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Attitude Dynamics with Limited Verbalisation Capabilities

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Abstract

This article offers a new perspective for research on opinion dynamics. It demonstrates the importance of the distinction of opinion and attitude, which originally has been discussed in literature on consumer behaviour. As opinions are verbalised attitudes not only biases in interpretation and adoption processes have to be considered but also verbalisation biases should be addressed. Such biases can be caused by language deficits or social norms. The model presented in this article captures the basic features of common opinion dynamic models and additionally biases in the verbalisation process. Further, the article gives a first analysis of this model and shows that precision as bias in the verbalisation process can influence the dynamics significantly. Presenting and applying the concept of area of influential attitudes the impact of each parameter (selective attitude, selective interpretation, and precision) is analysed independently. Some preliminary results for combined effects are presented.

Keywords: Opinion dynamics, attitude dynamics, verbalisation, selective attitude, selective interpretation, area of influential attitudes

1 Introduction

Consider a group of interacting individuals, e.g. researchers, each having a more or less positive attitude towards an object, e.g. universities. While the individuals talk about universities they may formulate their attitude, thus give their opinion. Because the individuals are not that narrow-minded, the other individuals' opinions may influence the own attitude. This causes a group-wide attitude and opinion formation process. Models about opinion dynamics are concerned with the patterns that are built by such formation processes. The models aim to explain what drives the tendency to polarization or convergence of opinions or generally what drives specific fragmentation patterns of opinions. Beside mathematical proofs (e.g. Hegselmann and Krause 2002) simulation is applied as a powerful tool to explore properties of complex opinion dynamic models.

Models of opinion dynamics frequently focus on the communication between individuals as the source for changes of opinions. The impact of other's opinions on the own one depends on factors like source of the opinion and differences between an agent's own and the perceived opinion (e.g. Deffuant 2001). Despite dealing with cognitive biases during perceptual processes many articles do not explicitly address the process of opinion formulation (e.g. Weisbuch *et al.* 2001, Hegselmann and Krause 2002, Deffuant *et al.* 2000, Deffuant 2001, and Holyst *et al.* 2001). An opinion is a verbalised attitude and the process of verbalizing an attitude can cause a bias, but the cited articles assume that the opinion is exactly the same as the attitude. This is the reason why many texts in opinion dynamics contrary to texts in marketing theory do not distinguish between attitudes that are at the beginning of the opinion formulation process and opinions that are at the end (Trommsdorff 1998). This lack of attention towards verbalisation processes leads to models based on the assumption that individuals can communicate the difference between an attitude of 0.5555 and 0.5556. It seems reasonable that during personal language-based communication between individuals such a high precision is not applied, therefore the assumption of infinite precision seems to be inappropriate. At the point of assuming finite precision we switch from continuous to discrete opinions, which can change the obtained results dramatically (Hegselmann and Flache 1998). But we do not switch completely because we assume continuous attitudes within the agents but allow communication only via discrete opinions. This article rebuilds a basic model of continuous opinion dynamics, which can be found in Deffuant *et al.* (2001) and Weisbuch *et al.* (2001) and rephrases it in the light of consumer behaviour theory. We try to verify the core results assuming restrictions for the verbalisation of attitudes.

Following this introduction section 2 introduces the difference between attitude and opinion. Based on attitude and opinion it presents basic concepts of biases in perceptual and verbalisation processes. Section 3 presents a formal model covering most of the biases explained before. The model has four essential parameters: number of individuals, selective attention, selective interpretation, and precision of opinions. Section 4 analyses changes to three of the four parameters. The article closes with a summary and an outlook to future research.

2 Basic concepts

Models of opinion dynamics aim to identify major forces of specific patterns of opinion evolution within a society. In case of consumers as individuals such processes are also elements of marketing research, in particular innovation diffusion research. Therefore, models of opinion dynamics are a vital part of several articles on innovation diffusion (e.g. Deffuant 2001, Chattoe and Gilbert 1999). In marketing theory changes of opinions are usually modelled as a result of personal experiences, social influences or cognitive efforts, i.e. reasoning about signals related to the object the opinion refers to (Fishbein and Ajzen 1975, Kroeber-Riel and Weinberg 1999, Shet *et al.* 1999). Models related to opinion dynamics frequently emphasize communication enabling a mutual, sometimes one-sided, exchange of opinions as the most important source for opinion formation. To make these models more compatible with marketing concepts this section links the term opinion to attitude and identifies four processes that are the foundation of a basic communication model.

2.1 Communication of attitudes

Attitudes (mental attitudes[1]) refer to cognitive states of individuals. These states are characterised by a learned predisposition to respond in a consistently favourable or unfavourable manner with respect to a given object (see Fishbein and Ajzen 1975, p. 6 and Shet *et al.* 1999, p. 388). Due to the actual context or due to a lack of individual abilities an individual's behaviour does not need to be is consistent with his or her attitudes. Despite that, in marketing theory, attitude is frequently used as a concept guiding an individual's behaviour.

According to Trommsdorff (1998), opinions are verbalised attitudes. Although it might be counterintuitive at the first glance, this conceptualisation meets the idea that attitudes are expressed by behaviour and observable by others but opinions are usually expressed by words[2]. An attitude verbalisation process gives the relation between attitudes that are internal states and opinions that are expressions able to be communicated via a communication channel.

As innovation diffusion models (and consumer behaviour models in general) aim to predict behaviour, attitude is the more interesting concept. Because changes in attitudes are frequently modelled as caused by communication, opinions become relevant as well. Therefore we want to extend classic opinion dynamic models by introducing a verbalisation process. That means we want to anticipate the difference of attitude and opinion (as they are interpreted in this article).

We start with a simple attitude communication model that anticipates the difference of attitude and opinion (see Figure 1). A communication process connects a sender with a receiver. This is done by a message that goes through a communication channel that is controlled by the environment. Four processes can be identified as most important. First, the attitude verbalisation process verbalises the sender's attitude towards an object. This process converts the cognitive state into an expression able to be communicated

[1]The Oxford Dictionary gives three meanings of the word attitude: (1) a way of thinking about something, (2) a way of behaving towards something, or (3) a position of the body (Hornby 1997). We use the word attitude with the meaning of mental attitude, thus follow the first interpretation.

[2]Even if opinions are not expressed by words their representation usually contains abstract symbols, i.e. figures or a meaningful position of the body.

via a communication channel, thus creates an opinion. Second, the opinion has to be transmitted as a message via a communication channel. The message received by the communication partner is interpreted as a perceived opinion. The receiver decides on the impact the other opinion has on its own attitude, i.e. whether she or he adopts the opinion or not[3]. The adoption of an opinion implies the establishment of an equivalent cognitive state, i.e. an attitude. Due to biases a perceived opinion can be different from the attitude that originally has been verbalised and transmitted.

