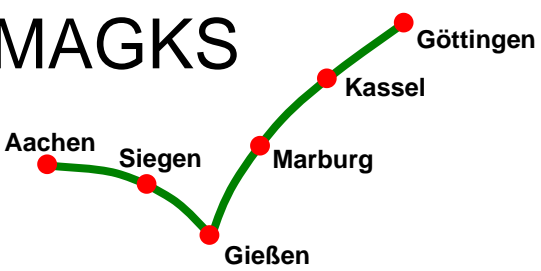MAGKS

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Matthias Göcke

Efficiency Wages and Negotiated Profit-Sharing under Uncertainty

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by Matthias Göcke

(University of Giessen)

Abstract: *Efficiency wage effects of profit sharing are combined with option values related to stochastic future profit variations. These option effects occur if the workers' profit share is fixed by long-term contracts. The Pareto-improving optimal level of the sharing ratio is calculated for two different scenarios. First, if the firm can unilaterally decide, the expected present value of net profits is maximised. Second, if the sharing ratio is based on bilateral Nash bargaining. Since a larger variation of revenues implies a higher redistribution of future profits, the inclusion of expected variations results in a lower worker's profit ratio in both scenarios.*

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Matthias Göcke

University of Giessen, VWL IV, Licher Str. 62

D - 39394 Gießen / Germany

Tel.: +49 641/99-22241

e-mail: Matthias.Goecke@wirtschaft.uni-giessen.de

Efficiency Wages and Negotiated Profit-Sharing under Uncertainty

1. Introduction

Since high wages can be a device to increase motivation and productivity of workers, some firms find it profitable to offer an extra pay in excess of the workers' reservation wage in order to elicit more effort. These *"efficiency wages"* may arise due to unobservable individual effort (in the so-called 'shirking version'): the firm (principal) – unable to monitor individual effort – can try to increase productivity of the employees (agents) through incentives based on the remuneration scheme. (For an overview see LEVINE [1989]). If this extra pay is not a fixed payment but based on *sharing the firm's profits with the employees*, these incentive effects are even amplified, since now the productivity benefits of cooperation partly flow to the workers, and because the marginal benefits of shirking are reduced. Thus, efficiency wages based on profit sharing (PS) may be an adequate instrument (see LIN, CHANG & LAI [2002] for an overview of the synthesis of profit sharing and efficiency wages). The positive productivity effects of giving a share of the profit as an extra payment to the workers are corroborated empirically (WADHWANI & WALL [1990], KRUSE [1992], SESSIONS [2008]). Extra Payments based on PS in addition to a fixed base wage can be used autonomously as an incentive instrument by a firm or can be the result of a bilateral bargaining process of the firm with a worker or with the union (for aspects of efficiency wages and bargaining see POHJOLA [1987] and SANFEY [1993]). Frequently, profit sharing schemes are fixed not only for the present period, but are based on a long-run commitment of the firm. However, the firm has to keep in mind that future profits are stochastically uncertain (KOSKELA & STENBACKA [2004]). Moreover, profit sharing is usually limited to positive profits, while negative profits will lead to a market exit of the firm and consequently to firing of the workforce. Thus, a commitment to a long-term PS scheme actually implies that a firm gives an option to the worker: If the future turns out to be advantageous, high profits will be shared with the worker, however, in an unfavourable future shared profits are downwards limited to zero. Since the negative realisations of the stochastic process of future profits are truncated, the money which is expected to be given to the workers as a share of the profits is the more, the larger is the expected revenue variation. Anticipation of this option effect by the firm leads to a more reluctant attitude towards PS by the firm, the more variable the revenues are. In this paper a simple model is presented describing these option/variation effects on voluntary "long-term" PS schemes based on unilateral firm decisions as well as on bilateral Nash bargaining.

The structure of the paper is as follows: In section 2. a simple formulation of positive productivity effects due to sharing profits is presented. In section 3. this productivity effect is integrated into a short-run scenario, where the relation between present revenues and Pareto improvement due to profit sharing is analysed. The consequences of uncertain future revenues in long-run PS schemes are addressed in section 4. Section 5. concludes.

2. A simple model of productivity effects due to profit sharing

A price-taking firm produces a final product using one unit of labour input (i.e. one employee). This worker has the opportunity to receive a standard market wage w if (s)he works under the conditions of the standard wage agreement without profit sharing, either in our firm or in another firm. This outside opportunity of the worker serves as the base wage w

in our analysis and is, as a numéraire, normalised to unity: $w = 1$. The firm has the opportunity to give the employee a share of its profit as an *extra* compensation in addition to the standard wage w in order to encourage the employee to work harder. The larger the worker's ratio λ of the profit, the higher is the incentive to work hard, and thus, the higher is the productivity of labour. As the reciprocal of productivity, the input coefficient is a decreasing function of the worker's sharing ratio λ.

(1) $\alpha = \alpha(\lambda, \cdot)$ with $\dfrac{\partial \alpha}{\partial \lambda} \leq 0$ $\lambda \in [0, 1]$

Since the standard wage w is normalised to unity, the input coefficient α equals the unit 'base labour costs' (excluding the worker's profit share). In order to keep the mathematics as simple as possible and to be able to calculate closed form solutions the following simple functional form of the labour input coefficient resp. the unit labour costs α is assumed:

(2) $\alpha = \alpha(\lambda, \eta) = \dfrac{1}{1 + \eta \cdot \lambda}$ $\eta \in [0, 1[$

> with: α : input coefficient of labour = unit labour costs
> λ : worker's sharing ratio of the profit (= share parameter)
> η : efficiency parameter of PS

The more effective profit sharing is – i.e. the more severe the monitoring problems without extra monetary incentives are –, the larger is the efficiency parameter η in eq. (2), i.e. the lower are the unit labour costs in a situation with profit sharing. Due to the normalisation of the base wage ($w = 1$) these 'base' unit labour costs are normalised to one for a situation without profit sharing [i.e. $\alpha(\lambda = 0) = 1$].

