Input price risk and optimal timing of energy investment: choice between fossil- and biofuels*

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July 12, 2002

Abstract

We consider energy investment, when a choice has to be made between fossil fuel and biomass fired production technologies. A dynamic model is presented to illustrate the effect of the different degrees of input price uncertainty on the choice of technology and the timing of the investment. It is shown that when the choice of technology is irreversible, it may be optimal to postpone the investment even if it would otherwise be optimal to invest in one or both of the plant types. We provide a numerical example based on cost estimates of two different power plant types.

JEL classification: D81, G31, Q42

Keywords: irreversible investment, price uncertainty, biomass, real options

*The work has been financed by the Nordic Energy Research, for which the authors are grateful.
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1 Introduction

Many countries have adopted the goal to enhance the role of renewable sources in energy supply. This is to a large extent motivated by the efforts to decrease greenhouse gas emissions arising from the combustion of fossil fuels. The European Commission published in 1997 a White Paper, which sets a target of increasing the share of renewable energy sources in the total energy consumption in the European Union from 6 to 12 percent by 2010 (European Commission, 1997). Biomass is there considered to be the most important energy source in meeting this target.[1] To be able to design concrete measures to reach such goals, it is therefore important to properly understand the factors affecting investments in new biomass fired plants.

The extent of investments in biomass is largely determined by its competitiveness relative to alternative fuel types. Biomass competes mainly with the different kinds of fossil fuels, particularly natural gas. In the literature, comparisons of the total production costs using different technologies are typically carried out using fixed estimates for the fuel costs (e.g. OECD, 1998, Kosunen and Leino, 1995). However, even with a thorough sensitivity analysis of the fuel costs, the uncertainties are not explicitly accounted for. Nevertheless, it is clear that the input price risk has an effect on the value of the energy production assets, and thus on the relative competitiveness of the fuel types. As the choice of fuel type for a particular project is in most cases irreversible, a rational investor considering an investment in a project that can be implemented using alternative technologies must take this uncertainty into account.

There are significant differences in the uncertainties associated with different fuel types. The fossil fuels are depletable resources, which have world market prices

[1] The White Paper describes biomass as being "...a widespread resource as it includes in addition to woody biomass and the residues of the wood working industry, energy crops, agricultral residues and agrofood effluents, manures as well as the organic fraction of municipal solid wastw or source, separated household waste and sewage sludge".

subject to a high degree of uncertainty. As the global reserves diminish, the prices are expected to rise.[2] Moreover, the reserves of such fuels are very unevenly distributed, and thus for most of the countries they have to be imported. This adds the risk of currency fluctuations. A further uncertainty is caused by the market power of the exporters. The OPEC countries have been successful in manipulating the oil prices for a long time. For natural gas, the lack of competition in supply may be even more severe in many cases: as gas requires a dedicated network over which the product is delivered, the number of suppliers may be very limited. The uncertainty is further increased by the unstable political situation in some of the major gas exporting countries.

Since biomass is renewable and usually produced domestically, its price is determined in a different way. There are in many cases abundant reserves, and the price is presumably based on the costs of growing, harvesting, and transporting, depending on the type of biomass. The characteristics that cause much of the uncertainty in the prices of fossil fuels, namely scarcity of reserves, currency risks, and market power, are not that significant. Therefore, there is reason to believe that the price of biomass should be more stable than the prices of fossil fuels.

This paper presents a dynamic model to study the choice between projects with different input price characteristics. We analyze the competitiveness of two alternative fuel types by looking at a representative investor who is considering an energy production investment, and faces the choice between a fossil fuel and a biomass fired plant. To make our point clear, we assume that the fossil fuel price evolves stochastically, while the biomass price is constant. The investment creates a given payoff stream, either as an explicit revenue flow from selling the output, or as a flow of avoided cost of purchasing the demanded energy service from elsewhere. It is assumed that the investment can be delayed without constraints, thus the

[2] For example, under Hotelling assumptions the price increases exponentially (see, e.g., Sweeney, 1993, for a review of the theory).

investor must choose both the timing of the investment and the type of the plant.

Methodologically the analysis is based on the theory of irreversible investment under uncertainty (see Dixit and Pindyck, 1994, for a review of the theory). This literature characterizes uncertainty through continuous time diffusion processes, usually the geometric Brownian motion. The basic theory points out that because investments are typically irreversible, the rational investor should wait with the investment until the net present value of the project exceeds a certain positive threshold level in order to compensate for the possibility that the value of the project turns in an unfavorable direction in the future. Our model has an additional irreversibility: at the moment of the investment, the investor also has to choose the type of the production technology. This choice can not be reversed later, even if the fossil fuel price evolves in the direction that would favour the opposite decision. The focus of our model is in the interaction between the timing decision of the investment and the simultaneous choice of the plant type.

The optimal solution to our model consists of two threshold levels for the fossil fuel price that trigger an investment: if the price drops to the lower threshold it becomes optimal to invest in the fossil fueled plant, while at the higher threshold it becomes optimal to invest in the biomass fired plant. Between the thresholds it is optimal to wait. The threshold levels can not be solved in closed form, but we derive the equations that can be used to solve them numerically.

The results indicate that as the choice of technology is irreversible, it is optimal to postpone the investment at a wide range of prices even if it would otherwise be optimal to invest in one or both of the plant types. In other words, the fuel price uncertainty associated with one plant type may postpone investments in alternative plant types as well, if the choice of input fuel is irreversible. This is due to the flexibility inherent in the real option that allows the choice between two technologies. This flexibility increases the value of the option, and because the investor has to give up this option at the moment of investment, it induces additional reluctance to invest. Even in the case where the biomass technology is chosen, the timing of the investment is triggered by the fossil fuel price development, because the two alternatives are interrelated through the real option, which is lost at the moment of investment.

