RUHR
ECONOMIC PAPERS

Jörg Franke

# Does Affirmative Action Reduce Effort Incentives?

## A Contest Game Analysis

technische universität dortmund

#185

# Imprint

## Ruhr Economic Papers

Published by

Ruhr-Universität Bochum (RUB), Department of Economics
Universitätsstr. 150, 44801 Bochum, Germany

Technische Universität Dortmund, Department of Economic and Social Sciences
Vogelpothsweg 87, 44227 Dortmund, Germany

Universität Duisburg-Essen, Department of Economics
Universitätsstr. 12, 45117 Essen, Germany

Rheinisch-Westfälisches Institut für Wirtschaftsforschung (RWI)
Hohenzollernstr. 1-3, 45128 Essen, Germany

## Editors



## Editorial Office



## Ruhr Economic Papers #185

Responsible Editor: Wolfgang Leininger



ISSN 1864-4872 (online) – ISBN 978-3-86788-208-8

Ruhr Economic Papers #185

Jörg Franke

# Does Affirmative Action Reduce Effort Incentives?

## A Contest Game Analysis

technische universität dortmund

## Bibliografische Informationen der Deutschen Nationalbibliothek

Die Deutsche Bibliothek verzeichnet diese Publikation in der deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über: *http//dnb.ddb.de* abrufbar.

ISSN 1864-4872 (online)
ISBN 978-3-86788-208-8

Jörg Franke[1]

# Does Affirmative Action Reduce Effort Incentives? – A Contest Game Analysis


## Abstract

*This paper analyzes the incentive effects of affirmative action in competitive environments modeled as contest games. Competition is between heterogeneous players where heterogeneity might be due to past discrimination. Two policy options are analyzed that tackle the underlying asymmetry: Either it is ignored and the contestants are treated equally, or affirmative action is implemented which compensates discriminated players. It is shown in a simple two-player contest game that a trade-off between affirmative action and high effort exertion does not exist. Instead, the implementation of affirmative action fosters effort incentives. Similar results hold in the n-player contest as well as under imperfect information if the heterogeneity between contestants is moderate.*



*JEL Classification: C72, D63, I38, J78*

*Keywords: Asymmetric contest; affirmative action; discrimination*

*April 2010*


---

*1 Technische Universität Dortmund. – I would like to thank Carmen Beviá for advice and several discussions. I also benefited from discussions with Caterina Calsamiglia and Miguel Angel Ballester, and from comments by Jordi Caballé, Luis Corchón, Matthias Dahm, Marta Ibarz, Wolfgang Leininger, Tom Palfrey, Pedro Rey-Biel, Heinrich Ursprung, and participants at several conferences and seminars. – All correspondence to Jörg Franke, TU Dortmund, Department of Economics and Social Science, Vogelpothsweg 87, 44227 Dortmund, Germany, e-mail: joerg.franke@tu-dortmund.de.*

# 1 Introduction

Affirmative action can be described as a public policy instrument that should ameliorate the adverse effects of discrimination on affected groups of individuals.[1] However, the implementation of affirmative action programs frequently gives rise to intense public discussions. One of the reasons for this controversy seems to be the fact that its implementation goes beyond formal equal treatment considerations by addressing discriminated groups directly which is, for example, reflected by phrases like 'positive discrimination', or 'preferential treatment' as synonyms for affirmative action. But even in contemporary societies in which formal equality is legally guaranteed and enforced, there exists empirical evidence of ongoing discrimination with respect to specific minority groups. Hence, although open discrimination is legally banned, some minority groups may be disadvantaged out of reasons for which they cannot be held ethically responsible.[2] In such cases in which formal 'equal treatment of equals'-legislation is ineffective because individuals are not ex-ante equal, the implementation of affirmative action policies could be justified on normative grounds; see Loury (1981) and Loury (2002).

However, opponents of affirmative action do not only criticize the, from their perspective, formal violation of the equal treatment principle but also they refer to potential adverse consequences with respect to effort incentives. The following statement by Thomas Sowell from his book "Affirmative Action Around the World" reflects this concern:

> Both preferred and non-preferred groups can slacken their efforts - the former because working to their fullest capacity is unnecessary and the latter because working to their fullest capacity can prove to be futile. [...] While affirmative action policies are often thought of, by advocates and critics alike, as a transfer of benefits from one group to another, there can also be net losses of benefits when both groups do less than their best. What might otherwise be a zero-sum game can thus become a negative-sum game. (Sowell (2004), p. 14)[3]

Hence, their might exist a trade-off between affirmative action (i.e. preferential treatment) and high effort exertion due to potential disincentive effects of those policies. This line of critique

[1] Discrimination is interpreted here as a disadvantage of a group of individuals in different social contexts that is based on some kind of exogenous marker, e.g. race, gender, or nationality, for which the members of these groups are personally not responsible. Alternatively, more shortly and less technical, discrimination can be described as "allowing racial identification [or gender, nationality etc.] to have a place in an individual´s life chances" (Arrows (1998), p. 91).

[2] This persistence of discrimination could, for instance, be interpreted as the consequence of historical discrimination that affects negatively the contemporaneous generation, e.g. if investment in human capital depends on the historical segregation of work and living places along races; see Lundberg and Startz (1998) for a theoretical model.

[3] Similar incentive arguments have also been applied in the legal discussion, for instance, by Justice Thomas in Grutter v. Bollinger, 539 U.S. 306 (2003).

is also addressed in Fryer and Loury (2005a), 'Myth No. 3', where it is stated that "confident a priori assertions about how affirmative action affects incentives are unfounded. Indeed, economic theory provides little guidance" (ibid., p. 153 f). The contest game, that is introduced in the next section, is an attempt to fill this gap in theoretical analysis by analyzing the incentive effects of affirmative action in the framework of a non-cooperative game between competing agents. In this contest game the implementation of affirmative action is modeled as a biased contest rule[4] where weak contestants are favored because ethical perception interprets their weakness as being the consequence of past discrimination. The alternative perception, i.e. holding the contestants ethically responsible for their heterogeneity, requires instead the implementation of an unbiased contest rule. Both policies are defined normatively as (procedural) restrictions with respect to the specification of the contest rule which imply different incentives for the individuals depending on the implemented policy option. The key question is therefore how individuals would react to the distortion of incentives that is induced by the two policies.

Contrary to Sowell´s prediction it is shown that in a two-player contest game the optimal individual response to the implementation of affirmative action is to increase individual effort level in comparison to the unbiased contest game (irrespective of the fact whether the individual is discriminated or not). Hence, affirmative action and high effort exertion are not conflictive objectives but rather complementary in this case. The reason for this result is intuitive: The implementation of affirmative action reduces discrimination related heterogeneity between individuals which results in a more balanced playing field. This induces higher competitive pressure which is directly translated into higher effort exertion by both players.[5]

However, relaxing the restriction on the number of players is not innocuous: The result for the two-player case can only be sustained in the n-player contest game if the underlying heterogeneity is not too severe because otherwise participation effects dominate incentive effects. Hence, if participation levels are similar under both policies (a precise condition is provided in

[4]The underlying game theoretic model is an asymmetric contest game with heterogeneous players. Asymmetric contest games are already mentioned in the seminal contribution by Tullock (1980), and are applied in different frameworks, for example, to analyze legal presumption in trials; see Bernardo et al. (2000), with the interpretation of prior probabilities; see Corchón (2000), or in a two-stage rent-seeking contest; see Leininger (1993).

[5]The result that favoring weak players in competitive situations can restore efficient outcomes is also observed in different contexts, e.g. in international trade theory, see McAfee and McMillan (1989), rank-order tournaments, see Lazear and Rosen (1981), or all-pay auctions with incomplete information, see Clark and Riis (2000).

the respective section) then the implementation of affirmative action is effort enhancing.

The contest model is formulated in general terms to reflect in a stylized way a variety of competitive situations in which affirmative action is implemented. Potential examples include, for instance, admission to high school or prestigious university programs, bonus tournaments within a firm, or even sport contests, where weak players are sometimes favored by the contest rules. Along these lines there exists a limited number of studies that analyze the consequences of affirmative action. Fu (2006) models college admission as a two-player all-pay auction under complete information and shows that favoring the discriminated player to some extent induces the maximal expected academic effort (interpreted as the expected test score) by both candidates. A similar conclusion is derived in Schotter and Weigelt (1992) who analyze, also experimentally, a two-player tournament with unobservable effort. However, those kind of contributions do not specify the normative objective of affirmative action, i.e. in these papers affirmative action is considered simply as a deviation from an unbiased 'equal treatment'-policy. This is a crucial difference to the contest model presented below because here the normative objective of affirmative action is explicitly defined and integrated into the model.[6] Additionally all mentioned studies restrict their analysis to the two-player case. Extending the set-up to more than two players might yield non-trivial results, as it is shown for the contest game as presented here.

