Characterizing the Optimal Composition of Government Expenditures

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ABSTRACT

This paper extends the neoclassical growth model with productive public capital by including an infrastructure efficiency index, which is assumed to depend on a public choice variable, in particular, the share of public spending allocated to productive public consumption. A golden rule for the allocation of public expenditure between productive consumption and investment is specified. Under this framework, the observed path for the stock of infrastructures and the proposed efficiency index in the US economy during the last fifty years have been close to optimal: a lower stock of infrastructures has been accumulated, but it has been used more efficiently.

RESUMEN

Este artículo extiende el modelo neoclásico de crecimiento con capital público productivo mediante la incorporación de un índice de eficiencia de las infraestructuras. Este índice se supone dependiente de una variable de elección del gobierno, en concreto, el porcentaje del gasto público destinado a consumo público productivo. Se propone una regla de oro para la distribución del gasto público entre consumo productivo e inversión. Bajo este contexto, las sendas temporales observadas en los últimos cincuenta años en la economía estadounidense para el stock de infraestructuras y el índice de eficiencia propuesto han sido cercanas a las sendas óptimas:se ha acumulado un menor stock de infraestructuras, pero también se ha utilizado de forma más eficiente.

1 Introduction

Literature that focuses on the macroeconomic effects of public expenditure has attached a productive character to the public investment[1] component, because it accumulates public capital, which is supposed to increase the productivity of private production inputs. From a theoretical perspective, relevant papers in the area are, among others, Barro (1990), Futagami et al (1993), Turnovsky and Fischer (1995), Corsetti and Roubini (1996), Glomm and Ravikumar (1997), Turnovsky (1996), Cassou and Lansing (1998), Lansing (1998) or Judd (1999). From an empirical perspective, Aschauer (1989), Holt-Eakin (1988), Munnell (1992), Ford and Poret (1991) or García-Milá and McGuire (1992)[2].

However, a few authors have stressed the decisive role played on long-run economic growth, not only by the stock of public capital but also by the efficiency with which public capital[3] is used. Hulten (1996) and Aschauer (2000) are central papers in this research area. Both authors study the convergence experiences for a sample of low and middle income countries. They extend the Mankiw, Romer and Weil (1992) model by introducing an explicit infrastructure effectiveness variable. That is, the decisive variable in their convergence equation is not the 'nominal' infrastructure stock but the intrastructure stock measured in 'efficient units', which is defined as the product of the nominal stock and an efficiency index[4]. They build the efficiency index from a set of physical indicators (mainline faults per 100 telephone calls, electricity generation losses as a percent of total system output, the percentage of paved roads in good condition, or diesel locomotive availability as percent of the total).

Hulten (1996) and Aschauer (2000) show that a large part of the differential growth rate between countries can be attributed to the difference in the effective use of infrastructure resources.

On the other hand, Devarajan et al (1996) study the effects on economic growth of the composition of public expenditure, which is measured through the split of government expenditures between public consumption[5] and investment. They examine a panel of developing countries and conclude that the effects on growth of both components of public expenditure (consumption and investment) depend not only on their physical productivity (i.e., the output elasticity of each component, in a Cobb-Douglas technology), but also on their initial share on total public spending. They find a positive relationship between GDP growth and the ratio of consumption to total expenditure, while the investment component is, at the margin, negatively related to GDP growth. They explain this result as an evidence of an excesive investment share in these countries. The authors claim: "The widespread recommendation to increase public investment's share of

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[1]Using the IMF's definition, public investment covers payments for the production of new durable goods, ie, goods with a life of more than one year, also named public capital, or the purchase of existing ones.

[2]See Munnell (1992), Gramlich (1994), Glomm and Ravikumar (1997) or Pérez (2001) for a survey.

[3]We will use the term 'public capital' and 'infrastructures' indistinctly.

[4]This type of transformation is usual in growth literature.

