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When Are Voluntary Export Restraints Voluntary? : A Differential Game Approach

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WHEN ARE VOLUNTARY EXPORT RESTRAINTS VOLUNTARY?: A DIFFERENTIAL GAME APPROACH*

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We revisit voluntariness of voluntary export restraints (VERs) in a differential game model of duopoly with sticky prices. We show that a VER set at the free trade level has no effect on equilibrium under open-loop strategies while the same policy results in a smaller profit for the exporting firm, i.e. it is involuntary under a *non-linear* feedback strategy. Moreover, we prove an extended proposition of Dockner and Haug (1991) on voluntariness of VERs under a linear feedback strategy.

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I. INTRODUCTION

Voluntary export restraints (VERs) have been adopted as a popular trade policy since they are expected to mitigate resisting pressures in the exporting country. According to the website of the World Trade Organization (WTO), 'The WTO Safeguards Agreement broke new ground in prohibiting 'grey area' measures and setting time limits ('sunset clause') on all safeguard actions.' That is, VERs are in principle prohibited, but they have still been observed around the world. A recent example is China's VER imposed on fibre exports to the United States the deadline of which was postponed to the end of 2008.

There are considerable works studying effects of VERs. In a seminal work, Harris (1985) proves that a VER set at the free trade level is voluntary in the sense that it gives rise to higher profits for both the home firm and foreign firms. Applying a conjectural variations approach, Mai and Hwang (1988) find that a VER raises (resp. lowers) the exporting firm's profit if the pre-VER equilibrium is more competitive (resp. collusive) than Cournot.[1] On the other hand, if the pre-VER equilibrium is Cournot, namely, the conjectural variations are zero, the VER does not change the exporting firm's profit. To sum, Mai and Hwang (1988) reveal that the sign of conjectural variations plays a relevant role for voluntariness of VERs (See Figure 1).

(Figure 1 around here)

Nevertheless, as Mai and Hwang (1988) admit, the conjectural variations approach contains no dynamic foundation, which is criticized in other literature as well, e.g. Eaton and Grossman (1986). The purpose of this paper is to overcome this difficulty by developing a differential game model.

There are two predecessors that share the same objective as ours. Applying a dynamic duopoly model developed by Simaan and Takayama (1978) and Fershtman and Kamien (1987), Dockner and Haug (1991) establish the following result.[2] A VER at the free trade level increases the exporting firm's profit, i.e. it is voluntary if the home and foreign firms choose a *linear* feedback strategy in free trade. The reasoning behind their finding is as follows. As shown by Fershtman and Kamien (1987), the Nash equilibrium under the linear feedback strategy is more competitive than the static Cournot outcome.[3] Hence, relating Mai and Hwang's (1988) observation to this feature of linear feedback Nash equilibrium, a VER will raise the foreign firm's profit.

Constructing a capital accumulation model of duopoly, Calzolari and Lambertini (2007, p. 3836) show that 'with substitute goods and quantity-setting firms, any VER hurts the firm employing this policy. Hence, contrary to the conclusions reached by Dockner and Haug (1991), VERs are not 'voluntarily' employed by Cournot firms.' What deserves attention in their finding is that open-loop and closed-loop solutions coincide so that the pre-commitment does not matter in their result. This paper returns to Dockner and Haug (1991) and fulfills a gap they leave. While they focus on *linear* feedback Nash equilibrium, we allow for other equilibria: open-loop and *non-linear* feedback Nash equilibria. We show that a VER has no effect in the open-loop Nash equilibrium and that it decreases the foreign firm's profit in one of the non-linear feedback Nash equilibria.[4] Moreover, we prove the validity of Dockner and Haug's (1991) proposition for an arbitrary rate of price adjustment and discount.[5] In short, there is a one-to-one relationship between the strategy in the differential game and the sign of conjectural variation. (See Figure 1) Our result, together with Dockner and Haug's (1991), will be useful since it provides Mai and Hwang (1988) with a dynamic foundation.