The rest of the paper is structured as follows. The contest game set up is introduced in the next section. Moreover, the respective policy options are formally defined and the corresponding policy weights are derived. The two-player contest game is analyzed in section 3, and its extension to more than two-players in section 4. Additionally, a specific example is provided to clarify derived results. This example is generalized to the case of imperfect information by the contest organizer in section 5. Section 6 contains some concluding remarks.

[6] In Fryer and Loury (2005b) a general equilibrium model with incomplete information is introduced where contestants compete in simultaneous pair-wise tournaments. In this framework group-sighted and group-blind affirmative action policies are compared without addressing explicitly the incentive effects of affirmative action versus unbiased tournament rules. For an empirical approach that also takes into account the reaction of affected individuals see Fryer et al. (2008).

## 2 The Model

Affirmative action instruments are usually applied in situations of competitive social interaction. The competitive structure of these situations can be captured by a contest game in which contestants compete for an indivisible prize by exerting effort. To clarify ideas the canonical example of affirmative action in university admissions is here briefly interpreted as a contest game. In the admission process applicants compete for a place in a university program based on their grade point average (GPA). As the admission officer might also take into account other personal characteristics, the outcome of the admission process will be probabilistic out of the perspective of the applicants, i.e. an applicant that has a marginal higher GPA than all her competitors for a specific place cannot be sure to be admitted with certainty. This feature is captured by a probabilistic contest success function, formally introduced below, where GPA is interpreted as individual effort exertion.

An applicant that intends to achieve a high GPA to increase the probability of being accepted faces a respectively higher disutility related with this attempt, i.e. obtaining good grades is laborious and different applicants might face different disutility for comparable GPA levels. If this heterogeneity can be traced back to reasons that lie beyond personal control (for instance, minority students could be disadvantaged because they did not have access to high quality schooling due to racial segregation of living places) then the implementation of affirmative action can be justified on normative grounds. In this case affirmative action favors the disadvantaged applicants in the admission process to achieve a level playing-field.[7] In the related contest game contestants are disadvantaged in the sense that they face different disutilities for exerting effort, i.e., achieving the same effort level for disadvantaged contestants is more costly. Affirmative action compensates for this disadvantage and is modeled as an asymmetric contest rule that is biased in favor of disadvantaged contestants.

As the contest model is sufficiently general to capture different competitive situations where affirmative action is applied, the formal set up will be based on the contest terminology instead of addressing explicitly the university admission example.

[7] For instance, minority students that applied for admission to the College of Arts and Sciences at the University of Michigan were granted a fixed bonus of 20 (out of 150) points. Later, this type of affirmative action policy was ruled as unconstitutional by the Supreme Court in Gratz v. Bollinger, 539 U.S. 244 (2003) because it was not a 'narrowly tailored' instrument.

## 2.1 The Contestants

Let $N = \{1, 2, \ldots, n\}$ denote the set of contestants that compete against each other for a prize with fixed value $V$. Each contestant $i \in N$ exerts an effort level $e_i \in \Re_+$ and takes the effort level of its rivals $e_{-i} = (e_1, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n) \in \Re_+^{n-1}$ as given. Contestants are heterogeneous with respect to their respective 'cost function' that captures the disutility of exerting effort.[8] It is assumed that this cost function is linear in $e_i$ and multiplicative in $\beta_i \in [1, \infty)$ for all $i \in N$:[9]

$$c_i(e_i) = \beta_i \, e_i \text{ for all } i \in N. \tag{1}$$

This specification implies that contestants can be ordered according to their marginal cost parameter $\beta_i$. The following order is assumed from now on to hold:

$$1 = \beta_1 \leq \beta_2 \leq \ldots \leq \beta_n.$$

The contestants perceive the outcome of the contest game as probabilistic. However, they can influence the probability of winning by exerting effort, i.e. the outcome depends on the vector of effort levels exerted by all individuals. The (potential) implementation of affirmative action is modeled as an asymmetric Contest Success Function (CSF), axiomatized in Clark and Riis (1998), that maps the vector of effort levels $e = (e_1, \ldots, e_n)$ into win probabilities for each contestant: $p_i(e) : \Re_+^n \to [0, 1]$:

$$p_i(e) = \frac{\alpha_i^P e_i^r}{\sum_{j \in N} \alpha_j^P e_j^r} \text{ for all } i \in N, \tag{2}$$

with $\alpha_i^P > 0$ for all $i \in N$ and $r \in (0, 1]$. The parameter $r$ measures the sensitivity of the outcome of the contest game with respect to differences in effort.[10] Additionally, each individual effort level is weighted by a positive parameter $\alpha_i^P$ that depends on the policy $P$,

[8] This heterogeneity might be the consequence of past discrimination. Moreover, the specified model is strategically equivalent to a contest game where, for instance, valuations of the prize are different among the contestants. The assumption that contestants are heterogeneous with respect to their cost functions is therefore without loss of generality.

[9] The lower bound on the distribution of marginal cost parameters, i.e. $\beta_1 = 1$, is made for analytical convenience without affecting results.

[10] The upper bound $r \leq 1$ guarantees the existence of an equilibrium in pure strategies.

formally defined in the next subsection.[11] If no contestant exerts positive effort it is assumed that none of the contestants receives the prize, i.e. $p_i(0, \dots, 0) = 0$ for all $i \in N$. Hence, the expected utility function has the following form:

$$u_i(e_i, e_{-i}) = p_i(e)V - c_i(e_i) \text{ for all } i \in N. \tag{3}$$

## 2.2 The Policy Options

It is assumed that the choice of policy $P$ is based on the normative perception of the heterogeneity of the contestants (i.e. the different cost functions)[12] which directly implies the normative objective of the respective policy option and therefore also governs the specification of the vector of individual effort weights $\alpha^P = (\alpha_1^P, \dots, \alpha_n^P)$. There are two potential interpretations for the source of this heterogeneity that result in two alternative policies.

The first interpretation holds the contestants ethically responsible for their respective cost function which implies that the probability to win the contest game (i.e. the CSF) should only depend on the vector of exerted effort. In other words, if a contestant $i$ exerts the same effort level as a contestant $j$ then both contestants should win the contest game with the same probability. This policy option would therefore treat the contestants equally with respect to their exerted effort level.

**Definition 1** *A policy is called* ***equal treatment approach*** *(ET) if:*

$$e_i = e_j \Rightarrow p_i(e) = p_j(e) \textit{ for all } i \neq j.$$

For the class of contest games as defined by the CSF in eq. (2) equal treatment implies that the policy weights $(\alpha_1^{ET}, \dots, \alpha_n^{ET})$ must be identical for all players:

$$\alpha_i^{ET} = \hat{\alpha}^{ET} \text{ for all } i \in N.$$

[11] Hence, in the terminology of Konrad (2002) the weighting factors that are specified by policy $P$ alter the 'productivity advantage' of the respective contestants.

[12] As the base model is formulated under complete information, the individual levels of marginal cost are common knowledge. An example with partial information is presented in section 5.

The last equation is derived based on the hypothetical assumption that $e_i = e_j$ and the definition of the CSF as given in eq. (2). Hence, as already revealed by its name, all players will be treated equally under policy $ET$.

Alternatively, policy $ET$ could also be interpreted as an anonymity principle because it postulates that the contest success function neither depends on the specific names nor on the exogenous characteristics of the players. As the CSF is homogeneous of degree zero, the $ET$ weights do not have any strategic effect on the contestants and could also be eliminated.[13] However, the outcome, i.e. expected equilibrium utility, of the contest game will indirectly depend on the characteristics of the players because weaker players will exert less effort in equilibrium.

