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A Contest Model of a Professional Sports League with Two-Sided Markets

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Abstract

This paper develops a model of a professional sports league with network externalities by integrating the theory of two-sided markets into a contest model. In professional team sports, leagues function as a platform that enables sponsors to interact with fans. In these league-mediated interactions, positive network effects operate from the fan market to the sponsor market, while negative network effects operate from the sponsor market to the fan market. Clubs react to these network effects by charging higher (lower) prices to sponsors (fans). Our analysis shows that the size of these network effects determines the level of competitive balance within the league. Traditional models, which do not take network externalities into account, under- or overestimate the actual level of competitive balance, which may lead to wrong policy decisions. Moreover, we show that clubs benefit from stronger combined network effects through higher profits. Finally, we derive policy recommendations for improving competitive balance by taking advantage of network externalities.

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1 Introduction

The professional team sports industry has a unique organizational structure. It is the only industry in which production is organized by leagues. This unique organizational structure is the result of the industry-specific production and competition process. Industry outsiders often tend to regard individual teams as firms and treat them as production units. Unlike an automobile firm, however, an individual team cannot produce a marketable product. Each team needs at least one opponent to play a match. However, even a match between two teams is not an attractive product. The individual matches must be upgraded by integrating them into an organized championship race. This upgrade, which gives each individual match additional value within the larger context of the championship race, is managed by the league.

From a sports perspective, each team within a league wants to win as many games as possible. Economically, however, teams are not so much competitors but are rather complementors. The quality or economic value of the championship race depends to a large extent on the level of competitive balance. Unlike Toyota and Ford, which prefer weak competitors in their industry, sports teams like Real Madrid, the New York Yankees, and the Dallas Cowboys benefit from having strong opponents within their leagues. A more balanced league usually produces a more attractive - that is, economically more valuable - product.

Economists have designed various models of sports leagues. In an early contribution, El-Hodiri and Quirk (1971) developed a dynamic decision-making model of a professional sports league and incorporated certain fundamental features of the North American sports industry such as the reserve clause, player drafts and the sale of player contracts among teams. They show that revenue sharing does not influence competitive balance and thus confirm the "invariance proposition".[1] Fort and Quirk (1995) derive similar results in an updated, static version of the El-Hodiri and Quirk model. Atkinson et al. (1988) contradict the invariance proposition and show that revenue sharing can improve competitive balance. In their model, Atkinson et al. adopt a pool-sharing arrangement and a club revenue function that depends on the team's performance and on the performance of all other teams. Their result is supported by Marburger (1997), who builds his model on the assumption that fans care about the relative and absolute quality of teams. Vrooman (1995) shows that sharing the winning-elastic revenue does not affect competitive balance, while sharing the winning-inelastic revenue does improve competitive balance. Késenne

[1]The "invariance proposition" goes back to Rottenberg (1956) and states that the distribution of playing talent between clubs in professional sports leagues does not depend on the allocation of property rights to players' services.

(2000) develops a two-team model consisting of a large- and a small-market club and shows that a payroll cap, defined as a fixed percentage of league revenue divided by the number of teams, will improve competitive balance as well as the distribution of player salary within the league.

The recent sports economics literature has suggested modeling a team sports league by making use of contest theory.[2] In his seminal article, Szymanski (2003) applied Tullock's (1980) rent-seeking contest to ascertain the optimal design of sports leagues. Based on a model of two profit-maximizing clubs and a club revenue function that depends on the relative quality of the home team, Szymanski and Késenne (2004) show that gate revenue sharing decreases competitive balance. This result is driven by the so-called "dulling effect." The dulling effect describes the well-known fact in sports economics that revenue sharing reduces the incentive to invest in playing talent. Dietl and Lang (2008) confirm this finding and further show that gate revenue sharing increases social welfare.

As this brief literature review shows, analytical models in sports are mainly focused on the effect of cross-subsidization schemes such as reserve clauses, revenue sharing and salary caps on competitive balance without taking into account that the clubs' competition provides the platform for the interaction of various market sides such as fans, advertisers and sponsors, the media, and merchandising companies.

The interaction of different market sides via an intermediary platform creates what is called a "multisided market." Each of the distinct market sides demands a specific good or service provided by the intermediary. Frequently, the market sides do not interact with each other directly; however, they exert network externalities on each other. These externalities influence the market's demand structure and the intermediary's pricing schemes. Fans demand competition and the experience of a live event, advertisers and sponsors demand an audience that they can inform about their products or services, the media demand an audience willing to pay for the use of their services, merchandising companies demand customers who want to buy their articles, etc. An interaction between two market sides only takes place because of the underlying sports event. Fans would hardly go to the stadium to look at advertisement billboards if there were not a match taking place in the stadium that featured their favorite team. Merchandising companies would sell many fewer fan articles if their products were not linked to an active sports team, and so on. These examples underline the importance of the clubs' competition to act as a platform for the different market sides that interact and exert network externalities on each other.

[2] The first approaches in contest theory were made by Lazear and Rosen (1981), Green and Stokey (1983) and Nalebuff and Stiglitz (1983).

Research related to multisided markets is flourishing and has been conducted on a broad range of topics and industries. For instance, software platforms (Evans et al., 2004), payment systems (Rochet and Tirole, 2002; Schmalensee, 2002; Wright, 2003, 2004), the Internet (Baye and Morgan, 2001; Caillaud and Jullien, 2003) and media markets (Crampes et al., 2009; Reisinger et al., 2009). More general models have been proposed by Rochet and Tirole (2003), Armstrong (2006), Armstrong and Wright (2007) and Belleflamme and Toulemonde (2009). Despite this large variety of applications, the theory of multisided markets has not yet been applied to sports leagues. This paper tries to fill this gap.

We add to the literature by contributing to two different strands of literature: on the one hand, the literature on multisided markets and on the other hand, the literature on analytical models of sports leagues. To the best of our knowledge, we are the first to integrate the theory of two-sided markets into a contest model of a professional team sports league. Our model can then be used as a basic framework to analyze the effect of different cross-subsidization schemes in team sports leagues.

This paper develops a model of a professional sports league with network externalities by integrating the theory of two-sided markets into a two-stage contest model. In professional team sports, leagues function as a platform that enables sponsors to interact with fans. In these league-mediated interactions, positive network effects operate from the fan market to the sponsor market, while negative network effects operate from the sponsor market to the fan market.[3] Clubs react to these network effects by charging higher (lower) prices to sponsors (fans). Our analysis shows that the size of these network effects determines the level of competitive balance within the league. Depending on the market potential of the sponsors, competitive balance increases or decreases with stronger combined network effects. Traditional models that do not take network externalities into account, thus under- or overestimate the actual level of competitive balance, which may lead to wrong policy implications. Moreover, we show that clubs benefit from the presence of network externalities because club profits increase with stronger combined network effects.

The paper is of interest to policy-makers in a professional team sports league because we derive policy recommendations indicating that one can improve competitive balance by taking advantage of network externalities. Our model shows that an increase in the market potential of sponsors produces a more balanced league because the small club will increase its talent investments more than the large club in equilibrium. An increase in the market potential of the sponsors can be achieved,

[3]See Becker and Murphy (1993) for a discussion on advertisements as a good or bad. For further analysis of advertisements see, e.g., Depken and Wilson (2004) and Reisinger et al. (2009).

for instance, through an increase in the quota for the amount of advertisements set by the league organization.