The rest of this paper is structured follows. Section II develops a basic model and derives the open-loop and non-linear feedback Nash equilibria in free trade. Section III characterizes the post-VER equilibrium and re-examines the effects of VERs. Section IV concludes. Appendix proves that the Dockner-Haug (1991) proposition assuming an infinite price adjustment survives arbitrary stickiness of prices.

II. A FREE TRADE EQUILIBRIUM

Consider a homogeneous product market of a country, say Home. The market is duopolized by one Home firm and one Foreign firm both of which compete in quantities. All the Foreign variables are asterisked. The inverse demand function is linear:

$$a - x - x^*, \quad a > 0, \tag{1}$$

where x is the Home firm's output and x^* is the Foreign firm's output. Each firm has an identical cost function, which is specified by $cx + x^2/2, a > c > 0$ and maximizes the discounted sum of profits by choosing the time profile of outputs. In the present setting, the current price p is a state variable. Given these specifications, the Home firm's profit maximization problem is formulated by

$$\begin{aligned} \max_{x} \quad & \int_0^\infty e^{-rt} \left(px - cx - \frac{x^2}{2} \right) \mathrm{d}t, \quad r > 0 \\ \text{s.t.} \quad & \dot{p} = s(a - x - x^* - p), \quad s > 0, \end{aligned} \tag{2}$$

where r and s are the constant rate of discount and the speed of price adjustment, respectively. Eq. (2) means that the price is sticky and rises (resp. declines) when the price implied by the demand is larger (resp. smaller) than the current price. In the rest of this section, we seek two solutions of this game.[6]

a) Open-loop Nash equilibrium

Let us begin with the open-loop Nash equilibrium. To this end, set up the Home firm's current value Hamiltonian:

$$H = px - cx - \frac{x^2}{2} + \lambda s(a - x - x^* - p),$$

where λ is the costate variable. Then, the first-order necessary conditions are (2) and

$$0 = p - c - x - \lambda s \tag{3}$$

$$\dot{\lambda} = \lambda(r + s) - x, \tag{4}$$

together with the transversality condition: $\lim_{t\to\infty} e^{-rt}\lambda p = 0$. Solving (3) for x yields

$$x = p - c - \lambda s. \tag{5}$$

Substituting (5) into (4), we have

$$\dot{\lambda} = \lambda(r + 2s) - p + c. \tag{6}$$

Letting λ^* be the Foreign firm's costate variable, an equation similar to (6) holds:

$$\dot{\lambda^*} = \lambda^*(r + 2s) - p + c. \tag{7}$$

Furthermore, substituting (5) and the Foreign firm's counterpart into (2), the price dynamics is rewritten by

$$\dot{p} = s(s\lambda + s\lambda^* - 3p + a + 2c). \tag{8}$$

The open-loop Nash equilibrium is characterized by (6)-(8). One can easily prove that the steady state in this system is saddle point stable since we have two positive and one negative eigenvalues. As in Dockner and Haug (1991), let us focus on the steady state in which $\dot{\lambda} = \dot{\lambda^*} = \dot{p} = 0$. Then, the price and output of each firm are obtained as follows.

$$p^O = \frac{(r+2s)a + 2(r+s)c}{3r+4s} \tag{9}$$

$$x^O = \frac{(r+s)(a-c)}{3r+4s}, \tag{10}$$

where superscript O stands for the open-loop Nash equilibrium.

b) non-linear feedback Nash equilibrium

While the open-loop strategy requires a pre-commitment, the feedback strategy does not. We derive feedback Nash equilibria by resorting to the technique developed by Tsutsui and Mino (1990) and Shimomura (1991).[7] It begins by defining the Home firm's Hamilton-Jacobi-Bellmann equation:

$$rV(p) = \max_x \left\{ px - cx - \frac{x^2}{2} + V'(p)s\left[a - x - x^*(p) - p\right] \right\}, \tag{11}$$

where $V(\cdot)$ is the Home firm's value function:

$$V(p) \equiv \max_x \left\{ \int_t^\infty e^{-r(s-t)} \left(px - cx - \frac{x^2}{2} \right) ds \;\middle|\; \dot{p} = s\left[a - x - x^*(p) - p\right] \right\}.$$

