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**Economic Growth and Distribution of Income**  
A Growth Model to Fit Ghanaian Data

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## ABSTRACT

The extent to which growth reduces poverty has been disputed for years, as has the controversy surrounding trade-offs between policies that seek growth and those that address equity. Structural models linking economic growth and the distribution of income and expenditure are relatively recent and have not been exploited. This paper exploits this literature by adapting and extending it to a multisector growth model with intermediate inputs, composite capital, and government revenue and expenditures, while accounting for income and expenditure, by quintile, of households in the modeled economy. The model is fit to Ghanaian data. We find that about 50 years are required to double income per worker, while lower- and upper-quintile levels of household income tend to converge modestly toward mean household income over time. Nevertheless, the dispersion of income remains relatively high in the long run. The sensitivity of these results to productivity shocks favoring agriculture shows that increasing labor productivity leads to growth with little change in the distribution of income relative to the base solution. Increasing land productivity and decreasing protection of the industrial sector do not alter the basic trends but do tend to cause the lower-income groups to fall further below “new” mean income and the higher-income group to exceed mean income relative to the base solution.

**Key words:** income distribution, economic growth

JEL classification: O11, O5, O15



# 1. INTRODUCTION

The extent to which growth reduces poverty has been disputed for 30 years (Deaton 2004), as has the controversy surrounding trade-offs between policies that seek growth and those that address equity (Shorrocks and van der Hoven 2004). Krueger (2008) argues that economic growth is essential for raising the living standards of the poor and paraphrases Amartya Sen's point: you cannot accomplish much by redistributing misery. Until relatively recently, however, growth models with endogenous savings provided no insight into how the distribution of expenditure and income over households evolves in transition to long-run equilibrium, and consequently, they could not provide much insight into this controversy.

This study builds on the work of Caselli and Ventura (2000),<sup>1</sup> who developed the basic analytical structure for a distributional analysis of income and expenditure using a single-sector growth model. We extend it to a three-sector growth model taken from Roe, Smith and Saracoglu (2010) and fit the model to Ghanaian data. Other links to the growth literature are Echevarria (1995, 1997, and 2000), Gollin, Parente, and Rogerson (2004), Irz and Roe (2005), and Roe and Smith (2008), among others. We extend the growth model literature by accounting for government expenditures and revenues from trade and indirect taxes, and we account for other features of a real economy such as the employment of intermediate inputs and the fact that a country's capital stock is the embodiment of outputs from virtually all sectors of the economy. The model is numerically solved forward in time. We adapt the Caselli and Ventura structure to link factor earnings in each of the three sectors to household income, which includes earnings from labor, capital, and land. Based on Ghanaian household survey data, we identify total income by quintile groups and sources of groups' income such as resource endowments and wage earnings from skilled and unskilled labor. From the forward solution of the model, the conceptual framework shows the exact links between the model's transition path to long-run equilibrium and factor earnings to each quintile group. We use this theory to show empirically the evolution of the distribution of income. Thus, our contribution is to provide a theoretical framework of economic growth and income distribution as well as an empirical examination of the linkage between economic growth and the evolution of households' income and expenditure for the case of Ghana.

The paper is organized as follows. A brief discussion of the Ghanaian economy and its performance is presented in the next section to help provide a point of departure for the empirical model. We then present the basic features of the growth model beginning with the optimizing behavior of households and how individual income can be distinguished from that of the representative household. This section provides the theoretical structure employed later to determine the empirical differences among household incomes in transition growth. The behavior of firms and the characterization of intra- and intertemporal equilibrium conclude the section. We then discuss briefly the data and our model calibration procedures.

The presentation of empirical results is divided into two major sections. The first focuses on the baseline model results for the period 2005 to 2035. We first discuss the basic economic forces of growth and factor returns. We find that only about 15 years are required to double gross domestic product (GDP), but more than 50 years are needed to double per capita GDP due to a high rate of population growth. The results show that primary agriculture (including food processing) experiences a declining share of GDP whereas the manufacturing and service sectors increase their share as capital deepening occurs over time. In the growth process, we find that although quintile level of income converges over time, considerable dispersion of income across quintiles exists even in the long run.

We then conduct three simulations, all of which favor an increase in agriculture's competitiveness for economywide resources. The simulations are designed in such a way that the magnitude of the positive shock leads to the same level of long-run utility for the representative (mean) household. In this way, the representative household is, in principle, indifferent to the magnitude or source of the shock

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<sup>1</sup> An earlier contribution in this regard is the paper by Chatterjee (1994).

chosen. Further, the magnitudes are chosen so that the basic fundamental forces of capital deepening on the various sectors of the economy do not alter the basic trends of the base solution. Thus, the forces driving the evolution of the economy in the base solution also prevail, albeit to differing degrees, in each of the simulations. The tendency to converge toward mean income also prevails in the simulations, but to varying degrees. The sensitivity of these results to productivity shocks favoring agriculture shows that increasing labor productivity leads to growth with little change in the distribution of income relative to the base solution. Increasing land productivity and decreasing protection of the industrial sector do not alter the basic trends, but they do tend to cause the lower-income groups to fall further below the “new” mean income and for the higher-income group to exceed mean income relative to the base solution.



## 2. BACKGROUND

Ghana has succeeded in sustaining an average annual growth in GDP per capita of more than 1.8 percent for the period 1985–2005. World Bank data show a rate of growth in per capita GDP of 4 percent for 2005–2006. If the latter rate is sustainable, Ghana could double per capita income in real terms in about 17 years, placing it close to middle-income-country status.

Highlighting faster growth in the agricultural sector as a driving force behind this success, Bogetić et al. (2007) discuss the vulnerability of the resource-based economy to external shocks and the challenge faced in improving infrastructure. Breisinger, Diao, and Thurlow (2009) evaluate the alternative growth options for Ghana by reviewing the countries that started at a similar per capita income position as Ghana and successfully attained middle-income status within Ghana's targeted time period of about 15 years. Their findings lead them to conclude that the dominant role of agriculture is likely to preclude the country from reaching its middle-income-status goal. They suggest that this large sector will tend to constrain growth in other sectors of the economy. The option that is mostly likely to support the targeted goal is the combination of growth across sectors.<sup>2</sup> This feature highlights the importance of agriculture in our analysis.

The question of income and expenditure distributions cannot, however, be deliberated independently of the analysis of the fundamentals of the economy. Decomposition of the economy into major sectors is essential to identify channels through which various resources are allocated with other sectors of the economy, including households. It is generally accepted empirically that the performance of the agricultural sector is a key determinant to economic growth, especially in developing nations, and hence its up- and downstream linkages are likely to have effects on the distribution of income. This link is mentioned by Bourguignon and Morrisson (1998). For African nations where the problem of persistent poverty and economywide growth often coexist, the question is whether poverty is locked in agriculture or whether agriculture locks in poverty. Christiansen, Demery, and Kuhl (2006), Kappel, Lay, and Steiner (2005), Kydd et al. (2004), and van den Berg and Ruben (2006) study various aspects of this question. In the case of Ghana, although no formal discussion of the relationship between poverty and agricultural performance is provided, Bogetić et al. (2007) indicate poverty reduction was linked to economic growth that was led by growth of the agricultural sector. However, Bussolo and Medvedev (2007) and Coulombe and Wodon (2007) suggest that the resources required for achieving the poverty reduction goal by the next decade can be substantial.

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<sup>2</sup> Diao and Pratt (2006) note the importance of foreign trade in this regard, so that lowering protection of an import-competing sector can stimulate growth of the agricultural sector through foreign trade opportunities.

### 3. THE MODEL

The conceptual framework is taken from Roe, Smith and Saracoglu (2010). The modeled environment is a small open economy that consumes and produces three distinct goods: manufactures, services, and agriculture. Households are endowed with the economy's resources, which include labor and capital that evolve with time and land that remains constant. Each of the three goods is produced by perfectly competitive firms. Firms employ primary factors and intermediate inputs. The manufacturing and agricultural goods are traded internationally, whereas the service good is traded only in the domestic economy. All three types of goods are allocated to final consumption, reinvested to increase the economy's stock of capital, and employed as intermediates of production. Any surplus or deficit of the manufacturing and agricultural goods is exported or imported at given world prices. Factors are traded among firms (domestically) so that clearing of the factor market yields a positive rental rate each period. International migration is not allowed, and domestic residents own the entire stock of capital.

#### Households

The economy consists of a large number of households, each of which has equal family size  $L(t)$  and is an infinitely lived dynasty. The population of each household grows at a constant rate  $n$ . Households, indexed by  $i = 1, 2, \dots, I$ , consume three types of goods, indexed by  $j$  equals  $m$  (manufactures),  $s$  (services), and  $a$  (agricultural). We assume a constant intertemporal elasticity of substitution utility function over a composite of these three types of consumption goods. Household  $i$  maximizes a weighted sum of future flows of utility given by

$$U_i = \int_0^{\infty} \frac{q_i(t)^{1-\theta} - 1}{1-\theta} e^{-(\rho-n)t} dt \quad (1)$$

where  $q_i$  is household  $i$ 's per worker level of composite good consumption (defined below),  $1/\theta$  is the elasticity of substitution, and  $\rho$  is the rate of time preference, where  $\theta > 0$ ,  $\rho > 0$ , and  $\rho > n$  are assumed. Parameters  $\theta$ ,  $\rho$ , and  $n$  are identical across the households in the economy. Household  $i$  receives income by providing the services of its labor, capital, and land in exchange for factor payments  $w$ ,  $r$ , and  $\pi$ , respectively, and consumes the three types of consumption goods using its income. Unspent income accumulates as an asset for future consumption. Household  $i$  owns capital in addition to assets that it can lend to and borrow from other households. Capital and loans are assumed to be perfect substitutes. Thus, the flow budget constraint for household  $i$  is given by

$$\frac{d(p_k(t)k_i(t))}{dt} = w(t)l_i + (r(t) - n)p_k(t)k_i(t) + \pi(t)H_i - \varepsilon_i(t) \quad (2)$$

where  $p_k$  and  $k_i$  are the price of capital and per worker capital, respectively, and  $\varepsilon_i$  is household  $i$ 's per worker expenditure.<sup>3</sup> We assume that labor,  $l_i$ , and land,  $H_i$ , are given exogenously and normalized such that  $(1/I)\sum_i l_i = 1$ , and  $(1/I)\sum_i H_i = 1$ . Time subscripts are omitted in the subsequent analysis whenever no ambiguity results.

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<sup>3</sup>The real rate of return for owners of capital equals  $r = \frac{r^k}{p_k} - \delta + \frac{\dot{p}_k}{p_k}$ , where  $r^k$  is the rental price for a unit of capital services. Thus, the budget constraint (2) can be written by  $p_k(t)\dot{k}_i(t) = w(t)l_i + (\frac{r^k(t)}{p_k(t)} - n - \delta)p_k(t)k_i(t) + \pi(t)H_i - \varepsilon_i(t)$ .

Recognizing the separability of the consumer's intratemporal choice problem from his or her intertemporal choice problem, the consumer's optimization can be divided into two stages. In particular, the household maximizes overall utility (1) by choosing a composite of consumption goods intertemporally while it chooses each type of consumption good intratemporally for a given value of the composite good. Therefore, preserving the richness of a typical Ramsey one-sector framework, the model is accompanied by interesting characteristics of a multisector economy in a simple fashion. Within a period, household  $i$  minimizes its expenditure subject to the utility function given by

$q_i = u(q_{mi}, q_{ai}, q_{si}) = Bq_{mi}^{\lambda_m} (q_{ai} - \gamma)^{\lambda_a} q_{si}^{\lambda_s}$ , where  $q_{ji}$  is consumption of good  $j$  in per worker terms by household  $i$ ,  $\gamma$  is the constant subsistence level of the agricultural good,  $B = \prod_j \lambda_j^{-\lambda_j}$ ,  $\sum_j \lambda_j = 1$ , and  $0 < \lambda_j < 1$ . Thus, the per worker level of total spending in household  $i$  is given by

$$\varepsilon_i = p_a \gamma + p_a^{\lambda_a} p_s^{\lambda_s} q_i \quad (3)$$

For our purposes here, per worker expenditure is separated into two components: spending on the basic agricultural good required to meet basic needs,  $p_a \gamma$ , and spending on the remaining consumption goods. We refer to the former as subsistence expenditure, which is identical for all households, and the latter as the supernumerary expenditure. Recognizing that the price of the manufacturing good is the numeraire, it is convenient to denote supernumerary expenditure  $p_a^{\lambda_a} p_s^{\lambda_s} q_i$  by  $\mu_i$ . Substituting (3) into (2) and solving the usual Hamiltonian problem with (1), we obtain the Euler equation in terms of the supernumerary expenditure as

$$\frac{\dot{\mu}_i}{\mu_i} = \frac{1}{\theta} ((r - \rho) - (1 - \theta) \lambda_s \frac{\dot{p}_s}{p_s}) \quad (4)$$

and the transversality condition for this economy is given by

$$\lim_{t \rightarrow \infty} k_i(t) e^{-\int_0^t (r(s) - n - \frac{\dot{p}_k}{p_k}) ds} = 0 \quad (5)$$

The household's level of per worker expenditure is obtained by using the differential equations (2), (4) and (5). The resulting household  $i$ 's (supernumerary) expenditure function is given by

$$\mu_i(t) = \xi(t)(k_i(t) + \omega_w(t)l_i + \omega_\pi(t)H_i - \omega_\gamma(t)) \quad (6)$$

where the following definitions apply:

$$\mu_i(t) = \varepsilon_i(t) - p_a \gamma,$$

$$\xi(t)^{-1} = \int_t^\infty p_k^{-1}(\tau) e^{-\left(\rho/\theta - n\right)(\tau - t) + \int_t^\tau \left(\frac{\dot{p}_k(v)}{p_k(v)} + \frac{1 - \theta}{\theta} \left(r(v) - \lambda_s \frac{\dot{p}_s(v)}{p_s(v)}\right) dv\right) d\tau},$$

$$\omega_w(t) = \int_t^\infty e^{-\int_t^\tau \left(r(v) - n - \frac{\dot{p}_k(v)}{p_k(v)}\right) dv} p_k^{-1}(\tau) w(\tau) d\tau,$$

$$\omega_\pi(t) = \int_t^\infty e^{-\int_t^\tau \left(r(v) - n - \frac{\dot{p}_k(v)}{p_k(v)}\right) dv} p_k^{-1}(\tau) \pi(\tau) d\tau,$$

and<sup>4</sup>

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<sup>4</sup> The expenditure function is derived in a similar way as derivation of the consumption function in the one-sector model. See chapter 2 in Barro and Sala-i-Martin 2004.

$$\omega_\gamma(t) = p_a \gamma \int_t^\infty p_k^{-1}(\tau) e^{-\int_t^\tau (r(v) - n - \frac{\dot{p}_k(v)}{p_k(v)}) dv} d\tau$$

The total wealth is denoted as  $\omega_i(\mathbf{t}) = k_i(\mathbf{t}) + \omega_w(\mathbf{t})l_i + \omega_\pi(\mathbf{t})H_i - \omega_\gamma(\mathbf{t})$  for future reference. The expenditure,  $\mu_i(t)$ , on the supernumerary goods is a fraction,  $\xi(t)$ , of total wealth  $\omega(t)$ , which consists of the net asset holdings, the present values of income from wage,  $\omega_w(t)$ , and land rent,  $\omega_\pi(t)$ , and the present value of subsistence expenditure,  $\omega_\gamma(t)$ .

