

E C O N O M I C S B U L L E T I N

On Forecasting Recessions via Neural Nets

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Abstract

In this research, we employ artificial neural networks in conjunction with selected economic and financial variables to forecast recessions in Canada, France, Germany, Italy, Japan, UK, and USA. We model the relationship between selected economic and financial (indicator) variables and recessions 1-10 periods in future out-of-sample recursively. The out-of-sample forecasts from neural network models show that among the 10 models constructed from 7 indicator variables and their combinations that we investigate, the stock price index (index) and spread between bank rates and risk free rates (BRTB) are most likely candidate variables for possible forecasts of recessions 1-10 periods ahead for most countries.

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1. Introduction

Prediction of recessions is an important as well an arduous job that requires careful model selection and underlying estimation algorithms. Estralla and Mishkin (1998) and Qi (2001) employed nonlinear time series models and artificial neural networks for predicting USA recessions because extensive empirical research² reveals that business cycle fluctuations are asymmetric.

Neural Networks have been applied successfully in engineering, medical science, business and economics because of their pattern recognition ability. For example, Kuan and White (1994) discussed neural networks and their applications in economics. Swanson and White (1995, 1997 a, 1997) found usefulness of neural network models in economic time series data pertaining to interest rate unemployment rate and GNP etc. Hutchinson et al. (1994) employed neural networks in option pricing, and Garcia and Gencay (2000), and Qi and Midala (1999) in stock market predictions. Similarly, Vishwakerma (1995), and Qi (2001) used neural networks for business cycle turning points and recessions respectively.

Despite substantial methodological advances there have been surprisingly few attempts at furthering our understanding on forecasting using data other than that of the U.S. time series. Therefore, in this study we seek to forecast recessions in Canada, France, Germany, Italy, Japan, UK, and USA using selected economic and financial variables and artificial neural networks. Although neural networks are under heavy criticism to over fit the data, following Kiani (2005) we carefully construct our neural network architecture to eliminate over fitting.

In the remaining paper, section 2 provides a brief description of neural networks employed in this paper and a procedure to evaluate ex-ante out-of sample forecasts whereas section 3 presents empirical results. Finally, section 4 incorporates brief conclusions.

2. Neural Network Models

Artificial neural networks (ANN) are powerful computational devices that can learn from examples and generalize these learnings to solve problems never seen before (Riley and Cooper, 1990). ANN modeling approach is useful for forecasters, and researchers who employ it especially in problems where data is available but the data generating process and its underlying laws are unknown. ANN are treated as nonlinear, nonparametric statistical methods due to which these are independent of the distributions of the underlying data generating processes (White 1989). A general form of neural networks model employed in this study is presented in Equation 1.

$$f(x) = sig \left[\alpha_0 + \sum_{j=1}^n \alpha_j sig \left(\sum_{i=1}^k \beta_{ij} x_i + \beta_{0j} \right) \right] + \varepsilon \quad (1)$$

where,

n is the number of hidden nodes in neural networks

k is the number of explanatory variables in neural networks

² Neftici (1984), Brunner (1997), Beaudry and Koop (1993), and Ramsey and Rothman (1996), Bidarkota (2000), and Kiani and Bidarkota (2004) including many others.

$$\text{sig}(a) = 1/(1 + e^{-a}), \{\alpha_j; j = 0,1,\dots,n\}$$

α_{ij} represents a vector of parameters from hidden to output layer unit.

$\beta_{ij} (i = 1,\dots,k); j = 0,\dots,n)$ denotes a matrix of parameters from the input to the hidden layers units and ε is the error term. The error term ε can be made arbitrary small if sufficiently many explanatory variables are included and if n is chosen to be large enough. However, the model may overfit if n is too large in which in-sample errors can be made very small but out-of-sample errors may be large. To avoid a tendency of neural network models to over fit the data, the criteria for selecting neural network architecture due to Kiani (2005) is employed.

