral, Salant and Woroch (1999) explore the conditions necessary for a low introductory price being optimal for a monopolist operating under network externalities. We show that a low introductory price is also optimal in a duopoly with competing networks. For a monopolist it is optimal to increase its price over time (Bensaid and Lesne, 1996). The same is true in our duopoly model. In fact, the introductory price of the Xbox in November 2001 was quite low (US\$ 299) and exactly matched the price of the Playstation2. Estimates suggest that Microsoft lost between US\$ 100 and \$ 125 per unit sold.<sup>2</sup> However, contrary to the theoretical prediction, the prices for Xbox and Playstation2 did not increase, but dropped further (Xbox: US \$ 149.99 on March 29, 2004; Playstation2: US \$149 on May 4, 2004). Price cuts by one of the two firms were usually countered by a subsequent equivalent price cut of the competitor.

There are many reasons why, firms in reality, may decrease prices over time: intertemporal price discrimination; reduced costs due to learning by doing; or scale economies are examples. The decreasing prices may be easily explained if these forces dominate the incentives to increase prices over time created by demand inertia. However, due to the multiple effects at work, it not possible to evaluate the effects of network externalities alone by just looking at observed pricing behaviour. We use a laboratory experiment

---

<sup>2</sup>See Schilling (2003), p 16.

to separate the effect of demand inertia from other effects. By eliminating the alternative factors mentioned above, which also play a role in dynamic pricing, we can be sure that the remaining effect is due to demand inertia or to idiosyncracies of oligopoly markets. Experimental oligopoly markets typically show a certain degree of collusion not explained by game theory. In order to separate the network effect from idiosyncratic collusion in a repeated oligopoly we run a control treatment with an identical market, but without demand inertia. We can isolate the effects of demand inertia on dynamic pricing by comparing prices in the demand-inertia treatment to the prices in the control treatment.

Reinhard Selten (1965) was the first to develop a comprehensive model of oligopolistic competition with demand inertia. We model demand inertia in a similar way. However, in Selten's model there is no direct strategic interaction in any particular period, as the period payoff does not depend on the prices of the competitors. The interaction is only indirect as the future demand is influenced by past price differences. Keser (1993, 2000) implements Selten's model in the laboratory and compares the observed play with the equilibrium prediction with the main objective of categorizing different patterns of behaviour. Keser is less interested in evaluating the effect of demand inertia, which in any case is difficult in her design, since the lack of immediate interaction does not allow for a control treatment where firms compete under the same conditions, but without demand inertia.<sup>3</sup> The ability to do so is crucial for our research question, as we want to compare markets with demand inertia created by network externalities with markets that do not have network characteristics, but are otherwise identical.

The remainder of the paper is structured as follows. In the next section we present our model. Section 3 derives some equilibrium predictions for the model. Section 4 describes the experimental setup, while section 5 reports the main results. We conclude

---

<sup>3</sup>Another problem for the isolation of demand-inertia effects is the inclusion of interest payments on early periods to simulate discounting. The effects of this design element and the inertia can not easily be separated.

in section 6. In the appendix we show how the demand function used can be derived from simple consumption decisions for goods with network externalities.

## 2 The model

In this section we develop a simple model of a market with network externalities. We reduce this market to its essentials and eliminate any other factor that could have an influence on dynamic pricing. We use a multi-period Bertrand duopoly with differentiated products. Market demand in each period is perfectly inelastic with a total market demand of  $a$  per period.<sup>4</sup> The market has a lifetime of  $T$  periods. Network externalities are captured by a state variable  $s_i^t$  - the share of past sales in the industry - which positively influences the individual period demand. The share of past sales is defined as

$$s_i^t := \frac{\sum_{k=1}^{t-1} q_i^k}{[t-1]a} \text{ for } t > 1, \quad (1)$$

where  $i \in \{1, 2\}$  denotes the firm,  $t$  gives the actual period, and  $q_i^k$  is the quantity sold in period  $k$  by firm  $i$ . Note that  $s_i^1$  cannot be defined by the expression above. So we need an initial condition which may reflect initial beliefs of the consumers about the quality of firms' products. Reputation, product reviews, and advertising may play a role.

The period sales of firm  $i$  are defined as

$$q_i^t := \max\{s_i^t a + p_j^t - p_i^t, 0\} \quad i, j \in \{1, 2\}, i \neq j, \quad (2)$$

where  $p_j^t$  and  $p_i^t$  are the prices. Both firms have the same degree of market power stemming from the consumers' preferences for the different goods varieties. Differences in market power at time  $t$  only arise from different market shares  $s_i^t$ , which only depend on past sales. The market shares are capturing the relative size of the network. We

---

<sup>4</sup>This rather restrictive assumption is not crucial for the qualitative results of the model, but will prove very useful for the identification of treatment effects in the experiment.

chose to link the benefits from the size of the network to the market share rather than to the absolute past sales for two reasons. Firstly, the marginal benefit of today's sales for tomorrow's market power is decreasing in the past sales. This reflects that the marginal benefit for the consumers from increases in network size is believed to be decreasing. Secondly, new consumers who decide in period  $t$  which brand to buy will put less weight on the nominal differences in network sizes if both networks are already large.

The current and future demand functions are common knowledge. So the two firms simultaneously choose prices  $p_i^t, p_j^t$  in each period  $t$  after having learned the market outcome in the previous period  $t - 1$ . They are fully aware of how the current sales will influence their future market power.

We show in the appendix that the demand functions above can be derived from consumer decisions for goods with network externalities, similar to the framework used in Katz and Shapiro (1985).

### 3 Some equilibrium predictions

In this section we will establish some equilibrium predictions. We will see that in spite of the simple structure of the model solving for the full equilibrium path is impractical. Therefore we will establish some qualitative results only. Later on we will use a computer algorithm to solve numerically for the equilibrium prices for the parameters used in the experiment.

This extensive-form game has many Nash Equilibria. However, to rule out equilibria that contain empty threats we add the requirement of subgame-perfectness. We therefore use backward induction and begin with the final period. We have to determine the optimal actions in the final period for both firms and all possible histories. All-payoff relevant history is captured by the market share. Therefore, the firms at period  $T$  will maximize the period payoff for a given market share. The payoffs are given by

$$\Pi_i^T := p_i^T [as_i^T - p_i^T + p_j^T] \quad i, j \in \{1, 2\}, i \neq j.$$

Then the first-order conditions give the following best response functions:<sup>5</sup>

$$b_i(p_j^T) = \frac{as_i^T + p_j^T}{2} \quad i, j \in \{1, 2\}, i \neq j, \quad (3)$$

which gives the optimal prices:<sup>6</sup>

$$p_i^{T*} = \frac{a[1 + s_i^T]}{3}, \quad (4)$$

$$p_j^{T*} = \frac{a[2 - s_i^T]}{3}. \quad (5)$$

We now turn to the penultimate period. At period  $T - 1$  the firms foresee what will happen in the last period depending on the prices they set in period  $T - 1$ . Put differently, arriving at period  $T - 1$  the firms know that their prices in  $T - 1$  will cause certain period outputs. They also know how these period outputs will influence their market share in period  $T$ . As they anticipate how they and their competitors will behave in the last period for market shares, they will set their prices such that the sum of the profits in periods  $T - 1$  and  $T$  will be maximized. Firm  $i$ 's aggregate profit is given by

$$\Pi_i^{T-1} + \Pi_i^T := \sum_{l=T-1}^T p_i^l [as_i^l - p_i^l + p_j^l].$$

Using the anticipated equilibrium prices  $p_i^{T*}$  and  $p_j^{T*}$  for the last period we get

$$\Pi_i^{T-1} + \Pi_i^T = p_i^{T-1} [as_i^{T-1} - p_i^{T-1} + p_j^{T-1}] + \left[ \frac{a(1 + s_i^T)}{3} \right]^2. \quad (6)$$

---

<sup>5</sup>The second-order conditions are obviously satisfied.