Selling the final product in the current period t, the firm receives the price p_t. The profit per unit of the product is $(p_t - \alpha)$. A single worker is able to produce more than one unit of the product if the productivity increases, i.e. if the input coefficient decreases: The production quantity of a single worker is $x_t = (1/\alpha)$ and the profit per employee is $x_t \cdot (p_t - \alpha)$ and – corrected by the 'dilution effect' due to the employee's profit sharing ratio λ – in period t the firm's current net profit R_t is:

(3) $R_t = (1 - \lambda) \cdot \dfrac{p_t - \alpha}{\alpha} = (1 - \lambda) \cdot [(1 + \eta \cdot \lambda) \cdot p_t - 1]$

In absence of any firing and hiring costs the firm will be active on the product market (and employ a worker), if the profit R_t is positive and leave the market if the profit is negative. The price level which triggers the market exit can be calculated via zero profit:

(4) $R_t = 0 \quad \Leftrightarrow \quad p_t = \alpha = \dfrac{1}{1 + \eta \cdot \lambda}$

> with: market exit and firing the worker if $p_t < \alpha$

The firm reacts with an firing strategy if the price level falls short of the unit labour costs α.

3. Determination of the worker's profit share in a "short-term" situation

We start with a situation, where the firm decides about PS as an incentive device only in the current period t. Thus, we have a *short-term* situation, where all relevant factors (e.g. the product price level p_t) are known. I.e. in the short run we have a situation with *certainty*.

3.1 Profit sharing as a Pareto improvement

Sharing the firm's profits brings about an incentive effect and increases the productivity of labour. Thus, paying an extra compensation exceeding the standard wage may be profitable for both – the worker and the firm. We assume that the worker may get a share of the profits as an efficiency wage *premium in addition* to the standard/market wage w, as an extra compensation for working harder. Since both, the firm and the worker, have to opt for PS voluntarily, contracts including shared profits as an efficiency premium will only result if both contracting parties are better off, i.e. if PS results in a *Pareto-superior* situation compared to a standard market wage compensation.

The firm will prefer profit sharing ($\lambda > 0$) to a situation with merely a standard wage compensation ($\lambda = 0$) if – after correcting for the 'profit dilution' due to the worker's share – an *additional* 'net' profit results for the firm. This *extra* net profit Z_t due to the incentive/productivity effect of sharing profits compared to an alternative without PS is to be differentiated concerning two different cases: case [a], where price level p_t is high, so that producing the product is profitable anyway, with or without PS. In this case [a] the two relevant opportunities of the firm are: being active on the market with PS compared to be active without PS. In case [b], the price level is low so that only the cost reducing efficiency effects of PS are ensuring profitability. In case [b] the alternatives are: activity with PS compared to not being active at all (with zero profit). Since $R_t(\lambda=0) = p_t - 1$, the borderline price between both cases is $p_t = 1$.

(5) $Z_t = R_t(\lambda > 0) - R_t(\lambda = 0) = (1-\lambda) \cdot [(1+\eta \cdot \lambda) \cdot p_t - 1] - D(p_t) \cdot [p_t - 1]$

  with a binary variable: $D(p_t > 1) = 1 \quad \Leftrightarrow \quad$ case [a]

  $D(p_t \leq 1) = 0 \quad \Leftrightarrow \quad$ case [b]

The firm prefers PS to a standard wage compensation, if the extra profit Z_t of PS is positive. If the price level is very high, the 'dilution effect' of sharing profits (which is negative from the firm's point of view) is larger than the labour cost reduction via the 'incentive effect'. Thus, a price ceiling exists, at which PS becomes unattractive for a very profitable firm. Hence, in case [a] ($p_t > 1$) and for a high price level, PS is only beneficial for the firm if $Z_t > 0$.

(6) $Z_t > 0 \Rightarrow$ firm prefers PS if $p_t < p_f$,   with $p_f = \dfrac{1}{1-\eta \cdot (1-\lambda)} \geq 1$

A worker will agree to a compensation including shared profits if the entire compensation exceeds the standard wage (w = 1) and if, additionally, a reimbursement for the disutility of working harder due to PS is paid. Since a higher λ is related to a lower labour input coefficient α, the disutility of working harder is positively related to the profit share λ. A disutility parameter ($\theta \geq 1$) is multiplied by productivity/effort parameters ($\eta \cdot \lambda$) in order to capture the disutility effect [$\theta \cdot (\eta \cdot \lambda)$]. The worker's *premium* L_t resulting from PS – corrected

by the disutility effect – has to be positive in order to make PS preferable for the worker. Thus, the worker's implicit lowest-price limit p_w at which (s)he will accept a PS contract instead of a standard wage contract is:

(7) $L_t = \dfrac{\lambda}{1-\lambda} \cdot R_t - \theta \cdot (\eta \cdot \lambda) \; = \; \lambda \cdot [\,(1+\eta \cdot \lambda) \cdot p_t - 1\,] - \theta \cdot \eta \cdot \lambda \; > 0$ with $\theta \geq 0$

   $L_t > 0 \; \Rightarrow$ worker prefers PS if $p_t > p_w$,   with $p_w = \dfrac{1+\eta \cdot \theta}{1+\eta \cdot \lambda} = \alpha \cdot (1+\eta \cdot \theta)$

The unit labour costs α multiplied by an 'effort cost' factor $(1+\eta \cdot \theta)$ determine the worker's price floor for favourable profit sharing.

