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by

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Is there a tendency for the rate of profit to fall? Econometric evidence for the U.S. economy, 1948-2007

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Abstract

The law of the tendential fall in the rate of profit has been at the center of theoretical and empirical debates within Marxian political economy ever since the publication of Volume III of Capital. An important limitation of this literature is the absence of a comprehensive econometric analysis of the behaviour of the rate of profit. In this paper, we attempt to fill this lacuna in two ways. First, we investigate the time series properties of the profit rate series. The evidence suggests that the rate of profit behaves like a random walk and exhibits "long waves" interestingly correlated with major epochs of U.S. economic history. In the second part, we test Marx's law of the tendential fall in the rate of profit with a novel econometric model that explicitly accounts for the counter-tendencies. We find evidence of a long-run downward trend in the general profit rate for the US economy for the period 1948-2007.

JEL Classification: B51, C22, E11.

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1 Introduction

Marx's claim in Volume III of Capital that there is a tendency for the general rate of profit to fall with the development of capitalism has spawned an enormous and growing literature

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often marked by bitter controversy and fruitful debate (Dobb, 1939; Sweezy, 1942, Gilman, 1957; Okishio, 1961; Shaikh, 1978; Wolff, 1979; Mandel, 1980; Roemer, 1981; Bowles, 1985; Foley, 1986; Michl, 1988; Shaikh, 1992; Duménil and Lévy 1993, 1995; Foley and Michl, 1999; Wolff, 2001; Duménil and Lévy, 2002a, 2002b; Wolff, 2003; Kliman, 2009).

The theoretical strand of this literature has focused on understanding the possible causes behind what Marx referred to as the law of the tendential fall in the rate of profit (LTFRP).² Recall that the rate of profit, in Marx (1993), is defined as

$$r = \frac{s}{c+v} = \frac{(s/v)}{1+(c/v)} = ek,$$

where r is the rate of profit, s the surplus value, v, the variable capital, c, the constant capital, e = s/v the intensity of exploitation (also referred to as the rate of surplus value) and k = 1/(1+(c/v)) the composition of capital (Foley, 1986). The early debate was focused on two crucial issues. The first issue pertains to whether the composition of capital falls with the development of capitalism, i.e., whether the increasing technical composition of capital translated into an increase in the value composition of capital. The second issue is whether the increase in the intensity of exploitation is swamped by the fall in the composition of capital, thereby leading to a fall in the rate of profit (Moseley, 1991).³ A third issue relating to choice of technique was added to this long-standing debate by Okishio's (1961) claim to have disproved the LTFRP. The subsequent theoretical literature can be fruitfully classified with reference to Okishio (1961), to our mind, into the following three strands.⁴ The first strand accepts the validity of the so-called Okishio Theorem, which is understood as having

¹This is a representative list; we make no claims about completeness or comprehensiveness.

²In this paper, we are concerned with studying long-run tendencies; hence, we will not refer to a separate strand of the literature, initiated by Weisskopf (1978), that studies cyclical fluctuations in the rate of profit.

³Marx used several different concepts of compositions of capital in his analysis. The technical composition of capital referred to the ratio of the "mass of the means of production employed ... and the mass of labour necessary for their employment". In modern parlance, that could be seen as the ratio of the stock of capital and the number of workers. The value composition of capital, on the other hand, referred to the ratio between constant capital and variable capital, c/v. The composition of capital, k, is a term used by Foley (1986); it is a transformation of the value composition of capital and is defined as: k = 1/(1 + (c/v))

⁴This classification is for the purposes of organizing our investigations; we make no claim to completeness or comprehensiveness.

"proved" that the LTFRP can never emerge as a significant tendency in a capitalist economy with profit-maximising entrepreneurs and viable technical change; prominent scholars in this strand include Romer (1981), Bowles (1985) and others. The second strand rejects the validity of the so-called Okishio Theorem in toto and instead believes that there is a secular tendency for the rate of profit to fall with capitalist development; prominent scholars in this strand are Shaikh (1978, 1987, 1992), Kliman (2007, 2009) and others. The third strand conditionally accepts the validity of the so-called Okishio Theorem, arguing that the key assumption that drives its result - fixed real wages - does not characterise the actual evolution of capitalism. Thus, neither a secular tendency for the profit rate to fall nor a secular tendency to increase can be a priori associated with capitalist development; prominent scholars in this strand are Foley (1986), Michl (1988), Moseley (1991), Duménil and Lévy (1993, 1995), Foley and Michl (1999), Duménil and Lévy (2003).⁵

Instead of engaging with this rich theoretical debate in any detail, in this paper our focus will be towards addressing a different but related question: what does the evidence show regarding the tendency of the general rate of profit to fall in the U.S.? The empirical strand of this vibrant literature has addressed this issue but without displaying the depth and sophistication of the theoretical literature. A major lacuna has been the dearth of serious econometric inquiry to inform an empirical analysis.⁶ A preponderance of empirical studies utilize only exploratory techniques (e.g., visual inspection of time series plots) in order to infer trends in the rate of profit (Gilman 1957; Wolff 1979, 2001, 2003; Duménil and Lévy 1993, 1995, 2002a, 2002b). While visual and exploratory techniques can be valuable starting points of empirical research, it is necessary to apply modern econometric methods for investigating trends in the rate of profit (e.g., an investigation of the time-series properties of the general rate of profit). It is this lacuna in the empirical literature on the LTFRP that

⁵Interestingly, returning to his work 40 years later, Okishio (2001) accepts that the key assumption of constant real wages is unrealistic. We would like to thank Iren Levina for pointing this out.

⁶To the best of our knowledge, Michl (1988) is the only exception.

we wish to addres.

The analysis in this paper proceeds in two steps. First, we conduct out detailed and systematic investigation of the time series properties of the general rate of profit in the U.S. economy using the Box-Jenkins approach to time-series analysis (Box and Jenkins, 1970) and complementing that approach with a battery of unit root tests. The results of this analysis suggests that the U.S. rate of profit is a random walk and exhibits "long waves" like any time series with stochastic trends, confirming the intuitive claims of Mandel (1980) and Shaikh (1992). Thus, our analysis imparts statistical substance to the long-standing claim about long waves in the profit rate series.

Using results about the non-stationarity of the profit rate, we proceed in the second part to econometrically test the LTFRP. We do so by estimating a novel time series regression model derived from Marx's analysis in Volume III of Capital. The novelty of our analysis derives from two aspects of our empirical approach. First, we control for the effects of what Marx had called "counter-tendencies". Second, we explicitly take account of non-stationary random variables in our statistical inference. To the best of our knowledge, both these aspects have not been adequately addressed in the existing literature.

While investigating the presence of a trend in the profit rate series, our regression model allows us to explicitly control for the effect of the counter-tendencies by treating them as regressors. Our approach, therefore, provides a rigorous test of the LTFRP as enunciated by Marx in Volume III of Capital. Without properly controlling for the effects of the counter-tendencies, Marx's hypothesis about the tendency of the profit rate to fall cannot be rigorously tested. By explicitly incorporating the counter-tendencies, the analysis of this paper makes a major contribution to the existing literature on the empirics of the LTFRP.

The importance of the issue of non-stationarity can be best understood by looking at Michl (1988), the paper closest to our work. Michl (1988) also tested for the presence of a negative time trend in the profit rate series within a regression framework. He also

recognized the importance of the effects of counter-tendencies in the analysis of the LTFRP and offered an illuminating discussion of the relative price of capital. This paper can be seen, therefore, as an effort to extend the analysis in Michl (1988). Michl (1988), it must be noted, had borrowed the methodology to test for the presence of a negative trend in the profit rate series from Feldstein and Summers (1977), who were responding to Nordhaus's (1974) finding about a falling tendency in the rate of profit. While Nordhaus's (1974) conclusions were based on visual inspection of the data rather than formal statistical tests, Feldstein and Summers (1977) tested the claim about a falling rate of profit with regression analysis. Fitting a time trend on the profit rate series, controlling for cyclical fluctuations with various measures of capacity utilization and controlling for serial correlation by the Cochrane-Orcutt method, Feldstein and Summers (1977) found the coefficient on the time trend to be statistically insignificantly. Michl (1988) reports similar results on a differently constructed profit rate series.

