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INTEREST RATE RISK MANAGEMENT - CALCULATING VALUE AT RISK USING EWMA AND GARCH MODELS

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1. The use of Value at Risk in the context of banking supervision

Value at Risk (VaR) has become the standard measure that financial analysts use to quantify market risk. The popularity that this instrument has achieved is due to its conceptual simplicity: VaR reduces the risk of any portfolio to just one number, the loss associated to a given probability. Certainly thise advantage permit one to take into account various kinds of cross-dependence between asset returns, fat-tail and non-normality effects. This make it to become a standard risk measure for banks, insurance companies, institutional investors and non-financial enterprises.

Since Value-at-Risk received its first wide representation in July 1993 in the Group of Thirty report, the numbers of users of Value-at-Risk have increased dramatically. Also the Value-at-Risk technique has gone through significant refinement it originally appeared. Theoretical research that relied on the Value-at-Risk as a risk measurement was initiated by Jorion (1997), Dowd (1998), and Saunders (1999). Proponents of VaR believe it will replace or at least complement less standardized techniques such as Asset and Liability Management and Stress testing. They also hoped that regulators, auditors, shareholders and management, will finally be speaking a common language with respect to risk.

VaR measures can have many applications, and is used both for risk management and for regulatory purposes. In particular, the Basel Committee on Banking Supervision (1996) at the Bank for International Settlements imposes to financial institutions to meet capital requirements based on VaR estimates. Providing accurate estimates is of crucial importance. If the underlying risk is not properly estimated, this may lead to a sub-optimal capital allocation with consequences on the profitability and the financial stability of the institutions. It it is also used by institutions in portfolio optimisation techniques to actively manage their risk. Regulators expect social benefits assuming that Value-at-Risk based risk management will reduce the likelihood of large-scale financial failures.

VaR is defined as the expected minimum loss of a portfolio over some time period for some level of probability. From a statistical point of view, VaR entails the estimation of a quantile of the distribution of returns. In other words, Value at Risk is the probability that returns or losses (ξ) are smaller than –VaR over a period of time (T):

$$P_{VaR} = P(\xi < -VaR) = \int_{-\infty}^{-VaR} P_T \cdot \xi \cdot d\xi ,$$

where P_T is the probability distribution of returns over the time horizon T.

Imputs to a VaR model include data on the bank's on and off balance sheet positions and on respective interest rates, exchange rates, equity and commodity positions. The measurement parameters include a historical time horizon, a holding period and a confidence interval that allows for prudend judgement of the optimal level of protection. The Basel Committee recommends a holding period of 10 days trading, an historical observation period of a minimum of one year and a 99th percentile confidence level.

2. The Value at Risk methodology

While VaR is an easy and intuitive concept, its measurement is a very challenging statistical problem. Although the existing models for calculating VaR employ different methodologies, they all follow a common general structure, which can be summarised in three points: mark-to-market the portfolio, estimate the distribution of portfolio returns, and compute the VaR of the portfolio.

The main differences among VaR methods are related to the way they address the problem of how to estimate the possible changes in the value of the portfolio. When the returns are normal, VaR is equivalent to using the variance as a risk measure. When risk is sensitive to rare events and extreme losses, we can build models based on simulation of VaR. When risks are recurrent, VaR can be estimated by using historical time series while for new situations, scenarios simulation or the construction of the theoretical models are needed. The existing models can be clasified into three categories:

- Nonparametric (Historical Simulation, Monte Carlo, Stress Scenarios);
- Parametric (RiskMetrics and GARCH);
- Semiparametric (Extreme Value Theory, CAViaR and quasi-maximum likelihood GARCH).

Non-parametric models are simulation or historical models. Among simulation approaches, we distinguish between full valuation and partial valuation models. A full simulation approach (Monte Carlo VaR, Historical Simulation, Stress Scenarios) creates a number of scenarios for the risk factors and then, for each scenario, performs a complete revaluation of the portfolio, thus giving the profit-loss distribution of the portfolio. A partial valuation approach uses simulations to create the distribution of risk factors but does not fully revalue the portfolio. Instead, it makes use of delta or delta-gamma approximations to obtain the portfolio value. The VaR is set equal to the percentile of the observed daily return distribution at the required level of confidence. The main drawback of these approaches is that extreme percentiles are difficult to estimate precisely without a large sample of historical data.