<sup>6</sup>Note that  $s_j^T = 1 - s_i^T$ .



Recall the definition of the market share and write  $s_i^T$  as a function of  $q_i^{T-1}$  and  $s_i^{T-1}$

$$s_i^T = \left[ \sum_{l=1}^{T-2} q_i^l + q_i^{T-1} \right] / a [T-1].$$

Using the demand definition from (2) for  $q_i^{T-1}$  and simplifying leads to

$$s_i^T = s_i^{T-1} + \frac{p_j^{T-1} - p_i^{T-1}}{a [T-1]}.$$

Replacing  $s_i^T$  in equation (6) and simplifying gives an expression for the aggregate profit, which includes the anticipated behaviour in the last period and which only depends on the prices in period  $T-1$ :

$$\begin{aligned} \Pi_i^{T-1} + \Pi_i^T &= p_i^{T-1} \left[ a s_i^{T-1} - p_i^{T-1} + p_j^{T-1} \right] \\ &+ \left[ \frac{p_j^{T-1} - p_i^{T-1} + a [1 + s_i^{T-1}] [T-1]}{3 [T-1]} \right]^2. \end{aligned} \quad (7)$$

We see that the anticipated profit for the last period (the second term of 7) depends negatively on the price chosen in period  $T-1$ . The first order condition is given by

$$-2p_i^{T-1} + p_j^{T-1} + a s_i^{T-1} - \frac{2 [p_j^{T-1} - p_i^{T-1} + a [1 + s_i^{T-1}] [T-1]]}{9 [T-1]^2} = 0,$$

which gives to the best response function

$$b_i(p_j^{T-1}) = \frac{p_j^{T-1} [1 - \omega] + a s_i^{T-1} - a [1 + s_i^{T-1}] [T-1] \omega}{2 - \omega}, \quad (8)$$

where

$$\omega = \frac{2}{9 [T-1]^2}.$$

Note that the reaction function taking into account the profit in the last period differs

from the reaction function a myopic firm would have by  $\omega$  being unequal to 0. The myopic reaction function can be obtained by setting  $\omega = 0$ . To see this take (8), set  $\omega = 0$ , and compare it to the reaction function for the last period (3) where firms play myopically. They are identical up to the subscript of the market share.

Inspection of (8) shows that firm  $i$  in equilibrium will set a price lower than the myopic price  $p_i^m(s_i^{T-1})$  in period  $T - 1$ .

**Proposition 1** *In every subgame perfect equilibrium we have  $p_i^{T-1*} < p_i^m(s_i^{T-1})$  and  $p_j^{T-1*} < p_j^m(s_i^{T-1})$ .*

**Proof.** Denote the best response functions for myopic players depending on the current market share in  $T - 1$  as  $b_i^m(p_j^{T-1})$  and  $b_j^m(p_i^{T-1})$ , respectively. If we can show that  $b_i^m(p_j^{T-1}) < b_i(p_j^{T-1})$  and  $b_j^m(p_i^{T-1}) < b_j(p_i^{T-1})$  hold for all  $s_i^{T-1}$  we can conclude that our claim is true, since all best response functions are obviously non-decreasing in the opponents price. As  $b_i^m(p_j^{T-1}) = b_i(p_j^{T-1})$  for  $\omega = 0$  and  $\omega > 0$  we must have  $b_i^m(p_j^{T-1}) > b_i(p_j^{T-1})$  if  $\partial b_i(p_j^{T-1})/\partial\omega < 0$  for all  $\omega$ ,  $s_i^{T-1}$ , and  $p_j^{T-1}$ . Differentiating (8) and simplifying gives

$$\frac{\partial b_i(p_j^{T-1})}{\partial\omega} = \frac{-p_j^{T-1} + a \left[ 2 - 2T + s_i^{T-1} [3 - 2T] \right]}{[2 - \omega]^2} < 0$$

for  $T \geq 2$ . Since the best response function of firm  $j$  is obtained by swapping indices only, the same holds for firm  $j$ . ■

In the next step is we show that the equilibrium prices are smaller in  $T - 1$  than in  $T$ , independent of the initial market share and the duration of the market  $T$ . While conceptually easy, this is tedious and does not create new insights. So we sketch the proof in the appendix only.

**Proposition 2** *In every subgame perfect equilibrium we have  $p_i^{T-1*} \leq p_i^T$  and  $p_j^{T-1*} \leq p_j^T \forall s_i^{T-1} \in [0, 1]$ .*

**Proof.** See appendix ■

The two propositions above tell us that the price in the penultimate period  $T - 1$  is a) below the myopic price and b) below the price in the final period  $T$ . The logic extends naturally to earlier periods, but the increased complexity of the algebra makes it impractical to solve for the prices in earlier stages. We will do this using a computer algorithm for the parameter values used in the experiment later on.

## 4 Experimental design

We conducted computerized laboratory experiments implementing markets with network externalities as defined above.<sup>7</sup> We also ran some control sessions of comparable markets without network externalities in order to isolate the effect network externalities have on dynamic pricing decisions. We asked students enrolled in the second-year “Microeconomics 2” at the University of Adelaide to participate. All 112 students enrolled in the course had the opportunity to participate, and 94 students attended the experimental session. The students were rewarded with a grade bonus of up to 10 percent on their final mark depending on the performance in the experiment. As “Microeconomics 2” is one of the more difficult courses at Adelaide University the subjects were highly motivated by the grade bonus. Subjects were trying hard to secure passing the course or to get one of the few distinctions.

Using students from a single course could be viewed as problematic since the sample is not randomly drawn. However, most economic experiments cannot guarantee the randomness of the sample. We can control for the background of students and their knowledge of economics by using the students of one course.<sup>8</sup> We are aware that this selection gives rise to problems when generalizing the results obtained in the experi-

---

<sup>7</sup>We used the computer programme Z-Tree (Fischbacher, 1999) to conduct the experiment. The Z-Tree code for the two treatments can be downloaded from the authors web site.