Maximizing the right-hand side in (11), the first-order condition for interior maximum is $sV'(p) = p - c - x(p)$. Substituting this into (11) and using the symmetry assumption that $x = x^* = x(p)$, we have an identity in p:

$$rV(p) = (p-c)x(p) - \frac{[x(p)]^2}{2} + [p - c - x(p)][a - 2x(p) - p]. \tag{12}$$

Differentiating both sides in (12) with respect to p and rearranging terms, the feedback strategy satisfies the following differential equation.

$$x'(p) = \frac{-rx(p) + (r+2s)p - sa - (r+s)c}{s[3x(p) - a + c]}. \tag{13}$$

Eq. (13) gives a number of candidates for feedback strategies. While (13) is extremely difficult to explicitly solve, it can be analyzed with the help of a diagram.

From (13) and (2), we have the following information.

$$\begin{aligned} x'(p) = 0 &\iff x(p) = \frac{(r+s)p - sa - (r+s)c}{r} \\ x'(p) = \infty &\iff x(p) = \frac{a-c}{3} \\ \dot{p} = 0 &\iff x(p) = \frac{a-p}{2}. \end{aligned}$$

Each of these relationships is depicted as a locus of $x'(p) = 0$, $x'(p) = \infty$ and $\dot{p} = 0$ in Figure 2.

(Figure 2 around here)

In the figure, there are two linear strategies x_1^L and x_2^L, only the former of which is asymptotically stable.

In what follows, let us focus on one non-linear feedback strategy x^N in the figure.[8] Someone may wonder why we focus on only x^N. While we have no compelling reason, the steady state supported x^N can be analytically characterized and its implication is often striking as shown in the literature, e.g. Tsutsui and Mino (1990) and Dockner and Long (1993). This strategy asymptotically converges to the steady state N at which x^N is tangent to the $\dot{p} = 0$ line. Note that $x = (a-p)/2$ holds in the steady state. Making use of this tangency condition and substituting $x = (a-p)/2$ into (13), we have

$$-\frac{1}{2} = \frac{-r \cdot \frac{a-p}{2} + (r+2s)p - sa - (r+s)c}{s\left(3 \cdot \frac{a-c}{2} - a + c\right)},$$

where the left-hand side is the slope of the $\dot{p} = 0$ line, and the right-hand side the slope of the strategy x^N evaluated at $x = (a-p)/2$. Solving this equation for p, the steady state price reached by x^N becomes

$$p^N = \frac{(2r+3s)a + 2(2r+s)c}{6r+5s}, \qquad (14)$$

where superscript N denotes the non-linear feedback Nash equilibrium. Substituting (14) into $x = (a-p)/2$, the steady state output x^N is

$$x^N = \frac{(2r+s)(a-c)}{6r+5s}. \qquad (15)$$

This completes describing the free trade equilibrium. The subsequent section considers the effect of a VER set at the free trade level under each of the strategies addressed above.

III. EFFECTS OF A VER

Suppose that the Home government imposes a VER, whereby the Foreign firm's export is fixed to x^O in the open-loop case and x^N in the non-linear feedback case. Taking these into account, the Home firm solves the following problem:

$$\begin{array}{ll} \max\limits_{x} & \displaystyle\int_0^\infty e^{-rt}\left(px - cx - \frac{x^2}{2}\right) \mathrm{d}t \\ \text{subject to} & \dot{p} = s(a - x - x^i - p), \quad i = O, N. \end{array}$$