The second interpretation is based on the perception that the contestants cannot be held ethically responsible for their heterogeneity, for instance, because it is the consequence of past discrimination. As heterogeneity affects the cost function of each contestant, fairness would require that two contestants that face equal disutility induced by the respectively chosen effort level (that could be different) should win the contest game with the same probability. The normative justification for this interpretation is the "moral intuition that two people incurring equal disutility deserve equal rewards", see Kranich (1994), p. 178.[14]

**Definition 2** *A policy is called* ***affirmative action*** *(AA) if:*

$$c_i(e_i) = c_j(e_j) \Rightarrow p_i(e) = p_j(e) \text{ for all } i \neq j.$$

For the class of contest games as defined by the CSF in eq. (2) the following relation with respect to the policy weights $(\alpha_1^{AA}, \ldots, \alpha_n^{AA})$ satisfies the definition of affirmative action:

$$\frac{\alpha_i^{AA}}{\beta_i^r} = \frac{\alpha_j^{AA}}{\beta_j^r} \text{ for all } i \neq j.$$

This relation can be derived by assuming that two contestants $(i, j)$ choose effort levels $(e_i, e_j)$

[13] In fact, in Skaperdas (1996), Theorem 2, the resulting CSF (specified by eq. (2) under policy $ET$) is axiomatized based on a conventional anonymity axiom, compare also footnote 15.

[14] In Kranich (1994) this quotation justifies a comparable normative restriction called 'equal-division-for-equal-work' in the framework of a two-player joint production economy based on sharing rules. In his model the interest is focused on the existence of efficient sharing rules that satisfy this principle and not on the their incentive effects. Note also that in the model presented here the terminology 'equal work' should be interpreted as equal disutility of effort as this is the relevant normative standard of comparison if contestants are not responsible for their different marginal cost parameters.

that induce identical disutility, $\beta_i\, e_i = \beta_j\, e_j$. This implies that $e_i = \frac{\beta_j}{\beta_i} e_j$. By Definition 2 the policy weights should be specified such that $p_i(e) = p_j(e)$. Simplifying this expression implies that $\frac{\alpha_i^{AA}}{\beta_i^r} = \frac{\alpha_j^{AA}}{\beta_j^r}$. As the CSF is homogeneous of degree zero, there is no loss in generality if the policy weights are normalized such that:

$$\alpha_i^{AA} = \beta_i^r \text{ for all } i \in N. \tag{4}$$

Policy $AA$ therefore generates a bias[15] of the contest success function in favor of discriminated contestants in such a way that both contestants have the same probability of winning the contest whenever they face the same disutility of effort. Note that this definition requires that the affirmative action bias is implemented multiplicatively through $\alpha_i^{AA}$ which increases the marginal efficiency of exerted effort for contestant $i$ and therefore changes the incentives for effort exertion.[16]

The definition of $AA$ as presented here can be considered as a principle that guarantees a notion of procedural fairness because it specifies normatively how the contest rule should be designed for specific (hypothetical) cases of chosen effort level. This determines uniquely (up to a multiplicative constant due to the fact that the CSF is homogenous of degree zero) the vector of $AA$ policy parameters. No consideration with respect to equilibrium behavior is needed for this interpretation. However, based on the equilibrium analysis in section 3 and 4 it can be shown that the $AA$ policy also induces identical expected equilibrium utility for all players.[17] Therefore, Definition 2 also satisfies an 'end-state interpretation' of affirmative action because it equalizes the expected outcome of the contest game. The normative argumentation for this type of alternative fairness interpretation is straight forward: If the contestants are perceived to be different because they are discriminated (for which they cannot be held ethically responsible)

[15] In Clark and Riis (1998) it is argued that the anonymity axiom of Skaperdas (1996) should be relaxed because "in many situations, however, contestants are treated differently (due to affirmative action programs for instance)" (Clark and Riis (1998), p. 201). The resulting CSF (axiomatized without anonymity axiom) has the same functional form as in eq. (2). Hence, Definition 2 can also be interpreted as a substitute of the anonymity axiom that entails now a specific normative restriction with respect to the asymmetry of the CSF based on the underlying heterogeneity of the contestants.

[16] This type of multiplicative affirmative action also has an interesting normative justification in a decentralized multi-contest situation, see Calsamiglia (2009). In this framework multiplicative affirmative action is the only policy that can equalize reward to effort.

[17] Under policy $AA$ all contestants choose equilibrium effort levels that induce identical equilibrium disutility. By Definition 2 this directly implies that in equilibrium also the probability to win the contest game must be identical for all contestants. Hence, they all obtain identical expected utility in equilibrium. This result holds for the two-player and also for the n-player contest game.

then the contest rule should be specified such that the outcome coincides with a hypothetical contest game where all contestants are de facto homogeneous.[18] This implies that expected utility in equilibrium should be identical for all contestants. The vector of policy weights as specified in eq. (4) satisfies this requirement. Hence, Definition 2 is robust with respect to the two presented alternative interpretations of normative fairness considerations.

In competitive situations in which affirmative action is implemented effort exertion can be interpreted as being socially valuable.[19] This is suggested by the fact that the incentive effects of those policies are considered in public discussions and legal disputes as mentioned in the introduction. In the quotation of Sowell, for instance, less effort of all participants is interpreted as being socially inferior. As the focus of this study is the positive analysis of the incentive effects of affirmative action, the two alternative policies $ET$ and $AA$ are compared based on a measure of total effort exertion, i.e. the sum of equilibrium effort that each policy generates.[20] As the equilibrium effort level of each contestant will depend on the ex-ante announced policy parameter $P$, the standard of comparison will therefore be defined as follows: $E_P^* = \sum_{i \in N} e_i^*(P)$ for $P \in \{ET, AA\}$. Additionally, the two policy options will be evaluated with respect to the individual reactions of the contestants by comparing equilibrium effort for individual contestants under the two policy regimes.

The timing of the complete contest game can be summarized in the following way: The heterogeneity of the contestants (i.e. different marginal cost parameters) is observed. Based on the ethical perception of this observation a policy option $P \in \{ET, AA\}$ is selected. The chosen policy option determines the vector of weighting parameters $\alpha^P = (\alpha_1^P, \ldots, \alpha_n^P)$ for the respective policy. Each contestant $i \in N$ exerts the utility-maximizing effort level $e_i^*(P)$, taking as given the effort levels of their rivals and the relevant weights induced by policy $P$. In the last step the exerted efforts are observed and the winner of the contest game is determined

[18]This normative part hinges crucially on the assumption that there is only one source of heterogeneity for which the contestants are either ethically responsible or not. In section 6 this assumption is relaxed in the sense that there are two sources of heterogeneity where only one is normatively relevant.

[19]This interpretation of exerted effort being socially valuable is the crucial difference to the extensive literature on rent-seeking contests (compare Konrad (2009) for a recent survey). There, exerted effort is usually interpreted as pure social waste that should be minimized. However, there also exist studies in this literature where total effort should be maximized, e.g. Gradstein and Konrad (1999). In the recent literature on sport contests effort, i.e. the performance of the athletes, has a similar interpretation; see Szymanski (2003).

[20]The selection effects that are generated by the implementation of affirmative action policies are not in the focus of this approach. Studies that address the selection effects of affirmative action are, for instance, Chan and Eyster (2003), as well as Chan and Eyster (2009).

according to the announced policy option. Finally, the total and individual equilibrium effort that is generated by each policy can be compared to shed lights on the induced incentive effects by the two policies.

## 3 The Two-Player Contest Game

Analyzing the two-player contest game is instructive in the sense that it provides the intuition why contestants are induced to exert more effort in equilibrium under the AA policy than under the ET policy. Additionally, the incentive enhancing effects of AA are (in contrast to the $n$-player contest game) valid without any extra assumption and the equilibrium in the two-player contest is characterized by simple first order conditions. Therefore, the two-player and the n-player contest game are analyzed in separate sections.