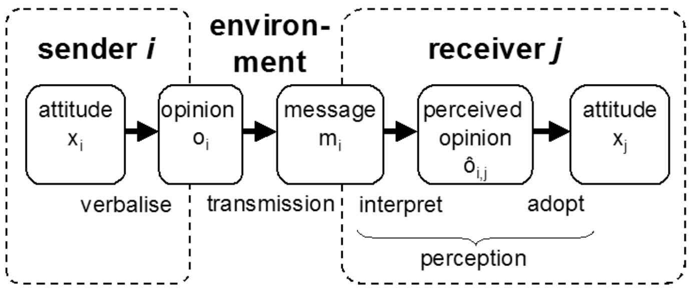


Figure 1: Communication Model

Biases in these processes, i.e. an incomplete understanding of messages, and their impact on the diffusion of information are issues for marketing communication research (Trommsdorff 1998, p. 283). Some of such biases analysed in consumer behaviour theory are presented within the next subsection. The perceptual processes have got the most attention by opinion dynamics researchers (and also in consumer behaviour theory, see for example Shet *et al.* 1999). Therefore, we first present the most important basic concepts related to perceptual processes. Biases in transmission processes are not considered. Then we develop a concept that captures biases in the verbalisation process. For the concepts some ideas for simulation are given.

2.2 Biases in the perceptual process: selective exposure, selective attention, and selective interpretation

Individuals are involved in many communications. To cope with the barrage of communications and communication opportunities humans become "selective" (Shet *et al.* 1999, pp 303-305). They ignore sources of information, hence communication opportunities, they pay less attention to specific information, and they may bias the perceived information.

Individuals tend to expose themselves to ideas that are in accordance with their interest, needs, and existing attitudes (Rogers 1995, p. 164). Based on past experiences about the content of specific sources this tendency to selective exposure leads to a selection of communication opportunities. An individual has different exposures to different communication channels and sources; it will not talk to everybody with the same probability.

[3]The interpretation process and the adoption process are also referred to as perceptual processes.

In several opinion dynamic models the selection of communication partners does not depend on past experiences, but depends on the static membership to different social networks[4] (e.g. Chattoe and Gilbert 1999, Deffuant *et al.* 2001, Deffuant *et al.* 2000, Weisbuch *et al.* 2001). In the basic model of Deffuant *et al.* (2001), which we build on, the communications are chosen arbitrarily.

If an agent receives information the source and the channel can lead to high or low attention towards this information, i.e. selective attention. Individuals with high attention may read a text completely but individuals with low attention stop after one or two paragraphs. While processing the information this attention can increase or decrease. If information is complex the selective attention influences the amount of information an individual extracts from a message (Kroeber-Riel and Weinberg 1999).

Homophily is another concept that explains different degrees of impact information have. If the communication partners are homophile, i.e. are similar in their ideas and behaviours, then the communication is efficient, thus the impact information have is higher (Rogers 1995). Assuming that an agent pays high attention to information that is similar to its own attitudes, selective attention may be modelled as a threshold that prevents an adoption of information if it is not sufficiently close to the own attitude. This concept is a fundamental idea of models in Deffuant *et al.* (2001) and Weisbuch *et al.* (2001). Of course the idea of threshold does not capture the idea of selective attention completely, but the common assumption in opinion dynamic literature of atomic messages does not support an incomplete perception of a message. It may be modelled such that low selective attention decreases the impact information has, which can be interpreted as taking less information into account, hence adopting the other's opinion incompletely.

Individuals have to decipher the message perceived from the communication channel. Doing that individuals tend to interpret the information so as to conform to his or her prior beliefs (Shet *et al.* 1999). If this effect of selective interpretation refers to a general perceptual distortion, as we assume, one should integrate a bias in the interpretation process. This implies that the difference of the other agent's opinion to the own attitude is reduced, thus an adoption becomes more likely but the adopted information is biased. The models presented in Deffuant *et al.* (2001) and Weisbuch *et al.* (2001) do not capture such effects.

If an individual has perceived one or more opinions it might adopt them. This means that the own attitude is changed in a way that it becomes similar to the perceived opinion. Usually the new information and the former attitude are weighted and the weights sum up to one (e.g. Deffuant *et al.* 2001, Hegselmann and Krause 2002). Then the weight is the impact an individual assigns to an opinion [5]. It can depend on the selective attention and exposure, hence on communication channel and communication partner as well as on the content of the message, and on the own attitude.

[4] The membership of an individual in a specific social network determines the frequency or likelihood of communication with other members of the same net, as well as the impact of the information transmitted within the social net (e.g. Deffuant 2001). An agent can be part of several social nets.

[5] The impact is not included as a parameter that will be varied during the analysis.

2.3 Biases in verbalisation process: limited precision

The biases in perceptual processes are subject to several simulations and are basic to consumer behaviour research. Biases in the process of verbalising attitudes, i.e. giving opinions, is less elaborated.

To build up a successful, i.e. efficient, communication the sender may take actions compensating the biases at receiver's side. To compensate the selective interpretation the sender may use exaggeration. To avoid negative consequences of selective exposure or selective attention the sender may shift its opinion intentionally towards the expected receivers opinion, like picking the receiver up where it is. Both, exaggeration and picking-up bias the verbalisation process but are too complex to be integrated into a simple opinion dynamics model.

The situation that sender and receiver share the same mind rarely occurs, hence both have to use a communication channel, which in information diffusion models is very often a symbolic or language based channel[6](e.g. Kroeber-Riel and Weinberg 1999). But human language applied by individuals usually covers only a limited number of expressions[7]; hence the precision of attitude descriptions is limited. The sender does not have simple mechanisms to avoid this.

There are some domains that make usage of attitude verbalisation. Marketing and sociology researchers frequently ask people for their opinion (e.g. Shet *et al.* 1999 and Kroeber-Riel and Weinberg 1999). If a perfect verbalisation, i.e. maximum precision, was possible, then the researchers would probably apply such mechanisms. Instead, they often employ scales that only have a few points (e.g. five or six points representing the range from "very bad" to "very good").

A restricted set of possible verbalisations cannot only be caused by the language limitations but also be caused by social norms. In a management meeting or for a court order it might not be acceptable to have more than two possible verbalisations (yes or no).

Because of language and social limitations the assumption of perfect verbalisation, as done in many articles on opinion dynamics, has to be relaxed. Based on the concepts presented in this section the next section develops a model of opinion dynamics, which in fact should be called attitude dynamics, that extends the basic model presented in Deffuant *et al.* (2001). Of course the basic model does not capture the state-of-the-art of opinion dynamics but it is a useful starting point to introduce a new concept as limited verbalisation capabilities. The basic model covers the concept of selective attention and assumes arbitrary one-to-one communication between the individuals. The extensions are related to selective interpretation, and more important, are related to biases in verbalisation processes.

[6]Diffusion models describe attitudes and opinions usually as a number (see for instance in Deffuant *et al.* 2001 and Deffuant 2001) and this number is communicated, thus a symbolic channel is used.

[7]Asking an individual for his or her opinion very common expressions are "very good", "good", "neutral" or "I do not know", bad, and "very bad", representing the continuum from a very strong tendency towards to a very weak tendency to react positively towards a given object.

3 Model

This section presents a model of attitude dynamics that captures the distinction of attitude and opinion, as well as the biases caused by selective attention, by selective interpretation, and by verbalisation restrictions. To keep the model simple concepts of selective exposure related to select sources of information and communication partners are not integrated. Finally, simulations should show how stable the properties derived by Deffuant *et al.* (2001) are against verbalisation biases.