There are several papers that are closely related to our work. Kobila (1990) considers the choice between hydro and thermal power generation under the assumption that hydro production has only completely irreversible capital costs, while gas production has only variable fuel costs. The focus is on the optimal timing to switch from gas to hydro in supplying a given demand unit at minimum cost. Even if the setting in that model is quite different from ours, especially in that it considers the optimal choice of technology from the point of view of a social planner, the model has technically many similarities to our paper. Kulatilaka (1993) and Brekke and Schieldrop (1999) are concerned with alternative technologies in energy production focusing on the value of flexibility to switch between two fuels. Kulatilaka (1993) considers the value of the option to switch between oil and gas in steam boilers, but does not consider the timing of the investment in such a plant. Brekke and Schieldrop (1999) have a setting more similar to ours, i.e. the timing of a power plant investment, when the investor must simultaneously choose the type of the plant. They consider two alternative inputs, oil and gas, but unlike us, they allow both fuel prices to follow separate stochastic processes, and also consider a flexible technology that allows switching between the two. This is more general than our setting, but makes it necessary to use approximation techniques in solving the model. Our model also differs from Brekke and Schieldrop (1999) in the characterization of the production process. They assume that once the plant has been built, it will be operated at all times even if the profit flow is negative. This assumption is likely to have an overestimating effect on the value of the flexible production technology in their model. In contrast, we allow the plant to be shut down whenever the profit flow would be negative. The theory of valuing such production assets has been developed in McDonald and Siegel (1985).

We use our model to provide a numerical example based on cost estimates of two types of power plants applicable in Finland. In Finland, already more than 20 % of energy consumed is produced using renewable energy sources. According to the Finnish action plan response to the EU White Paper on renewable energy sources, the goal is to increase this by 50 % by the year 2010 compared with the year 1995 (Ministry of Trade and Industry, 2000). This increase will be obtained almost entirely from biomass. Therefore, it is important to properly understand the factors affecting investments in such projects. This paper points out that when such a project can be alternatively implemented using some other technology, then the timing of the investment is directly influenced by uncertain factors associated with this alternative technology.

The paper is organized as follows. We first present the model in section 2. In section 3 we consider only the fossil fuel option. We derive the value of an operating fossil fueled plant that can be shut down and restarted costlessly. We also derive the optimal investment rule in such a plant when there is no alternative plant type available. In section 4 we proceed to consider the whole problem where both plant types are available as alternatives to each other. We derive the equations that must be solved numerically in order to get the optimal investment strategy. In section 5 we apply the model in an example case. Finally, section 6 concludes.

2 Model

We consider an investor, who has an opportunity to invest in a new energy production plant. There are two alternative plant types available: a fossil fuel and a biomass fired plant. We assume that the biomass price is constant, but the fossil fuel price is stochastic. Once the choice of the plant type has been made, the decision can not be reversed. The investor must thus choose both the timing of the investment and the type of technology to use.

We start by looking more closely at the fossil fuel option. The fossil fuel price, denoted P, is assumed to follow the geometric Brownian motion of the form:

$$\frac{dP}{P} = \alpha dt + \sigma dz, \tag{1}$$

where α and σ are constants reflecting the drift and volatility of the price process, dt is an infinitesimal time increment, and dz is the standard Brownian motion increment. We assume that the fluctuations of P are spanned by financial markets, in other words, there is a traded asset or a portfolio of assets with a price that correlates perfectly with P.[3] Let μ be the expected rate of return for this asset in equilibrium and denote by $\delta = \mu - \alpha$ the 'return shortfall'. Using the equivalent martingale measure, the gas price process is then:

$$\frac{dP}{P} = (r - \delta)\, dt + \sigma dz, \tag{2}$$

where r is the risk-free rate of return. Then, any contingent claim on P can be valued using equivalent risk neutral valuation, i.e. taking the expectation assuming that P follows (2) and discounting with the risk-free rate of return. The risk-aversion of the investor is thus accounted for by replacing the actual drift rate of the fossil fuel price by the certainty equivalent rate while retaining the risk-free rate of return as the discount rate (see Cox, Ingersoll, and Ross, 1985, or Dixit and Pindyck, 1994, chapter 4, for more details on the techniques). Alternatively, the value of the claim

[3] If this does not hold, then our results may be derived using dynamic programming, and assuming that the investor is risk neutral or uses a subjectively determined discount rate to discount risky cash flows.

can be derived using a standard arbitrage argument, which is the technique used throughout the book by Dixit and Pindyck.

The focus of the model is in the input price uncertainty. Therefore, we make the assumption that the output price is constant.[4] We denote by A the constant cash flows that the plant produces when it is operating. This cash flow may be interpreted as the income from selling the output, more precisely the electricity and/or heat price after taxes minus possible variable production costs other than fuel cost. Alternatively, we may interpret the model so that the investment is made in order to satisfy a given energy demand, and A is the avoided cost of purchasing the energy from some other source.

We further assume that the plant can be shut down and restarted costlessly. This means that whenever $P > A$, it is optimal not to produce, otherwise the plant earns $A - P$ per unit of time. The valuation of plants with a shut down option has been treated in McDonald and Siegel (1985).

We denote by $\pi_s(P)$ the total cash flow of an optimally operated fossil fueled plant:[5]

$$\pi_s(P) = \begin{cases} A - P, \text{ when } P < A \\ 0, \text{ when } P \geq A. \end{cases} \tag{3}$$

The investment in the plant incurs a fixed investment cost denoted I. Once the plant has been built, it will remain operational forever. The investment in such a plant is thus equivalent to swapping a fixed amount of money I to a perpetual stochastic cash flow stream $\pi_s(P)$. We will derive the value of such a cash flow stream in section 3.

[4] This assumption is, of course, a simplification of reality. It is made in order to keep the model tractable, but also in order to set the focus explicitely on the interaction of the investment timing and the simultaneous choice between risky and non-risky projects.

[5] We may also have a deterministic fixed flow cost component, but that is equivalent to increasing the investment cost by an appropriate amount (we do that in the example presented in section 5). We have adopted the current formulation to keep the notation as simple as possible.