For a data set with T observations (periods), the out-of-sample forecast for a given horizon h is constructed by first estimating the ANN (Equation 1) with data through period $t < T$, so that the last observation used in the estimation is $[y_t, x_{t-h}]$. Given the parameters $(\hat{\alpha}_t^h, \hat{\beta}_t^h)$ estimated with these data, the h -horizon forecast (made from vantage point t) is computed from

$$\hat{y}_{t+h} = \text{sig} \left[\sum_{j=1}^2 \hat{\alpha}_{j,t}^h \text{sig} \left(\sum_{i=1}^k \hat{\beta}_{ij,t} x_{i,t} + \hat{\beta}_{0j,t}^k \right) \right] \quad (2)$$

In evaluating model forecasts it is useful to consider suitable benchmark, or naïve, models. For example, this research considers static benchmark as of Qi (2001) that is based on a benchmark prediction equal to the historical frequency of recession; however, a linear probability model and a dynamic benchmark forecast is also considered that is based on the historical frequency of a recession only up to the time a forecast is made.

As specified in Equation 1 and 2 (or with the benchmark forecasts), the ANN output is a continuous variable on the $[0,1]$ interval. In actual predictive applications and unless probability-type forecasts are sufficient, such continuous output must be mapped either to 1 (recession is predicted) or 0 (no recession) according to some mapping threshold (MT) rule. Typically, the MT is assumed to be 0.5, which is reasonable if the ANN continuous output is considered to be the probability of a recession.

A practical problem with discrete dependent variable models in actual prediction is that, due to its data infrequency, the event of interest is often missed in prediction. For example, the event of forecast in the present case is forecasting recessions. That is, a forecast of all 0's might be deemed an accurate forecast by most moment-based statistics (e.g., sum of squared errors (SSE), or root mean squared error (RMSE)) when only a few 1's actually occur. Assuming that missing the event of interest pertains to type I error and predicting it when it does not occur results in type II error, in such a situation, a decision maker penalizes a Type I error (in this paper, missing a recession) more heavily than a Type II error (predicting a recession when it does not happen). One way to deal with this issue after predictive models are estimated is to consider different MT's for mapping continuous output to a 0 or a 1. For example, the medical and financial professions sometimes compute receiver-operator curves (ROC), which make explicit the tradeoff between Type I and II errors associated with each possible MT (e.g., Reiser and Faraggi, 1997). Alternatively, one might consider the usual MT = 0.5 rule in generating predictions since it seems most consistent with SSE minimization, but follow up with an explicit consideration of different Type I and II error costs. The present research uses this approach because it appears to

be more intuitive than the ROC method. Therefore, to evaluate our ANN and benchmark predictions in a manner that allows for different relative weighting of Type I and II errors, “SCORE” is computed using following Equation:

$$SCORE = 1 - \frac{C \left[\sum_t G_t - \sum_t G_t I(Y_t \geq 0.5) \right] + \left[\sum_t (1 - G_t) - \sum_t (1 - G_t) I(Y_t < 0.5) \right]}{C \sum_t G_t + \sum_t (1 - G_t)} \quad (3)$$

where G_t is valued at 1 if period t is a recession, else 0, Y_t is the continuous prediction of the model, $I(\cdot)$ is an indicator equaling 1 if its argument is true and 0 if false, and C is a constant that weights Type I errors (missed recessions) C times as highly as Type II errors (missed non-recessions). The left- and right-bracketed terms in the numerator are the number of incorrect recession and non-recession predictions, respectively. To normalize the metric between 0 and 1, the denominator is the maximum error cost (i.e., if every prediction were wrong). Finally, the fraction is subtracted from 1 so that higher *SCORE*s denote better models (like R-squared).

2.1. Estimation Issues

In the present paper, genetic algorithm is employed as an estimation algorithm which is considered to be the most reliable algorithm to estimate any nonlinear functional form, but it is slower to any other algorithm that could be used to approximate neural networks. Due to relatively large number of parameters and the nonlinearity inherent in the neural network model specification, the objective function is unlikely to be globally convex and thus can have many local minima. To ensure that global minimum is obtained, at the beginning of each recursive estimation, the neural network model is estimated 4 times and based on the values of the best parameter vector further approximation of the neural network is done. The model that generates the smallest sum of square error is used to make out-of-sample forecasts and its parameter estimates are used as initial values in the recursive estimation of neural network using `fminsearch`, which is simplex algorithm for several hundred thousand iterations. The benefit is that fewer units are wasted and the network converges faster compared to purely random initial parameter values.

