DISCUSSION PAPER SERIES

Discussion paper No.50

Optimal Environmental Policy for Waste Disposal and Recycling When Firms Are Not Compliant

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January, 2010

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Optimal Environmental Policy for Waste Disposal and Recycling When Firms Are Not Compliant*

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Abstract

We investigate, considering disposal and recycling activities after the consumption of products, the models that explicitly incorporate both the product market and the recycling market. In the field, the deposit-refund (D-R) policy has been discussed as an ideal policy to internalize disposal cost, which can result in the realization of the first-best policy. However, the possibility of firms' illegal disposal has been neglected. We introduce monitoring cost to prevent firms from disposing of collected residuals illegally and induce the second-best D-R policy. We find that the monitoring problem for firms brings about a variety in the optimal level of the refunds (which is typically be smaller than the first best level). Furthermore, we investigate an alternative policy that requires producers to take back residuals, and show how this policy works equivalently to the second-best D-R policy by applying the theory of tradable emission permits market. We find that the second-best system of this policy is the combination of the take-back requirement depending on the amount of each firm's outputs and initial exemption from that requirement.

Keywords: Deposit; Refund; Monitoring; Illegal Waste Disposal; Take-back requirement; Tradable rights

JEL Classification: H21; Q21; Q28

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*The author would like to thank Masahiro Okuno-Fujiwara for his numerous helpful and constructive suggestion. He would also like to thank Tomohiko Kawamori, Hideo Koide, Norimich Matsueda, Toshihiro Matsumura, Daisuke Shimizu, Mariko Onoda, and Kazuo Yamaguchi for their valuable comments. The author also gratefully acknowledge the Japan Society for the Promotion of Science (JSPS) Research Fellowships for Young Scientists and Grant-in-Aids for JSPS Fellows of the Ministry of Education, Culture, Sports, Science and Technology, Government of Japan.

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1 Introduction

The disposal of wastes generated after consumption involves certain costs: financial costs for appropriate disposal and physical costs as a result of the environmental problems caused by waste disposal. However, it is often observed that charging consumers directly for disposal costs is difficult due to the possibility of illegal waste disposal. In such a case, the disposal fee for consumers generates another cost for the monitoring and prevention of the illegal disposal of wastes. Choe and Fraser [4] explicitly introduced monitoring cost into their model and induced the second-best charge on waste collection. Since their primary aim was to investigate households' efforts toward waste reduction, they considered the non-market-based recycling activities that can not be subsidized. If recycling activities are market-based and so the subsidy on them is feasible, indeed, the deposit-refund (D-R) policy has been discussed as an ideal policy to internalize disposal cost and avoid the monitoring problem for consumers, which can result in the realization of the first-best policy (e.g. Dinan [5], Fullerton and Kinnaman [7], Fullerton and Wolverton [8, 9], Palmer and Walls [15], Walls and Palmer [19]).[1]

The D-R[2] policy is a combination of the tax (deposit) on products due to their disposal cost and a subsidy (refund) on recycling as a reward for the avoidance of disposal.[3] In this paper, we begin with the construction of a model that considers the disposal and recycling activities after the consumption of goods[4] and explicitly incorporates both the product market and the recycling market.[5] Using this model, we discuss the effect of the charge or penalty on waste disposal (Pigouvian tax) and the combination of tax on products and subsidy on recycling (the D-R policy) and further, compare it to the take-back requirement

[1] Fullerton and Wolverton [9] say that "The point (of the D-R policy) is to avoid the enforceability or measurement problems of a tax on pollution by applying the tax to observable market transactions $\cdots$ and simultaneously subsidizing other market transactions, $\cdots$."

[2] This term is often used to refer to the firms' voluntary deposit-refund system. Bohm and Russell [2] analyzed this system. Note that our D-R policy is a distinct one and is formulated by the authorities.

[3] Fullerton and Wolverton [9] call this policy a two-part instrument (2PI) in a more general context.

[4] In terms of ordinary environmental economics, wastes and recyclables correspond to emissions and abatements, respectively. Fullerton and Wolverton [8] point out and summarize this correspondence excellently.

[5] The abovementioned literatures treat recycling activities only in emission functions and do not consider abatement *markets* such as recycling markets. Calcott and Walls [3] pointed out the importance of modeling recycling markets explicitly.

in the recycling market (regulation). Then, we seek the optimal policy combinations.

All the past literature considers the non-compliance of households seriously; however, it neglects the possibility of firms' illegal waste disposal when the D-R policy is discussed. But firms as well as households can be non-compliant.[6] In the past analyses of the D-R policy, there is the tacit assumption that the time to give refunds is after the residuals have been converted to utilizable materials. Obviously, firms have no incentive to dispose of such valuable materials. However, in reality, the recycling market arises between the "collecting" process (segregating and collecting residuals after consumption) and the "re-processing" process (converting the collected residual into utilizable materials). The time to give refunds is usually when this recycling market is in existence, and giving refunds after the final reprocessing process is unlikely or costly since the residuals have already been converted or even if they have not been converted, the input for the reprocessing process constitutes inside information of firms. Unless all the existing residuals are valuable before the reprocessing process,[7] the firms have the incentive to dispose of the residuals that are collected to aim at obtaining refunds. As the main feature of our model, we introduce the recycling market and possibility of firms' illegal disposal after their transaction in this market as well as the possibility of households' illegal disposal after their consumption of products. Then, explicitly addressing the monitoring cost of illegal disposal, we induce the second-best D-R policy.

As a result, we find that while the deposit should always be equal to the disposal cost, the refund, which is the subsidy on collected residuals to encourage recycling, should vary (it should be smaller than or equal to the disposal cost). In particular, we find that the monitoring problem for firms can be avoided so long as the refund does not exceed the price of the recycling market, and this fact brings about a variety in the optimal level of the refunds.

As a further topic, we investigate the policy with take-back requirements and tradable emission permits. Under this policy, each firm is required to take back some amount

[6]There are many reports on firms' illegal disposal worldwide. For example, in Japan, according to the Ministry of Environment, in the latter half of 1990's, the illegally disposed of industrial wastes amounted to around 40 thousand tons almost every year.

[7]In such a case, the secondhand products market will arise and, finally, the utilities of the residuals will be lost.

of the post-consumption residuals, the amount being allocated according to the quantity of each firm's outputs, and firms can transact the gap between this requirement and the actual reprocessing as tradable rights. Fullerton and Wu [10] point out that the take-back requirement as well as D-R policy can result in the realization of the first-best policy[8] when there is no monitoring problem. However, as mentioned above, the actual reprocessing activity is difficult to monitor. We compare the policy with a take-back requirement to the D-R policy under the situation where firms are non-compliant.

We find that under this extension, to construct the equivalence to the second-best D-R policy, we must not only adjust the marginal amount of the take-back requirement for each firm's outputs but also give each firm some amount of initial exemption from the take-back requirement. Instead of the initial exemption, the marginal amount of the take-back requirement should be more than the actual additional residuals corresponding to each firm's additional production.

The chapter is organized as follows. In the next section, we begin with outlining the fundamental model and explain the first-best situation. In section 3, this model is extended by introducing monitoring problem of both households and firms. Then, we investigates the market behaviors with the tax-subsidy (D-R) policy under the possibility of illegal disposal and the second-best tax-subsidy policy is induced. In Section 4, the policy with take-back requirement is introduced and compared with the policy in Section 3. Section 5 concludes the paper.

2 Fundamental model

A pioneering work in the economics of recycling and waste management is provided by Vernon Smith (Smith [16]). He constructed a rudimentary model emphasizing the problem of post-consumption recycling and waste accumulation and found that the Pigouvian tax in this context is the charge on waste disposal. Dinan [5] reactivates the fields by focusing

[8]Since Fullerton and Wu [10] assume a representative firm, there is no need to combine the theory of tradable emission permits with the policy with a take-back requirement to show the equivalence between the two. We show this equivalence in the model where firms have heterogeneous technologies of both production and recycling by applying the theory of tradable emission permits. By using such a model, the process whereby firms reallocate the collecting residuals under the policy with take-back requirements is clarified.

on a policy failure for recycling activities. Fullerton and Kinnaman [7] and Palmer and Walls [15] pointed out that the D-R policy attains the first-best outcomes instead of the charge on waste disposal in a general equilibrium model and a partial equilibrium model, respectively. In this paper, by mainly reforming the model of Palmer and Walls [15][9], we construct the partial equilibrium model of good x, which explicitly incorporates both the product market and the recycling market. The conceptualization of our model is depicted in Figure 1.

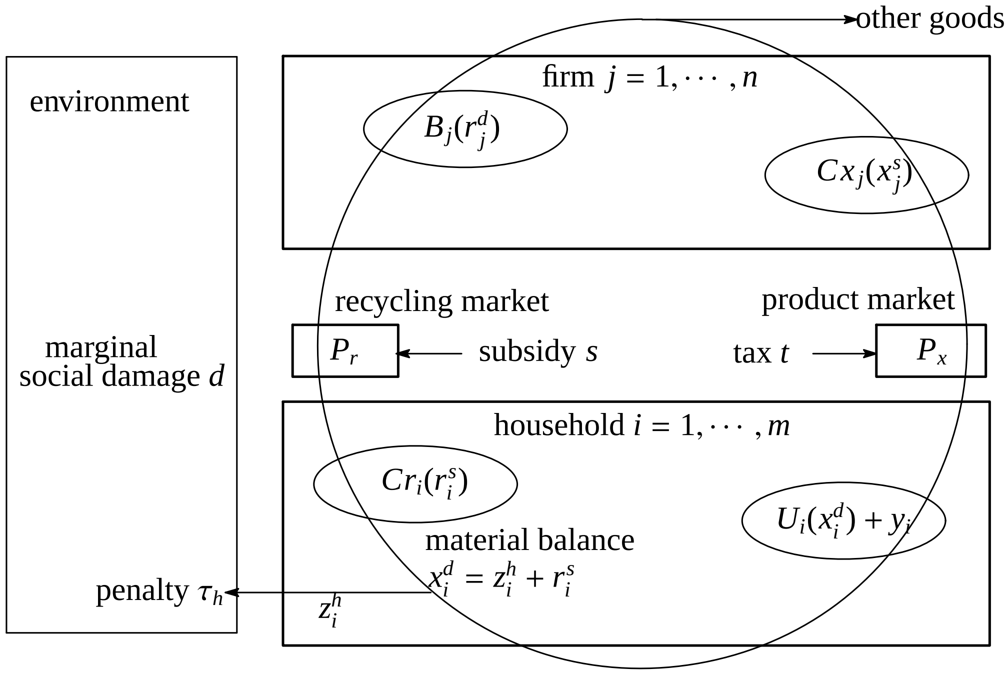

Figure 1: the model with a recycling market

2.1 The model with a recycling market

Suppose that given any policy for recycling and waste management, the authorities always keep an adequately high monitoring level in order to gain the information required to implement the policy, and put aside the monitoring cost for the present.