<sup>8</sup>We know exactly which courses the students have taken, and are aware of their performance in these courses.

ment. On the other hand, we believe that using students from an economics course - as opposed to students from different courses - does not have too severe drawbacks for our experiment. Since in reality price decisions are taken by people with backgrounds in commerce and economics we don't see a major problem in restricting the subject pool to students of a microeconomics course.<sup>9</sup> However, the usual caveat about using students as subjects applies.

Overall, 50 students played the duopoly market with network externalities (treatment NE), and 44 students were assigned to the control treatment, the duopoly market without network externalities (treatment No-NE). In both treatments the subjects played two supergames of ten periods each. The subjects knew that they were paired with the same opponent in both supergames.<sup>10</sup> In every period the subjects had to enter their price choice and a guess what price the opponent might choose. We restricted the valid prices to the range between 0 and 10. After both subjects had chosen their actions, they were informed about the market outcome (own price, opponent's price, and quantities sold) and their period profits. In the NE treatment we also displayed the new market share resulting from the actions taken.

In both treatments, the subjects were given detailed instructions containing payoff tables and examples of how to link choices and payoffs. In the NE treatment, we provided period-payoff tables for different market shares and explained how to extrapolate the payoffs for market shares between tabulated values. Additionally, we carefully explained how the market share evolved, depending on previous price choices. The instructions, which were both read aloud and given to the subjects in writing, can be found on the author's web page.

---

<sup>9</sup>We ran one session with graduate (Masters and Ph.D. students) in order to see if the degree of economic education has an influence. We did not find any striking differences in behaviour. However, the number of observations is not large enough to draw statistical inferences.

<sup>10</sup>This partner treatment design was chosen in order to obtain the maximum number of independent observations. The loss of control due to reputation effects is regarded as not problematic for our research question.

## 4.1 Parametrization and theoretical predictions

Implementation of the underlying model structure required a choice of parameter values. We set the total market demand  $a$  to 10. Additionally, we needed a starting value for the market share for the NE treatment. We decided to use a symmetric setting where the market shares are  $s_i^1 = s_j^1 = 1/2$ . Additionally, to avoid that the market share reacts too strongly in the first period we chose to set past sales to 10 units each. We can interpret this in two different ways. Firstly, we could say that there have been two periods of competition before the start of our experiment. Secondly, we could interpret this parameter choice as a reflection of the reputation of the firms, based on customers' experience with other goods this firm has produced.

The baseline duopoly - treatment No-NE - consisted of a market where the market shares are constant at  $1/2$  and do not depend on past sales. Note that the strategic situation in all periods of No-NE is identical to the situation in the last period of NE with market shares of  $1/2$ . Consequently, the predicted equilibrium prices for No-NE and all periods are given by equations (4) and (5). For our parameter values the optimal prices  $\bar{p}_1^*, \bar{p}_2^*$  are

$$\bar{p}_1^* = \bar{p}_2^* = 5$$

The derivation of the optimal price path for the NE treatment is much more complex. In order to solve for the equilibrium path, we have to conduct backward induction over 10 periods or use a dynamic programming recursive approach in the spirit of Selten (1965).<sup>11</sup> We used a computer algorithm to solve for the equilibrium-price path, which basically performs backward induction.<sup>12</sup>

Figure 1 shows the predicted equilibrium price path for the NE treatment together

---

<sup>11</sup>In our model the dynamic programming approach is more complex than in Selten (1965), because we have a duopoly market in each period. In Selten's model the firms are monopolists in each period who have to care for their future demand potentials, which depend on the past sales of all firms.

<sup>12</sup>The Mathematica code is available for download from the author's web site.

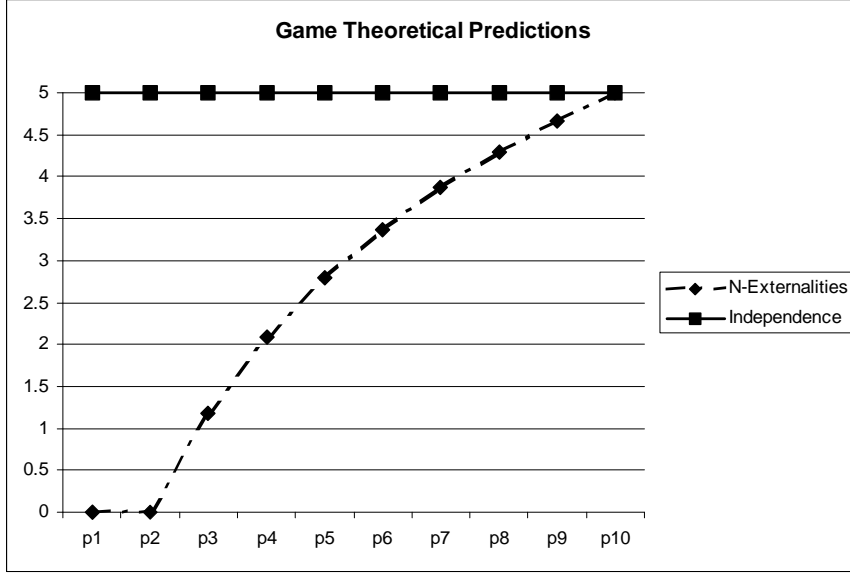


Figure 1: Predicted price paths for the different treatments

with the prediction for No-NE. Note that the symmetric initial market shares lead to a symmetric predicted price path, i.e. the competitors always choose the same price. We see that under NE we expect the price to increase from 0 at the start to 5 in the last period, which is the equilibrium price for the No-NE treatment. As play that deviates from the equilibrium path may lead to market shares different from 0.5, it is necessary to find a way of comparing play after a deviation from equilibrium with the optimal continuation from such a point. Here the assumption of a perfectly inelastic demand comes into play. Perfectly elastic demand has the implication that the average equilibrium price is independent of the history of play. Observe that equations (4) and (5) can be used to find the average price for the last period

$$\bar{p}^T = \frac{p_i^{T*} + p_j^{T*}}{2} = \frac{5 [1 + s_i^T]}{3} + \frac{5 [2 - s_i^T]}{3} = 5$$

The average price is independent of the history captured by the market share in period  $T$ . This insight tells us that a pair of firms in the NE treatment with an average price

below 5 prior to the final period is fighting for market share. A pair with an average price of 5 is playing myopically, while an average price above 5 can be interpreted as collusion.

The logic of the equilibrium average price being independent of the current market share in the NE treatment extends to earlier periods. This can be seen by investigating the outcomes of the computer algorithm used to solve the supergame, or by using a dynamic programming approach, which is contained in the appendix.