For the two-player contest game the expected utility function for policy $P \in \{ET, AA\}$ can be expressed as:

$$u_i(e_i, e_j) = \frac{\alpha_i^P e_i^r}{\alpha_1^P e_1^r + \alpha_2^P e_2^r} V - \beta_i e_i \text{ for } i = 1, 2;$$

where contestant 2 has a higher marginal cost parameter than contestant 1: $\beta_2 > \beta_1 = 1$. By Definition 1 and 2 the bias for contestant 1 is normalized to $\alpha_1^P = 1$ for $P \in \{ET, AA\}$. Solving first order conditions for a given policy parameter $P$ yields the candidate for equilibrium effort by player $i = 1, 2$:

$$e_i^*(P) = \frac{\alpha_i^P \alpha_j^P \beta_i^{r-1} \beta_j^r}{(\alpha_i^P \beta_j^r + \alpha_j^P \beta_i^r)^2} rV \qquad \text{for } i \neq j, \tag{5}$$

that would imply positive expected equilibrium utility by the assumption that $r \leq 1$:

$$u_i(e_i^*(P), e_j^*(P)) = \frac{(\alpha_i^P \beta_j^r)^2 + \alpha_1^P \alpha_2^P (\beta_1 \beta_2)^r (1-r)}{(\alpha_1^P \beta_2 + \alpha_2^P \beta_1)^2} V > 0. \tag{6}$$

A non-interior equilibrium in which a contestant exerts zero effort cannot exist because there always exists a profitable deviation for one of the contestants.[21] The second order conditions

[21] If both contestants would exert zero effort a deviating player $i$ will always win the contest with certainty by exerting a slightly positive effort level $\epsilon$: $u_i(\epsilon, 0) > u_i(0, 0) = 0$. If only one contestant $j$ would exert zero effort player $i$ can deviate profitably by decreasing her chosen effort level by a small amount $\epsilon$ because then she still wins the contest game with certainty: $u_i(e_i - \epsilon, 0) > u_i(e_i, 0)$ as long as $e_i - \epsilon > 0$.

can be expressed in the following way:

$$\frac{\partial^2 u_i(e_i,e_j)}{\partial e_i^2} = \frac{\alpha_1^P \alpha_2^P r V e_i^{2r-2} e_j^r}{(\alpha_1^P e_1^r + \alpha_2^P e_2^r)^3} \left[ \alpha_j^P (r-1) \left( \frac{e_j}{e_i} \right)^r - \alpha_i^P (r+1) \right] < 0,$$

which proves concavity because the expression in brackets is negative while the first expression is positive. Hence, the equilibrium is interior and unique. From eq. (5) it can also be noted that the relation between individual equilibrium effort levels is independent of the implemented policy because:

$$\frac{e_1^*(P)}{e_2^*(P)} = \beta_2 \quad \text{for } P \in \{ET, AA\}\,. \tag{7}$$

The two policy alternatives $ET$ and $AA$ can now be evaluated with respect to the sum of equilibrium effort $E_P^* = \sum_{i=1,2} e_i^*(P)$ that each policy generates. The following proposition states the result for the two-player contest game: The affirmative action policy as specified in Definition 2 will induce higher individual and also higher aggregated effort than the equal treatment policy. Hence, in the two-player contest game as specified here a trade-off between affirmative action and aggregate effort does not exist.

**Proposition 1** *In the two-player contest game (i) the sum of equilibrium effort, and (ii) individual equilibrium effort level for both contestants is higher under policy AA than under policy ET.*

*Proof*: Using eq. (5) and Definition 1 and 2, the inequality $E_{AA}^* > E_{ET}^*$ can be reduced to $\frac{rV}{4}\frac{\beta_2+1}{\beta_2} > \frac{rV\beta_2^r}{(1+\beta_2^r)^2}\frac{\beta_2+1}{\beta_2}$, which is always satisfied because it can be simplified to $(1-\beta_2^r)^2 > 0$. This establishes part (i) of the proposition.

Using the fact that the relation between the equilibrium effort levels remains constant, as stated in eq. (7), proves part (ii).□

This result can be attributed to the fact that the implementation of the $AA$ policy yields a contest game that is more balanced with respect to the characteristics of the contestants (the heterogeneity of the contestants is reduced by the biased CSF). As the contestants are more similar under $AA$, the competitive pressure is higher which implies higher equilibrium effort by both contestants.

In fact, the bias that is induced by $AA$ for the two-person contest game yields a level playing field, i.e. the contestants are as similar as possible under this set-up. Therefore, policy $AA$ also generates the maximal aggregated effort even for a contest game where the specification of the policy weights would not be restricted by any normative constraint. In other words, if the objective would solely be the maximization of total equilibrium effort by implementing an appropriate weight $\hat{\alpha_2}$ then this weight would coincide with the bias that is required by the $AA$ policy.[22]

**Proposition 2** *The policy option $AA$ generates the maximal sum of equilibrium effort in the two-player contest game.*

*Proof*: Consider the sum of equilibrium effort for an arbitrary parameter $\alpha_2$ that favors the discriminated contestant: $E^* = \frac{\alpha_2 \beta_2^r}{(\alpha_2+\beta_2^r)^2}\frac{\beta_2+1}{\beta_2} rV$. This expression is maximized for $\hat{\alpha_2} = \beta_2^r$ which coincides with $\alpha_i^{AA} = \beta_i^r$ for $i = 1, 2.\square$

The last two propositions reveal that in the above specified two-player contest there is neither a trade-off between the AA policy and aggregated effort nor individual effort exertion as both contestants will exert higher effort levels in equilibrium if they face affirmative action. In fact, as it was shown in Proposition 2, the affirmative action bias even leads to the highest possible level of total equilibrium effort under all possible policy weights that are in line with the CSF specified in eq. (2). In the next section it is analyzed if these results are also valid for contest games with more than two players.

## 4 The n-Player Contest Game

In the $n$-player contest game not all contestants will always actively participate in equilibrium, i.e. some contestants might prefer to exert zero equilibrium effort.[23] Therefore the derivation of the equilibrium and the proof of existence and uniqueness are more involved than in the two-player case. Moreover, an additional assumption is needed in the $n$-player contest game to

[22] Nti (2004) analyzes a similar 2-player contest game with heterogeneous valuations and a (linear) CSF of the form $p_i(e) = \frac{\alpha_i e_i + \gamma_i}{\sum_{i=1,2} \alpha_i e_i + \gamma_i}$. In this set-up, total equilibrium effort is maximized if $\gamma_1 = \gamma_2 = 0$ and the multiplicative parameters $(\alpha_1, \alpha_2)$ balance the heterogeneity of the valuations.

[23] This implies that the equilibrium in the n-player case might be non-interior and therefore cannot be characterized by first-order conditions. The approach that is instead applied is based on the notion of 'share functions' as defined in Cornes and Hartley (2005).

guarantee the existence of closed form solutions: The subsequent analysis is therefore restricted to linear CSFs with parameter $r = 1$.[24]

The expected utility of contestant $i$ in the n-player contest can be expressed as:

$$u_i(e_i, e_{-i}) = \frac{\alpha_i^P e_i}{\sum_{j\in N} \alpha_j^P e_j} V - \beta_i e_i \text{ for all } i \in N \text{ and for } P \in \{ET, AA\}. \tag{8}$$

The equilibrium of this contest game will be derived in the appendix, based on the observation that the contest game can be interpreted as an aggregative game with its convenient properties. The following equation provides an expression of equilibrium effort for those $m$ contestants of the set $M \subseteq N$ that are active, i.e. that exert positive equilibrium effort:

$$e_i^*(P) = \frac{1}{\alpha_i^P}\left(1 - \frac{\beta_i}{\alpha_i^P}\frac{(m-1)}{\sum_{j\in M}\frac{\beta_j}{\alpha_j^P}}\right)\frac{(m-1)V}{\sum_{j\in M}\frac{\beta_j}{\alpha_j^P}} \text{ for all } i \in M \text{ and } P \in \{ET, AA\}. \tag{9}$$

Set $M = (1, \ldots, m)$ is indirectly defined by the subsequent expression where $m$ denotes the contestant with the highest marginal cost that satisfies the following inequality:[25]

$$(m-1)\frac{\beta_i}{\alpha_i^P} < \sum_{j\in M}\frac{\beta_j}{\alpha_j^P} \text{ for all } i \in M \text{ and } P \in \{ET, AA\}. \tag{10}$$

Using the specification of the weights for the $AA$ and $ET$ policy and the characterization of the active set, the following lemma describes the set of participating contestants for each policy option.

**Lemma 1** *Under policy $ET$ the active set $M \subseteq N$ of contestants is implicitly defined by the following expression where $m$ is the maximum element of set $N$ that satisfies the inequality*

$$(m-1)\beta_i < \sum_{j\in M}\beta_j \textit{ for all } i \in M. \tag{11}$$

*Under policy $AA$ all contestants will be active.*

[24] Cornes and Hartley (2005) provide existence results for non-linear CSFs with $r \neq 1$ based on implicit equilibrium characterizations. They also show that there might exist multiple equilibria which makes the derivation of comparative static results intractable.

[25] The order of contestants in set $M$ and $N$ coincides, i.e. the contestants in set $M$ are also ordered as an increasing sequence with respect to their marginal cost parameter.