Parametric models such as delta-normal are based on statistical parameters such as the mean and the standard deviation of the risk factor distribution. Using these parameters and the delta of the position, VaR is calculated directly from the risk factor distribution. Models such as RiskMetrics (1996) and GARCH propose a specific parameterisation for the behaviour of prices. The main advantage of these methods is that they allow a complete characterisation of the distribution of returns and there may be space for improving their performance by avoiding the normality assumption.

Recently, alternative methods have been proposed to estimate Value at Risk, such as applications of Extreme Value Theory (Danielsson and deVries (1998) or Gourieroux and Jasak (1998)) and applications of regression quantile technique such as in Chernozhukov and Umantsev (2000) and Engle and Manganelli (1999). Extreme Value Theory seems to be a very general approach to tail estimation. The main strength is that the use of a GEV distribution to parameterise the tail doesn't seem to be a very restrictive assumption, as it covers most of the commonly used distributions. The

Conditional Autoregressive Value at Risk, or CAViaR model was introduced by Engle and Manganelli (1999) and models directly the evolution of the quantile over time, rather than the whole distribution of the portfolio.

The quality of a VaR model depends on its distributional assumption about the market risk factors and its valuation model. The empirical facts about financial markets are very well known, since the pioneering works of Mandelbrot (1963) and Fama (1965). They can be summarised as follows:

- the distribution of financial variables is leptokurtotic (it has heavier tails and a higher peak than a normal distribution);
- equity returns are typically negatively skewed;
- squared returns have significant autocorrelation, volatilities of market factors tend to cluster. This is a very important characteristic of financial returns, since it allows the researcher to consider market volatilities as quasi-stable, changing in the long run, but stable in the short period. Most of the VaR models make use of this quasi-stability to evaluate market risk.

3. The stochastic approaches used in modeling financial variables

The most used stochastic model to describe the evolution of the equity prices, commodity prices and exchange rates is *the Geometric Brownian Motion*. This model is going to outline how the risk factors will move across time. Also known as the Black Scholes Model with Zero Drift, this could be described as a random walk in continuous time. The movements in prices from one day to another are represented as a series of returns. The distribution of these continuously compounded returns at the end of any finite time interval is a Log Normal distribution. The evolution of the prices could be described as follows:

$$\frac{dP}{P} = \mu \cdot dt + \sigma \cdot dw,$$

where μ is the expected return, σ is the volatility and t is the time; *dw* is a Wiener process which can be described as follows $dw = \varphi(dt)^{\frac{1}{2}}$, where φ is a random variable with a normal distribution.

Applying Ito's lemma to the function above leads to:

$$\log\left(\frac{P + dP}{P}\right) = \left(\mu - \frac{\sigma^2}{2}\right) \cdot dt + \sigma \cdot dw$$

If the constant drift rate will be eliminated, than:

$$\log\left(\frac{P + dP}{P}\right) \approx \mathrm{N}(0, \sigma^2 dt),$$

which means that the expected returns are log-normally distributed.

Applying the first order Taylor approximation we will find that the expected returns are normally distributed:

$$\log\left(\frac{P + dP}{P}\right) = \log(P + dP) - \log(P) \approx \left(\log(P) + \frac{dP}{P}\right) - \log(P) = \frac{dP}{P} \approx \mathrm{N}(0, \sigma^2 dt)$$

In order to describe the evolution of the interest rates there are a series of models. The most used are: *the Vasiek model and the Cox-Ingersoll-Ross model (CIR) and the Hull-White model.* The stochastic evolution of the short-interest rate could be described by the following stochastic differential equation:

$$dr_t = \alpha_t(\theta_t - r_t)dt + \sigma_t r_t^{\gamma} dw_t,$$

where $\alpha_t, \theta_t, \sigma_t$ are the deterministic functions of time and w is a Brownian motion; α_t - represents the speed of pushing the interest rate towards its long run normal level; θ_t - is the long run interest rate; r_t - is the current level of interest rate; σ_t - is the instantaneous volatility of the interest rate.