The dynamics of the expenditure and income distribution are discussed in terms of the individual household's relative position to the corresponding mean at each period. The respective means are defined simply as the following values divided by the number of households:  $\varepsilon = (1/I) \sum_i \varepsilon_i$ ,  $q = (1/I) \sum_i q_i$ ,  $\mu = (1/I) \sum_i \mu_i$ , and  $k = (1/I) \sum_i k_i$ . Since the propensity to consume out of wealth is independent of individual characteristics, we obtain the economy's expenditure function by aggregating (6) over households. Thus, the expenditure function for the representative household is expressed as (6) above omitting the subscript for a household.

Similarly, averaging (2) over all households in the economy obtains the economy's flow budget constraint in per worker terms. The resulting economy's budget is given by the exact same form as (2), except for household  $i$ 's per worker assets and expenditure that are replaced by the corresponding means. The Euler condition (4) describes the growth of the average supernumerary spending in the economy after household index  $i$  is removed. Thus, we observe the average variables over all households in this economy as the behavior of the representative household in the Ramsey framework. This implies that once the behavior of the representative household is solved in the usual approach, the model allows us to characterize individual households and analyze the distributional effect of expenditure and income over time in a straightforward fashion.

### *Distinguishing Individual Expenditure from the Representative Household*

As can be seen in equation (4), household  $i$ 's supernumerary expenditure changes by the same proportion as that of other households in the economy over time. Since proportional changes in the price of service goods affect all households equally, the composite of consumption goods  $q_i$  also changes by the same proportion as other households. Therefore, household  $i$ 's relative position in the supernumerary expenditure as well as the composite of consumption goods spending remains the same over time. This result implies that when the intratemporal utility is a homothetic function accompanied by the framework of intertemporal utility function in (1), the expenditure of household  $i$  is in direct proportion to that of any other household so that there is no interhousehold dynamics in the distribution of the expenditure. This is the case for our empirical model for reasons noted below.

In the current setup of the consumer problem, however, the relative position of each household's total expenditure changes over time. The pattern of change is determined by the expenditure share of the two components in (3). The growth rate of expenditure is large when a higher share is associated with the supernumerary expenditure. The evolution of household  $i$ 's per worker expenditure relative to the representative household is shown by

$$\frac{d(\varepsilon_i / \varepsilon)}{dt} = \phi \left(1 - \frac{\varepsilon_i}{\varepsilon}\right) \quad (7)$$

where  $\phi = \frac{\dot{\varepsilon}}{\varepsilon} - \frac{\dot{\mu}}{\mu}$ .<sup>5</sup> Thus, for a given initial household expenditure relative to the mean, whether the relative expenditure of the  $i$ th household increases or decreases over time depends on the sign of  $\phi$ . The

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<sup>5</sup> For equation (7),  $d\varepsilon_i / dt = d\mu_i / dt$  for all  $i$  including the mean are used. Integrated form is given by

sign of  $\phi$  is negative when the mean expenditure increases as the economy converges to its long-run equilibrium from a level of initial capital stock that is less than its long-run equilibrium level. Otherwise,  $\phi$  is positive.

Therefore, in the case where the expenditure for the representative household is increasing,  $\phi < 0$ , households that initially spend less relative to the representative household will spend a smaller proportion relative to the representative household in the later periods. On the other hand, the households that consume more relative to the representative household will spend proportionally more in the later periods. In other words, the poor's expenditure (i.e., those whose expenditures are less than the representative household) does not grow as fast as the representative household over time. In this case, the poor's relative expenditure position (relative to the representative) deteriorates or declines in the later periods compared with the representative household. Because the growth rate of the supernumerary expenditure is the same for all households according to (4), the poor, who tend to have a higher expenditure share on the subsistence goods, experience a lower relative expenditure position over time.

The reverse argument holds when the value of  $\phi$  is positive. At the steady state, the value of  $\phi$  is zero and there is no evolution in the relative per worker expenditure level. Each household maintains its spending position relative to the representative household. This implies that there is no change in the relative expenditure position over time if preference is homothetic. When the value of parameter  $\theta$  is large, the poor's expenditure position tends to worsen at a slower rate than the case with a low value of  $\theta$ . This result is obtained because households are less willing to substitute consumption intertemporally. The greater the departure of  $\varepsilon_i$  from the mean, the larger the absolute value of the growth rate of expenditure.

In the present case, the series of household expenditure relative to the representative household is entirely determined by its initial expenditure position. Other agent-specific elements can be added to the expenditure decision (7) such as in the case of Caselli and Ventura (2000).

### *Distinguishing Individual Income from the Representative Household*

Whereas the evolution of relative expenditure is characterized in a simple fashion when the initial expenditure position is given, the dynamics of the asset distribution and the income distribution take additional heterogeneous elements into consideration. As indicated in the expenditure function (6), each household's expenditure is a function of its wealth, which consists of capital asset holdings and the present value of each source of income associated with a given endowment level. Because the growth rate of the supernumerary expenditure and the propensity to consume out of wealth are identical across all households, an individual household's total wealth grows at the same rate as all other households.

Thus, the evolution of the relative capital asset position is determined by the individual share of the capital asset in wealth as well as the dynamic behavior of total wealth and its components. In other words, a household that is endowed with a small share of capital assets improves (worsens) its relative asset position over time when the other sources of household wealth grow slower (faster) than the total wealth. Note that each household's relative (supernumerary) expenditure reflects the level of its total wealth, and the wealth share of each component corresponds to a fraction of the ratio of its relevant endowment to relative (supernumerary) expenditure.<sup>6</sup> Thus, a small (large) share of capital assets is a likely result of large (small) labor skill and landownership compared with (supernumerary) expenditure.

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$$\frac{\varepsilon_i(t)}{\varepsilon(t)} = \frac{\varepsilon_i(0)}{\varepsilon(0)} e^{-\int_0^t \phi(s) ds} + \int_0^t \phi(s) e^{-\int_s^t \phi(s) ds} d\tau$$

<sup>6</sup> That is, the total wealth equals  $\omega_i = \frac{w_i}{r} / \frac{w}{r}$ , the wealth share of wage equals  $\frac{\omega_{w_i} k_i}{\omega_i} = \frac{\omega_{w_i}}{w/r} \frac{k_i}{k}$ , and that of land rent equals  $\frac{\omega_{r_i} h_i}{\omega_i} = \frac{\omega_{r_i}}{r/h} \frac{h_i}{h}$ .

To see the linkage between the share of each component of wealth and the growth rate of capital assets, we use (2) and (6) for household  $i$  and the representative household, and derive the differential equation for the relative asset position of household  $i$  at period  $t$ :

$$\frac{d(k_i/k)}{dt} = \Omega_w(l_i - \frac{k_i}{k}) + \Omega_\pi(H_i - \frac{k_i}{k}) - \Omega_\gamma(1 - \frac{k_i}{k}) \quad (8)$$

where  $\Omega_w = (w - \xi\omega_w)/(p_k k)$ ,  $\Omega_\pi = (\pi - \xi\omega_\pi)/(p_k k)$ , and  $\Omega_\gamma = (p_a\gamma - \xi\omega_\gamma)/(p_k k)$ .<sup>7</sup> These terms are interpreted as the net savings out of wage income, rent income, and subsistence expenditure all as the capital asset share, respectively, for the representative household.<sup>8</sup> Thus, the heterogeneity among households of the initial asset position coupled with the characteristics of labor and land determines the dynamics of the relative position of the household  $i$ 's asset holdings over time. Equation (8) shows that large relative labor skill as well as landownership are likely to contribute to the improvement (deterioration) of the relative position of the capital asset when the net savings out of wage income and rent income from land are positive (negative). Whether the sign of the net savings out of each component is positive depends on the relationship among factor earnings, the interest rate, the price of capital, and the growth rate of labor force. This becomes an empirical question, which we address in a later section.<sup>9</sup>

Consider a special case in which there is no heterogeneity in labor skill and landownership with  $\theta = 1$ . In this case, the dynamics of per worker asset distribution is characterized by the difference in the average growth rates between the per worker capital assets and the supernumerary expenditure.<sup>10</sup> Suppose the interest rate is greater than the time preference rate so that the growth rate of the supernumerary expenditure is positive from (4). Then, the capital assets accumulate faster than the supernumerary expenditure; otherwise the transversality condition (5) is violated. Consequently, the relative asset position of a low asset holder improves whereas the relative position of a large asset holder worsens over time. Once the heterogeneity is extended to labor skill as well as landownership, the evolution of capital assets depends on which component overweighs other elements in (8).

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<sup>7</sup> First, differentiate  $k_i/k$  with respect to time and substitute household  $i$ 's budget (2) for  $\dot{k}_i$  and the representative household's budget for  $\dot{k}$ . Using (6) for household  $i$ 's supernumerary expenditure and likewise for the representative household and arranging the outcome, we obtain the result shown.

<sup>8</sup> Integrating (8), we obtain

$$\begin{aligned} \frac{k_i(t)}{k(t)} &= \frac{k_i(0)}{k(0)} e^{-\int_0^t (\Omega_w(s) + \Omega_\pi(s) - \Omega_\gamma(s)) ds} \\ &\quad + \int_0^t (\Omega_w(\tau)l_i + \Omega_\pi(\tau)H_i - \Omega_\gamma(\tau)) e^{\int_0^\tau (\Omega_w(s) + \Omega_\pi(s) - \Omega_\gamma(s)) ds} d\tau \end{aligned}$$

<sup>9</sup> The undetermined sign related to the net savings out of wage income is indicated in chapter 2 of Barro and Sala-i-Martin 2004.

<sup>10</sup> For this special case, we have

$$\begin{aligned} \frac{d(k_i/k)}{dt} &= \frac{1}{k}(\dot{k}_i - \dot{k} \frac{k_i}{k}) \\ &= \frac{1}{k}(w + \pi - p_a\gamma - (\rho - n)(\omega_w + \omega_\pi - \omega_\gamma))(1 - \frac{k_i}{k}) \\ &= \frac{1}{k}(w + \pi + (\rho - n)k - \varepsilon)(1 - \frac{k_i}{k}) \\ &= (\frac{\dot{k}}{k} - \frac{\dot{\mu}}{\mu})(1 - \frac{k_i}{k}) \end{aligned}$$

where equations (2) and (6) for household  $i$  and the representative household are used for the second line and equation (2) for the representative household is used for the third line. Finally, equations (2) and (4) for the representative household are used to derive equation (8). Note that all households whose capital asset holding is lower than the mean improve their asset position and the rest worsen their asset position if all households have the same amount of land and labor skill.

Define household  $i$ 's per worker level of income by  $m_i = wl_i + r^k k_i + \pi H_i$ , where  $r^k$  is the rental price for a unit of capital service.<sup>11</sup> Then, the relative position of income to the mean is given by

$$\frac{m_i}{m} = s_l l_i + s_H H_i + s_k \frac{k_i}{k} \quad (9)$$

where  $s_l = w/m$ ,  $s_H = \pi/m$ , and  $s_k = r^k k/m$  are the respective labor income share, capital income share, and land income share of the representative household. The position of the relative income is obtained by substituting the solution for the relative asset position from (8) into equation (9). This equation suggests that the level of relative capital assets may or may not be directly translated into the level of the relative income position depending on the average income shares of each income source. On the other hand, the dynamics of the relative income position is largely influenced by the evolution of the relative asset holdings when the income shares of each source are relatively constant over time.