The paper is structured as follows. In Section 2, we present our model with its notation and main assumptions. We specify fan and sponsor demand, the quality of the competition and club profits. In Section 3, we solve the two-stage game and derive the subgame-perfect equilibria. Section 4 highlights policy implications. Finally, Section 5 points out possible extensions and concludes the paper.

2 Model

We consider two clubs, denoted as 1 and 2, that compete in a professional team sports league. The clubs are asymmetric with respect to their market size - that is, there is one large-market club and one small-market club. Each club $i = 1, 2$ invests independently a certain amount $x_i \geq 0$ in playing talent to maximize its profits. We assume that talent is measured in perfectly divisible units that can be hired at a competitive labor market.

There are two groups of agents who are interested in the competition of the clubs, the fans and sponsors. Fans can consume sports competition in two ways: either by buying a ticket and going to the stadium or by purchasing a pay-TV license and watching the event on television. Either way, direct revenues for the clubs are generated in our simple model. Sponsors are attracted to the competition because sports events represent attractive levers that generate consumer interest. The fact that fans watch the competition draws the attention of sponsors who want to convince the fans to buy their products and services.

We consider the competition between the clubs as a platform that serves as the intermediary between fans on one market side and sponsors on the other market side. The attractiveness of a sports event for sponsors increases with the number of fans watching. The presence of sponsors, in turn, may have a negative effect on the attractiveness of the event for the fans. These indirect effects are modeled as network effects in the sponsor and fan demand functions.

The timing of the model features a two-stage structure:

1. Stage: Clubs invest independently in playing talent with the objective of maximizing their own profits. Talent investments determine the win percentages and thus the quality of the competition between the two clubs.

2. Stage: Given a certain quality of competition, fans decide what quantity of tickets/pay-TV licenses they want to purchase and sponsors decide what

amount of advertisements they want to place, taking into account the network effects that operate from one market side to the other. Each club then generates its own revenues dependent on the decisions of fans and sponsors.

In the paragraphs that follow we derive the demand functions of fans and sponsors under network effects and specify the quality of the competition. Finally, we derive club revenues, costs and profits.

2.1 Demand of fans and sponsors under network effects

The demand functions of the consumers (fans) and sponsors in stage 2 depend on the quality of the competition and are derived as follows:[4] we assume a continuum of fans and sponsors who differ in their valuation of a match with quality θ_i between the home team i and the away team j with $i, j = 1, 2,\ i \neq j$.[5] Fans and sponsors of club i have an individual valuation for quality that is measured for fans by ω_i^f and for sponsors by ω_i^s. For simplicity's sake, we assume that these preferences are uniformly distributed in $[0, m_i^f]$ for the fans and in $[0, m^s]$ for the sponsors.[6]

Without loss of generality, we assume that club 1 is the large-market club, with a higher drawing potential, and as a result, a bigger fan base than the small-market club 2, such that $m_1^f > m_2^f$. Furthermore, the parameter m^s represents the total market potential of the sponsors, or, in the case of a binding quota for sponsoring defined by the league authority, the sponsors' bounded market potential.[7] Even though parameter m^s is exogenously-given, the league authority could alter the market potential of sponsors by changing the quota on sponsoring.

Moreover, we assume a constant marginal utility of quality and define the net utility for fans as $\max\{\omega_i^f \theta_i - p_i^f, 0\}$ and that for sponsors as $\max\{\omega_i^s \theta_i - p_i^s, 0\}$. The price fans have to pay is denoted by p_i^f while p_i^s stands for the price for the sponsors. The price for fans can be interpreted as the gate price or the price for a pay-TV license, whereas the price for sponsors is the price they have to pay for advertisements. At prices p_i^f (and p_i^s), the fans (and sponsors) of club i who are indifferent to investing in the consumption of the match (and in advertising) are characterized by $\omega_i^{f*} = \frac{p_i^f}{\theta_i}$ (and $\omega_i^{s*} = \frac{p_i^s}{\theta_i}$). Hence, the measure of fans who purchase

[4] Our approach is similar to Falconieri et al. (2004), but we use a different quality function. Also see Dietl and Lang (2008), Dietl, Lang and Rathke (2009) and Dietl, Lang and Werner (2009).

[5] Note that quality θ_i represents the quality of the competition in the stadium of club i. The quality θ_i is specified below by equation (5).

[6] Note that the parameter m^s has no subscript, because there is only one homogeneous group of sponsors in the league offering advertisements to the two types of clubs.

[7] Under a quota on sponsoring one can imagine restrictions on where advertisements may be placed or on the specific types of companies that are allowed to appear as sponsors in a league.

tickets/pay-TV licenses at price p_i^f is $m_i^f - \omega_i^{f*} = m_i^f - \frac{p_i^f}{\theta_i}$. The corresponding measure of sponsors that offer advertising at price p_i^s is $m^s - \omega_i^{s*} = m^s - \frac{p_i^s}{\theta_i}$.

Taking into account that network effects operate from the fan to the sponsor market and vice versa, the demand function of fans at club $i = 1, 2$ is given by

$$q_i^f = m_i^f - \frac{p_i^f}{\theta_i} + n_s q_i^s, \tag{1}$$

and the demand function of sponsors that offer advertising at club $i = 1, 2$ is given by

$$q_i^s = m^s - \frac{p_i^s}{\theta_i} + n_f q_i^f, \tag{2}$$

where n_f and n_s stand for network effects exerted by fans and sponsors, respectively. Because the sponsor market and the fan market can coexist side by side, an additive combination of the two demand functions can be justified.[8] Note that network effects can be illustrated by a displacement of the demand functions q_i^f and q_i^s. In this respect, stronger network effects induce stronger displacement of the corresponding demand functions.

The network effects that operate from the fan market to the sponsor market are referred to as "fan-related network effects" and are denoted by n_f with $n_f \in [0, 1]$. We assume that the fan-related network effects are positive because more fans imply more publicity and thus have a positive effect on the demand in the sponsor market. On the other hand, the network effects that operate from the sponsor market to the fan market are referred to as "sponsor-related network effects" and are denoted by n_s. We assume that the sponsor-related network effects are negative because advertisement is considered to cause a certain disutility for fans and thus to have a negative effect on demand in the fan market. Therefore, $n_s \in [-1, 0]$.[9]

The combined network effects from fans and sponsors, denoted by η are given by

$$\eta := n_f + n_s.$$

A higher n_f (or a lower n_s) implies that the positive fan-related network effects are relatively more important than the negative sponsor-related network effects. Assuming that the positive fan-related network effects are at least not smaller than the negative sponsor-related network effects in absolute terms (i.e., $n_f \geq |n_s| \geq$

[8]See, e.g., Armstrong (2006) who uses similar demand functions.

[9]The utility effects of advertisements may be discussed. In this context, a first approach chooses the track of utility-reducing effects from advertisements. The disutility from advertisements can be drawn from the fact that fans go to the stadium to watch sports, not advertisements. In the case where the actual sports event is adapted to commercial requirements, e.g., special advertisement breaks, this becomes even more obvious. For further discussion of this aspect, see Becker and Murphy (1993), Depken and Wilson (2004) and Reisinger et al. (2009).