To sum up, in the case of effective incentives ($\eta > 0$), both the firm and the worker may be better off by profit sharing. An extra efficiency compensation exceeding the standard wage w is *Pareto-superior* if:

(8) $p_w < p_t < p_f \quad \Leftrightarrow \quad \dfrac{1+\eta \cdot \theta}{1+\eta \cdot \lambda} < p_t < \dfrac{1}{1-\eta \cdot (1-\lambda)}$

A sufficient condition for the existence of a real range $p_w < p_f$ with Pareto-superiority of PS is: $(\eta > 0 \wedge \theta < 1)$.

3.2 Firm alone decides about profit sharing

Since shared profits are given voluntarily by the firm as an efficiency wage premium in addition to the worker's opportunity wage w, we now assume that the firm has unilaterally the right to determine the level of the ratio λ of its profits which is given as a bonus to the worker. Correspondingly, the worker has the right to accept the PS efficiency wage contract based on the share level λ offered by our firm, or to turn down the offer and to opt for a standard/market wage contract.

If the firm wants to utilise the efficiency effect of PS it can decide upon the magnitude of the share λ of the profits which is given to the worker. The level of the worker's sharing ratio λ is chosen by the firm so as to maximise the value of the firm's net profit. This profit-maximising ratio λ_f^c can be calculated via the first order condition:

(9)   FOC: $\dfrac{\partial Z_t}{\partial \lambda} = 0 \quad \Rightarrow \quad \lambda_f^c = \dfrac{1 - p_t + \eta \cdot p_t}{2 \cdot \eta \cdot p_t} = \dfrac{1}{2} - \dfrac{1}{2 \cdot \eta} + \dfrac{1}{2 \cdot \eta \cdot p_t}$

   $\Leftrightarrow \; p_t = \dfrac{1}{1 - \eta + 2 \cdot \eta \cdot \lambda_f^c}$ restrictions: $p_t \in [\alpha, p_f] \; \Leftrightarrow \; \lambda_f^c \in [0, 1]$

The profit maximising ratio λ_f^c is decreasing in the product price p_t; and for $p_t = 1$ this ratio is ½. If the price level approaches the firm's "Pareto price ceiling" p_f, the profit-maximising λ_f^c approaches 0, and for prices above p_f sharing very high profits is not attractive for the firm due to the very expensive dilution effect. If the price equals the unit costs α, the profit is 0, and thus the (from the firm's point of view) optimal ratio λ_f^c is 1. However, the range of PS is limited by the acceptance of the worker. Substituting p_t by the workers "Pareto floor price" p_w in the FOC in equation (9) and solving for λ results in:

(10) $\quad p_t \geq p_w \quad \Leftrightarrow \quad \lambda_f^c \leq \lambda_{max}^c \qquad\qquad\qquad$ with $\quad \lambda_{max}^c = \dfrac{1-\theta+\eta\cdot\theta}{1+2\cdot\eta\cdot\theta}$

Profit-maximising PS is (relative to a standard wage compensation) an advantage for the worker if the price level exceeds p_w, which implies that the corresponding profit-maximising ratio is below λ_{max}^c. This ensures that the worker's profit share is large enough to compensate for her/his burden of working harder.

3.3 Negotiated profit sharing

Alternatively, the determination of the share parameter λ may be the result of a bilateral negotiation of the firm and the worker. We assume a Nash bargaining solution based on maximising the following Nash product N_t:

(11) $\quad N_t := Z_t \cdot L_t \quad$ with $\quad \max_{\lambda}(N_t) \quad$ FOC: $\dfrac{\partial N_t}{\partial \lambda} = 0 \quad \Rightarrow \quad$ solution: λ_N^c

The solution of the FOC in case [a] (with $p_t > 1$) is:

(11 a) $\quad \lambda_N^{c,[a]} = \dfrac{\frac{3}{8}p\eta + \frac{3}{8}\eta\theta - \frac{3}{4}p + \frac{3}{4} + \frac{1}{8}\sqrt{9p^2\eta^2 - 14p\eta^2\theta - 4p^2\eta + 4p\eta + 9\eta^2\theta^2 - 4\eta\theta p + 4\eta\theta + 4p^2 - 8p + 4}}{\eta p}$

The solution for case [b] ($p_t \leq 1$) was computed in closed form, however, the expression is to extensive. For a simple situation without any disutility effect the solution for case [b] is:

(11 b) $\quad \lambda_{N,(\theta=0)}^{c,[b]} = \dfrac{1}{8}\dfrac{3p\eta + 2 - 2p + \sqrt{9p^2\eta^2 - 4p\eta + 4p^2\eta + 4 - 8p + 4p^2}}{p\eta} \qquad$ for case [b] and $\theta = 0$

(11 c) $\quad$ The borderline between both cases [a] and [b] is: $\lambda_b^a = \frac{3}{8}\theta + \frac{3}{8} + \frac{1}{8}\sqrt{9\theta^2 - 14\theta + 9}$.