Consider next the alternative biomass option. This project is assumed to contain no risk, and thus produces a deterministic and known cash flow pattern. The assumption that there is no risk associated with the biomass plant is, of course, a simplification. It is made for several reasons. First, it keeps the model tractable. Second, it puts the focus on the choice between risky and non-risky projects. The model, as specified in the paper, can also be used to analyze the competitiveness of different plant types under a policy where the government, using appropriate subsidies, tries to ensure a riskless investment in renewable energy projects.[6]

We denote the net present value of the biomass project by V_R. It is obtained simply by summing all the cash flows and discounting them with the risk-free rate of return. V_R is assumed to be constant as long as the investment is not undertaken, in other words, there is no explicit value of waiting associated with the biomass project. To be an interesting alternative, the value of the project has to be positive, i.e. $V_R > 0$. The cash flows may also include investment or production subsidies by the government, as well as taxes. Therefore, the government can directly adjust V_R by designing a suitable tax/subsidy scheme. This can be used as an instrument for promoting investments in renewable biomass production. In section 5 we will discuss in more detail how the value of the project is composed in the context of an example case.

The basic theory says that an investment in such a project should be carried out if its net present value is positive. However, in our model this conventional investment rule would ignore the fact that when investing in a biomass plant, the investor loses the option to carry out the project using the alternative fossil fuel

[6] The EU White Paper "Energy for the future: renewable sources of energy" (1997) states: "The guiding principle for the Commission in assessing aid for the renewable energies (...) is that the beneficial effects of such measures on the environment must outweigh the distorting effects on competition. The commisison will consider appropriate modifications in favour of renewable energies in support of its policy in this area during the revision of the present guidelines taking into consideration the Council's Resolution on the Green Paper "Energy for the future : renewable sources of energy" which states that investment aid for renewables can, in appropriate cases, be authorised even when they exceed the general levels of aid laid down in those guidelines."

technology. Taking into account this option value implies that the timing of the investment is driven by the fossil fuel price development, even if the eventual choice would be to invest in the biomass plant.

3 Value of the fossil fueled plant

In this section, we ignore the biomass option, and consider the value of the fossil fueled plant and the optimal behavior of an investor who owns an option to invest in such a project. The techniques are adopted from Dixit and Pindyck (1994), who have uncertainty in the output price. The modifications to the case of input price uncertainty are straight-forward. Therefore, we have omitted some details in the derivations.

We denote by $V_s(P)$ the market value of an operational fossil fueled plant. Owning such a plant is equivalent to owning an asset that pays a perpetual stochastic income flow $\pi_s(P)$. This income flow, as given in (3), is a stochastic process driven by P. The dynamics of P are given under the martingale measure by (2). By a standard arbitrage argument, or alternatively applying dynamic programming and the "equivalent risk neutral valuation", the value of such an asset can be shown to satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 P^2 V_s''(P) + (r-\delta) P V_s'(P) - r V_s(P) + \pi_s(P) = 0, \tag{4}$$

where the primes denote the derivatives with respect to P.

We derive the solution to (4) in two parts, reflecting the form of $\pi_s(P)$ as given in (3). In the region where $P > A$, we have $\pi_s(P) = 0$, and the general solution is:

$$V_{s+}(P) = B_1 P^{\beta_1} + B_2 P^{\beta_2},$$

where:

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} > 1, \tag{5}$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0. \tag{6}$$

It is clear that the value of the plant must approach zero if the input price approaches infinity. Therefore we have the boundary condition $V_{s+}(\infty) = 0$, which implies $B_1 = 0$.

In the region $P < A$, we have $\pi_s(P) = A - P$, and the general solution is:

$$V_{s-}(P) = C_1 P^{\beta_1} + C_2 P^{\beta_2} + \frac{A}{r} - \frac{P}{\delta}.$$

If the input price approaches zero, it becomes more and more unlikely that the plant needs to be shut down in the near future. At the limit, therefore, the value of the plant must approach the value of the constant income flow A, and we get the boundary condition $V_{s-}(0) = \frac{A}{r}$. This implies $C_2 = 0$.

We have now the value of the plant for the two regions expressed separately with two free parameters, B_2 and C_1. To stitch the two parts together, two additional boundary conditions must be satisfied at the shut-down point $P = A$. There can be no jump either in the value of the plant or in its derivative: $V_{s+}(A) = V_{s-}(A)$ and $V'_{s+}(A) = V'_{s-}(A)$ (see, e.g., Dixit and Pindyck, 1994). These conditions, called the value matching and smooth pasting conditions, respectively, are in this case explicitly:

$$B_2 A^{\beta_2} = C_1 A^{\beta_1} + \frac{A}{r} - \frac{A}{\delta},$$
$$\beta_2 B_2 A^{\beta_2 - 1} = \beta_1 C_1 A^{\beta_1 - 1} - \frac{1}{\delta}.$$

These are linear in B_2 and C_1, and can thus be easily solved:

$$C_1 = \frac{A^{1-\beta_1}}{(\beta_1 - \beta_2)} \left(\frac{\beta_2}{r} - \frac{\beta_2 - 1}{\delta} \right),$$
$$B_2 = \frac{A^{1-\beta_2}}{(\beta_1 - \beta_2)} \left(\frac{\beta_1}{r} - \frac{\beta_1 - 1}{\delta} \right).$$

The value of the plant is then:

$$V_s(P) = \begin{cases} \frac{A^{1-\beta_1}}{(\beta_1-\beta_2)} \left(\frac{\beta_2}{r} - \frac{\beta_2-1}{\delta} \right) P^{\beta_1} + \frac{A}{r} - \frac{P}{\delta}, \text{ when } P < A \\ \frac{A^{1-\beta_2}}{(\beta_1-\beta_2)} \left(\frac{\beta_1}{r} - \frac{\beta_1-1}{\delta} \right) P^{\beta_2}, \text{ when } P \geq A. \end{cases}$$

Figure 1 illustrates $V_s(P)$ with different values of σ for a risk neutral investor.[7] The parameter values used are $r = \mu = 0.05$, $\alpha = 0.02$, $A = 1$. It can be seen that increasing uncertainty increases the value of the plant. This is because of the flexibility provided by the shut-down option. The possibility to shut-down the plant provides an insurance against unfavorable development of the gas price, but still gives full benefits of a favorable development. Therefore, such a plant benefits from volatile profitability conditions. More accurately, the same intuition can be confirmed by looking more closely at the formula for the profit flow. The profit flow, as given in (3), is a convex function of P, and thus, given a random P, its expected value is higher than its value at expected value of P, according to Jensen's inequality.

