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Abstract

The fund-flow approach to production theory was first proposed by Nicholas Georgescu-Roegen almost half a century ago. Since then, from time to time it has received attention, but, probably because of its analytical complexity and difficulty to deliver sound “operational conclusions”, it is now almost abandoned. The approach has been also recently criticized for its instrumental assumption of constant efficiency of funds, by emphasizing its limitations in addressing issues related to fixed capital depreciation.

The paper critically surveys Georgescu-Roegen’s original model, together with the later developments and modifications. It also discusses the recent criticisms. The conclusion is that, despite its drawbacks, the fund-flow approach has a “competitive advantage” in the actual description of production as a process unfolding in time and entailing a temporal coordination between different elements. In this respect, it seems that most of its fruitful applications have yet to come.

Keywords: Fund-flow model; Georgescu-Roegen; Production theory; Returns to scale; Technical coefficients.

JEL Classification: B29; B59; C67; D24; L23; O33.

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“It is vain to hope that the production theory may reach the stage when its general analysis will yield practical recipes.”

(Georgescu-Roegen, 1976, p.51)

1 Introduction

The fund-flow model of production was developed by [Georgescu-Roegen](#) (1969, 1970, 1971); it is based on the distinction between *funds* – “the *agents* of a process” of production – and *flows* – the elements “which are *used* or *acted upon* by the agents” (1971, p.230) – and treats production as a process unfolding in historical time. As such, the model has proved well-suited to analyze the actual organization of production, which requires temporal coordination and interaction between its elements.

The model has been taken up by some authors who have extended and/or partially modified the original framework in the attempt to make it more operational. In particular, [Tani](#) (1986) has provided a finer analytical description of the conditions, both in terms of requirements and patterns of activation of the elementary processes, for the *line production*. [Morrone](#) (1992) and [Piacentini](#) (1995) have developed specific matrices to synthetically represent the quantitative and temporal patterns of the production process. [Piacentini](#) (1995) has worked out *time-explicit cost functions*, i.e. cost functions where the time dimension of production is explicitly taken into account. Finally, building on Georgescu-Roegen’s original model, [Scazzieri](#) (1993) and [Landesmann and Scazzieri](#) (1996a,b) have developed a model where the production process is seen as a *network of tasks*.

The approach has also recently attracted a number of criticisms, because of its “inadequate” treatment of the problem of capital utilization ([Kurz and Salvadori](#), 2003) and the lack of “operational conclusions” ([Lager](#), 2000).

In the paper, I first review Georgescu-Roegen’s original model with the analytical refinements by [Tani](#) (1988) and [Mir-Artigues and González-Calvet](#) (2007) (Section 2). Then, I critically survey the later developments and extensions (Section 3). Finally, I discuss the criticisms, together with the pros and cons of the approach (Section 4). Section 5 concludes with some final remarks.

2 The standard framewok

The fund-flow model was first presented by Nicholas Georgescu-Roegen in the mid-sixties at the *Conference of the International Economic Association* (Rome, 1965).¹ Since then, it has appeared in some of his subsequent works

¹For an account of the life and the main contributions of Georgescu-Roegen see, for instance, [Maneschi and Zamagni](#) (1997).

with only minor modifications (Georgescu-Roegen, 1969, 1970, 1971, 1990).

2.1 Process as Change and differences among funds, flows and stocks

According to its inventor, the fund-flow model is an *analytical-descriptive* method to study the process of production. In order to study the production process, one should first recognize that “process is a particularly baffling concept, for process is Change or is nothing at all” (Georgescu-Roegen, 1971, p.211). As such, a process can never be defined, but only analytically delimited. “No analytical boundary, no analytical process” (1971, p.211). These boundaries must be both *spatial* – delimiting the frontier of the process – and *temporal* – determining the duration of the process.

Taking such boundaries as a datum, one can record at each instant in time what elements cross them, entering or leaving the process. In so doing, she can also draw up an exhaustive list of these elements.

They are divided by Georgescu-Roegen in two broad categories:.

- *funds*: the elements that enter *and* leave the process, providing certain services over a certain period of time. They are never physically incorporated in the product. So, for instance, in the process of shoes production, the workers, land, and capital equipment are all funds.²
- *flows*: the elements that either enter *or* leave the process, but not both; i.e. “elements which appear only as input or only as output” (Georgescu-Roegen, 1970, p.4). Some flows enter the process and are then “incorporated” in the product (e.g. the energy and the leather in the shoes production); whereas some others only leave it (the shoes and the waste generated by the production activity).

A fund is not a stock: while a stock can be accumulated or decumulated in one single instant, the use of a fund, i.e. its decumulation, requires time. To give an example, a bag of twenty candies is a stock: you can make twenty children happy today, tomorrow or make one children happy for twenty days. An electric bulb lasting one thousand hours is a fund: you cannot use it to light one thousand rooms for an hour at the same time.

While all stocks accumulate or decumulate in a flow, not all flows imply an increase or a reduction in a stock (e.g. electricity). Georgescu-Roegen strongly criticized the distinction between stocks and flows as commonly

²The classification is process-related: a fund in one process may well be a flow in another.

Georgescu-Roegen considers a fund also the bulk of semi-processed goods (*process-fund*). Although the inclusion has been criticised by Landesmann and Scazzieri (1996b, p.22) for the merely passive role of these goods in the process, as noted by Mir-Artigues and González-Calvet (2007), their exclusion might create problems of internal consistency in the representation of line productions. On the point see also Section 3.1.

meant in economics and crystallised in the so-called Fisher's (1896) dictum: "stock relates to a *point* in time, flow to a *stretch* of time". According to him, the mistake implied by this definition was the consequence of the "original sin" of mainstream economics: the adoption of a mechanistic perspective, where "Change consists of locomotion and nothing else". For him, a better definition of flow is "a stock spread out over an interval of time" (Georgescu-Roegen, 1971, p.223), where the stock is the "*quantum* of substance".

As for the difference between flows and fund services, Georgescu-Roegen stresses that no confusion can arise since the latter are expressed in terms of substance \times time, whereas the former in terms of a substance/time.

2.2 Analytical representation of the production process

Having identified the spatial and temporal boundaries of the process, one can analytically describe production by referring to the temporal patterns of entrance and exit of the "substances" crossing these boundaries. In this representation processes develop over time, and the punchline of the approach is in fact the explicit consideration of the time dimension of production.³

In particular, given a process temporally delimited from 0 to T ($t \in [0, T]$), by denoting with $I_k(t)$ ($O_k(t)$) a function of time expressing the cumulative amount of the element k that has entered (left) the process from 0 to t , the process can be analytically represented by the following vector of functions:

$$(-I_1(t)_0^T, \dots, -I_K(t)_0^T; O_1(t)_0^T, \dots, O_K(t)_0^T) \quad (1)$$

A more compact representation is:

$$(F_1(t)_0^T, \dots, F_K(t)_0^T) \quad (2)$$

where:

$$F_k(t) = O_k(t) - I_k(t). \quad (3)$$

The flow elements are identically represented by Eq. (1) and (2), since the cumulative output (input) functions of all the inflows (outflows) elements are identically nihil and therefore redundant. As for funds instead, the value of $F_k(t)$ (≤ 0) returns the degree of operation in the process of the fund k .⁴ To emphasize such difference, in case of funds Eq. (3) can be denoted with $U_k(t)$ ($\equiv F_k(t)$), and Eq. (2) rewritten as:

$$(F_1(t)_0^T, \dots, F_M(t)_0^T, U_1(t)_0^T, \dots, U_J(t)_0^T) \equiv (\mathbf{F}(t)_0^T, \mathbf{U}(t)_0^T) \quad (4)$$

³An attempt to model time-specific analysis within a neoclassical framework is Winston (1982). An explicit consideration of the time profile of in- and out-flows can be found also in Frisch's (1964) *phase diagrams*, although such diagrammatic tools do not enter in his core analytical framework.

⁴If $F_k(t) = 0$ at time t the fund k is not in operation. A negative value indicates that the fund is actively involved in the process.

where the first M elements are flows and the others funds ($M + J = K$).

In order to maintain a symmetry with the flow coordinates, for funds one might also use the cumulative amount of their services:⁵

$$S_j(t) = \int_0^t U_j(\tau) d\tau \quad (5)$$

and represent the production process as:

$$(F_1(t)_0^T, \dots, F_M(t)_0^T, S_1(t)_0^T, \dots, S_J(t)_0^T) \equiv (\mathbf{F}(t)_0^T, \mathbf{S}(t)_0^T). \quad (6)$$

Eq. (1), (2), (4) and (6) are all alternative analytical representations.

Assuming that the elements are ordered in such a way that the first element is the outflow of the output of interest (e.g. the outflow of shoes in the production process of shoes), the “catalogue of all *feasible* and *not-wasteful* recipes” (Georgescu-Roegen, 1971, p.236) can be represented by the following functional, i.e. a relation from a set of functions to a function:

$$\begin{aligned} Q(t)_0^T &= \Psi [F_2(t)_0^T, \dots, F_M(t)_0^T, U_1(t)_0^T, \dots, U_J(t)_0^T] \equiv \\ &= \Psi [\mathbf{F}_{-1}(t)_0^T, \mathbf{U}(t)_0^T] \end{aligned} \quad (7)$$

where $Q(t) \equiv F_1(t)$.

2.3 Elementary process and production systems

If one defines the *elementary process*, as “the process by which every unit of the product – a single piece of furniture or a molecule of gasoline – is produced” (Georgescu-Roegen, 1971, p.5), i.e. a process such that $Q(t) = 0$ for each $t \in [0, T)$ and $Q(T) = 1$, she soon realizes that most of the involved funds remain idle or underutilized during a great part of the process.

Georgescu-Roegen identifies three possible, not mutually exclusive, temporal arrangements of the elementary processes:

- i) *in series* – or *in sequence* (e.g. Mir-Artigues and González-Calvet, 2007) or *in succession* (e.g. Piacentini, 1995): the elementary processes are activated one after the other with no overlap in time;
- ii) *in parallel*: n elementary processes are carried out simultaneously, i.e. started at the same time and repeated once completed;
- iii) *in line*: n elementary processes are activated with some predetermined lag $\delta (\leq T)$, so that they partially overlap.⁶

⁵Given that $U_j(t)$ is only piecewise continuous, the integral should be defined piece by piece.

⁶Some authors (e.g. Tani, 1988; Mir-Artigues and González-Calvet, 2007) also distinguish a *conjoined activation* (or *functional processes* or *job shop processes*), in which, because funds are characterized by a certain degree of versatility and *different* elementary processes have some tasks in common, the funds jump between the stages of the different processes.

Given that each elementary process releases one unit of output every T units of time, if the elementary processes are activated in series the *scale of the process*, i.e. the amount of output per unit of time, is $1/T$.

In the arrangement in series there are two possible sources of inefficiencies. First, when indivisibilities exist for funds, some funds may be underutilized. So, for instance, if your oven can accommodate 100 biscuits and you employ it for one biscuit only, you are actually using $1/100$ of its capacity. Second, if the effective use of funds within the elementary process is not continuous, some funds may remain idle for some time. In the previous example, if the production process of biscuits lasts one hour and an half and you use the oven only to cook them, let's say for half an hour, the oven is actually idle for an hour.