The econometric model in Feldstein and Summers (1977) suffers from the problem of non-stationarity. If either the dependent variable (the rate of profit) or the regressors (the time trend and the capacity utilization rate) were non-stationary, then the standard errors reported in their paper would be incorrect and this would invalidate the statistical inference. Our analysis of the time series properties of the U.S. profit rate shows that the profit rate series is non-stationary. In formal statistical tests, we cannot reject the null hypothesis of a unit root in the rate of profit. This finding, therefore, implies that the conclusions in Feldstein and Summers (1977), and by extension in Michl (1988), are invalid. The econometric study of non-stationary time series was put on a solid foundation only towards the end of the 1980s with the pioneering work of Granger (1983), Phillips (1986), and Engle and Granger (1987). Since the profit rate, the time trend and several counter-tendencies are non-stationary random variables, a proper econometric treatment of Marx's hypothesis in a regression framework requires that we use the methodology of non-stationary analysis. This

methodology was not yet available to Feldstein and Summers (1977) and Michl (1988).

To theoretically motivate our empirical analysis, we draw a distinction between the short, medium and long run movements in the general rate of profit. Based on this three-fold distinction of the appropriate time frame of analysis we offer the following conjectures. Firstly, short run fluctuations in the profit rate are primarily driven by fluctuations in demand (as captured, for instance, by fluctuations in the rate of capacity utilization), fluctuations in the real wage rate and movements in the real interest rate (Duménil and Foley, 2008). Secondly, medium run fluctuations (the 3-4 decade long cycles or the so-called "long waves" à la Mandel (1980) and Shaikh (1992)) in the rate of profit are primarily driven by technological factors that impinge on the "counter-tendencies" that Marx had mentioned, and could be fruitfully explained by the theory of Marx-biased technical change (Foley and Michl, 1999; Duménil and Lévy, 2003). Finally, the long-run secular tendency (i.e., a negative time trend that runs through several medium-run cycles) for the rate of profit to fall is primarily driven by the long-run competitive pressures of the capitalist system that results in the inexorable increase in the mechanization of the production process, leading to an increase in the composition of capital (Shaikh, 1978, 1992) above and beyond the effect of the countertendecies. Based on this distinction in the time frame of analysis, we note that this paper is an attempt to investigate Marx's hypothesis of the secularly declining profit rate in the U.S. case that relates to the long run only.⁸ Irrespective of one's theoretical view on the matter, the fact remains that our paper uncovers an empirical regularity not recognized in the existing literature. As such, it is an empirical regularity that requires a theoretical explanation in future work.

The rest of the paper is organized as follows. The next section presents some arguments

⁷This long-run tendency of the composition of capital to increase was forcefully argued by Marx in Chapter 25 of Volume I of Capital and later used in his analysis of the LTFRP in Volume III of Capital.

⁸A slightly different, though similar, distinction between short, medium and long run analysis was made by Mosley (1991). Distinctions between short-run and long-run analysis figure in Duménil and Lévy (1993), and Foley and Michl (1999).

as to why the rate of profit might fall with capitalist development. Then, we conduct a systematic analysis of the profit rate series and establish its unit root character and present results of the estimation of the regression model to test the LTFRP. The last section offers our conclusions. Results pertaining to derivation of the distribution of the key test statistic in the text of the paper is presented in Appendix A, and details about the construction of the data series are presented in Appendix B.

2 Theoretical Arguments

In this section, we present some arguments, largely borrowing from the existing theoretical literature on the LTFRP, as to why the rate of profit *might* fall over time.

2.1 Materialized Composition of Capital

The first, an asymptotic, argument derives from Rosdolsky (1977) and Shaikh (1992). Let c_t stand for the value of constant capital (both fixed and circulating), v_t stand for variable capital, s_t stand for surplus value, all the quantities referring to their values in period t. Let $l_t = s_t + v_t$ stand for living labour, which creates the new value added in the production process. The rate of profit is defined as

$$r_t = \frac{s_t}{c_t} = \left(\frac{s_t}{l_t}\right) \left(\frac{l_t}{c_t}\right) = \frac{s_t}{v_t + s_t} \left(\frac{l_t}{c_t}\right) = \frac{(s_t/v_t)}{1 + (s_t/v_t)} \left(\frac{l_t}{c_t}\right).$$

With the development of capitalism both the rate of surplus value, (s_t/v_t) and the materialised composition of capital (c_t/l_t) increase over time, possibly at different rates. To capture the evolution of these two ratios over time, let

$$\frac{s_t}{v_t} = f(t), \quad f > 0$$

and

$$\frac{c_t}{l_t} = g(t), \quad g > 0, g' > 0.$$

Thus,

$$r_t = \frac{1}{1 + \frac{1}{f(t)}} \left(\frac{1}{g(t)} \right).$$

But, since f(t) > 0,

$$\frac{1}{1 + \frac{1}{f(t)}} \le 1 \quad \text{(for all t)};$$

hence

$$r_t \leq \frac{1}{g(t)}$$
.

Thus, the long-term trend in r_t is dominated by the growth of the materialised composition of capital, g(t). If the materialized composition of capital increases monotonically over time, i.e., if g' > 0, that will impart a long-term negative trend to the rate of profit. The most striking aspect of this argument is that it does not depend on the behaviour of the rate of surplus value. No matter how the rate of surplus value behaves over time (either increasing or decreasing or remaining constant), as long as the materialized composition of capital grows over time, the rate of profit will have a long-term negative trend.

2.2 Aggregate Labour Theory of Value

In the above exposition, the rate of profit was defined, following Shaikh (1992), as the ratio of the surplus value to the stock of constant capital. Marx (1993) defined the rate of profit as the ratio of surplus value to the total capital advanced, i.e., the sum of the constant and variable capital. We can extend Shaikh's (1992) argument to this case by looking at the following inequality:

$$\frac{s_t}{c_t + v_t} \le \frac{v_t + s_t}{c_t}.\tag{1}$$

⁹Though Marx did not offer a rigorous argument as to why the materialized composition of capital might increase with time, later authors like Shaikh (1978) and Duménil and Lévy (2003) have tried to work this out.

The relation holds as a strict inequality other than when $v_t = 0$, in which case the two sides become equal. Thus, other than the case when workers could be forced to work for free, the above inequality holds strictly.

Note that the left hand side of (1) has the rate of profit, as defined by Marx (1993). On the other hand, we have the reciprocal of the materialized composition of capital. If the progress of capitalist development imparts an upward trend to the materialized composition of capital, then the right hand side of the above inequality has a negative long run trend. This would, in turn, give a long run negative trend to the rate of profit.

Okishio (1961) had offered a critique of the above argument, which rested on the claim that Marx's definition of the rate of profit, s/(c+v), was incorrect, and hence, that the whole argument was invalid. To define the "correct" rate of profit, Okishio (1961) distinguished between basic and non-basic industries. Two kinds of industries were included in the category of basic industries: (1) those that produced wage goods, and (2) those that produced means of production, raw materials, or auxiliary materials for the wage goods industries. All the other industries were clubbed as non-basic industries. The intuition behind the distinction between basic and non-basic industries comes from Ricardo (2004) and derives from the claim that techniques of production in the non-basic industries do not affect the general rate of profit. If wage goods could be produced in a cheaper manner and real wages remained constant, that would reduce the amount of (direct and indirect) labour required to produce the fixed basket of commodities that make up the real wage, thereby increasing the general rate of profit. By the same logic, cheapening of the products of non-basic industries would not affect the real wage and so would not affect the general rate of profit.¹⁰

¹⁰ "If, therefore, by the extension of foreign trade, or by improvements in machinery, the food and necessaries of the labourer can be brought to the market at a reduced price, profits will rise. If, instead of growing our own corn, or manufacturing the clothing and other necessaries of the labourer, we discover a new market from which we can supply ourselves with these commodities at a cheaper price, wages will fall and profits rise; but if the commodities obtained at a cheaper rate, by the extension of foreign commerce, or by the improvement of machinery, be exclusively the commodities consumed by the rich, no alteration will take place in the rate of profits." (Ricardo, 2004, p. 132.) Okishio's (1961) mains results - first, that technical

There are three basic flaws in this argument. First, with the development of capitalism, real wages increase over time, rather than being stagnant. This is a well recognized empirical fact which is at variance with the basic assumption in Ricardo's (2004) and Okishio's (1961) argument. Second, in the presence of technology spillovers or externalities, technical change in the basic and non-basic industries cannot be plausibly assumed to be independent. Thus, technical change in one sector might not be restricted to change in the cost of production in that sector (basic industries, say) only; it might affect the cost of production in the other sector (non-basic industries). The interdependence of technical change in the basic and non-basic industries makes the Ricardian argument problematic. Third, the increase in the real wages of workers over time expands the bundle of commodities that make up the consumption basket of an average worker. Therefore, an ever larger part of non-basic industries become part of the category of basic industries. The boundary between basic and non-basic industries constantly shifts with time. Thus, while the distinction between basic and non-basic industries, as the industries producing respectively for workers and capitalists, might make sense in a static setting, it becomes analytically problematic in a long run, dynamic context. That is why, in his analysis of the long run tendencies of capitalism, Marx (1993) rejected the distinction between basic and non-basic industries and the argument based on that distinction.