0) the combined network effects η are not smaller than zero - i.e., $\eta \in [0,1]$.[10] Consequently, $\eta > 0$ describes a situation with positive combined network effects in which the positive fan-related network effects in absolute values are stronger than the negative sponsor-related network effects. If $\eta = 0$ then the combined network effects equal zero. In this case, we have either a situation without network effects (i.e., $n_f = n_s = 0$) or a situation with equalized network effects in which both individual network effects are equal in terms of absolute values (i.e., $n_f = -n_s$).

2.2 The quality of the competition

Following Dietl and Lang (2008) and Dietl, Lang and Werner (2009), we assume that the quality of the competition θ_i depends on two factors: the probability of club i's success and the uncertainty of outcome. Furthermore, we assume that both factors enter the quality function as a linear combination with equal weights, that is, the quality of the competition is represented by the combination of the win percentage and the uncertainty of outcome.[11]

We measure the probability of club i's success by the win percentage w_i of this club. The win percentage is characterized by the contest-success function (CSF), which maps the vector (x_i, x_j) of talent investment into probabilities for each club. We apply the logit approach, which is probably the most widely used functional form of a CSF in sporting contests, and define the win percentage w_i of club i as:[12]

$$w_i(x_i, x_j) = \begin{cases} \frac{x_i}{x_i + x_j} & \text{if } \max\{x_i, x_j\} > 0, \\ \frac{1}{2} & \text{otherwise,} \end{cases} \tag{3}$$

where $x_i \geq 0$ characterizes the talent investments of club $i = 1, 2$. Given that the win percentages must sum up to unity, we obtain the adding-up constraint: $w_j = 1 - w_i$ with $i, j = 1, 2$, $i \neq j$. In our model, we adopt the "Contest-Nash conjectures" $\frac{\partial x_i}{\partial x_j} = 0$ and compute the derivative of equation (3) as $\frac{\partial w_i}{\partial x_i} = \frac{x_j}{(x_i + x_j)^2}$.[13]

The uncertainty of outcome is measured by the competitive balance in the league. Following Szymanski (2003), Dietl and Lang (2008), and Vrooman (2008),

[10] Note that the possible positive (even though small) effect of advertising on consumers (see, e.g., Nelson, 1974 and Kotowitz and Mathewson, 1979) lowers the overall negative sponsor-related network effects such that it is therefore reasonable to assume that $n_f \geq |n_s| \geq 0$.

[11] We will see below that this specification of the quality function gives rise to a quadratic revenue function widely used in the sports economic literature.

[12] The logit CSF was generally introduced by Tullock (1980). It was subsequently axiomatized by Skaperdas (1996) and Clark and Riis (1998). An alternative functional form would be the probit CSF (e.g., Lazear and Rosen, 1981; Dixit, 1987) and the difference-form CSF (e.g., Hirshleifer, 1989). See Dietl et al. (2008) and Fort and Winfree (2009) for analyses concerning the effect of the discriminatory power in the CSF.

[13] See Szymanski (2004).

we specify competitive balance (CB) by the product of the win percentages, i.e.,

$$CB(x_i, x_j) = w_i(x_i, x_j) \cdot w_j(x_i, x_j) = \frac{x_i x_j}{(x_i + x_j)^2}, \tag{4}$$

with $i, j = 1, 2,\ i \neq j$. Note that competitive balance attains its maximum of 1/4 for a completely balanced league in which both clubs invest the same amount in talent such that $w_1 = w_2 = 1/2$. A less balanced league is then characterized by a lower value of CB.

With the specification of the win percentage given by equation (3) and competitive balance given by equation (4), club i's quality function θ_i as described above is derived as

$$\theta_i(x_i, x_j) = w_i(x_i, x_j) + w_i(x_i, x_j) \cdot w_j(x_i, x_j) = \frac{x_i(x_i + 2x_j)}{(x_i + x_j)^2}, \tag{5}$$

with $i, j = 1, 2,\ i \neq j$. A higher win percentage w_i of club i induces the quality of the competition θ_i to increase, albeit with a decreasing rate, which reflects the impact of competitive balance on the quality of the competition, i.e., $\frac{\partial \theta_i}{\partial w_i} > 0$ and $\frac{\partial^2 \theta_i}{\partial w_i^2} < 0$.[14]

2.3 Derivation of club revenues, costs and profits

Each club generates its own revenues such that total revenue R_i of club i is given by the sum of the revenue $p_i^f q_i^f$ generated by fans at the gate or through pay-TV licenses and the revenue $p_i^s q_i^s$ generated by advertisement contracts with the sponsors:[15]

$$R_i = p_i^f q_i^f + p_i^s q_i^s = \left[\left(m_i^f - q_i^f + n_s q_i^s \right) q_i^f + \left(m^s - q_i^s + n_f q_i^f \right) q_i^s \right] \cdot \theta_i, \tag{6}$$

with $\theta_i = 2w_i(x_i, x_j) - w_i(x_i, x_j)^2$. This club-specific revenue function, which is quadratic in the win percentage, is widely used in the sports economics literature. For instance, our revenue is consistent with the revenue functions used in Hoehn and Szymanski (1999), Szymanski (2003), Szymanski and Késenne (2004), Késenne (2006, 2007) and Vrooman (2007, 2008). Moreover, note that club i's revenues increase with the quality of the competition θ_i.

We further assume that talent investments x_i generate costs for club i according

[14] For analyses of competitive balance in sports leagues, see, e.g., Fort and Lee (2007) and Fort and Quirk (2009).

[15] Note that in the present model, we implicitly assume that there is decentralized broadcasting, i.e., each club generates its own revenues. For an analysis of centralized versus decentralized broadcasting, see Falconieri et al. (2004) and Gurtler (2007).

to a linear cost function given by $C(x_i) = c \cdot x_i$ where $c > 0$ is the marginal unit cost of talent.[16]

The profit function of club i is then given by revenues minus costs and yields

$$\pi_i(x_i, x_j) = R_i(w_i(x_i, x_j)) - c \cdot x_i. \tag{7}$$

3 Equilibrium Analysis

In the first stage, clubs decide on their investments in playing talent, considering the cost of talent and its effect on their win percentage. In the second stage, given the quality of the competition as determined in stage 1, fans and sponsors make their decisions about how much to invest in tickets/pay-TV licenses and advertisements, respectively. We apply backward induction to solve for the subgame perfect equilibria in this two-stage game.

3.1 Stage 2: Fans purchase tickets/pay-TV licenses and sponsors place advertisement

In this subsection, we characterize the point at which the pricing under network externalities is optimal such that clubs obtain maximum revenue. Clubs will take into account the relatedness of the fan and sponsor market and thus consider the consequences of the two distinct network effects on the pricing decisions and demand functions. Formally, club $i = 1, 2$ maximizes its revenue $R_i = p_i^f q_i^f + p_i^s q_i^s$ in stage 2 by taking the investment decisions made in stage 1 as given. Note that we assume that marginal costs for sponsors and fans are zero.