An arrangement in parallel can remove the first source of inefficiency. More precisely, let κ_j^* be the maximum number of elementary processes that a unit of a j -type fund can simultaneously process, in order to remove any excess capacity the number of elementary processes simultaneously activated must be equal to the *least common multiple* (\bar{n}) of the κ_j^* s for all the funds involved in the process. The *size of the process*, i.e. the number of elementary processes simultaneously activated, must be therefore equal to \bar{n} or a multiple of it, with a minimum number of j -type funds employed equal to \bar{n}/κ_j^* .⁷

While parallel production can actually deal with the first kind of inefficiency, it cannot address the second: the possible existence of periods of idleness for the funds. In order to reduce them, one needs to rearrange the processes in line. More precisely, given an elementary process that involves J different types of funds, with $d_{j1}, d_{j2}, \dots, d_{jh} \in [0, T]$ be the durations of the intervals of time in which the j -type fund is effectively used in the process, in order to completely remove the idleness of all the funds, one must activate T/δ elementary processes starting at *cycle time* intervals δ , that is the *greatest common measure* (or *divisor* in case of integers) of the d_{ji} s.⁸ To implement such line production one needs $\theta_j = \sum_i d_{ji}/\delta$ units of the fund

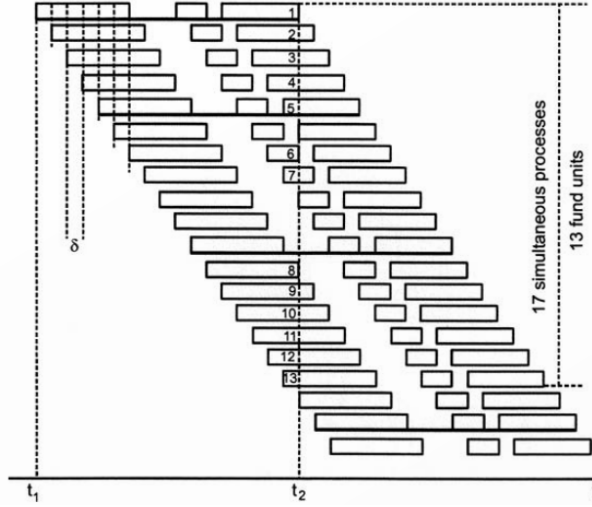
⁷Petrocchi and Zedde (1990) defines an *index of capacity utilization* as the ratio of the actual (κ_j) to the maximum (κ_j^*) number of elementary processes one unit of the j -type fund is involved in:

$$\gamma_j = \frac{\kappa_j}{\kappa_j^*}$$

In case of full employment this ratio is equal to one.

⁸The measure may also be a non integer number. A necessary condition is that the d_{ji} s and T are all *commensurable* numbers, i.e. their ratios are all rational numbers. Commensurability between two numbers a and b is in fact a necessary and sufficient condition for the existence of some real number c , and integers m and n , such that $a = m \times c$ and $b = n \times c$.

Furthermore, the requirement of commensurability is not as stringent as it may first appear. As stressed by Tani (1988), it is always possible to operate the process in line with an *ad hoc* lengthening of some stage. In particular, if we fix for each fund a maximum tolerable level of inactivity as a fraction of the total time (e_j), the problem of



Source: Mir-Artigues and González-Calvet (2007).

Figure 1: A line process with cycle time δ

j .⁹ (Figure 1 provides an example of such arrangement for the case of a simple process lasting 34 hours and involving only a fund fully used for three times, with time intervals respectively of 24, 4 and 10 hours. In this case the cycle time is equal to 2, in the *stabilized line process* there are 17 elementary processes acting simultaneously and 13 units of the fund continuously used.)

Given that δ is the maximum value compatible with the continuous use of funds, T/δ is the minimum size at which this condition holds. With this size, the output per unit of time is $1/\delta$.

the determination of the maximum lag δ can be set as follows:

$$\begin{aligned}
 & \max \delta \\
 \text{s.t. } & v_{js}\delta \geq d_{js} \quad s = 1, \dots, h_j, \quad j = 1, \dots, J \\
 & \sum_s (v_{js}\delta - d_{js}) \leq e_j \sum_s v_{js}\delta, \quad j = 1, \dots, J \\
 & v_{js} \in \mathcal{N}^+
 \end{aligned}$$

and it has always a solution.

⁹Although, to the best of my knowledge, none has noted it, in case of existence of excess capacity for the fund in some interval and assuming that the fund itself can simultaneously perform different phases of the elementary process provided that these simultaneous uses do not exceed its total capacity, the total number of units is actually smaller and equal to:

$$\theta_j = \left\lceil \sum_i \gamma_{ji} \frac{d_{ji}}{\delta} \right\rceil$$

where γ_{ji} is the index of capacity utilization of the fund j in the interval i and $\lceil \cdot \rceil$ is the ceiling function.

In order to get rid of both the sources of inefficiency – i.e. excess capacity coupled with fund indivisibilities *and* idleness – one can resort to the parallel activation of \bar{n} processes every δ units of time, or, alternatively, to an arrangement in line of the processes with a cycle time of δ/\bar{n} (Tani, 1986).

It follows that the *minimum efficient size* of the process is $T\bar{n}/\delta$, whereas \bar{n}/δ is its *minimum efficient scale*. Moreover, at every scale/size not multiple of these values, the overall efficiency decreases. This is nothing but the formal expression of the so-called *multiple principle* (or *Babbage’s (1835) factory principle*), according to which “efficiency reversals over certain ranges of increases in production levels can only be avoided if the scale increases take place in discrete jumps” (Landesmann, 1986, p.309). Although it is worth stressing that in the present framework the principle follows, not only because of the indivisibility of funds, but also because of the rigidities of the time profiles of their uses.

Finally, it is important to note that, in an arrangement in line, the different operations performed in the production process can always be assigned to the different funds to reach their *full specialization*, i.e. a division of the different phases among funds where each one performs a different operation. In fact, although this is not the only possible division compatible with the continuous use of the funds in the process,¹⁰ it is always feasible (Tani, 1986; Morroni, 1992). Fund specialization seems therefore to arise quite naturally from the arrangement in line and it is thus another factor behind the efficiency enhancing effect of the line production.¹¹

2.4 Indivisibility, decomposability and minimum efficient scale

The existence of an efficient scale of production and the multiple principle in the fund-flow model derives from the presence of both indivisibilities of inputs and rigidities of the time profile in the use of resources: “production elements tend to be combined, *at each given moment and for each given scale of production*, according to specific relations of complementarity, which allow a fairly narrow substitution range” (Morroni, 1992, p.143).

On the one side, the fund-flow approach stresses how the presence of both indivisible funds and *limitational factors* (Georgescu-Roegen, 1935, 1966) – i.e. inputs that are transformed in strict proportions during the production process – implies a low possibility of substitution among production elements. On the other side, it makes a distinction between *factor indivisibility* and

¹⁰If the durations of the periods of activity *and* idleness are all commensurable quantities, it is always possible to find a solution in which each unit performs all the operations of the elementary process (Tani, 1986).

¹¹In particular, as far as the labour fund is concerned, as first stressed by Smith (1776), specialization allows to speed up learning-by-doing and make process mechanisation easier. Moreover, it also allows to “separate tasks according to the degree of skill or strength required” by the funds (*Babbage’s principle*).

process indivisibility, emphasizing how the divisibility of production elements is a necessary but not sufficient condition for the divisibility of the process, and process indivisibility is in turn a necessary though not sufficient condition for the existence of increasing returns to scale.

As for factor indivisibility, [Morroni \(1992\)](#) distinguishes: (i) *economic indivisibility*, when one cannot exchange less than a given unit of a particular commodity; (ii) *technical indivisibility*, when a particular commodity cannot be divided, once it is exchanged, into amounts usable for production or consumption.

As for process indivisibility, a process is deemed indivisible if it is not possible to activate processes of smaller scale with the same proportions of inputs and outputs. In the fund-flow model, the definition must also consider the temporal pattern of production and restated as follows: a process $(\mathbf{F}(t), \mathbf{U}(t))$ is divisible if there is a $\eta > 1$ such that $(\frac{1}{\eta}\mathbf{F}(t), \frac{1}{\eta}\mathbf{U}(t))$ is a feasible process as well ([Mir-Artigues and González-Calvet, 2007](#), p.33).

It is worth first pointing out that “all individual processes whether in biology or technology follow exactly the same pattern: beyond a certain scale some collapse, others explode, or melt, or freeze. In a word, they cease to work at all. Below another scale, they do not even exist” ([Georgescu-Roegen, 1976](#), p.288).

In addition to this scale-dependent nature of many processes, there is another source of indivisibility related with the arrangement in line of the elementary processes. Indeed, in the case of line production systems, the organized process has only a quite limited range of efficient activation scales, even when all its elements are perfectly divisible. These rigidities come from the need to satisfy the time profile of the activation of the funds in the elementary process.

The temporal dimension in the model allows also to distinguish the character of divisibility from that of *decomposability* (or *fragmentability*) of processes – where an elementary process is decomposable if one can identify G *subprocesses* (or *stages*) of length T^g ($g = 1, \dots, G$) that can be separately activated; or, in formal terms, an elementary process $(\mathbf{F}(t), \mathbf{U}(t))$ is decomposable if there are G (> 1) subprocesses $(\mathbf{F}^g(t), \mathbf{U}^g(t))$, not all necessarily of the same length, such that:

$$\sum_g F_k^g(t) \equiv F_k(t), \quad \sum_g U_k^g(t) \equiv U_k(t), \quad \forall k \in \{1, \dots, K\}, t \in [0, T]$$

([Tani, 1976](#); [Mir-Artigues and González-Calvet, 2007](#)) – and to analyze the consequences of this feature on the minimum efficient scale/size of processes and factor requirements.

To do so, let us consider a generic decomposable elementary process whose subprocesses employ different fund elements each, and denote with δ^g the cycle time associated with the minimum efficient scale of activation

in line of the subprocess g . Since δ^g is the greatest common divisor of the intervals of fund activity in the subprocess, while δ the greatest common divisor of those intervals for the whole process, and the former intervals are a subset of the latter because of the assumption that each phase employs different types of funds, it follows that $\delta^g = \eta_g \delta$, with $\eta_g \in \mathcal{N}^+$. Hence, each subprocess cannot have a minimum efficient scale greater than the whole process.

Furthermore, given that the minimum requirements of type j funds employed in the subprocess g are $\theta_j^g = \sum_i d_{ji}/\delta^g$, we have:

$$\frac{\theta_j^g}{\theta_j} = \frac{\sum_i d_{ji}/\delta^g}{\sum_i d_{ji}/\delta} = \frac{\delta}{\delta^g} = \frac{1}{\eta_g},$$

Hence, even when the process in line associated with the complete elementary process is not divisible, the processes in line associated with the different subprocesses can be divisible and be therefore performed by different production units (Tani, 1976, 1986).¹²

2.5 Stabilized line production systems and production functions

It is interesting to study the conditions under which the fund-flow model and the standard representation of production by means of production functions tend to converge.

As pointed out by Georgescu-Roegen, this happens in the limiting case of a *continuous stabilized line production*.¹³ In this case, since production can be treated as instantaneous and funds used continuously, we have:

$$S_j(t) = \theta_j \cdot t \quad F_m(t) = f_m \cdot t \quad Q(t) = q \cdot t$$

where $f_m = F_m(\delta)/\delta$ is the flow rate of the inflow m in each cycle time and q is the flow rate of output.

The functional (7) then becomes:

$$(q t)_0^t = \Psi [(f_2 t)_0^t, \dots, (f_M t)_0^t, \theta_1, \dots, \theta_J]$$

which is “a very special functional: first, every function involved in it depends upon a single parameter and, second, the value of t is entirely arbitrary.”

¹²As stressed by Tani (1986), the decomposability of the elementary process is not a sufficient condition for the decomposability of the line process. What is necessary and sufficient is that, when in a phase of the elementary process a j -type fund is employed, the durations of the uses are all multiples of $c_j \delta$, where c_j is the number of elementary processes that can simultaneously use the same unit of the fund in the whole process arranged in line. This condition is always satisfied when each phase uses different types of funds.