De Jong (1975) has done pioneering work in application of the genetic algorithm to mathematical optimization. Later, genetic algorithm was applied in the optimization problems pertaining to biology, engineering and operation research (Goldberg, 1989). The first economic application of genetic algorithm was implemented due to Axelord (1987) and later by Maromon, McGartten and Sergeant (1990), and Dorsey and Walter (1995).

3. Empirical Results

3.1. Data Sources

Quarterly data on long term bond yield, bank rates, risk free rates (Treasury bill rates for all countries and money market rates for Japan), seasonally adjusted money supply, industrial production and real GDP for Canada, France, Germany, Italy, Japan, UK, and USA was obtained from October 2004 version of the International Financial Statistic’s CD-ROM. The stock prices for Canada, France, Germany, Italy, Japan, UK, and USA are S&P TSX composite Index, CAC 40, DAX 30, TOTMKT, Nikkei 225, FTSE 100, and S&P 500 Composite Indexes respectively

that were obtained from the DataStream. Table 2 show additional detail on data series employed for all the series for Canada, France, Germany, Italy, Japan, UK, and USA.

This paper investigates economic and financial variables (if any) that could be used to predict recessions using artificial neural network models for Canada, France, Germany, Italy, Japan, UK, and USA. Therefore, we employ 7 single indicator variables synonymous to Estrella and Mishkin (1998), and Qi (2001) which consist of interest rate variables, individual macroeconomic variables, interest rate spreads, stock price indexes and monetary aggregates in addition to 3 combined indicator variables. Table 1 show codes assigned to single as well as combined indicator variables that are employed to approximate ANN forecasts for various forecast horizons in future for Canada, France, Germany, Italy, Japan, UK, and USA series. The structure of these single and combined indicator variables is shown in Table 1 wherein codes 1–7 are assigned to the single indicator variables whereas the codes 8–10 represent combined indicator variables.

3.2. Out-of-Sample Forecast Evaluation

The neural network and benchmark forecasts developed in this study are evaluated using the Equation 3 procedure. Table 3 reports the *SCORE*'s for all the indicator variables and their combinations employed where Type I and Type II errors are assumed to have the same cost ($C = 1$ in Equation 3) for Canada, France, Germany, Italy, Japan, UK, and USA. With this selection, Table 3 ranks models identical to using RMSE calculated using model predictions that have been mapped to 0 or 1. Given our choice of mapping threshold ($MT = 0.5$), the static and dynamic benchmarks discussed earlier have the same accuracy; so only a single benchmark is reported. However, linear probability model was not able to beat the benchmark so we do not report these results and focus only on the results that are based on the dynamic benchmark. Like Qi (2001), only a few of the single-indicator models generate forecasts that beat the naïve benchmark. Though, accurate forecasts seem to span more distant horizons than Qi (2001), this may have been due to less over-fitting in our 2-hidden-node models as compared to Qi's 3 hidden nodes. Unlike Qi (2001), however, combined indicator variables did not seem to improve our accuracy over single indicator variables.

To provide some indication of how our models might perform in a situation where Type I and II errors are assigned different costs, Table 4 reports the *SCORE*'s using an arbitrary selection of $C = 10$ in Equation 3 for Canada, France, Germany, Italy, Japan, UK, and USA. Now, many models beat the benchmark. This is encouraging in that it seems highly likely that policy makers would want to penalize a misclassified recession more than a misclassified non-recession. Moreover, such error-costing may not necessarily have to be incorporated directly into the model estimation procedure.

3.3. Estimation Results on Forecast of Future Recessions

This paper seeks to predict possibility of future recessions in Canada, France, Germany, Italy, Japan, UK, and USA using selected indicator variables and their combinations. Table 3 show out of sample forecast evaluations for all the series that are evaluated using *SCORE* measure of forecast accuracy. In this Table, the numbers shown at different forecast horizons are those models whose out-of-sample forecasts are approximated using single/combined indicator variables. These forecasts are evaluated using *SCORE* accuracy measure, and dynamic benchmark of recession forecasts wherein missing recessions and missing non-recessions are equally penalized selecting $C = 1$. All the models reported in Table 3 have greater *SCORE*

than the relevant dynamic benchmarks³ of recessions. The dynamic benchmark of recession (not tabulated) changes over time and is different for different forecast horizons.