The equilibrium in prices and quantities in stage 2 is derived in the next lemma:

Lemma 1
In stage 2, the equilibrium prices and quantities for fans and sponsors of club $i = 1, 2$ are given by

$$(\widehat{p}_i^f, \widehat{q}_i^f) = \left(\frac{m_i^f (2 - n_f \eta) + m^s (n_s - n_f)}{(2-\eta)(2+\eta)} \theta_i, \frac{2m_i^f + m^s \eta}{(2-\eta)(2+\eta)} \right), \tag{8}$$

$$(\widehat{p}_i^s, \widehat{q}_i^s) = \left(\frac{m_i^f (n_f - n_s) + m^s (2 - n_s \eta)}{(2-\eta)(2+\eta)} \theta_i, \frac{m_i^f \eta + 2m^s}{(2-\eta)(2+\eta)} \right). \tag{9}$$

[16] By assuming a competitive labor market, the market clearing cost of a unit of talent is the same for every club. Moreover, for the sake of simplicity, we do not take into account non-labor costs and normalize the fixed capital cost to zero. See Vrooman (1995) for a more general cost function where clubs have different marginal costs or Késenne (2007) for a cost function with a fixed capital cost.

Proof. See Appendix A.1. ■

In equilibrium, fans will demand the quantity represented by $\widehat{q}_i^f$ and are willing to pay the price represented by $\widehat{p}_i^f$. Correspondingly, the sponsors will demand $\widehat{q}_i^s$ and pay $\widehat{p}_i^s$ for each unit of advertisement in equilibrium.[17]

In order to build the intuition, we consider a scenario in which the sponsors and the fans of club i have symmetric market potential - i.e., $m^s = m_i^f = m_i > 0$. In this scenario, equilibrium prices and quantities for sponsors and fans of club $i = 1, 2$ are given by

$$\widehat{q}_i^s = \widehat{q}_i^f = \frac{m_i}{2-\eta} \text{ and } \widehat{p}_i^s = \frac{m_i(1-n_s)}{2-\eta}\theta_i,\ \widehat{p}_i^f = \frac{m_i(1-n_f)}{2-\eta}\theta_i.$$

Note that due to the symmetry of the two markets, sponsors and fans of club i demand an equal quantity $\widehat{q}_i^f = \widehat{q}_i^s$ in equilibrium. We derive that stronger combined network effects η yield higher quantities for both fans and sponsors in equilibrium. This is intuitive because lower negative sponsor-related network effects and thus increased combined network effects lead to an increase in the demand of fans. Because of positive fan-related network effects, this induces an increase in demand on the part of sponsors. In contrast to the equilibrium quantities, the equilibrium prices differ between fans and sponsors. Sponsors pay a higher price in equilibrium than do fans - i.e., $\widehat{p}_i^s > \widehat{p}_i^f$ for all $n_f > 0 > n_s$. Note that the price $\widehat{p}_i^f$ for fans ($\widehat{p}_i^s$ for sponsors) is lower (higher), the stronger are the positive fan-related network effects n_f, whereas the price $\widehat{p}_i^f$ for fans ($\widehat{p}_i^s$ for sponsors) is lower (higher), the stronger are the negative sponsor-related network effects n_s.

Comparative statics for the general case with asymmetric market potential of fans and sponsors lead to the following proposition:

Proposition 1
(i) Equilibrium quantities ($\widehat{q}_i^f, \widehat{q}_i^s$) for fans and sponsors of club i increase (decrease) with stronger fan-related (sponsor-related) network effects, i.e., $\frac{\partial \widehat{q}_i^\mu}{\partial n_f} > 0$ and $\frac{\partial \widehat{q}_i^\mu}{\partial n_s} > 0$ with $\mu \in \{f, s\}$.
(ii) Given a certain quality of the competition θ_i equilibrium prices $\widehat{p}_i^f$ for fans ($\widehat{p}_i^s$ for sponsors) of club i decrease (increase) with stronger fan-related network effects, i.e., $\frac{\partial \widehat{p}_i^f}{\partial n_f} < 0$ and $\frac{\partial \widehat{p}_i^s}{\partial n_f} > 0$.

Proof. See Appendix A.2. ■

[17] Note that if the market potential of the sponsors is larger than that of the fans of club i, i.e., $m^s > m_i^f$, we must bound m^s from above such that $m^s < \overline{m}^s \equiv \frac{m_i^f(2-n_f\eta)}{n_f-n_s}$ in order to guarantee that $\widehat{p}_i^f > 0$.

Part (i) of the proposition shows that the stronger are the positive fan-related network effects, the higher is the equilibrium quantity demanded by fans and sponsors, whereas the higher is the disutility of the sponsors' advertisement for the fans, the lower is the equilibrium quantity demanded by fans and sponsors. It follows that the equilibrium demand for advertisements $\widehat{q}_i^s$ as well as for tickets/pay-TV licenses $\widehat{q}_i^f$ is higher in a situation in which the combined network effects are positive than in a situation in which the combined network effects are zero. The intuition is as in the case with symmetric market potential above. Ceteris paribus, a decrease in the disutility of the sponsors' advertisement directed toward the fans leads to an increase in fan demand and consequently, due to positive fan-related network effects, to an increase in the demand of the sponsors.

Note that fans of club i demand a higher quantity in equilibrium if their market potential is larger than that of the sponsors - i.e.,

$$\widehat{q}_i^f > \widehat{q}_i^s \Leftrightarrow m_i^f > m^s.$$

Part (ii) of the proposition shows that given a certain quality of the competition θ_i the equilibrium price $\widehat{p}_i^f$ for the fans of club i is lower, the stronger are the positive fan-related network effects n_f, whereas the opposite holds true for the equilibrium price $\widehat{p}_i^s$ for the sponsors. This result is in accordance with the special case of symmetric market potentials. Stronger fan-related network effects induce an increase in the demand function of the sponsors and yields, ceteris paribus, a decrease in the prices for sponsors. Thus, if club i decreases the price for the market with positive network effects (fan market), it enhances the positive effect on revenues. It follows that due to the positive network effects exerted by the fans on the sponsors, a revenue-maximizing club has an incentive to keep prices low on the market with the positive network effects, whereas in the market with negative network effects (the sponsor market), it has an incentive to charge higher prices.

Whether the equilibrium price for fans is higher than that for the sponsors depends on the relationship between the market potential of fans and sponsors and the particular network effects. Formally, we derive the following:

$$\widehat{p}_i^f < \widehat{p}_i^s \Leftrightarrow \frac{m_i^f}{m^s} < \frac{1-n_s}{1-n_f}. \tag{10}$$

Equation (10) shows that as long as the market potential of the fans relative to that of the sponsors is smaller than $(1-n_s)\,/\,(1-n_f)$, prices are higher in the sponsor market than in the fan market. Ceteris paribus, a decrease in the fan-related network effects renders the fan market less important (due to its lower network effects) and the right-hand side of the inequality decreases such that the inequality may not be satisfied anymore. In this case, equilibrium prices on the fan market may be higher than on the sponsor market. Note that if the market potential of the sponsor market is higher than the market potential of the fan market for club i (i.e. $m^s > m_i^f$) then independent of the network effects, prices will be higher in the sponsor market because $\frac{1-n_s}{1-n_f} > 1$ for all $1 \geq n_f \geq |n_s| \geq 0$.