For γ= 0 in the equation above we will obtain the *Vasiek model(1977)*. This means reversion assumption agrees with the economic phenomenon that interest rates are pulled back over time to the long run average value. When the interest rates increase, the economy slows down, and there is less demand for loans and a natural tendency for rates to fall. Besides its advantages such as being analytically tractable, the Vasicek model has several shortcomings. Since the short rate is normally distributed, for every t there is a positive probability that r is negative and this is unreasonable from an economic point of view. Because the nominal interest rate can not fall below zero as long as people can hold cash; it can become stuck at zero for long periods, however as when prices fall persistently and substantially. Another drawback of the Vasicek model is that it assumes γ= 0. This assumption implies the conditional volatility of changes in the interest rate to be constant.

To solve these inconvinnients *Cox, Ingersoll, Ross* proposed a model based on square root process. For γ= 0.5 in the equation above we will obtain the Cox, Ingersoll, Ross model (1985). CIR is an equilibrium asset pricing model for the term structure of interest rates which provides a complete characterization of the term structure that incorporates risk premiums and expectations for future interest rates. A process following such dynamics is traditionally referred to as square-root process. In a common sense, square-root processes are linked to non-central χ-square distributions.

In some situations the market's expectations about future interest rates involve time dependent parameters. The drift and diffusion terms could be defined as functions of time as well as being functions of r. The time dependence can arise from the cyclical nature of the economy, expectations concerning the future impact of monetary policies and expected trends in other macroeconomic variables. *Hull and White* extend the CIR model to reflect this time dependence. They add a time dependent drift β_t to the process for r, and allow both the reversion rate and the volatility factor to be functions of time:

$$dr = [\beta_t + \alpha_t(\theta_t - r_t)]dt + \sigma^{2_t} r_t^{\gamma} dw_t$$

4. The estimation of financial variables' volatility using EWMA and GARCH models

The successful implementation of VaR depends on the accurate estimation of the portfolio returns' distributions. While the normal distribution is widely used to forecast VaR, the asset returns are typically found to have fat-tails. This means that the VaR estimators based on the normal distribution are inefficient and lead to an underestimated risk.

To remedy this problem, we can analyse the tail behaviour in two ways. The first is to set up an *unconditional distribution* as a mixture of a normal distribution and another kind of distribution such as a normal-Poisson, a normal-lognormal or a Bernoulli-normal distribution, maintaining the assumption of homoscedasticity. The second is to use a *non-normal distribution* like the Student's t-distribution, a Laplace and a double exponential distribution or an exponential power distribution to capture the fat-tailed nature of most asset returns.

Practitioners have often dealt with time varying parameters by confining attention to the recent observations and ignoring those from the distant past. They developed models of time varying volatility like the GARCH model

(Generalised Auto-Regressive Conditional Heteroscedasticity) proposed by Engle and introduced by Bollerslev, and the EWMA model (Exponentially Weighted Moving Average) popularised by RiskMetrics department from J. P. Morgan.

EWMA model

In an EWMA model variances and covariances are modelled by using an exponentially moving average. The observations are given different weights, the most recent data getting the highest weight. The weights decline rapidly as we go back. The main advantage of the model is that it gives immediate reaction to the market crashes. The equation of the volatility can be described as follows:

$$\sigma_t^2 = (1-\lambda)\sum_{t=1}^{n}\lambda^2\left(R_t - \mu\right)^2 ,$$

where: σ_t^2 - is the standard deviation;

R_t - is the return at moment t;

μ – is the mean value of the distribution;

n – is the time horizon;

λ – is the exponential factor that shows the persistancy of volatility; it's value could change between 0 and 1;

1**-** λ – is a parameter that shows the speed with which a shock in the market is absorbed by the volatility.