The model is based on a set of interacting agents[8], i.e. a multi-agent system. There are N agents, each maintaining an attitude x_i. An attitude is a continuous variable between 0 and 1, inclusively. In each step of the simulation two arbitrary agents are selected[9]. These two agents communicate their opinions, more precisely they verbalise their attitudes. Both agents receive the other agent's opinion and may adopt this perceived opinion, thus modifying their current attitude x to create a new attitude x'.

Definition 1 *The attitude x'_j of agent j in the next step is changed on the base of the attitude x_j of the current step and the four communication-related processes that link agent j with agent i:*

$$x'_j = adopt_j\left(x_j, interpret_j\left(x_j, transmission\left(verbalise_i\left(x_i\right)\right)\right)\right)$$

The processes "interpret", verbalise", and "adopt" are within sender or receiver while the process "transmission" is part of the environment. As each process works on different inputs and produces different outputs these inputs and outputs are labelled and defined as follows.

Definition 2 *Assuming agent i communicating its attitude to agent j we define:*

$$\begin{aligned}
attitude\ x_i, x_j & \\
opinion\ o_i &= verbalise_i\left(x_i\right) \\
message\ m_i &= transmission\left(o_i\right) \\
perceived\ opinion\ \hat{o}_{i,j} &= interpret_j\left(x_j, m_i\right) \\
new\ attitude\ x'_j &= adopt_j\left(x_j, \hat{o}_{i,j}\right)
\end{aligned}$$

A prerequisite of communication about an attitude is verbalisation of the attitude, thus formulating the opinion. Here we assume that agents have the ability to mark their attitude on a scale from 0 to 1 with a given precision *prec*.

[8]The term agent refers to a formal representation of an individual. An analysis of agent paradigms can be found in Gilbert and Troitzsch (1999), Conte *et al.* (1998), and Burkhard (1995).

[9]Selecting communication partners arbitrary across the whole society is a strong assumption. But as we do not want to analyse the impact or the emergence of communication networks, this simplification emphasizes the focus on the impact of limited verbalisation capabilities.

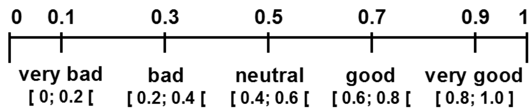


Figure 2: Values of opinion for a precision $prec = 5$.

The usage of a scale for verbalisation of an attitude implies that for each point on the scale, referred to as a *scale point*, there is a related interval of attitudes that are verbalised by this scale point. This interval is referred to as the *scale point interval*. Figure 2 exemplifies scale points and their related scale point intervals for a precision of 5. Except for the highest interval every interval includes its lower limit and excludes its higher limit. The highest interval includes both, higher and lower limit. Definition 3 formalises this concept.

Definition 3 *The verbalisation process is given by*[10]*:*

$$o_i = verbalise_i\left(x_i\right) = \begin{cases} \frac{\lfloor x_i \cdot prec_i \rfloor + 0.5}{prec_i} & if \quad x_i < 1 \\ 1 - \frac{1}{2 \cdot prec_i} & if \quad x_i = 1 \end{cases}$$

The communication channel is assumed to be perfect, in fact it is assumed that the receiver gets the message, as it is sent. Therefore, the transmission process is the identity.

Definition 4 *The transmission process does not bias the message, hence the process is given by identity:* $m_i = transmission\left(o_i\right) = o_i$

A message from agent i received by agent j has to be interpreted. This may cause selective interpretation, which is a bias that leads to an interpretation that is closer to the receiver's attitude than the sender has intended to be. As this bias is absolute, there are some messages that are interpreted to represent an opinion that is identical to the own agent's attitude.

Definition 5 *The perceived opinion* $\hat{o}_{i,j}$ *the agent* j *gets from the interpretation process depends on the received message* m_i *from agent* i *that is biased by selective interpretation* $seli_j$ *of agent* j*:*

$$\hat{o}_{i,j} = interpret_j\left(x_j, m_i\right) = \begin{cases} m_i - seli_j & if \quad m_i > x_j + seli_j \\ m_i + seli_j & if \quad m_i < x_j - seli_j \\ x_j & if \quad \left|m_i - x_j\right| \leqslant seli_j \end{cases}$$

When an agent has interpreted a message as the other agent's opinion it can adopt the perceived opinion. An adoption causes a change in the agent's attitude towards the perceived opinion. Selective attention causes a rejection of perceived opinions that are

[10] $\lfloor x \rfloor$ means that x is rounded down while $\lceil x \rceil$ means that x is rounded up to the next integer value.

not sufficiently close to the own attitude, thus it prevents the adoption. This is modelled via a threshold $sela_j$. The actual impact of the adoption on the the attitude is controlled by the impact (imp_j). The higher the impact the more does the attitude change towards the perceived opinion. The adoption process is defined similar to Deffuant *et al.* (2001).

Definition 6 *An adoption takes place with the impact (imp) if the perceived opinion is not too far from the own attitude.*

$$x'_j = adopt_j\left(x_j, \hat{o}_{i,j}\right) = \begin{cases} x_j + imp_j\left(x_j - \hat{o}_{i,j}\right) & if \quad |x_i - \hat{o}_{i,j}| \leqslant sela_j \\ x_j & if \quad |x_i - \hat{o}_{i,j}| > sela_j \end{cases}$$

4 Model Analysis

This section analyses properties of the model presented in the previous section. To derive some hypotheses we introduce the concept of *area of influential attitudes* and how this is influenced by the model's parameters. To demonstrate the mechanisms we first reproduce results obtained by other authors; then we derive hypotheses about the effects of biases in the verbalisation process. Simulations are used to test these hypotheses. Of course we cannot evaluate all properties of the model, but we will show some important features mainly derived from an analysis keeping the other parameters constant.

The simulations used in this work contain one or more groups of agents. Agents in one group differ only in their initial attitude. A group is described by (*num*, *seli*, *sela*, *imp*, *prec*, *min*, *max*), with *num* as the number of agents in the group. Parameters *seli*, *sela*, *imp*, and *prec* are defined as described in the model above. The initial attitudes of agents in a group are equally distributed along the interval given by *min* and *max*.[11].

4.1 Area of influential attitudes

The basic dynamic elements of the presented model are the communications between the agents. The attitude formation depends on the fact whether the agent adopts the perceived opinion or not. The more attitudes can influence an agent, the higher is the probability of changes of its own attitude. And the smaller the range of adopted attitudes is the smaller the range of the possible post-negotiation attitude.

To make the argumentation on the amount and value of influential attitudes more explicit we introduce the concept of *area of influential attitudes*, which covers all attitudes that may influence an agent. The area of influential attitudes depends on the agent's attitude

[11]For example, {{*8, 0.0, 0.5, 0.5, 5, 0, 1*},{*2, 0.1, 0.5, 0.5, 5, 0.5, 0.7*}} describes a multi-agent system with all ten agents having selective attention of 0.5 and an impact of 0.5. They can verbalise their attitudes on a five-point scale. The eight agents in the first group have no selective interpretation, while the two in the second group have that with a value of 0.1. The initial attitudes of the eight agents in the first group are given as 0, $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$, and 1. The two agents from the second group get the initial attitudes 0.5 and 0.7.

x, on its properties, i.e. selective attention and selective interpretation, and on the other agent's precision while verbalising its attitude. If an agent has the maximum selective attention ($sela = 1$), thus being very open-minded and not ignoring any other agent, and if the agent has no bias caused by selective interpretation ($seli = 0$) and if the precisions of the other agents are maximal ($prec \to \infty$) then the area of influential attitudes covers all possible attitudes from 0 to 1. Of course the area of influential attitudes is not equal to the number of influential attitudes, but for our analysis it has proven to be a useful tool[12]. The next paragraphs show how selective attention and selective interpretation influence the area of influential attitudes; it partly reproduces well-known results. The impact of precision is analysed in another section below.