Marx's (1993) rejection of the distinction between basic and non-basic industries seems to emerge from a deeper difference between his framework and Ricardo's (2004). For Marx, the labour theory of value operated at the aggregate level and not at the level of individual commodities, as Ricardo (2004) argued.¹¹ Thus, the labour theory theory of value merely asserted that the aggregate new value added in a given period of time represented the

progress in the non-basic industries will not alter the rate of profit, and second, that technical progress in the basic industries will necessarily increase the rate of profit - merely demonstrate Ricardo's propositions in a formal linear model of production.

¹¹We borrow this crucial insight from Foley (1986).

productive labour expended during that period of time, without making any claims about the relationship between the value of particular commodities and the labour contained in them. From this perspective, the distinction between necessary and surplus labour, again at the aggregate level, was rather more important than the distinction between basic and non-basic industries. The aggregate labour time of society could be thought of as being divided into two parts. One portion is devoted to the reproduction of its own material conditions and another to producing a surplus over and above what was needed for its own reproduction through time. While the first could be called necessary labour, the second part could be understood as surplus labour. Under capitalist social relations, the latter took the form of surplus value and expressed itself as profit. From an aggregate labour theory of value perspective, therefore, it made sense to define the rate of profit as the ratio of the aggregate surplus value and the stock of capital advanced at the aggregate level, as Marx did.

The distinction between basic and non-basic industries was not analytically important because it did not matter what commodity bundle went into the consumption basket of an average worker at any point in time; what mattered was the fraction of total social labour time that was needed to produce the material conditions for the reproduction of social labour. Since the consumption basket of an average worker changed and expanded over time the boundary between basic and non-basic industries continually shifted. But the division of total social labour time into necessary and surplus labour time remained intact, even as their ratio changed over time. Thus, in a long run dynamic context, it was an analytically superior strategy to focus on aggregate labour time and its division into necessary and surplus labour, as Marx (1993) did, rather than focus on the division between basic and non-basic industries as Ricardo (2004) did.

¹²With technological spillovers, similarly, it does not matter whether the process of technical change occurs in the basic or the non-basic industries. That is another reason, in a dynamic context, to question the validity of the argument based on this distinction.

Thus, Okishio's (1961) claim that Marx's definition of the rate of profit was incorrect is theoretically invalid. Marx's definition follows from an understanding of the labour theory of value at the aggregate level and is, in our opinion, not only correct but much better suited for long run dynamic analysis than the one that flows out of a Ricardian linear production model. Hence, as long as the materialized composition of capital, c/(v+s), has a tendency to increase over time, that will lead to a tendency for the rate of profit to fall over time. These arguments are meant to convey that it is plausible for the rate of profit to decline with the development of capitalism. In fact, Marx never argued that the empirically observed rate of profit will have a secularly declining trend. He was always careful to refer to the declining trend in the rate of profit as a "tendency" and to explicitly bring the important "counteracting influences", which work to reverse the tendency, into his analysis.¹³ The tendency operates at a high level of abstraction and will be visible only when the counteracting influences have been controlled for. But before we proceed to that task, we need to study the statistical properties of the profit rate series.

3 Time Series Analysis of the Rate of Profit

3.1 The Box-Jenkins Approach

The Box-Jenkins approach to time-series analysis consists of three analytical stages: model identification, model estimation, and diagnostic testing. We perform a Box-Jenkins analysis of the rate of profit in this section. In order to identify a tentative model, consider Figures (1) and (2). Figure (1) displays the lag plots for the rate of profit. Figure (2) the estimated autocorrelation function and the estimated partial autocorrelation function. In figure (1), each pane shows a bivariate scatter plot of r_t against r_{t-k} for $k \in \{1, 2, 3, 4, 5, 6\}$. For

¹³Referring to the "steadily falling general rate of profit", Marx asks his readers to remember that "this fall does not present itself in such an absolute form, but rather more in the tendency to a progressive fall." (Marx, 1993, p. 319).

example, the pane in the upper-left shows the scatter plot of r_t against r_{t-1} and initially suggests a strong correlation. Similarly, the pane in the first row and second column suggests that a correlation exists between r_t and r_{t-2} but this correlation is somewhat weaker.

[FIGURE 1]

In Figure (2), the sample autocorrelation function (A.C.F.) shows the estimated correlation coefficients between r_t and r_{t-k} . In other words, the sample A.C.F. plots the estimated coefficients obtained by a bivariate regression that fits a line to each pane in Figure (1) with O.L.S. The dashed lines indicate the bounds for statistical significance at the ten percent level. Indeed, the fact that the sample autocorrelation function exhibits a long decay suggests that this time-series is non-stationary. An examination of the sample A.C.F. for the first differences confirms the hypothesis of non-stationarity; moreover, there is good evidence that that the first differences are pure white noise. An estimated partial autocorrelation function shows the estimated coefficient obtained for r_{t-k} when z lags are included in the regression. Observe that only the first lag is statistically significant. Furthermore, there is no evidence to suggest that the data generating process includes moving average terms. In conclusion, the model identification stage of the Box-Jenkins procedure suggests that a good model for these data is ARIMA(1,1,0), viz., a random walk with drift.

[FIGURE 2]

In order to be conservative, we begin with the inclusion of three lags. Recall the mathematical form of an ARMA(3,0) model:

$$r_t = \beta_0 + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \beta_3 r_{t-3} + \varepsilon_t \tag{2}$$

where ε_t is assumed to be $i.i.d.(o, \sigma_{\varepsilon}^2)$. Since the maintained hypothesis is non-stationary, first differencing yields an estimating equation for ARIMA(3,1,0)

$$\Delta r_t = \beta_1 \Delta r_{t-1} + \beta_2 \Delta r_{t-2} + \beta_3 \Delta r_{t-3} + \gamma_t. \tag{3}$$

Note that $\beta_2 = \beta_3 = 0$ and $\beta_1 = 1$ are the restrictions for a random walk with drift. In the case of a pure random walk, we see that $\beta_0 = \beta_2 = \beta_3 = 0$ and $\beta_1 = 1$ and hence

$$r_t = r_{t-1} + \varepsilon_t \tag{4}$$

The maximum likelihood estimates for the parameters of various plausible A.R.I.M.A. models are reported in table (1). In accordance with the results of the identification analysis, our maintained hypothesis is that the data generating process is ARIMA(1,1,0). Nevertheless, caution dictates that an array of plausible models be considered in order to avoid specification errors.

[TABLE 1]

Table (2) reports the estimated value of the log-likelihood function, and the Bayesian Information Criterion (B.I.C.). The B.I.C. is a goodness of fit statistic and defined as -2ln(L)+kln(n) where ln(L) is the estimated value of the log-likelihood function, k denotes the number of parameters in the model, and n refers to the sample size. This information criterion insists that a model must be parsimonious and therefore penalizes over-parametrized models by a factor ln(n). Accordingly, a smaller B.I.C. implies a better model. This suggests the view that the true model is a random walk without drift.

This finding suggests that a revision of the maintained hypothesis is warranted. The rate of profit does not exhibit a stochastic trend of this form. Our new maintained hypothesis is that the data generating process is a random walk without drift, i.e., ARIMA(0,1,0). We test the new maintained hypothesis $H_0: \beta_0 = \beta_2 = \beta_3 = 0$ against the two-sided alternative with a likelihood ratio test but $\chi^2 = -0.42$ and we cannot reject the null hypothesis. This additional evidence therefore supports the view that the true model is a pure random walk.

[TABLE 2]

Finally, we conduct diagnostic tests of the null model by subjecting the residuals to a battery of tests in order to verify that the model is well-specified. In a well-specified model, the residuals are white noise and normally distributed. The results of our diagnostic tests are reported in Table 3 and Table 4. Table (3) reports the p-values of a Ljung-Box test for white noise, and both the Shapiro-Wilk and Jarque-Bera tests for normality. The evidence supports the hypothesis that ARIMA(0,1,0) is well-specified. The Ljung-Box test fails to reject the hypothesis that the residuals are white noise. The Shapiro-Wilk and Jarque-Bera tests of normality, furthermore, favours ARIMA(0,1,0). Finally, we conduct unit root tests and summarize the results in table (4) and fail to reject the null of unit root for the variables assumed to be non-stationary in our model.