Using a recursive substitution, the volatility can be rewritten as follows:

$$\sigma_t^2 = \lambda \cdot \sigma_{t-1}^2 + \left(1-\lambda\right)\cdot R_t^2$$

This approach has two important advantages. First of all, the volatility reacts faster to shocks in the market because recent data carry more weight then the distant past data. Secondly, the volatility declines exponentially after any large shock, as the weight of the shock observation falls as λ^{t-1}.

The most crucial part of the model is to choose the exponential factor. If it is a big number, the current variance effects will be small over total variance. The factor is based on investors' time horizon. The RiskMetrics uses a 0.94 value for daily volatility estimations and 0.97 for monthly volatility estimations. They use 480 time series as inputs from world market (money market rates, swap rates, foreign exchange rates, equity indices) generating 480 variance and 114.960 covariance forecasts. The closer the value λ to unity, the smoother the data series become. For EWMA calculation, the necessary number of days can be calculated by dividing the required accuracy to the factor value (both expressed logarithmically).

Taking in consideration what we have discussed above, the volatility for asset i at time t, can be written as follows:

$$\sigma_{i,t} = \sqrt{\frac{1-\lambda}{1-\lambda^n}\sum_{j=0}^{n}\lambda^j \cdot r_{i,j}}$$

The correlation between return forecasts can be construct in the same manner as performed for the volatility forecasts:

$$\sigma_{12,t}^2 = (1-\lambda)\sum_{j=1}^{T}\lambda^{j-1}\left(r_{1t} - \overline{r_1}\right)\left(r_{2t} - \overline{r_2}\right)$$

Next we compute one-day VaR forecasts for the bank's portfolio, using each of the EWMA estimators. The backtesting results demonstrate that, due to the flexibility of the parameters of the conditional distribution, the exponential distribution can properly capture the fat-tailedness characteristic of the asset return distributions. EWMA can also be regarded as a special case of GARCH as shown below.

GARCH model

GARCH model is widely used in financial markets researches but have many versions. It encompasses a broad class of models that estimate and predict the volatility and the correlations between different assets.The simplest GARCH (1,1) can be described as follows:

$$\sigma_t^2 = \gamma + \beta \cdot \sigma_{t-1}^2 + \alpha \cdot X_{t-1}^2 ,$$

where γ, α and β are the predicted parameters. α and β values show the persistence of the volatility, and α+β must be greater than 1. if the parameters are high, than the average volatility will be high. The parameter β is the same as λ (the exponential factor from the EWMA model), and α is the same as 1-λ (from the EWMA model). If γ=0 then the EWMA equation would be a special version of the GARCH model.

Going further, the GARCH(p,q) model is:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \cdot \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \cdot \sigma_{t-j}^2 \ ,$$

where ε_t is a white noise, which can be determined form the next equation:

$$R_t = \beta \cdot X_i + \varepsilon_t$$

Parameters α_0, α_i and β_j should be greater than 0, and in order to be an explosive process it must be satisfied the following condition:

$$0 \leq \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j \leq 1$$

This model has two crucial elements: the particular specification of the variance equation and the assumption that the standardised residuals are a random walk. The first element was inspired by the characteristics of financial data discussed above, and the second is a necessary device to estimate the unknown parameters. We can estimate γ, α and β by using the maximum likelihood method. Assuming that the returns are normally distributed and the mean of the returns is zero, than the likelihood of Ri being observed is the value of the probability density function, given by the next formulae:

$$f\left(R_i, \sigma_i\right) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \cdot \exp\left(\frac{-R_i}{2\sigma_i^2}\right)$$

In order to estimate VaR using the GARCH model, we should create the distribution of daily returns for the portfolio and then fit the model to these returns. The next step is to do a large number of simulations as many days ahead as the maximum time horizon that is of interest. Once the simulations are done, we can find the selected percentiles of the distribution of portfolio values within each simulated day.

In some cases there are aspects of the model which can be improved so that it can better capture the characteristics and dynamics of time series. There are some extensions at the basic GARCH model that makes it more flexible: Asimetric GARCH model, Exponential GARCH model, Integrated GARCH model, GARCH in Mean model. which responde in a different manner to the shocks in market.