Fig. 1 shows the sharing ratio λ (on the abscissa) as a response to different price levels $p = p_t$ (on the ordinate), for Nash bargaining (λ_N^c) and for net-profit maximisation (λ_f^c). Additionally, the Pareto-borders are depicted: The distance between the firm's price ceiling p_f and the worker's price floor p_w shows the range of Pareto-improvements by PS. This distance is the larger, the larger the PS-efficiency parameter η and the smaller the disutility parameter θ of working harder is. The Nash bargaining solution is "kinked" at $p = 1$, where the alternative cases [a] and [b] are changing. For every price level p the resulting sharing ratio under Nash bargaining λ_N^c exceeds the profit-maximising ratio λ_f^c. The higher the price level, the more dominant becomes the 'dilution effect', and so the profit-maximising sharing ratio as well as the Nash sharing ratio are decreasing in the price level.

Fig. 1: Simulation for net-profit maximisation of the firm and for Nash bargaining
$(\eta = 1/2, \theta = 1/4)$
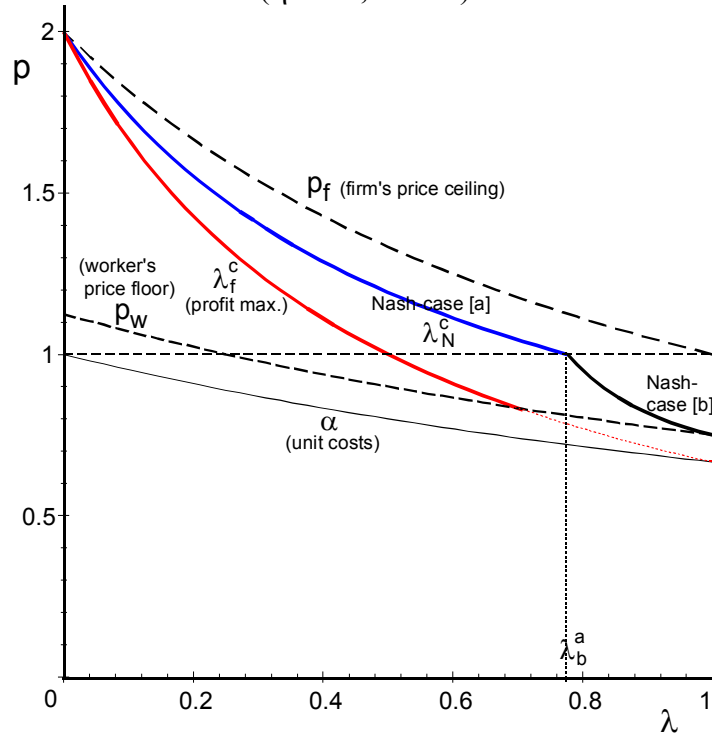
4. Uncertainty and "long-term" profit sharing contracts

Instead of determining the share parameter only for the current period t, a "long-term" PS contract fixes the ratio λ not only for the present period t but also for a second period t+1 (i.e. the "future") as well. However, since we allow for stochastic price variations, the future price level p_{t+1} and the future profit R_{t+1} is unknown at the moment when the PS ratio λ is permanently fixed by a contract.

4.1 A simple stochastic variation as an example

The following extension of the model is designed to illustrate the impact of stochastic variations of the future price level. However, in our simplistic model we merely analyse the effects of an expected *one-time* discrete binomial shock. By applying this very simple stochastic change, we are able to show how (and *in which direction*) the inclusion of option effects *qualitatively alter* our results concerning (1) the Pareto-superior price range under PS contracts, and (2) concerning the profit-maximising or the negotiated ratio λ of the profits a worker will receive. Thus, our model can be seen as a *simplistic example* provided to show the qualitative effects of sharing option values. These effects would remain in a more realistic/sophisticated framework (with e.g. ongoing uncertainty, under application of a time-continuous stochastic process). However, with our simple modelling, we are able to apply simple algebraic methods instead of using dynamic programming techniques, and we are able to calculate simple closed form analytical solutions.

We assume a binomial stochastic process: Both, the firm and the worker, expect a non-recurring single stochastic change with an absolute size of $\varepsilon \geq 0$, which can be either positive $(+\varepsilon)$ with probability W, or a negative $(-\varepsilon)$ change, with probability $(1-W)$.

(12) $\quad p_{t+1} = p_t \pm \varepsilon \quad \Rightarrow \quad E_t(p_{t+1}) = W \cdot (p_t + \varepsilon) + (1-W) \cdot (p_t - \varepsilon) = p_t + (2 \cdot W - 1) \cdot \varepsilon$

We assume that the planning horizon of the firm is two periods only: the current period t and the "future" (t+1).

4.2 Options of an active firm

The expected present value of the firm is the probability-weighted average of the present values of both stochastic ε-realisations. Concerning the present price level three possible situations are relevant.

Situation 1: ($\alpha + \varepsilon < p_t$). If in period t the price level exceeds ($\alpha + \varepsilon$), the firm will earn a positive profit in both periods, even in the case of a negative ($-\varepsilon$)-realisation. Thus, the firm will continue its activity in period t+1 in any case.

Situation 2: ($\alpha < p_t < \alpha + \varepsilon$). The firm is active in period t, Conditional on a positive ($+\varepsilon$)-change, which has the probability W, the firm will stay active in period t+1. Conditional on a negative ($-\varepsilon$)-realisation, which has the probability ($1-W$), the firm will use its option to leave the market in t+1, since then the price level is below the unit labour costs α.