¹³An arrangement in line with cycle time δ of an elementary process of duration T is stabilized after $T/\delta - 1$ cycles – or $(T - \delta)$ units of time.

(Georgescu-Roegen, 1970, p.6). Hence, production processes can be expressed with the function:

$$q = \Phi(f_2, \dots, f_M, \theta_1, \dots, \theta_J) \quad (8)$$

or, alternatively, with the function:

$$Q = \Theta(F_2, \dots, F_M, S_1, \dots, S_J) \quad (9)$$

or:

$$Q = \Lambda(F_2, \dots, F_M, \theta_1, \dots, \theta_J) \quad (10)$$

which closely resemble neoclassical production functions.

However, the “tacit presumption that the forms” (8) and (9) (or (10)) “are equivalent implies that returns to scale must be constant” (1970, p.2). Indeed, we have:

$$\begin{aligned} t \Phi(f_2, \dots, f_M, \theta_1, \dots, \theta_J) &= t \cdot q = Q = \Theta(F_2, \dots, F_M, S_1, \dots, S_J) = \\ &= \Theta(f_2 \cdot t, \dots, f_M \cdot t, \theta_1 \cdot t, \dots, \theta_J \cdot t) \end{aligned}$$

Since this relation must be true for any t , it must hold also for $t = 1$. Hence:

$$\begin{aligned} \Phi(f_2, \dots, f_M, \theta_1, \dots, \theta_J) &= \Theta(f_2, \dots, f_M, \theta_1, \dots, \theta_J) \\ &= \Lambda(f_2, \dots, f_M, \theta_1, \dots, \theta_J) \end{aligned}$$

It follows that $\Phi(\cdot) \equiv \Theta(\cdot) (\equiv \Lambda(\cdot))$ and it is a linear homogeneous function.

But, concludes Georgescu-Roegen, “of course, this does not mean that the factory process operates with constant returns to scale” and the “analytical imbroglio” behind production functions is thus brought to light: the homogeneity of the function results from “the tautology that if we double the time during which a factory works, then the quantity of every flow element and the service of every fund will also double. The issue of returns to scale pertains, instead, to what happens if the fund elements are doubled” (1970, p.7).

Therefore, a better representation of the process should make time explicit also in this case:

$$Q = \Theta(F_2, \dots, F_M, S_1, \dots, S_J; t) \quad (11)$$

or

$$Q = \Lambda(F_2, \dots, F_M, \theta_1, \dots, \theta_J; t) \quad (12)$$

3 Developments and modifications of the standard framework

By making the period of production and the patterns of factor use within it explicit, the fund-flow model allows the analysis of “real” time, i.e. historical time, in production models. The economic decisions involved in production

properly appear as far more complex than just choosing the right combination of factor inputs. Indeed, as it is made apparent by the model, these issues, also and above all, concern combining processes and single phases, and they thus touch organizational, temporal and qualitative aspects.

Nevertheless, because of its detailed description of production processes, the fund-flow model soon becomes too demanding, both in terms of analytical tractability and data requirement for practical uses, and can hardly deliver useful generalisations or “operational conclusions”.¹⁴

To overcome these problems, some scholars have modified the original formulation, reducing the analytical complexity while maintaining the basic insights. In particular, [Piacentini \(1995, 1997\)](#) has analyzed the effects on the average total cost of the different forms of arrangement of the elementary process, by assuming that the process itself can be divided into phases (Section 3.1). [Morrone \(1992\)](#) and [Piacentini \(1995\)](#) have developed useful tools to properly represent in a simple way the quantitative and temporal patterns of production (Section 3.2). [Mir-Artigues and González-Calvet \(2007\)](#) have borrowed tools from operational research and production management to study the basic features of line production (Section 3.3).

Finally, the model of production developed by [Scazzieri \(1993\)](#) and [Landesmann and Scazzieri \(1996a,b\)](#), where production process is conceived as a network of tasks, also strongly builds on Georgescu-Roegen’s model (Section 3.4).

3.1 Time-explicit cost functions and time-saving innovation

The different temporal arrangements of production processes, explicitly modeled in a fund-flow approach, clearly affect production costs.

Sticking to the original analytical framework, the cumulative cost incurred for fund services and inflows are:

$$E(t) = - \left(\sum_m p_m F_m(t) + \sum_{j=1}^J w_j S_j(t) \right)$$

where p_m is the price of the inflow m and w_j the price paid for the services of the j -type fund.

If the payments are made when the inputs enter the process, one needs to capitalize/discount them, as they refer to different moments in time. With a

¹⁴As pointed out by [Mir-Artigues and González-Calvet \(2007\)](#), the other criticism usually directed to the model, namely that the detailed representation of production processes pertains to the domain of engineering and it is therefore outside the scope of economics, is captious and cannot be accepted.

constant rate of interest (r), the total cost incurred at t is:

$$C(t) = \int_0^t e^{r(t-\tau)} dE(\tau) = - \left(\sum_m p_m \int_0^t e^{r(t-\tau)} F'_m(\tau) d\tau + \sum_{j=1}^J w_j \int_0^t e^{r(t-\tau)} U_j(\tau) d\tau \right) \quad (13)$$

This is the approach chosen by Tani (1986, Ch.9) and Zamagni (1993, Ch.8). However, this level of detail soon leads to intractability. Therefore, Piacentini (1989, 1995) makes some simplifying assumptions and studies the effects of the different possible arrangements of production processes on average costs.¹⁵

In particular, he assumes that production processes can be broken down into a sequence of *phases*, so to assume that the limiting case of a continuous stabilized line production (Section 2.5) holds in each of them. Then, he goes on studying the impact on average costs of the different forms of possible arrangements of elementary processes.

For an elementary process of length T_e with a single product, in case of *activation in series* the average cost, i.e. the ratio of total costs (C) over output (Q) in a given reference period (H) (e.g. total number of hours per annum), is:

$$c_s = \frac{C}{Q} = \frac{C}{H \frac{1}{T_e}} = \frac{\sum_j \sigma_j + \frac{H}{T_e} \sum_m p_m a_m}{H \frac{1}{T_e}} = \frac{T_e}{H} \sum_j \sigma_j + \sum_m p_m a_m \quad (14)$$

where σ_j is the (exogenous) cost of the availability of the fund j for the reference period (the year), $a_m = -F_m(T_e)$ is the technical coefficient for the inflow m and p_m its unit price.

If there are indivisible funds, the average cost corresponding to the *activation in parallel* that removes all the excess capacities is:

$$c_p = \frac{C}{\frac{H}{T_e} \bar{n}} = \frac{\sum_j \frac{\bar{n}}{\kappa_j^*} \sigma_j}{\frac{H}{T_e} \bar{n}} + \sum_m p_m a_m = \frac{T_e}{H} \sum_j \frac{1}{\kappa_j^*} \sigma_j + \sum_m p_m a_m \quad (15)$$

where κ_j^* is the maximum number of elementary processes that a unit of fund j can process at the same time and \bar{n} the least common multiple of the κ_j^* s. Because $\kappa_j^* \geq 1$ for all $j \in \{1, \dots, J\}$, it follows that $c_p \leq c_s$.¹⁶

¹⁵An analysis of time-explicit cost functions is also in Petrocchi and Zedde (1990).

¹⁶Piacentini (1995) then goes on working out for each fund a *parameter of saturation*, claiming that the average unit cost in case of non-full capacity operation (c'_p) is directly proportional to c_p , with a constant of proportionality equal to the maximum of such coefficients. The claim is in general false.

Finally, in case of *activation in line*, the average cost that corresponds to the temporal organisation of elementary processes that eliminates the idle

Indeed, Piacentini starts from the unit cost for full capacity utilization (c_p) defined as:

$$c_p = \frac{T_e}{N} \sum_j \frac{N}{n_j} \sigma_j = \frac{T_e}{N} \sum_j \mu_j \sigma_j$$

where n_j is our κ_j^* , i.e. “the number of elementary processes simultaneously executable” by the fund j ; N is our \bar{n} , i.e. the “minimum common multiple of the n_j s”; “the ratio $\mu_j = N/n_j$ will give the number of each fund factor j which should be installed in order to have full utilization” (1995, p.475); finally, for the sake of simplicity, flows are not considered and the reference period assumed of unit length ($H = 1$).

Then, he says:

to introduce underutilization of capacity, we can bring in a parameter of *saturation* defined as:

$$\Omega_j = \frac{\mu_j n_j}{\mu_j n'_j} \geq 1 \quad \text{for } j = 1, 2, \dots, k$$

where n'_j is the average number of elementary processes *effectively* carried out by the funds of type j operating in parallel. Final output will now be constrained by effective productivity of the fund with the *worst* value for saturation, $X = N/\Omega' \times 1/T_e$, where $\Omega' = \max_j (\Omega_j)$. Unit cost becomes:

$$c'_p = \frac{\sum_j \mu_j \sigma_j}{N/\Omega' 1/T_e} = \Omega' c_p$$

(Piacentini, 1995, p.477).

The flaw of the argument lies in the fact that, in general, the number of funds employed in the full-capacity utilization case (μ_j) can differ from the number of units that may be actually employed in the other cases (μ'_j). In fact, the unit cost for non full capacity utilization is:

$$c'_p = \frac{T_e}{n} \sum_j \left[\frac{n}{n_j} \right] \sigma_j = \frac{T_e}{n} \sum_j \mu'_j \sigma_j$$

where n is the actual number of elementary processes simultaneously activated.

On the contrary, if we impose this equality by assumption ($\mu_j = \mu'_j$), it follows that:

$$\Omega_j = \frac{\mu_j n_j}{\mu_j n'_j} = \frac{N}{\mu_j n'_j} = \frac{N}{\mu_j \frac{n}{\mu_j}} = \frac{N}{n}$$

and, for this special case, it is true that $c'_p = \Omega' c_p$, but $\Omega_j = \Omega_i$ for each i, j and the reference to the maximum is therefore redundant.

In general, instead, we have:

$$\frac{c'_p}{c_p} = \frac{T_e \sum_j \frac{\mu'_j}{n} \sigma_j}{T_e \sum_j \frac{\mu_j}{N} \sigma_j} = \frac{\sum_j \sigma_j \frac{1}{n'_j}}{\sum_j \sigma_j \frac{1}{n_j}} = \frac{\sum_j \frac{\sigma_j}{\sum_j \sigma_j} \frac{1}{n'_j}}{\sum_j \frac{\sigma_j}{\sum_j \sigma_j} \frac{1}{n_j}}$$

Hence, although c'_p is directly proportional to c_p , the constant of proportionality is equal to the ratio between the weighted harmonic means of n_j and n'_j . Such ratio is always less than or equal to $\Omega' = \max_j (n_j/n'_j)$, which is thus an upper bound:

$$c'_p \leq \Omega' c_p.$$

time for all the funds is:

$$c_l = \frac{C}{H^{\frac{1}{\delta}}} = \frac{\delta}{H} \sum_j \theta_j \sigma_j + \sum_m p_m a_m \quad (16)$$

where $\theta_j = \sum_i d_{ji}/\delta$ are the units of j needed to perform the T_e/δ elementary processes in the stabilized line production. By denoting with $d_j (= \sum_i d_{ji} \leq T_e)$ the time of effective utilization of the fund j in the elementary process, Eq. (16) can be rewritten as:

$$c_l = \frac{T_e}{H} \sum_j \frac{d_j}{T_e} \sigma_j + \sum_m p_m a_m \quad (17)$$

Since $d_j \leq T_e$ for all $j \in \{1, \dots, J\}$, it follows that $c_l \leq c_s$.