There are n firms ($j = 1, 2, \cdots, n$) producing homogeneous products according to the strictly increasing and strictly convex cost function $Cx_j(x_j^s) : \mathbb{R}_+ \mapsto \mathbb{R}_+$, where x_j^s is firm j's supply of the products. The products are transacted in the product market and

[9] Their model treats the recycling market implicitly.

consumed by m households ($i = 1, 2, \cdots, m$). $U_i(x_i^d) + y_i$ is household i's utility function taking the quasi-linear form, where x_i^d is household i's demand for the products. Note that y_i is household i's demand for numeraire. $U_i : \mathbb{R}_+ \mapsto \mathbb{R}$ is strictly increasing and strictly concave.

Products generate the same amount of residuals as their consumption[10]. Household i chooses the method of disposal of the residuals. One way is to supply them to firms for recycling; thus, r_i^s is the quantity of residuals disposed of in this manner. Returning the residuals to firms is costly, and the cost function of this recycling activity is $Cr_i(r_i^s) : \mathbb{R}_+ \mapsto \mathbb{R}_+$, [11] which is strictly increasing and strictly convex. The other way is to convert residuals into direct wastes from households z_i^h. $d \in \mathbb{R}_{++}$ is the unit damage from this direct waste disposal (including both external disposal cost and environmental damage). Thus, if this direct disposal is not adequately charged or penalized, it will have negative externalities. The direct wastes are monitored by the authorities and the unit charge or penalty of these direct wastes is $\tau_h \in \mathbb{R}_+$.[12] We suppose that the households separate the residuals according to simple material balance, $x_i^d = r_i^s + z_i^h$. [13]

The residuals collected for recycling are transacted in the recycling market. Firm j demands the residuals at the amount r_j^d. Firm j can gain a benefit by recycling according to a strictly concave function, $B_j(r_j^d) : \mathbb{R}_+ \mapsto \mathbb{R}$. Note that we allow B_j to be negative since it contains the cost to reprocess the residuals. [14] To facilitate our analysis, we assume that

[10]To restrict our attention, we neglect the numeraire's residuals.

[11]This cost is interpreted as the transaction cost in the collection process for recycling. Usually, intermediate traders or recyclers participate in the collection process for the consumed products. However, for simplicity, we suppose that the entire cost in this process is passed on to households. If all the markets for intermediate traders are competitive, this supposition is not essential.

[12]If the direct wastes from the households are not monitored, $\tau_h = 0$. Thus, the households can freely dispose of the wastes into the environment.

[13]Many studies in this field use the concept of material balances. In particular, Eichner and Pething [6] construct the model that pays full attention to material balances. The above simplification of the material balance does not essentially work to induce our result. A more complex formulation can be used instead of the simple material balance, which is used only to facilitate our computation.

[14]In our model, B_j is assumed to be exogenously given but it can be interpreted in various ways. More concretely, B_j is defined by $B_j' = P - C'(r_j^d)$, where P is the exogenously given price and $C(r_j^d)$ is the reprocessing cost. If the firm sells its recycled materials in some other market, P is the market's price. If the firm uses recycled materials as inputs for its own products, P is the price of perfectly substitutable virgin inputs v_j^d. Note that if we assume the production function of the form $x_j^s = f(r_j^d + v_j^d)$ and the interior solution of v_j^d, we can treat B_j and Cx_j independently.

$B_j'(0) > 0$[15] and that $\lim_{r\to\infty} B_j'(r) = -\infty$.[16]

The prices in the product market and the recycling market are $P_x \in \mathbb{R}_{++}$ and $P_r \in \mathbb{R}$, respectively. When $P_r > 0$ ($P_r < 0$), the firms (households) pay for recycling.[17] We assume that the households and firms are the price takers with regard to all the prices. The authorities can use the tax $t \in \mathbb{R}$ (deposit) in the product market, and subsidy $s \in \mathbb{R}$ (refund)[18] in the recycling market.

Consider the utility maximization problem of household i:

$$\max_{x_i^d, y_i, r_i^s, z_i^h} U_i(x_i^d) + y_i.$$

$$\text{s.t.} \quad P_x x_i^d + y_i + \tau_h z_i^h + Cr_i(r_i^s) \leq I_i + P_r r_i^s$$
$$x_i^d = z_i^h + r_i^s.$$

where I_i is household i's income. Because of our interest, we presume $x_i^d > 0$, $r_i^s > 0$, and $z_i^h > 0$. In particular, the assumption that $z_i^h > 0$ is important since this assumption focuses our attention on the case where even if the market is under the optimal policy, it is not desirable to attain zero direct wastes (zero emission) under current recycling technologies. Owing to the first-order condition with respect to y_i and z_i^h, the Lagrange multiplier of the first constraint equals one and that of the second constraint equals τ_h. Taking these into consideration, the first-order conditions are

$$U_i'(x_i^d) = P_x + \tau_h, \tag{1}$$

$$P_r + \tau_h = Cr_i'(r_i^s). \tag{2}$$

[15] If there are recycling activities in spite of the absence of a policy to encourage them, to judge that $B_j'(0) > 0$ is accurate, but note that the converse is not true since even if $B_j'(0) > 0$, the collecting cost Cr is needed.

[16] We assume this assumption to exclude the case where $B_j{}'$ decreases asymptotically. Indeed, without this assumption, we can show all of our propositions. However, to consider the lower bound of $B_j{}'$, we must prepare an additional troublesome case in our analysis. Thus, we exclude this case to make our analysis clear.

[17] P_r can be negative since the household may pay for recycling to avoid paying the penalty for its disposal.

[18] It is more precise if our setting with regard to recycling cost is stated in terms of the time to levy the subsidy. We divide the entire cost of the recycling process into two parts: the cost expended before levying the subsidy and the cost expended after levying the subsidy. Cr_i represents the former and the latter is included in B_j. If we consider the realistically feasible time to levy the subsidy, in many cases, these two (i.e., Cr_i and some part of B_j) will correspond to the collecting cost and reprocessing cost, respectively. Therefore, in the body of this chapter, we use these terms.

Consider the profit maximization problem of firm j:

$$\max_{x_j^s, r_j^d} [(P_x - t)x_j^s - Cx_j(x_j^s)] + [B_j(r_j^d) - (P_r - s)r_j^d].$$

Then, after presuming $x_j^s > 0$ and $r_j^d > 0$, the first-order conditions for firms are

$$P_x - t = Cx_j'(x_j^s), \tag{3}$$

$$B_j'(r_j^d) = P_r - s. \tag{4}$$

In this model, social welfare W is defined by

$$W \equiv \sum_{i=1}^{m}[U_i(x_i^d) - P_x x_i^d] + \sum_{i=1}^{m}[P_r r_i^s - Cr_i(r_i^s)] - \tau_h \sum_{i=1}^{m} z_j^h \tag{5}$$

$$+ \sum_{j=1}^{n}[(P_x - t)x_j^s - Cx_j(x_j^s)] + \sum_{j=1}^{n}[B_j(r_j^d) - (P_r - s)r_j^d] \tag{6}$$

$$+ \tau_h \sum_{i=1}^{m} z_j^h + t\sum_{j=1}^{n} x_j^s - s\sum_{j=1}^{n} r_j^d - d\sum_{i=1}^{m} z_j^h. \tag{7}$$

(5) represents the consumer surplus including the payment for direct disposal. (6) is the producer surplus. Note that the second brackets of (5) and (6) are the surplus related to recycling activities. (7) is the authorities' income and expenditure and the environmental damage from direct disposal. The authorities maximize the welfare under the conditions for clearing the product and recycling markets and the material balance:

$$\begin{aligned} \max_{x_i^d, r_i^s, x_j^s, r_j^d, z_i^h} W &= \sum_{i=1}^{m}[U_i(x_i^d) - Cr_i(r_i^s)] + \sum_{j=1}^{n}[B_j(r_j^d) - Cx_j(x_j^s)] - d\sum_{i=1}^{m} z_i^h \\ \text{s.t.} \quad & \sum_{i=1}^{m} x_i^d = \sum_{j=1}^{n} x_j^s, \quad \sum_{j=1}^{n} r_j^d = \sum_{i=1}^{m} r_i^s, \quad \sum_{i=1}^{m} z_i^h = \sum_{i=1}^{m} x_i^d - \sum_{i=1}^{m} r_i^s, \end{aligned} \tag{8}$$

where W has been arranged by using the market-clearing conditions. After canceling z_i^h by the material balance and the Lagrange multipliers of the market-clearing conditions, we obtain the first-order conditions for the social optimal:

$$U_i'(x_i^d) = Cx_j'(x_j^s) + d, \tag{9}$$

$$B_j'(r_j^d) + d = Cr_i'(r_i^s). \tag{10}$$

Therefore, by comparing conditions (9) and (10) with conditions (1)–(4) in private markets, we find that the tax and subsidy $t = s = d$ can attain the first-best outcomes as well as the waste charge $\tau_h = d$. The former is the D-R policy and the latter is the Pigouvian tax.

2.2 Demand, supply, and equilibrium

To explicitly observe what happens in the product market and the recycling market, let us consider the demand and supply functions of products and residuals more formally. By (1) and (3), we obtain the demand and supply functions of products:

$$x_i^d(P_x;\tau_h) = {U_i'}^{-1}(P_x+\tau_h), \tag{11}$$

$$x_j^s(P_x;t) = {Cx_j'}^{-1}(P_x-t). \tag{12}$$

Therefore, the aggregate demand and supply functions of products are

$$X^d(P_x;\tau_h) = \sum_{i=1}^{m} {U_i'}^{-1}(P_x+\tau_h), \tag{13}$$

$$X^s(P_x;t) = \sum_{j=1}^{n} {Cx_j'}^{-1}(P_x-t). \tag{14}$$

Suppose that $X^d(P_x;\tau_h)$ and $X^s(P_x;t)$ are differentiable in a relevant range.

We obtain households' supply and firms' demand of residuals from (2) and (4), respectively, as below.

$$r_i^s(P_r;\tau_h) = {Cr_i'}^{-1}(P_r+\tau_h), \tag{15}$$

$$r_j^d(P_r;s) = {B_j'}^{-1}(P_r-s). \tag{16}$$

Therefore, the aggregate supply and demand functions of residuals are

$$R^s(P_r;\tau_h) = \sum_{i=1}^{m} {Cr_i'}^{-1}(P_r+\tau_h). \tag{17}$$

$$R^d(P_r;s) = \sum_{j=1}^{n} {B_j'}^{-1}(P_r-s), \tag{18}$$

Suppose that $R^d(P_r;s)$ and $R^s(P_r;\tau_h)$ are differentiable in a relevant range.

For the later analysis, it is often convenient to use the inverse supply function of residuals. This is obtained from (17) as

$$P_r(R^s;\tau_h) = R^{s-1}(P_r;\tau_h) \tag{19}$$

for all $R^s > 0$.