Since this is a single agent's optimal control problem, one can solve it with the maximum principle. Let us set up the current value Home firm's Hamiltonian:

$$H = px - cx - \frac{x^2}{2} + \lambda s(a - x - x^i - p).$$

The resulting optimality conditions are

$$0 = p - c - x - \lambda s \tag{16}$$

$$\dot{\lambda} = \lambda(r + s) - x \tag{17}$$

$$\dot{p} = s(a - x - x^i - p) \tag{18}$$

$$0 = \lim_{t\to\infty} e^{-rt}\lambda p.$$

Solving (16) for x yields

$$x = p - c - \lambda s. \tag{19}$$

Substituting (19) into (17) and (18), the reduced system becomes

$$\begin{bmatrix} \dot{\lambda} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} r + 2s & -1 \\ s^2 & -2s \end{bmatrix} \begin{bmatrix} \lambda \\ p \end{bmatrix} + \begin{bmatrix} c \\ s(a + c - x^i) \end{bmatrix}.$$

The steady state proves saddle-point stable since the determinant of the coefficient matrix is negative, i.e. we have one positive and one negative eigenvalues. In the steady state, the endogenous variables are obtained as follows.

$$p^i_{VER} = \frac{(r+2s)(a+c-x^i)-sc}{2r+3s} \tag{20}$$

$$x^i_{VER} = \frac{(r+s)(a-c-x^i)}{2r+3s}, \tag{21}$$

where subscript VER refers to the VER equilibrium.

Noting that since the Foreign firm's output must be fixed to the free trade level, its post-VER profit exceeds the free trade profit if and only if the post-VER price is larger than the free trade price. That is, we have only to check whether the imposition of a VER raises the price.

Let us begin by examining voluntariness of VER in the open-loop equilibrium. This is stated in:

Proposition 1 *When the open-loop strategy is chosen in the free trade equilibrium, a VER has no effect on the equilibrium.*

Proof: Substituting (10) into (20), the post-VER price is

$$p^O_{VER} = \frac{(r+2s)a+2(r+s)c}{3r+4s},$$

which coincides with the free trade price given by (9). Accordingly, the Foreign firm makes the same profits under free trade and a VER. **Q.E.D.**

In contrast, when feedback strategies are allowed for, the impacts of a VER will prove to depend on whether the strategy is linear in the state variable or not. Before proceeding to the non-linear feedback strategy, let us restate Dockner and Haug' (1991) proposition which is based on the linear feedback strategy:[9]

Proposition 2 (Dockner and Haug, 1991) *When the linear feedback strategy is chosen in the free trade equilibrium, a VER raises the Foreign firm's profit.*

At this stage, the voluntariness of a VER always follows as long as feedback strategies chosen, i.e. pre-commitment is ruled out. However, the above result highly depends on the linearity of the feedback strategy and exactly the opposite result can be established if we consider the *non-linear* feedback strategy given by x^N in Figure 2. This is summarized in:

Proposition 3 *When the non-linear feedback strategy given by x^N in Figure 1 is chosen in the free trade equilibrium, a VER lowers the Foreign firm's profit.*

Proof: Substituting (15) into (20), the post-VER price associated with strategy x^N becomes

$$p_{VER}^N = \frac{4(r+s)(r+2s)a + (8r^2+16rs+7s^2)c}{(6r+5s)(2r+3s)}.$$

Then, comparing this price with the free trade price p^N in (14), we see that

$$\frac{p_{VER}^N}{p^N} = \frac{4(r+s)(r+2s)a + (8r^2+16rs+7s^2)c}{(2r+3s)[(2r+3s)a+2(2r+s)c]} < 1,$$

that is, imposing a VER lowers the price and the Foreign firm's profit. **Q.E.D.**

Propositions 1-3 provide a simple classification on the voluntariness of a VER as summarized in Figure 1. The upper region collects the result of Mai and Hwang (1988) based on the conjectural variations approach. The lower region corresponds to our results. Making use of this figure, there is a one-to-one relationship between the sign of conjectural variations and the strategy considered in differential games.