Furthermore, we derive from (8) and (9) that in a situation without network effects (i.e., $n_f = n_s = 0$), club i maximizes its revenue by making the quantity sold to fans directly proportional to the quantity sold to sponsors with $\widehat{q}_i^f = \left(m_i^f / m^s\right) \cdot \widehat{q}_i^s$.[18] That is, fans consume a m_i^f/m^s-fold amount of tickets/pay-TV licenses relative to the advertisements placed by the sponsors, and the pricing decisions with respect to fans and sponsors are independent of each other. Finally, we see that equilibrium prices for fans (sponsors) are lower (higher) in a situation with positive combined network effects than in a situation in which combined network effects equal zero.

By substituting equilibrium prices and quantities of fans and sponsors from (8) and (9) in the revenue function (6), we compute the revenue of club $i = 1, 2$ as

$$\widehat{R}_i = \kappa_i \cdot \theta_i = \kappa_i \frac{x_i(x_i + 2x_j)}{(x_1 + x_2)^2}, \tag{11}$$

with

$$\kappa_i := \frac{\left(m_i^f\right)^2 + (m^s)^2 + m_i^f m^s \eta}{(2-\eta)(2+\eta)}.$$

Note that the revenue function given by (11) satisfies the properties of the revenue function proposed by Szymanski and Késenne (2004).

In the next lemma, we derive some useful properties of the function κ_i which will be exploited in the subsequent analysis:

Lemma 2

We consider $\kappa_i(\eta)$ as a function of the combined network effects η and derive the following properties:

$$\kappa_1(\eta) > \kappa_2(\eta) \textit{ and } \frac{\partial \kappa_1(\eta)}{\partial \eta} > \frac{\partial \kappa_2(\eta)}{\partial \eta} > 0.$$

Proof. See Appendix A.3. ■

It follows from Lemma 2 that given a certain quality of competition equal for both clubs - i.e., $\theta_1 = \theta_2$ - the revenue of the large club will be higher than the

[18]Note that this relationship holds true also in a situation in which combined network effects are zero.

revenue of the small club. Moreover, revenues for both types of clubs increase with stronger combined network effects, where the increase is stronger for the large club than for the small club.

3.2 Stage 1: Clubs invest in playing talent

In stage 1, club i maximizes its profits by anticipating the decisions made in stage 2. By substituting club revenues (11) into the profit function (7), we derive the maximization problem of club $i = 1, 2$ in stage 1 as

$$\max_{x_i \geq 0} \left\{ \widehat{R}_i(x_i, x_j) - cx_i \right\} = \frac{\left(m_i^f\right)^2 + (m^s)^2 + m_i^f m^s \eta}{(2-\eta)(2+\eta)} \cdot \theta_i(x_i, x_j) - cx_i, \qquad (12)$$

with $\theta_i = \frac{x_i(x_i+2x_j)}{(x_1+x_2)^2}$. The solution to this maximization problem is given in the next lemma:

Lemma 3
In stage 1, the equilibrium talent investments of clubs $i = 1, 2$ are given by

$$\widehat{x}_i = \frac{2\kappa_i\kappa_j \left[\kappa_i(\kappa_i + 3\kappa_j) - (\kappa_i\kappa_j)^{1/2}(3\kappa_i + \kappa_j)\right]}{c(\kappa_i - \kappa_j)^3}, \qquad (13)$$

with $\kappa_i = \frac{\left(m_i^f\right)^2 + (m^s)^2 + m_i^f m^s \eta}{(2-\eta)(2+\eta)}$ *and* $i, j = 1, 2, i \neq j$.

Proof. See Appendix A.4. ■

Both types of clubs invest a positive amount $\widehat{x}_i > 0$ in playing talent. Moreover, the large club invests more in talent than does the small club (i.e., $\widehat{x}_1 > \widehat{x}_2$) because the marginal revenue of talent investments is higher for the former type of club due to the larger market potential of its fans.[19] Note that the investments of both clubs are influenced by the network effects exerted by fans and sponsors. Again, the extent to which fans and sponsors indirectly influence each other determines the decision of each club to invest in playing talent.

Substituting the equilibrium investments (13) in the CSF function (3) yields the following equilibrium win percentages:

$$(\widehat{w}_1, \widehat{w}_2) = \left(\frac{\kappa_1}{\kappa_1 + (\kappa_1\kappa_2)^{1/2}}, \frac{\kappa_2}{\kappa_2 + (\kappa_1\kappa_2)^{1/2}} \right). \qquad (14)$$

By analyzing the impact of network effects on the win percentages, we can establish the following proposition:

[19] Grossmann and Dietl (2009) show that in a dynamic two-period model of a sports league, it is possible that the small-market club invests more than the large-market club.

Proposition 2

Stronger combined network effects η induce the large (small) club to decrease (increase) its win percentage in equilibrium and thus produce a more balanced league if and only if the market potential of the sponsors is sufficiently small. Formally, $\frac{\partial \widehat{w}_1}{\partial \eta} < 0$ and $\frac{\partial \widehat{w}_2}{\partial \eta} > 0 \Leftrightarrow (m^s)^2 < m_1^f m_2^f$.

Proof. See Appendix A.5. ■

The proposition shows that if the market potential of the sponsors is sufficiently small, then the win percentage of the large (small) club is lower (higher), the stronger are the positive network effects that operate from fans to sponsors (or equivalently, the higher the disutility of the sponsors' advertisement for the fans). The rationale for this result is that in the case of low market potential on the part of the sponsors (i.e., $(m^s)^2 < m_1^f m_2^f$), higher combined network effects induce the small club to increase its equilibrium investments more than the large club. This results from the positive impact the network effects exert on the incentives of the small club to invest. The opposite, however, holds true if the market potential of the sponsors is sufficiently large. In this case, competitive balance decreases through stronger combined network effects.

Thus, a league in which the positive network effects that operate from the fan market to the sponsor market are stronger than the negative network effects that operate from the sponsor market to the fan market may be characterized by a higher degree of competitive balance than a league in which combined network effects are zero. The opposite holds true if the market potential of the sponsors is sufficiently large. In this case, competitive balance decreases through stronger combined network effects.

Furthermore, note that the quality of the competition $\widehat{\theta}_i$ in equilibrium can be expressed in terms of κ_i as

$$\widehat{\theta}_i = \widehat{w}_i + \widehat{w}_i \widehat{w}_j = \frac{\kappa_i \left(2\kappa_j + \sqrt{\kappa_i \kappa_j}\right)}{\left(\kappa_i + \sqrt{\kappa_i \kappa_j}\right)\left(\kappa_j + \sqrt{\kappa_i \kappa_j}\right)}.$$

A direct consequence of Proposition 2 is that stronger network effects imply a lower (higher) quality of competition for the large (small) club if and only if the market potential of the sponsors is sufficiently small. Formally, $\frac{\partial \widehat{\theta}_1}{\partial \eta} < 0$ and $\frac{\partial \widehat{\theta}_2}{\partial \eta} > 0 \Leftrightarrow (m^s)^2 < m_1^f m_2^f$.[20]

[20] Note that the match quality for the large (small) market club decreases (increases) if and only if the league becomes more balanced. As we know from Proposition 2, a more balanced (unbalanced) league emerges in the case of sufficiently low (high) market potential on the part of the sponsors.

The impact of network effects on club profits is established in the following proposition:

Proposition 3
Stronger combined network effects increase profits for both the small and the large club.