To explain what economic variables contribute to changes in the relative position of income, we differentiate the equation for the relative income position above and express it as follows:

$$\frac{\dot{m}_i}{m_i} - \frac{\dot{m}}{m} = \frac{w}{w} \left( \frac{l_i}{m_i/m} - 1 \right) s_l + \frac{\pi}{\pi} \left( \frac{H_i}{m_i/m} - 1 \right) s_H + \left( \frac{r^k}{r^k} + \frac{k}{k} \right) \left( \frac{k_i/k}{m_i/m} - 1 \right) s_k + \frac{k_i/k}{m_i/m} \left( \frac{\dot{k}_i}{k_i} - \frac{\dot{k}}{k} \right) s_k \quad (10)$$

The difference in proportional changes of income between household  $i$  and the representative household can be decomposed into four components: (direct) contributions from labor and land as well as asset holdings and an additional effect from the proportional change in the asset holdings relative to the representative household. Each component of the direct contributions from the factors of income is expressed as the growth rate of earnings multiplied by its weight  $\left( \left( \frac{l_i}{m_i/m} - 1 \right) s_l, \left( \frac{H_i}{m_i/m} - 1 \right) s_H, \text{ and } \left( \frac{k_i/k}{m_i/m} - 1 \right) s_k \right)$ , and these weights sum to zero. Each of these weights measures the deviation of the individual household's factor income share from the representative household. For instance, an increase in wage has a positive effect on the improvement of household  $i$ 's income position when the household labor income share is larger than the average labor income share. This is equivalent to household  $i$ 's (relative) labor skill that is larger than the ratio of its income to the representative household (i.e.,  $l_i > m_i/m$ ). The contribution from wage to the difference in income growth is significant when the difference between labor skill and the relative income position is large. A similar argument holds for the relationship between the relative income position and landownership.

The weights that are attached to the growth rate of earnings sum to zero. Thus, part or all direct effects from the wage and land rent are cancelled by the direct effect of capital assets. Thus, the evolution of the relative asset position, as shown by the last term in (10), has the most important role for the determination of the relative income position.

The direct effect on the change of the relative income from labor and landownership in the first two components of equation (10) is attributed to the interrelationship of the factor intensity among sectors.<sup>12</sup> In addition, an increase in the relative asset position that depends on labor, land, and initial asset holdings contributes to the improvement of the relative income position. As a special case where there is no heterogeneity in labor and land with  $\theta = 1$ , the growth rate of relative income is determined by a

<sup>11</sup> See footnote 3.

<sup>12</sup> When the production function is given by a Cobb-Douglas form such as in the present case, the factor prices are a function of the price of service good. Thus, the components of the differential equation (10) can be written as  $\frac{\dot{w}}{w} = \epsilon_{p_s}^w \frac{\dot{p}_s}{p_s}$ ,  $\frac{\dot{\pi}}{\pi} = (\epsilon_{p_s}^{\pi} \epsilon_{p_s}^w + \epsilon_{r^k}^{\pi} \epsilon_{p_s}^r) \frac{\dot{p}_s}{p_s}$ , and  $\frac{\dot{r^k}}{r^k} = \epsilon_{p_s}^{r^k} \frac{\dot{p}_s}{p_s}$ , where  $\epsilon_b^a = \frac{\partial \ln(a)}{\partial \ln(b)}$  means the  $b$ 's elasticity of  $a$  (for example,  $\epsilon_{p_s}^w = \frac{\partial \ln(w)}{\partial \ln(p_s)}$  means the (service) price elasticity of wage.) Similarly,  $\epsilon_{p_s}^{r^k}$ ,  $\epsilon_{p_s}^{\pi}$ , and  $\epsilon_{p_s}^w$  are (service) price elasticity of interest rate, wage elasticity of land rent, and interest elasticity of land rent, respectively. These elasticity terms are constant, and each sign is determined by factor intensity in the sectors.

simple form.<sup>13</sup> In this case, the poor's relative position of income improves and the high income's relative position worsens along with that group's asset position when capital deepening occurs.

As (7) and (10) suggest, the evolution of the distribution of income can have features quite different than the expenditure distribution, due in part to the important role played by asset holdings. However, the poor quality of distribution data often forces empirical studies to overlook the difference between these distributional concepts and to use one for the other interchangeably. As the theory above suggests, these distributions convey different information that can lead to a misinterpretation of the poor's relative standing and misleading predictions as to how policy interventions are likely to affect their status. Given Ghanaian survey data and the model of the economy developed below, we empirically implement this theoretical structure and provide a complete numerical characterization of the evolution of the distribution of income and expenditures in a later section.

## Firms

Firms producing the manufactured,  $Y_m$ , and the service,  $Y_s$ , goods are assumed to employ technology that is constant-returns-to-scale Cobb-Douglas in primary factors of production, labor  $L_j$  and capital  $K_j$ , and Leontief in intermediate factors,  $Y_{ij}$ :

$$Y_j \leq \min \left\{ \mathbf{F}^j (e^{xt} L_j, K_j), \frac{Y_{aj}}{\sigma_{aj}}, \frac{Y_{mj}}{\sigma_{mj}}, \frac{Y_{sj}}{\sigma_{sj}} \right\}, \quad j = m, s \quad (11)$$

where  $\sigma_{ij}$ ,  $i = m, s, a$ , and  $j = m, s$  is the amount of  $Y_{ij}$  required to produce one unit of  $Y_j$ . In the case of agriculture, the technology is

$$Y_a \leq \min \left\{ \mathbf{F}^a (e^{xt} L_a, K_a, e^{\eta t} H), \frac{Y_{aa}}{\sigma_{aa}}, \frac{Y_{ma}}{\sigma_{ma}}, \frac{Y_{sa}}{\sigma_{sa}} \right\}, \quad (12)$$

where  $H$  is the sector's resource-specific endowment (e.g., land). The exogenous rate of technological change specific to agriculture  $\eta$  (or the effective rate of land augmentation) is assumed to equal the growth of the workforce  $n$ , plus that Harrod rate of growth  $x$  in effective labor supply to prevent the model's differential equations from becoming nonautonomous, thus simplifying the model's numerical solution.

Next, we normalize each technology by the economywide effective labor force  $A(t)L(t)$ . For the manufacturing and service sector, we derive the corresponding total cost functions per economywide effective labor subject to their respective technology (11) and (12), and simply state the result as

$$\left( C^j (\hat{w}, r^k) + \sum_{i=m,a,s} \sigma_{ij} p_i \right) \hat{y}_j, \quad j = m, s \quad (13)$$

where  $\hat{y}_j = Y_j/A(t)L(t)$ ,  $\hat{w}$  is the effective wage rate ( $\hat{w} = w e^{-xt}$ ), and  $r^k$  is the return to capital and includes depreciation  $\delta$ . The unit cost of intermediate inputs is given by  $\sum_{i=m,a,s} a_{i,j} p_i$ , where  $a_{i,j}$  are output prices. For future reference, the value-added price for each of these two sectors is defined as

<sup>13</sup> Assuming  $l_i = H_i = 1$  and  $\theta = 1$  and arranging the differential equation for the relative income, we obtain  $\frac{d(m_1/m_2)}{dt} = \left( \frac{m}{m} - \frac{j^k}{r^k} - \frac{\mu}{\mu} \right) \left( 1 - \frac{m_2}{m} \right)$ .



$$p_{vj} = p_j - \sum_{i=m,a,s} \sigma_{i,j} p_i, \quad j = m, s.$$

It is well known that  $C^j(\hat{w}, r^k)$  is nondecreasing and homogeneous of degree one in its arguments.

The agricultural sector receives  $p_a$  for a unit of product sold, and pays  $\hat{w}$  for labor,  $r^k$  for capital,  $\pi$  for the rental for land, and  $p_i$ ,  $i = m, a, s$  for intermediate factors. Competition among firms ensures that total revenues after labor, capital, and intermediate factor payments are just sufficient to cover the rent to land. Consequently, the maximization of firm profits subject to (12) when land remains specific to agriculture leads to the following restricted value-added function<sup>14</sup> per economywide unit of effective labor  $A(t)L(t)$ :

$$\Pi(p_{va}, \hat{w}, r^k) H. \quad (14)$$

Agriculture's value-added price is  $p_{va} = p_a - \sum_{i=m,a,s} a_{ia} p_i$ . The land rental rate per  $A(t)L(t)$  is  $\Pi(\cdot)$ . The function  $\Pi(\cdot)$  is homogeneous of degree one in its arguments, nondecreasing in  $p_{va}$ , and nonincreasing in  $\hat{w}$  and  $r^k$ .

### Specification of Composite Capital

We model composite capital by combining the incremental outputs of all three sectors of the economy in a least-cost manner, presuming some underlying technology. For the purposes here, we choose constant-returns-to-scale Cobb-Douglas technology,

$$F(Y_{mk}, Y_{ak}, Y_{sk}),$$

with positive marginal physical products, although other choices of functional forms such as Leontief or constant elasticity of substitution with substitution elasticities less than unity are alternatives. The  $Y_{jk}$  is the quantity of sector  $j$  output at each instant of time allocated to the production of an increment composite capital. We assume the quantities  $Y_{jk}$  are combined in a least-cost manner to minimize unit cost:

$$c^k(p_m, p_a, p_s) \equiv \min_{(Y_{jk})} \left\{ \sum_{j=1}^3 p_j Y_{jk} : 1 \leq F(Y_{mk}, Y_{ak}, Y_{sk}) \right\}. \quad (15)$$

In equilibrium,  $p_k = \tilde{p}_k(p_s) = c^k(p_m, p_a, p_s)$  is the price of composite capital. The evolution of  $p_k$  is determined by  $p_s$  since the prices  $p_m$  and  $p_a$  are exogenous. Shepherd's lemma applied to  $c^k(\cdot)$  yields the cost-minimizing amount of  $Y_{jk}$  used to produce a unit of capital. The total cost for each  $t$  per worker of producing an increment of "new" capital is

$$c^k(p_m, p_a, p_s) (\dot{k} + k(\delta + n)).$$

Notice that the rate of change in the price of composite capital equals the product of the share of the home good  $\lambda_{sk}$  in the total cost of composite capital and the rate of change in the price of the home good  $\dot{p}_s / p_s$ . That is,

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<sup>14</sup> Solving the profit-maximization problem using (12) gives  $(B_a p_{va})^{\frac{1}{\beta_3}} \hat{w}^{-\frac{\beta_1}{\beta_3}} (r^k)^{\frac{\beta_2}{\beta_3}} H$ , where  $\eta = n + x$  is used.

$$\frac{\dot{p}_k}{p_k} = \frac{d}{dt} \log(p_k) = \frac{c_{p_s}^k(p_m, p_a, p_s) p_s \dot{p}_s}{p_k p_s} = \frac{Y_{sk} p_s \dot{p}_s}{p_k p_s} = \lambda_{sk} \frac{\dot{p}_s}{p_s}. \quad (16)$$

Below, we substitute  $\lambda_{sk}(\dot{p}_s / p_s)$  for  $\dot{p}_k / p_k$  in the Euler condition (4).

In units of effective labor, the total amount of sectoral output per effective worker allocated to composite capital is given by

$$\hat{y}_{jk} = \tilde{y}^{jk}(p_s) \left( \hat{k} + \hat{k}(x+n+\delta) \right) = c_{p_j}^k(p_m, p_a, p_s) \left( \hat{k} + \hat{k}(x+n+\delta) \right), \quad j = m, a, s \quad (17)$$

where, to lower notational clutter, we define

$$\tilde{y}^{jk}(p_s) \equiv c_{p_j}^k(p_m, p_a, p_s), \quad j = m, a, s$$

since the price variables  $p_m$  and  $p_a$  are exogenous. We now discuss the model's equilibrium conditions.

## Equilibrium

A competitive equilibrium for this economy is a list of sequences of positive prices  $\{p_s, \hat{w}, r^k\}_{t \in [0, \infty)}$  of outputs and inputs, household consumption plans  $\{\hat{q}_m^*, \hat{q}_a^*, \hat{q}_s^*\}_{t \in [0, \infty)}$ , and production plans  $\{\hat{y}_m^*, \hat{y}_a^*, \hat{y}_s^*, \hat{k}_m^*, \hat{k}_a^*, \hat{k}_s^*, l_m^*, l_a^*, l_s^*\}_{t \in [0, \infty)}$  given initial resource endowments  $\{K(0), L(0), H\}$  such that the discounted present value of household utility is maximized, firms maximize profit subject to their technology at each instant of time  $t$ , and markets clear for all inputs and the outputs. In addition, the no-arbitrage condition between the values of capital and land and the transversality condition (5) are satisfied. It is convenient to characterize equilibrium into an intra- and an intertemporal component.

### Intratemporal Equilibrium

Given the endogenous sequence  $\{\hat{k}, \hat{\varepsilon}\}_{t \in [0, \infty)}$ ,<sup>15</sup> intratemporal equilibrium is given by the quintuple sequence of positive values  $\{\hat{w}, r^k, \hat{y}_m, \hat{y}_s, p_s\}_{t \in [0, \infty)}$  that satisfy the following five equations for each  $t$ : zero profit in sectors<sup>16</sup>  $m, s$ ,

$$C^j(\hat{w}, r^k) = p_{vj}, \quad j = m, s \quad (18)$$

labor market clearing,

$$\sum_{j=m,s} C_{\hat{w}}^j(\hat{w}, r^k) \hat{y}_j - \Pi_{\hat{w}}(p_{va}, \hat{w}, r^k) H = 1 \quad (19)$$

capital market clearing,

$$\sum_{j=m,s} C_{r^k}^j(\hat{w}, r^k) \hat{y}_j - \Pi_{r^k}(p_{va}, \hat{w}, r^k) H = \hat{k} \quad (20)$$

<sup>15</sup> This definition of equilibrium assumes that subsistence level  $\gamma = 0$  and, thus,  $\varepsilon(t) = \mu(t)$  in order for the model to allow nonzero values of Harrod rate of growth  $x$ .