With regard to accuracy, the risk managers should be concerned with whether the model's ex-post performance is compatible with the theoretically desired level. The regulatory capital-adequacy framework also provides an incentive to develop efficient models, that offer enough coverage in relation to the risk so that the supervisors' requirements can be met with the minimum amount of capital that is required to be held**.**

5. Case study: Calculating VaR for interest rate risk management

In order do determine the exposore of a bank's portfolio to the interest rate risk we will calculate the VaR indicator of a hypothetical portfolio of financial instruments in lei and euro for a 10 days horizon. The stochastic variables which compose the interest rates will be modelled by EWMA and GARCH models.

We will suppose that the loan portfolio of a bank is composed by positions exposed to the interest rates - ROBOR, LIBOR and EURIBOR:

- loans with one year maturity – 5.000.000 EUR, at EURIBOR (1 month) + 2% margin;
- loans with one year maturity – 3.000.000 EUR, at LIBOR (1 month) + 3% margin;
- loans with one year maturity – 3.000.000 LEI, at ROBOR (1 month) + 3% margin;

VaR will be calculated on daily data from 01.01.2007 to 24.12.2008. In order to determine the exposures in EUR we will use the interest rates communicated by BNR for 2007 and 2008.

The moments of the three series distributions will be those presented below:

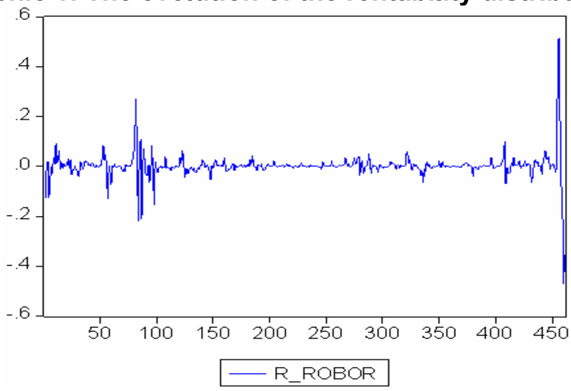
Table 1: The distributions' moments and the Jarque Berra Test's results

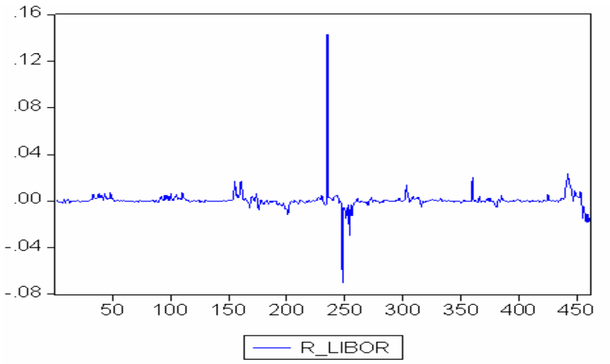
	ROBOR	LIBOR	EURIBOR
Mean	0,001662	0,000571	0,000581
Median	0,000696	0,000153	0,000001
Maximum	0,514035	0,143552	0,142813
Minimum	-0,468803	-0,070598	-0,061919
Standard Deviation	0,054407	0,008526	0,008429
Skewness	1,507226	8,974777	9,542545
Curtosis	52,84460	8,974777	185,7350
Jarque Berra Test	47793,44	635313,9	644183,4