We now seem the reason for the above one-to-one relationship. As Fershtman and Kamien (1987) point out, the linear feedback strategy induces each firm to produce more than Cournot. Hence, this case corresponds to negative conjectural variations. In contrast, the non-linear feedback strategy x^N is known to approximate the collusive solution as Tsutsui and Mino (1990) illustrate. The intuition behind this outcome is substantially the same as the Folk Theorem in repeated games; strategy x^N has a role similar to the trigger strategy in repeated games. As a result, the equilibrium supported by this strategy is more collusive than Cournot and corresponds to positive conjectural variations. Finally, the open-loop strategy mimics the static Cournot solution as shown in Fershtman and Kamien (1987) and thus the VER has no effect in the open-loop case.

IV. FINAL REMARKS

We have developed a differential game model to explore when a VER is voluntarily accepted by the foreign firm. As we have identified, whether a VER raises the exporting firm's profit crucially depends on the strategy chosen in free trade. When the open-loop strategy is taken, a VER has no effect on equilibrium. In contrast, if one particular *non-linear* feedback strategy is considered, a VER lowers the exporter's profit, namely, a VER is involuntary. Recalling Dockner and Haug's (1991) result that the *linear* feedback strategy leads to voluntariness of a VER, Mai and Hwang's (1988) insightful finding with conjectural variations survives a model with a dynamic foundation based on dynamic games.

We have admittedly left much unexplored. First, we have assumed away any trade barrier such as transport costs and import tariffs for a technical reason. Incorporating these may make the result richer. Second, it is worth reconsidering Syropoulos' (1996) result of Pareto-improving VERs, which is based on a static setting. Given our arguments, his result is guessed to survive open-loop strategies while the robustness will be unclear under feedback strategies. These studies are left as our future research agenda.

APPENDIX: PROOF OF THE EXTENDED DOCKNER-HAUG (1991) PROPOSITION

Dockner and Haug (1991), who show voluntariness of VERs under the linear feedback strategy, confine attention to a special case with $s \to \infty$, which is later criticized by Calzolaria and Lambertini (2007).[10] In response to their critique, this appendix proves the validity of the Dockner-Haug proposition for an arbitrary s.

Supposing a linear feedback strategy such that $x(p) = \alpha p + \beta$ where α and β are undetermined coefficients, the auxiliary equation (13) becomes

$$\alpha = \frac{-r(\alpha p + \beta) + (r + 2s)p - sa - (r + s)c}{s[3(\alpha p + \beta) - a + c]},$$

which has an alternative expression:

$$\left[3s\alpha^2 + r\alpha - (r+2s)\right] p + (3s\alpha + r)\beta + sa + (r+s)c - s\alpha(a-c) = 0.$$

α is determined so that the quadratic equation multiplied by p is zero. Then, we have[11]

$$\begin{aligned} \alpha &= \frac{-r+\sqrt{\Delta}}{6s} \\ \Delta &\equiv r^2 + 12s(r+2s) > 0. \end{aligned} \tag{22}$$

On the other hand, β is determined by setting the other terms to zero as follows.

$$\beta = \frac{s\alpha(a-c) - sa - (r+s)c}{3s\alpha + r}. \tag{23}$$

Substituting (22) and (23) into $x(p) = \alpha p + \beta$ yields the closed form of the asymptotically stable feedback strategy. In the steady state where $\dot{p} = a - p - 2(\alpha p + \beta) = 0$ holds, the pre-VER price is

$$\begin{aligned} p^L &= \frac{a - 2\beta}{2\alpha + 1} \\ &= \frac{\left(\sqrt{\Delta} + 5r + 12s\right) a + 2\left(\sqrt{\Delta} + 5r + 6s\right) c}{3\left(\sqrt{\Delta} + 5r + 8s\right)}, \end{aligned} \tag{24}$$

where superscript L refers to the linear feedback Nash equilibrium.