<sup>16</sup> Zero profits in agriculture are implied by  $\Pi(p_{va}, \hat{w}, r^k)$  equaling the rental rate of sector-specific resources required for the rental market to clear.

and clearing of the market for the home good including the intermediate demands and the contribution to composite capital demand,

$$\frac{\lambda_s(\hat{\mathcal{E}})}{p_s} = \hat{y}_s(1 - \sigma_{ss}) - \sigma_{sm}\hat{y}_m - \sigma_{sa}\hat{y}_a - \tilde{y}^{sk}(p_s) \left( \dot{\hat{k}} + \hat{k}(x + n + \delta) \right) \quad (21)$$

where

$$\hat{y}_a = \Pi_{p_{va}}(p_{va}, \hat{w}, r^k)H. \quad (22)$$

Notice that given the sequence,  $\{\hat{k}, \hat{\mathcal{E}}\}_{t \in [0, \infty)}$ , the dimensionality of the intratemporal conditions permits the calculation of the remaining endogenous variables  $\{\hat{w}, r^k, \hat{y}_m, \hat{y}_s, p_s\}_{t \in [0, \infty)}$  and consequently household consumption per effective worker  $\{\hat{q}_m, \hat{q}_a, \hat{q}_s\}_{t \in [0, \infty)}$  and resource allocations of capital and labor  $\{\hat{k}_m, \hat{k}_a, \hat{k}_s, l_m, l_a, l_s\}_{t \in [0, \infty)}$ .

### *The Steady State and Equations of Motion*

The characterization (18) to (21) is easily expressed in reduced form as four equations that, together with the budget constraint and the Euler equation, permit the derivation of the steady state and two equations of motion. Use the zero profit equations (18) to express  $r^k$  and  $\hat{w}$  as a function of value-added prices  $p_{vm}$  and  $p_{vs}$ . Because the price of the home good is the only endogenous price, we express this result as

$$\hat{w} = \tilde{w}(p_s) = W(p_{vm}, p_{vs}) \quad (23)$$

and

$$r^k = \tilde{r}(p_s) = R(p_{vm}, p_{vs}) \quad (24)$$

to lower notational clutter. Substitute (23) and (24) into the factor market-clearing equations, which are linear in  $\hat{y}_m$  and  $\hat{y}_s$ , to obtain the reduced-form supply functions

$$\hat{y}_m = \tilde{y}^m(p_s, \hat{k}) \quad (25)$$

and

$$\hat{y}_s = \tilde{y}^s(p_s, \hat{k}). \quad (26)$$

Substituting (23) and (24) for  $\hat{w}$  and  $r^k$  into (22) yields the reduced-form supply function of the agricultural good, which we express as

$$\hat{y}_a = \tilde{y}^a(p_s)H \quad (27)$$

For later reference, denote the value added by as

$$\tilde{\pi}(p_s) = \Pi(p_{va}, \tilde{w}(p_s), \tilde{r}(p_s)) \quad (28)$$

If a steady state exists, the Euler equation (4) for the representative household, expressed in units of effective labor, implies the root,  $p_s^{ss}$ , satisfies

$$\frac{\tilde{r}(p_s)}{\tilde{p}^k(p_s)} - \delta - \rho - \theta x = 0$$

This root is the price of the home good in the steady state. Given  $p_s^{ss}$ ,  $r^{k,ss}$  and  $\hat{w}^{ss}$  are easily calculated.

To obtain a steady-state value of the capital stock per effective worker,  $\hat{k}^{ss}$ , expenditure  $\hat{\varepsilon}$  must be expressed as a function of  $\hat{k}$ . First, substitute the supply functions (25), (26), and (27) into the home good market-clearing condition for  $\hat{y}_m$ ,  $\hat{y}_s$ , and  $\hat{y}_a$ , respectively. Solve for expenditure  $\hat{\varepsilon}$  and express the result as

$$\hat{\varepsilon} = \tilde{\varepsilon}(p_s, \hat{k}) = \quad (29)$$

$$\frac{p_s}{\lambda_s} \left( \tilde{y}^s(p_s, \hat{k}) \left( 1 - \frac{1}{a_{ss}} \right) - \frac{1}{a_{sm}} \tilde{y}^m(p_s, \hat{k}) - \frac{1}{a_{sa}} \tilde{y}^a(p_s) H - \tilde{y}^{sk}(p_s) (\hat{k}(x+n+\delta)) \right)$$

Finally, substitute (29) and the relevant reduced forms for  $\hat{w}$  and  $r^k$  into the budget constraint (2) for the representative household, expressed in effective worker terms. The result is the differential equation

$$\dot{\hat{k}} = \tilde{K}(p_s, \hat{k}) = \frac{1}{\tilde{p}^k(p_s)} = (\tilde{w}(p_s) + \tilde{r}(p_s)\hat{k} + \tilde{\pi}(p_s)H - \tilde{\varepsilon}(p_s, \hat{k})) - \hat{k}(x+\delta+n) \quad (30)$$

Given  $p_s^{ss}$ , the root  $\hat{k}^{ss}$  for  $\dot{\hat{k}} = 0$  is the level of the capital stock per worker satisfying the steady-state solution.

To solve for the transition path, an additional differential equation expressing  $\dot{p}_s$  as a function of  $\hat{k}$  and  $p_s$  is needed. We obtain this equation by drawing upon the market-clearing condition for the home good (29) and the Euler equation (31),

$$\frac{\dot{\hat{\varepsilon}}}{\hat{\varepsilon}} = \frac{1}{\theta} \left( \frac{r^k}{p_k} - \delta - \rho - \theta x - (1-\theta)\lambda_s \frac{\dot{p}_s}{p_s} + \frac{\dot{p}_k}{p_k} \right) \quad (31)$$

Time differentiate the home good market-clearing equation (29) to obtain

$$\dot{\hat{\varepsilon}} = \tilde{\varepsilon}_{p_s}(p_s, \hat{k}) \dot{p}_s + \tilde{\varepsilon}_{\hat{k}}(p_s, \hat{k}) \dot{\hat{k}}$$

Use the Euler equation to replace  $\dot{\hat{\varepsilon}}$ , substitute (30) for  $\dot{\hat{k}}$ , and solve for  $\dot{p}_s$  to obtain

$$\dot{p}_s = \tilde{P}(p_s, \hat{k}) = \frac{\tilde{\varepsilon}(p_s, \hat{k})\tilde{r}(p_s)/\tilde{p}_k(p_s) - \delta - \rho - \theta x - \theta\tilde{\varepsilon}_{\hat{k}}(p_s, \hat{k})\tilde{K}(p_s, \hat{k})}{\theta\tilde{\varepsilon}_{p_s}(p_s, \hat{k}) - \tilde{\varepsilon}(p_s, \hat{k})(\lambda_{sk} - (1-\theta)\lambda_s)/p_s} \quad (32)$$

If a steady state exists, the numerator of this equation is zero.

The time elimination method is used to solve these two equations numerically to obtain the sequence  $\{\hat{k}^*, p_s^*\}_{t \in [0, \infty)}$ . Substituting this sequence into the reduced-form system (23) to (29) leads to values for all remaining endogenous variables.

## Comparative Statics

The comparative static properties of the model are similar to properties discussed in the simpler model by Roe and Smith (2008). We state the properties without providing proof to economize on space. The factor rental rate equations (23) and (24) are homogeneous of degree one in value-added prices  $p_{v_m}$  and  $p_{v_s}$  and exhibit Stolper-Samuelson-like properties. In the case of Ghana, the home good sector  $j=s$  is labor intensive relative to sector  $j = m$ . If capital deepening occurs so that  $\dot{r}^k / r^k \leq 0$  and  $\dot{w} / w \geq x$ , then we claim that the price of the home good tends to converge from below to its long-run value,  $\dot{p}_s / p_s \geq 0$ . Consequently, the rates of change in value-added prices are  $\dot{p}_{vm} / p_{vm}$  and  $\dot{p}_{vs} / p_{vs} \geq 0$  since the home good's input-output coefficients for  $j = m, s$  are fractions,  $0 \leq a_{sj} \leq 1$ . Therefore, in transition growth, sector  $j = m$  experiences a negative internal terms-of-trade effect compared with the home good sector. As we see below, the magnitude of these effects for the case of Ghana is relatively small. Agriculture's value-added price  $p_{va}$  also declines, but its decline relative to sector  $m$  depends upon the relative magnitude of the input-output coefficients  $a_{sm}$  and  $a_{sa}$ . These negative terms-of-trade effects on the traded-good sectors can be lowered by any changes in the real economy that translate into an increased home good scale parameter  $B_s$  in (11), an increase in the sector's rate of labor augmenting technological change, or a decrease in the input-output coefficients  $a_{sj}$ ,  $j = m, a$ . We return to this point later.

In transition growth, capital deepening occurs at a rate in excess of  $x \leq \dot{k} / k$ . The supply functions (25) and (26) thus exhibit Rybczynski-like effects in transition. Because the Ghanaian manufacturing sector is the most capital intensive, capital deepening, all else constant, gives rise to  $\dot{y}_m / y_m \geq \dot{y}_s / y_s$ . The case of agriculture is indeterminate. Whether the negative effect of  $\dot{w} / w \geq x$  dominates the positive effect of  $\dot{r}^k / r^k \leq 0$  in (28) depends both on its share of labor in total costs relative to capital in total costs, and on the rate of change in factor prices. Moreover, the effects of these changes on agricultural supply and factor demand can be nonmonotonic. In the long-run equilibrium, given the restriction that  $\eta = n + x$ , all sector supplies grow at rate  $x$  per worker.

## 4. DATA AND MODEL CALIBRATION

Fitting the model to data requires a number of steps. We aggregate input-output data from the recent social accounting matrices (SAMs) provided by Breisinger et al. (2009). These data allow us to estimate production function parameters appearing in (11) and (12), the input-output coefficients, and the parameters of the expenditure function (3). Estimates of the intertemporal elasticity of substitution,  $1/\theta$  in (1), the rate of time preference  $\rho$ , and the rate of depreciation  $\delta$  are based on the literature.

We conduct a growth-accounting exercise to determine total factor productivity, that is, Solow’s residual. From this residual, we estimate the Harrod rate of technological change, denoted by  $x$ . From this exercise we also obtain an estimate of the country’s stock of capital over the 1971-to-2005 period. The capital stock estimate for the year 2005 helps to establish the country’s “distance” from its long-run equilibrium. These parameters are difficult to estimate precisely, and they vary over time. Consequently, a number of sensitivity analyses are performed over the range of changes in the estimates of the stock and technological change parameter. Our point estimate of factor productivity is reported in Table 1. Using factor shares based on the SAMs, we find a geometric mean estimate of the Harrod rate of technological change over the period 1971 to 2005 to average about 1.1 percent. This Harrod rate corresponds to a Solow total factor productivity measure of 0.79 percent per annum. These estimates tend to be in the midrange of values reported in the literature for many other countries (see for example Martin and Mitra [2001]).

**Table 1. Behavioral and technical parameters of the model**

Inter- Temporal Elasticity	Rate of Time Preference	Harrod Rate of Labor Augmenting Tech. Change	Growth Rate of Labor Force	Rate of Capital Depreciation
$1/\theta$	$\rho$	$x$	$n$	$\delta$
0.794	0.049	0.011	0.026	0.03

The social accounting matrix used for this analysis appears in Table 2. The service sector is the most labor intensive of the three sectors, whereas the industrial sector is the most capital intensive. The data suggest that capital accounts for 7.7 percent of the agricultural sector’s value added, the smallest share in the three sectors. As the comparative static properties discussed in the previous section suggest, factor intensities affect the direction of transition of the nontraded-good price and growth rate of sectoral output and factor use. In the process of growth, as wages rise and the cost of capital falls, unit cost in agriculture will rise (albeit at diminishing rates) more rapidly than in manufacturing because labor is a larger share of total agricultural cost. The nontraded-good sector, while more labor intensive than agriculture, can partially compensate for the rise in the cost of labor by the increases in home good price. Together, these adjustments can cause agriculture’s share in GDP to fall.

Intermediate demand accounts for about 32 percent of agriculture’s gross output, and 56 percent and 42 percent, respectively, for the case of manufacturing and the home good sector. The country’s own output is the most important intermediate input, and such pattern of intermediate demand is similar as in the other countries. The relatively large share of home good in total intermediate demand suggests that the performance of the service sector can have potentially important effects on the country’s internal terms of trade in the process of growth.

The empirical model departs from the analytical model presented above by the inclusion of government. Government consumption accounts for more than 17 percent of GDP, which the data suggest is spent on the home good. Fiscal revenues less than government expenditures are handled as lump-sum transfers to households. The data show that the households saved a relatively small 4.1 percent of factor earnings. That low saving rate should receive further attention, but we do not pursue it here. Excluding

savings, households allocate about 33 percent of income to food, 45 percent to manufactured goods, and 22 percent to services (home good).

Giovannini (1985) concludes that the intertemporal elasticity of substitution should be less than one, implying  $\theta > 1$ . Consumers are more motivated to smooth consumption over time the larger is  $\theta$ . We chose  $\theta = 1.26$  in the model. The time rate of discount  $\rho$  and depreciation  $\delta$  imply an interest rate in the steady state of 6.3 percent. This rate is in the upper range of rates found in the literature.

We fit the model to the data discussed above, and solve for the model's endogenous variables using the equations characterizing equilibrium and the time elimination method to solve the differential equations (30) and (32) subject to initial conditions.