Selective attention. Figure 3(a) marks the area of attitudes that are adopted under the assumption of no selective interpretation ($seli = 0$) and a positive selective attention ($sela > 0$). In contrast to attitude a_1, attitude a_2 is not adopted because it differs too much from the agent's own attitude x. The higher the selective attention the larger is the area of influential attitudes.

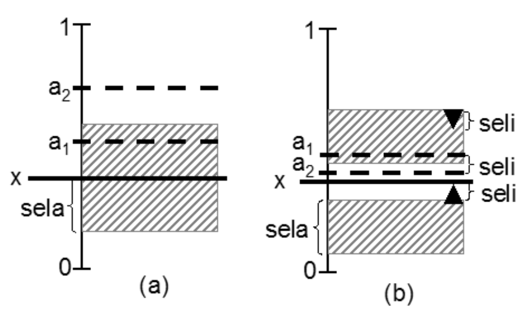


Figure 3: Area influential attitudes: (a) Due to selective attention an agent having the attitude x adopts a_1 but ignores a_2. (b) Due to selective interpretation an agent having the attitude x perceives the original a_1 as a_2 and the original a_2 as being equal to x. Both, a_1 and a_2 are adopted.

Selective Interpretation. Figure 3(b) demonstrates the case with positive selective interpretation ($seli > 0$) and the same selective attention as in Figure 3(a). It can be seen that the area is expanded compared with case (a) but close to the own attitude x there is a white hole. This white area marks all attitudes that after verbalising and transmitting are interpreted as representing the own attitude, thus causing no change in the own attitude. The arrows express the fact that all influential attitudes are biased during their verbalisation, transmission, and interpretation. Here for instance, attitude a_1 is interpreted as attitude a_2, while a_2 is treated as equal.

Precision. Consider one agent's attitude, the impact of precision on the area of influential attitudes can be twofold. First, if a scale point is in the area of influential attitudes

[12]We can use the simple tool of area of influential attitudes only because we exclude every concept that distinguishes agents by other attributes than the verbalised attitude.

and the related scale point interval is partly outside, then the area is expanded. This is due to the fact that the attitudes that are in the scale point interval but outside the original area of influential attitudes are verbalised as being within the area. Second, if the scale point interval is partly inside the area but the scale point is outside then the area of influential attitudes is cut. If both, scale point and the related scale point interval, are outside or both are within the original area of influential attitudes the area is not influenced by this scale point. Figure 4 exemplifies the expansion as well as the reduction; in this case the overall effect is an expansion of the area of influential attitudes leading to the known effects on fragmentation. Because these effects depend on the attitude an agent has, the impact of scale points on the area of influential attitudes can be different for several agents. The arrows in Figure 4 mark the fact that, as in the case of selective interpretation, the influential attitudes are biased while they are verbalised, transmitted, and interpreted; here they are biased by verbalisation, they are set to the corresponding scale point.

The next subsections apply the concept of area of influential attitudes to derive dynamic properties of the model presented in this article.

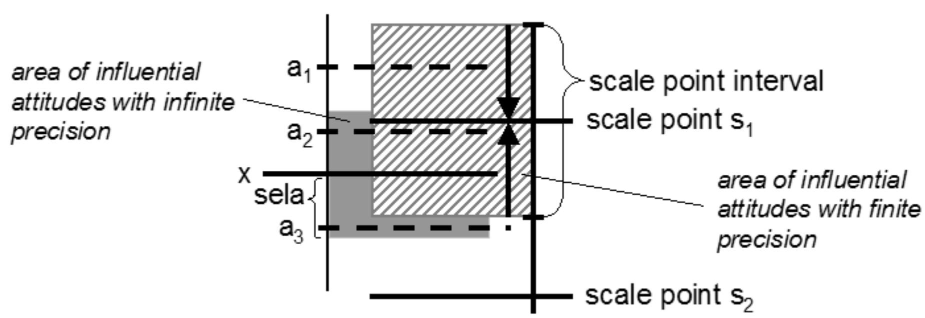


Figure 4: Area of influential attitudes and precision: Due to the finite precision an agent having the attitude x perceives attitude a_1 and a_2 as s_1 and a_3 as s_2. Contrary to infinite precision a_1 now influences the agent and a_3 does not.

4.2 Selective attention

After introducing the idea of the area of influential attitudes we will analyse the patterns of the system's dynamics concerning changes of selective attention keeping selective interpretation zero ($seli = 0$) and having the same selective attention and impact rate imp for all agents. Then we weaken the assumption and allow different selective attentions for different agents.

A communication between two agents, which leads to an adoption of a perceived opinion by at least one agent, results in an attitude configuration where both agents' attitudes are closer to each other than before. In combination with an upper and lower limit of the range of attitudes and many agents an infinite number of randomly chosen communications will lead to a stable configuration. Such a *stable configuration* is characterised by the fact that for each communication in this configuration either both attitudes are perceived as being equal or an adoption is prevented by the selective attention.

Within a stable configuration one can identify clusters. *Clusters* are disjunctive sets of agents such that each agent has only agents in its own set whose perceived opinion is not ignored due to a too big difference to the own attitude. In other words, all agents that have attitudes in the area of influential attitudes of another agent are in the same cluster as this agent.

Homogeneous societies. For homogeneous societies with agents that only differ in their initial attitude, the description of a stable configuration gives the maximum number of clusters of attitudes as in proposition 1, i.e. how many different attitudes are present in the stable configuration. Proposition 1 utilises the fact that between two clusters there has to be at least a difference of $sela + \varepsilon$, else an agent from one cluster can adopt a perceived opinion from another cluster, which then means that the clusters by definition have not been clusters[13].

> **Proposition 1** *For many agents ($N \to \infty$), assuming for every agent an infinite precision and no selective interpretation ($seli = 0$) the maximal number of clusters of attitudes is $\lceil \frac{1}{sela} \rceil$. The expected number of clusters of attitudes is $\lfloor \frac{1}{2 \cdot sela} \rfloor$.*

The smaller the selective attention is, the smaller is the area of influential attitudes and the less adoptions can take place. Thus more clusters of attitudes can evolve. The number of attitudes actually influencing an agent depends on the initial distribution and on the sequence of communications. To reach the maximum number it needs very specific communication patterns and specific configurations of initial attitudes[14]. For a random distribution and random patterns of communication Deffuant *et al.* (2001) derive the mean number of clusters as in proposition 1. The occasion of getting the maximum number is less likely the more agents are in the system.