**Proposition 3** *The equilibrium prices have the form  $p_i^{t*} = \gamma^t + \phi^t s_i^t$  resulting in an average equilibrium price of  $\bar{p}^t = \gamma^t + \phi^t/2$ , which is independent of the current market share.*

**Proof.** See appendix ■

Consequently, we can say that a pair in the NE treatment with an **average** price higher than the calculated equilibrium price (for equal market shares) do not sufficiently take the future profits into account. This judgment can be made independently of whether the previous play was on the equilibrium path or not. So market shares that differ from the equilibrium market share due to prior off-equilibrium play, do not prevent a judgement about how their prices compare to the equilibrium price level.

The independence of the average prices from the history of play makes the interpretation of our results possible and enables us to compare prices between the No-NE treatment, where market shares are fixed at 0.5, and the NE treatment, where market shares other than 0.5 may occur due to off-equilibrium play.

## 5 Results

In what follows we present our main results. The three basic questions will be:

1. How do prices evolve over time compared to the theoretical prediction for different treatments?

2. How do the prices differ among treatments?
3. How competitively do subjects behave under different treatments?

The first question is mainly concerned with the stylized fact that prices for commodities in the real world decrease after they are introduced, while a reduced model of network externalities predicts increasing prices. The second question asks whether network externalities have any influence on pricing behaviour at all, while the third question asks if we can infer the impact of network externalities on the degree of competition from the observed price choices.

## 5.1 Evolution of prices

The evolution of chosen prices does not even roughly approximate the game theoretical prediction in both treatment.<sup>13</sup> While the deviation from Nash Equilibrium in the non-network treatment can be attributed to tacit collusion, it is not clear a priori why prices do not follow the predicted path in the presence of network externalities. We shortly comment on the evolution of the prices in the No-NE treatment, before discussing the results for the NE treatment in more depth.

### *Treatment No-NE*

Looking at the average prices in the standard Bertrand duopoly with differentiated products (Figure 2) we see that as in other experimental studies the average prices are above the Nash Equilibrium prediction for the whole time.<sup>14</sup> However, prices decrease over time, illustrating slowly eroding tacit collusion. Note that although play (particularly in early periods) exhibits considerable tacit collusion, the subjects by no means come close to the joint profit maximizing outcome, which required both players to choose the maximum price of 10. A remarkable result in our No-NE treatment, which is also

---

<sup>13</sup>The raw data and the stata programmes for data analysis can be downloaded from the author's web site.

<sup>14</sup>See Huck, Normann and Oechssler (2000) for an example.



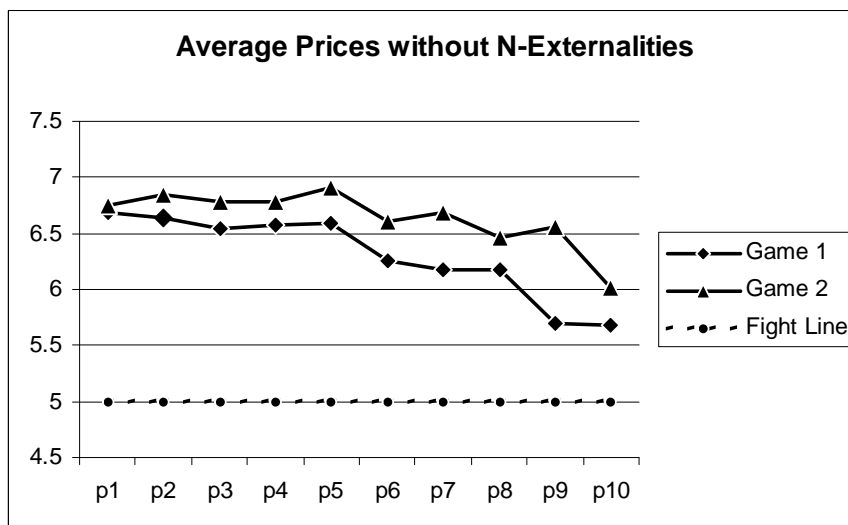


Figure 2: Price paths in the No-NE treatment

observed in repeated social dilemma experiments, is the existence of a restart effect. As the subject pairs stay the same for both 10-period supergames and the individual periods are independent, the whole experiment is theoretically equivalent to 20 independent periods of duopolistic competition. However, subjects perceive the game differently. After the first 10 periods of play, the announcement of the beginning of a new 10-period game causes the average price to return to the level of the first period in game 1.<sup>15</sup> This can be interpreted as the restart acting as a cue for the subjects to newly try to establish co-operation. With the experience from the first game, the subjects are more successful in sustaining tacit collusion in game 2. The average prices in game 2 are higher than in game 1. For the first 5 periods of game 2 the average prices stay at the restart level (or even slightly above) before the typical erosion of cooperation occurs, with a particularly strong end effect experienced.

<sup>15</sup>There is no statistically significant difference of average prices within pairs between the period 1 prices in games 1 and 2. However, pairs increase their prices significantly between the last stage of game 1 and the first stage of game 2 (one-sided Wilcoxon matched-pairs signed-ranks test,  $p < .01$ ).

In order to test that the trend of declining prices is not only present in the aggregate, but occurs also within individual pairs, we used a Wilcoxon matched-pairs signed-ranks test. For both games the average price per pair is significantly higher in period 1 than in period 6 (one-sided Wilcoxon matched-pairs signed-ranks test,  $p < .01$  for game 1 and  $p < .04$  for game 2). The average prices per pair are significantly higher in period 6 than in period 10 in game 1 ( $p < .03$ ), while the difference in game 2 shows only weak significance ( $p < .09$ ).

**Result 1** *In the Bertrand duopoly without network externalities, we find some tacit collusion, which is eroding partially over time. Cooperation is stronger in game 2 and the erosion of collusion is weaker and starts later than in game 1.*

#### *Treatment NE*

Prices in the experimental markets with network externalities are very different from the prediction as Figure (3) shows. Prices in the early rounds are much higher than subgame-perfection predicts. However, prices are never above 5, which documents the absence of tacit collusion in the stage games. Additionally, in early periods for both supergames, prices decline rather than increase. This is in strong contrast to the prediction. In game 1 the average price of pairs in period 1 is significantly higher than in period 6 ( $p < .01$ ) and in period 10 ( $p < .01$ ). Between periods 6 and 10 there is no statistically significant change. In game 2 the increased experience does not change the decreasing prices in the early periods. Pairs set significantly higher prices in period 1 than in period 6 ( $p < .04$ ). For later periods the competitors seem to increase prices a bit. The difference shows only weak significance, though ( $p < .09$ ).