[TABLE 3]

In conclusion, our evidence suggests that the null model is well-specified. Our assertion is supported by the Bayesian information criterion and various diagnostic tests. The B.I.C. suggests that a random walk without drift yields the best fit to these data. Diagnostic tests, furthermore, suggested that there is relatively strong evidence that residuals of this model are white noise. There is also relatively strong evidence that the residuals are normal. Stationarity testing support the claim that the rate of profit has a unit root. In other words, we find evidence of a stochastic trend in the rate of profit series for the US economy for the period 1948-2007.

[TABLE 4]

3.2 Long Waves in the Rate of Profit

Any time series which is characterised by unit root nonstationarity is known to display significant persistence; this persistence in the time series of the unit root nonstationary random variable can impart to it the character of "long waves". When such a series starts to decline, it continues to do so for a considerable period of time; moreover, it persists at the low levels for a while before beginning a reverse movement. Similarly, when it begins its ascent, it continues on the upward movement, again, for a significant number of periods.

Following a long tradition of economists who have studied long waves of aggregate economic activity under capitalism, Mandel (1980) and Shaikh (1992) have conjectured that the long waves of aggregate economic activity might be related to long waves of the general rate of profit. Does the general rate of profit display long waves? Figure (3) presents evidence to answer this question in the affirmative.

[FIGURE 3]

Figure (3) plots the general rate of profit for the US economy. The figure is the time series plot of the rate of profit computed by Duménil and Lévy (1993) running from 1869 to 2007, the longest time series of consistent and reliable estimates of the general rate of profit for the US economy. This is the series that we have used for our analysis in the previous section of this paper. The plot has been supplemented by its Lowess trend (Cleveland, 1979).

Examining the Lowess trend, it is apparent that the trend in the profit rate series displays significant "long waves". Taking the movement of the trend, we observe a declining trend in the general rate of profit in the US economy from the mid-1860s to the mid-1910s. The profit rate, then, displays an upward trend till the early 1960s, to be followed by another round of decline to 2007. Specifically, movements in the U.S. profit rate may be delineated into four phases. Firstly, we observe a downward trend during the period 1869 – 1894. This movement coincides with the Depression of the 1890s. Secondly, there is no strong trend in the rate of profit from 1894 until the onset of the Great Depression. Thirdly, there is a substantial decline coincident with the Great Depression and a substantial upward trend coincident with WWII. Subsequently, the rate of profit exhibits a tendency to fall. The

profit rate series displays considerable persistence and, therefore, on balance there initially appears to be good reason for supposing that there is a stochastic trend in these data but the hypothesis of a deterministic trend appears implausible.¹⁴ In conclusion, the evidence confirms Mandel's (1980) and Shaikh's (1992) conjecture about "long waves" in the general rate of profit. How these long waves in the rate of profit is related to the long waves of aggregate economic activity is, of course, a separate issue, one that we do not investigate in this paper. Having established the statistical properties of the profit rate series for the US economy, let us now turn to an investigation of Marx's hypothesis about the LTFRP.

4 Tendency of the Rate of Profit to Fall

4.1 Introduction

To get started, let us briefly recall Marx's simple and powerful argument, outlined by Marx in Volume III of Capital (Marx, 1993), regarding the LTFRP. Marx starts his argument by noting that the driving force of capitalism is the relentless search for surplus value. While the early phase of capitalism is characterized by the relentless search for increasing absolute surplus value, i.e., by increasing the length of the working day while keeping the real wage rate constant, the later phase is characterized by the search for increasing relative surplus value. This is because labour, in its perpetual struggle against capital, wins an important victory in putting an upper limit on the length of the working day. From then on, the search for surplus value primarily takes the form of the drive to increase relative surplus value. The drive to increase relative surplus value, moreover, lies at the heart of the enormous technological dynamism of capitalism, compared to earlier modes of production, and is objectively enforced through the incessant competition between capitalists to reduce

¹⁴Does the seeming absence of a deterministic time trend in the profit rate series, as evidenced by a visual inspection of the profit rate time series plot, imply an empirical refutation of the LTFRP? We think not, as we argue in greater detail in the next section.

the costs of production.

Competition between capitalists to reduce the costs of production, and thereby increase surplus value and profit, often moves through the route of labour-saving technical progress. In other words, capitalists, in their bid to reduce the cost of production, increasingly replace labour with non-labour inputs to production. The replacement of workers with machines has another, oft neglected, dimension: power to control various aspects of the production process. The contradiction between labour and capital manifests itself not only as a struggle over the division of the value added between wages and surplus value, but also as a struggle to control various aspects of the production process like intensity and pace of labour, working conditions relating to safety of workers, recess frequency and duration, pace and direction of technological change, etc. The constant tussle between labour and (representatives of) capital to control the various aspects of the production process is as old as capitalist social relations. Mechanization, i.e., replacement of workers by machines, is a potent tool in the hands of the capitalist class in their conflict with labour: a machine, after all, is much easier to dominate than a recalcitrant worker. This political dimension of mechanization was highlighted by Marx in his discussion of skilled workers and engineers in England (p. 563, Marx, 1994) and remains largely valid even today.

This increasing mechanization of the production process, on the one hand, enormously increases the productivity of labour and facilitates the extraction of larger amounts of (relative) surplus value. On the other hand, the increasing replacement of labour with non-labour inputs is reflected in the fall in the share of total capital outlays supporting constant capital as opposed to variable capital, leading to a fall in what Marx called the composition of capital. This reduces, per unit of capital outlay, the amount of labour available for exploitation by capital, i.e., the production of surplus value. If the rate of surplus value remains constant, this fall in the composition of capital can lead to a fall in the rate of profit.

Having outlined the argument for the tendency for the rate of profit to decline over time,

Marx immediately notes the existence of powerful "counter tendencies" in real capitalist economies, which act to slow down or even reverse the tendency he highlighted. In particular, there are five counter tendencies that Marx specifically mentions. There is the increasing exploitation of labour, which could increase the rate of surplus value; the cheapening of the elements of constant capital due to the increasing productivity of labour; the deviation of the wage rate from the value of labour-power; the existence and increase of overpopulation; and the cheapening of consumption and capital goods through imports¹⁵

All these counter tendencies act against the tendency for the rate of profit to fall. Hence they must be explicitly incorporated into the analysis, and their effects on the trend of profitability must be controlled for before arriving at any conclusion about whether the rate of profit displays a statistically significant declining trend over time. Hence, if a time series plot of the general rate of profit, as for instance in figure (3), does not display a negative time trend, that is not evidence against Marx's hypothesis.

4.2 Empirical Test

Marx's hypothesis about the tendency for the rate of profit to fall, as outlined in Volume III of Capital (Marx, 1993) and nicely summarized in Sweezy (1942) and Foley (1986), is remarkably well suited for a restatement in the language of modern econometrics. Hence it is amenable to rigorous empirical testing using modern statistical tools. To see this, note that Marx's hypothesis can be restated as follows: under capitalism, there is a tendency for the rate of profit to fall after controlling for (a) the increasing exploitation of labour, (b) the cheapening of the elements of constant and variable capital either due to increasing productivity of labour or due to imports, ¹⁶ (c) the deviation of the wage rate from the value of labour-power, and (d) the existence of overpopulation. This can be immediately put into

¹⁵Marx mentions a sixth counteracting influence: increase in share capital. It is not very clear how this factor enters into the analysis of the LTFRP and so, following Foley (1986), we ignore it.

¹⁶Note that we are combining the second and fifth counteracting influence into one.

a time-series regression framework, as outlined below.

Though there is an enormous literature which has attempted to theoretically and empirically study the tendency for the rate of profit to fall in capitalism, to the best of our knowledge, none has looked at the matter in this way. Most, if not all, empirical studies have merely plotted a time series of the rate of profit and fitted a trend (linear or polynomial) and have attempted to see if there is evidence for a statistically significant downward trend. The evidence seems to suggest, as displayed in figure (3), that there are periods when there is a pronounced downward trend but periods when there is none.

Though the existing approaches offer valuable insights, they do not test *Marx's hypothesis*. Marx's hypothesis, as indicated above, related to the trend of the profit rate only after the counter tendencies had been taken into account, only after they had been controlled for, in the language of modern econometrics. Existence or non-existence of a downward trend, without controlling for the counter tendencies, is not a valid test of Marx's hypothesis.