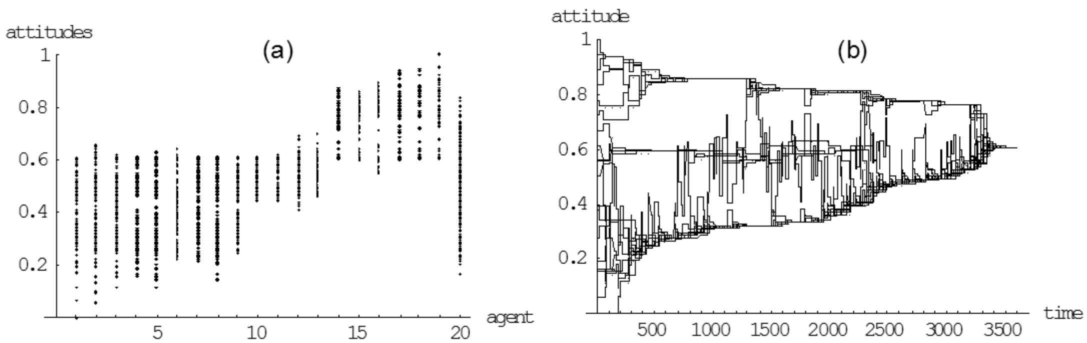
Heterogeneous societies. In a heterogeneous scenario, when agents have different selective attentions, one can observe pseudo stable configurations. *Pseudo stable configurations* are configurations that have a large subset of agents that is stable. This means that only very few agents are the reason for further dynamics. Therefore the dynamics are slowly and seldom. If the number of agents with high values of selective attentions is relatively small, then in the short run the low values determine the clustering, but in the long run it collapses to the clusters according to the high values of selective attention. Figure 5(b) shows the case of a heterogeneous society. One open-minded agent influences

[13] The symbol ε is a very small value that has to be integrated because the area of influential attitudes includes its limits.

[14] For example, the initial configuration of two relatively open-minded agents ($sela = 0.9$) that have the extreme initial attitudes 0.05 and 0.95 leads to two stable clusters, which is above the expected number of clusters (1).

the narrow-minded agents such that they step-by-step get closer and finally collapse into one attitude. Figure 5(a) shows that the open-minded agent (agent 20) has the widest range of attitudes adopted during the simulation. Deffuant *et al.* 2000, Weisbuch *et al.* 2001 and Hegselmann and Krause 2002 obtain similar results.

Proposition 2 *For many agents ($N \to \infty$), assuming for every agent an infinite precision and no selective interpretation ($seli = 0$) as well as heterogeneous selective attentions ($sela_i \neq sela_k > 0$) the maximal number of clusters of attitudes is in the long run $\left\lceil 1/\max_i (sela_i) \right\rceil$. The expected number of clusters of attitudes is $\left\lfloor 1/\left(2 \cdot \max_i (sela_i)\right) \right\rfloor$.*



{{19, 0.0, 0.16, 0.4, 1000, 0.0, 1.0}, {1, 0.0, 0.42, 0.4, 1000, 0.5, 0.5}}

Figure 5: Dynamics with heterogeneous selective attentions: (a) shows all attitudes each of the 20 agents adopts during the simulation run, while (b) shows the sequential simulation run. Contrary to agents 1 to 19 that are narrow-minded with an equally small selective attention, agent 20 is open-minded, i.e. has a higher selective attention.

4.3 Selective Interpretation

The results related to the selective attention are common ground in articles about opinion dynamics, but what about selective interpretation. As shown above selective interpretation modifies the area of influential attitudes and as we have seen this area significantly influences the fragmentation of attitudes. Because a positive selective interpretation expands the area of influential attitudes one gets a smaller expected number and also a smaller maximum number of clusters. Because attitudes that differ slightly from the own attitude are perceived as being equal one can also expect that the clusters do not collapse into one value but cover a range of attitudes. The range, referred to as the *cluster range*, depends on the value of selective interpretation and is given as $2 \cdot seli$. The proposition 1 can be extended as in proposition 3.

Proposition 3 *For many agents ($N \to \infty$), assuming for every agent an infinite precision and a positive selective interpretation ($seli > 0$) the maximal number of clusters of attitudes is $\left\lceil \frac{1}{seli+sela} \right\rceil$. The expected number of clusters is $\left\lfloor \frac{1}{2\cdot(sela+seli)} \right\rfloor$.*

Figures 6(c) and (d) present simulation runs that show the predicted behaviour. The clusters do not converge completely. In (c) one can see a tendency to fewer clusters; only by chance one agent survived as a third cluster. The expansion of the area of influential attitudes should also increase the likelihood of adoption and hence decreasing the time until stable configurations emerge. This is easy to see by comparing the time axes of (b) and (d).

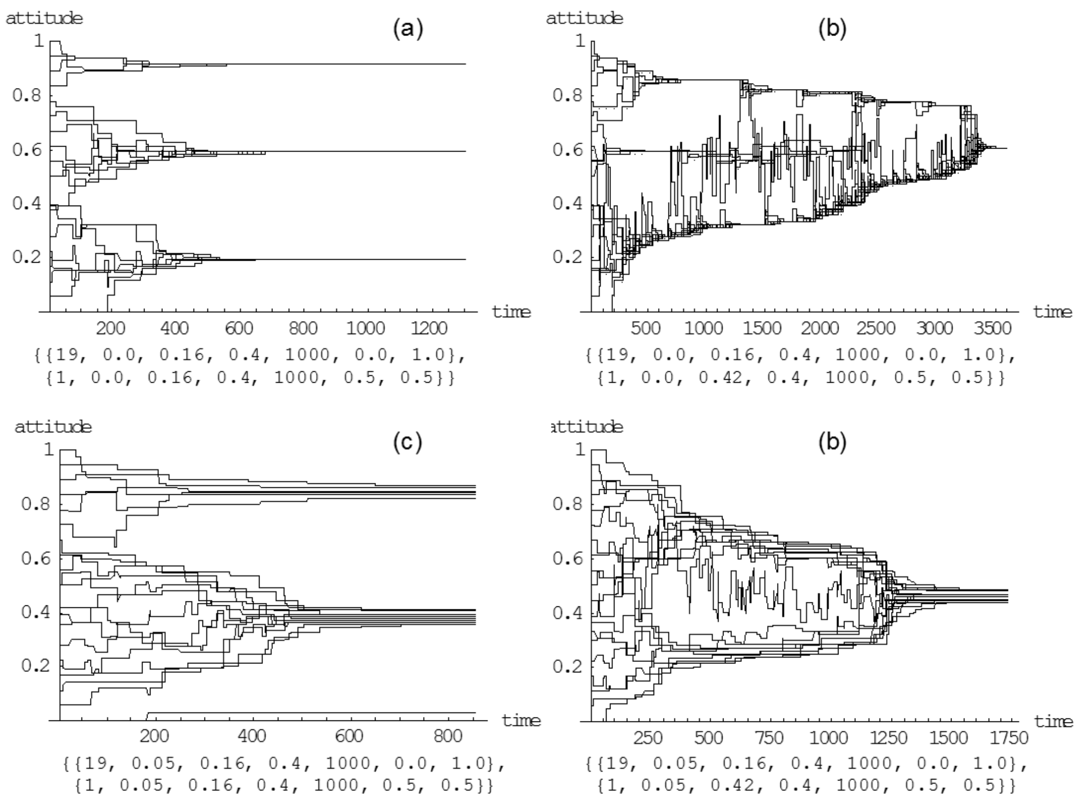


Figure 6: Zero and positive selective interpretation with homogeneous and heterogeneous selective attentions: (a) is a simulation run with 20 agents that only differ in their initial attitude, but all have no selective interpretation, a selective attention of 0.16, and an impact rate of 0.4. Simulation run (b) is the same as (a) but with one agent being open-minded, i.e. having a selective attention of 0.4 (see also Figure 5). Simulation run (c) is the same as (a) but with a selective interpretation of 0.05. Simulation run (d) changes (c) such that there is one open-minded agent as in (b).