[5]Public consumption includes expenditures such as wages and salaries of public employees or payments for the purchase of non-durable goods. It includes expenditures on: i) economic services (transport, communication, electricity, agriculture, etc); ii) social services (education, health, etc); iii) general government services (general public administration, defense, public order and safety, etc.).

the budget in developing countries could be misleading. Several components of current expenditure, such as operations and maintenance, may have higher rates of return than capital expenditure." (Devarajan et al (1996), page 338).

This paper links the works of Hulten (1996) and Aschauer (2000), on the one hand, and Devarajan *et al.* (1996) on the other, by analyzing: i) the role played in the aggregate production process by the efficiency with which infrastructures are used; ii) the role played in the efficient use of infrastructures by the public consumption expenditure allocated to the operation and maintenance of the infrastructure stock. The reason to consider 'productive' this type of public consumption is that it seems sensible to assume that the services offered by infrastructures to the private inputs is the result of a productive process in which some components of public investment and consumption take part together. For instance, a hospital or a school could not be productive without expenditures on doctors' or teachers' wages. The same way, a highway could become useless without expenditures that keep it in good conditions of use. The part of public consumption allocated to the operations and maintenance of existing infrastructures will be named 'productive public consumption'.

In our model, productive public consumption could substitute for public capital accumulation: the reallocation of public expenditure towards productive consumption leads to a lower infrastructure accumulation, but also to a more efficient use of it, beeing eventually able to increase the available level of infrastructure in efficiency units (or effective infrastructure).

The aim of this paper is to obtain the optimal allocation of public exhaustive spending (the portion of public expenditure included in GDP, that is, public consumption plus public investment) between investment, i.e., expenditures that accumulate physical infrastructures, and productive consumption, i.e., expenditures for operation and maintenance of infrastructures. From now on, the term *public spending* when used in the paper will make reference to exhaustive public spending, that is, excluding transfers.

Our paper differs from this previous literature in two fundamental issues:

i) With respect to Devarajan et al (1996), we specify a technology of production for effective infrastructure, where an efficiency index is included.

ii) Hulten (1996) and Aschauer (2000) construct an infrastructure efficiency index by aggregating physical indicators of public capital performance. Hence, their efficiency index is exogenous to public policy. On the contrary, the efficiency index in our paper depends on a public choice variable, the part of public consumption allocated to the operations and maintenance of existing infrastructures (i.e., productive public consumption), in relative terms to total public investment.

The normative results of the model are related to the north-american recent history (1952 to 2001), see figure 1. On the one hand, the observed ratio of nominal infrastructure over private capital has decreased uninterruptedly since the last sixties, and, many authors claimed that the observed decline contributed significantly to the slowdown in US labor productivity growth during the seventies and eighties. On the other hand, the realized path for the infrastructure efficiency index defined in the paper, which is an increasing function of the share of public spending allocated to productive consumption, increased significantly after 1968, as a consequence of the reallocation of public spending in favor of productive consumption. Under the normative results of the model we could conclude that

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the more efficient use of infrastructures after 1968 has partly compensated the decline in the stock of nominal infrastructures. As a consequence, the observed decline in the ratio of nominal infrastructures over private capital for the US economy would not explain, by itself, the productivity slowdown process. Using a very different framework, Cassou and Lansing (1998) also conclude that the decline in that ratio explains only a minor part of the productivity slowdown.

[**INSERT FIGURE 1**]

In fact, we show that the observed evolution in figure 1 for the US economy is consistent with the normative results of the model. The increasing path observed in the efficiency index, together with the declining ratio of nominal infrastructures to private capital, is compatible with the transition towards an optimal allocation of government spending between investment and productive consumption.

The main contributions of the paper are:

i) Extending the neoclassical growth model with productive public capital by including an infrastructure efficiency index, which is assumed to depend on a public choice variable, in particular, the share of public spending allocated to productive public consumption.

ii) Specifying a golden rule for the allocation of public expenditure between productive consumption and investment.

iii) To show that, under this framework, the observed path for the stock of infrastructures and the efficiency index in the US economy during the last fifty years have been close to optimal.