The equilibrium prices, $P_x^*(t;\tau_h)$ and $P_r^*(s;\tau_h)$, are defined by (13) and (14) and (18) and (17), respectively to satisfy the following market clear conditions of the product market and the recycling market:

$$X^d(P_x^*(t;\tau_h);\tau_h) = X^s[P_x^*(t;\tau_h);t] \equiv X^*(t;\tau_h), \tag{20}$$

$$R^s(P_r^*(s;\tau_h);\tau_h) = R^d[P_r^*(s;\tau_h);s] \equiv R^*(s;\tau_h). \tag{21}$$

The last equivalent terms, $X^*(t;\tau_h)$ and $R^*(s;\tau_h)$, define the aggregate equilibrium outcomes. Note that $P_r^*(s;\tau_h) = P_r(R^*(s;\tau_h);\tau_h)$ by the definition of the inverse supply function (19). Then, each firm's equilibrium outcomes, $x_j^{s*}(t;\tau_h)$ and $r_j^{d*}(s;\tau_h)$, and each household's equilibrium outcomes, $x_i^{d*}(t;\tau_h)$ and $r_i^{s*}(s;\tau_h)$, are defined by putting equilibrium prices into (12) and (16) and (11) and (15), respectively.

$$x_j^{s*}(t;\tau_h) \equiv x_j^s[P_x^*(t;\tau_h);t] \quad j = 1,2,\cdots,n, \tag{22}$$

$$r_j^{d*}(s;\tau_h) \equiv r_j^d[P_r^*(s;\tau_h);s] \quad j = 1,2,\cdots,n, \tag{23}$$

$$x_i^{d*}(t;\tau_h) \equiv x_j^d[P_x^*(t;\tau_h);\tau_h] \quad i = 1,2,\cdots,m, \tag{24}$$

$$r_i^{s*}(s;\tau_h) \equiv r_j^s[P_r^*(s;\tau_h);\tau_h] \quad i = 1,2,\cdots,m. \tag{25}$$

Comparative statics using (20) and (21) yield the following result.

Lemma 1. *Suppose that $x_j^{s*}(t;\tau_h), x_i^{d*}(t;\tau_h) > 0$ and $r_i^{s*}(s;\tau_h), r_j^{d*}(s;\tau_h) > 0$ for all i, j. Then,*

$$\frac{\partial P_x^*}{\partial t} = \frac{\sum_{j=1}^n (1/Cx_j''(x_j^{s*}))}{\Delta_1} > 0, \tag{26}$$

$$\frac{\partial P_x^*}{\partial \tau_h} = \frac{\sum_{i=1}^m (1/U_i''(x_i^{d*}))}{\Delta_1} < 0, \tag{27}$$

$$\frac{\partial X^*}{\partial t} = \frac{\partial X^*}{\partial \tau_h} = \frac{\sum_{j=1}^n (1/Cx_j'') \sum_{i=1}^m (1/U_i'')}{\Delta_1} < 0. \tag{28}$$

and

$$\frac{\partial P_r^*}{\partial s} = \frac{-\sum_{j=1}^n (1/B_j''(r_j^{d*}))}{\Delta_2} > 0, \tag{29}$$

$$\frac{\partial P_r^*}{\partial \tau_h} = \frac{-\sum_{i=1}^m (1/Cr_i''(r_i^{s*}))}{\Delta_2} < 0, \tag{30}$$

$$\frac{\partial R^*}{\partial s} = \frac{\partial R^*}{\partial \tau_h} = \frac{-\sum_{i=1}^m (1/Cr_i'') \sum_{j=1}^n (1/B_j'')}{\Delta_2} > 0. \tag{31}$$

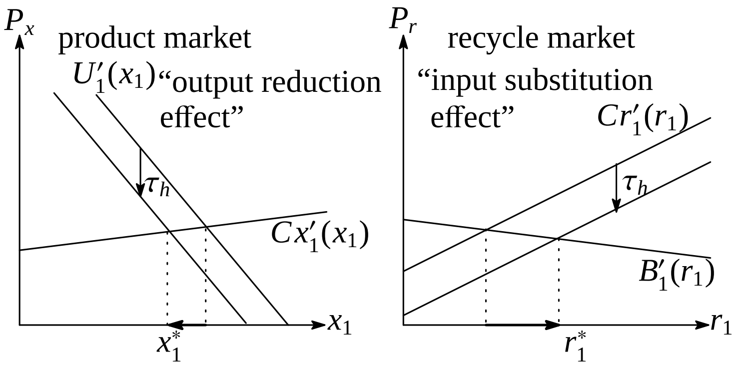

Figure 2: penalty on waste disposal

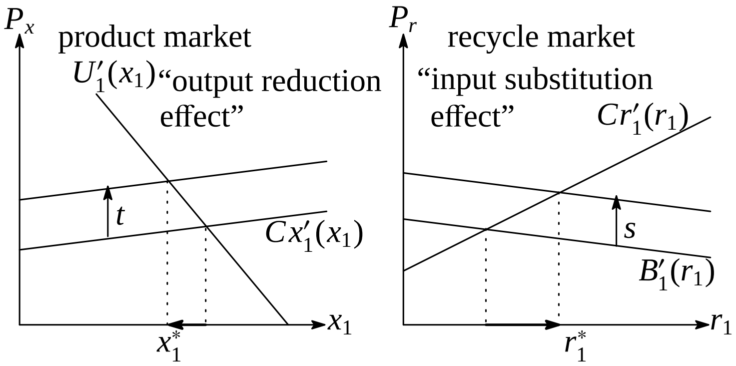

Figure 3: the deposit-refund policy

where $\Delta_1 = \sum_{j=1}^{n}(1/Cx_j''(x_j^{s*})) - \sum_{i=1}^{m}(1/U_i''(x_i^{d*})) > 0$ *and* $\Delta_2 = \sum_{i=1}^{m}(1/Cr_i''(r_i^{s*})) - \sum_{j=1}^{n}(1/B_j''(r_j^{d*})) > 0$.

From this lemma, we can gain an insight into the reaction of the households and the firms to the policy instruments. (28) and (31) provide the mathematical decomposition which represents impacts of the output reduction effect and the input substitution effect explained below.

Figures 2 and 3 depict the product market and recycling market. To make the discussion clear, we depict the figure supposing that $n = m = 1$ (one representative household and one representative firm). (28) and (31) indicate that penalty on waste disposal τ_h simultaneously has two effects– both on product demand and on recycling supply. On the other hand, tax t and subsidy s have one effect respectively– on product supply and on recycling demand, respectively. In the product market, as seen in the left panel of Figure 2 (Figure 3), households (firms) become aware that they must bear the penalty (bear the tax) for each additional unit of their consumption (production). This reduces the demand (supply) of products and contributes to the reduction of waste disposal. This is called the "output reduction effect." In the recycling market, as seen in the right panel of Figure 2 (Figure 3), households (firms) can avoid the penalty (gain the subsidy) for each additional unit of their recycling. This increases the supply (demand) for recycling. Thus, recycling is encouraged, which results in smaller amounts of waste. This is referred to as the "input substitution effect.[19]"

2.3 Illustration of the first-best policy

We illustrate the optimal use of penalty on waste disposal τ_h, tax on products t, and subsidy on recycling s. By using Figure 4, which depicts the product market and the recycling market in one figure, we see the socially optimal state and the market failure of this model. A unit of product imposes a unit disposal damage d on the social welfare and a unit of recycling saves a unit disposal damage d. Hence, the marginal social cost of production is $Cx_j{}'(x_j^s) + d$ and the marginal social benefit of recycling is $B_j{}'(r_j^d) + d$. In the optimal state, as seen in (9) and (10), these values should balance the marginal utility of production and the marginal cost of recycling, respectively. Thus, the first-best quantity of products is the length OC; that for recycling, OB; and for waste disposal, BC. The two shaded triangles in figure 4 represent the dead weight loss caused when $\tau_h = t = s = 0$. Under such a circumstance, the economy neglects the disposal damage d and considers only the production cost Cx_1' and recycling benefit B_1'. Then, the quantity of products is the length OD; that for recycling, OA; and for waste disposal, AD. This results in the overproduction of products, the underproduction of recycling, and the over emission of wastes as compared to the first-best levels.

The charge on wastes τ_h, which is the Pigouvian tax on waste disposal, internalizes the cost and benefit of waste disposal simultaneously since it causes both the output reduction effect and the input substitution effect. We can accomplish the first-best outcomes by using charge on wastes $\tau_h = d$ as seen in figure 4. Notice that the combination of tax t and subsidy s also internalizes the cost and benefit of waste disposal instead of the charge

[19]Walls [17] introduced these terms into the economics of waste and recycling management.

on wastes τ_h. Since tax t causes the output reduction effect and subsidy s causes the input substitution effect, they reduce the dead weight loss by internalizing the cost of disposal and the benefit of avoidable disposal, respectively. When $t = s = d$, the dead weight loss disappears and the outcomes reach the first-best level, as seen in figure 4.

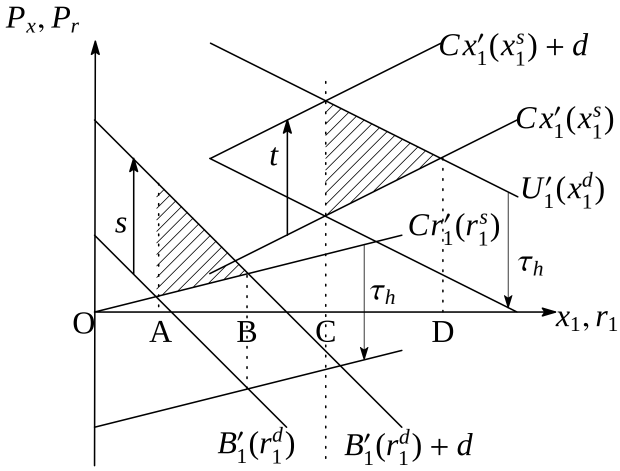

Figure 4: D-R policy and charge on waste disposal. x_1: quantity of products. r_1: quantity of recycling. P_x: price of products. P_r: price of recycling. Cx_1: production cost. U_1: utility from consumption. Cr_1: recycling cost. B_1: benefit from recycling. d: unit damage from waste disposal.

3 Monitoring problem

3.1 Setting

We will introduce the possibility of firms' illegal disposal and monitoring cost of both households' and firms' disposal activities into a partial equilibrium model of good x in Section 2. The modified conceptualization of our model is depicted in Figure 5. Similar to the model in Section 2, there are n firms and m households who transact the products in the product market and residuals in the recycling market. Their cost functions and utility (benefit) functions are same in Section 2.