**Result 2** *In contrast to the theoretical prediction, in the NE treatment the average price per pair decreases in the first half of both supergames. Average prices in early periods are close to the myopic Bertrand Equilibrium.*

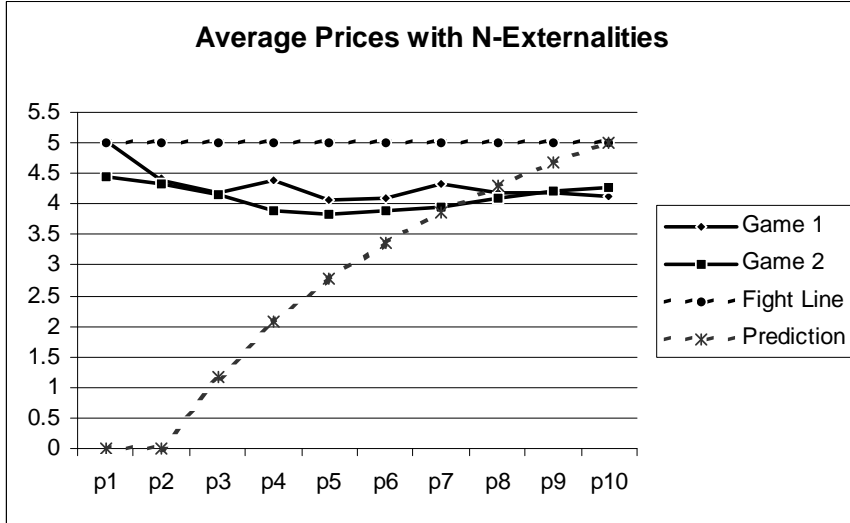


Figure 3: Price paths in the NE treatment

Our interpretation of this observed behaviour is the following. As humans are not able to perform backward induction over a many stages (e.g. Selten, 1978 or Brandts and Figueras, 2003) and our supergame is quite complex, the subjects in game 1 start off near the stage game equilibrium and use a rule of thumb. This rule of thumb seems to consist of a heuristic that balances the trade-off between increasing the market share and forgoing short-term profit. The model prediction that a higher present market share should increase the price chosen is turned into the opposite by the subjects. A subject that puts a higher value on the market share in its heuristic will play more aggressively independent of the present market share and will choose a lower price. However, the market share depends negatively on the past prices. So if our interpretation of subjects using heuristics is correct we should observe a negative correlation between current market share and price chosen for a given expected price of the opponent. It is important to take the opponent's expected action into account because without doing this we may misinterpret a relatively high price as myopic, while - given the expectation that the opponent will set a very high price - it is in fact intended to be very aggressive. In order

to test this we created a variable that measures the deviation from the myopic best response to the guessed price of the competitor. This variable captures the intention of a player. The lower this variable is the more aggressively the subject intends to behave.

Period	2	3	4	5	6	7	8	9	10
Game 1	-**	-	-*	+	-	-*	-***	-*	-**
Game 2	-***	-***	-	-***	-***	-*	-**	-	-

\* 10% level, \*\* 5% level, \*\*\* 1% level

Table 1: Correlation between intention to fight and market share

Table 1 shows that the correlation between the deviation from the myopic best response to the guessed price and the market share is significantly negative for many periods and never significantly positive. Put differently, subjects who have obtained a high market share already try to increase their market share further, instead of cashing in on their market power, as game theory would predict. Over all the sign of the correlation coefficient is only positive for period 5 in game 1 (highly insignificant with  $p = .39$ ). Note that in the first supergame the motive of fighting for market share is even dominant in the final period, where this cannot be explained by any future profit consideration. This illustrates that subjects pursued gaining a high market share as a goal per se rather than doing so to increase future profit opportunities.

The period 10 average prices in the NE treatment are below the myopic average price of 5 for both supergames (4.12 in game 1 and 4.28 in game 2). In sharp contrast to this the average prices in the No-NE treatment lie above 5 (5.58 in game 1 and 6.02 in game 2). The picture becomes even clearer when we look at the intended play. In the No-NE treatment subjects choose prices close to the best response to the anticipated price of the opponent in period 10. On average the chosen prices are .11 below the best response in game 1 and only .05 below the best response in game 2. In the NE treatment the intended play shows that subjects were still fighting for market share in the final

period. In game 1 the chosen prices were on average .64 below the best response to the anticipated price of the opponent. The difference in game 2 was smaller, but still substantial, with average prices being .46 below the best response.

**Result 3** *Subjects use a heuristic that puts certain weights on short-term profit and on market share rather than backward induction as a behavioural rule. The deviation from the myopic best responses to the opponent's expected price are negatively correlated with the current market share. This is consistent with a heuristic and incompatible with backward induction.*

## 5.2 Network externalities and competitiveness

Figures 4 and 5 compare the average prices in the treatments for game 1 and 2. It is obvious that the prices in the treatment with network externalities are consistently lower than in the No-NE treatment ( $p < .01$  for periods 1 to 18 and  $p < .025$  for the remaining two periods, one-sided Mann-Whitney U-test). We observe that the price differences are greater in game 1 (roughly 1.7) than in game 2 (between 1.8 and 3).

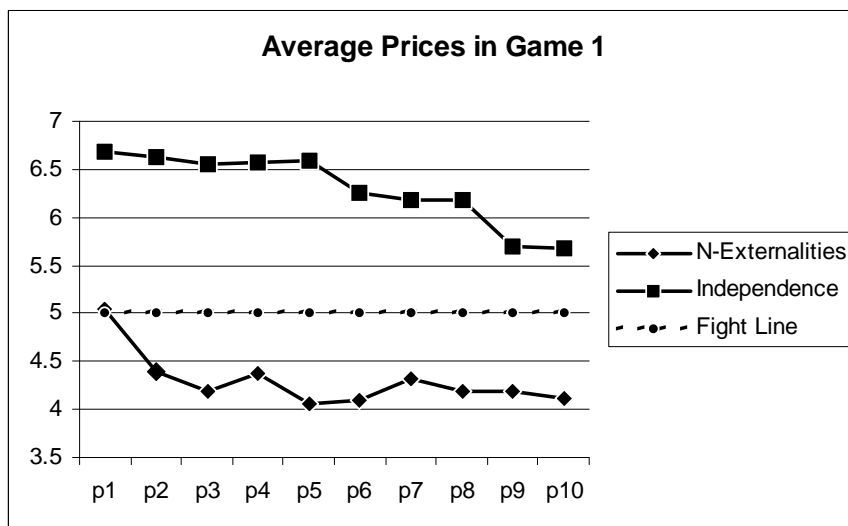


Figure 4: Price differences game 1

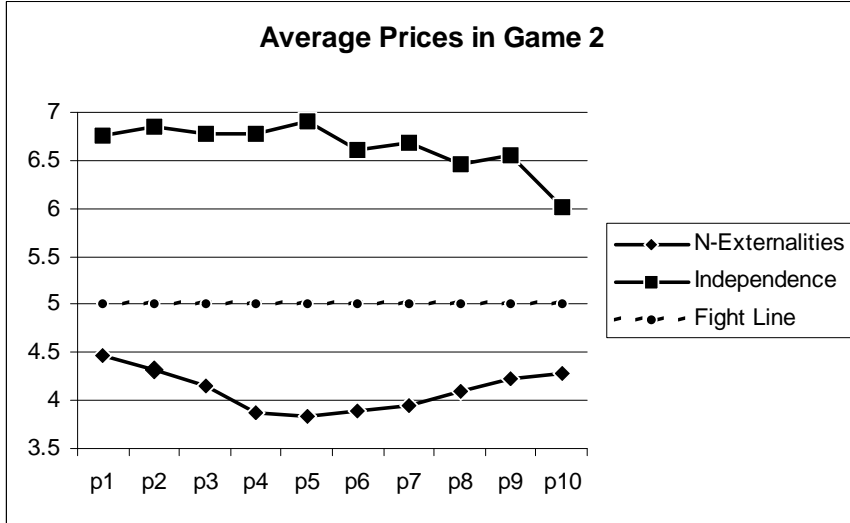


Figure 5: Price differences in games 2

**Result 4** *Average prices in the No-NE treatment are significantly higher than in the NE treatment for both games. The price differences among treatments are greater in game 1 than in game 2..*