Proof. See Appendix A.6. ■

The proposition shows that the profits of the small and the large club increase if the positive network effects that operate from the fan market to the sponsor market increase or equivalently if the negative sponsor-related network effects decrease. Thus, the two types of clubs benefit from stronger network effects. To see the intuition behind this result, remember that the profits of club i in equilibrium are given by $\widehat{\pi}_i = \kappa_i \widehat{\theta}_i - c\widehat{x}_i$, and thus, the partial derivatives with respect to combined network effects η yield $\frac{\partial \widehat{\pi}_i}{\partial \eta} = \frac{\partial \kappa_i}{\partial \eta}\widehat{\theta}_i + \kappa_i \frac{\partial \widehat{\theta}_i}{\partial \eta} - c\frac{\partial \widehat{x}_i}{\partial \eta}$. Through stronger combined network effects, both types of clubs face higher costs due to a higher investment level in playing talent. On the other hand, stronger combined network effects have a positive effect on equilibrium quantities $\left(\widehat{q}_i^f, \widehat{q}_i^s\right)$ and prices $\left(\widehat{p}_i^f, \widehat{p}_i^s\right)$ such that club revenues for both types of clubs increase. The higher club revenues compensate for the higher costs, and thus, club profits increase. Note that the positive effect on club revenues due to stronger combined network effects holds true even though the quality of the competition $\widehat{\theta}_i$ will decrease for the large (small) club if the market potential m^s of the sponsors is sufficiently small (large).

4 Policy Implications

In the following proposition, we suggest a policy measure that may contribute to improving competitive balance in a team sports league.

Proposition 4
The league authority can improve competitive balance by increasing the market potential m^s of the sponsors.

Proof. See Appendix A.7. ■

The proposition shows that an effective measure for improving competitive balance is to increase the market potential of the sponsors. The intuition behind the proposition is as follows. Remember that clubs generate revenues from fans and sponsors, where the amount of sponsorship revenues also depends on the amount of fans affiliated with a certain club (see Lemma 1). In equilibrium, the revenues generated from the sponsors' advertisements are higher for the large club than for the small club due to the larger market potential from the fans of the large club. An increase in the quota for the amount of advertising for the sponsors increases both clubs' revenues. Due to the decreasing returns to scale of sponsors' advertising the increase in revenues, however, is stronger for the small club than for the large club. It follows that the incentives to invest in playing talent are higher for the small club than for the large club, which causes the former type of club to increase its equilibrium talent investments more than the latter type of club. As a result, the win percentage of the large (small) club decreases (increases) and a more balanced league emerges. Note that an increase in the market potential of the sponsors could be achieved through an increase in the quota for the volume of advertisements set by the league organization. For instance, the league could allow sponsoring to appear on game jerseys or other areas that had been free of advertisements before.

Another aspect of interest is the choice of sponsors that a club makes. Based on the above analysis, it seems reasonable to assume that there are situations in which a club may decline a potentially attractive offer by a sponsor. This might be the case if this sponsor has a very negative reputation and thus exerts very strong negative network effects on the fan market. This may reduce revenues generated by the fans to such an extent that the advertising-related additional revenue cannot compensate for the loss in fan-related revenue. Under such circumstances, it may be profitable for a club to decline supposedly attractive offers by sponsors.

5 Conclusion

In this paper, we develop a contest model of a professional team sports league with two market sides. The competition between the clubs is the platform between fans on one market side and sponsors on the other market side. Positive network effects operate from the fan market to the sponsor market, and negative network effects operate from the sponsor market to the fan market.

Our analysis shows that a revenue-maximizing club has an incentive to keep prices low in the market with positive network effects (fan market) and charge a higher price in the market with negative network effects (sponsor market). The reason is that low prices on the fan market enhance the positive effect on club revenues due to the positive network effects that operate from the fan market to the sponsor market. Note that an increase in the demand in the fan market leads (through positive fan-related network effects) to an increase in the demand on the sponsor market. If, however, clubs charged high prices in the market with positive network effects, they would inhibit the positive effect on their revenues. Moreover, increased network effects induce an alteration of the demand function in the other market. Stronger fan-related network effects induce an increase in sponsor demand, whereas stronger sponsor-related network effects induce a decrease in fan demand.

We further derive that network externalities crucially affect competitive balance in a sports league. In particular, we show that stronger combined network effects induce both clubs to increase their talent investments in equilibrium. If the market potential of the sponsors is sufficiently small, the increase in talent investments of the small club will be stronger than that of the large club because the small club will benefit more from stronger network effects than will the large club. As a result, the win percentage of the small club increases and the win percentage of the large club decreases in equilibrium, yielding a more balanced league. We conclude that it is important to incorporate network effects into the analysis of team sports leagues. Depending on the market potential of the sponsors, traditional analyses of sports leagues that do not take network effects into account may under- or overestimate the actual level of competitive balance in a league. Based on these predictions, traditional analyses may therefore suggest the wrong policy implications. For instance, they may suggest the implementation of measures to increase competitive balance, which may not be necessary because the league may already be sufficiently balanced. Finally, our model shows that both types of clubs benefit from the presence of network externalities because club profits always increase with stronger combined network effects. This result holds true even though costs increase for both types of clubs due to higher talent investments. The higher club revenues, however, compensate for the higher costs, such that club profits always increase.

Taking a closer look at major team sports leagues worldwide, one can find a number of phenomena that can be explained by our model. For example, the differences in match attendance and average ticket prices between national leagues in European football are accompanied by strong divergences in sponsor-related revenues. While matchday income (e.g., ticket sales and the like) makes up a higher percentage of revenues in the English Premier League than in the German Bundesliga, sponsorship is far more important in the latter. This fact mirrors the trade-off between fan-related and sponsor-related revenues. The quota for sponsorship in many North American major leagues represents another example; even though teams might be able to obtain higher revenues by increasing the amount of sponsoring/advertisements, the majority of teams refrains from posting advertisements on jerseys.[21]

Our model can be used as a basic framework to analyze the effect of different

[21] Note that teams in the National Football League (NFL) are allowed to post a sponsor on their jerseys. Only a small proportion of teams, however, makes use of this opportunity.

cross-subsidization schemes in team sports leagues. For instance, an interesting avenue for further research would be the extension of our model to a league with a revenue-sharing arrangement (either gate revenue sharing or pool revenue sharing). Revenue-sharing arrangements have been introduced to improve competitive balance and are common in professional sports leagues all around the world. The most prominent example is probably the salary cap operated by the National Football League (NFL), where the visiting club secures 40% of the locally earned television and gate receipt revenue. Major League Baseball (MLB) has a revenue-sharing agreement whereby all the clubs in the American League put 34% of their locally generated revenue (gate, concessions, television, etc.) into a central pool, which is then divided equally among all the clubs. The inclusion of some form of revenue sharing in the model with two-sided markets could yield further important implications for the governance of team sports leagues.

Another possible extension is the incorporation of so-called intra-side network externalities into our model. Intra-side network externalities are network effects that operate on one market side. In our setting, one may think that fans positively value the presence of other fans on their market side - e.g., due to a better atmosphere in the stadium - whereas sponsors may experience disutility if there are other sponsors on their same market side because advertisers compete for visibility. In this respect, it would be interesting to analyze the effect of positive and/or negative intra-side network effects on prices, quantities, competitive balance and club profits.