Situation 3: ($\alpha - \varepsilon < p_t < \alpha$). In the current period the price level p_t is below the cost level α. However, the firm will enter the market in t+1 if a positive ($+\varepsilon$)-change will occur, and will (with probability W) earn positive future profits.

In order to be able to use a single general notation for all three situations, a formulation using an average price is applied. In situation 2 ($\alpha < p_t < \alpha + \varepsilon$), the firm is active in period t with probability 1 and in period t+1 with probability W. We calculate a weighted average price level over both periods, conditional on market activity. The weights are 1 for the present and the probability W multiplied by a discount/weighting factor δ for future variables. This probability weighted average price level pa_2 is:

(13) $\quad$ situation 2 ($\alpha < p_t < \alpha + \varepsilon$): $\quad pa_2 = \dfrac{1}{1 + \delta \cdot W} \cdot [1 \cdot p_t + \delta \cdot W \cdot (p_t + \varepsilon)] = p_t + \dfrac{\delta \cdot W \cdot \varepsilon}{1 + \delta \cdot W}$

In situation 1 ($p_t > \alpha + \varepsilon$), the firm is active in both periods t and t+1 with probability 1. The probability weighted average price level pa_1 is:

(14) $\quad$ situation 1 ($p_t > \alpha + \varepsilon$):

$$pa_1 = \frac{1}{1+\delta} \cdot [p_t + \delta \cdot W \cdot (p_t + \varepsilon) + \delta \cdot (1-W) \cdot (p_t - \varepsilon)] = p_t + \frac{\delta \cdot (2 \cdot W - 1) \cdot \varepsilon}{1 + \delta}$$

In situation 3 ($\alpha - \varepsilon < p_t < \alpha$) the firm is inactive in period t, and with probability W active in t+1. Conditional on market activity, the probability weighted average price level pa_3 is:

(15) $\quad$ situation 3 ($\alpha - \varepsilon < p_t < \alpha$): $\quad pa_3 = \dfrac{1}{W} \cdot [W \cdot (p_t + \varepsilon)] = p_t + \varepsilon$

The expected present value V_1 of the firm in situation 1 is the current profit R_t plus the discounted expected profit in the next period $E_t(R_{t+1})$:

(16) situation 1 $(p_t > \alpha + \varepsilon)$:

$$E_t(R_{t+1}) = (1-\lambda) \cdot [\,(1+\eta \cdot \lambda) \cdot E_t(p_{t+1}) - 1\,]$$

$$= R_t + (1-\lambda) \cdot (1+\eta \cdot \lambda) \cdot (2 \cdot W - 1) \cdot \varepsilon$$

$$\Rightarrow \quad V_1 = R_t + \delta \cdot E_t(R_{1,t+1})$$

$$= (1+\delta) \cdot R_t + \delta \cdot (1-\lambda) \cdot (1+\eta \cdot \lambda) \cdot (2 \cdot W - 1) \cdot \varepsilon$$

$$\Leftrightarrow \quad V_1 = (1+\delta) \cdot (1-\lambda) \cdot [\,(1+\eta \cdot \lambda) \cdot pa_1 - 1\,]$$

Expected present value V_2 of the firm in situation 2 is the current profit R_t plus a positive future profit in case of a $(+\varepsilon)$-realisation, with probability W:

(17) situation 2 $(\alpha < p_t < \alpha + \varepsilon)$:

$$E_t(R_{t+1}) = W \cdot (1-\lambda) \cdot [\,(1+\eta \cdot \lambda) \cdot (p_t + \varepsilon) - 1\,]$$

$$\Rightarrow \quad V_2 = R_t + \delta \cdot E_t(R_{t+1})$$

$$= (1+\delta \cdot W) \cdot R_t + \delta \cdot W \cdot (1-\lambda) \cdot (1+\eta \cdot \lambda) \cdot \varepsilon$$

$$\Leftrightarrow \quad V_2 = (1+\delta \cdot W) \cdot (1-\lambda) \cdot [\,(1+\eta \cdot \lambda) \cdot pa_2 - 1\,]$$

Expected present value V_3 of the firm in situation 3 is a zero current profit plus a positive future profit in case of a $(+\varepsilon)$-change:

(18) situation 3 $(\alpha - \varepsilon < p_t < \alpha)$:

$$V_3 = \delta \cdot E_t(R_{t+1}) = \delta \cdot W \cdot (1-\lambda) \cdot [\,(1+\eta \cdot \lambda) \cdot (p_t + \varepsilon) - 1\,]$$

$$\Leftrightarrow \quad V_3 = \delta \cdot W \cdot (1-\lambda) \cdot [\,(1+\eta \cdot \lambda) \cdot pa_3 - 1\,]$$

A general formulation representing all situations $(i = 1, 2, 3)$ of the expected present value is:

(19) $V_i = C_i \cdot (1-\lambda) \cdot [\,(1+\eta \cdot \lambda) \cdot pa_i - 1\,]$

with a constant C_i $(i = 1, 2, 3)$: $C_1 = (1+\delta)$, $C_2 = (1+\delta \cdot W)$, $C_3 = \delta \cdot W$

4.3 "Long-term" PS contracts and Pareto improvement

A risk-neutral firm will prefer profit sharing $(\lambda > 0)$ to a situation with merely a standard wage compensation $(\lambda = 0)$ if – after correcting for the employee's profit share – an *additional* "net" expected present value Z_i results for the firm. In the alternative without PS $(\lambda = 0)$ the expected present value V_i is positive for $pa_i > 1$. Thus, the net gain Z_i from PS is:

(20) $Z_i = V_i(\lambda > 0) - V_i(\lambda = 0) = C_i \cdot (1-\lambda) \cdot [\,(1+\eta \cdot \lambda) \cdot pa_i - 1\,] - D(pa_i) \cdot [\,pa_i - 1\,]$

with a binary dummy: $D(pa_i > 1) = 1 \quad \Leftrightarrow \quad$ case [a]

$$D(pa_i \leq 1) = 0 \quad \Leftrightarrow \quad \text{case [b]}$$

The firm prefers PS if the extra present value Z_i of PS is positive:

(21) $\quad Z_i > 0 \quad$ if $\quad pa_i < \dfrac{1}{1 - \eta \cdot (1 - \lambda)} \quad \Rightarrow \quad$ firm prefers PS if $pa_i < p_f$

The workers are risk-neutral as well and have the same expectations as the firm. A worker agrees to PS if the expected compensation exceeds the standard wage compensation plus a reimbursement for the disutility of working harder. The worker's expected *premium* L_i resulting from PS has to be positive. Thus, the worker's implicit lowest-price limit at which (s)he will accept a PS contract instead of a standard wage contract is:

(22) $\quad L_i = C_i \cdot \{ \lambda \cdot [(1 + \eta \cdot \lambda) \cdot pa_i - 1] - \theta \cdot \eta \cdot \lambda \} > 0$

$\quad\quad \Rightarrow \quad$ worker prefers PS if $\quad pa_i > p_w = \dfrac{1 + \eta \cdot \theta}{1 + \eta \cdot \lambda} = \alpha \cdot (1 + \eta \cdot \theta)$

Analogous to the short-term case under certainty (from section 3.), both the firm and the worker can be better off by PS. A "long-term" contract with a time invariant fixed profit share λ is *Pareto-superior* if:

(23) $\quad p_w < pa_i < p_f \quad \Leftrightarrow \quad \dfrac{1 + \eta \cdot \theta}{1 + \eta \cdot \lambda} < pa_i < \dfrac{1}{1 - \eta \cdot (1 - \lambda)}$

If the firm unilaterally can decide upon the magnitude of the time-invariant ratio λ of the profits which is given to the worker, λ is chosen by the firm so as to maximise the expected present value Z_i:

(24) $\quad$ FOC: $\quad \dfrac{\partial Z_i}{\partial \lambda} = 0 \quad \Rightarrow \quad \lambda_f^u = \dfrac{1 - pa_i + \eta \cdot pa_i}{2 \cdot \eta \cdot pa_i} = \dfrac{1}{2} - \dfrac{1}{2 \cdot \eta} + \dfrac{1}{2 \cdot \eta \cdot pa_i}$

$\quad\quad \Leftrightarrow \quad pa_i = \dfrac{1}{1 - \eta + 2 \cdot \eta \cdot \lambda_f^u}$

This is similar to the solution of the short-term case in eq. (9), if the present period price p_t is replced by the weighted average price pa_i.

Alternatively, the determination of the sharing ratio λ may be the result of a bilateral negotiation of the firm with the worker. We again assume a Nash bargaining based on maximising the following "long-term" Nash product N_i:

(25) $\quad N_i := Z_i \cdot L_i \quad$ with $\quad \max_{\lambda}(N_i) \quad$ FOC: $\dfrac{\partial N_i}{\partial \lambda} = 0 \quad \Rightarrow \quad$ solution: λ_N^u

Again, the long-term solution λ_N^u is analogous to the short-term solution in eq. (11), if the current price $p = p_t$ is replaced by the weighted average price pa_i. The short-term case – without any uncertainty about the current price – can be interpreted as a special case of situation $i = 1$, with a zero uncertainty level: $\varepsilon = 0$. Thus, the general solution for all cases is given by eqs. (19) to (25). A graphical representation of all situations $(i = 1, 2, 3)$ would

resemble Fig. 1, if instead of the current price $p = p_t$ the average price level pa_i is stated on the ordinate. A representation of all situations in one diagram based on the current price level p_t would shift all graphs vertically downwards by an extent of s_i, depending on the situation:

$$(26) \quad p_t = pa_i - s_i \qquad\qquad \text{with:} \quad s_1 = \frac{\delta \cdot (2 \cdot W - 1) \cdot \varepsilon}{1 + \delta} \ , \ s_2 = \frac{\delta \cdot W \cdot \varepsilon}{1 + \delta \cdot W} \ , \ s_3 = \varepsilon$$