As well as the households separate the residuals according to simple material balance, $x_i^d = z_i^h + r_i^s$, firm j can directly dispose of the collected residuals instead of completing the reprocessing process after it demands the residuals at the amount r_j^d. r_j^c represents the quantity of residuals completely and appropriately reprocessed by firm j; z_j^f, the amount of

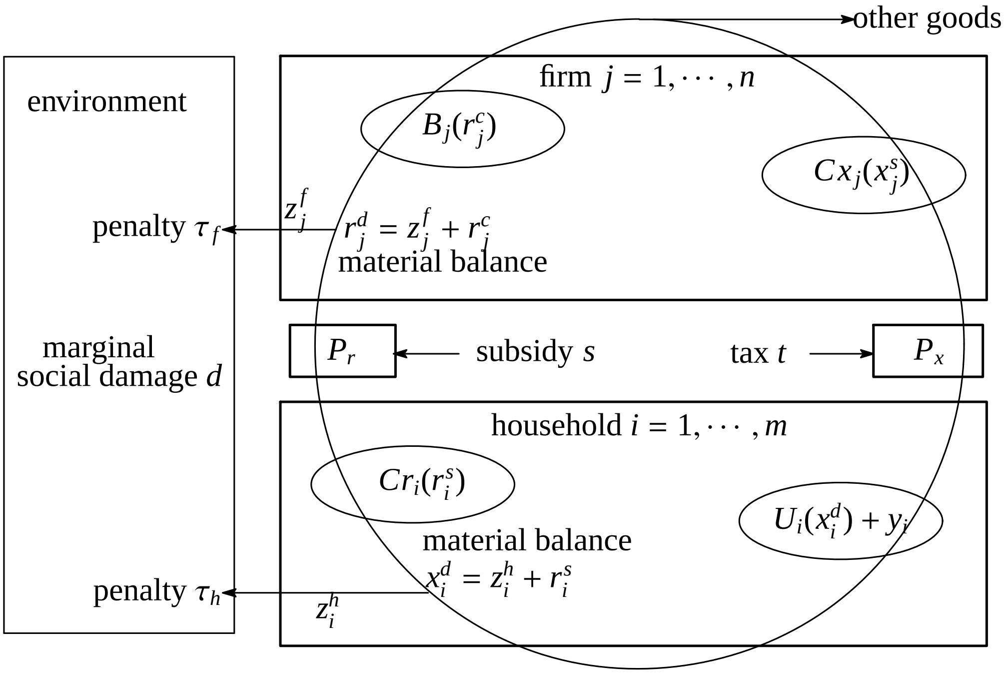

Figure 5: the model with monitoring problem

firm j's direct waste disposal. Again, we suppose the simple material balance $r_j^d = z_j^f + r_j^c$. Firm j can gain the benefit by reprocessing r_j^c according to the function, $B_j(r_j^c)$, which is defined in Section 2 (Just the argument is replaced with r_j^c). We denote the aggregate quantities $Z_h \equiv \sum_{i=1}^m z_i^h$, $R^c \equiv \sum_{j=1}^n r_j^c$ and $Z_f \equiv \sum_{j=1}^n z_j^f$.

Similarly to the unit expected penalty of the direct wastes from the households $\tau_h \in \mathbb{R}_+$, the direct wastes from the firms are also monitored by the authorities and the unit expected penalty is $\tau_f \in \mathbb{R}_+$. The direct disposal activities of both households and firms generate some social cost with a constant marginal external damage of $d \in \mathbb{R}_{++}$. Thus, if these disposal activities are not adequately charged or penalized, they will have negative externalities.

We assume that only the market-based information of the firms, that is, $(x_1^s, \cdots, x_n^s)$ in the product market and $(r_1^d, \cdots, r_n^d)$ in the recycling market, is observable and verifiable for the policy maker. Thus, we assume that the market-based information to impose the tax and give the subsidy is perfect. Since it is difficult to obtain information about each household's consumption and disposal activities, we suppose that the information of the households, $(x_1^d, \cdots, x_n^d)$ or $(r_1^s, \cdots, r_n^s)$, is unobservable. The amount of each firm's complete recycling $(r_1^c, \cdots, r_n^c)$ cannot be observed because this constitutes firms' inside information. Therefore, the households can dispose of their residuals directly by saying that they have not consumed the products or have recycled them appropriately, and the firms can, deceiving the authorities by pretending to reprocess the residuals. However, the authorities can monitor which household or firm directly disposes of the residuals. In other words, a part of z_i^h and z_j^f can be made observable and verifiable by the monitoring.[20]

The authorities control the expected penalties of households' and firms' direct disposal, τ_h and τ_f, by way of monitoring. Expected penalties are determined by multiplying the unit penalties by the probabilities of detecting disposal activities.[21] To increase these factors, more powerful monitoring is needed.[22] It costs $\Gamma_h(\tau_h) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ and $\Gamma_f(\tau_f) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ to monitor the households' and firms' disposal, respectively, where Γ_h and Γ_f are convex and strictly increasing functions.[23]

Under these settings, after using the conditions for clearing two markets, $X \equiv \sum_{i=1}^m x_i^d = \sum_{j=1}^n x_j^s$ and $R \equiv \sum_{j=1}^n r_j^d = \sum_{i=1}^m r_i^s$, we can define and arrange social welfare W as

$$W = \sum_{i=1}^{m}[U_i(x_i^d) - Cr_i(r_i^s)] + \sum_{j=1}^{n}[B_j(r_j^c) - Cx_j(x_j^s)] - d[\sum_{i=1}^{m} z_j^h + \sum_{j=1}^{n} z_j^f] - \Gamma_h(\tau_h) - \Gamma_f(\tau_f). \quad (32)$$

3.2 Behaviors of firms

Since the model is modified by introducing the possibility of firms' illegal disposal, the utility maximization problem of each household is the same as is described in Section 2. On the other hand, the profit maximization problem of firm j is modified as below.

$$\begin{aligned} \max_{x_j^s, r_j^d, r_j^c, z_j^f} & [(P_x - t)x_j^s - Cx_j(x_j^s)] + [B_j(r_j^c) - \tau_f z_j^f - (P_r - s)r_j^d] \\ \text{s.t.} \quad & r_j^d = z_j^f + r_j^c. \end{aligned} \quad (33)$$

[20]To verify z_j^f (z_i^h), the authorities may monitor r_j^c $(x_i^d$ and $r_i^s)$ instead of z_j^f (z_i^h).

[21]To introduce monitoring into our model, we mainly follow the formulation adopted by Choe and Fraser [4].

[22]To raise the probability of detection, more searching and observation, which increases the monitoring cost, are needed. Furthermore, we assume that the cost to enforce someone to pay the penalty gets higher as the unit penalty is raised or that the feasible penalty have upper bound. In reality, firms' penalty for illegal wastes usually constitutes the duty to return to the original state and compensation for victims. But it is often difficult to levy the penalty because of the incentive to evade it and the relevant firm's low ability to pay. See Becker [1] for additional details about monitoring cost.

[23]Our setting includes the simple linear formulation of monitoring cost, that is, $\Gamma(\tau) = \gamma\tau$, which is often used; for examples, in the context of environmental economics on monitoring, see Lee [12], etc.

Similarly to Section 2, we presume $x_j^d > 0$, $r_j^d > 0$. Then, the first-order conditions with respect to x_j^d and r_j^d are

$$P_x - t = Cx_j{}'(x_j^s), \tag{34}$$

$$\mu_j = P_r - s, \tag{35}$$

and further, the first-order conditions with respect to r_j^c and z_j^f are

$$B_j{}'(r_j^c) \leq \mu_j \quad \text{with equality if } r_j^c > 0, \tag{36}$$

$$-\tau_f \leq \mu_j \quad \text{with equality if } z_j^f > 0, \tag{37}$$

where μ_j is the Lagrange multiplier of the constraint. This is considered as the marginal shadow cost to dispose of residuals for firms.

Figure 6 depicts the product market and recycling market (depicted by supposing that $m = n = 1$). If all the collected residuals are supposed to be recycled completely ($r_1^d = r_1^c$), the firm's demand curve for residuals is $P_r = B_1' + s$ by (36) and (35). On the other hand, if all the collected residuals are supposed to be disposed of directly ($r_1^d = z_1^f$), the firm's demand curve of residuals is $P_r = s - \tau_f$ by (35) and (37). By tracing the upper curve between the marginal benefit of recycling ($B_1' + s$) and that of direct disposal ($s - \tau_f$), we obtain the firm's demand curve of residuals, represented by the bold line ℓ_1. On this curve, the firm recycles the collected residuals from point O till A and disposes of them directly beyond the point A.

From this figure, we can gain an insight into how the households and firms react to policy instruments. Similarly to Section 2, from the first-order conditions, we find the supply curve of residuals (ℓ_2), the demand curve of products (ℓ_3), and the supply curve of products (ℓ_4). Consequently, under the policy set in Figure 6, the equilibrium outcomes $x_1^d = x_1^s$ are determined to be length OC; $r_1^d = r_1^s$, length OB; z_1^h, length BC; r_1^c, length OA; and z_1^f, length AB. We illustrate the case where both the household and firm emit direct wastes. On the other hand, if we have $\tau_h = 0$ and $s = 0$; $x_1^d = x_1^s$ is found to be length OC'; $r_1^d = r_1^s = r_1^c$, length OA'; z_1^h, length A'C'; and $z_1^f = 0$, at the equilibrium. While τ_h and s encourage the reduction of wastes and the promotion of recycling, they can make direct disposal more beneficial for firms than complete recycling, if the monitoring level

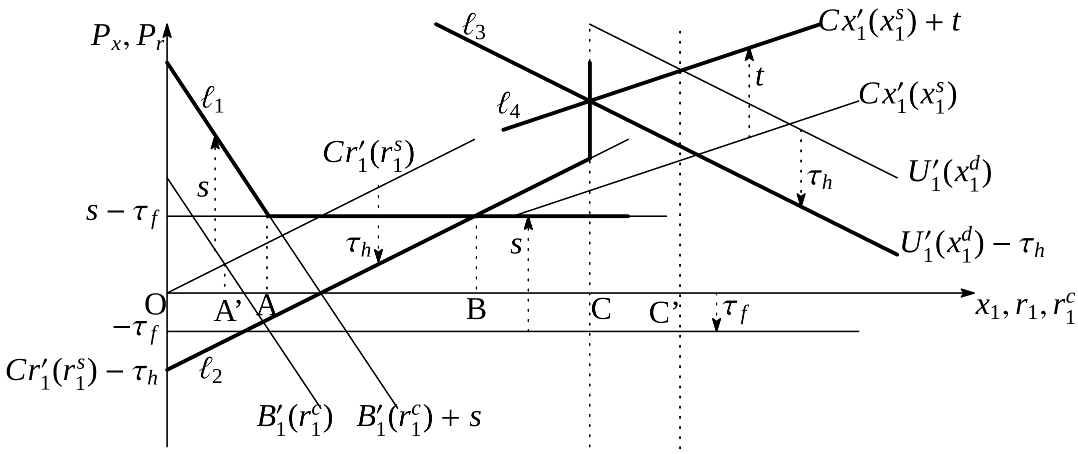

Figure 6: Reactions to the policy instruments

τ_f is not enough. To encourage recycling, the authorities intend to provide some incentive for recycling by subsidizing collected residuals or levying penalty on households' wastes. The direct effect of this incentive, however, works on the collecting activities, not on the reprocessing activities. Therefore, this policy can make firms collect more residuals than the amount of residuals that they would like to reprocess. In other words, it is excessive encouragement (relative to monitoring) that provides firms with the incentive to directly dispose of residuals.

Then, to induce the optimal policy, we can exclude the situations where the firms directly dispose of their residuals. As stated above, excess encouragement provides firms with the incentive to directly dispose of residuals. Thus, if there is some direct disposal from firms, we can prevent it by reducing τ_h or s. Moreover, we can adjust the reduction level so that firms' direct wastes are converted into household wastes and other things remain constant. This saves the collecting cost of residuals, resulting in the improvement of the welfare. The following lemma states this fact. Let $Z_f^*(t, s; \tau_f, \tau_h)$ be the equilibrium outcome of firms' aggregate direct disposal under a policy set $(t, s; \tau_f, \tau_h)$.

Lemma 2. *When $Z_f^*(\bar{t}, \bar{s}; \bar{\tau}_f, \bar{\tau}_h) > 0$ for a given policy set $(t, s; \tau_f, \tau_h) = (\bar{t}, \bar{s}; \bar{\tau}_f, \bar{\tau}_h)$, there exists an alternative policy set which improves welfare.*

Proof. See the appendix. Q.E.D.