As equilibrium effort is given by eq. (9), the two policies can now be compared with respect to the aggregated equilibrium effort $E_P^* = \sum_{i \in M} e_i^*(P)$ that they induce. However, Lemma 1 already reveals that the comparison between the two policy options will not be as straight forward as in the two-player contest game because total equilibrium effort depends on the distribution of the cost parameter that determines the active set.

The following notation will simplify the characterization of the relevant distribution for a subset $J = (1, \ldots, j) \subseteq N$ of contestants: The arithmetic mean of the cost parameters of agents of set $J$ will be denoted as $\bar{\beta}_J = \frac{1}{j} \sum_{i \in J} \beta_i$ (where $\bar{\beta} = \bar{\beta}_N$ to facilitate notation), and the harmonic mean respectively as: $\beta_J^H = \left[\frac{1}{j} \sum_{i \in J} \frac{1}{\beta_i}\right]^{-1}$.

The subsequent proposition states the condition under which policy $AA$ generates higher aggregated effort.

**Proposition 3** *In the n-player contest game the sum of equilibrium effort levels is higher under policy $AA$ than under policy $ET$ if and only if:*

$$\frac{\bar{\beta}_M}{\beta_N^H} > \frac{\frac{m-1}{m}}{\frac{n-1}{n}}. \tag{12}$$

*Proof*: Calculation of the sum of equilibrium effort for each policy under consideration of lemma 1 yields $E_{AA}^* = \frac{n-1}{n^2} V \sum_{i \in N} \frac{1}{\beta_i}$ and $E_{ET}^* = \frac{m-1}{\sum_{i \in M} \beta_i} V$. Reformulating the inequality $E_{AA}^* > E_{ET}^*$ leads to condition (12).□

The following intuitive explanation is provided for condition (12) to hold which is afterwards clarified by a numerical example. As already observed in the two-player contest game, $AA$ in general induces higher competitive pressure because contestants are more similar than under $ET$. Increasing the number of active contestants therefore yields higher total effort for both policies because this implies more intense competition. However, inducing heavily discriminated contestants to participate comes at a non-negligible cost, especially for the $AA$ policy, because by Lemma 1 all participants will be active under $AA$. This effect is less profound for $ET$ because highly discriminated contestant will not participate under $ET$.

*Numerical Example*: Consider the following contest game with three contestants that have marginal costs of $(\beta_1, \beta_2, \beta_3) = (1, 2, 2)$. The underlying dispersion is measured by the coefficient of variation (defined as $CV = \sigma(\beta)/\bar{\beta}$) which is in this case $CV \approx 0.2828$. For these parameters $AA$ will generate $E^*_{AA} \approx 0.4444$ that is higher than the aggregated effort under $ET$ which is $E^*_{ET} = 0.4$. If a fourth contestant with $\beta_4 = 2.43$ (which yields nearly the same level of dispersion $CV \approx 0.2828$) is added the difference between $AA$ and $ET$ becomes even more profound: $E^*_{AA} \approx 0.4522$ versus $E^*_{ET} \approx 0.4038$ and $E^*_{AA} - E^*_{ET} \approx 0.0483$. As expected, total effort is higher under both policies because the (active) fourth player contributes effort as well and induces more competition than in the 3-player contest. Moreover, this effect is higher for the $AA$ policy. However, if the fourth contestant is highly discriminated ($\beta_4 = 10$) this would imply a decline of total effort in the case of $AA$: $E^*_{AA} \approx 0.3938$. This decline is less intense in case of $ET$ because here the fourth player will not participate, i.e. $E^*_{ET} = 0.4$ as in the 3-player contest game. Comparing both values shows that for this four player constellation the result of the policy analysis has been reversed because now $E^*_{AA} < E^*_{ET}$.

This example demonstrates that the key factor for the outcome of the policy comparison is the distribution of the discrimination parameter in combination with the number of contestants. In general it can be stated that either a low number of contestants or a sufficient low dispersion makes it more probable that $AA$ will induce more total effort than $ET$ because then the set of active contestants tends to be similar for both policies.[26] The exact relation between the distribution of discrimination parameters and the number of players is described by the inequality in Proposition 3 in combination with the characterization of the active set in eq. (10).[27]

Two additional remarks with respect to the relation between the results for the two-player and the n-player contest should be in order at this point. First, Proposition 3 does not contain a result with respect to the existence of a vector of policy weights that would maximize total equilibrium effort. This issue is a complex mathematical problem which is discussed in more detail in section 6. Second, applying Proposition 3 to a two-player contest game with $r = 1$ would yield the same result as Proposition 1 because condition (12) holds irrespective of

[26]The observation that affirmative action might imply a distortion of the participation decision of individuals (which could finally dominate the effect of increased competitive pressure) has also empirical relevance: In an econometric analysis of bid preferences in highway procurement auctions Marion (2007) shows that preferential treatment implies a decline in competitive pressure because some non-preferred bidders switched to procurement auctions without bid preference program.

[27]Note, that the left hand side of condition (12) is lower than one for $m$ small and larger than one for $m$ large where $m$ is determined according to condition (10). Inspection of the right hand side reveals that it is always lower or equal to one. This confirms the qualitative statement that condition (12) is likely to hold if the number of contestants is relatively small or the distribution is not too dispersed.

the distribution of cost parameters in the two-player case: For policy options $AA$ and $ET$ both contestants will exert positive equilibrium effort, i.e. set $M$ and $N$ coincide. Therefore, Proposition 3 is satisfied without further restriction because condition (12) can be reduced to $\bar{\beta} > \beta_N^H$ which is always satisfied.

Due to the implicit formulation of individual equilibrium effort (which depends on the active set of contestants) a direct comparison of individual equilibrium effort under both policies in the n-player contest is only possible for given distributions of the cost parameters. Hence, the additional assumption of full participation by all contestants under both policies shall be considered to get further insights into individual equilibrium behavior. The assumption of full participation would imply that the dispersion of cost parameters is sufficiently low such that also under policy $ET$ all contestant would be active.

Although total equilibrium effort for this case is higher under policy $AA$ versus policy $ET$ (without any further conditions) as in the two-player case, the result with respect to individual equilibrium effort is different: In the n-player contest game the set of contestants that individually exert higher equilibrium effort under policy $AA$ than under $ET$ is restricted to contestants with either very low marginal cost or higher than average marginal cost.

**Proposition 4** *If all contestants in the n-person contest game are active under policy ET, then (i) the sum of equilibrium effort levels is higher under policy AA, and (ii) the individual equilibrium effort is higher under AA for all contestants with marginal cost parameter* $\beta \in \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$ *or* $\beta \in \left[1, \frac{1}{n-1}\bar{\beta}\right)$ *as long as this set is non-empty, i.e. for distributions where* $n-1 < \bar{\beta}$. *Individual equilibrium effort is lower for contestants with* $\beta \in \left(max\left\{1, \frac{1}{n-1}\bar{\beta}\right\}, \bar{\beta}\right)$. *For contestants with* $\beta \in \left\{\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right\}$ *the individual equilibrium effort is identical under both policies.*

*Proof*: If all contestants are active then set $M$ and $N$ coincide, and condition (12) simplifies to $\bar{\beta} > \beta_N^H$. This inequality is always satisfied which proves the first part of the proposition.
For the second part the following inequality has to be analyzed: $e_i^*(AA) > e_i^*(ET)$. Simplifying this expression yields after some algebra the following inequality:

$$\left(\sum_{j\in N}\beta_j\right)^2 - n^2\beta_i\left(\sum_{j\in N}\beta_j - (n-1)\beta_i\right) > 0. \tag{13}$$

This inequality is satisfied if $\beta_i \in \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$, where the upper bound comes from the assumption of full participation under the optimal $ET$ policy. It is also satisfied if $\beta_i \in \left[1, \frac{1}{n-1}\bar{\beta}\right)$ under the condition that this set is non-empty. The left hand side of eq. (13) is equal to zero for $\beta \in \left\{\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right\}$. Continuity of the left hand side of eq. (13) in $\beta_i$ implies the condition for the reversed inequality where the expression $max\left\{1, \frac{1}{n-1}\bar{\beta}\right\}$ accounts for the fact that the set $\left[1, \frac{1}{n-1}\bar{\beta}\right)$ could be empty. □

Proposition 4 states that there are two cut-off values of marginal cost parameters that partition the set of contestants into three (or potentially two) subsets of contestants that react differently to the implementation of policy AA. For contestants with higher than average cost parameter the implementation of AA is effort enhancing because they are favored under AA relatively more than the other contestants. For contestants with intermediate cost parameters (lower than average) policy AA is not beneficial because they are favored relatively less than all other contestants. Finally, for contestants with very low marginal cost parameters the effect of increased competitive pressure might come into play (if $n - 1 < \bar{\beta}$) because their natural advantage under policy ET is now reduced under policy AA. Hence, their optimal response is to increase effort exertion under policy AA.