Moreover, the steady state output before a VER is obtained by substituting (22)-(24) into $\alpha p + \beta$:

$$x^L = \frac{\left(\sqrt{\Delta} + 5r + 6s\right)(a-c)}{3\left(\sqrt{\Delta} + 5r + 8s\right)}. \tag{25}$$

As noted in the main text, it suffices to check whether the price increases before and after a VER in order to examine whether a VER raises the Foreign firm's profit. To this end, taking the ratio between p^L_{VER} in (20) evaluated at x^L (post-VER price) and p^L (pre-VER price), we have

$$\begin{aligned} \frac{p^L_{VER}}{p^L} &= \frac{3\left(\sqrt{\Delta} + 5r + 8s\right)\left[(r+2s)\left(a + c - x^L\right) - sc\right]}{(2r+3s)\left[\left(\sqrt{\Delta} + 5r + 12s\right) a + 2\left(\sqrt{\Delta} + 5r + 6s\right) c\right]} \\ &= \frac{2(r+2s)\left[\left(\sqrt{\Delta} + 5r + 9s\right) a + \left(2\sqrt{\Delta} + 10r + 15s\right) c\right] - 3s\left(\sqrt{\Delta} + 5r + 8s\right) c}{(2r+3s)\left[\left(\sqrt{\Delta} + 5r + 12s\right) a + 2\left(\sqrt{\Delta} + 5r + 6s\right) c\right]}. \end{aligned}$$

Subtracting the denominator from the numerator and rearranging terms yield

$$(\text{numerator}) - (\text{denominator}) = s\left(\sqrt{\Delta} - r\right)(a - c) > 0,$$

which allows us to conclude that $p^L_{VER} > p^L$ and that the VER under the linear feedback strategy raises the Foreign firm's profit. This completes the proof of the Dockner-Haug proposition extended to an arbitrary s.

REFERNCES

Calzolari, G. and Lambertini, L. 2007, 'Export Restraints in a Model of Trade with Capital Accumulation', *Journal of Economic Dynamics and Control*, vol. 31, pp. 3822-3842.

Cellini, R. and Lambertini, L. 2004, 'Dynamic Oligopoly with Sticky Prices: Closed-Loop, Feedback and Open-Loop Solutions', *Journal of Dynamical and Control Systems*, vol. 10, pp. 303-314.

Dockner, E.J. and Haug, A.A. 1991, 'The Closed-Loop Motive for Voluntary Export Restraints', *Canadian Journal of Economics*, vol. 24, pp. 679-685.

Dockner, E.J. and Long, N.V. 1993, 'International Pollution Control: Cooperative versus Noncooperative Strategies', *Journal of Environmental. Economics and Management*, vol. 24, pp. 13-29.

Dockner, E.J., Jorgensen, S., Long, N.V. and Sorger, G. 2000, *Differential Games in Economics and Management Science*, Cambridge University Press, Cambridge.

Eaton, J. and Grossman, G.M. 1986, 'Optimal Trade and Industrial Policy under Oligopoly', *Quarterly Journal of Economics*, vol. 101, pp. 383-406.

Fershtman, C. and Kamien, M.I. 1987, 'Dynamic Duopolistic Competition with Sticky

Prices', *Econometrica*, vol. 55, pp. 1151-1164.

Harris, R. 1985, "Why Voluntary Export Restraints Are "Voluntary"', *Canadian Journal of Economics*, vol. 18, pp. 799-809.

Itaya, J. and Shimomura, K. 2001, 'A Dynamic Conjectural Variations Model in the Private Provision of Public Goods: A Differential Game Approach', *Journal of Public Economics*, vol. 81, pp. 153-172.

Kamien, M.I. and Schwartz, N.L. 1991, *Dynamic Optimization: the Calculus of Variations and Optimal Control in Economics and Management*, North-Holland, Amsterdam.

Mai, C. and Hwang, H. 1988, 'Why Voluntary Export Restraints Are Voluntary: An Extension', *Canadian Journal of Economics*, vol. 21, pp. 877-882.