From Lemma 2, under the optimal policy, we can concentrate our discussion on such policies that can attain zero direct waste disposal from firms. To make the model tractable, from this point until the end of this paper, we make the assumption that the model has a unique optimal policy with respect to $(t, s; \tau_f, \tau_h)$[24].

4 Second-best tax-subsidy policy

In this section, we induce the optimal tax-subsidy policy to internalize the disposal cost and monitoring cost. When the authorities maximize the welfare, we can show that the D-R policy, which takes monitoring cost into account, arises as the optimal policy.

For this, we solve the optimal taxation problem. This is solved by two steps. First, given policy variables, market equilibrium outcomes are derived. Second, given these outcomes, welfare is maximized with respect to tax, subsidy, and the expected penalties of direct disposal.

4.1 Equilibrium outcomes

As the first step, we define the equilibrium outcomes and investigate how these outcomes are influenced by policy variables. By lemma 2, in order to induce the optimal policy, it is adequate if we investigate the policies that can attain zero illegal wastes from firms. Therefore, we suppose that the authorities always monitor firms at an adequately high level for that. To induce such a monitoring policy, first, we assume the demand (18), which is defined under the supposition that $Z_f = 0$, is the real demand of residuals for defining the equilibrium outcome. Under this supposition, since we assume that z_i^h exhibits an interior solution, the equilibrium outcomes of products and residuals can described as $\{x_j^{s*}(t; \tau_h), r_j^{d*}(s; \tau_h)\}_{j=1}^n$ and $\{x_i^{d*}(t; \tau_h), r_i^{s*}(s; \tau_h)\}_{i=1}^m$, and the aggregate outcomes of them as $\{X^*(t; \tau_h), R^*(s; \tau_h)\}$, which are precisely defined in Section 2.2.

Now, we seek the monitoring policy that realizes the abovementioned supposition. How should the authorities monitor firms such that they attain zero direct wastes from firms? For this, the authorities should maintain the monitoring level high enough according

[24]Note that since $r_j^d = r_j^c$ under the optimal policy from Lemma 2, the equilibrium under the optimal policy must exhibit an interior solution with respect to (x_j^s, r_j^d, r_j^c) for all j and (x_i^d, r_i^s, z_i^h) for all i, that is, all the equilibrium outcomes except for z_i^f are assumed to be positive.

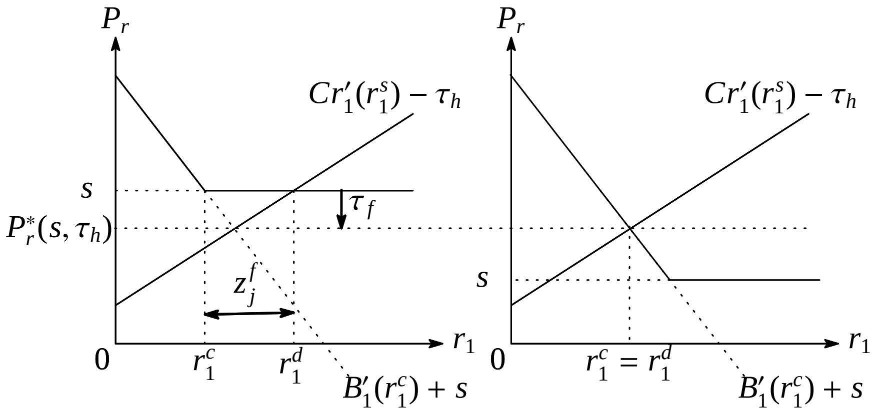

Figure 7: Optimal monitoring rule

to the *optimal monitoring rule* $\tau^*_f(s;\tau_h)$ defined as

$$\tau^*_f(s;\tau_h) = \begin{cases} s - P^*_r(s;\tau_h) & \text{if} \quad s > P^*_r(s;\tau_h), \\ 0 \quad \text{if} \quad s \leq P^*_r(s;\tau_h). \end{cases} \tag{38}$$

Lemma 3. *If $\tau_f \neq \tau^*_f(s,\tau_h)$, we can improve the welfare.*

The proof is plausible if we see the Figure 7 (depicted by supposing that $m = n = 1$). When $s > P^*_r$, as seen in the left panel of the figure, we must have $z^f_j > 0$ if there is no monitoring for firms ($\tau_f = 0$). Thus, by Lemma 2, we can improve the welfare as far as τ_f is not enough large. To be $z^f_j = 0$ under this situation, the monitoring level must increase to the levels such that satisfy $s - \tau_f \leq P^*_r$; equivalently $\tau_f \geq s - P^*_r$. Since the monitoring cost is strictly increasing in τ_f, $\tau_f > s - P^*_r$ is not optimal. When $s \leq P^*_r$, as seen in the right panel of the figure, we have $z^f_j = 0$ even if $\tau_f = 0$. Therefore, $\tau_f > 0$ is not optimal due to the positive monitoring cost (the boundary case).

When the authorities maintain the monitoring at a level that is high enough according to the optimal monitoring rule, that is, $\tau_f = \tau^*_f(s;\tau_h)$, we can write the equilibrium outcome of complete recycling as $r^{c*}_j(s;\tau_h) = r^{d*}_j(s;\tau_h)$ for all j and the aggregate outcome of them as $R^{c*}(s;\tau_h) = R^*(s;\tau_h)$. The equilibrium outcome of households' wastes $Z^*_h(t,s;\tau_h)$ can be defined by the entire economy's material balance. That is, $Z^*_h(t,s;\tau_h) = X^*(t;\tau_h) - R^*(s;\tau_h)$, because now, we have $Z^*_f(t,s;\tau^*_f(s;\tau_h),\tau_h) = 0$ owing to the optimal monitoring rule.

4.2 Welfare maximization

Next, given these equilibrium outcomes and the optimal monitoring rules, we will proceed to the second step, in which we determine the policy set to maximize welfare. The welfare maximization problem for the authorities is described as follows:

$$\max_{t,s;\tau_h} W^*(t,s;\tau_h) \equiv \sum_{i=1}^{m}[U_i(x_i^{d*}(t;\tau_h)) - Cr_i(r_i^{s*}(s;\tau_h))] + \sum_{j=1}^{n}[B_j(r_j^{d*}(s;\tau_h)) - Cx_j(x_j^{s*}(t;\tau_h))] - d[X^*(t;\tau_h) - R^*(s;\tau_h)] - \Gamma_h(\tau_h) - \Gamma_f(\tau_f^*(s;\tau_h)). \quad (39)$$

First of all, the monitoring problem for households should be prevented by the zero penalty of households' waste disposal.

Proposition 1. *Under the optimal policy, the authorities always set $\tau_h = 0$.*

Proof. See the appendix. Q.E.D.

Therefore, henceforth, we focus on the case where $\tau_h = 0$ and omit τ_h from the argument if it is unnecessary.

Remark 1 With regard to the policy for households, we always have $\tau_h = 0$. The monitoring problem for households should be prevented by free direct waste disposal. Note that this is true even if the monitoring cost for households is less than that for firms.[25] If $\tau_h > 0$, the supply curve of recycling shifts to the right and their price falls. This encourages firms not only to reprocess the residuals but also to dispose of them directly. In other words, the incentive for direct disposal spills over by way of the price of residuals. Consequently, both households and firms must be monitored to effectively operate the penalty on households' direct disposal. Therefore, using only the policy for firms saves the trouble of monitoring households. This result supports the concept of extended producer responsibility (EPR) proposed by the OECD.[26]

[25]If the market-based subsidy to some firms is not feasible, it is possible that $\tau_h > 0$ in the optimal policy, e.g., in the case that there are some unobservable very small businesses engages in recycling activities. Households' efforts with regard to waste reduction investigated in Choe and Fraser [4] is essentially what is observed in this case. Thus, the subsidy level of the policy with monitoring induced in Proposition 2 is consistent with the second-best charge on waste collection induced by them.

[26]See the OECD [14] for past discussions on EPR.

To solve the problem (39), we divide the situation into two cases according to the pattern of the optimal monitoring rule. One case is that where the authorities implement some monitoring for firms, $\tau_f^*(s) > 0$. This case arises if and only if $s > P_r^*$ by the definition of the optimal monitoring rule. The other case is that in which the authorities do not monitor the firms, $\tau_f^*(s) = 0$. This case arises if and only if $s \leq P_r^*$. Hence, before we begin the analysis, we translate the conditions that determine whether or not $s > P_r^*$ into the terms of subsidy. For this, we can find a threshold subsidy level $\hat{s}$.

Lemma 4. *There uniquely exists $\hat{s} > 0$, $s \lesseqgtr P_r^*(s)$ if and only if $s \lesseqgtr \hat{s}$.*

Proof. See the appendix. Q.E.D.

If we set the subsidy as smaller than $\hat{s}$, the price to take back the residuals is higher than the subsidy. Since the unit gain from direct waste disposal for firms is $s - \tau_f$, it is not beneficial to take back and directly dispose of such residuals even if $\tau_f = 0$. As a result, the firms do not have an incentive to directly dispose of collected residuals even without monitoring. Therefore, $\hat{s}$ is the upper limit of the subsidy level, which does not require firms to be monitored. In other words, under the optimal monitoring rules, $\tau_f^*(s) > 0$ if and only if $s > \hat{s}$. Therefore, we use $\hat{s}$ to describe the two cases mentioned above.

Now, we are ready to investigate the welfare maximization problem for the authorities. First, suppose that the authorities implement some monitoring for firms, that is, $s > \hat{s}$ under the optimal policy. In this case, the first-order conditions of the problem (39) with respect to t and s are

$$\frac{\partial W^*(t,s)}{\partial t} = \sum_{i=1}^{m} \frac{\partial x_i^{d*}}{\partial t} U_i' - \sum_{j=1}^{n} \frac{\partial x_j^{s*}}{\partial t} Cx_j' - d\frac{\partial X^*}{\partial t} = 0, \tag{40}$$

$$\frac{\partial W^*(t,s)}{\partial s} = \sum_{j=1}^{n} \frac{\partial r_j^{d*}}{\partial s} B_j{}' - \sum_{i=1}^{m} \frac{\partial r_i^{s*}}{\partial s} Cr_i' + d\frac{\partial R^*}{\partial s} - \left[1 - \frac{\partial P_r^*}{\partial s}\right]\Gamma_f' = 0. \tag{41}$$

We assume that the second-order conditions are globally satisfied when $s > \hat{s}$.[27] From (40)–(41), we obtain the optimal tax t^M and subsidy s^M.

[27]For example, if we suppose the quadratic cost functions and the quadratic utility and benefit functions, we can easily see that these are satisfied.

Proposition 2. *Suppose the authorities implement some monitoring for firms. Then, the optimal tax and subsidy are* (t^M, s^M)*, which satisfies*

$$t^M = d, \ s^M = d - A(s^M),$$

where

$$A(s^M) \equiv -\frac{\Gamma_f'(\tau_f^*(s^M))}{\sum_{j=1}^n [1/B_j''(r_j^{d*}(s^M))]} > 0.$$

We refer to this policy set as the policy with monitoring.[28]

Proof. See the appendix. Q.E.D.