The setting in the No-NE treatment is relatively collusion friendly, while in the NE treatment the network externalities introduce an additional competitive element - the struggle for market share. So the result that competition is more fierce in a market with modest network externalities than in one without is not surprising. More surprising is that the price difference in the market does not strongly decrease, the closer we get to the end of the product lifetime.<sup>16</sup>

An interesting question is to compare the predicted average effects network externalities have on the distribution of rents with the experimental outcome. Are the consumers getting a relatively better deal out of the additional competition due to network externalities in theory or in the experiment? A measure is the relative benefit of the network

<sup>16</sup>There is an end effect in game two. The price difference shrinks in the last round. However, in early periods where we expect the gap to narrow with a high rate, the gap increases.

externalities in the experimental sessions compared with the theoretical benefit. As we used a perfectly inelastic demand we cannot use consumer surplus as a measure.<sup>17</sup> However, we can compare the profits the firms make in theory and experimental practice. Since the quantities in theory and in the experiment are constant for all rounds, we can use the average price per round as an indication of how much potential consumer surplus the firms were able to transform into profits. Table 2 shows the average prices over all rounds and firms. We see that the absolute benefit for consumers (the average price difference between No-NE and NE) is in the same range for theory and the two experimental games (2.27 versus 2.0 and 2.53, respectively); but in the No-NE treatment collusion with high average prices mean that the relative benefit of increased competition is smaller in the experimental NE market than theory predicts.

	Theory	Game 1	Game 2
No-NE	5.00	6.30	6.64
NE	2.73	4.30	4.11
Benefit of NE absolute	2.27	2.00	2.53
Benefit of NE relative	45.5%	31.8%	38.1%

Table 2: Average prices over all rounds

**Result 5** *Increased competition due to network externalities reduces average prices approximately by the amount the theory predicts. The relative price-reduction is smaller in the experimental markets.*

## 6 Conclusion

Markets with network externalities are characterized by demand inertia. This demand inertia creates an incentive for firms producing competing products to set low introduct-

<sup>17</sup>Note that in our model due to the perfectly inelastic demand collusion does not cause any efficiency loss.

ory prices for their products as they seek to increase the size of their network. Optimal prices increase when the market matures as the incentive to fight for market share gets weaker as the market gets to the end of the product cycle. In reality we regularly do not observe increasing prices when markets with network externalities mature. However, this could be the result of more dominant countervailing effects, such as intertemporal price discrimination, increasing returns to scale due to learning by doing, or decreasing demand. In order to be able to determine the effect of demand inertia created by network externalities on dynamic prices we conducted a laboratory experiment. Our experimental setup was designed to isolate the effects of demand inertia by removing all other influence factors. As a benchmark we ran a control treatment where network externalities were absent. This set up enabled us to isolate the effect of demand inertia on dynamic pricing.

We found that as theory predicts, the average prices are lower if network externalities are present. However, average prices under network externalities in accordance with the real world decrease if subjects are not experienced. Theory predicts increasing prices. Even if subjects gain some experience prices still decrease over time in young markets. They only increase slightly when markets mature. We attribute this deviation from the theoretical prediction to the inability of subjects to conduct backward induction over a long and rather complicated supergame. We suggest that subjects instead use a rule of thumb that mitigates the trade-off between current profit and future profit potential depending on the market share. This rule of thumb seems to be surprisingly stable over time. Intended aggressiveness of play is positively correlated with the market share throughout the supergame. This means that people who play aggressively at the beginning of the game do not cash in on their obtained high market share in later periods, but stick to their rule of thumb and continue to play aggressively. This suggests that people are not only not able to perform backward induction, but have also problems to at least intuitively follow the logic of intertemporal profit maximization.



## References

- [1] Baake, Pio and Annette Boom (2001), “Vertical Product Differentiation, Network Externalities, and Compatibility Decisions.”, *International Journal of Industrial Organization*, 19, pp 267-284.
- [2] Belleflamme, Paul (1998), “Adoption of Network Technologies in Oligopolies.” *International Journal of Industrial Organization*, 14, pp 673-699.
- [3] Bensaid, Bernhard and Jean-Philippe Lesne (1996), “Dynamic Monopoly Pricing with Network Externalities.” *International Journal of Industrial Organization*, 14, pp 837-855.
- [4] Brandts, Jordi and Neus Figueras (2003), “An Exploration of Reputation Formation in Experimental Games.” *Journal of Economic Behaviour & Organization*, 50, pp 89-115.
- [5] Cabral, Louis M.B., Salant, David J. and Glenn A. Woroch (1999), “Monopoly Pricing with Network Externalities.” *International Journal of Industrial Organization*, 17, pp 199-214.
- [6] De Bijl, Paul W.J. and Sanjeev Goyal (1995), “Technological Change in Markets with Network Externalities.” *International Journal of Industrial Organization*, 13, pp 307-325.
- [7] Economides, Nicholas (1996), “The Economics of Networks.” *International Journal of Industrial Organization*, 16, pp 415-444.
- [8] Urs Fischbacher (1999), “z-Tree - Zurich Toolbox for Readymade Economic Experiments - Experimenter’s Manual.”, Working Paper Nr. 21, Institute for Empirical Research in Economics, University of Zurich.