A Appendix

A.1 Proof of Lemma 1

In stage 2, club $i = 1, 2$ maximizes its revenue $R_i = p_i^f q_i^f + p_i^s q_i^s$, by taking the investment decisions made in stage 1 as given. Formally, club i solves the following maximization problem:[22]

$$\max_{(q_i^f, q_i^s) \geq 0} R_i = p_i^f q_i^f + p_i^s q_i^s = \left[(m_i^f - q_i^f + n_s q_i^s) q_i^f + (m^s - q_i^s + n_f q_i^f) q_i^s \right] \theta_i. \quad (15)$$

The reaction functions are derived as

$$q_i^f \left(q_i^s \right) = \frac{1}{2} \left(m_i^f + (n_f + n_s) q_i^s \right) \text{ and } q_i^s \left(q_i^f \right) = \frac{1}{2} \left(m^s + (n_f + n_s) q_i^f \right).$$

Note that there is a positive relationship between the quantities demanded by sponsors and fans in equilibrium because if the combined network effects are positive, i.e., $n_f + n_s > 0$.

Solving this system of reaction functions, yields the following equilibrium quantities for club i

$$\widehat{q}_i^f = \frac{2m_i^f + m^s \eta}{(2-\eta)(2+\eta)} \text{ and } \widehat{q}_i^s = \frac{m_i^f \eta + 2m^s}{(2-\eta)(2+\eta)}.$$

Substitution into prices $p_i^f = \left(m_i^f - \widehat{q}_i^f + n_s \widehat{q}_i^s \right) \theta_i$ and $p_i^s = \left(m^s - \widehat{q}_i^s + n_f \widehat{q}_i^f \right) \theta_i$ yields

$$\widehat{p}_i^f = \frac{m_i^f (2 - n_f \eta) + m^s (n_s - n_f)}{(2-\eta)(2+\eta)} \theta_i \text{ and } \widehat{p}_i^s = \frac{m_i^f (n_f - n_s) + m^s (2 - n_s \eta)}{(2-\eta)(2+\eta)} \theta_i.$$

This completes the proof of the lemma.

A.2 Proof of Proposition 1

(i) In order to show that equilibrium quantities $(\widehat{q}_i^f, \widehat{q}_i^s)$ for fans and sponsors of club i increase (decrease) with stronger fan (sponsor) network effects, we compute

$$\frac{\partial \widehat{q}_i^f}{\partial n_f} = \frac{\partial \widehat{q}_i^f}{\partial n_s} = \frac{4m_i^f \eta + m^s (4 + \eta^2)}{\left[(2-\eta)(2+\eta) \right]^2} > 0 \text{ and } \frac{\partial \widehat{q}_i^s}{\partial n_f} = \frac{\partial \widehat{q}_i^s}{\partial n_s} = \frac{4m^s \eta + m_i^f (4 + \eta^2)}{\left[(2-\eta)(2+\eta) \right]^2} > 0,$$

for all $m_i^f > 0$, $m^s > 0$, $1 \geq n_f \geq |n_s| \geq 0$ and $\eta \in [0, 1]$.

[22] In our setting it is an equivalent approach if clubs first maximize revenues with respect to quantities and then derive equilibrium prices or vice versa.

(ii) In order to show that, given a certain quality of the competition θ_i, equilibrium prices $\widehat{p}_i^f$ for fans ($\widehat{p}_i^s$ for sponsors) of club i decrease (increase) with stronger fan-related network effects, we compute

$$\begin{aligned}
\frac{\partial \widehat{p}_i^f}{\partial n_f} &= \frac{m_i^f\left(n_s\eta^2 - 4n_f\right) + m^s\left[4 + \eta(n_f - 3n_s)\right]}{\left[(2-\eta)(2+\eta)\right]^2} < 0, \\
\frac{\partial \widehat{p}_i^s}{\partial n_f} &= \frac{m_i^s\left(4n_f - n_s\eta^2\right) + m_i^f\left[4 + \eta(n_f - 3n_s)\right]}{\left[(2-\eta)(2+\eta)\right]^2} > 0,
\end{aligned}$$

for all $m_i^f > 0$, $m^s > 0$, $1 \geq n_f \geq |n_s| \geq 0$ and $\eta \in [0,1]$.

This completes the proof of the proposition.

A.3 Proof of Lemma 2

We consider $\kappa_i(\eta)$ as a function of η and derive the following properties:

$$\begin{aligned}
\kappa_1(\eta) - \kappa_2(\eta) &= \frac{\left(m_1^f - m_2^f\right)\left(m_1^f + m_2^f + m^s\right)}{(2-\eta)(2+\eta)} > 0, \\
\frac{\partial \kappa_i(\eta)}{\partial \eta} &= \frac{\left(m_i^f\eta + 2m^s\right)\left(2m_i^f + m^s\eta\right)}{\left[(2-\eta)(2+\eta)\right]^2} > 0, \\
\frac{\partial \kappa_1(\eta)}{\partial \eta} &> \frac{\partial \kappa_2(\eta)}{\partial \eta}.
\end{aligned}$$

for all $m_1^f > m_2^f > 0, m^s > 0$ and $\eta \in [0,1]$. This completes the proof of the lemma.

A.4 Proof of Lemma 3

In stage 1, club $i = 1,2$ maximizes its profit by anticipating the optimal behavior in stage 2. Thus the maximization problem of club $i = 1,2$ in stage 1 is given by

$$\max_{x_i \geq 0} \pi_i = \kappa_i\theta_i(x_i, x_j) - cx_i = \kappa_i\frac{x_i(x_i + 2x_j)}{(x_1 + x_2)^2} - cx_i,$$

The first-order conditions for this maximization problem yield[23]

$$\frac{\partial \pi_1}{\partial x_1} = \frac{2\kappa_1 x_2^2}{(x_1 + x_2)^3} - c = 0 \text{ and } \frac{\partial \pi_2}{\partial x_2} = \frac{2\kappa_2 x_1^2}{(x_1 + x_2)^3} - c = 0,$$

[23]It is easy to verify that the second-order conditions for a maximum are satisfied.

with $\kappa_i = \frac{\left(m_i^f\right)^2 + (m^s)^2 + m_i^f m^s \eta}{(2-\eta)(2+\eta)}$. Solving this system of equations, yields

$$\begin{aligned}
\widehat{x}_1 &= \frac{2\kappa_1\kappa_2\left[\kappa_1(\kappa_1+3\kappa_2)-(\kappa_1\kappa_2)^{1/2}(3\kappa_1+\kappa_2)\right]}{c(\kappa_1-\kappa_2)^3}, \\
\widehat{x}_2 &= \frac{2\kappa_1\kappa_2\left[-\kappa_2(3\kappa_1+\kappa_2)+(\kappa_1\kappa_2)^{1/2}(\kappa_1+3\kappa_2)\right]}{c(\kappa_1-\kappa_2)^3}.
\end{aligned}$$

This completes the proof of the lemma.