The policy set in Proposition 2 and Proposition 1 is the system of imposing a tax (deposit) on the products, the reduced subsidy (refund) on taken-back residuals, and the zero penalty for household waste to avoid monitoring for households. If the marginal monitoring cost Γ_f' is infinitesimal, the induced policy is $t^M = s^M = d$. This is consistent with the first-best Deposit-Refund policy discussed in many studies (see Section 2). However, if the marginal monitoring cost is not zero, the optimal unit refund is smaller than the amount corresponding to the unit deposit. The reason for this is as follows. If the authorities adopt the policy wherein a subsidy is offered for collected residuals to encourage recycling activities, it could be beneficial for the firms to dispose of the collected residuals directly (in some cheaper but illegal and inefficient way) to aim at obtaining the subsidy. Thus, the authorities must bear monitoring costs to prevent such negative activities. Therefore, the subsidy should be reduced from the first-best level upon considering this second cost.

Next suppose that the authorities do not implement monitoring for firms, that is, $s \leq \hat{s}$ under the optimal policy. In this case, while the first-order condition with respect to t is the same as (40), the first-order condition with respect to s is modified as

$$\frac{\partial W^*(t,s)}{\partial s} = \sum_{j=1}^n \frac{\partial r_j^{d*}}{\partial s} B_j'(r_j^{d*}) - \sum_{i=1}^m \frac{\partial r_i^{s*}}{\partial s} Cr_i'(r_i^{s*}) + d\frac{\partial R^*}{\partial s} \geq 0 \text{ with equality if } s < \hat{s}. \quad (42)$$

[28]Note that A is the information in recycling market since we have the relation

$$\frac{\Gamma_f'}{\sum_j (1/B_j'')} = \frac{\Gamma_f'}{dR^d/dP_r} = \frac{P_r \Gamma_f'}{R}\left(\frac{dP_r/P_r}{dR^d/R}\right) = \frac{P_r \Gamma_f'}{R\eta_R^d}.$$

where η_R^d represents the elasticity of the aggregate demand of residuals.

By (40) and (42), we obtain the optimal tax and subsidy in this case, which is denoted by (t^N, s^N) to distinguish it from the policy with monitoring in Proposition 2.

Proposition 3. *Suppose the authorities do not implement monitoring for firms. Then, the optimal tax and subsidy are* (t^N, s^N)*, which satisfies*

$$t^N = d, \; s^N = \begin{cases} d & \text{if } d < \hat{s}, \\ \hat{s} & \text{if } d \geq \hat{s}. \end{cases}$$

We refer to this policy set as the policy without monitoring.

Proof. See the appendix. Q.E.D.

The optimal tax on products is same as in the case of Proposition 2. We, however, obtain peculiar results with regard to the optimal subsidy. To obtain s^N, we must consider two further cases in this situation: (i) $d < \hat{s}$ and (ii) $d \geq \hat{s}$. Recall that the authorities can use subsidy without monitoring as far as the subsidy does not exceed $\hat{s}$. On the other hand, if we fully encourage the recycling activities without considering the monitoring problem, the subsidy should be equal to the externality of direct disposal d. Therefore, when (i) $d < \hat{s}$, we have $s^N = d$ because there is no need to worry about monitoring. This case indicates that even if we consider the monitoring problem for firms, the first-best D-R policy is feasible when the recycling is so highly developed that the price of residuals is higher than the full-encouragement subsidy $s = d$ (See Lemma 4). On the other hand, when (ii) $d \geq \hat{s}$, if the authorities set the subsidy at the first-best level, they must implement monitoring. Thus, to encourage recycling activities, as far as possible to avoid monitoring, $s^N = \hat{s}$ is desirable. In this case, the subsidy should equals the price of the recycling market by Lemma 4. In other words, the authorities can not subsidize the recycling activities more than the subrogation of collecting cost.[29]

Now, is it desirable to implement monitoring for firms or not?

Proposition 4. *The policy with monitoring is the optimal policy if and only if* $d > \hat{s} + A(s^M)$.

[29]For example, in Japan's "Act for recycling of containers and package," the local governments rather than the firms are responsible for collecting residuals. According to the results of Proposition 3, this policy is rational if the government regards the monitoring cost of the firms as too great to implement the monitoring but the externality of direct disposal is higher than the collecting cost.

cases	$d > \hat{s} + A(s^M)$	otherwise	
		$d \geq \hat{s}$	$d < \hat{s}$
tax t	d		
subsidy s	$d - A(s^M)$	$\hat{s}$	d
monitor τ_f	$\tau_f^*(s^M) > 0$	0	
monitor τ_h	0		

Table 1: Optimal Policy Set

Proof. See the appendix. Q.E.D.

Our results with regard the second-best policy are somewhat complicated. For convenience, Table 1 summarizes the results of Proposition 1 to 4. Lastly, in this section, we provide some supplemental comments about the optimal tax-subsidy policy.

Remark 2 In any case, regardless of the state of affairs with regard to recycling, the optimal pre-tax on a unit product due to the reason of future disposal is its marginal disposal cost in equilibrium. We presume the situation where $Z_h^* > 0$ under the optimal policy. Thus, if attention is paid to the additional production in equilibrium, the structure where direct waste disposal is the only option for disposal[30] appears in our model. Thus, we always have $t = d$ since the tax in our model works as the advanced disposal fee on products due to the disposal cost.

Remark 3 The optimal subsidy level on residuals to encourage recycling should vary: the level of full encouragement $s = d$, the level subsidizing only the collecting cost $s = \hat{s}(\leq d)$, and the moderate level considering monitoring cost $s = d - A(> \hat{s})$. Which level is desirable is determined according to the relation among monitoring cost, disposal cost, and recycling technology. Figure 8 depicts the pattern of the optimal subsidy.[31] The horizontal axis represents the disposal cost d. When the disposal cost is small ($d < \hat{s}$) relative to given recycling technology, since the price of recycling market $P_r^*(s)$ is higher than the subsidy s even under the full encouragement $s = d$, the first-best policy can be achieved

[30] Wertz [20] constructed such a model, showed that the disposal fee should equal the corresponding disposal cost, and analyzed its effects on the demand for products.

[31] This figure is depicted in the case of the quadratic cost functions and the quadratic utility and benefit functions.

without monitoring. When the disposal cost gets larger ($d > \hat{s}$), if the full encouragement $s = d$ is implemented, $P_r^*(s)$ is less than s and thus, the firms have the incentive to dispose the collected residuals. Therefore, the input substitution effect should be "weakened" to avoid the monitoring problem of them. At first, when $d < \hat{s} + A$, the authorities set $s = \hat{s}$ (and achieve $P_r^*(s) = s$) not to bear monitoring cost for the firms. However, if the disposal cost gets further larger, the difference between t and s expands. This difference generates distortion since the waste reduction is biased toward the output reduction effect. Therefore, finally, when $d > \hat{s} + A$, the authorities set $s > \hat{s}$ and implement monitoring considering the trade off between the abovementioned distortion and monitoring cost.

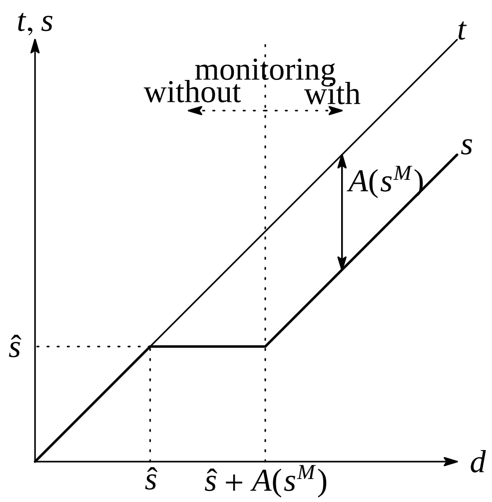

Figure 8: Second-best tax and subsidy

5 Policy with a take-back requirement and tradable rights

In the previous section, we analyzed the D-R policy. Indeed for economical analysis, it is convenient to consider tax and subsidy as benchmarks, but in actual policies to encourage recycling, the authorities often regulate the firms and require them to take back their products' residuals. In this section, we consider the case wherein authorities implement a policy with such a take-back requirement. More concretely, each firm has the responsibility to collect residuals depending on the amount they produce. To make this policy feasible, the authorities must know the amount of each firm's products and collecting residuals. In our paper, we assumed that $(x_1^s, \cdots, x_n^s)$ and $(r_1^d, \cdots, r_n^d)$ are observable. Therefore, the authorities can enforce the take-back requirement of residuals.[32]

Consider the situation where all firms have homogeneous technology for production and recycling, and the monitoring problem is put aside for the present. Under this situation, it is evident that even in our model, as Fullerton and Wu [10] showed, the policy with a take-back requirement works well in the settings in which the authorities only require firms to take back appropriate quantity of residuals[33] and each firm recycles exactly the amount it collects.

Our model, however, allows the heterogeneity of firms' production and recycling technology. Thus, the recycling depending on each firm's outputs is not efficient, because each firm's recycling technology need not be related to the production technology. Therefore, a market for the resale of the collected residuals among firms must arise. In this market, collecting residuals will be transacted from the firms that produce much but have lower recycling technology to the firms that produce little but have higher recycling technology. Hence, it is also important for the authorities to prepare an appropriate system for such transactions and not just enforce the take-back requirement. To formalize this market, we apply the theory of tradable emission permits market, because of tractability and elegancy of formulation and the possibility of variety in interpretation for real systems[34].

5.1 Behaviors of firms

Even when the authorities use the policy with a take-back requirement, the behaviors of households is the same as is described in Section 2. Noting this, we begin to investigate the behaviors of firms.

We consider the system to trade the collecting residuals as follows. At the outset, each firm has the responsibility to take back residuals in proportion to the amount of its own output. αx_j^s represents the amount that firm j is required to take back. $\alpha \in \mathbb{R}_+$ is a policy parameter and this can be interpreted as the momentousness of firms' responsibility to their

[32]In this paper, residuals are anonymous. This means that the residuals are not distinguished with regard to the firm that produced them. Hence, it is the quantity of taking-back residuals that is required.

[33]The authorities should put firms under obligation to take back $x - W^*/n$ of residuals, where x is the common quantity of each firm's outputs and W^* is the desirable amount of direct disposal. Readers will be able to understand this easily form the rest of analysis of this section.

[34]In real world, the United Kingdom has the tradable credit scheme applied to packaging waste. See Walls [18] in detail.

products' residuals. If a firm collects more residuals than is required, that is, $r_j^d - \alpha x_j^s > 0$, the firm can contract to take over other firms' responsibility verifiably. In other words, the firm can sell the right to be exempted from the responsibility to take back the same amount of residuals as it has excessively collected. Based on our context, we call this right tradable rights of exemption from take-back requirement (TRET). Conversely, if a firm's collection of residuals is less than the responsibility, that is, $r_j^d - \alpha x_j^s < 0$, the firm must buy TRET at the amount of deficiency. Summing up, $r_j^d - \alpha x_j^s$ represents firm j's supply of TRET; and $\alpha x_j^s - r_j^d$, the demand. The authorities also supply TRET. Similar to the TRET that the firms supply, if the firms hold this, they can avoid the responsibility of collecting residuals by the amount that they hold. The authorities supply these rights by some means independent from firms' control variables.[35] The authorities control the total amount of TRET that they supply, $Z_T \in \mathbb{R}_+$.