This last effect is different from the previous one, because it comes from a more efficient use of resources in time rather than from the traditional effect of scale. Piacentini (1995) refers to it as *temporal economies*, emphasizing that, “although activation in line doubtless implies high volumes of production, these are the result of a higher speed of “throughput” of inputs within the process rather than of “scale” meant as aggregation of productive capacity at a given moment of time.” (1995, p.476).¹⁷

¹⁷Also in the case of activation in line, Piacentini (1995) works out the relation between the average unit cost for the case of “perfect coordination” and the case of existence of miscoordinations, and also in this case his argument is not flawless.

Indeed, he first writes down the unit cost for the case of smooth operations over time:

$$c_l = \delta \sum_j \Phi_j \sigma_j$$

where $\Phi_j (= t_j/\delta)$ is our θ_j , i.e. the number of j funds needed in a stabilized line production process, and, for the sake of simplicity, flows are not considered and a unit reference period is assumed ($H = 1$).

Then, he states:

any event causing irregularities in the strict observance of the cycle time will disrupt the synchronic operation of the line process. The effective time of service by one particular fund for each item in process, t'_j , will tend to overrun the technical times (t_j) because of irregularities in parts supply flows, ‘waiting’, etc. consequent to disruption. The consequence is at this point similar to those considered for the case of unbalanced capacity among phases: effective output within the reference period will be constrained by the productivity of the “bottleneck” fund, i.e. the fund with the worst value for the saturation index:

$$\Omega'' = \max_j(\Omega'_j) = \max_j\left(\frac{t'_j}{\Phi_j \delta}\right) \geq 1$$

Average cost will become:

$$c'_l = \frac{\sum_j \Phi_j \sigma_j}{\min_j(\Phi_j/t'_j)} = \frac{\sum_j \Phi_j \sigma_j}{1/\Omega'' \delta} = \Omega'' c_l$$

These are the effects of the different possible arrangements of elementary processes on average costs. Now, making the hypothesis of a process split into its component phases, leaving aside inflows, denoting with σ_i the cost of the availability of the bundle of funds needed in the phase i , and with t_i the time needed to perform such phase, the average cost of each phase is:

$$c_i = t_i \frac{\sigma_i}{H} \quad (18)$$

But the unit cost of a process performing in succession the different phases will be $\sum_i c_i$ only in case of *balanced production*, i.e. $t_i = t_j$ for each i, j . In the more general case in which $t_i \neq t_j$ for some i, j , the cost is:

$$c = \sum_i \frac{\sigma_i}{H / \max_i(t_i)} = t_{\max} \sum_i \frac{\sigma_i}{H} \quad (19)$$

In this situation the pace of production is constrained by the productivity of the slowest phase: “for phases upstream of the “bottleneck”, accumulation of a stock of unfinished products which cannot be further processed would be wasteful; phases downstream, on the other hand, are directly constrained by the “bottleneck”. The possible volume of production will thus become

Saturation indexes are thus seen to be a simple device for parameterizing cost increases with respect to the efficiency hypothesis. (Piacentini, 1995, p.478).

However, the final claim is false and no such simple relation exists between the unit costs for the case of smooth (c_l) and non smooth operations (c'_l) in processes arranged in line. To understand why, let us note that, with a constant cycle time (δ), the average unit cost in the latter case becomes:

$$c'_l = \delta \sum_j \Phi'_j \sigma_j = \delta \sum_j \left\lceil \frac{t'_j}{\delta} \right\rceil \sigma_j \quad (> c_l)$$

For the particular case in which $\lceil t'_j / \delta \rceil = t'_j / \delta$ ($\forall j \in \{1, \dots, J\}$), it follows that:

$$\frac{c'_l}{c_l} = \frac{\delta \sum_j \Phi'_j \sigma_j}{\delta \sum_j \Phi_j \sigma_j} = \frac{\sum_j \frac{t_j}{\delta} \frac{t'_j}{t_j} \sigma_j}{\sum_j \Phi_j \sigma_j} = \sum_j \frac{\Phi_j \sigma_j}{\sum_j \Phi_j \sigma_j} \Omega''_j \leq \Omega''$$

that is: although c'_p is directly proportional to c_p , the constant of proportionality is equal to a weighted average of the saturation indexes, and not to their maximum.

Moreover, in order to compare the two cases, one might also assume the constancy of the units of funds actually employed in the two cases ($\Phi_j = \Phi'_j$). This assumption implies that the cycle time must change so as to satisfy the following relations:

$$\frac{t_j}{\delta} = \left\lceil \frac{t'_j}{\delta'} \right\rceil, \quad \forall j \in \{1, \dots, J\}$$

where $\delta' > \delta$ and the unit cost becomes:

$$c'_l = \delta' \sum_j \Phi_j \sigma_j \quad (> c_l).$$

H/t_{\max} ; output of the other phases will have to be adjusted ex post to the latter through proportional reductions in effective hours of operation” (Piacentini, 1995, p.476).

Moreover, with respect to the unit cost of each phase (Eq. (18)), two possible and *distinct* sources of cost reduction can be identified: i) a decrease in the price/quantity of the factor bundle needed to perform the phase (σ_i); ii) the reduction in the phase production time (t_i).

Accordingly, one can classify (process) innovations in two broad classes: i) *factor-saving*; and ii) *time-saving*.¹⁸ As pointed out by Piacentini (1997), learning-by-doing should be properly viewed as a source of time-saving, rather than factor-saving technical progress.

Moreover, from Eq. (19) it follows that a time-saving innovation in a single phase is actually effective only if it falls on the slowest phase and as long as it does not create a new bottleneck, i.e. a new phase that becomes the most lengthy one.

Finally, as for input flows, besides the cost of the flows embodied in each unit of output ($\sum_m p_m a_m$), one should also consider the *opportunity cost* of the *circulating capital* (or *process-fund*), i.e. the value of semi-processed goods that must already be available when a stabilized process in line is started and still remain, as work in progress, when it is stopped. While the temporal dimension of the process does not affect input flows as such – indeed, no matter the temporal arrangement of production, one always needs $\sum_m p_m a_m$ for each unit of the product, as clearly emerges from the comparison of Eq. (14), (15) and (17) –, it affects instead the volume of the process-fund and so the extent of the opportunity cost associated with it.¹⁹ Such cost should be included as a component of the average cost and clearly depends on time: “any increase in t_i owing to “waiting time” or miscoordination will proportionally increase “work-in-process” cost. The significance of organizational improvements such as “just in time” operation, where the required inputs become available exactly at the moment of their active immission in the process, is clearly evidenced” (Piacentini, 1995, p.479).

The presence of both aspects, physical inputs and time, in the relation between process-fund-related economies and the length of the production process has led Morroni (1992) to build a different classification of technical change. To better analyze the typologies and determinants of temporal economies, he has provided a detailed account of the time profile of the production process (summarized in Table 1), where a distinction is made between: i) the “breaks due to periods of time in which semi-finished products

¹⁸The distinction is in Piacentini (1997), who however identifies three different kinds of innovation: i) *capital-saving*; ii) *labor-saving*; and iii) *time-saving*. I merged the first two.

¹⁹So, for instance, with the simplifying assumption that all flows enter the process at the beginning of each phase, the process-fund will be equal to $\sum_{i,m} p_m a_{mi} t_i$, where a_{mi} is the coefficient of the inflow m in the phase i .

lie in technical inventories for maturing or settling”; ii) “all interruptions in the use of funds for organizational reasons, such as breaks due to differences in the productive capacity on individual process phases, or to internal movement times” (1992, p.73). The rationale of the distinction is that the former (*technical inventories*) are an integral part of the process, while the latter (*organizational inventories*) depend on its actual organization.

Accordingly, Morroni distinguishes three forms of technical change: a) *time-saving* (or *technical-inventory-cost-saving* and/or *goods-in-progress-cost-saving*), “if it reduces the total process time, by decreasing total net process time and/or technical inventory breaks”; b) *organizational-inventory-saving*, “if it reduces the quantity of semifinished goods in organizational inventories”; c) *inputs-saving*, “if it reduces the quantity of input flows or of services of funds which enter the elementary process directly”. With respect to *b*, he observes that this is “an intermediate form, between *a* and *c*, because it allows both a cut in the length of the production process (the duration), as well as in the quantities of inputs (semi-finished goods in organizational inventories)” (1992, p.78-79).

The previous classification properly emphasizes the difference between: the innovations, mainly of organizational nature, that reduce working capital without altering the process time; and the other innovations, mainly of technical nature, that instead shorten this time. However, the classification and the related definitions have some flaws.

First of all, Morroni terms the time-saving technical change also *technical-inventory-cost-saving* and/or *goods-in-progress-cost-saving*, arguing that “it causes a decrement in costs by decreasing technical inventories and/or total interest on monetary capital locked-up for the semi-finished goods in progress” (1992, p.83). This could be misleading: if, on the one hand, a shortening of production length always decreases the process-fund, thus reducing costs; on the other hand, the main channel through which time-saving technical change impacts on costs is by increasing the *speed of rotation* of flows, so augmenting output in the given reference period. Therefore, it seems better to refer to all the improvements that tend to reduce the length of the process only as time-saving technical change, treating the related reduction in the working capital as a by-product.

Second, Morroni defines *inputs-saving* technical change as the technical change that decreases “the quantity of input flows or of *services* of funds which enter the elementary process directly”. (1992, p.78, emphasis added). But the reference to fund services in the definition is misleading: given that these services are measured in terms of substance \times time, each and every time-saving innovation actually reduces them. Therefore, in the definition of the factor-saving technical change it seems better referring to the *quantity* of funds actually employed, the total cost of their availability for the reference period or, at least, the amount of their services *per unit of time*.

Nevertheless, Morroni’s account of the time profile of the production

Table 1: The time profile of production in [Morrioni \(1992\)](#)

Time to collect customer orders and transfer them to production lines	<i>Response Time^a</i>			Time to deliver the product
	Delivery time of raw materials	<i>Gross Duration</i>		
		<i>Duration</i>		
		Pauses for shift regulations, working conditions and seasonal variations	<i>Working Time</i>	
<i>Process Time</i>				
	Organizational inventories ^b	Technical inventories ^c	<i>Net Process Time^d</i>	

^a“This is the time lapse between the order being received and the delivery of the finished product” ([Morrioni, 1992](#), p.73). In a pure *pull* system – i.e. production started upon activation by demand downstream – it is always higher than gross duration; in a *push* system – i.e. production executed before demand and finished products stocked in warehouses – it can be significantly lower than duration.

^bBreaks due to: i) differences in the productive capacity of individual process phases; ii) internal movement times.

^cBreaks due to periods of time in which semi-finished goods lie in inventories for maturing or settling, as an integral part of the process.

^d*Net Process Time* (or *Gross Machine Time*): the time needed to produce a unit of output, excluding all interruptions apart from i) loading; ii) set-up; and iii) maintenance breaks of funds.

Financial Time: period of time between the payment for raw materials and the sale of the product.

process turns out to be particularly useful when what is actually at stake is the identification of the possible sources of time-saving innovations and their nature. Indeed, organizational changes mostly impact on *duration* and *working time*, via the reduction of organizational inventories, but they seldom affect *net process time* or *process time*, which are instead reduced mainly by technical innovations. On the contrary, the *gross duration* of the process is influenced also by factors, such as delivery time of raw materials, that can be under the control of different agents and are strongly affected by improvements in transportation and communications. Finally, *response time*, i.e. “the time lapse between the order being received and the delivery of the finished product” (Morrone, 1992, p.73), strongly depends on the actual organization of production. So, for instance, in a pure *pull* system, where production starts upon activation by demand downstream, the response time is necessarily higher than the gross duration. On the contrary, in a *push* system, i.e. a system in which production is executed before demand arises and the finished products are stocked in warehouses, the response time is not directly related to the gross duration, at least in the short run, and may be significantly smaller than the latter.