[7] Risk neutrality is assumed in this figure in order to avoid confusing different aspects of the model. If the firm would be risk-averse, different risk-adjustments would be appropriate for different uncertainty levels. The qualitative nature of the figure would remain unchanged.

The higher the uncertainty, the larger is this difference. As the value of the plant is the expected value of all future profit values (under the martingale measure), it is obvious that increasing the volatility, while keeping the mean unchanged, increases the value of the plant.

Figure 1 *here*

Next, we consider the optimal behavior of an investor who has an opportunity to invest in such a fossil fueled plant. We denote by $F_s(P)$ the value of the option to invest in the project. Again, a standard arbitrage argument can be used to show that, as long as not exercised, the value of the option must satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 P^2 F_s''(P) + (r-\delta) P F_s'(P) - r F_s(P) = 0. \tag{7}$$

Note that equation (7) differs from (4) in that the income flow term, $\pi_s(P)$, is missing. This is because the investment option does not pay any cash flows as long as the investment has not been carried out.

The general solution to (7) is:

$$F_s(P) = D_1 P^{\beta_1} + D_2 P^{\beta_2},$$

with β_1 and β_2 given by (5) and (6). It is obvious that the value of the option to invest in a fossil fueled plant is decreasing in the fuel price. At the limit, where the fuel price approaches infinity, the option must lose its value altogether, leading to the condition $F_s(\infty) = 0$. This implies that $D_1 = 0$.

It is a standard result in the literature that the optimal investment rule can be expressed as a threshold level such that whenever the fuel price is below this level, it is optimal to invest, and otherwise it is optimal to wait. Further, the option value must satisfy the value-matching and smooth-pasting conditions at the optimal investment trigger. Denoting the threshold level by P^*, these are:

$$F_s(P^*) = V_s(P^*) - I, \tag{8}$$

$$F_s'(P^*) = V_s'(P^*). \tag{9}$$

At the point where it is optimal to invest, it is also optimal to run the plant. Therefore, it must be that $P^* < A$.[8] Thus, $V_s(P^*) = \frac{A^{1-\beta_1}}{(\beta_1-\beta_2)}\left(\frac{\beta_2}{r} - \frac{\beta_2-1}{\delta}\right)(P^*)^{\beta_1} + \frac{A}{r} - \frac{P^*}{\delta}$, and conditions (8) and (9) are explicitly:

$$D_2(P^*)^{\beta_2} = \frac{A^{1-\beta_1}}{(\beta_1-\beta_2)}\left(\frac{\beta_2}{r} - \frac{\beta_2-1}{\delta}\right)(P^*)^{\beta_1} + \frac{A}{r} - \frac{P^*}{\delta} - I, \tag{10}$$

$$\beta_2 D_2(P^*)^{\beta_2-1} = \beta_1\frac{A^{1-\beta_1}}{(\beta_1-\beta_2)}\left(\frac{\beta_2}{r} - \frac{\beta_2-1}{\delta}\right)(P^*)^{\beta_1-1} - \frac{1}{\delta}. \tag{11}$$

These can be used to solve for the unknown parameter D_2 and the optimal investment trigger P^*. From (10) we have:

$$D_2 = (P^*)^{-\beta_2}\left[\frac{A^{1-\beta_1}}{(\beta_1-\beta_2)}\left(\frac{\beta_2}{r} - \frac{\beta_2-1}{\delta}\right)(P^*)^{\beta_1} + \frac{A}{r} - \frac{P^*}{\delta} - I\right].$$

Substituting this in (11) gives:

$$\left[\frac{A^{1-\beta_1}}{(\beta_1-\beta_2)}\left(\frac{\beta_2}{r} - \frac{\beta_2-1}{\delta}\right)(P^*)^{\beta_1}\right]\left(\frac{\beta_2-\beta_1}{P^*}\right) + \frac{\beta_2}{P^*}\left(\frac{A}{r} - I\right) + \frac{1-\beta_2}{\delta} = 0.$$

This must be solved numerically to get P^*, the optimal investment threshold. Figure 2 illustrates the optimal investment rule. The investment cost is $I = 3$ and

[8] It is obvious that it can not be optimal to invest in a plant if it will be shut-down immediately. In such a case, postponing the investment by a short interval would earn the time value for the investment cost, while it would not change any other cash-flows because the plant would be shut-down during the interval.

the fuel price volatility is $\sigma = 0.1$. Otherwise, the parameter values are as in figure 1. The optimal investment threshold is $P^* = 0.61$. If the fuel price is below this, it is optimal to invest, otherwise it is optimal to wait. It can be seen in the figure that the value of the investment option, $F_s(P)$, is connected smoothly with the value of the plant minus the investment cost at the optimal investment threshold, as required by the value matching and smooth pasting conditions.

Figure 2 here

4 Optimal investment timing and choice of fuel

In this section we derive the solution to the original problem, where the investor must choose both the timing of the investment and the type of production technology. The opportunity to invest has a positive value, because it entails an option, but no obligation, to undertake the project. Even if it is not optimal to carry out the investment now, it may be so in the future. Since the fossil fuel price is the only source of uncertainty in our model, the value of the investment option is a function of P. We denote thus by $F(P)$ the value of the option to make an irreversible investment in either a fossil fuel or a biomass plant at any time in the future. Again, the value of such an option must, by an arbitrage argument, satisfy the differential equation:

$$\frac{1}{2}\sigma^2 P^2 F''(P) + (r-\delta) PF'(P) - rF(P) = 0.$$

The general solution to this is:

$$F(P) = E_1 P^{\beta_1} + E_2 P^{\beta_2}.$$

It is clear that the lower the fossil fuel price, the more attractive is the investment in the fossil fueled plant. On the other hand, it must be that the higher the fossil fuel price, the more attractive becomes the alternative biomass plant. The optimal solution to the investor's problem can thus be expressed as two threshold levels for the fossil fuel price. When the price is below the lower threshold, it is optimal to invest in the fossil fueled plant. On the other hand, when the price is above the higher threshold, it is optimal to invest in the biomass plant. Between the two trigger levels it is optimal to wait. We denote the lower threshold by P^G and the higher by P^R.