**Table 2. Social accounting matrix, Ghana in billions of 2005 cedi**

	Activities			Commodities			Primary Resources			Institutions					total
	Agriculture	Industry	Services	Agriculture	Industry	Services	Labor	Capital	Land	Households	Government	Taxes	Accumulation	Trade	
Activities															
Agriculture				49,730											49730
Industry					56,248										56248
Services						57,716									57716
Commodities															
Agriculture	4,155	5,741	910							26,313				12,611	49730
Industry	5,528	21,710	10,468							36,216			2,827		76750
Services	6,027	3,966	12,655							17,602	16,473		992		57716
Primary Factors															
Labor	24,997	14,034	24,997												64029
Capital	2,643	10,797	8,686												22125
Land	6,775														6775
Institutions															
Households							64,029	22,125	6,775						92929
Government												16,473			16473
Taxes	-395				7,891							8,978			16473
Accumulation												3,820			3820
Trade					12,611										12611
	49,730	56,248	57,716	49,730	76,750	57,716	64,029	22,125	6,775	92,929	16,473	16,473	3,820	12,611	



## 5. DISCUSSION OF THE MODEL'S BASELINE RESULTS

Results are presented in tables 3 through 6. The model predicts a rate of growth in real GDP for the year 2005 of 4.5 percent, which is close to the 4.6 percent average rate of growth the country experienced over the 2000–2005 period. The economy approaches the midpoint long-run equilibrium in 2030, where growth in real GDP per worker is about 1.2 percent per annum. Gross domestic product per worker, in real terms, increases by a factor of 1.5 over the 25-year period between 2005 and 2030, increasing from 9,483 cedi to about 14,196 cedi per worker in 2030 (both in 2005 cedi, Table 3, column 1). This indicates that the country requires 51 years to double per worker GDP, although total GDP doubles in 17 years. Table 3 shows that wages account for about 70 percent of total income in 2005, while returns to capital account for 23 percent and land the remaining 7 percent. By 2030, the capital share in total income rises to 28.4 percent, while the proportion of income accounted for by wages falls to about 67 percent, and land falls to 4.6 percent.<sup>17</sup>

The change in structure of the economy over the 30-year period is significant compared with the past 30 years (see Table 4). Industry's share of GDP rises from 26 to 40 percent, the service sector share remains relatively constant, and agriculture declines from 36.7 to 23.6 percent (Table 4, columns 1 to 3). Commensurate with these changes are the corresponding changes in the share of labor and capital employed by the three sectors (Table 4). The share of the country's labor force employed in agriculture falls, while the share employed in industry rises. The share of labor employed in the service sector remains relatively constant. Table 6 shows, unlike the industrial and service sectors, the rate of growth in agriculture's gross output increases over time, thus converging from below to its long-run rate of growth. This occurs as agriculture substitutes capital for labor that is being made more expensive due to its rising productivity in other sectors of the economy. Agriculture's direct contribution to growth in gross output thus increases over time. Its indirect contribution to growth comes about from its being a supplier of labor to the rest of the economy, a provider of indirect inputs to the other sectors, and a demander of other sectors' output including capital. Agricultural exports also permit industrial good imports.

The structural features of these changes can be explained by drawing upon the Rybczynski-like effects caused by an increase in the capital stock related to the growth in labor services, and by the Stolper-Samuelson-like effects that link the changes in the price of the service good to changes in factor payments. As capital deepening occurs, the most capital-intensive sector of the economy (industry) experiences a positive Rybczynski-like effect on growth in output (Table 6, columns 1 and 3). Industrial gross output grows by 7.6 percent in 2010, declining to about 4 percent by 2030 as the rate of capital deepening slows. As the industrial sector employs a larger share of the economy's total capital services (about 60 percent, Table 4, column 7), its productivity of labor rises, which makes profitable the employment of more labor at a higher wage rate. The service sector is the most labor intensive and experiences a positive Rybczynski-like effect on growth in output from the growth in labor services (Table 6, column 4). However, since the growth in capital stock converges from above to the growth in labor services, if all else were constant, capital deepening in the industrial sector would increase its marginal value product of labor relative to the growth in the marginal value product of labor in the service sector.

Due to the service sector's relative labor intensity, firms in this sector need to employ proportionately more labor than other sectors to meet the growth in the service sector brought about by the growth in household income shown in Table 3. Although capital deepening is causing the unit cost of capital to decline, the relatively small share of capital in the total cost of producing the service good does not provide sufficient incentive for firms to increase output. Consequently, the market price for services

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<sup>17</sup> A reviewer noted the model predicts a rate of transition of labor from agriculture and a rate of increase in the share of industry in GDP over a 30-year period that exceeds their respective rates over the 1970–2005 period. Intersectoral resource transfers may be constrained by a number of factors not captured by the model. For example, the skill of rural workers may need upgrading for employment outside the sector, and adjustment costs may slow the speed of capital deepening. Nevertheless, the model should capture the major forces of transition, but it may overestimate the speed of transition.

increases in order for firms to compete for the labor and capital necessary to increase output. The real price of the service good increases modestly (Table 5) with the growth in gross service sector output that is initially around 4.3 percent (Table 6, column 1) but declines to 3.7 percent as the long-run equilibrium is approached.

The increase in the price of the service good has multiple effects. The rise in service price has Stolper-Samuelson-like effects to contribute to the rise in the wage rate. Another effect is the direct and indirect effect on the internal terms of trade. The direct effect is the rise in price of the service good relative to the prices for industrial and agricultural goods. The indirect effect is through the service good employed as an intermediate in industry and agriculture, which causes the value-added price in those sectors to decline. The employment of the service good as an intermediate accounts for 12 and 7 percent of gross output in the agricultural and industrial sectors, respectively. The negative terms-of-trade effect on agriculture thus exceeds that of industry. While the decline in terms of trade is small (Table 5, columns 2 and 3), the rise in the price of the service good can be seen as tending to pull resources out of traded-good production. An implication is that policies to increase the efficiency of home good production can be expected to release resources for traded-good production, with agriculture the major beneficiary owing to its relative dependence on service as an intermediate input. The effect will be to raise returns to those resources that traded-good production employs intensively relative to the home good sector.

Agriculture faces a decline in its value-added price driven by the decline in the cost of capital services. As shown in Table 6, this causes the growth in agricultural output to lag behind growth in the other sectors. The decline in value-added price and rise in the wage rate have a negative effect on the growth in agricultural output that is larger in the early stages of growth (Table 6), whereas the decline in the cost of capital services has a small positive effect in the 2005–2015 period. As the economy approaches its long-run equilibrium, the net negative effect of these forces on growth in output declines, causing the growth in agriculture’s gross output to converge to its long-run rate of growth. The competition for resources from other sectors causes the share of the economy’s labor force employed in agriculture to decline from about 39 percent in 2005 to about 25 percent by 2035. Whereas agriculture’s share of total capital service employment in the economy declines (Table 4, column 8), the level of capital employed per agricultural worker increases. Consequently, land productivity increases so that farm profits, measured as returns to agriculture’s sector-specific resources, also increase. Returns to the sector’s specific resources increase by about 13 percent per farm worker at the midpoint to the long-run equilibrium.

These fundamental changes in the economy’s transition to long-run growth have profound effects on the distribution of income. The distribution of income is driven by changes in factor payments over time, households’ initial asset holdings (labor, capital, and land), and the accumulation of assets in the process of growth.

**Table 3. Evolution of GDP and factor earnings per worker and saving to GDP in thousands of 2005 cedi, model results**

Year	GDP/ Worker	Capital/ Worker	Wage Earning/ Worker	Capital Earnings/ Worker	Land Rent/ Worker	Expend- iture/ Worker	Total Saving to GDP
2005	9483	20930	6551	2248	684	6023	0.249
2010	10701	28162	7248	2835	618	6655	0.180
2015	11672	33163	7840	3230	603	7198	0.149
2020	12539	37015	8387	3542	610	7703	0.134
2025	13369	40271	8923	3816	630	8196	0.125
2030	14196	43238	9464	4075	657	8693	0.121
2035	15040	46093	10020	4331	689	9205	0.118

**Table 4. Change in economy structure and resource allocation, model results**

Year	Sector Share in GDP			Labor Share in			Capital Share in		
	Industry	Ag.	Service	Industry	Ag.	Service	Industry	Ag.	Service
2005	0.257	0.367	0.376	0.210	0.385	0.404	0.472	0.119	0.410
2010	0.341	0.294	0.360	0.284	0.315	0.402	0.558	0.085	0.357
2015	0.376	0.263	0.353	0.315	0.284	0.401	0.589	0.073	0.338
2020	0.394	0.248	0.349	0.331	0.269	0.400	0.603	0.067	0.329
2025	0.403	0.241	0.348	0.339	0.260	0.400	0.610	0.064	0.325
2030	0.407	0.236	0.347	0.344	0.256	0.400	0.614	0.063	0.323
2035	0.410	0.234	0.346	0.346	0.254	0.400	0.616	0.062	0.322

**Table 5. Evolution of prices, model results**

Year	Price of	Value-Added Price of		
	Service Good	Industry	Agriculture	Service
2005	1.0009	0.4414	0.6919	0.5843
2010	1.0141	0.4405	0.6903	0.5946
2015	1.0209	0.4400	0.6895	0.5999
2020	1.0245	0.4397	0.6891	0.6028
2025	1.0266	0.4396	0.6888	0.6043
2030	1.0277	0.4395	0.6887	0.6052
2035	1.0283	0.4395	0.6886	0.6057

**Table 6. Growth in gross output, and the percentage point contributions of prices, factors, and technological change, model results**

Industry	Growth in	Contributions to Growth			
		Gross Output	Value-Added Price	Capital Stock	Effective Labor
Year					
2005	0.1592	-0.1670	0.4322	-0.1060	
2010	0.0758	-0.0613	0.2211	-0.0839	
2015	0.0542	-0.0291	0.1609	-0.0775	
2020	0.0456	-0.0151	0.1356	-0.0748	
2025	0.0416	-0.0082	0.1233	-0.0735	
2030	0.0395	-0.0045	0.1169	-0.0728	
2035	0.0384	-0.0025	0.1134	-0.0725	
Agriculture	Growth in	Contributions to Growth			
Year		Gross Output	Value-Added Price	Wage Effect	Interest Rate Effect
2005	-0.0064	-0.0026	-0.0479	0.0071	0.0370
2010	0.0156	-0.0013	-0.0235	0.0035	0.0370
2015	0.0258	-0.0007	-0.0124	0.0018	0.0370
2020	0.0309	-0.0004	-0.0067	0.0010	0.0370
2025	0.0336	-0.0002	-0.0037	0.0006	0.0370
2030	0.0351	-0.0001	-0.0021	0.0003	0.0370
2035	0.0359	-0.0001	-0.0012	0.0002	0.0370
Service	Growth in	Contributions to Growth			
Year		Gross Output	Value-Added Price	Capital Stock	Effective Labor
2005	0.0429	0.1563	-0.2248	0.1114	
2010	0.0402	0.0740	-0.1561	0.1224	
2015	0.0388	0.0382	-0.1266	0.1271	
2020	0.0380	0.0206	-0.1121	0.1295	
2025	0.0376	0.0114	-0.1045	0.1307	
2030	0.0373	0.0063	-0.1004	0.1314	
2035	0.0372	0.0036	-0.0982	0.1318	

## 6. ANALYSIS OF DISTRIBUTIONS

### Numerical Procedures

We turn to the numerical implementation of the theoretical framework discussed in Section 3 to analyze the relationship between the dynamics of the economy and the behavior of individual households' expenditure as well as income. We first examine the factors' contribution and the evolution of household expenditure and income in the model's baseline. This analysis is followed by a set of simulations in which different types of shocks are given to the economy to assess the corresponding changes in the distributional behavior of expenditure and income.

An advantage of this methodology is the capability of analyzing the dynamics of the distribution in expenditure and income within the representative agent model. As provided in conditions (4) and (8), accompanied by their integrated forms, the dynamics of distributions for individual households are characterized in a straightforward fashion once the behavior of the representative household is known. A main difference between empirical distributional studies and the present study is to characterize household expenditure in the same conceptual context as its income. A key element is the behavior of accumulative assets (capital in this case) by individual households. Because of accumulation, returns to capital assets, a driving force to capital deepening, decline along transition growth. As condition (6) implies, the evolution of wages and land rents influences agents' choice regarding asset accumulation.

Although asset data are difficult to obtain in general, it is possible to conjecture the asset accumulation behavior from expenditure or income when the household's initial endowment is known. Whereas the income distribution is derived from the expenditure distribution data combined with the household endowment for the case of Ghana, the procedure is simply reversed when only income distribution data are available.

The primary source of data for the distribution analysis is the Ghana Living Standards Survey 5, 2005/06. For each quintile group, per capita expenditure,<sup>18</sup> labor skill, and landownership are shown in Table 7 as an index when the mean expenditure, mean labor skill, and mean landownership is normalized to one.<sup>19</sup> Labor skill above the mean that is one (e.g., 1.007) is associated with higher relative expenditure (1.061). Income of landownership is relatively high in the middle-expenditure groups and low at both ends of the expenditure groupings. This indicates that many farmers with productive land form the middle quintile groups, while the poor as well as the urban residents tend to obtain the majority of their income as wage earners with little income from land. This dispersion affects the dynamics of the wealth distribution. We see in the asset distribution dynamics below that what matters to the evolution of the capital assets of the average household in the  $i$ -th quintile category are the wealth shares of these endowments (see equation (6) ).

**Table 7. Relative expenditure, labor, and landownership by quintile group (the mean for each variable is one)<sup>20</sup>**

	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
Expenditure	0.20141	0.42880	0.67805	1.06062	2.62996
Labor skill	0.36217	0.53258	0.64199	1.00723	2.45499
Landownership	0.79544	0.96167	1.21347	1.04221	0.98718

Source: Data provided by IFPRI.

<sup>18</sup> The per capita term is used because no per worker values are available. In other words, we assume per worker value is proportional to the corresponding per capita term.

<sup>19</sup> The labor cash income data for the first and second jobs in agriculture and nonagriculture are used as the indicator for the labor skill index. The total value of crop production is used as the indicator of landownership.

<sup>20</sup> Overall mean is 3,877,356 cedis per person, per year in 2005. Labor skill and landownership are an index when the mean is set to one.

We assume a constant level of labor skill as well as landownership over time for each household in the economy.<sup>21</sup> For each quintile group, we focus on the behavior of the hypothetical household within each group that spends the group average amount and owns the group average level of labor skill and landownership. We omit the phrase “the group average household” for each quintile group in order to avoid confusion with the average over all households. However, it is important to keep in mind that there is an evolution of the intragroup distribution in expenditure as well as income for each group. In this study, however, we focus only on group averages.