4.4 Precision

This section will analyse the effect of different precisions while agents verbalise their attitudes. For simplification the analysis is restricted to cases with no selective interpretation, but based on the insights about the impact of selective interpretation on the area of influential attitudes it is easily extendable.

As agents are forced to verbalise their attitudes as opinions and both do not have to be identical, an agent might adopt a perceived opinion that differs more from the own attitude than the other agent's attitude actually does. This can lead to the over adoption which has occurred when two agents communicate and an agents pre-communication attitude x_i shifts over the attitude a_j of another agent to the post-communication attitude x_i'. This effect is specific for biases in verbalisation processes. An example can be seen in Figure 7(d). The same figure also shows a consequence of over adoption, there are configurations that result in final attitudes for all agents that are above the highest initial attitude.

> **Proposition 4** *For a finite precision ($prec < \infty$) the initial order of attitudes of agents involved in a communication can be inversed after the communication.*

For the whole agent society the effects of the verbalisation restriction is difficult to analyse. This is mainly due to the strong dependency on the initial attitude distribution and on the communication sequence. For simplification we distinguish three important scenarios and assume an equal distribution of attitudes among many agents. Additionally we give the attitude distribution leading to the most extreme effect, i.e. the maximum number of clusters[15].

Scenario I: Low precision. The first scenario represents the cases such that at the beginning each agent has at most one scale point in its area of influential attitudes. This can be represented by the condition $prec \leqslant \lfloor 1/(2 \cdot sela) \rfloor$.

If the scale has only a small number of scale points, then the smaller the selective attention is the more agents experience a cut of their area of influential attitudes. This leads to a high number of clusters. In fact, there are agents in this scenario between two scale points that have an empty area of influential attitudes. These agents will not change their attitude at all, even if they talk with an agent having an attitude very close to their own. The other agent is forced to verbalise its attitude as a scale point far away. Therefore the maximal number of clusters equals the number of agents, especially if all agents have an initial attitude outside the areas that are set by scale point plus/minus selective attention around each scale point. To get the average number we take into account that in average $prec \cdot \lfloor 2 \cdot sela \cdot N \rfloor$ agents lie within such areas and build clusters according to the scale points. The other agents are not influenced, hence being a cluster itself (see proposition 5).

[15]The minimum number of clusters is 1 for all cases, especially in the case of all agents having the same initial attitude.

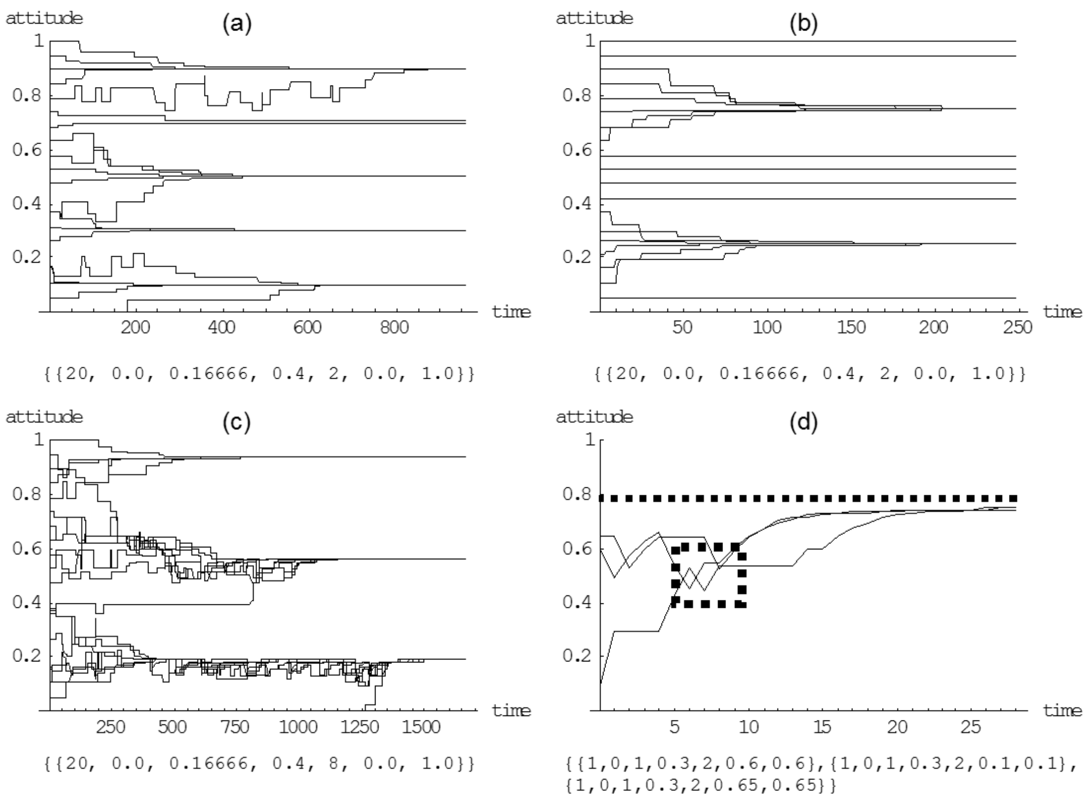


Figure 7: (a), (b), and (c) represent the three important scenarios for the analysis of the impact of precision: low, medium, and high precision. The setting of all three simulation runs is: 20 agents, no selective interpretation, a medium selective attention of 0.16666, and an impact rate of 0.4. Precision *prec* is 2 in (a), 5 in (b), and 8 in (c). Simulation run (d) exemplifies over adoption.

In scenario I every agent has at most one scale point in its area of influential attitudes. Therefore an agent changes its attitude in one and only one direction or it does not change it at all. This leads to an easy way to calculate all attitudes an agent might adopt during a simulation run. An agent can only form attitudes that are in the range between its initial attitude and its attitude in the final stable configuration. The latter is given by the scale point it has in its area of influential attitudes. Proposition 5 summarises the results for scenario I.

Proposition 5 *For many agents ($N \to \infty$) assuming a low precision ($prec \leqslant \lfloor 1/(2 \cdot sela) \rfloor$), no selective interpretation ($seli = 0$), and a positive selective attitude ($sela > 0$):*

- *The maximal number of clusters of attitudes is N.*
- *The expected number of clusters is $prec + N - prec \cdot \lfloor 2 \cdot sela \cdot N \rfloor$.*
- *For every step from x to x' the difference $x - x'$ has the same sign or equals zero (shift the attitude only in one direction).*
- *For every scale point s the difference $s - x$ has for every simulation step the same sign or equals zero (no crossing of scale points).*
- *If $x - o(x) > sela$, with x as the attitude of an agent and $o(x)$ as its verbalised attitude, then the agent will never adopt a perceived opinion (some agents might never adopt another attitude).*
- *If $x - o(x) \leqslant sela$, with x as the attitude of an agent and $o(x)$ as its verbalised attitude, then the attitudes the agent adopts during the simulation are within the interval $[x, o(x)]$.*

Figure 7(a) shows a simulation run that fits to scenario I. Proposition 5 predicts a number of ten clusters, which is exactly the number as reached in simulation[16]. One can also see that every agent moves only into one direction.