- [9] Huck, Steffen, Hans-Theo Normann and Joerg Oechssler (2000), “Does Information about Competitors’ Actions Increase or Decrease Competition in Experimental Oligopoly Markets?”, *International Journal of Industrial Organization*, 18(1), pp 39-57.
- [10] Katz, Michael L. and Carl Shapiro (1994), “Systems Competition and Network Effects.” *Journal of Economic Perspectives*, 8(2), pp 93-115.
- [11] Katz, Michael L. and Carl Shapiro (1994), “Product Introduction with Network Externalities.”, *Journal of Industrial Economics*, 40(1), pp 55-83.
- [12] Katz, Michael L. and Carl Shapiro (1986), “Technology Adoption in the Presence of Network Externalities.” *Journal of Political Economy*, 94(4), pp 822-841.
- [13] Katz, Michael L. and Carl Shapiro (1985), “Network Externalities, Competition, and Compatibility.” *American Economic Review*, 75(3), pp 424-444.
- [14] Keser, Claudia (2000), “Cooperation in Symmetric Oligopoly Markets with Demand Inertia.” *International Journal of Industrial Organization*, 18, pp 23-38.
- [15] Keser, Claudia (1993), “Some Results of Experimental Duopoly Markets with Demand Inertia.”, *Journal of Industrial Economics*. 41(2), pp 133-151.
- [16] Kristiansen, Eirik Gaard (1996):“R&D in Markets with Network Externalities.” *International Journal of Industrial Organization*, 14, 1996, pp 769-784.
- [17] Liebowitz, S. J. and Steven Margolis (1994),“Network Externality: an Uncommon Tragedy.”, *Journal of Economic Perspectives*, 8(2), pp 133-150.
- [18] Schilling, Melissa A. (2003), “Technological Leapfrogging: Lessons from the U.S. Video Game Console Market.” *California Management Review*, 45(3), pp 6-32
- [19] Selten, Reinhard (1978), “The Chain-Store Paradox.” *Theory and Decision* 9, pp 127-159.

- [20] Selten, Reinhard (1965), “Spieltheoretische Behandlung eines Oligopolmodells mit Nachfragetraegheit.” Zeitschrift fuer die gesamte Staatswissenschaft (JITE), 121, pp 301-324 and 667-689.
- [21] Witt, Ulrich (1997): “‘Lock-in’ vs. ‘Critical Masses’ - Industrial Change Under Network Externalities.” International Journal of Industrial Organization, 15, pp 753-773.

## A Derivation of the inverse demand function

In this appendix we demonstrate how the specific demand function used in the text can be derived from simple (specific) preferences for differentiated goods and network sizes. For comparable preferences we get similar results for the inverse demand functions. We chose this specific setting to keep the inverse demand functions as simple as possible.

Assume that consumers purchase one unit of the good per period. Every period  $a$  consumers are active in the market. They decide which brand to buy by comparing the net surplus the goods are creating. As the goods are not homogeneous the consumers *ceteris paribus* prefer one of the brands. Denote the surplus a certain consumer  $k$  derives from consuming the product of firm  $i$  as  $\theta_i^k$ . Define the additional surplus  $\nu$  for consumer from the network of product  $i$  as half the average sales of good  $i$  per past period:

$$\nu_i := \sum_{l=1}^{t-1} q_i^l / 2(t-1)$$

This can be interpreted in two ways:

1. The consumer learns some quality aspects of the good from the number of previous sales.
2. The consumer cares only for number of sales in the actual period (valued at 1/2

monetary unit each) and forms expectations according to the average past sales.<sup>18</sup>

Then the total net surplus of consuming good  $i$  is given by:

$$CS_i^k := \nu_i^k + \theta_i^k - p_i$$

So consumer  $k$  will purchase good  $i$  whenever  $CS_i^k > CS_j^k$  or

$$\Delta\theta^k > p_i - p_j - \sum_{l=1}^{t-1} [q_i^l - q_j^l] / 2[t-1]$$

where  $\Delta\theta^k$  is given by  $\theta_i^k - \theta_j^k$ . Suppose that the differences between values  $\Delta\theta^k$  for the differentiated products is distributed uniformly on the interval  $[-a/2, a/2]$ . Then for given prices and network sizes the number of consumers buying good  $j$  is given by

$$\begin{aligned} q_j^t &= aF\left(p_i - p_j - \sum_{l=1}^{t-1} [q_i^l - q_j^l] / 2[t-1]\right) = \\ &= -1/2a + p_i - p_j - \sum_{l=1}^{t-1} [q_i^l - q_j^l] / 2[t-1] \end{aligned}$$

Recall that  $\sum_{l=1}^{t-1} q_i^l = at - \sum_{l=1}^{t-1} q_j^l$ , since past total sales of all brands are equal to  $ta$ .

Replacing  $\sum_{l=1}^{t-1} q_i^l$  gives

$$\begin{aligned} q_j^t &= p_i - p_j + \frac{1}{t-1} \sum_{l=1}^{t-1} q_j^l = \\ &= as_j^t + p_i - p_j, \end{aligned}$$

which is the inverse demand function we use. The demand for firm  $i$  is just  $a - q_j^t = as_1^t + p_j - p_i$ .

---

<sup>18</sup>In the equilibrium of a symmetric duopoly these expectations are rational and we have a rational expectation equilibrium as both firms sell  $a/2$  in every period.

## B Proof of proposition 1

This proof is conceptually easy, but quite tedious. We only sketch the proof and omit some intermediate calculations.

**Proof.** We use the best response functions (2) in order to compute the equilibrium prices in the penultimate stage. This gives the following equilibrium price for player  $i$  :

$$p_i^{T-1*} = a \frac{-41 + 3s_i^{T-1} [T-1]^2 [9T-11] + 3T [41 + T [9T-35]]}{3 [T-1] [23 + 27T [T-2]]}$$

Note that  $p_j^{T-1*}$  is found by just replacing the index. Using those equilibrium prices and the law of motion for the market share we can compute the equilibrium price in the final period as a function of the market share in period  $T-1$  and subtract the equilibrium price obtained above:

$$p_i^{T-1*} - p_i^{T*} = \frac{a}{3} \left[ \frac{1}{T-1} - \frac{6 [2s_i^{T-1} - 1] [T-1]}{23 + 27T(T-2)} \right]$$

Further inspection shows that this converges to 0 when  $T$  approaches infinity. Additionally, the roots for  $T$  are all smaller than 2 ( $T_{1/2} = 1 \pm 2/\sqrt{33 - 12s_i^{T-1}}$ ). As  $T \in [2, \infty)$  we know that  $p_i^{T-1*} - p_i^{T*} \geq 0$  for all  $s_i^{T-1}$  if we find a valid  $T$  such that  $p_i^{T-1*} - p_i^{T*} > 0$  holds. For  $T = 2$  we get  $p_i^{T-1*} - p_i^{T*} = a [29 - 12s_i^{T-1}] / 69 > 0$ . ■

## C Dynamic programming approach to prove proposition 2

In this section we outline the dynamic programming solution of the NE-game. We assume functional forms for prices and continuation payoff and show that these assumptions are correct. The derivation of the average price in the main text uses these functional forms.