We denote the competitive price of the TRET by P_T. Then, the profit maximization problem of firm j is

$$\begin{aligned} \max_{x_j^s, r_j^d, r_j^c, z_j^f} & [P_x x_j^s - Cx_j(x_j^s)] + [B_j(r_j^c) - \tau_f z_j^f - P_r r_j^d] - P_T(\alpha x_j^s - r_j^d) \\ \text{s.t.} \quad & r_j^d = z_j^f + r_j^c. \end{aligned} \tag{43}$$

After presuming $x_j^s > 0$ and $r_j^d > 0$, the first-order conditions are

$$P_x - \alpha P_T = Cx_j{}'(x_j^s), \tag{44}$$

$$\mu_j = P_r - P_T, \tag{45}$$

$$B_j{}'(r_j^c) \leq \mu_j \quad \text{with equality if } r_j^c > 0, \tag{46}$$

$$-\tau_f \leq \mu_j \quad \text{with equality if } z_j^f > 0. \tag{47}$$

These conditions are the same as those under the D-R policy (34)-(37) except that t is replaced by αP_T and s is replaced by P_T. If a firm raises its output by one unit, the amount required for collection increases by α units. Thus, the firm must buy additional α units of TRET. Hence, the price of TRET multiplied by α plays the role of output tax. If a firm raises its collection by one unit, it can reduce its demand of TRET or increase its supply

[35] For example, the TRET supplied by the authorities can be initiated by a grandfathering system or auction.

of TRET by one unit. Therefore, the price of TRET also plays the role of subsidy in the collection of residuals.

5.2 Welfare comparison

We will examine whether the policy with a take-back requirement is as effective as the second-best D-R policy described in Subsection 4.2. First of all, we can check the following situation never attain the welfare level in the second-best D-R policy.

Lemma 5. *When $Z_f > 0$ or $\tau_h > 0$ in a policy with take-back requirement, the second-best D-R policy is always superior to this policy.*

Proof. See the appendix. Q.E.D.

Therefore, we focus on the situation where $\forall j, z_j = 0$[36] and $\tau_h = 0$.[37] In this situation, by replacing (t, s) with $(\alpha P_T, P_T)$ and fixing $\tau_h = 0$,[38] we can describe the equilibrium prices and outcomes given the price of TRET by using those in Subsection 2.2.

The price of TRET is determined by the market clear condition,

$$\alpha X^*(\alpha P_T) - R^*(P_T) = Z_T. \tag{48}$$

From this condition, we can show the following.

Lemma 6. *If $P_T > 0$, (α, Z_T) and $(\alpha P_T, P_T)$ are one to one by (48).*

Proof. See the appendix. Q.E.D.

Therefore, although the true control variables are the policy variables, α and Z_T, we can treat αP_T and P_T as the control variables instead of them. Write Z_T as satisfying (48) in the form $Z_T = Z_T(\alpha P_T, P_T)$.

[36] On the tradable emission permit (TEP) market when firms are non-compliant, see Malik [13] for a more advanced discussion. The study discusses the problem wherein the existence of non-compliant firms can make the TEP market suboptimal. However, in our model, we can avoid such a problem, since firms are potentially non-compliant but in the equilibrium under the optimal policy, they are compliant.

[37] The existence of free penalty for households' wastes is another difference from Fullerton and Wu [10]. In their model, firms themselves directly dispose of residuals instead of the households when there is a take-back requirement. In our model, however, such a situation cannot be optimal since extra collecting cost Cr is introduced.

[38] Therefore, we omit τ_h from the argument.

By viewing control variables in this way, we can obtain the welfare maximization problem entirely dual to the problem (39) by fixing $\tau_h = 0$.[39] Then, if we replace t and s in Section 4 with αP_T and P_T in this section, respectively, we can get a perfectly parallel optimal solution. Hence, we obtain the following Proposition.

Proposition 5. *The optimal policy with a take-back requirement works equivalently to the optimal D-R policy.*

However, observe that $t^M > s^M$ or $t^N > s^N$ if $d > \hat{s}$ in Section 4; therefore, similarly $\alpha^M P_T^M > P_T^M$ and $\alpha^N P_T^N > P_T^N$ if $d > \hat{s}$, where superscript "M" represents the optimal value with monitoring of each valuable, and superscript "N" represents the optimal value without monitoring of each valuable. Thus, in each case, the required momentousness of responsibility to the residuals of additional production, α^M and α^N, must be more than actual additional residuals 1.

It may sound strange to assert that $\alpha^M > 1$, because this regulation seems to require firms to collect non-existent residuals. However, this is not the case, because to fill the gap, the authorities supply more TRET than the wastes that are actually directly disposed of by households. From (48) and $\alpha^M > 1$, $Z_T(\alpha^M P_T^M, P_T^M) \equiv Z_T^M = \alpha^M X^M - R^M > Z_h^M \equiv X^M - R^M$, where $X^M \equiv X^*(\alpha^M P_T^M)$ and $R^M \equiv R^*(P_T^M)$. The relation between take-back requirement and actual residuals are depicted in Figure 9. If $Z_T^M = Z_h^M$, the take-back requirement must exceed the rest of the actual residuals other than the wastes directly disposed of. By setting $Z_T^M > Z_h^M$, as we can seen at the point E in the figure, these two matches each other in equilibrium. The same discussion holds true for $\alpha^N > 1$.

Proposition 6. *When $d > \hat{s}$, we must have $\alpha > 1$ and $Z_T > Z_h$ under the optimal policy.*

In reality, Z_T will be distributed in an output-based form rather than in the form of TRET. More concretely, the amount of take-back requirement $\alpha X - Z_T$ is zero when $X = Z_T/\alpha$. Then, if firms produce away from these points, the requirement rises and falls at the rate α. In other words, the initial exemption points of the requirement, which are equal

[39]Note that since P_T plays the role of subsidy, to attain zero direct disposal from firms, we should adopt the optimal monitoring rule $\tau_f^*(P_T)$, which is defined in Subsection 4.1, and to check the pattern of the optimal monitoring rule, we can use the subsidy level $\hat{s}$.

to Z_T^M/α^M (if the points are summed up), based on firms' output, are offered to each firm. When $d > \hat{s}$, from $\alpha > 1$ and $R > 0$, this initial-exemption level is greater than the optimal wastes, that is, $Z_T/\alpha = X - R/\alpha > X - R = Z_h$ (See the Figure 9).

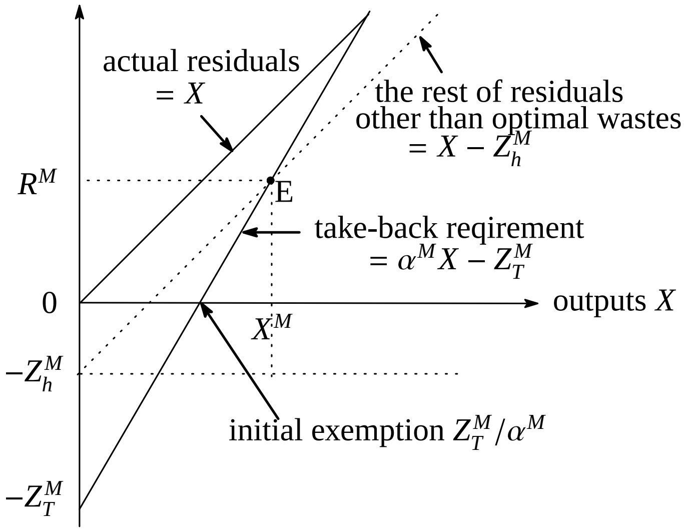

Figure 9: Structure of take-back requirement

6 Concluding remarks

We investigated the optimal D-R policy with monitoring cost under the circumstance where illegal disposal on the part of both households and firms can occur and the policy instruments are the tax on products, the subsidy on collected residuals, and monitoring for households and firms. In this system, the authorities should set the penalty for household waste to zero and monitor firms instead of households. The deposit should always be equal to the disposal cost since this is the advanced fee on products due to the disposal cost, but the refund, which is the subsidy on collected residuals to encourage recycling, should vary according to the relation among monitoring cost, disposal cost, and recycling technology and typically be smaller than the first best level.

Further, we compared the policy with a take-back requirement to the D-R policy. We find that the second-best system of this policy is the combination of the take-back requirement depending on the amount of each firm's outputs and initial exemption from that requirement. Although, even if there exists a monitoring problem, the two policies work equivalently, the marginal momentousness of the take-back requirement, should typically be larger than the actual additional waste generation. In this case, the initial exemption of take-back requirement should be given to the firms and the amount of this exemption is larger than the optimal wastes.

Finally, we point out the limitation of our model and the possibility of future research. First, we focus on the situation where the zero emission is not optimal and the benefit function of recycling is given exogenous. Owing to these assumptions, we can treat the prices of product market and recycling market independently, and decompose the output reduction effect and the input substitution effect. Second, we used a very simple formulation with regard to monitoring. More generally, Γ_h or Γ_f can depend on the amount of violation, z_i^h or z_j^f. If we use such an extended formulation, positive violation arises in the equilibrium under the optimal policy. This affects the price of TRET and the policy with a take-back requirement will be less effective. Third, we focused on one kind of good in the partial equilibrium model. However, more generally, if the other goods have externality with regard to disposal cost, it is possible that the internalizing disposal cost of good x causes more externality in the other goods. Therefore, it is necessary to consider the relations with the other goods. In reality, we should prepare a list on the waste generation rates of various kinds of goods that have high substitutability with each other,[40] and simultaneously internalize the disposal cost. Fourth, we investigated the situation where the residuals of each firm's products are anonymous. But sometimes, especially in the case where there are several famous makers in the market, the authorities can trace down which firm's products the illegally disposed residuals are. Using this type of monitoring, the authorities can require firms to take back the residuals of their own products in the name of EPR.[41] This may be cheaper than monitoring which firm illegally disposes of the residuals. Therefore, when this type of monitoring is feasible, we can predict the possibility that the policy with EPR is supported over the D-R policy.

[40] For example, in Japan, such a list is made for the policy to recycle packages and containers.

[41] Ino [11] analyzes the residuals that are not anonymous in an oligopoly market and the policy with EPR.

Appendix

Lemma 2

Proof. First, we describe the demand in the recycling market. From (35), (36), and (37), the demand correspondence of r_j^d is described as

$$r_j^d(P_r; s, \tau_f) = \begin{cases} B_j'^{-1}(P_r - s) & \text{if } P_r - s \geq -\tau_f \\ [\hat{r}_j(\tau_f), \infty) & \text{if } P_r - s = -\tau_f, \end{cases} \tag{49}$$

where $\hat{r}_j(\tau_f) \equiv B_j'^{-1}(-\tau_f)$. Therefore, the aggregate demand correspondence of collected residuals is described

$$R^d(P_r; s, \tau_f) = \begin{cases} \sum_{j=1}^n B_j'^{-1}(P_r - s) & \text{if } P_r - s \geq -\tau_f \\ [\hat{R}(\tau_f), \infty) & \text{if } P_r - s = -\tau_f, \end{cases} \tag{50}$$

where we define $\hat{R}(\tau_f) \equiv \sum_{j=1}^n \hat{r}_j(\tau_f)$. This demand curve is described in Figure 10.