3.2 Synthetic and operational representations of technologies in a fund-flow approach

Piacentini’s device of a logical breakdown of the production process into a set of phases, for which it is reasonable the assumption of a continuous stabilized line production, turns out to be particularly useful in a synthetic representation of the production process and of technical progress, consistent with a fund-flow approach. As said in Section 2.5, in this case one can avoid using functionals and describe the phases only by means of flow rates, fund units and time durations, thus employing a model of production resembling the input-output framework.²⁰

In particular, in order to analytically represent an elementary process made up of I phases, Piacentini (1987, 1989, 1995, 1996) specifies three elements: i) a vector of production times by phase, (t_1, \dots, t_I) , with t_i the time required to complete the phase i ; ii) a flows/phases matrix \mathbf{f} , whose generic element f_{mi} is the flow rate per unit of time of the outflow (inflow) m in the phase i ; and iii) a funds/phases matrix Θ , where the element θ_{ji}

²⁰It goes without saying that, as also stressed by Piacentini (1995), despite the fact that the breakdown into phases is “instrumental to the applicability of a discrete parameterization”, the specification and identification of the different phases also strongly depends on the specific purpose of the analysis and the specificities of the case.

measures the units of fund j employed in phase i :

$$\mathbf{t} = (t_1 \quad \dots \quad t_I) \quad \mathbf{f} = \begin{pmatrix} f_{11} & \dots & f_{1I} \\ \vdots & \ddots & \vdots \\ f_{M1} & \dots & f_{MI} \end{pmatrix} \quad \Theta = \begin{pmatrix} \theta_{11} & \dots & \theta_{1I} \\ \vdots & \ddots & \vdots \\ \theta_{J1} & \dots & \theta_{JI} \end{pmatrix} \quad (20)$$

This representation “allows our recipe of the production process to be enhanced by means of information on the temporal scanning of inflows and outflows of the process, while traditional information on limitational input/output ratios is preserved” (1995, p.472). And one may also include “intermediate” products, which, for a balanced process, would appear with the same value but opposite signs in the adjacent columns of the matrix \mathbf{f} , or add a vector to represent the process-fund.²¹

With the same aim, i.e. to simplify and operationalize the fund-flow model, a different conceptual scheme is put forward by Morroni (1992, 1996, 1999). While the building block of Piacentini’s analysis is the concept of “phase”, Morroni instead relies on the notion of *stage* of a decomposable process (Section 2.4), where “an elementary process is *decomposable* if it is possible to identify *individual intermediate stages* (or subprocesses) separable in time and space, and which are linked by the fact that the product of one stage is an input to (at least) one other stage” (1992, p.68).

In his empirical analysis, Morroni (1992) summarizes the relevant information of production processes by means of two tables, detailed at the level of the single stage: i) the *quantitative-temporal matrix* \mathbf{A}_{pt} (Table 2), which shows, for a *given total process time*, “the dated input and output flows, and fund services, required by an elementary technical unit (or a chain of elementary technical units) to produce one economically indivisible unit of the product emerging from an organized elementary process” (1992, p.86-87); and ii) the *organizational scheme* (Table 3), that “summarizes or develops

²¹Piacentini (1987, 1997) also extends his analytical framework to *multiproduct operations*. In so doing, he defines two other matrices: the phases/products matrix (\mathbf{T}_P) and the switching time matrix ($\mathbf{T}_C^{(i)}$):

		Product			
Phase	1	...	N		
I	t_{11}	...	t_{1N}	t_1	
⋮	⋮	⋱	⋮	⋮	
I	t_{I1}	...	t_{IN}	t_I	
	t_{01}	...	t_{0N}		

		Product			
Product	1	2	...	N	
1	0	$t_{12}^{(i)}$...	$t_{1N}^{(i)}$	
2	$t_{21}^{(i)}$	0	...	⋮	
⋮	⋮	⋮	⋱	⋮	
N	$t_{N1}^{(i)}$	$t_{N2}^{(i)}$...	0	

The generic element of the former, t_{in} , gives the direct cycle time for application of phase i to product n . The column sum of the same matrix returns the total production time of the correspondent product, whereas the row sum gives the phase time. The generic element of the second matrix, $t_{nn'}^{(i)}$, gives the time needed to switch from the production of product n to the production of product n' in the phase i .

data provided by the production matrix”, also giving “further information on the time profile and the dimension of scale of the elementary process considered” (1992, p.93).²²

With respect to this framework, besides the criticisms raised in the previous section on the choice to focus on the total fund services rather than on their quantities or services per unit of time, it should be stressed that, apart from an explicit consideration of the process-fund in the last three rows, the quantitative-temporal matrix suggested by Morroni (1992) is ultimately no different from a traditional input-output matrix at a stage level. But, on the one side, the choice of the stage as the “atom” of the analysis does not justify in itself the validity of such representation: nothing assures that a decomposition of the process in its constituent stages actually reduces its complexity. On the other side, time as such does not enter directly in the representation: first of all, the “index” of the matrix is actually the only reference to time in the quantitative-temporal matrix – too little to justify the adjective “temporal” in the name; second, the real description of the time profile of production is in the Block B of the organizational scheme, but it has no direct connections with the previous description of production in terms of flows and fund services. The only place where time enters in the picture is the separate account of organizational inventories, which indirectly measures the extent of the unbalances among the stages in the production process. Definitely too little.

However, it is also true and must be emphasized that, although not perfectly consistent with a fund-flow perspective and quite demanding in terms of data, Morroni’s analytical framework can provide researchers with a detailed picture of production processes which can be very useful in the microeconomic analysis of process innovations.

3.3 Fund-flow model and production management

The analysis of the conditions for an arrangement in line of the production process to reduce idle times and increase fund productivities in the fund-flow model shows strong connections with some of the issues that *production* (and *inventory*) *management* usually treats in a engineer-oriented perspective (e.g. Vonderembse and White, 1991). Within the field of *operational research*, the latter commonly deals with the problem of time optimization of line processes. This has spurred some scholars to explore these analysis in trying to find some useful crossing between the two fields, and, in particular, to take advantage of the results of the latter in developing time-explicit economic

²²Morroni’s (1992; 1996) studies are one of the few “real” empirical applications based on the fund-flow approach. He has also contributed to develop a computer program – *Kronos Production Analyser* (Moriggia and Morroni, 1993; Morroni and Moriggia, 1995) – to help researchers collecting, organizing and analyzing production data in a way consistent with his theoretical framework.

Table 2: Example of a quantitative-temporal matrix for a simple process made of three stages

Production elements	Intermediate Stage (IS)			Total
	1	2	3	
Output IS1	a_{11}	$-a_{12}$		0
Waste IS1	a_{21}			a_{21}
Output IS2		a_{32}	$-a_{33}$	0
Waste IS2		a_{42}		a_{42}
Output IS3			a_{53}	a_{53}
Waste IS3			a_{63}	a_{63}
Input IS1	$-a_{71}$			$-a_{71}$
Input IS1, IS2, IS3	$-a_{81}$	$-a_{82}$	$-a_{83}$	$-(a_{81} + a_{82} + a_{83})$
\vdots				\vdots
Services of fund IS2		$-s_{12}$		$-s_{12}$
Services of fund IS1, IS3	$-s_{21}$		$-s_{23}$	$-(s_{21} + s_{23})$
\vdots				\vdots
Organization inventories	$-a_{K+1,1}$	$-a_{K+1,2}$	$-a_{K+1,3}$	$-a_{K+1,4}$
Technical inventories	$-a_{K+2,1}$	$-a_{K+2,2}$	$-a_{K+2,3}$	$-a_{K+2,4}$
Elements in progress	$-a_{K+3,1}$	$-a_{K+3,2}$	$-a_{K+3,3}$	$-a_{K+3,4}$

Table 3: The organizational scheme

A	<i>Output^a</i>	1	Internal production per day
		2	External production per day
		3	Production sold per day
B	<i>Time</i>	1	Net process time
		2	Process time
		3	Working time
		4	Duration
		5	Response time
		6	Gross duration
C	<i>Labour</i>	1	(who) Number of workers by occupation, shifts, sex, age, education
		2	(how) Tasks, jobs and skills by occupation
		3	(where) Employees/machine ratio
D	<i>Plant</i>	1	Machines by type (number, time and intensity of use)
		2	Adaptability (variations in quantity produced)
		3	Flexibility (variations in product mix, minimum batch)
E	<i>Demand</i>	1	Quantitative variations
		2	Qualitative variations
F	<i>Quality of products in progress</i>	1	Average incidence of defective intermediate products

Source: [Morroni \(1992\)](#).

^aEach row of the scheme is divided into $I + 1$ columns. Each of the first I columns provides information for the correspondent intermediate stage. The last column gives information for the process as a whole.

models of production.

Some references to production management are in Piacentini (1997), who addresses the issue of *optimal lot-sizing* (Nahmias, 2008) in analyzing the relation between production costs and switching times. However, the first seminal analysis of the differences and similarities in the treatment of production processes in the fund-flow approach and production management can be found in Mir-Artigues and González-Calvet (2007).

The authors focus on the issue of *assembly line design* in production management (e.g. Scholl, 1999), where the problem of reduction of process duration and idle times for funds is usually treated as a problem of assignment of a given set of *tasks* (I), temporally ordered on the base of a *precedence graph*, to a number of *workstations*.

In this *balancing problem* – usually quite complex to solve and that requires sophisticated mathematical algorithms for its solution – one must first consider the *feasibility problem* for each proposed assembly line and then its optimisation. The outcome is a certain assignment of the tasks to a certain number of workstations ($J \leq I$), performing the related tasks in a common interval of time, the *cycle time* (c).

This interval comprises both the *service time* (ς_j) and the *balancing delay time* ($c - \varsigma_j$), where the former is divided in two components: the *effective working interval* (or *transformation work time*) and the *non-processing time*, i.e. the time required “to move tooling, load and unload jigs, test the product, convey the output from one workstation to another and so on” (Mir-Artigues and González-Calvet, 2007, p.98).²³

This seems in many respects the natural framework to study the bottlenecks emerging in interlinked activities, with the related inducement mechanisms of innovation emphasized by Rosenberg (1976), and probably most of its fruitful applications to economic theory have yet to come, although, needless to say, these engineer-oriented models are sometimes too complex and specific, with very few useful generalisations or strong economic implications.

²³With respect to each design, one can then compute the *balance delay ratio*, i.e. an index of the relative efficiency:

$$BR = 1 - \frac{\sum_j \varsigma_j}{cJ}$$

and the *smoothness index*, i.e. an index of the degree of homogeneity in the distribution of work between the different workstations:

$$SX = \sqrt{\sum_j (c - \varsigma_j)^2}.$$

3.4 Production as a network of tasks

More grounded in economic theory is instead the model developed by Landesmann (1986), Scazzieri (1993) and Landesmann and Scazzieri (1996a,b), who confess their intellectual debt to Georgescu-Roegen’s fund-flow approach.

Their analysis of production is *process-based*, i.e. based on the general characteristics of production conceived as a process. As Georgescu-Roegen before, they start from the identification of the dynamic features of the concept of process, which they claim are: i) *sequentiality*, since the relationship between two of its stages is always unidirectional; ii) *non-stationarity*, because, “whatever analytical process description ... we choose and whatever process stage we identify, it will always be possible to find an interval long enough within that process so that the precise sequence of stages will not be repeated”; iii) *temporally boundedness*, given that the description of “a process must always include the specification of an initial and a final stage” (1996b, p.193).

Then they move to the analysis of the constituent elements of the production process, which they classify in: i) *agents* (or *funds*); ii) *tasks*, i.e. the elementary operations (or bundles of elementary operations) performed by the agents; and iii) *materials*, i.e. the flows entering the process and undergoing the process of transformation.