At the optimal investment thresholds, the value-matching and smooth-pasting conditions must again be satisfied. Since we now have two such levels, there are altogether four conditions. On the other hand, there are two free parameters (E_1 and E_2) and two threshold levels to solve. We have thus four unknowns and four equations. The boundary conditions are:

$$\begin{aligned} F\left(P^G\right) &= V_s\left(P^G\right) - I, \\ F'\left(P^G\right) &= V_s'\left(P^G\right), \\ F\left(P^R\right) &= V_R, \\ F'\left(P^R\right) &= 0. \end{aligned}$$

As mentioned in the previous section, it must be so that at the point where it is optimal to invest in the fossil fueled plant, it is also optimal to run the plant, i.e. $P^G < A$. Therefore the conditions are explicitly:

$$
\begin{aligned}
E_1\left(P^G\right)^{\beta_1}+E_2\left(P^G\right)^{\beta_2} &= \frac{A^{1-\beta_1}}{(\beta_1-\beta_2)}\left(\frac{\beta_2}{r}-\frac{\beta_2-1}{\delta}\right)\cdot \\
&\quad \left(P^G\right)^{\beta_1}+\frac{A}{r}-\frac{P^G}{\delta}-I, \qquad (12)\\
\beta_1 E_1\left(P^G\right)^{\beta_1-1}+\beta_2 E_2\left(P^G\right)^{\beta_2-1} &= \beta_1\frac{A^{1-\beta_1}}{(\beta_1-\beta_2)}\left(\frac{\beta_2}{r}-\frac{\beta_2-1}{\delta}\right)\cdot \\
&\quad \left(P^G\right)^{\beta_1-1}-\frac{1}{\delta}, \qquad (13)\\
E_1\left(P^R\right)^{\beta_1}+E_2\left(P^R\right)^{\beta_2} &= V_R, \qquad (14)\\
\beta_1 E_1\left(P^R\right)^{\beta_1-1}+\beta_2 E_2\left(P^R\right)^{\beta_2-1} &= 0. \qquad (15)
\end{aligned}
$$

These should be solved to get E_1, E_2, P^G and P^R. Equations (14) and (15) are linear in E_1 and E_2, and can be solved to get:

$$
\begin{aligned}
E_1 &= V_R\beta_2\frac{\left(P^R\right)^{-\beta_1}}{-\beta_1+\beta_2},\\
E_2 &= -\beta_1\left(P^R\right)^{-\beta_2}\frac{V_R}{-\beta_1+\beta_2}.
\end{aligned}
$$

Substituting these in (12) and (13) results in two equations for P^G and P^R that must be solved numerically:

$$
\begin{aligned}
0 &= V_R\frac{\beta_2}{(P^{R\beta_1})(-\beta_1+\beta_2)}\left(P^G\right)^{\beta_1}-\frac{\beta_1}{P^{R\beta_2}}\frac{V_R}{-\beta_1+\beta_2}\left(P^G\right)^{\beta_2}\\
&\quad +\frac{A}{(A^{\beta_1})(\beta_1-\beta_2)}\left(P^G\right)^{\beta_1}\left(-\frac{\beta_2}{r}+\frac{\beta_2}{\delta}-\frac{1}{\delta}\right)-\frac{A}{r}+\frac{P^G}{\delta}+I, \qquad (16)\\
0 &= \beta_1 V_R\frac{\beta_2}{(P^{R\beta_1})(-\beta_1+\beta_2)}\left(P^G\right)^{\beta_1-1}-\beta_2\frac{\beta_1}{P^{R\beta_2}}\frac{V_R}{-\beta_1+\beta_2}\left(P^G\right)^{\beta_2-1}\\
&\quad +\beta_1\frac{A}{(A^{\beta_1})(\beta_1-\beta_2)}\left(P^G\right)^{\beta_1-1}\left(-\frac{\beta_2}{r}+\frac{\beta_2}{\delta}-\frac{1}{\delta}\right)+\frac{1}{\delta}. \qquad (17)
\end{aligned}
$$

However, (16) and (17) are only necessary conditions for optimal P^G and P^R, and it turns out that there may be more than one solution to them. When solving numerically for the investment thresholds, some attention must be paid to checking the optimality of the result. An alternative way to solve the problem is to use only the value-matching conditions (12) and (14) to obtain E_1 and E_2 as functions of P^G and P^R, and then choose the threshold prices P^G and P^R in order to maximize the value of the investment option, $F(P)$.

Figure 3 illustrates the situation. This figure is obtained using parameter values that will be commented more in the next section. The threshold prices are $P^G = 4.28$ (euro/MWh) and $P^R = 6.32$. Between the thresholds, the value of the option to invest, $F(P)$, is greater than the net value of either of the projects, $V_s(P) - I$ and V_R for the fossil fuel and biomass plants, respectively. It can also be seen that the value matching and smooth pasting conditions are again satisfied at the optimal investment thresholds.

Figure 3 *here*

5 Numerical example

In this section we use the model to provide a numerical example in which an investor is considering to build a power production plant at some specific site. The investment decision involves two critical choices; the timing and the type of power plant. The timing can be chosen freely, but the plant is limited to be either natural gas or biomass fired. We focus on some comparative statics. More specifically, we look at how the degree of gas price uncertainty and the magnitude of investment subsidies to the biomass plant affect the optimal investment rule. The investment decision is based on a number of parameter values. The assumptions we have made about these values are described in the following subsection.

5.1 Parameter values

We assume that the risk-free interest rate is $r = 0.05$. Because we want to analyze how the degree of gas price uncertainty affects investment, it is most natural to assume that the investor is risk neutral, otherwise we would have to make assumptions about the risk-adjustments to use at different degrees of uncertainty. Due to the risk neutrality assumption, the risk-free interest rate is equal to the risk adjusted rate of return, i.e. $r = \mu$. The value for the expected growth rate of the price of natural gas is chosen to be $\alpha = 0.02$. This gives us $\delta = \mu - \alpha = 0.03$.