A numerical simulation of the Nash bargaining solution is shown in Fig. 2 by a (p, λ)-diagram and the corresponding profit-maximising solution is illustrated in Fig. 3. The reaction of the share parameter λ on the current price level is discontinuous at the borderline between the three relevant situations: the unit costs-curve α, separating situation 2 and 3, and at the curve for $(\alpha + \varepsilon)$, separating situation 1 and 2. As it is obvious in both figures, the vertical shift of the relations between p_t and the resultant profit ratio λ_N^u or λ_f^u is equivalent to a shift of all these graphs to the left, eventually resulting in lower sharing ratios. In situation 1, with a high price level, the firm will continue to employ the worker in the future even in case of a negative $(-\varepsilon)$-change. Though the level of future profits is uncertain, however, in situation 1 continuing the activity is certain. In situation 2, where the price is below $(\alpha + \varepsilon)$, the firm will exit the market and fire the worker in case of a negative $(-\varepsilon)$-realisation, however, for a positive $(+\varepsilon)$-change the profits will be shared. In this situation 2 the firm does not only share current profits, but with the long-term PS contract the firm additionally gives the worker an option to share high future profits in case of $(+\varepsilon)$ – and simultaneously ensures not to share low profits (or even losses) if in the future a negative $(-\varepsilon)$-realisation will occur. Since the negative realisation with low profits is truncated, and since only very profitable realisations are resulting in effective production and actual profit sharing, in situation 2 the firm is more reluctant to fix a high sharing ratio λ in a long-term PS contract. This option effect dominates in situation 3, where the firm is currently not employing the worker, but with a chance of future employment if a $(+\varepsilon)$-realisation will eventuate. In situation 3, via permanently fixing λ in advance by a long-term PS scheme, the firm has no advantage for the current period, but only gives the worker an option to participate in future profits, conditional on the case that these profits are high. Consequently, under uncertainty the firm is very reluctant to fix high sharing ratios for future employees. In situations 2 and 3, where these options on future profits are relevant, the shift downwards (resp. to the left) is the stronger the larger the absolute level of uncertainty ε is. Furthermore, in situations 1 and 2, the firm prefers a low sharing ratio, if a positive future is very probable, i.e. if the probability W of a $(+\varepsilon)$-change is large. Moreover, with a larger ε, situations 2 and 3 are prevailing, since the distance between the borderlines α and $(\alpha + \varepsilon)$ increases. On the contrary, situation 1 prevails if the level of uncertainty ε is relatively low. A comparison to a situation with a lower level of uncertainty ε is illustrated in Fig. 4.

Fig. 2: Illustration of "long-term" Nash bargaining λ_N^u under uncertainty
$$(\eta = 1/2, \theta = 1/4, W = 1/2, \varepsilon = 1/2, \delta = 9/10)$$
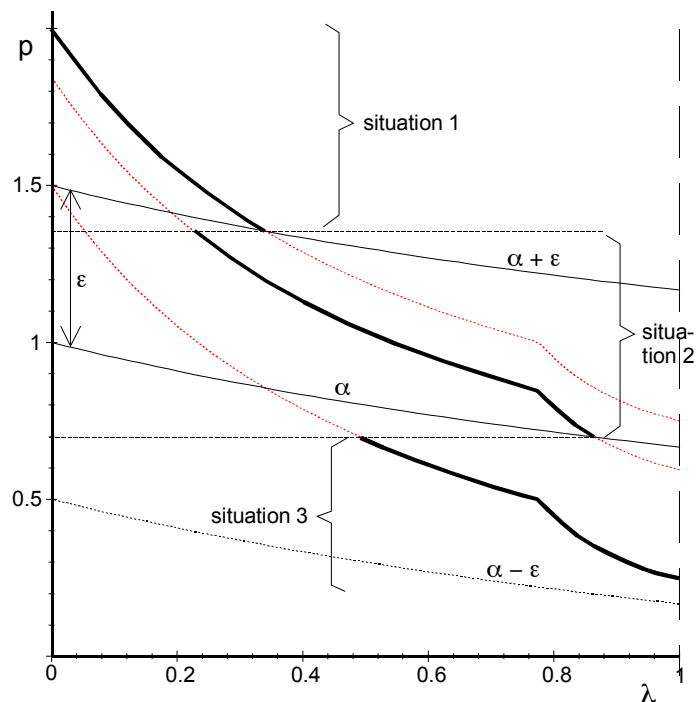

Fig. 3: Illustration of "long-term" net-profit maximisation λ_f^u under uncertainty
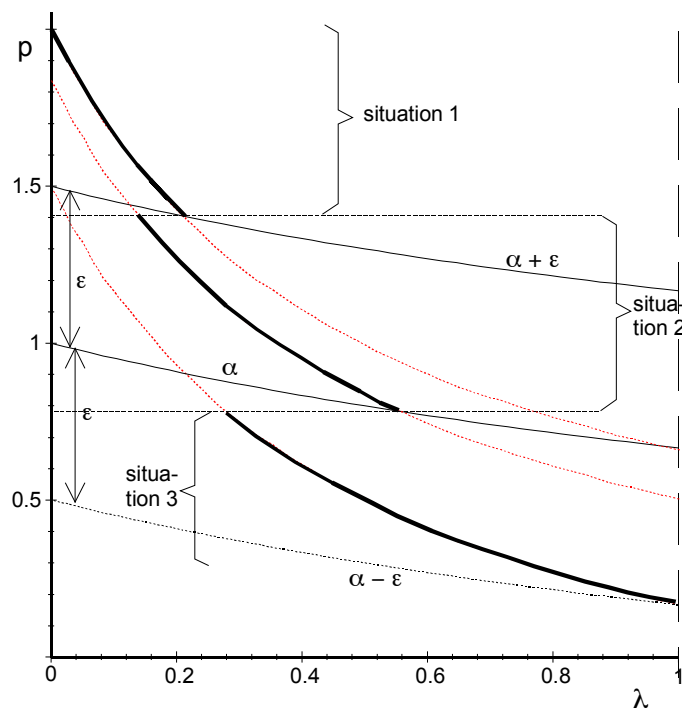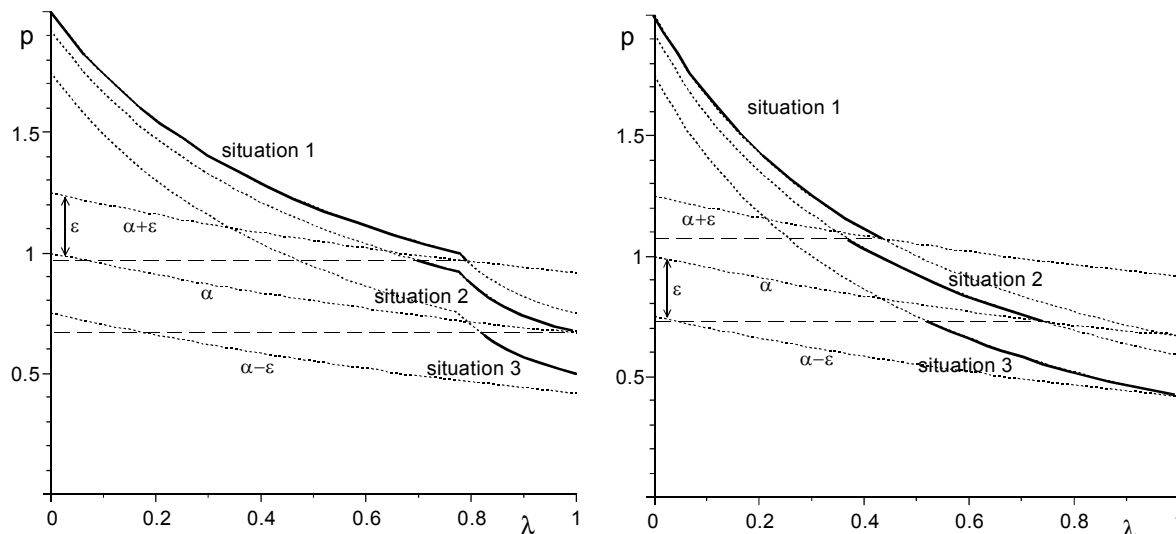$$(\eta = 1/2, \theta = 1/4, W = 1/2, \varepsilon = 1/2,)$$