Accordingly, there are three possible descriptions of the production process – a specific pattern of coordination among funds; a network of interrelated tasks; a sequence of transformations undergone by materials – and these distinct and interrelated dimensions entail different and interlocked issues, namely: the determination of the temporal and spatial coordination patterns among the funds; the structuring and sequencing of tasks; and the analysis of the stocks and flows of the work-in-process materials moving from one stage to the other.

As for the first dimension, i.e. the coordination among productive agents, most of Landesmann and Scazzieri’s analysis is molded upon the fund-flow model, stressing problems arising from the rigidities in the time profile of fund utilization and their indivisibility, as well as the indivisibility of the process as such.

The link with the second dimension of the production process, i.e. the network of tasks, is in the *relative task adequacy* of the funds. According to the authors, given the set of elementary operations to be performed in the process, one can measure the *performance* of each fund with respect to each operation. Because this performance is a “multidimensional concept, ... in order to arrive at an overall performance indicator, different performance criteria, such as accuracy, speed, etc. have to be weighted. ... Given a particular weighting scheme, an ordering of fund inputs in terms of *relative task adequacy* with respect to particular tasks can be obtained” (1996b, p.197-198). Such ordering is taken into account in the *job specification*

programme, i.e. a mapping from the set of funds to the set of tasks to be performed.²⁴

These set of tasks and their arrangement constitute the second dimension of the production process and define what has to be performed in the process and how. The tasks can be either *simple*, with a one-to-one correspondence with the elementary operations, or *complex*, i.e. resulting from the strict interaction among several elementary operations. In this vein, production processes can be grouped on the base of the similarities of the tasks to be performed.

Landesmann and Scazzieri stress that the different tasks may be complementary in the sense that they must be performed sequentially, but the actual precedence relations among them can derive also from “the nature of the available fund inputs and the issue of capacity (or capability) utilization which requires a particular sequencing of tasks. Or it could lie in the nature of the material in process” (1996b, p.196).

This leads to the third element of the production process, namely the material in process, and to the related possible description of the process as a sequence of transformations of this material, which allows a decomposition of the process into *transformation stages* and a representation of production as “a system of pipeline denoting the timing and sequential arrangement of the different stages in that transformation process” (1996b, p.204).²⁵

In the present theoretical framework, it is so readily made apparent that “the sheer complexity of coordination problems, together with other features such as the durability of fund agents, the irreversibility in the direction of learning processes, and the fact that work-in-process materials have definite characteristics which can only be changed with advances in knowledge about materials and about the processes using such materials, makes any specific form of production organisation relatively difficult to introduce and explains its relative durability over time.” (1996b, p.219)

Therefore, in the analysis of the actual production processes one should take into account the organizational constraints associated with the rigidities and indivisibilities of fund and processes, as well as the limited knowledge on: the capabilities and the utilization patterns of a given capabilities structure; the existing materials and the feasible transformation processes; the “process anatomy” in terms of feasible task specification and arrangement. It follows that the case of *perfect synchronization* of all the three levels of operation

²⁴Drawing on Landesmann (1986), in Landesmann and Scazzieri (1996b) fund-inputs are defined as “bundle of capabilities”. Accordingly, each fund is defined as a vector of measurable capabilities in the n -dimensional capabilities set \mathcal{C} and one has to relate each of these vectors to the elementary operation in order to arrive to the vector of performance of the funds with respect to the task.

²⁵As noted by Landesmann and Scazzieri, the material-in-process dimension of production processes has been considered mainly by those economists that have analyzed the relationship between production and time, such as, among the others, von Böhm-Bawerk (1891), Hicks (1973) and Löwe (1976).

(i.e. agents, tasks and materials) should be regarded as a very special one.

4 The original aim of the fund-flow model and the pros and cons of the approach

Georgescu-Roegen developed the fund-flow model “as a substantial illustration of the harm caused by the blind symbolism that generally characterizes a hasty mathematization”. In particular, it was originally meant to show that the mainstream theory of production was disregarding “a basic requirement of science; namely, to have as clear an idea as possible about what corresponds in actuality to every piece of our symbolism” (1970, p.1).

According to him, the elementary features of the production process hidden by the “blind symbolism” beneath the representation of the production process by means of production functions were basically two, namely: i) production is not instantaneous, but it develops in a sequential (i.e. historical) time; ii) the production process entails two distinct elements: funds – the agents of the process, whose services should be expressed in terms of substance \times time – and flows – the elements acted upon by the agents, to be measured in terms of substance/time.

Although Georgescu-Roegen also partly analysed the implications of his approach for other related neoclassical concepts (e.g. marginal productivity and optimization), that was not his main aim. Indeed, such issues were hardly treated in his later work. Moreover, he did not mention at all the Cambridge capital controversy (Harcourt, 1972; Stiglitz, 1974; Birner, 2002; Han and Schefold, 2006) and, as far as the problem of capital aggregation was concerned, he simply observed incidentally that, although, “as a highly abstract simile, the standard form of the Neoclassical production function – as a function of K , the cardinal measure of homogenous ‘capital’, and H , the cardinal measure of homogenous ‘labor’ – is not completely useless”, “it is absurd ... to hold on to it in practical applications – as is the case with the numberless attempts at deriving it from cross-section statical data”, since “the K_i in these data are not all qualitatively identical and, hence, have no common measure” (1971, p.244).²⁶

4.1 On the “sameness” of funds

In his model, Georgescu-Roegen makes however one crucial assumption, namely, that each fund-element that leaves a production process is the *same*

²⁶Commenting this sentence, Kurz and Salvadori (2003) reply that this “common measure” actually exists and it is the price. Anyway, Georgescu-Roegen was well aware of this. Indeed, immediately after the quoted passage he remarks how “capital and labour may be rendered homogeneous but only if they are measured in money”; hence, “cost is the only element that counts in this problem”, where the problem is that of explaining the reaction of production techniques to prices (Georgescu-Roegen, 1971, p.244).

element that has entered it, or, at least, that we can treat it so by assuming that its level of efficiency is kept constant over the production cycles.

He was aware of how much “heroic” this step was and also of the analytical issues it entails, but he nevertheless concluded that “the merits of the fiction are beyond question” (1971, p.229).²⁷

What is more, he plainly considered the analytical possibility of representing the used funds – i.e. tired workers and worn-out equipment – as by-products of the production process, treating them as different commodities and so reducing fixed to circulating capital. But he decided not to follow this representation. As he argued:

... an analytical picture in which the same worker (or the same tool) is split into two elements would undoubtedly complicate matters beyond description. The reason why these complications have not upset the various other analytical models currently used in natural and social sciences is that the issue of qualitative change of qualitative change has been written off *ab initio* by various artifices. ... (Nevertheless) we should expect an economist to make room in his analytical representation of a production process for ... the wear and tear. ... But in doing so he resorts to evaluating depreciation in money terms according to one of the conventional rules set up by bookkeepers. The solution is not only arbitrary, but also logically circuitous: it presupposes that prices and the interest rate, which in fact are influenced by production, are independent of it.

An inspection of the basic models of production (in real terms) reveals however, that none includes the tired worker or the used tool among their coordinates. In addition to the formal complications already mentioned, there are other reasons which command the economist to avoid the inclusion of these elements in his analytical representations of a process. The economist is interested first and last in *commodities*. ...

Even though there is no fast and general rule for determining what is and what is not a commodity, by no stretch of imagination could we say that tired workers and used tools are commodities. They certainly are *outputs* in every process, yet the aim of the economic production is not to produce tired workers and worn-out equipment. Also, with a few exceptions – used automobiles

²⁷ Funny enough, this idea of “capital equipment *being kept as a constant fund by the very process in which it participates*” is a blatant violation of the Entropy Law, whereas much of Georgescu-Roegen’s work can be read instead as an attempt to make economic theory consistent with this law. And he was well aware of this too: “a process by which something would remain indefinitely outside the influence of the Entropy Law is factually absurd” (1971, p.229).

and used dwellings are the most conspicuous ones – no used equipment has a market in the proper sense of the word and, hence, no ‘market price’. Moreover, to include tired workers and used tools among the products of industry would invite us to attribute a cost of production to such peculiar commodities. Of course, the suggestion is nonsense. Economics cannot abandon its commodity fetishism any more than physics can renounce its fetishism of elementary particle or chemistry can renounce that of molecule. (Georgescu-Roegen, 1971, p.217-218)

It is worth making some comments on this long quotation. First of all, the above claim that no economic model included tired workers or used tools among the products is actually false. As pointed out by Kurz and Salvadori (2003), at that time there were models that allowed for used tools in a joint-product framework, namely those by von Neumann (1945) and Sraffa (1960),²⁸ and Georgescu-Roegen knew for sure the former, because he had referred to it before (see Georgescu-Roegen, 1966, p.311).

Second, apart from the issues of arbitrariness and logical circularity in the solution to the problem of depreciation, the arguments Georgescu-Roegen provides for ruling out used machineries or tired workers are rather weak. In particular, he argues that tired workers and used tools cannot be considered commodities because i) their production is not the aim of the process; ii) they have no proper market and thus no market price; iii) they have no “real” cost of production. However, all these arguments can be disproved. First, as stressed by Mir-Artigues and González-Calvet (2007), even if no production process is actually meant at the production of used equipment for sale, they are nevertheless by-products for which there is always a second-hand market or a scrap market. Moreover, even when no such market exists, there are still book values. As regards the supposed production cost we are invited to attribute to the used equipment only because they are included among the outputs, it suffices to notice that, among the output of each production process there are almost always outflows (e.g. waste or other emissions) which may have a value, positive or negative, although no attached “production cost”. And this value actually depends on the whole system of production and consumption (Kurz and Salvadori, 1995, 2003).

The arguments put forward by Georgescu-Roegen to support his “fiction” are therefore unconvincing. “They were so unconvincing that they only added to the confusion. And, moreover, they led to the entire model falling into disrepute. Nevertheless, after a critical review of the arguments given by Georgescu-Roegen, it is not difficult to obtain a clearer understanding of the weaknesses of the model: *all of them come from the limitations of the partial equilibrium approach*. This is the theoretical framework that sustains the funds and flows model” (Mir-Artigues and González-Calvet, 2007, p.40).

²⁸Sraffa (1960) credits Torrens (1815) with having first suggested this logical device.

But this fiction, with the related partial equilibrium framework that sustains it, has also some merits. which, far from being “beyond question”, should be plainly discussed.

In what follows, I first survey the main drawbacks of the fund-flow model, as pointed out by [Kurz and Salvadori \(2003\)](#) and [Lager \(2000, 2009\)](#), and mostly coming from the assumption of a perennial maintenance of the original efficiency for funds (Section 4.2). Then, I discuss the limitations of the alternative framework which these authors regard as always superior, namely, the *flow-flow approach* (Section 4.4), thus making the comparative merits of Georgescu-Roegen’s approach to production theory apparent.

4.2 Limitations of the model

The limitations of the fund-flow model connected with the crucial assumption of the “economic invariableness” for funds have been thoroughly analysed by [Kurz and Salvadori \(2003\)](#).

They first stress that, if such invariableness must be understood as keeping each and every durable means of production at the original level of efficiency, it may be both technically unfeasible and economically unviable. Moreover, such assumption excludes *ipso facto* from the analysis important issues concerning fixed capital, namely: i) the choice of the economic lifetime of a durable means of production; ii) the choice of its pattern of utilization over time.

As regards the former, the authors emphasize that it is only by assuming a decreasing or changing efficiency profile in capital goods that issues of premature truncation, i.e. the possibility of a machine becoming economically obsolete before the end of its technically feasible lifetime, can arise.