**Step 1: solve the last stage.** Optimal prices are

$$p_i^{T*} = \frac{a [1 + s_i^T]}{3} \quad (9)$$

$$p_j^{T*} = \frac{a [1 + s_j^T]}{3} \quad (10)$$

Stage payoffs are

$$\Pi_i^{T*} = [p_i^{T*}]^2 = \left[ \frac{a [1 + s_i^T]}{3} \right]^2 \quad (11)$$

$$\Pi_j^{T*} = [p_j^{T*}]^2 = \left[ \frac{a [1 + s_j^T]}{3} \right]^2 \quad (12)$$

**Step 2: The law of motion for the state variable**

$$s_i^{t+1} = s_i^t + \frac{p_j^t - p_i^t}{a [1 + t]} \quad (13)$$

**Step 3: Guessing functional forms for the recursion**

$$p_i^{t*} = \phi^t s_i^t + \gamma^t \quad (14)$$

$$\hat{\Pi}_i^t = \Phi^t [s_i^t]^2 + \Gamma^t s_i^t + \Lambda^t \quad (15)$$

The functional forms proposed do definitively work for the last round:

$$\begin{aligned}
\phi^T &= \frac{a}{3} \\
\gamma^T &= \frac{a}{3} \\
\Phi^T &= \left[\frac{a}{3}\right]^2 \\
\Gamma^T &= \frac{2a}{3} \\
\Lambda^T &= \left[\frac{a}{3}\right]^2
\end{aligned}$$

**Step 4: Bellman equation**

$$\hat{\Pi}_i^t = \pi_i^t + \hat{\Pi}_i^{t+1} \quad (16)$$

Differentiating (16) with respect to  $p_i$  and using (13) gives:

$$\begin{aligned}
\frac{\partial \hat{\Pi}_i^t}{\partial p_i^t} &= -2p_i^t + p_j^t + as_i^t + \frac{\partial s_i^{t+1}}{\partial p_i^t} \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}} = \\
&= -2p_i^t + p_j^t + as_i^t - \frac{1}{a[1+t]} \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}}
\end{aligned} \quad (17)$$

We can use the recursion relation from (15) to write the first-order conditions (17) as

$$\frac{\partial \hat{\Pi}_i^t}{\partial p_i^t} = -2p_i^t + p_j^t + as_i^t - \frac{2\Phi^{t+1} \left[ s_i^t + \frac{p_j^t - p_i^t}{a[1+t]} \right] + \Gamma^{t+1}}{a[1+t]} = 0$$

**Step 5: Solving for the prices** By solving for the parameters of the optimal prices we find:

$$\begin{aligned}
p_i^{t*} &= s_i^t \frac{Y^{t+1} [aY^{t+1} - 2\Phi^{t+1}]}{3Y^{t+1} - 4\Phi^{t+1}} + \\
&\quad \frac{2a\Phi^{t+1}Y^{t+1} + [2\Phi^{t+1} + 3\Gamma^{t+1}] [Y^{t+1}]^2}{4\Phi^{t+1}Y^{t+1} - 3[Y^{t+1}]^3} \\
&\quad + \frac{-a[Y^{t+1}]^3 - 4\Phi^{t+1}[\Phi^{t+1} + \Gamma^{t+1}]}{4\Phi^{t+1}Y^{t+1} - 3[Y^{t+1}]^3}
\end{aligned} \tag{18}$$

where  $Y^t = at$  and recursively

$$Y^t = Y^{t+1} - a. \tag{19}$$

The proposed functional form is correct, since  $p_i^{t*}$  is an affine function of the market share. We can get the coefficient for the optimal price from (18):

$$\phi^t = \frac{Y^{t+1} [aY^{t+1} - 2\Phi^{t+1}]}{3Y^{t+1} - 4\Phi^{t+1}} \tag{20}$$

$$\begin{aligned}
\gamma^t &= \frac{2a\Phi^{t+1}Y^{t+1} + [2\Phi^{t+1} + 3\Gamma^{t+1}] [Y^{t+1}]^2}{4\Phi^{t+1}Y^{t+1} - 3[Y^{t+1}]^3} \\
&\quad + \frac{-a[Y^{t+1}]^3 - 4\Phi^{t+1}[\Phi^{t+1} + \Gamma^{t+1}]}{4\Phi^{t+1}Y^{t+1} - 3[Y^{t+1}]^3}
\end{aligned} \tag{21}$$

Note that by definition from equation (14) the average price  $\bar{p}^t$  is independent from the current market share if our functional forms are correct:

$$\bar{p}^t = \phi^t/2 + \gamma^t \tag{22}$$

**Step 6: The equilibrium motion for the market share** from equations (13) and (14) we can derive the equilibrium market share recursively:

$$s_i^{t+1} = s_i^t + \frac{[1 - 2s_i^t] \phi^t}{Y^{t+1}} \tag{23}$$

This tells us that the game has a steady state at a market share of 1/2.



**Step 7: Check the functional form for the profits** Note that the future profit only depends on the market share. So we have to find out how the future profit varies with the market share. Differentiating the total future profit at time  $t$  from (16) with respect to  $s_i^t$  gives:

$$\frac{\partial \hat{\Pi}_i^t}{\partial s_i^t} = \frac{\partial \pi_i^t}{\partial s_i^t} + \frac{\partial \pi_i^t}{\partial p_i^t} \frac{\partial p_i^t}{\partial s_i^t} + \frac{\partial \pi_i^t}{\partial p_j^t} \frac{\partial p_j^t}{\partial s_i^t} + \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}} \frac{\partial s_i^{t+1}}{\partial s_i^t}$$

Using our previous results and assumptions about functional forms from (14), (15), and (23) makes it possible to develop the previous equation

$$\begin{aligned} \frac{\partial \hat{\Pi}_i^t}{\partial s_i^t} &= ap_i^t + [-3p_i^t + p_j^t + as_i^t] \phi^t + \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}} [1 - 2\phi^t] = \\ &= ap_i^t + [-4\phi^t s_i^t + \phi^t - 2\gamma^t + as_i^t] \phi^t + \frac{\partial \hat{\Pi}_i^{t+1}}{\partial s_i^{t+1}} [1 - 2\phi^t] = \\ &= [-4\phi^t s_i^t + \phi^t - 2\gamma^t + 2as_i^t] \phi^t + a\gamma^t + \\ &\quad [1 - 2\phi^t] [2\Phi^{t+1} s_i^{t+1} + \Gamma^{t+1}] \\ &= [-4\phi^t s_i^t + \phi^t - 2\gamma^t + 2as_i^t] \phi^t + a\gamma^t + \\ &\quad [1 - 2\phi^t] [2\Phi^{t+1} [s_i^t + [1 - 2s_i^t] \phi^t] + \Gamma^{t+1}] \end{aligned} \quad (24)$$

Our result from (24) is linear in  $s_i^t$ . So we can see that integration leads to the form we proposed. We get the following recursive relationships:

$$\Phi^t = [1 + 4[\phi^t - 1] \phi^t] \Phi^{t+1} + a\phi^t - 2[\phi^t]^2 \quad (25)$$

$$\Gamma^t = [1 - 2\phi^t] \Gamma^{t+1} + a\gamma^t + \phi^t [\phi^t + 2\Phi^{t+1} - 4\phi^t \Phi^{t+1} - 2\gamma^t] \quad (26)$$

With the recursive relations (20), (21), (19), (23), (25), and (26) we have defined the subgame-perfect equilibrium-behaviour of the firms. Furthermore, we have shown that the functional forms assumed are correct and the average price (defined in 22) is inde-

pendent from the current market share.