Using The Nelson and Siegel Model of The term Structure in Value at Risk Estimation

Pilar Abad
D. Economía Aplicada
Universidad de Vigo
D. Econometría, Estadística y Ec. Española
Universidad de Barcelona
pabad@uvigo.es

Sonia Benito
D. Fundamentos de Análisis Económico II
Universidad Complutense de Madrid
D. Análisis Económico II
UNED
soniabm@ccee.ucm.es

Over the past decade, no other tool in financial risk management has been used as much as Value at Risk (VaR). VaR is an estimate to determine how much a specific portfolio can lose within a given time period at a given confidence level. Nowadays, in order to improve the performance of VaR methodologies, researchers have suggested numerous modifications of traditional techniques. Following this tendency, this paper explores the use of the model proposed by Nelson and Siegel (with the aim to estimate the term structure of interest rate, TSIR) to implement a simulation to calculate the VaR of a fixed income portfolio. In this approach the dimension of the problem is reduced as the price of the portfolio depends on a vector of four parameters. Subsequently, we can use Monte Carlo simulation techniques to generate future scenarios in these parameters and use them to re-evaluate the portfolio. The resulting changes in portfolio value are arranged and the appropriate percentile is determined to provide the VaR estimate. Despite the fact that this approach theoretically facilitates the calculation of VaR on fixed income portfolios, we show that the PROBLEM in practise ignores price sensitivities. So this method cannot therefore be used to calculate VaR on fixed income portfolios.
Consequently, the purpose of this paper is to ask and to answer the following question: Can the model proposed by Nelson and Siegel to estimate term structure be used to calculate the VaR? Despite that theoretically it should be possible; we show this is not the case.

1. Value at Risk.
Value at Risk (VaR) has emerged as a major tool for measuring market risk, and nowadays it is used by banks and regulators. VaR is an estimate to show how much a certain portfolio can lose within a given time period, at a given confidence level. Despite its conceptual simplicity, the measurement of VaR is a statistical problem. This has given rise to substantial literature including statistical descriptions of VaR and examinations of different modelling issues and approaches.
Formally, a VaR measure is defined as the upper limit of the one-sided confidence interval:

$$\Pr\left[\Delta P_t(\tau) < VaR_t\right] = \alpha \qquad (1)$$

where α is the confidence level and $\Delta P_t(\tau)$ is the relative change in the portfolio value over the time horizon τ. Therefore, the VaR values are obtained from the probability distribution of changes in portfolio value. The existing models for calculating VaR mainly differ in the way they construct the probability density function of portfolio value. The traditional techniques of approximating this distribution are the parametric method (analytical), historical simulation (non-parametric), Monte Carlo simulations (stochastic simulation) and the stress-testing (scenario analysis).[1]

The parametric approach makes strong assumptions about the returns distribution. Most of the time it assumes that the returns are distributed normally. Below this assumption the changes in the portfolio value are distributed normally too. So, all we have to do to calculate VaR is to estimate the returns variance and covariance matrix in the portfolio. On the contrary, the non-parametric one doesn't offer strong assumptions on the distribution of the returns. The essence of historical simulation is to allow the data to speak for itself as far as possible, and use the recent empirical distribution of return to VaR estimate. The Monte Carlo approach simulates normally distributed future scenarios using historical variances in risk factor returns and uses them to re-evaluate the portfolio. Finally, a stress test examines the implications when and where the abnormal unexpected worst-case scenario does materialise.

2. Approach to VaR estimation in fixed income portfolios

In this paper we examine a new approach to VaR estimation for a fixed income portfolio. The approach discussed in this section is a Monte Carlo simulation model. Our approach is based on the Nelson and Siegel (1987) model, proposed to estimate the zero coupon yield curve. The yield curve or term structure of interest rates (TSIR) forms the basis for the valuation of all fixed income instruments. The price of a fixed income security can be calculated as the net present value of the stream of cashflows. Each cashflow has to be discounted using the zero coupon interest rate for the associated term to maturity. Therefore, the price of a portfolio of fixed income depends on a vector of zero coupon rates, which depends on a vector of parameters. Thus, the dimension of the problem is reduced by assuming that the price of the portfolio depends on a vector of parameters.