The results derived so far are now clarified based on an example of a n-player contest with binary distribution of cost parameters, i.e., there are two groups with identical group members.[28] The simplified model presented here is also the starting point of the generalized model in the next section where the informational requirements of the contest organizer are relaxed.

#### An Example

The set of $n$ contestants is partitioned into two groups $A$ and $B$ that each consists of $n_A \geq 2$ and $n_B \geq 2$ members. The members of each group are identical, i.e. they face the same marginal cost parameter: $\beta_i = \beta_A = 1$ for all $i \in A$ and $\beta_i = \beta_B$ for all $i \in B$ where $\beta_B > 1$. In line with section 2 the policy weighting factor are specified as: $\alpha_i^{ET} = \hat{\alpha}^{ET}$ and $\alpha_i^{AA} = \beta_i$ for all $i \in N$.

The active set under policy $ET$ is determined in the following way (under policy AA all con-

[28] The implementation of affirmative action policies is usually not based on individual characteristics, but on group membership, e.g. minority, sex, race, etc. The reason for this grouped treatment might be incomplete information with respect to individual discrimination (addressed in the next section), or simply the fact that group members are sufficiently homogeneous to treat them similar (addressed in this section).

testants will be active): Denote the number of active contestants of group $A$ by $m_A$, and $m_B$ for group $B$. Starting with the less discriminated group $A$, all members of $A$ are active because condition (10) reduces to $1 < \frac{m_A}{m_A - 1}$ which is trivially satisfied for all $m_A \leq n_A$. Considering group $B$, condition (10) becomes $\beta_B < \frac{n_A + m_B \beta_B}{n_A + m_B - 1}$ which can be simplified to $\beta_B < \frac{n_A}{n_A - 1}$. This inequality does not depend on $m_B$ which implies that it either holds for all or for none of the members of $B$. Hence, the following two cases are possible:

1. If $\beta_B < \frac{n_A}{n_A - 1}$ then both groups are active under $ET$.

2. Otherwise, only members of group A are active under $ET$.

For the two cases the aggregated equilibrium effort level under policy $AA$ and $ET$ is now compared. In case 1 all contestants are active which coincides with the special case considered in Proposition 4. Hence, Proposition 4 can be applied directly to conclude that $AA$ induces higher aggregated effort than $ET$. To compare individual equilibrium effort condition (13) has to be analyzed: As $\beta_A = 1$ and $\beta_B > \bar{\beta}$ this implies that $\beta_B \in \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$ and that $\beta_A \in \left[1, \frac{1}{n-1}\bar{\beta}\right)$ as long as this interval is non-empty. This is the case if $n - 1 < \bar{\beta}$ which for this example simplifies to: $\beta_B > ((n_A + n_B)^2 - 2n_A)/n_B - 1$. Hence, if this last inequality is satisfied all individuals will individually exert higher equilibrium effort under $AA$. Otherwise, only group B members will increase their individual effort.

For case 2 Proposition 3 is applicable to compare the aggregated equilibrium effort[29] under policy $AA$ and $ET$. Condition (12) simplifies here to the following expression:

$$\beta_B < \frac{n_A(n_A + n_B - 1)}{n_A(n_A + n_B - 2) - n_B} \equiv \beta^*. \tag{14}$$

The intuitive explanation for condition (12) to hold, as provided in the last section, is that either the level of dispersion is sufficiently low or the number of contestants is relatively small. For the case considered here this can be verified explicitly based on eq. (14).[30] In fact, it is satisfied if either $\beta_B$ is low in comparison to $\beta^*$ (which coincides with low dispersion), or if $n_A$

[29]The analysis of individual equilibrium effort under the two policies is obvious in this case: Members of group B trivially exert more effort under policy AA (because they are not active under $ET$). This also implies that members of group A exert individually less effort under $AA$ than under $ET$.

[30]It should also be noticed that condition (14) is not trivial in the sense that satisfying inequality $\beta_B < \frac{n_A}{n_A - 1}$ would automatically imply condition (14). In fact, the opposite relation holds, i.e. $\beta^* > \frac{n_A}{n_A - 1}$. Hence, it is possible that the sum of equilibrium effort is higher under policy $AA$ than under policy $ET$ although not all contestants are active.

and $n_B$ are sufficiently low (it can be checked that $\beta^*$ is decreasing in $n_A$ and $n_B$).

Eq. (14) can also be used to show that the dispersion of the cost parameter becomes more important with the fraction of discriminated group members: Denote the relative proportion of discriminated contestants by $\gamma = \frac{n_B}{n_A}$. As $\beta^*$ is decreasing in $\gamma$, a relative high proportion of discriminated contestants implies a low $\beta^*$ which makes it less likely that condition (14) will hold. In other words, if the group of discriminated contestants is relatively small it is more probable that aggregated equilibrium is higher under $AA$.

# 5 A Contest Organizer with Partial Information

In this section the previous example is generalized by relaxing two assumptions: First, homogeneity within groups and, second, complete information of the contest organizer with respect to individual characteristics of the contestants. Hence, contestants again face different individual marginal costs which is common knowledge among themselves but not for the contest organizer. However, the contest organizer has, by assumption, information about the group membership of each contestant and the average marginal cost in each group, denoted by $\bar{\beta}_A = \frac{1}{n_A}\sum_{j\in A}\beta_j$ for group A and $\bar{\beta}_B = \frac{1}{n_B}\sum_{j\in B}\beta_j$ for group B, respectively. Group $B$ is assumed to be disadvantaged on average: $\bar{\beta}_B > \bar{\beta}_A$.

For this framework the specification of policy $ET$ according to Definition 1 remains as before ($\alpha_i^{ET} = \hat{\alpha}^{ET}$ for all $i \in N$) because it is defined for all contestants identically. However, the definition of affirmative action has to be modified in this case because Definition 2 required complete information of the contest organizer. Therefore the limited information of the contest organizer has to be taken into account, i.e. affirmative action policy weights must be based on the average (group-specific) level of discrimination:

**Definition 3** *A policy is called* ***affirmative action*** *($AA'$) in a contest game with a partially informed contest organizer if:*

$$\bar{\beta}_A e_i = \bar{\beta}_B e_j \Rightarrow p_i(e) = p_j(e) \textit{ for } i \in A, j \in B. \tag{15}$$

The specification of the vector of weighting parameters $\alpha^{AA'} = (\alpha_1^{AA'}, \ldots, \alpha_n^{AA'})$ can be derived similarly as in Definition 2 which yields the following specification of weights:

$$\alpha_i^{AA'} = \bar{\beta}_i \text{ for all } i \in \{A, B\}\,. \tag{16}$$

Hence, the contest organizer will specify only two different weighting factors which is due to the restricted knowledge that she faces.

An alternative interpretation of this limited information case would be to assume two sources for the heterogeneity of the contestants: One source, for which the contestants are not held responsible from a normative perspective (i.e. the discrimination of group B on average terms), and a second individual source, for which the contestants are held ethically responsible, e.g. the so called ´expensive tastes'. An example would be the following cost function: $c_i(e_i) = (\bar{\beta}_A + \gamma_i)\, e_i$ if $i \in A$ (analogously for $i \in B$) where the idiosyncratic (taste) parameter $\gamma_i$ is symmetrically distributed around zero with the restriction that $|\gamma_i| < \bar{\beta}_A - 1$ to guarantee that marginal costs are positive and larger than unity. The objective of affirmative action is then limited to balance solely the difference between $\bar{\beta}_A$ and $\bar{\beta}_B$, which is due to discrimination between the two groups, and not the individual differences due to taste parameters $\gamma_i$ for all $i \in N$.[31]

The comparison between policy $ET$ and $AA'$ is complex for this kind of set-up because not all contestants will always be active under $AA'$.[32] However, a condition that guarantees higher effort exertion under $AA'$ than under $ET$ would depend (in a similar way like in Proposition 3) on the number of active contestants under each policy option and on the underlying distribution of marginal cost parameters in both groups.[33]

Focusing on the same case as in the last section where all contestants are active under both policy options yields the following result:

[31] I thank Caterina Calsamiglia for suggesting this interpretation.
[32] Lemma 1 does not hold anymore in this framework.
[33] The condition for $E^*_{AA'} > E^*_{ET}$ that would correspond to the one stated in Proposition 3 is:

$$\frac{\bar{\beta}_{M_{AA'}}}{\beta^H_{M_{ET}}} > \frac{\frac{m_{ET}-1}{m_{ET}}}{\frac{m_{AA'}-1}{m_{AA'}}},$$

where $M_P$ denotes the active set under policy $P \in \{ET, AA'\}$. Note, that policy $AA'$ still balances (at least to some extent) the underlying heterogeneity of the contestants which implies that (weakly) more agents will be active under $AA'$ versus $ET$. Hence, the right-hand-side of the inequality will still be lower than one. Making statements about the left-hand-side without knowing the exact distribution of the marginal cost parameters is more difficult because the order of active contestants in set $M_{ET}$ and $M_{AA'}$ might not coincide.