4.3 The Empirical Model

To test Marx's hypothesis about the LTFRP, we use the following econometric model:

$$\log r_t = \alpha + \beta t + \gamma_1 z_{1t} + \gamma_2 z_{2t} + \gamma_3 z_{3t} + \gamma_4 z_{4t} + u_t, \tag{5}$$

where α is a constant, u_t is the error term, r_t is the rate of profit, z_{1t} is a measure of the exploitation of labour by capital, z_{2t} is a measure of the deviation of the wage rate from the value of labour-power and z_{3t} is a measure of the overpopulation in the economy, z_{4t} is a measure of the relative price of constant capital and t represents a deterministic time trend. Thus, this specification, flowing from Marx's account of the LTFRP in Volume III of Capital, explicitly takes account of the counteracting influences that could be expected to reverse the tendency of the rate of profit to fall over time.

Two regressors in the above equation require special discussion: the measure of the relative price of capital and the determinstic time trend. Inclusion of the measure of the relative price of constant capital follows the discussion in Michl (1988); it attempts to capture the cheapening of the elements of constant capital relative to the elements of variable capital, both due to technological progress and imports. Since the value composition of capital, c/v, is formed by the ratio of the value of constant to the value of variable capital, changes in the relative price of capital to consumer goods, which captures the relative rates of technical change in the two sectors, will be a relevant counteracting influence ceteris paribus.

The deterministic time trend in the above equation does not imply that the passage of time per se affects the output-capital ratio; rather, the passage of time is a proxy for the accumulation of capital, and it is the process of capital accumulation that tends to depress the output-capital ratio over time. Of course, the process of capital accumulation will not always lead to a fall in the output-capital ratio; it is only a particular pattern of technical change that often accompanies capital accumulation, referred to by Foley and Michl (1999) as Marx-biased technical change, that will lead to a fall in the productivity of capital. There are substantial periods in the life of capitalist economies when specific economic and political factors counteract the tendency for Marx-biased technical change; thus, it is precisely these factors that temporarily counter the underlying tendency for the rate of profit to fall. But, it was Marx's claim that if these counteracting factors had been removed from the picture, it would be possible to detect the underlying tendency.

Thus, the above specification attempts to capture Marx's idea that the process of capital accumulation under capitalism is often accompanied by a fall in the output-capital ratio, which, in turn, leads to a fall in the rate of profit. It is only when the process of technical change leads to a relatively large fall in the price of capital goods that capital accumulation is not accompanied by a fall in the output-capital ratio.

How would the regressors impact on the dependent variable? The intensity of exploitation can be expected to be positively related to the rate of profit; thus, the coefficient on z_{1t} can be expected to be positive. A positive deviation of the real wage from the value of labour power would decrease surplus value and would thus reduce the rate of profit; hence, the coefficient on z_{2t} can be expected to be negative. Increase in the relative surplus population can be expected to reduce the bargaining power of workers, leading to higher profits; thus, the coefficient on z_{3t} can be expected to be positive. Increase in the price of capital goods (i.e., elements of constant capital) relative to the price of consumer goods would lead to an increase in the value composition of capital and lead to a fall in the rate of profit; thus, the coefficient on z_{4t} could be expected to be negative.

The crucial issue, of course, is to test whether the coefficient on the time trend is negative.

Thus, the crucial issue is to test the following null hypothesis

$$H_0: \beta = 0$$

against the alternative

$$H_1: \beta < 0;$$

if the null is rejected then that would provide evidence in favour of Marx's hypothesis.

To motivate the econometric exercise, let us return to figure (3). As we have already indicated, the pattern in the trend of the profit rate series can be characterized as displaying "long waves". But the presence of these long waves do not settle the question about the possibility of rate of profit to fall because the Lowess trend has not been constructed after taking account of the counter-tendencies. The question, therefore, that we wish to investigate, following Marx's suggestion in Volume III of Capital, is whether these long waves in the rate of profit hides an underlying negative time trend. It was, we believe, Marx's contention that if the effects of what he referred to as the "counteracting influences" were taken out of the time series of the rate of profit, the underlying long-term negative time

trend would emerge; this would provide evidence of the tendency for the rate of profit to fall. The econometric model that we have outlined is meant to test this key proposition from Marx's analysis of capitalism.

4.3.1 The Test Statistic

Recall that the econometric model that we wish to use to investigate Marx's hypothesis about the falling rate of profit is the following:

$$\log r_t = \alpha + \beta t + \gamma_1 z_{1t} + \gamma_2 z_{2t} + \gamma_3 z_{3t} + \gamma_4 z_{4t} + u_t, \qquad t = 1, 2, \dots, T$$
(6)

where $\alpha = (\alpha_1 + \alpha_2)$ is a constant, $u_t = \varepsilon_{1t} + \varepsilon_{2t}$ is an error term, r_t is the rate of profit, z_{1t} is a measure of the exploitation of labour by capital, z_{2t} is a measure of the deviation of the wage rate from the value of labour-power and z_{3t} is a measure of the overpopulation in the economy, z_{4t} is a measure of the price of constant capital and t represents a deterministic time trend.

While we can estimate the parameters of the model by ordinary least squares (OLS), we will need to address serious statistical issues if we wish to carry out legitimate inference on the parameter estimates. Standard methods of inference, involving the t statistic, will not work because the parameters do not have standard distributions. Therefore, considerable effort will need to be devoted to deriving the distribution of the estimators of test statistic constructed out of those estimators to make statistically valid inference.

The major theoretical problem arises from the fact that the model in (6) involves variables with very different statistical properties. There are, in fact, three different kinds of variables in the model in (6): (a) stationary random variables (like the deviation of the wage from its trend and the measure of the intensity of exploitation); (b) the deterministic time trend; (c) unit root non-stationary random variables (like the measure of the relative price of constant capital, the overpopulation in the labour market). Hence, the rates of convergence of the

estimators of the different coefficients in (6) will be different; thus, the estimators will not have standard distributions and standard t and F tests will not work. This problem can be addressed using the method outlined in Sims, Stock and Watson (1990) and Hamilton (1994).

To proceed, recall that we wish to test, with reference to (6), the following null hypothesis

$$H_0: \beta = 0$$

against the alternative

$$H_1: \beta < 0.$$

Let $\hat{\beta}$ be the OLS estimator of β . To express the (non-standard) distribution of $\hat{\beta}$, we will need some notation. To begin, note the following about the regressors in (6): z_{1t} and z_{2t} are zero-mean stationary random variables; z_{3t} and z_{4t} are unit root nonstationary random variables. Let $var(z_{1t}) = \sigma_1^2$, $var(z_{2t}) = \sigma_2^2$ and $cov(z_{1t}, z_{2t}) = \sigma_{12}$; further, let

$$z_{3t} = z_{3t-1} + u_{3t},$$

where $u_{3t} \sim (0, \sigma_3^2)$, and the long run variance of u_{3t} is λ_3 ; similarly, let

$$z_{4t} = z_{4t-1} + u_{4t}$$

where $u_{4t} \sim (0, \sigma_4^2)$, and the long run variance of u_{4t} is λ_4 . Suppose, further, that the error term in (6) has an MA(∞) structure to allow for general serial correlation:

$$u_t = \psi(L)\varepsilon_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j},$$

where $\sum_{j=0}^{\infty} j |\psi_j| < \infty$, ε_t is an i.i.d. sequence with mean zero and variance γ_0 , and finite fourth moment. Let the long-term variance of u_t be denoted by λ , where

$$\lambda = \gamma_0 \sum_{j=0}^{\infty} \psi_j.$$

Let W(.) denote standard Brownian motion, and Q denote the following (6×6) symmetric matrix,

$$\begin{pmatrix} 1 & 1/2 & 0 & 0 & \lambda_3 \int_0^1 W(r) dr & \lambda_4 \int_0^1 W(r) dr \\ 1/2 & 1/3 & 0 & 0 & \lambda_3 \int_0^1 W(r) dr & \lambda_4 \int_0^1 W(r) dr \\ 0 & 0 & \sigma_1^2 & \sigma_{12} & 0 & 0 \\ 0 & 0 & \sigma_{12} & \sigma_2^2 & 0 & 0 \\ \lambda_3 \int_0^1 W(r) dr & \lambda_3 \int_0^1 r W(r) dr & 0 & 0 & \lambda_3^2 \int_0^1 [W(r)]^2 dr & (1/2)(\lambda_5^2 - \lambda_3^2 - \lambda_4^2) \int_0^1 [W(r)]^2 dr \\ \lambda_4 \int_0^1 W(r) dr & \lambda_4 \int_0^1 r W(r) dr & 0 & 0 & (1/2)(\lambda_5^2 - \lambda_3^2 - \lambda_4^2) \int_0^1 [W(r)]^2 dr & \lambda_4^2 \int_0^1 [W(r)]^2 dr \end{pmatrix}$$

let A denote the following (6×1) vector,

$$A = \begin{pmatrix} \lambda W(1) \\ \lambda \{W(1) - \int_0^1 W(r) dr \} \\ \sigma_1 \sqrt{\gamma_0} W(1) \\ \sigma_2 \sqrt{\gamma_0} W(1) \\ (1/2)(\lambda^2 - \sigma_3^2) \int_0^1 [W(r)]^2 dr - \lambda^2 [W(1)]^2 \\ (1/2)(\lambda^2 - \sigma_4^2) \int_0^1 [W(r)]^2 dr - \lambda^2 [W(1)]^2 \end{pmatrix}$$
(8)