But the hypothesis of a constant efficiency profile also impinges upon the possibility to analyze the optimal patterns of utilization of durable capital goods. Indeed, given the assumption of constant efficiency for funds, in the fund-flow model the maximum degree of utilization consistent with a given endowment is always the optimal one. This in fact led Georgescu-Roegen to the following statement: “the economics of production reduces to two commandments: first, produce by the factory system and, second, let the factory operate around the clock” (1970, p.8). However, as pointed out by Kurz and Salvadori, once the hypothesis of a constant efficiency for funds is relaxed, it might well be the case that the optimal degree of utilization differs from the maximal one, since it depends on several factors, such as, the efficiency profile of durable goods and the time variability of input and output prices.

If the constant efficiency hypothesis does not hold, the fund-flow approach may fail to identify the cost-minimizing technique. This is readily shown by the authors using the von Neumann-Sraffa approach to fixed capital through the analysis of the steady-state equilibrium in a simple example of a *pure*

fixed capital system – i.e. no joint production in finished goods with durable means of production – and production processes lasting one period (Kurz and Salvadori, 1995, Ch. 7-9). In this approach, a *flow-flow* description of technology is adopted and each process k is represented as:

$$\begin{pmatrix} \mathbf{a}_k \\ l_k \end{pmatrix} \rightarrow \mathbf{b}_k \quad (21)$$

where the vectors $\mathbf{a}_k, \mathbf{b}_k \in \mathcal{R}^N$ are, respectively, inputs and outputs of the N products in the process and the scalar l_j is the labour input. The fixed capital is reduced to circulating capital by treating the old machines left at the end of each period as different goods from the ones that entered production at the beginning of the period. And the available processes are represented in a compact form as:

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{l} \end{pmatrix} \rightarrow \mathbf{B} \quad (22)$$

where $\mathbf{A} = (\mathbf{a}_1 \dots \mathbf{a}_K)$, $\mathbf{l} = (l_1 \dots l_K)$, and $\mathbf{B} = (\mathbf{b}_1 \dots \mathbf{b}_K)$.

After having showed that, in their framework, the fund-flow approach can be misleading for the problem of the choice of the cost-minimizing technique, Kurz and Salvadori stress that this problem “cannot generally be answered without taking into consideration the economic environment, in particular, whether the economy is growing and at what rate” (2003, p.501).

4.3 The fund-flow model as a special case of the flow-flow model

In Kurz and Salvadori (2003), the fund-flow approach is actually showed to be more restrictive than the von Neumann-Sraffa approach, because the former cannot properly deal with the problem of fixed capital depreciation.

The point is taken up by Lager (2000, 2009), who starts from an extension of the latter that can deal with production processes lasting more the one period. In particular, Lager (2000) moves from Eq. (21) and represents a production process as follows:

$$\left(\begin{pmatrix} \mathbf{a}_{k0} \\ l_{k0} \end{pmatrix}, \begin{pmatrix} \mathbf{a}_{k1} \\ l_{k1} \end{pmatrix}, \dots, \begin{pmatrix} \mathbf{a}_{k,T_k-1} \\ l_{k,T_k-1} \end{pmatrix} \right) \rightarrow (\mathbf{b}_{k1}, \mathbf{b}_{k2}, \dots, \mathbf{b}_{kT_k}) \quad (23)$$

where \mathbf{a}_{kt} (\mathbf{b}_{kt}) is the vector of the inflows (outflows) in the process k in period t , l_{kt} is the labour input during the same period, the process lasts T_k periods and, since production requires time, $\mathbf{a}_{kT_k} = \mathbf{b}_{k0} = \mathbf{0}$.

So, each process is described as a series of dated quantities of inflows and outflows in a discrete time environment, and the previous Eq. (21) can be considered a special case of Eq. (23). Lager calls the general case represented by this equation a *flow-input flow-output* process, whose special cases are:

- a *flow-input point-output* process:

$$\left(\left(\begin{array}{c} \mathbf{a}_{k0} \\ l_{k0} \end{array} \right), \left(\begin{array}{c} \mathbf{a}_{k1} \\ l_{k1} \end{array} \right), \dots, \left(\begin{array}{c} \mathbf{a}_{k,T_k-1} \\ l_{k,T_k-1} \end{array} \right) \right) \rightarrow \mathbf{b}_{kT_k}$$

- a *point-input point-output* process:

$$\left(\begin{array}{c} \mathbf{a}_{k0} \\ l_{k0} \end{array} \right) \rightarrow \mathbf{b}_{kT_k}$$

Then he notes that any generic flow-input flow-output process lasting T_k periods can be always broken down into $(T_k - 1)$ point-input point-output processes of unit duration by introducing additional intermediate goods connecting the time series of the processes, that is:

$$\underbrace{\left(\begin{array}{c} \mathbf{a}_{k0} \\ \mathbf{0} \\ l_{k0} \end{array} \right)}_{k_1} \rightarrow \left(\begin{array}{c} \mathbf{b}_{k1} \\ \mathbf{e}_1 \lambda_1 \end{array} \right), \quad \underbrace{\left(\begin{array}{c} \mathbf{a}_{k1} \\ \mathbf{e}_1 \lambda_1 \\ l_{k1} \end{array} \right)}_{k_2} \rightarrow \left(\begin{array}{c} \mathbf{b}_{k2} \\ \mathbf{e}_2 \lambda_2 \end{array} \right), \quad \dots, \quad \underbrace{\left(\begin{array}{c} \mathbf{a}_{k,T_k-1} \\ \mathbf{e}_{T_k-1} \lambda_{T_k-1} \\ l_{k,T_k-1} \end{array} \right)}_{k_{T_k}} \rightarrow \left(\begin{array}{c} \mathbf{b}_{kT_k} \\ \mathbf{0} \end{array} \right) \quad (24)$$

where \mathbf{e}_i is a vector of dimension $(T_k - 1)$ with the i th element equal to one and all the other elements equal to zero. And Eq. (24) is considered by Lager an equivalent vertically disintegrated point-input point-output representation of (23).

In this framework, Georgescu-Roegen's fund-flow approach is represented as the special case in which fixed capital lasts "forever". In particular, by assuming that the vectors of inputs and outputs are ordered in such a way that the first elements are, respectively, circulating capital (\mathbf{a}_k^c) and "real" outputs (\mathbf{b}_k^q), while the others are fixed capital inputs (\mathbf{a}_k^f) and used machines (\mathbf{b}_k^f), Eq. (23) can be rewritten as:

$$\left(\left(\begin{array}{c} \mathbf{a}_{k0}^c \\ \mathbf{a}_{k0}^f \\ l_{k0} \end{array} \right), \dots, \left(\begin{array}{c} \mathbf{a}_{k,T_k-1}^c \\ \mathbf{a}_{k,T_k-1}^f \\ l_{k,T_k-1} \end{array} \right) \right) \rightarrow \left(\left(\begin{array}{c} \mathbf{b}_{k1}^q \\ \mathbf{b}_{k1}^f \end{array} \right), \dots, \left(\begin{array}{c} \mathbf{b}_{kT_k}^q \\ \mathbf{b}_{kT_k}^f \end{array} \right) \right) \quad (25)$$

where the idea of perennial machines is captured by the following two conditions:

$$\sum_{\tau=1}^t (\mathbf{b}_{k\tau}^f - \mathbf{a}_{k,\tau-1}^f) \leq \mathbf{0} \quad \forall t \in \{1, 2, \dots, T-1\} \quad (26)$$

$$\sum_{\tau=1}^T (\mathbf{b}_{k\tau}^f - \mathbf{a}_{k,\tau-1}^f) = \mathbf{0} \quad (27)$$

Condition (26) states that the total amount of machines listed among the outputs during the interval $[1, t]$ cannot be greater than the amount of them entering the process within the interval $[1, t - 1]$, while condition (27) imposes that all the machines entered the process actually will leave it at the end.

Lager also considers the possible alternative representation in which funds elements are described by their services. Within his framework, this can be formalized as follows:

$$\left(\left(\begin{array}{c} \mathbf{a}_{k0}^c \\ \theta_{k0} \\ l_{k0} \end{array} \right), \dots, \left(\begin{array}{c} \mathbf{a}_{k,T_k-1}^c \\ \theta_{k,T_k-1} \\ l_{k,T_k-1} \end{array} \right) \right) \rightarrow \left(\mathbf{b}_{k1}^q, \dots, \mathbf{b}_{kT_k}^q \right) \quad (28)$$

where the vector θ_{kt} represents the units of funds (“perennial” capital goods) employed in the interval $[t, t + 1]$:

$$\theta_{kt} = \sum_{\tau=0}^t \mathbf{a}_{k\tau}^f - \sum_{\tau=1}^t \mathbf{b}_{k\tau}^f \quad (29)$$

Lager emphasizes that the previous representation clearly reveals that in the fund-flow model fixed capital is treated like Ricardian land, i.e. a natural resource with “*original* and *indestructible* powers”, and this implies that its price is determined by the present value of the rental rates paid for it, as he actually shows.

4.4 “Technical” coefficients and the comparative merits of the fund-flow approach

Besides proving that the fund-flow approach cannot properly deal with issues related to fund depreciation, both Kurz and Salvadori (2003) and Lager (2000) claim that the von Neumann-Sraffa approach to production theory is always superior to the fund-flow one. Indeed, as we saw, Lager (2000) considers the former only a specific case of the latter, to which any flow-input flow-output process can be actually reduced via the logical device of vertical disintegration; while Kurz and Salvadori (2003) state that one cannot “identify any aspect which can be tackled using the latter, but not the former” and therefore “this is enough to decide in favor of the flow-flow approach” (2003, p.499).

In what follows I will try to show that this is not the case. On the contrary, the von Neumann-Sraffa approach suffers from serious limitations too, where they mostly come from being it a time-discrete model which fully relies on “technical” coefficients to represent production processes. Hence, there are cases where a fund-flow description might be more suited, notably all those in which, given the time span of the analysis and its partial equilibrium perspective, the assumption of constant efficiency for fund is not too unrealistic.

Let us start with Lager's (2000) claim that the fund-flow model is only a specific case of his own specification of the von Neumann-Sraffa approach. As we saw, Lager's model treats what he calls flow-input flow-output processes as a discrete set of vectors of dated inputs and outputs (Eq. (23)). In fact, given the discrete nature of the model, in case of processes entailing practically continuous flows (e.g. electricity, emissions, etc..), this is a very rough approximation of reality. In this case, what is actually recorded is:

$$a_{kj,t} = I_{kj}(t+1) - I_{kj}(t) = \int_t^{t+1} I'_{kj}(\tau) d\tau \quad (30)$$

$$b_{kj,t} = O_{kj}(t+1) - O_{kj}(t) = \int_t^{t+1} O'_{kj}(\tau) d\tau \quad (31)$$

where $I_{kj}(t)$ ($O_{kj}(t)$) is the cumulative input (output) of j in process k at time t and $I'_{kj}(t)$ ($O'_{kj}(t)$) the correspondent instantaneous rate of flow.²⁹

²⁹The mental straitjacket generated by these discrete time models is well shown by the following example. In discussing the impact of the time profile of production processes on observed input-output coefficients, Lager states:

The second conceptual problem concerns the definition of the matrices of 'technical' coefficients. A coefficient a_{ij} of the matrix \mathbf{A} is defined as a technical magnitude and indicates the quantity of commodity i used up per unit of output j . Because production requires time, we also need to specify production time. This is the time-span which elapses between the utilization of inputs and the point of time at which output is produced. This time-span is $\mu_{ij} > 0$ periods, such that, to produce $q_j(\tau + \mu_{ij})$ outputs at time $\tau + \mu_{ij}$, an amount of $\nu_{ij}(\tau) = a_{ij} q_j(\tau + \mu_{ij})$ is required at time τ . Given total outputs of commodity j produced within one period, say one year, with:

$$q_{j,t} = \int_t^{t+1} q_j(\tau) d\tau$$

and given inputs of circulating capital used within that period, i.e.:

$$\nu_{ij,t} = \int_t^{t+1} \nu_{ij}(\tau) d\tau = a_{ij} \int_t^{t+1} q_j(\tau + \mu_{ij}) d\tau$$

we may calculate observed input-output coefficients:

$$a_{ij}^* = \frac{\nu_{ij,t}}{q_{j,t}} = a_{ij} \frac{\int_t^{t+1} q_j(\tau + \mu_{ij}) d\tau}{\int_t^{t+1} q_j(\tau) d\tau}$$

It follows that the observed IO coefficients will generally reflect technological conditions and will also be affected by the time profile of the output flows. (Lager, 1997, p.359)

I fully agree with Lager as far as the non technical nature of technical coefficients is concerned (a point on which I will return later), but I would like to stress that, in spite of the fact that he assumes continuous time, as demonstrated by the integrals, he nevertheless represents output flows as stocks at different points in time, instead of using cumulative output functions and instantaneous rate of flows, a far more appropriate description for the processes releasing their outputs in an almost continuous way.