Scenario II: Medium precision. Due to a very low precision in the first scenario every agent has at most one scale point in its area of influential attitudes. Inequality $\lfloor 1/(2 \cdot sela) \rfloor < prec \leqslant \lfloor 1/sela \rfloor$ is the assumption of the second scenario, which represents the fact that every agent has at least one and at most two scale points in its area of influential attitudes. Additionally, if an agent has an attitude close to a scale point, it has exactly one scale point in its area of influential attitudes. The meaning of close is determined by being less than $sela - 1/(2 \cdot prec)$. If this difference becomes very small then the probability of coming sufficiently close to a scale point is very small, hence the probability of many agents with oscillating attitudes increases. As the number of oscillating agents increases when selective attention increases or precision decreases such changes increase the time taken until a stable configuration may be reached. As we will see later this is not the only reason why a convergence might be delayed.

Because every agent has a scale point in its area of influential attitudes and for many agents with equally distributed initial attitudes there are agents for each scale point that have only this one in their area of influential attitudes the maximum number of clusters

[16] In Figure 7(a) there is one data line identical with the time axis.

and the expected number is the same and equals *prec*. The simulation run presented in Figure 7(b) supports this. The same figure also shows that agents can move into two directions but do not cross a scale point.

Proposition 6 *For many agents ($N \rightarrow \infty$), assuming a medium precision ($\lfloor 1/(2 \cdot sela_i) \rfloor < prec < \lfloor 1/sela_i \rfloor$), no selective interpretation ($seli = 0$), and a positive selective attitude ($sela > 0$):*

- *The maximal number of clusters of attitudes as well as the expected number of clusters is given by the precision.*
- *For every simulation step the difference of any scale point and an agent's attitude has the same sign or equals zero. If it equals zero then this holds forever (no crossing of scale points).*
- *If the area of influential attitudes of the agent's initial attitude covers only one scale point then the range of possibly formed attitudes of this agent is between its initial attitude and its verbalisation.*
- *If the area of influential attitudes of the agent's initial attitude covers two scale points s_1 and s_2 ($s_1 < s_2$) and it converges to s_j with $j \in \{1, 2\}$ then the range of possibly adopted attitudes is given by the following interval* $[\min(s_j, s_2 - sela), \max(s_j, s_1 + sela)]$.

Scenario III: High precision. The third scenario assumes a high precision, thus it includes the case with infinite precision. Because the area of influential attitudes can cover many scale points, in fact at least two (even if it is at a scale point), an agent's attitude changes do not have to have a unique direction. The same reason leads to the occasion that agents' attitudes can cross scale points, which is intuitive, especially in the case of infinite precision. Therefore, the range of attitudes that can be adopted by an agent during a simulation run cannot easily be determined based on selective attention, impact, and selective interpretation, as can be done in scenario I and II.

As this scenario covers the case of infinite precision and the proposition 1 only depends on a sufficient high precision, the mean and maximum number of clusters is the same as in proposition 1 (see Figure 7(c)). Simulations have demonstrated that this result depends heavily on the fact that there are many agents with initial attitudes that are equally distributed over the range of possible attitudes. The smaller the number of agents is, the higher the probability of getting more than the predicted number of clusters.

Proposition 7 *For many agents ($N \rightarrow \infty$), assuming high precision $\lfloor 1/sela \rfloor \leqslant prec < \infty$, no selective interpretation ($seli = 0$), and a positive selective attitude ($sela > 0$), proposition 1 holds.*

In scenarios II and III attitude changes do not have a unique direction. Together with the effect of over adoption one can observe a cyclic over adoption. A cyclic over adoption for two agents describes the following effect. Two agents communicate their pre-communication attitudes and adopt the perceived opinions. Due to limited precision over adoption happens. Although the agents do not adopt the other agent's opinion completely they over adopt due to verbalisation biases. This leads to a post-communication attitude that is similar to the partner's pre-communication attitude. If this over adoption happens for both agents the agents switch their positions. Every next communication does the same. Figure 8 shows a simulation run exemplifying cyclic over adoption. The cyclic over adoption is not restricted to two agents. Similar effects can be observed with many agents. Cyclic over adoption can prevent a stable configuration.

Because the parameter impact (*imp*) determines the degree of adoption, i.e. the difference that an agent moves towards the other agent, the cyclic over adoption effect also depends on this parameter. The area that is crossed during cyclic over adoption decreases while the impact becomes smaller.

Agents participating in a cyclic over adoption express different opinions along the time. Therefore such a specific configuration can prevent or delay stable configurations and they can be pseudo stable for ever.

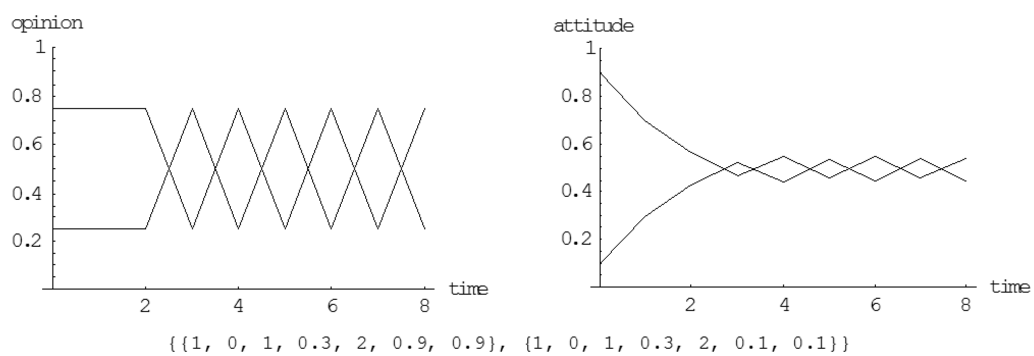


Figure 8: Cyclic over adoption: Simulation run with two agents, both without selective interpretation, both adopting every perceived opinion, both assigning an impact of 0.3 to perceived opinions, but each with another initial attitude.

4.5 Precision and the number of clusters

The presented results for different precisions show that there is no one single rule for the number of clusters one can expect, even if the number of agents is assumed to be very high and if the initial attitudes are equally distributed. Figure 9 shows approximately how the number of clusters depends on the precision.

4.6 Precision and selective interpretation

Although this article only analyses the effects of changes in a single model parameter Figure 10 shows some simulation runs with positive selective interpretation, small selective attention and several degrees of precision.

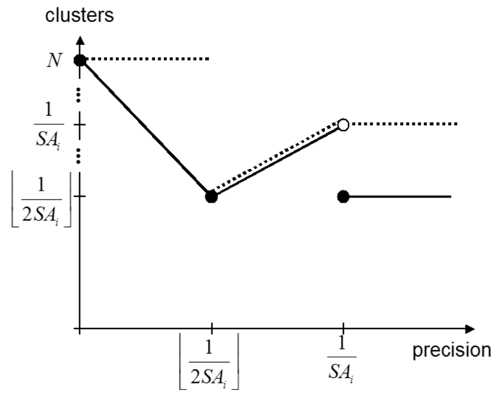


Figure 9: Number of clusters depending on precision for an infinite number of agents with equally distributed initial attitudes. The dotted line marks the maximal number of clusters in a stable configuration.