Section 2 presents the model. Section 3 discusses the calibration of model parameters. Section 4 computes the competitive equilibrium, and defines the static golden rule for the public spending composition. Section 5 analyzes the model transition after a permanent reallocation of government spending. Section 6 defines the golden rule for the composition of government expenditures taking into account the transitional dynamics of the model (we will refer to this as 'dynamic golden rule'), and compare it to the US ratio of infrastructures to private capital. Conclusions are included in section 7.

2 Model

A version of the neoclassical growth model is used, where productive public capital is included. The economy consists in a government, a representative firm and a representative household. The only produced good can be used as a consumption good or as an investment good.

Firms and households act competitively. Government behaviour is summarized by three ratios: i) percentage of public expenditure allocated to productive consumption; ii) percentage of public expenditure on output; iii) tax rate on total income.

2.1 Households

Households make consumption-investment decisions so as to maximize their lifetime utility. The standard isoelastic form for the single period utility function is used. Leisure is

not included in the utility function[6], implying an inelastic household's labor supply which is normalized to unity.

The agent finances his/her consumption (c_t) and investment (i_t) with the labor and capital incomes received from the firm as a payment for his/her labor supply (n_t) and the renting of his/her capital stock (k_{t-1}). The government establishes a uniform and constant tax rate (τ) on both types of income, as well as a certain amount of lump sum transfers (TR_t).

The optimization problem is, therefore:

$$\underset{\{c_t, n_t, k_t\}_{t=0}^{\infty}}{Max} \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma} - 1}{1-\sigma} \right) \tag{1}$$

$$s.t. \quad :$$

$$c_t + i_t \leq (1-\tau)\left[w_t n_t + r_t k_{t-1}\right] + TR_t \ , \tag{2}$$

$$i_t = k_t - (1-\delta_k)k_{t-1} \ , \tag{3}$$

$$k_0 \ given \ ,$$

where w_t is the wage, r_t is the return on capital renting, $\delta_k \in (0,1)$ is the private capital depreciation rate, $\beta \in (0,1)$ is the rate of time preference, and $1/\sigma$ is the intertemporal elasticity of substitution. Equation (3) is the law of motion for private capital, which is predetermined in period t to the value reached by the end of period $t-1$.

Household takes taxes, transfers, wages and interest rate as given, because all of them are determined out of his/her control[7]. Households are the owner of firms, but dividends are not included in the individual budget constraint because firms' profits are zero in equilibrium.

2.2 Government

The public sector raises taxes on households' labor and capital incomes, and uses the revenues to finance public expenditures, which are determined as a constant percentage of aggregate output.

Tax revenues and public expenditures are given by (in aggregate per capita terms):

$$T_t = \tau \ (w_t N_t + r_t K_{t-1}) \ ; \ \tau \in (0,1) \tag{4}$$

$$G_t = \gamma \ Y_t \ ; \ \gamma \in (0,1) \tag{5}$$

where T_t and G_t are, respectively, tax revenues and public spending; Y_t is aggregate output; $w_t N_t$ and $r_t K_{t-1}$ are labor and capital incomes; τ is the tax rate on income and γ is the ratio of public expenditure to output.

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[6]The aim is to isolate, in the policy experiments, the welfare gains due to increments in private consumption.

[7]The competitive household is negligible with respect to the economy size, and hence, is not able to alter the equilibrium wage or interest rate levels; on the other hand, the tax rate and transfers are chosen by the government.

Transfers are determined to guarantee that the budget balances in every period:

$$TR_t = T_t - G_t . \tag{6}$$

Public expenditures are allocated to: i) expenditures for accumulation of physical infrastructure (investment, Ig_t), ii) consumption expenditures for operation and maintenance of physical infrastructure (productive consumption, Cg_t). In per capita aggregate terms:

$$Cg_t = \Phi_t\, G_t = \Phi_t\, \gamma\, Y_t\ ;\ \ \Phi_t \in (0,1) \tag{7}$$
$$Ig_t = (1 - \Phi_t)\, G_t = (1 - \Phi_t)\, \gamma\, Y_t\ , \tag{8}$$

where Φ_t is the portion of public spending allocated to productive consumption; from now on, we will refer to variable Φ_t as public expenditure composition. Such composition is time-varying, as the $t-$index denotes. Both types of public spending have a productive nature.