The first step is to solve equation (7) numerically. From that solution we obtain the expenditure distribution over time for the initial relative expenditure of group  $i$  to the mean given by data. Our assumption that preferences are homothetic causes each group’s expenditure to grow proportional to the mean expenditure over time.<sup>22</sup> Nonhomothetic preferences, as shown in the conceptual framework, are required to obtain group deviations from mean expenditure. Given the fact that the relevant differential equations are autonomous or time independent, we numerically compute the relative asset position to the mean as well as income position for each quintile group through the budget constraint.

### Dynamics of Capital Asset and Income Distributions

We now use the results obtained from the transitional growth for the economy as a whole (discussed in Section 5) to construct the income distribution.<sup>23</sup> As defined by (9), relative income is a weighted average of each income source (labor, capital, and land) held by the group  $i$  average household. Thus, the dynamics of the relative income position of the group  $i$  household is largely influenced by the evolution of the relative position of asset holdings since the income share ( $s_l, s_H, s_k$ ) of each source is relatively constant over time. In the present case, the income from land rent is dominated by the other sources of income since the mean income share of land rent in total income is approximately 5 percent, compared with 73 percent and 22 percent for the wage and capital income shares, respectively (Table 3, columns 3 to 5). Thus, although the capital asset holding has the most influence on determining relative income growth, it has limited influence for determining the level of total relative income due to the large share of wage earnings.

The relative asset position ( $k_i / k$ ) is determined by the individual share of capital assets in wealth as well as the dynamic behavior of total wealth and its components. As can be seen from the expenditure equation (6), each household’s expenditure is a function of its wealth, which consists of capital asset holdings and the present value of each source of income associated with a given endowment level. Since the average present value of each income source is determined by the economywide variables, which are independent of individual heterogeneity, the behavior of the capital asset holdings for each household is easily calculated once the evolution of its expenditure is known for the entire period. Because the propensity to consume out of wealth ( $\xi(t)$ ) in equation (6) is identical across all households, each household’s relative expenditure reflects the level of its total wealth, and the wealth share of each component corresponds to a fraction of the ratio of its relevant endowment to relative expenditure.<sup>24</sup> In particular, a large ratio of labor and landownership to expenditure suggests a large wealth share attributable to wage and land rent that are given as economywide variables.

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<sup>21</sup> The conceptual model implies a no-arbitrage condition between capital and land that allows the price of land to adjust over time so that households are indifferent between holding an additional unit of land or an additional unit of capital. This precludes the need for urban households to replace some of their capital assets with land or for households owning land in the initial period to sell land in later periods to acquire capital. The model does not allow for households to forego consumption to accumulate human capital, which makes the constant-level-of-skill assumption more problematic.

<sup>22</sup> The World Income Inequality Database reports the distribution based on Ghanaian consumption for 1987, 1989, 1992, and 1998. No monotonic behavior of consumption relative to the mean is observed for any class during the reported period.

<sup>23</sup> Income for the current study is pretransfer income in spite of the need to model lump-sum transfers from households to government so that fiscal revenues balance expenditures.

<sup>24</sup> Recall that  $\mu_i = \varepsilon_i$  for the present case. See footnote 6.

Table 7 shows that the wealth level increases as the relative expenditure position increases from the lower to the upper quintile groups. Total wealth of the lowest quintile is only 20.1 percent of the mean and 7.6 percent of the highest quintile. Labor and landownership as the ratio to expenditure tend to be larger for the lower quintiles. In particular, as can be seen from the first two columns of Table 7, the relative expenditure position for the first and second quintile groups is lower than the group's own labor skill as well as landownership. This implies that the share of wealth accounted for by wage and land rent for these two lower quintile groups is large and, correspondingly, the share accounted for by their capital asset holdings is small. The reverse prevails for upper quintile groups. The wealth share of their capital assets is large while components of wealth accounted for by wage and land rents are relatively small.

The individual household's total wealth grows at the same rate as that of all other households due to the assumption of homothetic preferences. A small share of capital assets in a household's total wealth leads to a larger growth rate of capital accumulation when the total wealth grows faster than the components of the wealth other than capital assets. In other words, the household with a small share of capital assets foregoes relatively more expenditure to increase its capital asset holdings. In the present case, the evolution of total wealth and its components is similar to that of total income and the corresponding per worker earnings.

## Baseline Results

Table 8 shows the relative asset and relative income position of each quintile group for the selected years up to 2035. The main results shown in the table are the increase over time in income relative to the mean household in the first two columns representing quintile groups 1 and 2, and a decline in the columns for quintile groups 4 and 5. This indicates that transition growth, which causes the income of the mean household to increase, causes a "convergence" toward mean income of the two lowest quintiles and of the two highest quintiles. Only the third quintile experiences "divergence" from the mean income, as shown by the sequence 0.85 to 0.83.

**Table 8. Capital asset and income relative to the mean, baseline results for selected years**

Capital Asset Holdings					
Year	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
2005	0.04342	0.35428	1.55203	1.53779	1.51212
2010	0.20445	0.46572	1.46176	1.44529	1.42248
2015	0.26425	0.50708	1.42784	1.41074	1.38981
2020	0.29196	0.52624	1.41200	1.39467	1.37486
2025	0.30609	0.53601	1.40389	1.38646	1.36729
2030	0.31364	0.54123	1.39954	1.38207	1.36326
2035	0.31779	0.54410	1.39715	1.37965	1.36106

Income <sup>a/</sup>					
Year	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
2005	0.33668	0.53272	0.85417	1.10794	2.16766
2010	0.35734	0.54590	0.84397	1.09822	2.15375
2015	0.36611	0.55146	0.83890	1.09374	2.14896
2020	0.37039	0.55415	0.83621	1.09146	2.14697
2025	0.37262	0.55555	0.83474	1.09024	2.14603
2030	0.37382	0.55631	0.83392	1.08957	2.14556
2035	0.37449	0.55672	0.83347	1.08920	2.14531

<sup>a/</sup>These values are proportional to GDP/worker reported in Table 3. ( e.g., the income of the household of the 1st quintile for year 2035 is 0.37449\*15040.)

We observe from Table 7 that the lowest quintile spends only 20.1 percent of the mean expenditure even though it is endowed with 36.2 percent of the mean labor skill and 79.5 percent of the mean landownership. This low percentage of mean expenditure accompanied by the large endowment of labor and land relative to the mean household suggests that the group has a relatively low level of asset holdings. Because the share of the noncapital components in their total wealth is large, these households choose to improve their relative asset position by accumulating assets at a rate faster than the mean capital accumulation. This increase in assets causes their relative income position to improve over time. The income position reflects the dominant effect of wage earnings on its relative income. Since the stock of land cannot accumulate and the growth in effective labor is exogenous (i.e., the model does not consider an agent investing in labor skills), the household with these characteristics has an incentive to forgo expenditures to accumulate capital assets at a pace faster than the mean household.

The second quintile experiences an evolution of asset and income earnings similar to the lowest quintile. In contrast, the remaining groups' asset positions deteriorate over time relative to the mean, and thus their income position worsens over time. The third quintile spends 67.8 percent of mean expenditures and is endowed with the highest landownership among the groups. This group owns 21 percent more land than the mean. However, this group is endowed with a low level of labor skill that is only 64.2 percent of the mean. Because the wage earnings account for a relatively small share in total wealth compared with the lower quintile groups, the third quintile's capital asset holding is relatively large for its level of expenditure. Despite a relatively high level of capital asset holdings, the income position of this group ranges from 0.85 to 0.83, indicating it is lower than the mean due to the low wage earnings. That is, the large level of assets and declining capital payments, which are a driving force of income dynamics, cause this group's income position to not "keep up" with the income position of the mean household.

Contrary to the third quintile, the two upper quintiles are endowed with a high level of labor skill, and consequently they benefit from capital deepening that causes wages to rise. However, their relative income decreases over time because of their relative asset deterioration that is attributed to their smaller share of noncapital assets in total wealth. In particular, these two upper groups' share of wage earnings in total wealth is as small as that of the third quintile,<sup>25</sup> and thus, with a very minor role of land rents in total wealth, the upper quintiles hold capital assets that are similar in value to those of the third quintile. Their high level of labor skill causes wage earnings to have a dominant effect on the level of total income relative to the mean. The fifth quintile's total income exceeds by a factor of about 2.1 that of the mean income household.

In summary, the relative capital asset positions of the first two quintiles improve over time by accumulating, whereas the relative asset positions of the remaining quintiles deteriorate over time, all measured relative to the mean. Consequently, transition growth "pulls" both lower and upper quintile groups closer to the mean capital asset, while the higher quintiles are also pulled marginally closer to the mean asset. Transition growth also pulls all groups closer to the mean income except for the third quintile.<sup>26</sup> In the case of the third quintile, despite its level of income being lower than the mean, its income position over time deteriorates (relative to the mean) due to its low labor skill that results in a large share (and thus, a low growth rate) of capital asset holdings to augment labor income. Clearly, the distribution of income has been modified by economic growth, but the basic pattern prevails in the long run.

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<sup>25</sup> See footnote 6.

<sup>26</sup> We could examine the past income behavior by calibrating backward from the initial period, 2005, for the validation purpose. According to the World Income Inequality Database (2c), income distribution data are available for Ghana in 1987, 1989, and 1992. For the period between 1987 and 1989, the lowest three groups deteriorate their position while the upper two groups improve their position. However, for the period between 1989 and 1992, all of the groups improve their position at the cost of the deterioration of the fifth-quintile group. The quality rating that is given to these data is "for observations where both the income concept and the survey are problematic or unknown."

## 7. SENSITIVITY OF DISTRIBUTIONS TO ECONOMIC SHOCKS

The sensitivity of the income and expenditure distributions discussed above is explored by considering three different shocks that commence in year 2005 and remain unchanged thereafter. These are

1. an increase in the productivity of land;
2. an increase in the Harrod rate of labor productivity growth; and
3. a decrease in the relative rate of protection of the nonfarm sector.

The shocks are chosen to increase agriculture's competitiveness for economywide resources. Importantly, the magnitude of each shock is chosen to yield the same identical value of long-run household utility. In this way, each shock is neutral in the sense that in the long run, the representative agent, while being made better off than in the base solution, is indifferent between the three shocks considered. Prior to focusing on the detailed effects of each shock, we provide a brief summary of distribution results.

### An Overview

The dynamic behavior of asset and income flows due to each simulated shock trends in the same direction as the baseline solution. The lower two and upper two quintiles converge toward mean household income over time, while the third quintile's position remains relatively unchanged. These results obtain because the simulations do not alter the basic fundamental forces underlying the country's growth path. However, the asset and income positions of each group, with the exception of the fourth quintile, are further from the "new" mean associated with each shock than they are from the mean of the base solution.

These basic results are summarized in Figure 1. The vertical axis reports a group's position relative to the mean, where "1" indicates that a bar reaching this height is mean income. The horizontal axis shows the bars for each of the quintile categories for the initial year 2005 and year 2035. Hence, there are five sets of bars for each of these two periods. For each period, there are four bars, one depicting the base solution, and the other three depicting the land shock (sim1), the factor productivity shock (sim2), and decreasing the protection of the nonfarm sector (sim3). First, consider the time dimension of changes in income by quintile. Comparing the height of the base solution bar for year 2005 with the base solution bar for year 2035, we see that the 2035 bar is higher than the 2005 for the first three quintiles. This is the convergence toward the mean discussed in the previous section. We see little change in the fourth quintile as the base period bar for 2005 is about equal in height to the bar for 2035. The fifth group's base solution bar is slightly lower for year 2035 than for year 2005.

Next, consider the land productivity shock simulation. Over time, we see the bar for year 2035 to be higher than the bar for year 2005 for the first two quintiles, indicating convergence toward the mean. Comparing the height of the bar with the base solution, we see that the land shock bar is shorter than the base solution bar for these two lowest quintiles. This result indicates that these two groups' income is further below the "new" mean income compared with the base. Observe the fifth quintile. The bar for the land shock exceeds the fifth group's bar for the base solution in 2005 and in 2035. Thus, the land shock causes this group's income to exceed the "new" mean income to a greater extent than in the base solution. However, this group's bar for year 2035 is slightly below its corresponding bar for 2005, which indicates convergence toward the "new" mean.

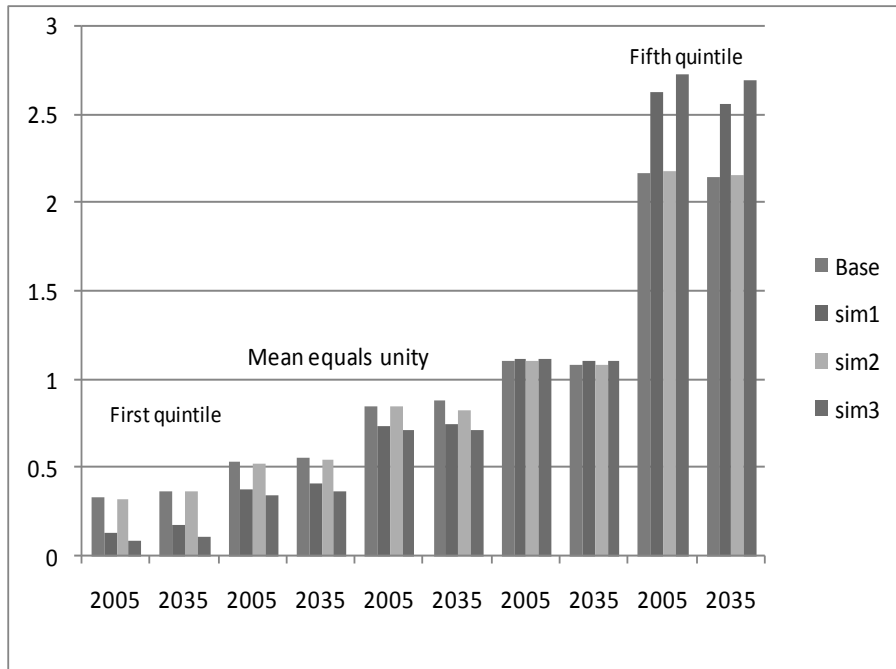
We proceed in the same manner for the remaining two simulations. We observe that the height of the bar for the labor productivity shock (sim2) is almost always the same height as the base period bar for all quintile groups. This result suggests that this shock increased mean income within each quintile in virtually equal proportion to its base period mean. The main exception is the third quintile, where, for this group, we observe some divergence from the "new" mean since the bar for 2035 is slightly lower than the corresponding bar for 2005.