Rowat, C. 2007, 'Non-Linear Strategies in a Linear Quadratic Differential Game', *Journal of Economic Dynamics and Control*, vol. 31, pp. 3179-3202.

Rubio, S.J. and Casino, B. 2002, 'A Note on Cooperative versus Non-Cooperative Strategies in International Pollution Control', *Resource and Energy Economics*, vol. 24, pp. 251-261.

Shimomura, K. 1991, 'Feedback Equilibria of a Differential Game of Capitalism', *Journal of Economic Dynamics and Control*, vol. 5, pp. 317-338.

Simaan, M. and Takayama, T. 1978, 'Game Theory Applied to Dynamic Duopoly Problems with Production Constraints', *Automatica*, vol. 14, pp. 161-166.

Syropoulos, C. 1996, 'On Pareto-Improving Voluntary Export Restraints', *International Journal of Industrial Organization*, vol. 14, pp. 71-84.

Tsutsui, S. and Mino, K. 1990, ‘non-linear Strategies in Dynamic Dupolistic Competition with Sticky Prices’, *Journal of Economic Theory*, vol. 52, pp. 136-61.

Wirl, F. 1996, ‘Dynamic Voluntary Provision of Public Goods: Extension for non-linear Strategies’, *European Journal of Political Economy*, vol. 12, pp. 555-560.

Footnotes

1. Harris (1985) assumes Bertrand competition, which is implied by a negative conjectural variation.
2. Allowing for an arbitrary number of firms in the Fershtman-Kamien (1987) model, Cellini and Lambertini (2004) compare the open-loop, closed-loop memoryless and feedback equilibria.
3. For the equilibrium concepts in differential games, see Kamien and Schwartz (1991) and Dockner et al. (2000).
4. non-linear feedback strategies are known to yield a drastic difference in implications. Examples include Tsutsui and Mino (1990) in industrial organization, Dockner and Long (1993) in environmental economics and Wirl (1996) and Itaya and Shimomura (2001) in public economics.
5. The result Dockner and Haug (1991) prove rests on the assumption that the price adjustment speed is infinity or the discount rate is zero, which is severely criticized by Calzolari and Lambertini (2007).
6. The below subsections admittedly contain nothing new since Fershtman and Kamien (1987), Tsutsui and Mino (1990), and Dockner et al. (2000) have already developed these arguments. Hence, only the minimum essence is sketched.
7. As long as one focuses on linear feedback strategies, the familiar method of guessing a value function is also used. However, this method is not applicable to non-linear strategies.
8. Itaya and Shimomura (2001), Rubio and Casino (2002), and Rowat (2007) identify the conditions for x^N to be chosen.
9. Dockner and Haug (1991) prove this result by setting $s \to \infty$ on which leads Calzolari

and Lambertini (2007, p. 3836) to claim that 'restricting the analysis to the case of instantaneous price adjustment prevents Dockner and Haug from producing a general assessment of the feasibility of VERs for the general case where prices are sticky'. However, Appendix shows the validity of Dockner and Haug's (1991) result for an arbitrary s.

10. See footnote 5.

11. The negative candidate for α is eliminated since it violates asymptotic stability, which is confirmed from Figure 1. Only x_1^L which corresponds a linear feedback strategy with a positive α converges to the steady state whereas x_2^L with a negative α diverges. More detailed arguments are found in Fershtman and Kamien (1987).