One might reply that the approximation could be made less severe by reducing the interval of the “discrete jump”. And this is true, but, at the same time, it is also true that, the less the time span of the jump, the greater the number of elements needed to describe the process in Lager’s representation. In particular, it is important to note that, halving the interval entails, for each process, doubling the number of vectors of inputs and outputs and also doubling the elements in each vector aimed at capturing the “durable means of production”, given that each durable element leaving a stage of each process may in principle be treated as a different element with respect to the one that has entered it. Moreover, the less the time span of the interval in the point-input point-output process, the less untenable Georgescu-Roegen’s hypothesis of sameness for funds, and thus, the more the latter model is a suitable description of the process stage.

Strictly speaking, a description of production as a series of point-input point-output processes is not even a representation in terms of flows, because in order to represent a flow one has to consider an *interval* of time – even if infinitesimal – and not a *point* in time. This is explicitly, though quite incidentally, recognized by Lager himself, when he says that in the von Neumann-Sraffa models inputs and outputs are measured as “stocks at a point in time” (1997, p.370).

Hence, Lager’s suggestion should be retained as a very rough approximation of reality. In fact, if his model had to be applied literally, it would not be just “hardly (to) find data for a rigorous application” (Lager, 2000, p.249), but simply impossible. What one can do is instead to conceive point observations as an approximation of the inputs or outputs between two point observations (Eq. (30) and (31)). Indeed, this is usually the way these models are interpreted.

Moreover, when we move from the description of the single process to the description of the whole system and the interdependencies among the different processes, as in Kurz and Salvadori (2003), there is another important assumption to be considered: all processes must have the same (unit) time duration. When this is not the case, “processes of longer duration (have) to be broken down into single processes of unit duration introducing if necessary intermediate products as additional goods” (von Neumann, 1945, p.2). If we interpret input-output coefficients in these models simply as an ex post accounting of intersectoral transactions, this idea of temporal rescaling does not raise any issue. But if we instead assume constant returns to scale – as we need to if we want to apply linear algebra to solve the problem of the choice of technique or find the intensities of operation of the different processes (see, for instance, Kurz and Salvadori, 1995, 2003) – the temporal rescaling can generate inconsistencies in all the cases in which the processes cannot be fragmented.³⁰

³⁰Let us note in passing that this reduction of all the processes to the same duration,

This is not to mention the related but distinct assumption of divisibility (of both elements and processes) behind the hypothesis of constant returns to scale. As we saw, the fund-flow model clearly shows that the relation between inputs and outputs is hardly constant and not even continuous.

In fact, on a closer examination, it shows something more, namely, that the relation captured by an input-output coefficient a_{ij} – i.e. the quantity of commodity i “used up” per unit of output j – can be considered quite stable if i is a flow in Georgescu-Roegen’s sense, i.e. an input to be physically incorporated in the product, given that these inputs are usually *limitational factors*, but not so when i is instead a fund, which is never physically incorporated in the product. Indeed, as clearly stated by Piacentini (1995):

Input-output coefficients ... provide ratios for the *limitational complementarities* among flows within a process, but they omit the parameter – we would call it the *speed of rotation* of flows – without which quantitative information on scale, and qualitative information on efficient resource use, cannot be adequately derived (Piacentini, 1995, p.465, emphasis in original)

The “technical” coefficients of funds are not stable, because they crucially depend on the speed of rotation of flows; a piece of information utterly ignored in the input-output framework. The fund-flow model shows that this speed of rotation is affected by the actual arrangement of production processes in time, and the possibility to implement such an arrangement is in turn affected by the overall scale of the organized process.

Kurz and Salvadori claim that their analytical framework “does not do away with Georgescu-Roegen’s important distinction between the ‘agents of a process’ of production ... and its flow elements”, because there is “no presumption that by analytically reducing fixed capital to circulating, the former becomes substitutable against the elements of circulating capital as conventionally defined” (2003, p.496). But the point is not the complementarity between circulating and fixed capital, but rather the instability of the derived coefficients for the latter. And such instability is not the

so excluding *ipso facto* from the analysis issues connected with temporal coordination, makes extremely difficult to understand the benefit involved in the lagged activations of the different processes in order to increase the utilization of fund elements.

A hint of the conceptual problems involved in capturing the very same idea of line production systems within a von Neumann-Sraffa framework is provided by the following example. Kurz and Salvadori (2003) cite a passage from Georgescu-Roegen (1970):

He added that “the economics of production reduces to two commandments: first, produce by the factory system (i.e. by arrangement in parallel) and, second, let the factory operate around the clock” (Georgescu-Roegen, 1970, p.8). (Kurz and Salvadori, 2003, p.498)

Here the explanatory note in brackets (i.e. by arrangement in parallel), which has been added by the authors, is actually wrong: the peculiarity of a factory system is not an arrangement in parallel, but in line!

result of processes of “substitution” between circulating and fixed capital, as commonly meant in economics, but rather of changes in the speed of rotation.

But that is not all. There is a particular fund – or stock, given that its classification is debatable (e.g. Landesmann and Scazzieri, 1996b) – missing in the von Neumann-Sraffa representation of production processes: the process-fund. When a new process in line is activated the “pipeline” has to be fulfilled. After that, the duration of production processes is greatly reduced. This strongly affects input-output coefficients. One might say that, given the long-run perspective of the analysis, one could look only at the coefficients prevailing in the stabilized processes. Nonetheless, it is important to note that this stock (or fund) cannot be treated as any other stock, that can be reduced without altering the functioning of the system in the steady-state, but must be maintained above a certain level, although this entails a cost. A reduction of the duration of production processes, besides reducing input-output coefficients of funds in a given period, can actually reduce this stock. Such decrease is *in itself* a benefit, but a flow-flow approach fails to capture this aspect.

Finally, I would like to stress that what is really needed for a fund-flow approach to work is not an hypothesis of “perennial” maintenance of the original efficiency for funds, as stated by its critics. What we need to assume is simply that the hypothesis of “sameness” for funds holds for a *certain period of time* or until a *certain level of wear and tear*, so we can treat them as the same good within that period or below that threshold. If these goods were actually treated as different after this period or above this level nothing would change in the analytical apparatus of the model.

Clearly, there would be some degree of arbitrariness in choosing the period or the level, but this arbitrariness is simply connected with the discrete nature of the choice involved, and it is no higher and possibly lower than the arbitrariness present in all the models based on the von Neumann-Sraffa approach.

5 Concluding remarks

The fund-flow approach to production theory, put forward by Nicholas Georgescu-Roegen almost half a century ago, was originally aimed at showing the harm produced by the “blind symbolism” that characterizes the “hasty mathematization” of economics. The model makes the temporal structure of production explicit and initiates the formal analysis of the patterns of coordination among the factors of production in economics. Georgescu-Roegen identifies the different possible arrangements of processes in time – in series, in parallel, and in line – and formally studies the relation between the division of labour and production efficiency. In this respect, he realizes that

the assembly line and the factory system, which allow to strongly reduce the idleness of factors, “deserves to be placed side by side with money as the two most fateful *economic* innovations for mankind” (Georgescu-Roegen, 1970, p.8, emphasis added).

This paper was intended at critically and, as far as possible, exhaustively reviewing the contributions on the fund-flow approach.

I first summed up Georgescu-Roegen’s (1970; 1971) original formulation with the analytical refinements by Tani (1986), emphasizing the important implications of the model for production theory, namely: the discontinuities in the relation between average cost and output; the difference between factor divisibility and process divisibility; the notion of process decomposability.

Then I dealt with some suggested extensions and modifications of the original framework, which are mostly intended to “operationalize” the model. In particular, I analyzed the idea of *time-explicit cost functions* put forward by Piacentini (1995), critically reexamining some of his results about simple expressions for a “distance” of actual processes with respect to the fully efficient case. I also surveyed two suggested synthetic representations of technologies consistent with a fund-flow approach: the one put forward by Piacentini (1995, 1997), based on the breakdown of the production process into *phases*; and the analytical framework developed by Morroni (1992), where the analytical atom is the *stage*. With respect to the latter, I made a few critical remarks about the way he treats the temporal aspects of production in his framework.

The analysis of the optimal temporal arrangement of production processes to increase efficiency and reduce idle time of funds is also the subject of *production* and *inventory management*. Some scholars (e.g. Mir-Artigues and González-Calvet, 2007) have tried to engage these fields in the fund-flow approach. This looks quite promising and seems the natural framework to study the bottlenecks induced by process innovations in interlinked activities. There are nevertheless some problems entailed by the engineer-oriented nature of the models, quite often too specific and analytically complex.

I also reviewed Landesmann and Scazzieri’s (1996b) analysis of production, which heavily draws on Georgescu-Roegen’s approach, although it is less formal and more broad in scope. These authors conceptualize production processes as multilayer networks, to be studied along three distinct but connected dimensions: agents (or funds), tasks and materials. Their analysis is aimed at harmonizing several different contributions to production theory in one single consistent framework. In this respect, it seems quite useful to frame the different approaches to production theory, including the fund-flow one.

Finally, I analyzed some recent criticisms raised against the fund-flow model, namely those by Kurz and Salvadori (2003) and Lager (2000), who emphasize the “inadequate” treatment of the problem of capital utilization in the fund-flow approach, due to the “fiction” of a constant efficiency for

funds.

These authors are indeed right in this respect. But they also claim that the alternative von Neumann-Sraffa approach is always superior. In order to prove the contrary, I pointed out some of the strong limitations of the latter approach, namely: the discrete treatment of time in the model; the non technical nature of “technical” coefficients; the strong instability of these coefficients when worked out for funds; the fact that the model lacks crucial information, as the speed of rotation and the process-fund. And all these drawbacks are absent in a fund-flow representation of production processes. So, it is true that, as stressed by Kurz and Salvadori, “there are several problems concerning fixed capital which cannot be investigated in terms of a formalism in which fixed capital does not wear out”; but the subsequent statement that one cannot “identify any aspect which can be tackled using the latter (the fund-flow approach), but not the former (the flow-flow approach)” and that it is “enough to decide in favor of the flow-flow approach” is untenable (2003, p.496).

On the contrary, although not suitable to analyze the reciprocal influences between economic sectors or the optimal pattern of utilization of fixed capital, the fund-flow approach can give us invaluable insights on the organizational aspects of production processes, as processes unfolding in time and requiring coordination between their elements. Aspects such as the temporal coordination among the phases and the different patterns of activation, that are related with different scales and continuity to process operations, can be analyzed.

In particular, the approach can help enhancing our understanding of the possible sources and forms of technical change; and, in this respect, it seems particularly important the conceptual category of time-saving technical change put forward by Piacentini (1997), complementary to the traditional categories of capital-saving and labour-saving innovations. And probably some of the most useful applications of the fund-flow approach to production theory have yet to come.

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