In Table 3, columns 2–8 show the candidate variables for predicting recessions respectively for Canada, France, Germany, Italy, Japan, UK, and USA. In this Table, for example, column 1 rows 2 and 3 shows forecasts approximated from ANN models using indicator variable that bears code 5 is able to predict Canada recessions both at forecast horizons 2 and 3. Likewise, row 4 in column 2 show that the models encompassing variables that carry codes 4, 7, and 9 are able to predict Canada recessions four quarters ahead in future out of sample. The results for the remaining countries i.e. France, Germany, Italy, Japan, UK, and USA series are presented in a similar manner.

Table 4 show codes for the single and combined indicator variables that are constructed and represented in Table 1. The numbers shown in this Table show single and combined indicator variables whose forecasts are approximated using ANN that are evaluated using SCORE measure of forecast accuracy when missing recessions is penalized 10 times higher than missing non-recessions choosing $C = 10$. The choice of selecting such a penalty is arbitrary which gives policymakers an option to exercise this penalty to the level they might like depending on their situation. The codes representing various models constructed from single and combined indicator variables that are shown in Table 4 beat relevant dynamic benchmarks of recessions.

The summarized results shown in Table 3 reveal that the single indicator variables *index* and *BRTB* are the most likely candidate variables that predict recessions at different horizons in most of the series with over 80 percent SCORE forecast accuracy⁴. Similarly, the indicator variable *BondLT*, *RF*, and *MI* are less likely candidate variables to predict recessions at different horizons in a few of the series studied.

3.4. Discussions on Results

The single indicator variables *RF*, *BondLT*, *MI*, *Spread*, *BRTB* and *index* are the candidate variables for predicting recession forecasts at different forecast horizons for Canada, France, Germany, Italy, Japan, UK, and USA. A single indicator variable *IPG* is an exception that is unable to predict recessions in any of the countries studied. However, the combined indicator variables *RF&BondLT* and *BRTB&Spread* are also candidate variables for predicting recessions in most countries.

As proposed by Qi (2001), the present paper carefully selects the estimating algorithm and reduces the biased nodes in the neural networks for mitigating the over fitting issues associated with neural networks. This allows to predict recession 1-10 periods in future for most countries studied including USA as against Qi (2001) who predicted USA recessions 1–4 periods in future, and in addition, only 2 biased nodes are used in the neural networks as against Qi (2001)

³While considering the dynamic benchmark of recession forecasts we employ historical frequency of recessions only up to the point the forecast is made whereupon it is updated for the next forecast horizon and so on until the last observation of the series is attained.

⁴Due to space constraint Tables 3 and 4 show summary of the results that are approximated using single indicator variables and their combinations 1–10 periods in future. However, detail results can be obtained from the author upon request.

who used 3 nodes in her neural network. Moreover, for evaluating forecast accuracy of predictions from neural nets approximations, contrary to Qi (2001), dynamic benchmarks and linear probability models are employed for determining accuracy of predictions from the models.

4. Conclusions

Empirical literature reveals that business cycles are asymmetric, therefore, artificial neural networks that are highly flexible form of nonlinear models are employed to investigate predictability of future recessions (if any) for Canada, France, Germany, Italy, Japan, UK, and USA one to ten periods in future using a number of indicator variables and their combinations.

The out-of-sample results indicate that among the 7 indicator variables and their combinations that are investigated, the variable *index* and *BRTB* are the most likely candidate variables that predict recessions at different horizons in most of the countries with over 80 percent SCORE forecast accuracy. Similarly, the indicator variable *BondLT*, *RF*, and *MI* are less likely candidate variables to predict recessions at different horizons in a few of the countries one to ten periods in future. Qi (2001) concluded that US recessions are predictable using neural networks one to four periods in future although most researchers missed it except Lahiri and Wang (1996) and Filrado (1999). However, the neural network models employed in the present research are able to beat static benchmarks as of Qi (2001) as well as other benchmarks such as linear probability models. Moreover, we also employed dynamic benchmark of prediction and a measure (SCORE) that allows determining the forecast accuracy in percentage terms.