With these notations in place, we can now state the distribution of $\hat{\beta}$ as

Proposition 1 If $\hat{\beta}$ is the OLS estimator of β in (6), then

$$T^{3/2}(\hat{\beta} - \beta) \xrightarrow{L} \frac{1}{|Q|} \sum_{j=1}^{6} p_{2j} a_j,$$

where $\stackrel{L}{\longrightarrow}$ denotes weak convergence, and for $j = 1, 2, \dots, 6$,

$$p_{2j} = (-1)^{2+j} |Q_{j2}|,$$

where |A| denotes the determinant of any matrix A, and Q_{j2} is the matrix formed by deleting row j and column 2 of the symmetric matrix, Q, in (7), and a_1 represents the first element, a_2 the second element, ... and a_6 the sixth element of the (6 × 1) vector, A, in (8), and |Q|is the determinant of the matrix Q.

Comments: For a proof see Appendix A; since this random variable has a non-standard distribution, we need to compute critical values by Monte Carlo methods every time we wish to make valid statistical inferences. Numerically computing the distribution of the test statistic $T^{3/2}(\hat{\beta} - \beta)$ involves working with unknown parameters: $\sigma_1, \sigma_2, \sigma_{12}, \sigma_3, \sigma_4, \gamma_0, \lambda, \lambda_3, \lambda_4, \lambda_5$. Since these parameters are unobservable, we use consistent estimators of each in our computation.

4.3.2 Data and Results

Results of estimating (6) by OLS using annual U.S. macroeconomic series for the period 1948-2007 are presented in Table 5. Note that because of the non-standard distribution of the estimators, we have not reported standard errors for the point estimates. The variables used in the analysis are: the rate of profit, the intensity of exploitation, the deviation of the real wage rate from the value of labour-power, a measure of overpopulation in the economy and the relative price of capital stock.

The variables have been measured as follows. The intensity of exploitation is computed by an application of the Hodrick Prescott filter to the productivity of labour. We computed the trend of labour productivity and remove this trend from the series. A detailed discussion of this variable is provided in an appendix. Briefly, this variable acts as a surrogate for the intensity of exploitation not due to mechanization (e.g., speeding-up of production). The

relative price of fixed capital is the ratio of a price deflator for the fixed capital stock to the consumer price index. The deviation of the wage from the value of labour-power has been computed as the deviation of the real wage rate series from its trend, where the trend has been computed using the Hodrick-Prescott filter.¹⁷ The procedure is identical to that described above for the intensity of exploitation. Finally, overpopulation is measured as one minus the civilian employment population ratio. We enclose a table of descriptive statistics below and the construction of the variables is discussed in detail in an appendix.

Table 4 reports p-values associated with two standard unit root tests, the Augmented Dickey-Fuller and the Phillips-Perron test, on the variables in the model. In both cases, the null hypothesis is that the variable under consideration is unit root non-stationary while the alternative is that the variable is stationary. The p-values in Table 4 suggest that the following two variables are stationary: the intensity of exploitation and the deviation of the real wage from the value of labour power; it suggest that the following variables are unit root nonstationary: the rate of profit, the measure of overpopulation and the relative price of the capital stock. This confirms the assumption underlying the results in Proposition 1.

For the purposes of this study, which is to test whether the evidence supports Marx's hypothesis regarding the tendency of the rate of profit to fall over time, the crucial parameter is β , the coefficient on the time trend. Since the dependent variable in (6) is the logarithm of the profit rate, the parameter β has the following interpretation: $100 \times \beta$ gives the annual percentage change in the rate of profit. From Table 5, we see that the coefficient on the time trend has a negative sign, taking the numerical value of -0.003. Thus it implies that for the period 1948-2007, the general rate of profit in the U.S. has been falling about 0.3% per annum.

To test whether the negative coefficient on the time trend is statistically significant, we

¹⁷We assume, in this analysis, that the long-run trend in the real wage rate gives us a measure of the value of labour-power.

compute the test statistic corresponding to result in Proposition 1 as: $T^{3/2} \times \hat{\beta}$. Since, for our sample, T = 60 and the point estimate of β is -0.003, the value of the test statistic turns out to be -1.516. From Table 6 we see that, using the distribution of the test statistic under the null hypothesis (that the coefficient on the time trend is zero), we can reject the null hypothesis in favour of the alternative at the 1 percent significance level. Hence, the evidence seems to support Marx's hypothesis regarding the tendency for the rate of profit to fall for the U.S. economy for the period 1948-2007.

[TABLE 5]

When we look at the estimates of the other regressors we note that the signs on all of them are along expected lines. The effect of the intensity of exploitation on the rate of profit is positive, as expected: if the intensity of exploitation increases, that can be expected to increase the rate of surplus value and thereby increase the rate of profit. The sign on the deviation of real wages from the value of labour power is negative: when there is a positive deviation of the real wage from the value of labour power, that reduces the rate of surplus value and thus decreases the rate of profit. Along expected lines, the effect of overpopulation on the rate of profit is positive: an increase in the overpopulation reduces the bargaining power of labour, pushing up the surplus value and rate of profit. The rate of profit is impacted negatively by the relative price of capital stock: when elements of the capital stock become more expensive relative to wage goods, the value composition of capital rise ceteris paribus; this puts a downward pressure on the rate of profit.

Note that we cannot make any statements about the significance of these regressors. This is because the coefficients on these regressors do not have standard distributions; hence, standard t-values are not meaningful. Moreover, since we were primarily interested in testing the significance of the time trend variable, we have not computed the critical values for the other regressors. Hence, we are not in a position, in this paper, to make any statements

about the statistical significance of the effects of the other regressors. The fact that the signs are along expected lines suggest that we should take up the issue of significance in future research work.

[TABLE 6]

5 Conclusion

Marx's claim in Volume III of Capital regarding the tendency for the general rate of profit to fall has spawned an enormous literature. Although the theoretical strand of the literature has focused on understanding the causes of this tendency, this paper has focused on empirically testing Marx's hypothesis. A major lacuna has been the dearth of serious econometric inquiry to inform the empirical analysis. As we noted earlier, a preponderance of studies utilize only exploratory techniques such as visual inspection of time series plots.

Starting with a systematic investigation of the statistical properties of the profit rate series, we arrive at the conclusion that the rate of profit displays unit root non-stationarity. Our initial inspection of the sample autocorrelation function suggests that the rate of profit is non-stationary. We estimated an array of models, ranging from ARIMA(3,1,0) to ARIMA(0,1,0), and conclude that ARIMA(0,1,0) provides the best fit for these data. Diagnostic testing does not lead us to reject this model. Of course, any time series with unit root nonstationarity is known to display considerable persistence. Such persistence can impart "long waves" into the series. When such a series begins a decline, this fall continues for some time before a reversal of the trend. Likewise, when beginning its ascent, it continues to rise for a substantial number of periods. Following a tradition of economists that have studied long waves under capitalism, some scholars have speculated that long waves of aggregate economic activity might be related to long waves of the general rate of profit.

Using the nonstationarity of the profit rate series and explicitly accounting for the

counter-tendencies that Marx had mentioned in Volume III of Capital, we build a novel econometric model to test Marx's hypothesis. Our formulation specifies that the expected rate of profit is correlated with the intensity of exploitation, the cheapening of the elements of constant and variable capital, the deviation of the wage rate from the value of labour power, the existence of overpopulation in the labour market, and a deterministic time trend. Most empirical studies have simply examined time series plots and fit a trend to these data. However, existence or nonexistence of a downward trend is not a valid test of Marx's hypothesis unless the counter-tendencies are appropriately controlled for. While we can estimate the parameters of our model by ordinary least squares, we confront serious statistical difficulties related to the assumptions that ensure the optimality of the standard estimator. The usual methods of inference (e.g., involving the t-statistic) will not be valid. Hence, we have devoted considerable effort to deriving the valid null distribution of the estimators. The major theoretical problem that we confronted is that the regressors have different rates of convergence which invalidates the usual inferential procedures. Although some regressors are stationary random variables, others are unit-root non-stationary.