We further assume that the electricity price is 20 EUR/MWh. In order to relate this assumption to the real world, we observe that the average system spot price of electricity at the Nordpool power exchange so far this year (April 2002) is 20.23 EUR/MWh. In 2001 and 2000 the average prices were 23.15 EUR/MWh and 12.75 EUR/MWh, respectively.[9]

The parameter values more directly connected to the two alternative plants are based on Kosunen and Leino (1995), who provide cost data for different plants applicable in Finland. The actual data for the biomass plant is for a plant with a capacity of 150 MW, which uses wood chips as its input. The price of wood chips is reported to be 11.33 EUR/MWh. The plant has 60 employees working full time. The average yearly wage is set to be 42000 EUR per person. The fossil fuel plant is a natural gas fired plant with a capacity of 300 MW. This plant has 35 employees earning the same average wage. In our example we want to compare two alternative projects with the same production capacity. Because the data provided by Kosunen and Leino are for two plants of different capacity, we assume that the data for the biomass plant can be scaled up to represent a plant of 300 MW.

The investment cost of the natural gas fired plant is in the report estimated to be 173.7 mill. EUR. For the 150 MW biomass plant, the estimate is 137.17 mill.

[9] The numbers are taken from http//:www.nordpool.no.

EUR. We assume that doubling the capacity will, due to economies of scale, less than double the investment costs. Therefore, we assume the investment costs of the biomass plant to be 250 mill. EUR. We also assume that the 300 MW plant can be operated using the same number of personnel as the 150 MW plant. In addition, we assume that both types of plants have a fixed flow of other operation and maintenance costs equal to 2.5 mill. EUR/year. We assume that the plants can operate continuously during the year, i.e. 8760 hours per year, and that they have an infinite lifetime.[10] Thus, the plants have a production potential of 2.628 million MWh/year for ever. When operating, the revenue flow generated by a 300 MW plant in annual units is $A = 52.56$ mill. EUR.

Given these parameters the value of the biomass can be calculated. It turns out that the value is clearly negative: $V_R \approx -880$ mill. EUR. This illustrates how far such a plant is from being profitable at such a low electricity price.[11] For the biomass plant to constitute an alternative to the investor it must have a positive value. Therefore, we assume that the authorities provide an investment subsidy that makes the value of the biomass plant positive, more specifically $V_R = 50$ mill. EUR. Remember that, due to the assumptions of the model, this value contains no risk.

The gas fired plant has a fixed cost flow per year equal to 7.91 mill. EUR. This includes personnel costs (1.47 mill. EUR/year), other operation and maintenance costs (2.5 mill. EUR/year), and the fixed delivery cost of natural gas of 1094 EUR/MW/month. In the formulation we presented in section 2, the discounted sum of these costs must be included in the investment cost I.

Before performing the comparative statics analysis we make an illustrative solv-

[10] Kosunen and Leino (1995) use a time horizon of 20 years and run-time of 6500 hours per year in their cost estimates. We have made our assumptions to simplify the exposition. These simplifications do not affect the qualitative nature of the results.

[11] It should be noted that also the natural gas fired plant would have a clearly negative net present value at such a low electricity price if the natural gas price value is given a realistic current estimate.

ing of the model assuming that the volatility of the natural gas price process is $\sigma = 0.1$. We also assume that the price of natural gas is the only variable cost of the natural gas plant. The threshold prices for the two different investment alternatives are obtained by solving (16) and (17). From the specific parameter values assumed above we get a threshold price for the natural gas fired plant equal to $P^G = 8.39$ EUR/MWh, and the threshold price for the biomass plant equal to $P^R = 12.39$ EUR/MWh. However, these values correspond to the price of the quantity of natural gas needed to produce one MWh or electricity. Therefore, to report them in actual natural gas price units, as it is usually reported, one must remember that the energy content of 1 MWh of natural gas is not enough to produce 1MWh of electricity. Kosunen and Leino (1995) reports an "efficiency rate", meaning the amount of natural gas energy needed to produce 1 MWh of electricity, in the natural gas plant to be 1.96. Therefore, the actual natural gas price threshold levels are $P^G = 4.28$ EUR/MWh and $P^R = 6.32$ EUR/MWh.

This is illustrated graphically in figure 3, which was already commented in the previous section. V_R is the value of the biomass plant, $V_s(P) - I$ is the value of the natural gas plant less the investment cost, and $F(P)$ is the value of the investment option. The interpretation of the thresholds is that if the price of natural gas gets below 4.28 EUR/MWh, the investment in the natural gas plant is undertaken, while if the price rises above 6.32 EUR/MWh, the investment in the biomass plant is undertaken. For natural gas prices in the interval between these two thresholds, the option value of waiting is larger than the value of either of the investment alternatives.

It should be noticed that the threshold levels in this example are quite low. Kosunen and Leino (1995) use a static price estimate of 8.25 EUR/MWh when calculating the cost of producing electricity in natural gas plants. Therefore, the application of our model at that natural gas price level would suggest an immediate investment in the biomass plant. Behind this result is the low value for the electricity price that we used. This makes investment in a new natural gas plant unattractive, but on the other hand it leads us to the assumption of a very high investment subsidy for the biomass plant in order to make V_R positive and thus make biomass a relevant alternative. Therefore, the example would suggest that the prospects for the natural gas plant look so bad that it is optimal to invest in the heavily subsidized biomass plant.

5.2 Effect of uncertainty

The degree of uncertainty in the natural gas price development is difficult to estimate so it should be interesting to look at its effect on the investment problem. The result of solving the model for uncertainty levels in the interval between $\sigma = 0$ to 0.3 is shown in figure 4.

As can be seen in the figure, the threshold price for the natural gas plant, P_G, is falling within the whole interval. There are, in fact, two counteracting effects. On one hand, the increased volatility increases the value of the plant[12], which increases the attractiveness of investment at a given price level. On the other hand, the value of waiting is also increased, which decreases the attractiveness of an immediate investment. The figure shows that the latter effect dominates.