For the last simulation, a reduction in trade protection of the industrial sector, we observe "short" bars relative to the base for the first three quintiles. This result indicates that the income of the mean



household for each of these groups is further below the “new” mean income than it was below the mean income of the base solution. Consider the highest-income group. We see its income to be higher than the “new” mean to a larger degree than it was above the base period mean. The fact that the height of its bar for 2035 is slightly lower than its bar for year 2005 indicates convergence toward the “new” mean over the 2005–2035 period.

**Figure 1. Comparison of base solution and simulated shocks on the distribution of income, by quintile, for the initial year 2005 and ending year 2035 (mean income is normalized to unity)**



## A Positive Shock to Land Productivity

### *Effects of the Shock on the Economy*

Land productivity is increased by 7.2 percent of base period productivity. This positive productivity shock increases the vertical chain’s competitiveness, from primary agriculture through food processing, for economywide resources. Growth in agriculture’s gross output rises from 1.56 percent per annum in 2010 of the base solution to 2.6 percent in the simulation. After 2010, growth of agricultural output declines, and by year 2030, the output is only 3 percent higher than its level given by the base solution for 2030. Agriculture’s increased competition for resources comes at the expense of the industrial sector, which is relatively capital intensive. An implication is that the country’s stock of capital is now somewhat less valuable or productive than in the initial period. The shock to agriculture causes the share of industry in GDP to decline from the 34 percent reported in Table 4 for year 2010 to 25 percent, and from the 41 percent reported in Table 4 for year 2030 to 29 percent. The service sector share of GDP is only slightly higher in this case because of the demand-side effect—that is, higher income causes expenditures on the service good to rise. Service output increases by 2 percent in 2010 from its baseline same-year level, and by 1 percent in 2030.

Of more importance for our analysis here is the effect of the shock on factor payments, and hence on the distribution of income. Overall, the share of the economy’s resources employed in the industrial

sector declines, and rises in the other sectors. With relatively cheaper capital and more costly labor than for comparable years of the base solution, some substitution of capital for labor occurs. Because the agricultural sector is almost as labor intensive as the home good sector, and the least capital-intensive sector in the economy, the productivity shock causes agriculture to employ a relatively large quantity of labor that it must bid away from the other two sectors. Consequently, wages rise relative to the baseline by about 5 percent initially, and then continue to rise but at a declining rate relative to the base solution. This rising cost of labor causes the industrial sector to release some capital to the other two sectors, with most of the released capital being employed in the agricultural sector. However, since the industrial sector is capital intensive, the amount of capital it releases is more than the other sectors find profitable to employ at the former interest rate. The release of capital causes the returns to capital and the interest rate to be lower than in the base solution. Compared with the base solution, returns to capital decline by about 6.6 percent initially and continue to decline by 12 percent by 2010 and 16 percent in the long-run equilibrium.

As a result of these adjustments, agriculture’s share of total labor employed is 26 percent higher than that in the baseline solution for 2010 and 40 percent higher than the baseline in the long run. Because capital is relatively cheaper, agriculture increases the use of capital, and the share of capital employed in the agricultural sector rises about 8.5 percent above that of the base solution in year 2010, reaching 13 percent, and 6.3 percent higher than that in the base solution for 2030. Likewise, the service sector also increases its share of capital from 36 percent to 42 percent and from 32 percent to 39 percent for the same respective points in time. The share of labor employed in the home good sector is virtually unchanged. We thus see the positive shock to the agricultural sector as causing it to compete for resources formerly employed in the industrial sector as opposed to those employed in the service sector.

The effect of these adjustments on total factor income per worker is to raise income by 7.5 percent from the baseline year initially, by 7.3 percent in 2015, and by 6.9 percent in the long run (Table 9). Capital returns fall, while returns to land, an agriculture sector–specific factor, rise by about 21 percent in the initial period relative to the baseline reported in Table 3, and reach 41 percent more than the baseline level in the long run (Table 9, column 4).

**Table 9. Effect of land productivity on factor earnings relative to the baseline**

Year	Total Factor Income	Wage Payments	Returns to Capital	Land Rents
2005	1.0750	1.0502	0.9335	1.2050
2010	1.0748	1.0274	0.8808	1.2982
2015	1.0726	1.0156	0.8607	1.3499
2020	1.0710	1.0091	0.8506	1.3799
2025	1.0700	1.0053	0.8447	1.3975
2030	1.0695	1.0031	0.8412	1.4080
2035	1.0692	1.0018	0.8391	1.4142

### *Analysis of Distributions*

The positive land productivity shock increases the welfare (utility) of all households, relative to the “new” mean household income. The upper two quintiles’ incomes are above the mean to a greater degree than they are above the mean in the base solution. The incomes of the first three quintiles are further below this “new” mean income than they are from the mean income of the base solution. This result is shown by the small numbers of income in the first three columns of Table 10 compared with the first three columns of corresponding numbers in Table 8. The numbers of income for the two highest quintiles in Table 10 are

larger than the corresponding numbers of Table 8. Thus, the two highest quintiles benefit the most from an increase in land productivity.

The dynamic behavior of households' income distribution is determined by the distribution of land and skilled labor reported in Table 7, the evolution of resource payments shown in Table 9, and in the case of capital, its accumulation over time. Table 10 shows the evolution of the relative asset and income position, respectively, for each quintile group. The general dynamics, or direction of change, of the individual relative positions are unchanged from the baseline solution except for the third quintile. The first two and last two quintiles converge toward mean income over time. The third group's income level remains almost unchanged (or improves only slightly) relative to mean income, whereas in the base solution above, the third group experienced a deterioration relative to the mean. This group is able to "hold" its position relative to the mean because its relatively large land holdings pressure the relative assets downward and, thus, lead to the improvement of the relative asset position.

The negative values in the first two columns of Table 10 indicate that households in these categories have an incentive to increase their borrowing (incurring debt) from other households since capital is relatively cheap compared with the base solution.<sup>27</sup> This is mainly due to the adjustment of their asset positions to the productivity shock. In particular, the first and second quintiles incur debt that amounts to, respectively, 132.4 percent and 65.7 percent of the mean level of capital assets initially, and 83.3 percent and 30.4 percent in 2035. These two groups earn income from other resources that are sufficient to pay interest and some principal each period of time. Because there is no risk of default, they pay no risk premium. The third quintile reduces its capital asset holdings to approximately 95.9 percent initially and 97.8 percent in the long run due to the rise in land productivity.<sup>28</sup>

We apply the logic used to explain the dynamics of the previous section. The positive shock to land productivity causes total wealth for all groups to increase at the same rate. However, individual quintile groups adjust their holdings of capital assets differently, depending on their respective share of capital assets in their total wealth. The lower quintiles whose share of noncapital assets in total wealth is large benefit from the increase in land rents due to the land productivity shock. Thus, the share of capital asset holdings in total wealth is lower than in the baseline model. Over time, these lower quintile groups increase their asset holdings (or decrease their level of borrowing) at a rate that is higher than the mean household, although the level of their asset holdings is far below the mean. The upper quintiles increase their asset holdings at a rate that is less than the mean rate, although their level of asset holdings far exceeds the mean. The third group is "in the middle" in terms of behavior. Its level of land holdings is relatively high, so the increased rental income from land reduces its incentive to increase capital holdings relative to the mean. The households that hold relatively little land and benefit little from the rise in land rental income are compensated by increasing their holding of capital assets over time, relative to the mean.

The overall net effect of the shock is to make all agents better off in welfare, but initially the shock also "spreads out" the distribution of income. Over time, with the exception of the third group, the model predicts a convergence in the direction of the mean that is similar as in the baseline.

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<sup>27</sup> Debt is interhousehold lending where borrowing households earn sufficient wage and land rental income to remunerate interest payments and, in the longer run, payment of principle.

<sup>28</sup> The model presumes formal and informal institutional arrangements permit borrowing among households that can be allocated to household expenditures. If borrowing is not possible, the distribution results are changed but not the supply-demand conditions of the economy.

**Table 10. Effect of land shock on relative capital assets holdings and relative income**

Capital Asset Holdings					
Year	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
2005	-1.32376	-0.65734	0.95916	1.66101	4.35854
2010	-1.04436	-0.45594	0.96968	1.58194	3.94658
2015	-0.93142	-0.37452	0.97391	1.54996	3.78008
2020	-0.87901	-0.33675	0.97587	1.53512	3.70284
2025	-0.85316	-0.31811	0.97683	1.52780	3.66474
2030	-0.84002	-0.30864	0.97732	1.52408	3.64538
2035	-0.83324	-0.30376	0.97757	1.52216	3.63539

Income					
Year	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
2005	0.12964	0.37953	0.74435	1.11562	2.62968
2010	0.15369	0.39608	0.74465	1.10947	2.59498
2015	0.16473	0.40368	0.74468	1.10659	2.57920
2020	0.17015	0.40741	0.74467	1.10516	2.57149
2025	0.17290	0.40930	0.74466	1.10443	2.56760
2030	0.17431	0.41027	0.74465	1.10405	2.56560
2035	0.17505	0.41077	0.74464	1.10386	2.56456

## A Positive Shock to Labor Productivity

### *Effects of the Shock on the Economy*

The exogenous rate of Harrod productivity in the baseline is 1.74 percent per annum, the rate at which a unit of labor increases the services supplied to the market at each instant in time, and equivalent to the Solow total factor productivity measure of 1.24 percent per annum. A positive shock of 4.13 percent to the Harrod rate yields the same long-run level of utility as the 7.2 percent shock to land productivity considered in the preceding simulation.

This shock augments the supply of labor services and consequently increases the competitiveness for resources in those sectors (agriculture and services) that are relatively labor intensive. The shock tends to lower the cost of intermediates employed by all sectors, and it makes the production of capital less costly compared with the baseline solution. The manufacturing sector faces increased competition for resources employed in the other two sectors because they benefit from the Rybczynski-like effect caused by the increased supply of labor services since each unit of physical labor is not more productive. Output per worker in agriculture is 0.32 percent more than that in the baseline in 2015 and 0.78 percent more by 2055 (see Table 3 for baseline values). For the same comparisons, the home good sector ranges from 0.05 percent in 2015 to 0.43 percent more than that in the baseline. The industrial sector's output per worker declines by 0.43 percent from its baseline level in 2015 and by 0.22 percent in 2055. Otherwise, the basic trends of sector share in GDP are similar to those of the base solution. GDP share of industrial and service sectors rises, whereas agricultural share in GDP declines.

The share of the workforce employed in agriculture is higher than in the base solution. Nevertheless, this share trends downward over time whereas the share employed in the industrial sector trends upward. The share of labor in the service sector remains virtually unchanged from the baseline level. The increase in the supply of effective labor per unit of initial-period capital increases the

productivity of capital in the agricultural and the service sectors. Additional capital per worker is mostly employed in the agricultural and service sectors, causing the shares of the economy’s capital employed in those sectors to exceed their corresponding levels in the base solution.

The main effects of these shocks are to increase labor and land payments relative to their base values, but such gains are modest (Table 11). Total factor income relative to base values grows slowly, exceeding base level in 2035 by about 1.4 percent. Land rental payments tend to grow more rapidly, exceeding their base level by 1.4 percent in 2015 and by 2.7 percent in the long run. Wage payments also increase modestly, ranging from 0.2 percent higher than the baseline level in 2015 to about 1.0 percent in the long run. Payments to capital are slightly less than the baseline level through 2025, and then surpass it thereafter by a small amount. This rather modest shock to augment labor services shortens the time for doubling income per worker from 51 years to 49 years.

**Table 11. Effect of shock to total factor productivity on factor earnings relative to base**

Year	Total Factor Income	Wage Payments	Returns to Capital	Land Rents
2005	1.0000	1.0000	1.0000	1.0000
2010	1.0024	1.0006	0.9965	1.0081
2015	1.0046	1.0019	0.9965	1.0136
2020	1.0068	1.0036	0.9976	1.0178
2025	1.0090	1.0056	0.9991	1.0213
2030	1.0112	1.0076	1.0009	1.0244
2035	1.0135	1.0098	1.0029	1.0272

### *Analysis of Distributions*

The increase in total factor productivity tends to have a more neutral effect on the distribution of income in the sense that all household groups tend to be as “close” to the “new” mean income as they are in the base solution. The deviation from the baseline solution is relatively evenly spread across resources relative to the previous simulation. This more uniform pattern causes the wealth share of each component to remain close to the baseline solution. Thus, the income share of each factor component tends to maintain a position that is close to the base solution, although the share of interest income is slightly lower than the baseline because of the lower rate of capital accumulation per worker through 2020, after which the stock of capital per worker exceeds the baseline solution. The slower capital accumulation effect is counterbalanced by the wage increases as well as rising land rents.

Table 12 shows that the relative position of capital assets and income of each quintile group is very close to the baseline solution, unlike the case of the land productivity increase. Using the same reasoning as the previous case, we observe that the first and second quintiles reduce their level of asset holdings, but the degree of change is modest. The lowest group incurs small debt while the second group decreases its assets to 31.8 percent in the initial period, and 27.6 percent and 51.4 percent, respectively, in 2035. The relative asset position of the third quintile is very close to the baseline solution; the difference in its relative asset position between this simulation and the baseline result is less than 1 percent.