The Nelson and Siegel formulation specifies a parsimonious representation of the forward rate function given by:

$$r(t) = \beta_0 + \beta_1 e^{\frac{t}{\tau}} + \beta_2 \frac{t}{\tau} e^{\frac{t}{\tau}} \tag{2}$$

where $\beta = \{\beta_0, \beta_1, \beta_2, \tau\}$ are the parameters to be estimated. The forward rate function can be integrated to obtain the relevant spot rate function:

$$r(0,t_i) = \beta_0 + \beta_1 \frac{\tau}{t_i}(1 - e^{\frac{t_i}{\tau}}) + \beta_2 \frac{\tau}{t}\left[1 - e^{\frac{t_i}{\tau}}(1 + \frac{t_i}{\tau})\right] \tag{3}$$

Then the present value of an *m*-period bond making a series of coupon payments *c* every period and with redemption value *N* is:

$$P_t = \sum_{i=1}^{m-1} (c \times N) e^{-m_i r(0,m_i)} + (c+1) \times N e^{-m_k (r(0,m_k))} \tag{4}$$

Substituting equation (3) in equation (4) we can write the price of a bond as a function of four parameters β. In a similar way we can write the portfolio value as a function of these parameters.

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[1] See Jorion (2000) to get more information about Value at Risk.

To implement our approach, a statistical model of the parameters must be selected. Then, we can use Monte Carlo simulate techniques to generate future scenarios in these parameters and use them to re-evaluate the portfolio. The resulting changes in portfolio value are sorted and the appropriate percentile is determined to provide de VaR estimate.

3. The problem: Illustration

The proposed method to calculate VaR in fixed-income portfolios is as follows. Firstly the stochastic process is specified by which the four parameters appearing in the Nelson and Siegel model follow, and to which we use autoregressive integrated moving average process [2]. Secondly, we use the said models to simulate the parameters of the model in a one-day horizon and so obtaining simulations of the value of a bond portfolio in said horizon. The VaR(α%) will be the percentile α% of the distribution of changes in the portfolio value.

Although the aforementioned procedure allows us the extract reasonable simulations of the parameter changes in the Nelson and Siegel model, the simulated changes in the interest rates are not considered reasonable from a theoretical point of view as the unconditional moments of the simulated changes are totally different to those shown in the historical changes in the interest rates. Therefore, the procedure proposed in this essay leads us to inappropriate value simulations in a bond portfolio.

The problem emerges thus: the sensibility of interest rates to changes in the parameters of the Nelson and Siegel model is seen not to be constant in time and basically depends on the level of the interest rates. This leads us to observe that results of the TSIR are noticeably different when subject to similar changes in the parameters of the model. To demonstrate this we carried out the following exercise.

We selected the Spanish TSIR from the public debt market dated December 28, 2001 (see figure 1).[3] At this date, the zero coupon rates at periods of 1, 3, 5, and 10 years were 3.2%, 4.0%, 4.0% and 5.2% respectively. Using the aforementioned procedure we can simulate the values of the parameters of the Nelson and Siegel model at a one-day forecast and use said values to simulate the interest rates of the Spanish TSIR in the same forecast (see figure 1). On that day the simulated rates at 1, 3, 5, and 10 years were 2.9%, 3.0%, 3.2% and 3.9%. It can be observed that, in the short term (less than a year) coupon zero rates dropped by around 25 basic points and the rates at medium and long term dropped greater than 100 basic points.

Secondly we chose an estimated rates curve from January 1, 1999 (see figure 2). On this day the coupon zero rates in periods of 1, 3, 5 and 10-years were at respective levels of 2.9%, 3.1%, 3.4% and 4.0%. From the estimated parameters of January 1, 1999 and employing the changes in the simulated in the previous exercise we simulate a new temporary structure in a one period horizon (see figure 2). On this occasion, the short term coupon zero rates dropped around 30 basic points and in medium and long term between 30 and 60, which is noticeably lower than in the previous exercise. It is also interesting to see

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[2] To simulate the aleatory component of these models we have employed different methods; parametric simulation, supposing that estimated model residues follow a normal distribution and bootstrap techniques which consist of extracting aleatory realizations of the estimated historical residues. Additional in both cases also we have simulated supporting the structure of correlations of the residues.

[3] The TSIR for the secondary market for Spanish public debt was obtained from a zero coupon interest rate curve as proposed by Nelson and Siegel (1987).

that whilst on December 28, 2001, there is a change in the profile of the TSIR rates curve in contrast to that seen on the same day in January 1999 where the simulated betas produced a parallel movement of the TSIR.