Figures 10(a) and (b) demonstrate a behaviour that is compatible with previous findings. Low precision leads to the expected number of clusters but the range of the clusters is influenced by selective interpretation. In 10(c) one can see that some attitudes oscillate between two scale points. The positive selective interpretation prevents a complete convergence to the scale points and it biases the scale points towards the agent's attitude. Hence, in this case, the agents continuously have two scale points in their area of influential attitudes, thus oscillating between both scale points. In 10(d) one can see a simulation run that shows another interesting characteristic. No agent adopts a scale point as its attitude, but their attitudes converge at the limits of the range of attitudes, which they had perceived as equal if they would adopt the scale point. The simulation run presented in Figure 10(e) seems to combine these effects. Simulation runs with a precision of at least $\lceil 1/sela \rceil$ show the same patterns as with an infinite precision, as presented in Figure 10 (f).

5 Conclusion

The previous sections have presented a model and its analysis that captures communication-related concepts from consumer behaviour literature, i.e. selective interpretation, selective attention and attitude verbalisation processes. In consumer behaviour literature the attitude verbalisation process is not deeply anticipated and in opinion dynamic models its is rarely addressed. But the model analysis has shown that especially for the case of medium precision the impact on opinion dynamics is significant and should be explored more detailed. Because opinion leaders and multipliers, e.g. mass media and consulting institutions, can choose the scale that they express their attitudes on, the results of this article are interesting for practitioners, too.

Scenario II, which assumes a precision such that at least one but at most two scale points are in the area of influential attitudes, seems to be very interesting. This scenario

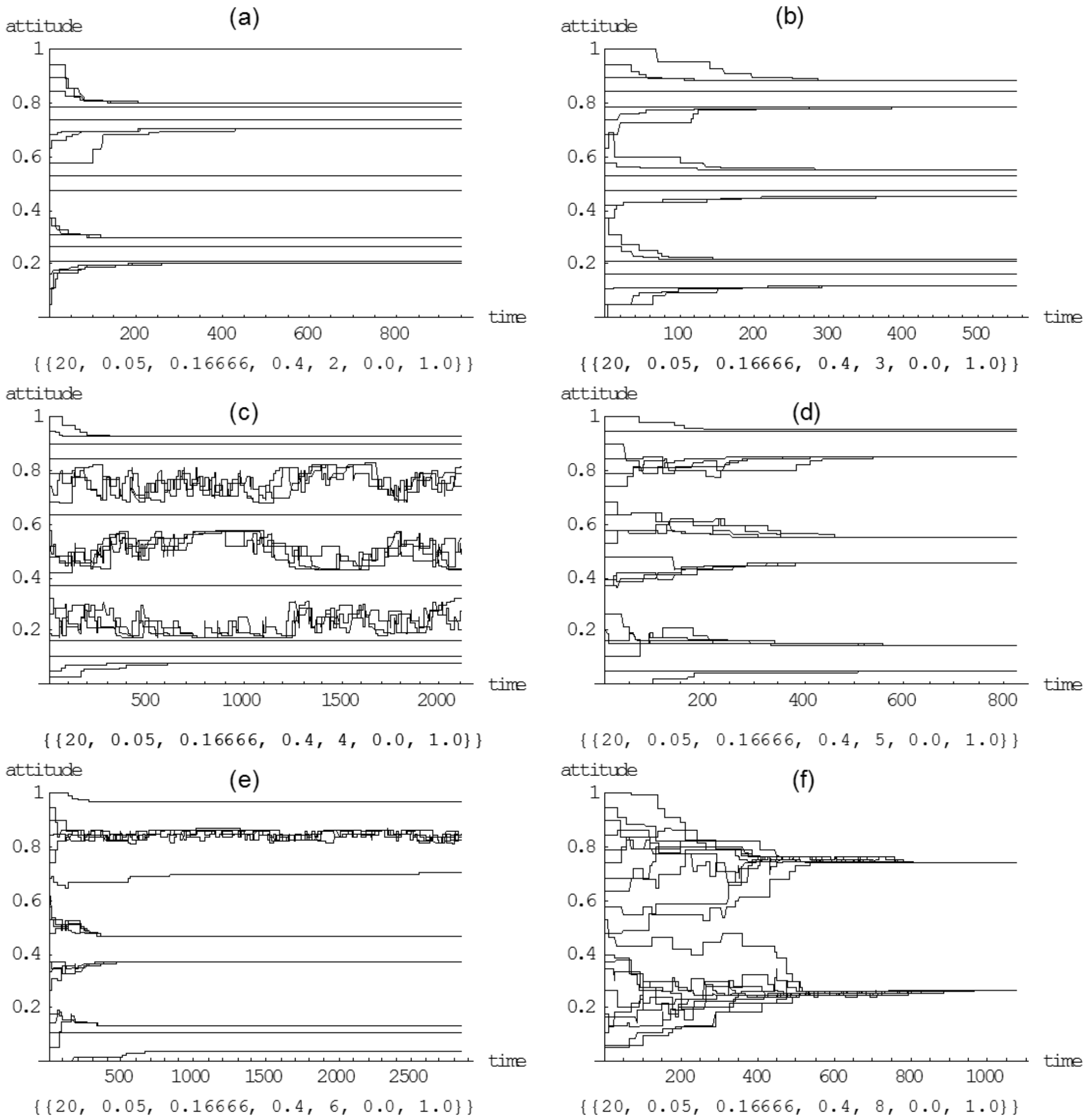


Figure 10: Limited precision and selective interpretation: Each simulation run is with 20 agents having a selective interpretation of 0.05, a selective attention of 0.16, and an impact of perceived opinions of 0.4. The simulations runs differ in the value of precision: (a) $prec = 2$, (b) $prec = 3$, (c) $prec = 4$, (d) $prec = 5$, (e) $prec = 6$, and (f) $prec = 8$.

shows patterns that contradict the findings of Deffuant *et al.* (2001) and Weisbuch *et al.* (2001). The number of clusters is significantly higher than predicted by those authors. Additionally one can see that in combination with selective interpretation medium precision shows complex fragmentation patters (see Figures 10 (c)-(f)), which base on the over adoption effect and the resulting cyclic over adoption.

Cyclic over adoption as demonstrated in Figure 8 does not seem to be intuitive for human behaviour. Individuals may develop mechanisms that prevent such cyclic behaviour. These mechanisms should be explored in more detail.

The analysis in this article only investigates the isolated impact of one model parameter but does not analyse the combination of them. Figure 10 shows by example simulations that complex dynamic patterns can arise if selective interpretation and limited precision is coupled. After these first impressions and results that justify additional effort, one should further improve the model and the tools of analysis, i.e. area of influential attitudes.

If the model and analysis is mature then one can extend the model to capture more concepts that are relevant for marketing research, for instance Mowen and Minor (1998) state that consumers overemphasise negative signals in mouth-to-mouth communication. Also strategic biases by the sender, that are intended to prevent biases by the receiver, may be integrated into the model.

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