The threshold for the biomass plant, P_R, is rising quite steeply as uncertainty increases. This is because there are now two parallel effects: the increased volatility increases both the value of waiting and the value of the gas plant relative to the biomass plant, thus making investment in biomass plant less attractive.

The effect on the difference in the threshold levels, i.e. the interval within which it is optimal to wait, is thus increasing with the uncertainty. The high uncertainty about the price of natural gas increases the option value of waiting. It becomes more

[12] Because of the flexibility effect explained in connection of figure 1.

likely that there will be periods where the natural gas plant is very profitable due to a very low natural gas price, and thus it is optimal to wait for further information at a wide range of prices.

Figure 4 *here*

5.3 Effect of the value of the biomass plant

We have argued that the national authorities may want to subsidize investments in renewable energy production. In fact, this is actually being done in many countries.[13] The value of projects utilizing renewables can be significantly influenced by government subsidy schemes. Motivated by this, we have solved the model for different values of the biomass plant. Figure 5 shows the investment thresholds for the two alternative technologies corresponding to values of V_R from 0 to 300 million euro. The volatility of the natural gas price is fixed at $\sigma = 0.1$.

Figure 5 *here*

We see from the figure that the difference between the price thresholds decreases with increasing values of the renewable project. Thus, when the value of the renewable project is low, there is a wide interval of coal prices where it is optimal to wait. In the limit, where the value of the biomass plant decreases towards zero, the threshold level triggering investment in biomass plant increases towards infinity. This is because even a small option value of waiting is sufficient to exceed the value of the project. As long as the investor has to make an irreversible choice between the fossil fueled plant and the renewable project, the value of the biomass

[13] Norway may serve as an illustration as the Norwegian authorities provide investment subsidies to the building of wind turbines. In addition, a number of technologies utilising renewable energy sources enjoy exemption from a specific investment tax that are put on conventional technologies (Factsheet, 2001).

plant must be relatively high in order to exceed the option value of waiting for more information. This type of effects, induced by uncertainties associated with investment projects, should be taken into account when discussing the framework for governmental subsidization of environmentally friendly energy production.

6 Conclusions

A characteristic feature of energy production is that there are many alternative technologies available. Different technologies have different properties in terms of the cost structure and associated uncertainties. In this paper we have pointed out that the irreversibility in the choice of production technology, combined with uncertainties in the input prices, have important implications on the optimal investment behavior. We have formalized this by modeling an investor who is considering an energy production investment. The key assumption is that there are two alternative plant types that the investor must choose between: a fossil fuel and a biomass fired plant. The fossil fuel price is assumed to be stochastic, while the biomass price as well as the output price are assumed constant. The fact that the choice of the plant type is assumed to be irreversible leads to the result that the fossil fuel price development drives the timing of the investment and the eventual choice of the plant type. It is shown that it may be optimal to postpone the investment at a wide range of fossil fuel prices, because of the value of information about the future profitability of the fossil fueled plant gained by waiting. Increased input price volatility is demonstrated to widen this waiting range, while increased value of the biomass plant reduces it.

We have used a simplified characterization of the plant types. The main feature is that the fossil fueled plant relies on an input with a stochastic price and is allowed to shut-down whenever production is not profitable, while the biomass plant relies on a deterministic input price. This is enough to provide the main insights on the nature of the problem. However, for an actual valuation of potential plants and derivation of optimal investment rules in real cases, some features of the model would require refining. For example, the assumptions of constant biomass and output prices are clear simplifications. Also, some other stochastic process instead of the geometric Brownian motion for the fossil fuel price could be more appropriate. On the technological side, some shut-down and restarting costs could be realistic. However, to apply our methodology on such an accurately specified case with above mentioned refinements would require more tailored numerical solving methods. Salahor (1998), Bradley (1998), Laughton (1998), and Baker et al. (1998) set some general guidelines for applying modern asset pricing methods (that our model also represents) in real applications.

On the other hand, even if the model as presented in this paper is restricted to a specific setting, the general idea can be seen from a broader perspective. There are many other investment settings where an irreversible choice has to be made between alternative modes. Even more broadly, the irreversibility of the choice between different actions may appear in other kind of contexts. One example could be the optimal choice and timing of an policy regime against some environmental problem. If there are uncertainties specific to different policies, then this may induce a value of waiting for further information delaying the optimal timing to implement the policy.

There are some possible extensions to the model that could be analyzed with further work. One possibility would be to relax partly the irreversibility of the fuel choice. In reality, it is in some cases possible to change the input fuel of a plant with an additional investment. Allowing switching at a given investment cost would have some effect on the values of both plants, and would also have some effect on the investment thresholds. The solving would in principle be similar, but would consist of more boundary conditions and free parameters. On the other hand, in reality there are plants available that can utilize more than one fuel type. Introducing an additional plant that can use both fuel types, but requires a higher investment cost is another possible extension. This would lead to a larger number of different price regions, in each of which it would be optimal either to wait or to invest in one of the three different plants. The effect of the degree of uncertainty would be particularly relevant in this context.

From the point of view of economic theory, the obvious restriction of the model is that it does not account for competitive aspects. The assumption of constant output price isolates the firm under consideration completely from the actions of other firms. However, taking competitors into account would require dynamic modelling of the demand evolution. Assuming a stochastically evolving demand function, however, would add a second stochastic process to the model. Solving the optimal investment problem for a firm that takes the output price as an exogenous stochastic process in addition to stochastic input price would require solving a certain free boundary partial differential equation. Solving this numerically could, however, be an interesting future research topic. The results of Leahy (1993) and Baldursson and Karatzas (1997) would then make it possible to directly derive the rational expectations competitive equilibrium from the solution of the firm that sees an exogenous output price process. The resulting endogenous price process would characterize customers paying higher output prices at times when the "social" value of waiting would be high. The formalization of this intuition could be a possible future research topic.