**Proposition 5** *If all contestants are active under policy $ET$ and $AA'$ in a contest game with a partially informed contest organizer, then (i) the sum of equilibrium effort levels is higher under $AA'$ than under $ET$, and (ii) the individual equilibrium effort of contestant $i$ is higher under $AA'$ than under $ET$ if and only if*

$$\beta_i < \tfrac{n}{(n-1)} \tfrac{\bar{\beta}_A \bar{\beta}}{(\bar{\beta}+\bar{\beta}_A)}, \ \textit{for } i \in A, \quad \textit{or} \tag{17}$$

$$\beta_i < \tfrac{n}{(n-1)} \tfrac{\bar{\beta}_B \bar{\beta}}{(\bar{\beta}+\bar{\beta}_B)}, \ \textit{for } i \in B. \tag{18}$$

*Proof*: For the first part the following inequality has to be analyzed: $E^*_{AA'} > E^*_{ET}$. If all contestants are active, this inequality can be reduced to $n_A n_B(\bar{\beta}_A - \bar{\beta}_B)^2 > 0$ which is always satisfied by assumption.

For the second part the individual equilibrium effort has to be compared. Starting with a member of group A, the inequality $e^*_{i\in A}(AA') > e^*_{i\in A}(ET)$ can be reduced to $\beta_{i\in A} < \frac{(n_A+n_B)\bar{\beta}_A\bar{\beta}}{(n_A+n_B-1)(\bar{\beta}+\bar{\beta}_A)}$ with the analogous expression for members of group B.□

The first part of Proposition 5 is intuitive because policy $AA'$ levels the playing field at least to some extent: The discriminated group is favored on average such that the heterogeneity between the groups is lower under $AA'$ than under $ET$. This ameliorates the disincentive effects due to the differences in cost functions between the two groups. The assumption on full participation implies then increased competitive pressure between the two groups which results in higher total effort exertion under policy AA'.

However, contrary to the full information case individual equilibrium effort increases only for those contestants whose marginal costs are below a specific cut-off parameter. The reason is that under policy $AA'$ the individual weighting factors are constant and not proportional to the individual level of discrimination/marginal cost (as it was the case under policy $AA$ with a fully informed contest organizer). Therefore, policy $AA'$ remains relative ineffective for those contestants with a high level of discrimination. However, under $AA'$ higher competitive pressure between the groups still has incentive augmenting effects for contestants with relatively low marginal costs that will increase equilibrium effort under $AA'$ where the exact threshold level is given by the two inequalities in Proposition 5.

## 6 Concluding Remarks

The implementation of affirmative action policies is a controversial topic in public policy discussion. In particular the potential distortion of the incentive structure of competitive situation is frequently criticized without providing well-founded theoretical arguments. The model presented here is an attempt to analyze this allegation based on a simple contest game framework. The novelty of this approach is that it is at first focused on a normative derivation of the objective of affirmative action (which also determines its form of implementation) and secondly on the positive analysis of its consequences. As the implementation of affirmative action affects the way how efforts are weighted in the contest game, contestants will react to this bias in the contest rule. Hence, the consequences of the implementation of affirmative action can be analyzed with respect to the equilibrium effort that this policy induces.

Using this approach it is shown that for the two-player contest game a trade-off between affirmative action and high effort exertion does not exist and that both objectives are in fact correlated. The result for the $n$-players case and the case with a partially informed contest designer is more ambiguous: A trade-off between affirmative action and high effort exertion is unlikely to exist if the participation decisions of the contestants are not altered substantively through the implementation of the affirmative action policy.

The existence of weighting factors that maximize total effort is only addressed for the two player case. The extension of this result to the n-player contest game is technically involved for two reasons: The feasible set of policy weights is not compact and the objective function, i.e. the sum of equilibrium effort of all active players, might be non-smooth due to the fact that contestants become active depending on the chosen weighting factors. This problem is attacked in Franke et al. (2009) based on techniques from bi-level mathematical programming with equilibrium constraints. It is shown that the vector of effort maximizing weighting factors exists and reduces the heterogeneity of contestants to some extent but not to the full extent as in the two-player contest game (comp. Proposition 2). Hence, for more than two players the optimal vector of weighting factors will be a mixture between the ET and the AA policy which also takes into account the fact that inducing weak players to participate implies a disincentivating effect for contestants with relatively low marginal cost.

Nevertheless, the general idea of how the implementation of affirmative action affects the incentives with respect to effort exertion is straight-forward: Discrimination is a source of heterogeneity between individuals in competitive situations. The implementation of appropriate affirmative action ameliorates (at least in the aggregate) this heterogeneity and makes individuals more similar. This increases competitive pressure and therefore induces higher effort by all participants as long as participation decisions are not substantially affected.[34] A condition on the distribution of the heterogeneous characteristics is provided that guarantees that the last mentioned qualification is satisfied.

A final comment should be related to potential empirical validations of the derived results. With the exception of bid preferences in public procurement auctions, see Marion (2007), suitable data on these issues seems to be rather scarce.[35] This suggests two approaches to address this lack of empirical data: A first approach is the analysis of similar competitive situation which are governed by rules that balance the heterogeneity of competitors and where sufficient data is available. Possible examples are sport contests that sometimes incorporate those kind of balancing rules (e.g. in horse races, golf, or formula one races), although their implementation is usually not motivated by normative concerns.[36] A second potential approach is to conduct experiments where the incentive effects of affirmative action policies can be analyzed in a controlled environment like, for instance, in Calsamiglia et al. (2009).[37] Both approaches are work in progress and suggest (at a preliminary stage) that balancing the ex-ante heterogeneity of participants does not have adverse incentive effects which is in line with the theoretical results derived from the contest model as provided here.

[34] This argumentation must not be restricted to the specific model of contest games considered here. Che and Gale (2000) show for a two player set up that the effort reducing effect of asymmetries, the so called 'preemption' effect, also exists for difference-form contests that include all-pay auctions as a special case.

[35] A recent exception is Hickman (2009) where affirmative action in college admission is analyzed. This approach is based on an auction-theoretic framework and adresses also quota versus lump-sum policies are counterfactually compared.

[36] An empirical analysis of amateur golf tournament in Franke (2010) suggests that performance in leveled tournaments is significantly higher than in unbalanced tournaments. Similar results are derived for professional tennis in Sunde (2009).

[37] In this experiment pairwise real effort tournaments between school children are conducted where disadvantaged children are favored by the tournament rules. Additionally, the incentive effects of different degrees and types of affirmative action instruments are addressed in this experimental study.

## Appendix: Equilibrium in the n-Player Contest Game

To construct the share function of contestant $i$, her expected utility function is transformed such that the contest game can be interpreted as an aggregative game. The transformed utility function of contestant $i$ can be expressed as $\pi_i(z_i, Z)$, where $Z = \sum_{i \in N} z_i$. The following transformation is considered that is strategically equivalent to eq. (8): $z_i = \alpha_i^P e_i$, which can be inverted to $e_i = z_i/\alpha_i^P$ for all $i \in N$. The resulting transformed expected utility function for contestant $i$, which has the aggregative game property, is of the following form:

$$\pi_i(z_i, Z) = \frac{z_i}{Z} - \delta_i^P z_i \text{ for all } i \in N \text{ and for } P \in \{ET, AA\}, \tag{19}$$

where $\delta_i^P = \frac{\beta_i}{\alpha_i^P V}$ and $Z$ defined as above. This transformed contest game is now covered by the model in Cornes and Hartley (2005). The share function can therefore be constructed in an analogous way by deriving the first order condition:

$$z_i \left( \frac{Z - z_i}{Z^2} - \delta_i^P \right) = 0 \text{ for } z_i \geq 0. \tag{20}$$

The best response $z_i^*$ of player $i$ can be expressed in terms of the aggregated equilibrium effort:[38] $z_i^*(Z) = \max\{Z - \delta_i^P Z^2, 0\}$. Finally, define player $i$'s share function as her relative contribution

$$s_i(Z) = \frac{z_i^*(Z)}{Z} = \max\{1 - \delta_i^P Z, 0\}. \tag{21}$$

In equilibrium the aggregated effort $Z^*$ is implicitly defined by the condition that the individual share functions sum up to one:

$$S(Z^*) = \sum_{i \in N} s_i(Z^*) = 1. \tag{22}$$

Theorem 1 in Cornes and Hartley (2005) states that a solution to this equation exists and is unique by observing that the aggregated share function $S(Z)$ is continuous and strictly decreasing for positive $Z$, and that it has a value higher than one for $Z$ sufficiently small and equal to zero for $Z$ sufficiently large.