Fig. 4: "Long-term" Nash bargaining λ_N^u and profit maximisation λ_f^u

with a low level of uncertainty ($\varepsilon = 1/4$)

5. Conclusions

In this contribution a very simple model of profit sharing as an entrepreneurial instrument to create incentive-based productivity effects was presented. If efficiency gains result, a remuneration contract including shared profits as a premium pay in addition to the market wage is Pareto-superior: By sharing the efficiency gains, both parties, the firm and the worker, are better off compared to a standard wage regime. Furthermore, the efficiency gains due to sharing profits may result in stimulation of labour demand and employment, since the firm's costs are reduced – though the worker receives a higher overall compensation. However, the focus of this paper is to combine efficiency effects of profit sharing with the impact of an option value which is based on the *expected variation of stochastic future profits*, if a *long-term profit sharing scheme* is ex-ante determined. An *optimal* remuneration policy was presented for two scenarios: First, the firm unilaterally offers a premium based on sharing profits in order to *maximise the firm's profits*, and second, a bilateral *Nash bargaining* solution was computed. In both cases option value effects have to be considered by the firm when permanently determining an optimal instrumental level of the profit sharing ratio given to the worker. The inclusion of expected future revenue variations results in a lower worker's profit sharing ratio – since a larger variation of revenue implies a higher redistribution of profits from firm to worker if a positive revenue change will occur in the future. In the case of a favourable future revenue development very high profits must be shared with the workers. In contrast, negative future outcomes are truncated, since future losses will not be shared because the firm uses its option to fire a worker in a loss situation, and since the worker has the option to leave the firm and to work elsewhere for the standard market wage. This is anticipated by the firm and results in a lower worker's sharing ratio which the firm is willing to fix in a long-term wage contract if the sharing ratio is ex-ante determined and held constantly over a period of time.

References

ANDERSON, S., DEVEREUX, M. [1989], Profit Sharing and Optimal Labour Contracts. *Canadian Journal of Economics*, 89, 425–433.

CHANG, J.-J. [2006], Profit Sharing, Risk Sharing, and Firm Size: Implications of Efficiency Wages. *Small Business Economics*, 27, 261-273.

ELBAUM, B. [1995], The Share Economy with Efficiency Wages. *Industrial Relations*, 34, 299-323.

KOSKELA, E., STENBACKA, R. [2004], Profit Sharing and Unemployment: An Approach with Bargaining and Efficiency-Wage Effects. *JITE – Journal of Institutional and Theoretical Economics*, 160, 477-497.

KRUSE, D.L. [1992], Profit Sharing and Productivity: Microeconomic Evidence from the United States. *Economic Journal*, 102, 24-36.

LEVINE, D.I. [1989], Efficiency Wages in Weitzman's Share Economy. *Industrial Relations*, 28, 321-334.

LIN, C.-C., CHANG, J.-J., LAI, C.-C. [2002], Profit sharing as a worker discipline device. *Economic modelling*, 19, 815-828

POHJOLA, M. [1987], Profit Sharing, Collective Bargaining and Employment. *JITE – Journal of Institutional and Theoretical Economics*, 143, 334-342.

SANFEY, P. J. [1993], On the Interaction between Efficiency Wages and Union-Firm Bargaining Models, *Economics Letters*, 41, 319–324.

SESSIONS, J.G. [2008], Wages, supervision and sharing. *Quarterly Review of Economics and Finance*, 48, 653-672.

WADHWANI, S.B., WALL, M. [1990], The Effects of Profit Sharing on Employment, Wages, Stock Returns and Productivity: Evidence from UK micro-data. *The Economic Journal*, 100, 1-17.

ANFANG VERBORGEN

ENDE VERBORGEN