In this econometric setting, we make certain assumptions about the regressors that are relatively robust. For example, we assume that overpopulation will be a non-stationary random variable and in particular a random walk without drift. We applied stationarity tests to the regressors, and the results are consistent with our assumptions. Moreover, the error term in our full model has a general moving average structure that captures general serial correlation. These assumptions allow us to derive the null distribution of the OLS estimator. However, since this random variable has a nonstandard distribution, we computed critical values by Monte Carlo methods. The key finding of this paper is that the deterministic trend is negative and statistically significant at the one percent level. Indeed, the tendency of the rate of profit to fall is given a precise econometric meaning: the rate of profit declines at a rate of approximately 0.3 percent per annum after controlling for the counter-tendencies.

This finding establishes the relationship between the inexorable mechanization of capitalist production and the tendency of the rate of profit to decline.

Appendix A

In this appendix, we will prove the claim in Proposition (1). To proceed, note, from the text of the paper, that we use the following econometric model to investigate Marx's hypothesis about the falling rate of profit:

$$\log r_t = \alpha + \beta t + \gamma_1 z_{1t} + \gamma_2 z_{2t} + \gamma_3 z_{3t} + \gamma_4 z_{4t} + u_t, \qquad t = 1, 2, \dots, T$$
(9)

where α is a constant, u_t is an error term, r_t is the rate of profit, z_{1t} is a measure of the exploitation of labour by capital, z_{2t} is a measure of the deviation of the wage rate below the value of labour-power and z_{3t} is a measure of the overpopulation in the economy, z_{4t} is a measure of the price of constant capital and t represents a deterministic time trend.

Recall the following about the regressors in (9): z_{1t} and z_{2t} are zero-mean stationary random variables; t is a time trend; z_{3t} and z_{4t} are unit root nonstationary random varibles. Let $var(z_{1t}) = \sigma_1^2$ and $var(z_{2t}) = \sigma_2^2$; further, let

$$z_{3t} = z_{3t-1} + u_{3t},$$

where $u_{3t} \sim \text{i.i.d.}(0, \sigma_3^2)$, and

$$z_{4t} = z_{4t-1} + u_{4t},$$

with $u_{4t} \sim \text{i.i.d.}(0, \sigma_4^2)$.

The model in (9) can be written as

$$y_t = x_t' \omega + u_t,$$

where x_t is the 6×1 vector given by

$$x'_t = [1 \quad t \quad z_{1t} \quad z_{2t} \quad z_{3t} \quad z_{4t}],$$

and ω is the vector of coefficients given by

$$\omega = [\alpha \quad \beta \quad \gamma_1 \quad \gamma_2 \quad \gamma_3 \quad \gamma_4].$$

If $\hat{\omega}$ is the ordinary least squares (OLS) estimator for ω , then

$$\hat{\omega} - \omega = \left[\sum_{t=1}^{T} x_t x_t' \right]^{-1} \left[\sum_{t=1}^{T} x_t u_t \right].$$

Letting the summation run from t = 1 to t = T, we can write out the elements of the (6×6) symmetric matrix $\left[\sum_{t=1}^{T} x_t x_t'\right]$ as

$$\begin{bmatrix} \sum 1 & \sum t & \sum z_{1t} & \sum z_{2t} & \sum z_{3t} & \sum z_{4t} \\ \sum t & \sum t^2 & \sum tz_{1t} & \sum tz_{2t} & \sum tz_{3t} & \sum tz_{4t} \end{bmatrix} = \begin{bmatrix} \sum t & \sum t^2 & \sum tz_{1t} & \sum tz_{2t} & \sum tz_{2t} & \sum tz_{3t} & \sum tz_{4t} \\ \sum t & \sum tz_{1t} & \sum tz_{1t} & \sum tz_{1t} & \sum tz_{2t} & \sum tz_{1t} & \sum tz$$

Similarly, we can write out the elements of the (6×1) vector $\left[\sum_{t=1}^{T} x_t u_t\right]$ as

$$\begin{bmatrix} \sum u_t \\ \sum t u_t \end{bmatrix} = \begin{bmatrix} \sum z_{1t} u_t \\ \sum z_{2t} u_t \end{bmatrix} .$$

$$\sum z_{2t} u_t \\ \sum z_{3t} u_t \end{bmatrix}$$

Since the different elements of A have different rates of convergence, following Hamilton (1994), we will use the following scaling matrix:

Thus

$$S(\hat{\omega} - \omega) = S\left[\sum_{t=1}^{T} x_t x_t'\right]^{-1} \left[\sum_{t=1}^{T} x_t u_t\right] = \left(S^{-1} \left[\sum_{t=1}^{T} x_t x_t'\right] S^{-1}\right)^{-1} \left(S^{-1} \left[\sum_{t=1}^{T} x_t u_t\right]\right).$$

Let W(.) denote standard Brownian motion, and \Longrightarrow denote weak convergence; then,

$$S^{-1} \left[\sum_{t=1}^{T} x_t x_t' \right] S^{-1} =$$

where Q =

and

$$S^{-1}\left[\sum_{t=1}^{T} x_t u_t\right] \Longrightarrow A,$$

where

$$A = \begin{pmatrix} \lambda W(1) \\ \lambda \{W(1) - \int_0^1 W(r) dr \} \\ \sigma_1 \sqrt{\gamma_0} W(1) \\ \sigma_2 \sqrt{\gamma_0} W(1) \\ (1/2)(\lambda^2 - \sigma_3^2) \int_0^1 [W(r)]^2 dr - \lambda^2 [W(1)]^2 \\ (1/2)(\lambda^2 - \sigma_4^2) \int_0^1 [W(r)]^2 dr - \lambda^2 [W(1)]^2 \end{pmatrix}$$

Thus,

$$S(\hat{\omega} - \omega) = \begin{pmatrix} T^{1/2}(\hat{\alpha} - \alpha) \\ T^{3/2}(\hat{\beta} - \beta) \end{pmatrix}$$

$$T^{1/2}(\hat{\gamma}_1 - \gamma_1) \\ T^{1/2}(\hat{\gamma}_2 - \gamma_2) \end{pmatrix} \Longrightarrow Q^{-1}A,$$

$$T(\hat{\gamma}_3 - \gamma_3)$$

$$T(\hat{\gamma}_4 - \gamma_4)$$

where the (6×6) matrix Q and the (6×1) vector A is as denied above. Under the null hypothesis that $\beta = 0$, we, therefore, have

$$T^{3/2}(\hat{\beta}) \Longrightarrow \frac{1}{|Q|} \sum_{j=1}^{6} p_{2j} a_j,$$

where \Longrightarrow denotes weak convergence, and for $j=1,2,\ldots,6,$

$$p_{2j} = (-1)^{2+j} |Q_{j2}|,$$

where |D| denotes the determinant of any matrix D, and Q_{j2} is the matrix formed by deleting row j and column 2 of the symmetric matrix, Q, in (7), and a_1 represents the first element, a_2 the second element, ... and a_6 the sixth element of the (6×1) vector, A, in (8), and |Q|is the determinant of the matrix Q. This proves the claim of Proposition (1).

Appendix B: The Augmented Duménil and Lévy Data Set, 1948-2007

The empirical analysis in this paper uses data for the period 1948-2007 from Duménil and Lévy (2008) and augments it with data from some other sources. The following variables have been used in our empirical analysis: the net profit rate, the intensity of exploitation, the deviation of the real wage from the value of labour power, the surplus population in the labour market, and the relative price of capital.

The net profit rate series has been directly taken from The Duménil and Lévy (2008). It is defined as the ratio of the net domestic product minus the wage bill and the net stock of fixed capital.