[38] It should be obvious that best response- and also the share functions depend on the policy parameter $P$. But as the finally implemented policy does not affect the proof of equilibrium existence and uniqueness, it is suppressed in this section for notational convenience.

Equation (21) already indicates that contestants with a high level of $\delta$ might have an equilibrium share of zero, i.e. they might prefer to stay non-active. Note that due to the definitions of policies $AA$ and $ET$ the order of the contestants according to $\delta_i^P$ coincides[39] for both policies with the order based on marginal costs because $\delta_1^P \leq \delta_2^P \leq \ldots \leq \delta_n^P$.

Based on this observation the set of active contestants $M \subseteq N$ can be characterized, i.e. the $m$ players with strict positive share in equilibrium. From eq. (21) it is obvious that in equilibrium $Z^* < 1/\delta_i^P$ for all $i \in M$. Combining eq. (21) and (22) yields $Z^* = \frac{m-1}{\sum_{j \in M} \delta_j^P}$. The last two expressions yield the condition that indirectly defines the set $M \subseteq N$ of active contestants that consists out of those $m$ contestants with the lowest $\delta$ values that satisfy the following inequality:

$$(m-1)\delta_i^P < \sum_{j \in M} \delta_j^P \text{ for all } i \in M \text{ and for } P \in \{ET, AA\}\,. \tag{23}$$

From the definition of the share function in eq. (21) the equilibrium effort level of contestant $i$ can be calculated as $e_i^*(P) = z_i^*/\alpha_i^P = s_i(Z^*)Z^*/\alpha_i^P$. Inserting the expression for $Z^*$ leads to eq. (9).[40]

## References


Arrows, K. J. (1998): "What Has Economics to Say About Racial Discrimination?" *Journal of Economic Perspectives*, 12, 91–100.

Bernardo, A. E., E. Talley, and I. Welch (2000): "A Theory of Legal Presumptions," *Journal of Law, Economics and Organization*, 16, 1–49.

Calsamiglia, C. (2009): "Decentralizing Equality of Opportunity," *International Economic Review*, 50, 273–290.

Calsamiglia, C., J. Franke, and P. Rey-Biel (2009): "The Incentive Effects of Affirmative Action in a Real-Effort Tournament," *Mimeo.*


[39]Under policy $AA$ the order of individuals satisfies the prescribed order in a trivial sense because $\delta_i^{AA} = \delta_j^{AA}$ for $i \neq j$.

[40]Stein (2002) derives a similar expression in a rent-seeking framework where the contestants are heterogeneous with respect to valuations instead of marginal costs.

Chan, J. and E. Eyster (2003): "Does Banning Affirmative Action Lower College Student Quality?" *American Economic Review*, 93, 858–872.

——— (2009): "The Distributonal Consequences of Diversity-Enhancing University Admissions Rules," *Journal of Law, Economics, and Organization*, 25, 499–517.

Che, Y.-K. and I. Gale (2000): "Difference-Form Contests and the Robustness of All-Pay Auctions," *Games and Economic Behavior*, 30, 22–43.

Clark, D. J. and C. Riis (1998): "Contest Success Functions: An Extension," *Economic Theory*, 11, 201–204.

——— (2000): "Allocation Efficiency in a Competitive Bribery Game," *Journal of Economic Behavior and Organization*, 42, 109–124.

Corchón, L. C. (2000): "On the Allocative Effects of Rent Seeking," *Journal of Public Economic Theory*, 2, 483–491.

Cornes, R. and R. Hartley (2005): "Asymmetric Contests with General Technologies," *Economic Theory*, 26, 923–946.

Franke, J. (2010): "The Incentive Effects of Leveling the Playing Field: An Empirical Analysis of Amateur Golf Tournaments," *Mimeo.*

Franke, J., C. Kanzow, W. Leininger, and A. Väth (2009): "Effort Maximization in N-Person Contest Games," *CESifo Working Paper No. 2744.*

Fryer, R. G. and G. C. Loury (2005a): "Affirmative Action and Its Mythology," *Journal of Economic Perspectives*, 19, 147–162.

——— (2005b): "Affirmative Action in Winner-Take-All-Markets," *Journal of Economic Inequality*, 3, 263–280.

Fryer, R. G., G. C. Loury, and T. Yuret (2008): "An Economic Analysis of Color-Blind Affirmative Action," *Journal of Law, Economics, and Organization*, 24, 319–355.

Fu, Q. (2006): "A Theory of Affirmative Action in College Admissions," *Economic Inquiry*, 44, 420–428.

Gradstein, M. and K. Konrad (1999): “Orchestrating Rent Seeking Contests,” *Economic Journal*, 109, 536–545.

Hickman, B. R. (2009): “Effort, Race Gaps and Affirmative Action: A Game-Theoretic Analysis of College Admissions,” *Typescript, University of Iowa Department of Economics.*

Konrad, K. (2002): “Investment in the Absence of Property Rights; The Role of Incumbency Advantages,” *European Economic Review*, 46, 1521–1537.

——— (2009): *Strategy and Dynamics in Contests*, Oxford University Press.

Kranich, L. (1994): “Equal Division, Efficiency, and the Sovereign Supply of Labor,” *American Economic Review*, 84, 178–189.

Lazear, E. P. and S. Rosen (1981): “Rank-Order Tournaments as Optimum Labor Contracts,” *Journal of Political Economy*, 89, S. 841–864.

Leininger, W. (1993): “More Efficient Rent-Seeking: A Münchhausen Solution.” *Public Choice*, 75, 43–62.

Loury, G. C. (1981): “Is Equal Opportunity Enough?” *American Economic Review, Papers and Proceedings*, 71, 122–126.

——— (2002): *The Anatomy of Racial Inequality*, Cambridge, London: Harvard University Press.

Lundberg, S. J. and R. Startz (1998): “On the Persistence of Racial Inequality,” *Journal of Labor Economics*, 16, 292–323.

Marion, J. (2007): “Are Bid Preferences Benign? The Effect of Small Business Subsidies in HighwayProcurement Auctions,” *Journal of Public Economics*, 91, 1591–1624.

McAfee, P. and J. McMillan (1989): “Government Procurement and International Trade,” *Journal of International Economics*, 26, 291–308.

Nti, K. O. (2004): “Maximum Efforts in Contests with Asymmetric Valuations,” *European Journal of Political Economy*, 20, 1059–1066.

SCHOTTER, A. AND K. WEIGELT (1992): "Asymmetric Tournaments, Equal Opportunity Laws, and Affirmative Action:Some Experimental Results," *Quarterly Journal of Economics*, 107, S. 511–39.

SKAPERDAS, S. (1996): "Contest Success Functions," *Economic Theory*, 7, 283–290.

SOWELL, T. (2004): *Affirmative Action Around the World*, New Haven, London: Yale University Press.

STEIN, W. E. (2002): "Asymmetric rent-seeking with more than two contestants," *Public Choice*, 113, 325–336.

SUNDE, U. (2009): "Heterogeneity and Performance in Tournaments: A Test for Incentive Effects Using Professional Tennis Data," *Applied Economics*, 41, 3199–3208.

SZYMANSKI, S. (2003): "The Economic Design of Sporting Contests," *Journal of Economic Literature*, 41, 1137–1187.

TULLOCK, G. (1980): "Efficient Rent Seeking," in *Towards a Theory of the Rent Seeking Society*, ed. by J. Buchanan, R. D. Tollison, and G. Tullock, Texas A&M University Press, 97–112.