The present research employs neural networks that are carefully developed that encompass minimum possible biased nodes as was proposed by Qi (2001). That is why these models were able to predict recession 1-10 quarters in future as against Qi (2001) whose models predicted USA recessions 1-4 periods in future. However, future work in this area might need inclusion of additional variables and some new models in such type of studies.

5. References

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Table 1: Structure of Indicator Variables

Code	Variable	Description
<i>Single Indicator Variable Models</i>		
<i>Interest Rates</i>		
1	<i>RF</i>	Risk free rate (3-month Treasury bill market equivalent bond rate)
2	<i>BondLT</i>	Long term bond rate
<i>Individual Macroeconomics Variable</i>		
3	<i>IPG</i>	Industrial Production Growth
<i>Monetary Aggregates</i>		
4	<i>M1</i>	M1 Money Supply, seasonally adjusted
<i>Spread</i>		
5	<i>Spread</i>	10-year Treasury bond rate less risk free rate
6	<i>BRTB</i>	Bank rate less free rate
<i>Stock Prices</i>		
7	<i>Index</i>	Relevant Stock Market Index
<i>Combined Indicator Variable Models</i>		
8	<i>RF&BondLT</i>	<i>T-bill and BondLT</i>
9	<i>BRTB&Spread</i>	<i>BRTB and Spread</i>
10	<i>RF&Spread</i>	<i>Risk free rate and Spread</i>

Notes on Table 1

1. Column 1 shows codes assigned to various indicator variables, column 2 names of the indicator variables and the column 3 shows description of the single as well as combined indicator variables.
2. The Table comprises of seven single indicator variables that consists of two interest rate variables, two spread variables, individual macroeconomic variables, and the relevant stock prices in addition to three combined indicator variables.

Table 2: Quarterly Data for Selected Macro financial Variables

Variable	Canada	France	Germany	Italy	Japan	UK	USA
<i>Risk Free Rate</i>	1965:1-2004:2	1970:1-2004:2	1975:3-2004:4	1977:1-2004:2	1965:1-2004:2	1965:1-2004:2	1965:1-2004:2
<i>LT bond yield</i>	1965:1-2004:2	1965:1-2004:2	1965:1-2004:2	1965:1-2004:2	1966:4-2003:4	1965:4-2004:4	1965:1-2004:4
<i>Industrial Production</i>	1965:1-2004:2	1965:1-2004:2	1965:1-2004:2	1965:1-2004:2	1965:1-2004:2	1965:1-2004:2	1965:1-2004:2
<i>Money supply</i>	1965:1-2004:2	1978:1-1998:4	1965:1-1990:4	1974:2-1998:4	1965:1-2004:4	1969:3-2004:2	1965:1-2004:2
<i>Stock Price Index</i>	1969:1-2004:1	1987:3-2003:4	1965:1-2003:2	1973:1-2003:3	1965:1-2004:2	1978:2-2003:3	1965:1-2003:3
<i>Bank rates</i>	1971:1-2004:2	1965:1-2004:2	1977:3-2004:4	1982:1-2003:4	1965:1-2003:2	1966:3-2004:2	1965:1-2004:2
<i>GDP</i>	1966:1-2004:2	1965:1-2004:2	1965:1-2004:2	1965:1-2004:1	1965:1-2004:2	1965:1-2004:1	1965:1-2004:2

Table 3: Out-of-Sample Forecasts at Horizons 1–10 with *SCORE* Forecast Accuracy (When $C = 1$)

Forecast Horizons	Canada	France	Germany	Italy	Japan	UK	USA
Horizon 1	--	--		--		--	--
Horizon 2	5	--	2	--	6	--	--
Horizon 3	5	1	2	--	--	--	--
Horizon 4	4, 7, 9	--	8	--	--	--	--
Horizon 5	7	--	6	2	8	--	--
Horizon 6	--	--	--	--	--	--	--
Horizon 7	--	--	--	7	--	--	--
Horizon 8	--	--	7	--	--	--	6, 7
Horizon 9	--	--	--	--	--	7, 9	4, 5
Horizon 10	1	--	6	7	--	--	--