Figure 1. Term Structure Interest Rate
(December 28, 2001)

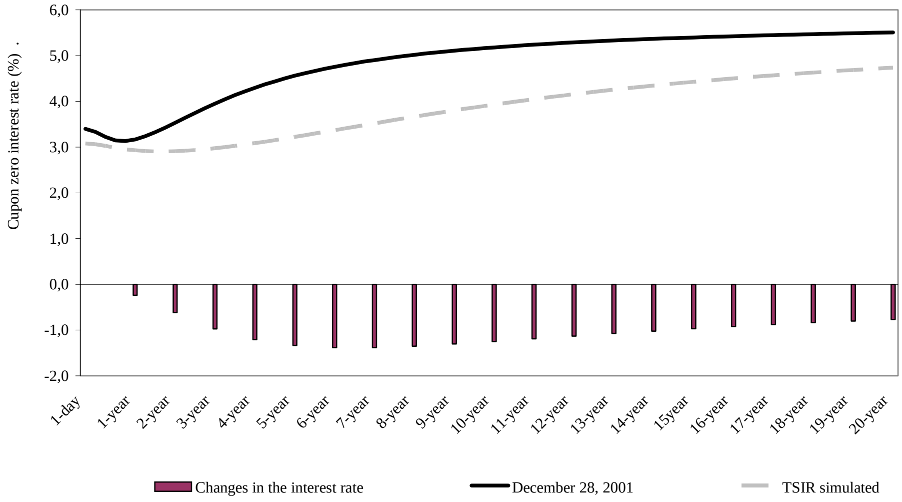
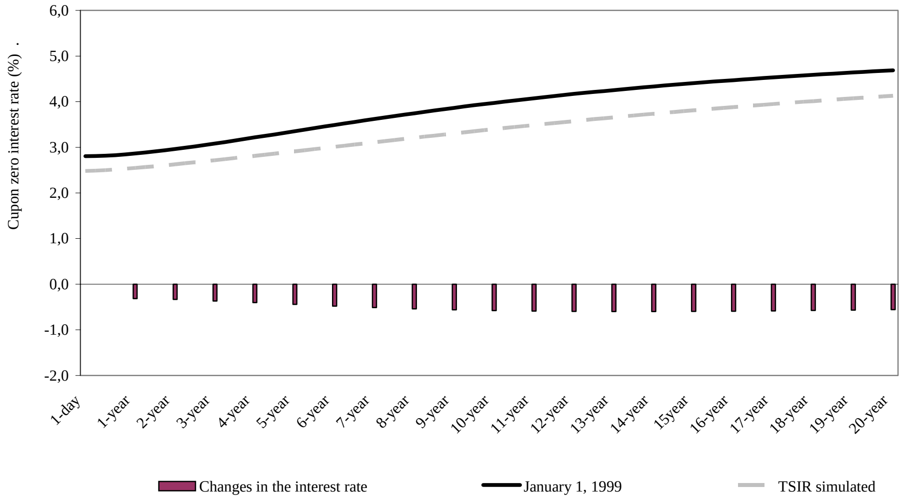


Figure 2. Term Structure Interest Rate
(January 1, 1999)



This exercise demonstrates an important fact. Similar changes in the parameters generate substantial changes in the STIR. This is to say that the answer from the TSIR when faced with changes in the parameters of the Nelson and Siegel model depend on both the level and the profile the yield curve adopts. This is the reason why it is not possible to use Monte Carlo simulation methods to calculate the VaR of a fixed-rate portfolio when running from a Nelson and Siegel model (1987).

4. Conclusion

Due to the increase in VaR methodology in reference to risk management, we have shown in this article that from a Nelson and Siegel model (1987) it is *not* possible to obtain a suitable measurement of VaR of a fixed-rate portfolio if its estimation is to employ bootstrapping methods. Hence, the reason for this is that the sensibility of the interest rates to changes in the parameters in the Nelson and Siegel model is not constant in time and basically depends on both the level and the profile the yield curve adopts. This causes the reaction of the TSIR to be greatly different when faced with similar changes in the parameters of the model. Therefore, portfolio value simulations obtained from simulated parameters are not deemed reasonable from a theoretical point of view.

References

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