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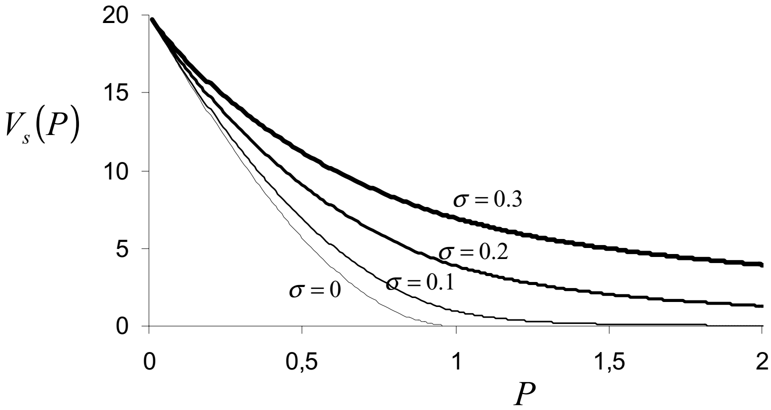

Figure 1: Value of the fossil fuel plant at different volatility levels.

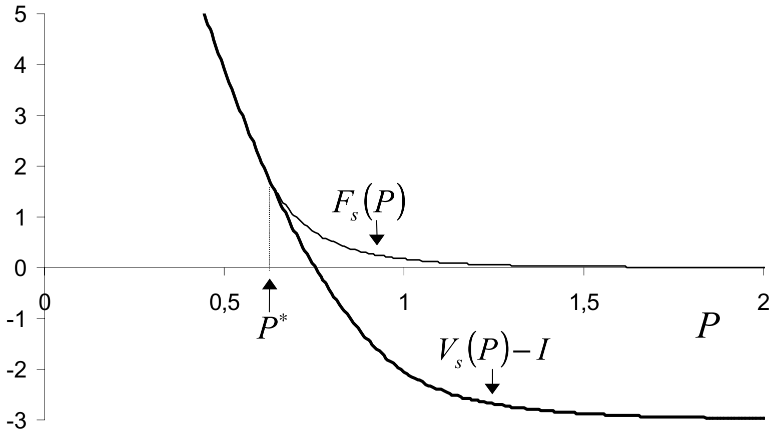

Figure 2: Optimal price threshold to invest in a fossil fuel plant.

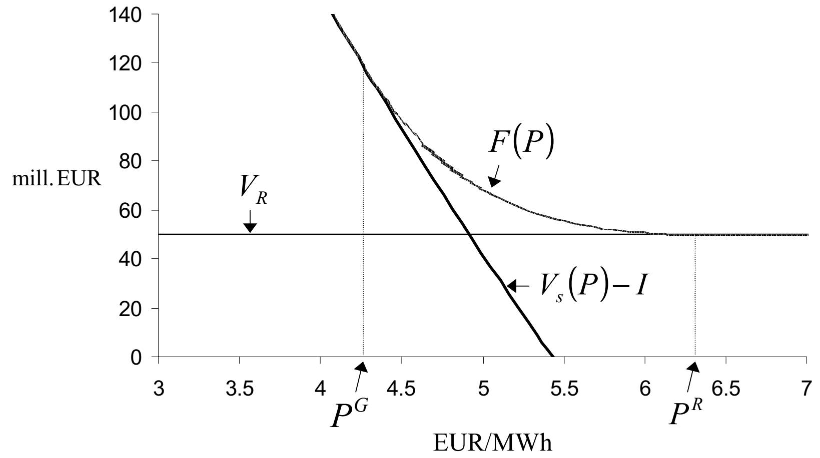

Figure 3: Optimal investment thresholds.

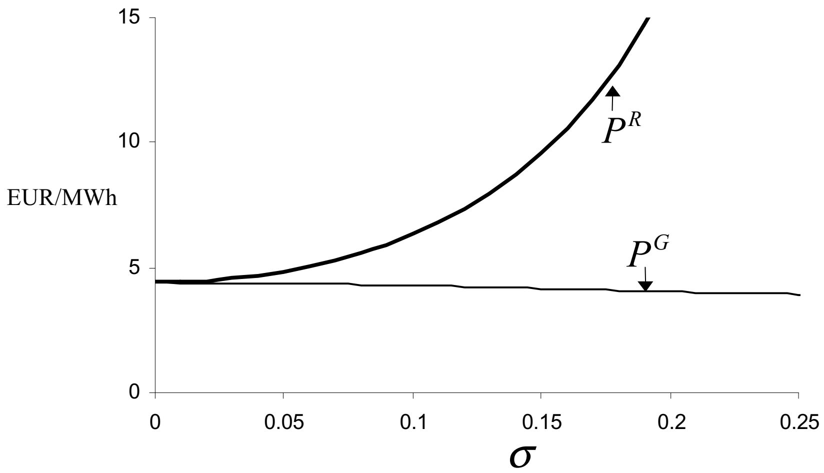

Figure 4: Investment thresholds as functions of fossil fuel price volatility.

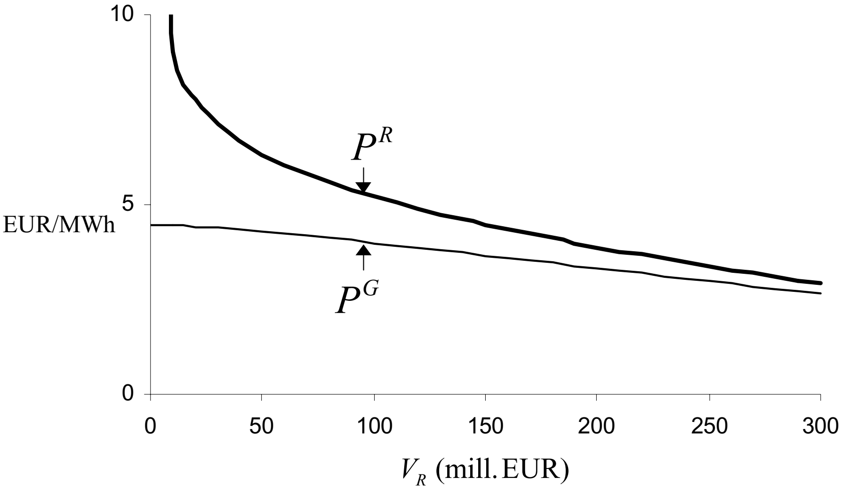

Figure 5: Investment thresholds as functions of the value of the renewable project.