By a similar argument used to explain the previous simulation, the two upper quintiles improve their relative asset positions for a given period of time compared with the baseline solution. However, the changes from the baseline result in terms of distance from the mean are small throughout transition growth. For each time period, the fourth quintile generally increases its capital asset position relative to the mean by less than 1 percent while the fifth quintile increases its position by approximately 6 to 7 percent compared with the baseline result. In terms of the growth rate of relative asset accumulation, the asset position of the three upper quintiles deteriorates while the two lower quintiles improve their positions over time as in the case of the baseline. Once again, we obtain a convergence toward a more rapid growth in mean income.

Because the evolution of the relative asset holdings is the main driving force for the dynamics of the relative income when the income shares of each component hardly change, the relative income positions deviate little from the baseline result. The relative income position for the lowest quintile changes the most among the groups, and its position relative to the mean is lower in the simulation than that in the baseline. The range of difference between the simulation result and the baseline for the second quintile is 1.1 to 1.2 percent. For the upper quintile groups, the reverse is the case, but much smaller differences are observed.

**Table 12. Effect of shock to total factor productivity on relative capital asset holdings and relative income**

Capital Asset Holdings					
Year	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
2005	-0.00516	0.31805	1.52552	1.53955	1.62161
2010	0.15959	0.43251	1.44249	1.44944	1.51559
2015	0.22125	0.47533	1.41108	1.41556	1.47644
2020	0.24982	0.49517	1.39642	1.39981	1.45846
2025	0.26434	0.50525	1.38894	1.39178	1.44936
2030	0.27208	0.51062	1.38494	1.38750	1.44453
2035	0.27630	0.51355	1.38276	1.38517	1.44190
Income					
Year	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
2005	0.32773	0.52604	0.84924	1.10824	2.18789
2010	0.34858	0.53945	0.83986	1.09881	2.17246
2015	0.35750	0.54514	0.83519	1.09447	2.16685
2020	0.36185	0.54791	0.83273	1.09226	2.16441
2025	0.36412	0.54934	0.83139	1.09109	2.16323
2030	0.36534	0.55012	0.83065	1.09045	2.16262
2035	0.36601	0.55054	0.83024	1.09009	2.16229

## A Reduction in Protection of the Industrial Sector

### *Effect of the Shock on the Economy*

A 2.3 percent decrease in the relative rate of protection of industry is required to obtain the same level of long-run utility as obtained by the other two shocks. Such a decrease in protection causes industrial output to fall by 4.2 percent from its baseline level in 2005, by 10.5 percent in 2015, and by 12.5 percent in the long run. It also causes an improvement in agriculture's terms of trade—agricultural output increases by 2.6 percent from baseline value in 2005, 5.3 percent in 2015, and 6.22 percent in the long run. Service output remains virtually unchanged. The industrial sector's share of total capital employed declines by 3 percent from the baseline in 2005 and by 7.4 percent in the long-run base share (see Table 4 for baseline results). The share of capital employed in agriculture increases by 5.4 percent from baseline value initially, and rises to 14 percent in the long run. The home good sector also experiences an increase in its share of capital employed. Of course, agriculture increases the share of labor employed relative to the baseline; the average increase is more than 7 percent of the baseline values.

With relatively fewer resources employed in the industrial sector than in the baseline solution for corresponding points in time, capital stock is 5.7 percent below the baseline solution in the long run (see Table 3, column 2 for baseline results). The economy attains a steady-state equilibrium with the quantity of all goods consumed exceeding their baseline levels. Households increase their consumption levels for two reasons: one is due to lowered prices for goods consumed, and the other is due to a marginally lower incentive to save for purposes of increasing the capital stock of the protected industrial sector. Relative to the baseline result, the saving-to-GDP ratio declines by 10 percent in the initial period, and by 6.0 to 4.7

percent over 2015–2035. Consequently, households have an incentive to forego consumption in each year relative to the baseline. The level of household expenditure relative to the corresponding baseline results rises by about 23 percent in the initial period, and then declines in transition by 17 percent in 2015 and 15 percent in the long run (Table 13).

**Table 13. Effect of a decline in protection of the industrial sector on factor earnings relative to the baseline**

Year	Total Factor Income	Wage Payments	Returns to Capital	Land Rents	Expenditure
2005	1.0108	1.0304	0.8822	1.2458	1.2291
2010	0.9667	0.9982	0.7914	1.4006	1.1913
2015	0.9466	0.9824	0.7589	1.4859	1.1720
2020	0.9355	0.9733	0.7432	1.5327	1.1607
2025	0.9306	0.9694	0.7359	1.5612	1.1558
2030	0.9275	0.9668	0.7316	1.5768	1.1525
2035	0.9257	0.9653	0.7291	1.5858	1.1506

A focus on factor income as an indicator of utility can be misleading in this case. Real total factor income and its components are expressed in Table 4 relative to the base solution. Notice first that trade liberalization does not increase the level or rate of growth in total factor income relative to the base solution. The growth in wage income remains the major contributor to income growth, accounting for almost 70 percent of income growth. However, the growth rate in wage income is lower than that in the baseline, which is a Stolper-Samuelson–like phenomenon that can be explained by the reduced-form equations (23) and (24). The decline in protection causes two opposing effects—a downward pressure on the rental rate of capital and an upward pressure on wages. Although wages rise as they do in the baseline, the net effect of these forces is to lower the rate of growth of wages relative to the baseline. Growth in capital stock owned by households contributes about 23 percent to the growth in factor income, whereas the decline in the rate of return to capital has virtually no effect. The decline in protection of the industrial sector leads to a more profitable agricultural sector (Table 13). Although households are better off, the time required to double income per worker is not appreciably changed from the base solution of 51 years.

### *Analysis of Distributions*

Although lowering the protection of the industrial sector has unique economic implications compared with the other simulations, the general behavior of the distributive dynamics is very similar to that of the other two cases, especially to the case of the land productivity increase. Both cases (lowering industry protection and boosting land productivity) create a disproportional increase in the share of wealth related to land rent accompanied by a modest increase in the share of wealth accounted for by wage income. Thus, the groups that hold more land and labor tend to experience an improvement in their asset and income positions.

The most substantial changes involve the lowest two quintile groups. According to Table 14, the first quintile has an incentive to incur debt that is 69.7 percent larger than the amount of the mean capital asset holding initially, and 34.8 percent larger in 2035. As a result, the lowest quintile reduces its relative position of income to 8.8 percent of the “new” mean initially and 11.5 percent by 2035. Similarly, the second quintile incurs debt that is 94.5 percent of the mean level of capital assets initially and 68.6 percent in 2035. Its relative income declines to 34.8 percent of the mean initially and 36.6 percent by 2035. Since the second quintile is endowed with a higher level of labor skill and landownership than the lowest quintile, the latter receives more benefit to its relative income from the increase in wage and land rent, at the same time alleviating the negative impact of its large debt due to the lower income share of capital income. The reduction of the tariff rate also imposes a downward pressure on the relative income position for the third quintile throughout the period of transition to long-run growth. However, the change

in the dynamic behavior of this group from the baseline solution is explained in the same fashion as with the case of land productivity increase.

Contrary to these lower quintile groups whose relative capital is reduced compared with the baseline, the fifth quintile increases its capital holdings by a factor of 5.19 times the mean in the initial period and 4.64 times larger in 2035. This group increases its relative income by a factor of 2.73 larger than the mean initially and 2.69 larger in 2035. This highest quintile is followed by the second-highest group. This group shows a similar pattern in the evolution of capital assets and income. However, because of the exceptionally low wealth share of landownership, the deviation of the relative income from the baseline result for the fifth group is not as significant as the change in the relative capital assets position.

**Table 14. Effect of a decline in protection of the industrial sector on relative capital asset holdings and relative income**

Capital Asset Holdings					
Year	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
2005	-1.69737	-0.94460	0.75793	1.68677	5.19429
2010	-1.51491	-0.81169	0.77762	1.64041	4.90579
2015	-1.43008	-0.74990	0.78676	1.61886	4.77167
2020	-1.38775	-0.71907	0.79133	1.60810	4.70476
2025	-1.36589	-0.70314	0.79368	1.60254	4.67020
2030	-1.35440	-0.69477	0.79492	1.59962	4.65203
2035	-1.34830	-0.69033	0.79557	1.59807	4.64239

Income					
Year	1st Quintile	2nd Quintile	3rd Quintile	4th Quintile	5th Quintile
2005	0.08849	0.34752	0.71398	1.11443	2.73434
2010	0.10182	0.35680	0.71494	1.11132	2.71389
2015	0.10852	0.36147	0.71540	1.10974	2.70365
2020	0.11200	0.36389	0.71563	1.10892	2.69835
2025	0.11383	0.36516	0.71575	1.10849	2.69557
2030	0.11480	0.36584	0.71581	1.10826	2.69409
2035	0.11532	0.36620	0.71584	1.10813	2.69330



## 8. CONCLUSION

Our study fits to Ghanaian data an intertemporal three-sector growth model depicting interconnections of the economy through various channels. The study's unique contribution is to analytically and empirically derive the intertemporal implications to the distribution of household income by quintile groupings in transition growth to a long-run equilibrium. The study also provides insights into the question of whether the current growth of the economy supports middle-income-country status and to what extent the structure of the economy changes over time, and whether such growth affects income growth by quintile group.

The baseline model result shows that the capital stock-to-GDP ratio increases from about 2.2 to 3.06 over the period 2005 to 2035, without the country attaining the middle-income status of \$980 per capita. By 2035, the model predicts an income in 2005 cedi per worker of 15,040,000 (about \$1,654/worker or \$725/capita). This result suggests that the country's potential to increase household income in transition growth is rather limited. The model predicts that more than 50 years are required to double per worker GDP, though total GDP may double in 17 years. These results are similar to those of Breisinger, Diao and Thurlow (2009), who also questioned the ability of the country to attain middle-income status over a similar interval of time.

The income distribution implications of transition growth are encouraging in that, over the 2005–2035 period, the lower two and upper two quintile income categories converge modestly toward rising mean household income. The data suggest that the income earned by the lowest quintile is only 33.7 percent of the mean level of household income initially. The model predicts modest improvement over time, with that group's income rising to 37.4 percent of the mean income over 30 years. The richest quintile earns 2.17 times the mean income initially, but that group's income position deteriorates modestly, declining to 2.15 times the mean income over the same period of time. Thus, the model's forecast of transition growth suggests that growth is "pro-poor," although the improvement in income distribution is modest.

Because agriculture is such a large part of the Ghanaian economy, three simulations were performed. The simulations are designed in a special manner. First, each simulation considers an economic shock that increases agriculture's competitiveness for resources. The shocks simulated consist of increasing land productivity, increasing the exogenous rate of Harrod neutral technical change augmenting labor, and reducing the tariff protecting the industrial sector. Second, the level of each shock is chosen in such a way that each yields the same long-run (or steady-state) level of utility. In this way, the level of the shock studied is neutral in the sense that each leads to the same long-run level of well-being.

The simulations show that competition for resources caused by boosting land productivity causes industry's share of GDP to decline and the service sector's share of GDP to remain virtually unchanged compared with the baseline result. Increasing the rate of factor productivity contributes to the expansion of the agricultural and service sectors (the two most labor-intensive sectors of the economy) by increasing the shares of the economy's resources that are allocated to those sectors. Lowering the tariff protecting the industrial sector has effects similar to increasing the productivity of land. All three shocks improve well-being, but the magnitude of the shocks, such as a 7.2 percent increase in land productivity, is not sufficient for the country to gain middle-income status by 2035.

The shocks did not alter the basic underlying economic forces of transition growth. Industrial share in GDP increases as labor transitions from agriculture to the industrial and service sectors of the economy. Accompanying that transition is capital deepening in all sectors of the economy, rising wages and land rents. Land rents (or farm profits) typically decline in early transition but rise later as the growth in wages (a cost of production in agriculture) moderates. The rise in the price of nontraded (or service) goods contributes to a decline in the internal terms of trade for the two internationally traded goods, thus inducing the nontraded-goods sector of the economy to pull some resources from the internationally traded goods sector.

Subject to the shares of labor income, capital income, and land income in total income across quintile groups, the key to the evolution of income distribution over time is the households' saving behavior that allows capital asset accumulation. The model shows that such evolution is determined by the components of household wealth.

The three simulations indicate that the patterns of income distribution over time will not change much with additional interventions in the economy. Compared with the baseline, each of the three simulations, while leading to a higher level of utility for the mean household in the long run, generates the result in which the lower quintile groups reduce their relative income positions compared with the mean by increasing their debt position or depleting their capital assets, whereas the upper groups improve their relative income by accumulating a larger holding of capital assets. Of the three simulations, the case of the increase in total factor productivity deviates from the baseline result most insignificantly since the dynamics of factor payment are similar to the baseline result. These results are limited in many ways. To mention a few, the skills of workers are exogenous; the model presumes no adjustment costs associated with the rate of capital deepening; and credit markets among households, and among households and firms, are complete with no risk of default.

Nevertheless, the model suggests the presence of rather fundamental constraints to growth that limit the country's ability to reach middle-income status. These constraints are embodied in the parameters of our model and cannot be identified without further investigation. They include (1) the production function scale parameters, (2) our estimate of the country's stock of capital in 2005 and its relatively low savings rate; and (3) input-output coefficients. Briefly, the scale parameters of the sectoral production functions suggest the efficiency with which inputs translate into outputs on a per unit of output basis. Ghana's scale parameters are smaller than we see for other countries that we have modeled. This observation suggests that there are some unknown structural limitations to growth that further research should attempt to identify and further investigate.

The country's capital-to-output ratio should, in principle, lead to high output, using the capital output ratio of Organization for Economic Cooperation and Development countries as a guide. For advanced economies, the ratio is about 3.2–3.5, which Ghana approaches in 2035, but with only a fraction of the output level. Finally, the intermediate sectoral input coefficients for the off-diagonal elements, particularly with regard to the amount of service resources other sectors demand per unit of output, may suggest rigidities in transportation, or concentrated down- and upstream markets that extract excessive rents.

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