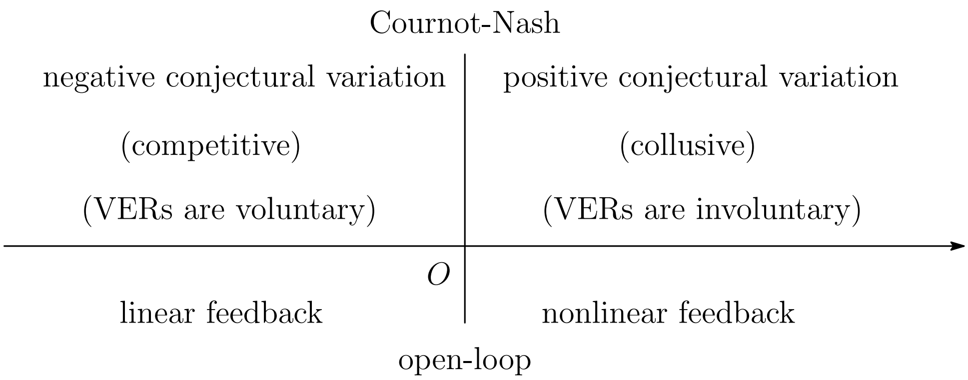

Figure 1: Strategies in a differential game and conjectural variations

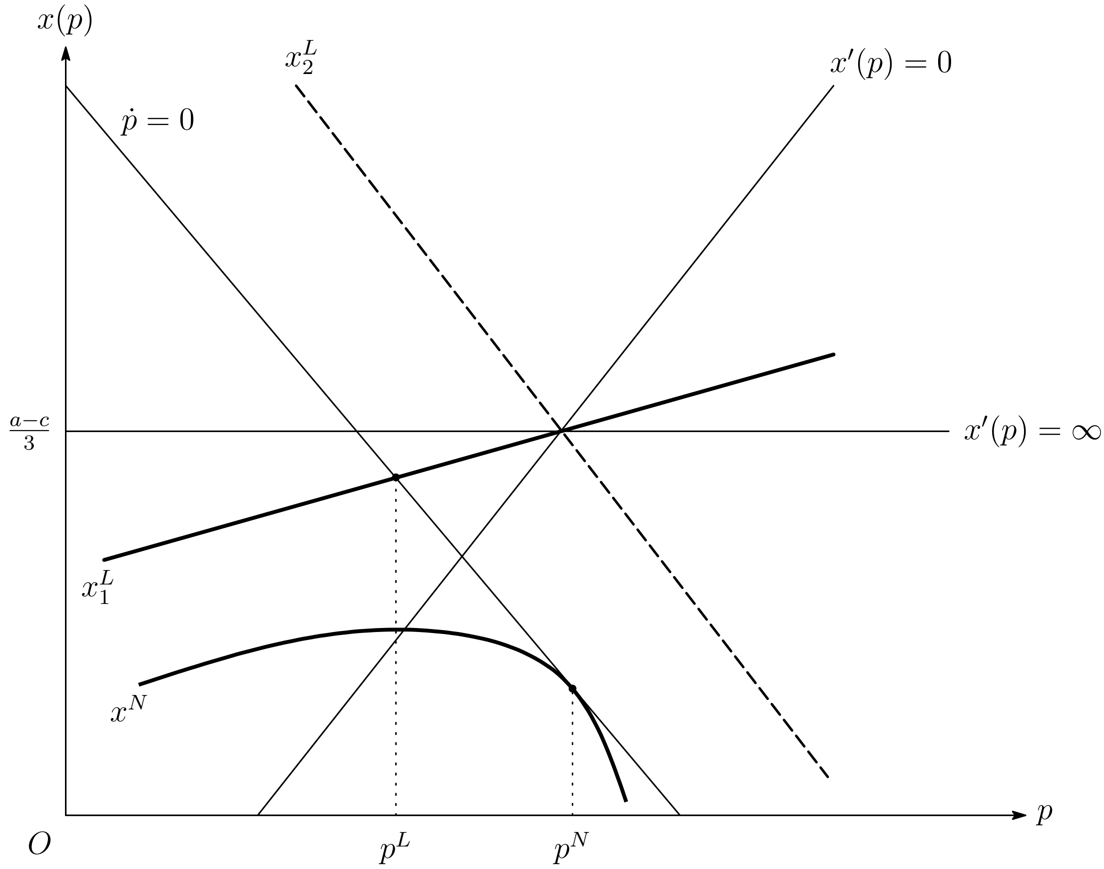

Figure 2: Equilibrium strategies