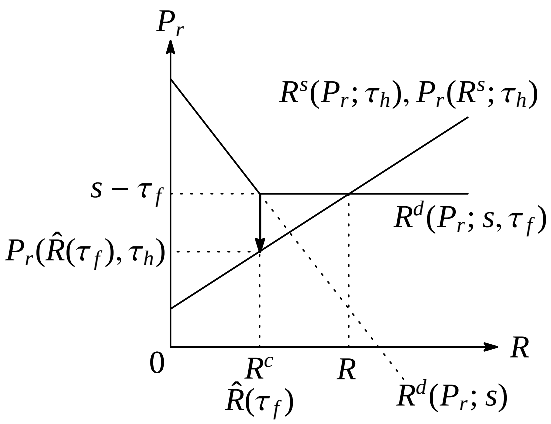

Figure 10:

Next, noting that the equilibrium outcomes are defined just by replacing the demand curve of recycling in Section 2 with the demand curve induced above, we will alter the policy instruments. Suppose that $Z_f^*(\bar{t}, \bar{s}; \bar{\tau}_f, \bar{\tau}_h) = \bar{Z}_f > 0$. We depict the starting state of each case in Figure 10. In this case, the following must be satisfied: $P_r(\hat{R}(\bar{\tau}_f); \bar{\tau}_h) < \bar{s} - \bar{\tau}_f$ and $R^c = \hat{R}(\bar{\tau}_f)$. Reset $s = \bar{s} - \{(\bar{s} - \bar{\tau}_f) - P_r(\hat{R}(\bar{\tau}_f); \bar{\tau}_h)\}$. The downward arrow in the figure represents this policy alteration. Then, $s - \bar{\tau}_f = P_r(\hat{R}(\bar{\tau}_f); \bar{\tau}_h)$. Therefore, $Z_f^* = 0$ and still, $R^c = \hat{R}(\bar{\tau}_f)$. So, $\Delta R = \bar{Z}_f$, where $R \equiv \sum_{j=1}^n r_j^d$ and Δ represent the difference between the starting state and the state after the alteration of the policy. Since s does not affect the supply and demand of products, $\Delta R = \Delta Z_h$. Because the monitoring level is unchanged, the monitoring cost is constant. Thus, $\Delta W = Cr(\bar{Z}_f + \hat{R}(\bar{\tau}_f)) - Cr(\hat{R}(\bar{\tau}_f)) > 0$. Q.E.D.

Proposition 1

Proof. Suppose that the policy set $(t, s; \tau_h) = (\bar{t}, \bar{s}; \bar{\tau}_h)$ is optimal (the solution of problem (39)), but $\bar{\tau}_h > 0$. Note that we have $\tau_f = \tau_f^*(\bar{s}; \bar{\tau}_h)$ according to the optimal monitoring rule. Then, the equilibrium outcomes under this policy set are $(X, R, Z_h, Z_f) = (X^*(\bar{t}; \bar{\tau}_h), R^*(\bar{s}; \bar{\tau}_h), X^*(\bar{t}; \bar{\tau}_h) - R^*(\bar{s}; \bar{\tau}_h), 0)$.

Consider the case where the authorities reset the policy $(t, s; \tau_h) = (\bar{t}+\bar{\tau}_h, \bar{s}+\bar{\tau}_h; 0)$ under the optimal monitoring rule $\tau_f = \tau_f^*(\bar{s}+\bar{\tau}_h; 0)$. Then, the equilibrium outcomes in this case are exactly the same as those before the resetting. This is because $X^*(\bar{t}; \bar{\tau}_h) = X^*(\bar{t} + \bar{\tau}_h; 0)$ by (28), $R^*(\bar{s}; \bar{\tau}_h) = R^*(\bar{s} + \bar{\tau}_h; 0)$ by (31), and $Z_h^* = X^* - R^*$ by $Z_f^* = 0$ owing to the optimal monitoring rule. Furthermore, since $\partial(s - P_r^*)/\partial s = \partial(s - P_r^*)/\partial \tau_h$ by (29) and (30), we must have $\tau_f^*(\bar{s}; \bar{\tau}_h) = \tau_f^*(\bar{s} + \bar{\tau}_h; 0)$ by the definition of the optimal monitoring rule.

Since only the monitoring cost for households is changed and other things are constant after the alteration of the policy set, $W^*(\bar{t} + \bar{\tau}_h, \bar{s} + \bar{\tau}_h; 0) - W^*(\bar{t}, \bar{s}; \bar{\tau}_h) = \Gamma_h(\bar{\tau}_h) - \Gamma_h(0)$. Since $\Gamma_h(\bar{\tau}_h) > \Gamma_h(0)$ by $\bar{\tau}_h > 0$, the monitoring cost for households is saved by the policy resetting. This improves the equilibrium welfare — a contradiction. Q.E.D.

Lemma 4

Proof. First, we will show that $\hat{s} > 0$ which satisfies $\hat{s} = P_r(\hat{s})$ exists. By (18), $\forall s$, $R^d(s; s) = \sum_j B_j'^{-1}(0) = R^d(0; 0) > 0$, where the existence of $B_j'^{-1}(0) > 0$ comes from the assumptions that $B_j'(0) > 0$ and that $\lim_{r\to\infty} B_j'(r) = -\infty$. Set $s = P_r(R^d(0, 0)) > 0$. Then, $P_r^*(s) = s$ since the recycling demand $R^d(P_r; s)$ and the recycling supply $P_r(R^s)$ intersect at $(R^d(0, 0), P_r(R^d(0, 0)))$. Thus, we obtain $\hat{s} = P_r(R^d(0, 0))$.

Next, ascertain the fact that $s - P_r^*(s)$ is strictly increasing in s. When $R^*(s) > 0$, from (29),

$$\frac{\partial s - P_r^*}{\partial s} = \frac{\sum_{i=1}^m (1/Cr_i'')}{\sum_{i=1}^m (1/Cr_i'') - \sum_{j=1}^n (1/B_j'')} > 0.$$

Since $\hat{s} - P_r^*(\hat{s}) = 0$, the uniqueness of $\hat{s}$ and claim that $s \gtreqless P_r^*(s)$ if and only if $s \gtreqless \hat{s}$ immediately follows from this fact. Q.E.D.

Proposition 2

Proof. Plug the first-order conditions for products markets into (40) and solve the equation with respect to t. Then, we obtain $t = d$.

The next step is to induce the optimal subsidy with monitoring. Suppose that $s > \hat{s}$; then, the authorities need to implement some monitoring for firms. Plug into the first-order conditions for the recycling market into (41) and solve the equation with respect to s using (29) and (31). Then, we obtain the requested result. Q.E.D.

Proposition 3

Proof. Since the first order condition with respect to t is (40), we can find that the optimal tax is still $t^N = d$ by exactly the same way as is the case in the proof of Proposition 2.

The next step is to induce the optimal subsidy without monitoring. Note that the interior solution, $R^*(s) > 0$, is assumed. By substituting the households' and firms' first-order conditions for the recycling market, we rewrite the left hand side of (42) as $[P_r^*(s) - s - P_r^*(s) + d]\partial R^*/\partial s = (d-s)\partial R^*/\partial s$. Thus, by (31), the first-order condition (42) is reduced as

$$d - s \geq 0, \text{ with equality if } s < \hat{s}. \tag{51}$$

(i) Consider the case $d \geq \hat{s}$. If $s < \hat{s}$, we must have $d < \hat{s}$ since $s = d$ by (51). This cannot occur in this case. Meanwhile, $s = \hat{s}$ satisfies (51) since $d \geq \hat{s}$.

(ii) Consider the case $d < \hat{s}$. If $s = \hat{s}$, we must have $\hat{s} \leq d$ since $s \leq d$ by (51). This cannot occur in this case. Meanwhile, $s = d$ satisfies (51) since $d < \hat{s}$. Q.E.D.

Proposition 4

Proof. Observe that since each equilibrium outcome is continuous in s by the maximum theorem and $\tau_f^*(s)$ is also continuous in s by its definition and Lemma 4, W^* is, in particular, at $s = \hat{s}$, continuous in s.

Suppose that $s^M > \hat{s}$, that is, $d > \hat{s} + A(s^M)$, where s^M is defined in Proposition 2. In this case, since $d > \hat{s}$, the optimal policy without monitoring s^N is $\hat{s}$, which is defined in Proposition 3. Since we suppose that the second-order condition is globally met when $s > \hat{s}$, $W^*(t^M, s^M) - W^*(t^N, s^N) = W^*(d, s^M) - W^*(d, \hat{s}) > 0$ if $s^M > \hat{s}$ by the continuity of W^* at $\hat{s}$.

Otherwise, there arises the case where $s^M \leq \hat{s}$ or the case where s^M does not exist, $W^*(d, s) - W^*(d, \hat{s}) < 0$ for all $s > \hat{s}$. This is because if there exists $\bar{s} > \hat{s}$ such that $W^*(d, \bar{s}) - W^*(d, \hat{s}) > 0$, $W^*(d, s)$ is increasing in s when $s \geq \bar{s}$ since s^M, which satisfies $\partial W^*/\partial s = 0$, does not exist in the region where $s > \hat{s}$ — this contradicts the existence of the optimal policy with positive direct disposal. Therefore, the optimal policy without monitoring is the global optimal policy by the continuity of W^* at $\hat{s}$. Q.E.D.

Lemma 5

Proof. Under the policy with a take-back requirement in the statement, suppose that $P_T = \bar{P}_T$. By setting $t = \alpha\bar{P}_T$ and $s = \bar{P}_T$, we can replicate the same outcome in the D-R policy. From Lemma 2 and Proposition 1, the optimal D-R policy is superior to this policy. Q.E.D.

Lemma 6

Proof. Fix (α, Z_T). Then, the left-hand side of (48) is strictly decreasing in P_T since

$$\begin{aligned}\frac{\partial[\alpha X^*(\alpha P_T) - R^*(P_T)]}{\partial P_T} &= \alpha^2 \frac{\partial X^*(\alpha P_T)}{\partial(\alpha P_T)} - \frac{\partial R^*(P_T)}{\partial P_T} \\ &= \frac{\sum_{j=1}^{n}(1/Cx_j'') \sum_{i=1}^{m}(1/U_i'')}{\sum_{j=1}^{n}(1/Cx_j'') - \sum_{i=1}^{m}(1/U_i'')} - \frac{-\sum_{i=1}^{m}(1/Cr_i'') \sum_{j=1}^{n}(1/B_j'')}{\sum_{i=1}^{m}(1/Cr_i'') - \sum_{j=1}^{n}(1/B_j'')} < 0\end{aligned}$$

where the third expression is obtained from (28) and (31). Therefore, for given (α, Z_T), P_T and thus αP_T must be uniquely determined by (48). Conversely, fix $(\alpha P_T, P_T)$. Obviously, α is uniquely determined by $\alpha = \alpha P_T/P_T$. Then, Z_T must also be uniquely determined by (48) since for given (α, P_T), the left-hand side of (48) is strictly decreasing in P_T. Q.E.D.

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