The intensity of exploitation is computed by an application of the Hodrick Prescott filter to the productivity of labour. Using this technique, we extracted the trend of labour productivity. Since variations in labour productivity are conceptualized as the sum of technological

 $^{^{18} {\}rm The~Dum\acute{e}nil~and~L\acute{e}vy}$ (2008) data set is available at http://www.jourdan.ens.fr/levy/uslt4x.txt

changes in the production process and variations in the intensity of labour exploitation that are independent of technology, the deviation of labour productivity from its trend serves as a surrogate for the latter term. Variations in the intensity of labour exploitation that are independent of technology might arise due to shifts in the collective power of labour. For instance, various political variables might shift the intensity of labour independent of technology. Hence, we measure the intensity of exploitation by the deviation of the productivity of labour in a particular year from its trend. Labour productivity, in turn, is defined as the ratio of real net domestic product (chained 2000 millions of dollars) and the number of hours worked (expressed in millions of hours).

The relative price of fixed capital is the ratio of an implicit price deflator for the fixed capital stock to the consumer price index. The implicit price deflator for the net stock of private fixed assets is computed in two steps using the formulae in the NIPA Guide (2005). In the first step the chained dollar value of the stock of fixed assets is computed as: chained dollar value = (chain-type quantity index * current dollar value in 2005)/100, where data for the chain-type quantity index of fixed assets is available from NIPA Fixed Assets Table 6.2, the base year is 2005 and the current dollar value of the fixed asset stock is taken from NIPA Fixed Assets Table 6.1. In the second step the implicit price deflator is computed as: implicit price deflator = (current dollar value * 100)/ chained dollar value.

As has been noted earlier, if the rate of technological progress in the capital goods sector is faster than the rate of technical progress in the overall economy that would reduce the price of capital goods faster than the price of other goods. This might act as a countervailing force to the tendency for the rate of profit to fall (Michl, 1988). Note, however, that our denominator differs from that of Michl (1988). The appropriate logic for capitalists does not consist of a comparison between the price of capital and final goods and services. Rather, the appropriate comparison is between the price of fixed capital and wage goods; hence we use the CPI instead of the GDP deflator.

The deviation of the wage from the value of labour-power has been computed as the deviation of the real wage rate series from its trend, where the trend has been computed using the Hodrick-Prescott filter. The procedure is identical to that described above for the intensity of exploitation. The wage variable is the nominal hourly wage in the data set of Duménil and Lévy and this has been deflated using the consumer price index. Finally, overpopulation is measured as one minus the civilian employment population ratio available from the Federal Reserve Bank of St. Louis.

[TABLE 7]

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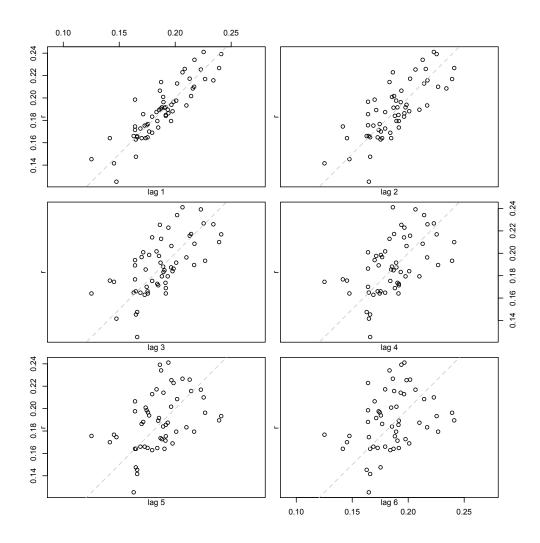


Figure 1: Lag Plots of the U.S. Profit Rate, 1948-2007

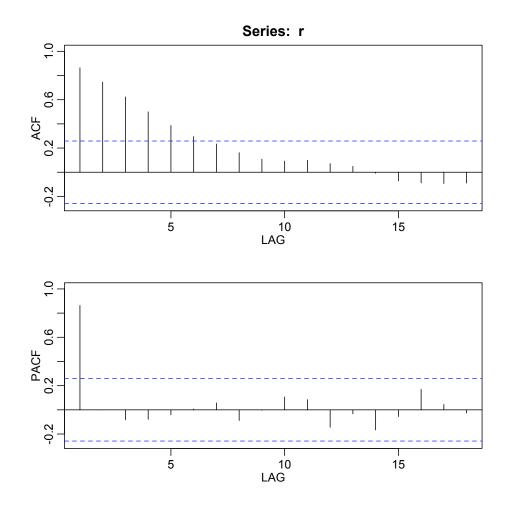


Figure 2: Sample Autocorrelations and Partial Autocorrelations of the Rate of Profit

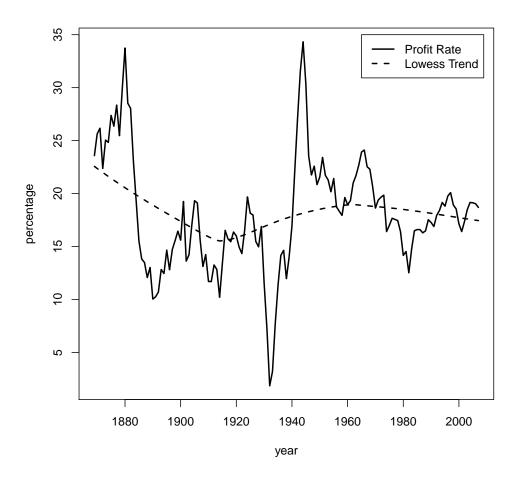


Figure 3: Long Waves in the Profit Rate

Table 1: ARIMA models: Parameter Estimates

Model	b_1	b_2	b_3	s
ARIMA(3,1,0)	0.0032	-0.0122	-0.0854	0.0117
ARIMA(2,1,0)	0.0009	-0.0121	N/A	0.0118
ARIMA(1,1,0)	0.0007	N/A	N/A	0.0118
ARIMA(0,1,0)	N/A	N/A	N/A	0.0118

Table 2: ARIMA models: Goodness of Fit Statistics

Model	Log Likelihood	B.I.C.
ARIMA(3,1,0)	178.5609	-344.9927
ARIMA(2,1,0)	178.3526	-348.6192
ARIMA(1,1,0)	178.3484	-352.6537
ARIMA(0,1,0)	178.3483	-356.6967

Table 3: ARIMA models: p-Values for Diagnostic Statistics

Model	Shapiro-Wilk	Jarque-Bera	Ljung-Box
ARIMA(3,1,0)	0.096	0.1470	0.9998
ARIMA(2,1,0)	0.1796	0.1895	0.9245
ARIMA(1,1,0)	0.1967	0.2017	0.9246
ARIMA(0,1,0)	0.1971	0.2019	0.9247

Table 4: P-Values for Unit Root Tests

Variable	Dickey-Fuller	Phillips-Perron
Rate of Profit	0.536	0.508
intensity of exploitation	0.000	0.000
Wage Deviation	0.000	0.000
Overpopulation	0.183	0.338
Relative Price of Capital	0.417	0.748

Table 5: Estimation^aResults for the US

	1948-2007
CONSTANT	5.109
TIME TREND	-0.003***
INTENSITY OF EXPLOITATION	0.271
DEV OF WAGE FROM VALUE OF LP	-0.172
OVERPOPULATION	1.710
RELATIVE PRICE OF CAPITAL b	-1.854

^a Dependent variable is log of the profit rate; the regression has been estimated by OLS with annual data for the period indicated. Details of the data set can be found in Appendix B.

^{***} Significant at the 1% level, where significance refers to the test statistic defined in Proposition 1 and computed using the reported estimate for the coefficient on the time trend and the relevant sample size.

^b For these regression, we use 1- the employment-population ratio as a proxy for the level of overpopulation in the economy.

^c Please note that we have *not* computed the standard errors of the estimators of any of the coefficients other than the one on the time trend; thus, we have not tested the statistical significance of any of the other regressors. This is because the main purpose of the analysis in this paper was to test the significance or otherwise of the negative time trend.

Table 6: Critical Values of Test Statistic

1%	2.5%	5%	10%	50%
-0.448	-0.378	-0.319	-0.249	0.001

Table 7: Descriptive Statistics of the Augmented Duménil and Lévy Data

Variable	Minimum	1st Quantile	Median	Mean	3rd Quantile	Maximum
ROP^a	0.125	0.172	0.188	0.189	0.203	0.241
EXP^b	-0.614	-0.123	0.0238	0.000	0.141	0.344
DEV^c	-0.199	-0.0568	-0.006	0.000	0.050	0.244
EMP^d	0.356	0.377	0.421	0.408	0.432	0.447
PKK^e	0.479	0.511	0.564	0.556	0.587	0.652

^a Rate of profit.

^b Intensity of exploitation.

^c Deviation of real wage from the value of labour power.

 $[^]d$ 1- Employment-population ratio.

^e Relative price of capital.