Notes on Table 3

1. The Table presents indicator variables that are candidate variables for forecasting recessions at different forecasting horizons in Canada, France, Germany, Italy, Japan, UK, and USA. The construction of these indicator variables is given in Table 1.
2. The results on recession forecasts along horizons 1–10 period ahead in future for each of the indicator variable and the combined indicator variables for all the countries are obtained from a number of neural network model approximations that are compared with static, dynamic and linear probability model.
3. The results presented in this Table are evaluated using SCORE measure of forecast accuracy.
4. In this Table for example, row 2 column 2 shows that the single indicator variable 5 (*Spread*) is a candidate variable for predicting Canada recessions two periods ahead) in future (at forecast horizon 2. The remaining number shown in this Table can be explained in a similar manner.
5. The SCORE forecast accuracy for this Table assumes that Type I error costs as much as Type II errors ($C=1$).
6. Based on MT rule described in section 2 above, the dynamic benchmark stays well above 0.5 for forecast horizons 1-10 in future.
7. The sign "--" indicates that no variable was able to be a candidate for predicting recessions at the given horizon.

Table 4: Out-of-Sample Forecasts at Horizons 1–10 with *SCORE* Forecast Accuracy (When $C = 10$)

Forecast Horizons	Canada	France	Germany	Italy	Japan	UK	USA
Horizon 1	3, 4, 5, 7		1, 10	1, 8, 10	4	2, 3, 5, 7	1, 4, 6, 8, 9,10
Horizon 2	1, 3, 4, 8, 9	4, 5	8	1, 9	4, 6	4, 8	1, 4, 5, 6 4, 5, 6, 7, 8, 9,
Horizon 3		1, 3, 5	2, 6, 8	1, 2, 8, 10	4, 5	2, 10	10
Horizon 4	2, 4, 9	5, 8	2, 6, 8, 9, 10	1,5, 10	2, 4	--	2, 3, 5, 6, 7, 10 2, 3, 4, 6, 7, 9,
Horizon 5	2, 4, 5, 7, 9	7	1, 2, 6, 8, 9, 10	1, 5, 6, 8, 9	8	5, 10	10 2, 3, 4, 5, 6,9,
Horizon 6	3, 7	1,	1, 2, 6, 8, 10 1, 2, 5, 6, 7, 8,	1, 5, 6, 8, 9	--	5	10 1, 2, 3, 4, 5, 6,
Horizon 7		1, 2, 5	10	1, 5,6, 8, 9	3, 8	4	7, 9, 10 1, 2, 3, 4, 5, 8,
Horizon 8	4	2, 3, 5	1, 4, 6, 7, 10	1,8, 9	2, 5	5, 6, 7, 8, 10	9, 10
Horizon 9	3, 7, 9, 10	3, 5,6	1, 6, 8, 9, 10	1, 2, 6, 10	2, 5, 8	5, 6, 7, 8, 9	1, 3, 4, 5, 6, 7,
Horizon 10	1, 2	3, 5	1, 2, 6, 7, 9, 10	6, 7	2, 8	1, 3, 5, 7, 9	1, 2, 3, 5, 6, 7, 8, 10

Notes on Table 4

1. See notes on Table 3.
2. Compared to Table 3, this Table shows that the indicator variables and their combinations that are candidate variables for predicting recessions in additional forecast horizons in future for all the countries. For example the number shown in column 2 row 1 show that the indicator variables 3 (*IPG*), 4(M_1), 5 (*Spread*), and 7 (*Index*) are the candidate variables for predicting Canada recessions at one-step ahead in future (forecast horizon 1). Since we use quarterly data each horizon equals one quarter.
3. The SCORE forecast accuracy for this Table assumes that Type I error costs as much as Type II errors ($C=10$). This type of error costing can help policymakers to penalize miss-classified recessions compared to miss-classified non-recessions or vice versa.