A.5 Proof of Proposition 2

In order to prove that stronger network effects induce the large (small) club to decrease (increase) its win percentage in equilibrium if and only if the market potential of the sponsors is sufficiently small, we proceed as follows. We write $\frac{\partial \kappa_i(\eta)}{\partial \eta} = \kappa_i'(\eta)$. According to Lemma 2, we know that $\kappa_1(\eta) > \kappa_2(\eta)$ and $\kappa_1'(\eta) > \kappa_2'(\eta) > 0$. Thus, we compute

$$\frac{\widehat{w}_1}{\widehat{w}_2} = \frac{\kappa_1(\eta)}{\sqrt{\kappa_1(\eta)\kappa_2(\eta)}} > 1.$$

Now, we will show that $\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial\eta} < 0$ and thus $\frac{\partial \widehat{w}_1}{\partial \eta} < 0$ and $\frac{\partial \widehat{w}_2}{\partial \eta} > 0$:

$$\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial\eta} = \frac{\kappa_1(\eta)\left[\kappa_1'(\eta)\kappa_2(\eta)-\kappa_1(\eta)\kappa_2'(\eta)\right]}{2\left[\kappa_1(\eta)\kappa_2(\eta)\right]^{3/2}} < 0 \Leftrightarrow \frac{\kappa_1(\eta)}{\kappa_2(\eta)} > \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)}.$$

With $\kappa_i(\eta) = \frac{\left(m_i^f\right)^2 + (m^s)^2 + m_i^f m^s \eta}{(2-\eta)(2+\eta)}$, it holds

$$\frac{\kappa_1(\eta)}{\kappa_2(\eta)} = \frac{\left(m_1^f\right)^2 + (m^s)^2 + m_1^f m^s \eta}{\left(m_2^f\right)^2 + (m^s)^2 + m_2^f m^s \eta} \text{ and } \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)} = \frac{\left(m_1^f\eta + 2m^s\right)\left(2m_1^f + m^s\eta\right)}{\left(m_2^f\eta + 2m^s\right)\left(2m_2^f + m^s\eta\right)}.$$

We conclude that

$$\frac{\kappa_1(\eta)}{\kappa_2(\eta)} > \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)} \Leftrightarrow (m^s)^2 < m_1^f m_2^f.$$

This completes the proof of the proposition.

A.6 Proof of Proposition 3

For expositional sake, we provide a formal proof for a linear revenue function. The proof for a quadratic revenue function is mathematically equivalent but notational very cumbersome. We therefore stick to the case of linear revenues. In case of linear revenues, the profit function of club i is given by

$$\pi_i = \kappa_i w_i - x_i,$$

such that the equilibrium investments $\widehat{x}_i$ and win percentages $\widehat{w}_i$ yield

$$\begin{aligned}
(\widehat{x}_1, \widehat{x}_2) &= \left(\frac{\kappa_1^2 \kappa_2}{(\kappa_1 + \kappa_2)^2}, \frac{\kappa_1 \kappa_2^2}{(\kappa_1 + \kappa_2)^2} \right), \\
(\widehat{w}_1, \widehat{w}_2) &= \left(\frac{\kappa_1}{\kappa_1 + \kappa_2}, \frac{\kappa_2}{\kappa_1 + \kappa_2} \right).
\end{aligned}$$

Equilibrium profits $\widehat{\pi}_i$ of club i are thus computed as

$$\widehat{\pi}_i = \frac{\kappa_i^2}{\kappa_1 + \kappa_2}.$$

The derivative with respect to network effects η is given by

$$\frac{\partial \widehat{\pi}_i}{\partial \eta} = \frac{\kappa_i(\eta) \left[(\kappa_i(\eta) + 2\kappa_j(\eta)) \kappa_i'(\eta) - \kappa_i(\eta) \kappa_j'(\eta) \right]}{(\kappa_i(\eta) + \kappa_j(\eta))^2}.$$

We derive

$$\frac{\partial \widehat{\pi}_1}{\partial \eta} > 0 \Leftrightarrow \kappa_1(\eta) > \kappa_2(\eta) \text{ and } \kappa_1'(\eta) > \kappa_2'(\eta) > 0,$$

whereas

$$\frac{\partial \widehat{\pi}_2}{\partial \eta} > 0 \Leftrightarrow \frac{2\kappa_1(\eta) + \kappa_2(\eta)}{\kappa_2(\eta)} > \frac{\kappa_1'(\eta)}{\kappa_2'(\eta)}.$$

However, one can show that the last inequality is always fulfilled with

$$\kappa_i = \frac{\left(m_i^f\right)^2 + (m^s)^2 + m_i^f m^s \eta}{(2 - \eta)(2 + \eta)},$$

in combination with $m_1^f > m_2^f > 0$, $m^s > 0$ and $\eta \in [0, 1]$. This completes the proof of the proposition.

A.7 Proof of Proposition 4

In order to prove that a larger market potential m^s of the sponsors increases the competitive balance in the league, we proceed as follows. We consider

$$\kappa_i(m^s) = \frac{\left(m_i^f\right)^2 + (m^s)^2 + m_i^f m^s \eta}{(2 - \eta)(2 + \eta)},$$

as a function of m^s and write $\frac{\partial \kappa_i(m^s)}{\partial m^s} = \kappa_i'(m^s)$. We derive the following properties:

$$\begin{aligned}
\kappa_1(m^s) - \kappa_2(m^s) &= \frac{\left(m_1^f - m_2^f\right)\left(m_1^f + m_2^f + m^s\right)}{(2-\eta)(2+\eta)} > 0, \\
\kappa_i'(m^s) &= \frac{m_i^f \eta + 2m^s}{(2-\eta)(2+\eta)} > 0, \\
\kappa_1'(m^s) &> \kappa_2'(m^s).
\end{aligned}$$

for all $m_1^f > m_2^f > 0, m^s > 0$ and $\eta \in [0,1]$. We know that competitive balance can be expressed in terms of $\kappa_i(m^s)$ as

$$\frac{\widehat{w}_1}{\widehat{w}_2} = \frac{\kappa_1(m^s)}{\sqrt{\kappa_1(m^s)\kappa_2(m^s)}} > 1$$

Now, we will show that $\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial m^s} < 0$ and thus $\frac{\partial \widehat{w}_1}{\partial m^s} < 0$ and $\frac{\partial \widehat{w}_2}{\partial m^s} > 0$:

$$\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial m^s} = \frac{\kappa_1(m^s)\left[\kappa_1'(m^s)\kappa_2(m^s) - \kappa_1(m^s)\kappa_2'(m^s)\right]}{2\left[\kappa_1(m^s)\kappa_2(m^s)\right]^{3/2}} < 0 \Leftrightarrow \frac{\kappa_1(m^s)}{\kappa_2(m^s)} > \frac{\kappa_1'(m^s)}{\kappa_2'(m^s)}$$

We derive

$$\frac{\kappa_1(m^s)}{\kappa_2(m^s)} = \frac{\left(m_1^f\right)^2 + (m^s)^2 + m_1^f m^s \eta}{\left(m_2^f\right)^2 + (m^s)^2 + m_2^f m^s \eta} \text{ and } \frac{\kappa_1'(m^s)}{\kappa_2'(m^s)} = \frac{m_1^f \eta + 2m^s}{m_2^f \eta + 2m^s}$$

and can show that $\frac{\kappa_1(m^s)}{\kappa_2(m^s)} > \frac{\kappa_1'(m^s)}{\kappa_2'(m^s)}$ holds for all $m^s > 0$. We conclude that competitive balance increases with a larger market potential of the sponsors, i.e., $\frac{\partial(\widehat{w}_1/\widehat{w}_2)}{\partial m^s} < 0$. This completes the proof of the proposition.

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