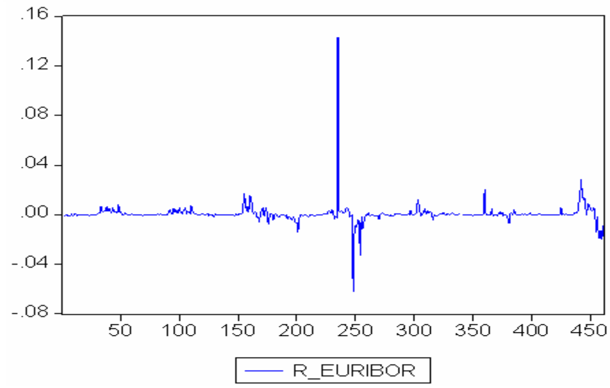
Applying the ADF and the Philipe-Peron Tests it will be observed that the series composed of the interest rates' rentabilities have one unit roots, which means that it is needed a first order differentiation in order to become stationary. Applying the Jarque Berra Test will be obtained leptokurtic distributions, more sharped than the normal ones, for all of the interest rates, a fact shown by the curtosis coefficient. Analysing the skewness coefficient we will observe that the distributions are shifted to the left, compared with the normal distribution.

According to all these factors, the distribution of the rentabilities presents fat tails, which correspond to the extreme variations that took place on the money market. In the last year e series of extreme negative values have taken place. This can lead to an overestimation of VaR, especially that the method describes the maximum expected loss. Here appears the "volatility clustering" phenomena, which can be remedied by the homoscedasticity models EWMA and GARCH.

Graphic 1: The evolution of the rentability distribution







The high level of the interest rates' volatility, observed from the graphics above, leads to many problems in the estimations of the future evolution based on the historical data. In order to correctly estimate the VaR of the portfolio we will determine the volatility by using GARCH models. The first step is to do some simulations, than to find the quantiles of the distribution for each simulation.

Calculating the volatility of the interest rates by GARCH model the results below will be obtained:

Interest rate	Standard deviation
EURIBOR	0,0000708
LIBOR	0,0000727
ROBOR	0,0029601

In order to model the evolution of the interest rates in a stochastic manner we have to know the correlations between the three variables. The correlation matrix calculated on the historical data will be:

$$R = \begin{pmatrix} 1.0000000 & 0,0000710 & 0,0000094 \\ 0,0000710 & 1.0000000 & 0,0000125 \\ 0,0000094 & 0,0000125 & 1.0000000 \end{pmatrix},$$

where the variables' order is: LIBOR, EURIBOR, ROBOR.

In establishing the probability by which can be calculated the maximum loss of the portfolio it will be used a confidence coefficient α=2,33 which correspond to a probability of 99%, the recommendation of BNR. In calculating the daily VaR it will be used the next relationship:

$$VaR_i = -V_{i,0} \times \alpha \times \sigma_i ,$$

where $V_{i,0}$ - is the market value of the portfolio, and σ_i - represents the volatility. Applying the above formula we will determine the maximum potential losses of the portfolio.

Maximum potential daily losses:

Interest rate	Volatility	Exposure	Daily VaR (99%)
EURIBOR	0,0000708	18.500.000	3.051,8340
LIBOR	0,0000727	11.100.000	1.880,2401
ROBOR	0,0029601	30.000.000	206.910,9900

From the table above it is observed that the maximum possible daily loss (206.910,9900 lei) can be caused by the exposures on ROBOR, which is followed by the exposures on EURIBOR, the maximum possible loss in the last case is 3.051,8340 lei daily.

Because the interest rates are correlated it will be necessary to determine the daily VaR for the entire portfolio, taking into consideration the correlations between the interest rates:

$$VaR_{pf}^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} VaR_i \times VaR_j \times \rho_{ij}$$

We will obtain a daily VaR value for a day of 12.000,0095 lei. In order to determine the VaR for a 10 days time horizon, we will apply the next formulae:

$$VaR_{pf,h} = VaR_{pf} * \sqrt{h}$$

and it will be obtained 37.947,362 lei, which represents the maximum possible loss of the portfolio for the next ten days.

Conclusions

Value at Risk (VaR) has become the standard measure in quantifying the market risk and it is also used for regulatory purposes. Although VaR is an easy and intuitive concept, its measurement is a very challenging statistical problem, because the return distributions are not constant over time. The asset returns are typically found to have fat-tails. This means that the VaR estimators based on the normal distribution are inefficient and lead to an underestimated risk. To remedy this problem, we can analyse the tail behaviour by using EWMA and GARCH models, which deal with time varying parameters by confining attention to the recent observations.

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