The law of motion for the nominal infrastructure stock, in per capita aggregate terms, is:

$$Kg_t = (1 - \delta_g)\, Kg_{t-1} + Ig_t\ , \tag{9}$$

where $\delta_g \in (0,1)$ is the infrastructure depreciation rate and Kg_{t-1} is the nominal infrastructure stock accumulated by the end of period $t - 1$.

2.3   Firms

The single firm in the economy uses three types of productive inputs: i) private capital stock, rented to households at a unit cost of r_t, ii) labor, obtained from households at a wage w_t, and iii) infrastructure stock in efficiency units, obtained from the government without any direct remuneration.

The productive role of infrastructures, from a theoretical and an empirical perspective, has been extensively analyzed. For example, from a theoretical point of view, Barro (1990), Futagami et al (1993), Baxter and King (1993), Ambler and Paquet (1996), Cassou and Lansing (1998), or Lansing (1998).

However, several empirical papers have also presented evidence of the important role played by the efficiency with which nominal infrastructures are used (see Hulten (1996) and Aschauer (2000)). This paper proposes a theoretical framework which is consistent with this empirical evidence.

The firm maximizes profits every period:

$$\underset{\{N_t, K_{t-1}\}}{Max}\ Y_t - w_t N_t - r_t K_{t-1} \tag{10}$$
$$s.t.\ :\ Y_t = N_t^{1-\alpha}\, K_{t-1}^{\alpha}\, E_t^{\alpha_g}\ , \tag{11}$$

where Y_t is aggregate output, N_t and K_{t-1} are the firms' labor and capital demands, for which they pay a wage (w_t) and a rental rate (r_t), and E_t is the available effective

infrastructure level in period t. The production technology is assumed to be of Cobb-Douglas[8] type, with constant returns on private factors (because no direct remuneration for public productive activity is assumed, so that labor and capital rents exhaust all national income).

On the other hand, decreasing returns on capital stocks are assumed:

$$\alpha + \alpha_g < 1$$

which exclude the possibility of endogenous growth. Calibration supports this assumption which is not crucial as long as the analysis is focused in the study of ratios.

Next a functional form for the effective infrastructure level is proposed, according to the following points.

First, like Hulten (1996) or Aschauer (2000), effective infrastructures are obtained by multiplying nominal infrastructures by an efficiency index, that is:

$$E_t = \Psi_t \, Kg_{t-1} \; . \tag{12}$$

where Ψ_t is the index of infrastructure efficiency, Kg_{t-1} is per capita aggregate stock of nominal infrastructure which is available for production in period t (according to law of motion (9))[9]. Note that nominal infrastructure (Kg_{t-1}) is observable but the efficiency index (Ψ_t) is not observable and, hence, neither the efficient infrastructure (E_t).

Hulten (1996) or Aschauer (2000) do not identify a theoretical motive for the differences in countries' efficiency index, that is, they treat such index as exogenous to the governments choice. On the contrary, we assume that the efficiency index depends on productive public consumption, i.e., the part of public expenditures employed in the use and maintenance of the physical infrastructures. More precisely, we assume that the services offered by the infrastructures to the private inputs are the result of a productive process in which investment and productive consumption take part together. For example, school buildings (investment) and teachers' wages (consumption) are both necessary for educational services.

However, it seems more adequate to assume that the efficiency effect of productive consumption depends on the relative size of consumption with respect to the investment flow (Cg_t/Ig_t). Equivalently, the ratio of consumption to exhaustive expenditure (Cg_t/G_t) results appropiate.

Nevertheless, the ratio Cg_t/G_t, previously denoted by Φ_t, is more convenient because it falls between 0 and 1, whilst the ratio Cg_t/Ig_t is not bounded. See **figure 2,** where similarity between the time paths for both ratios is displayed.

[**INSERT FIGURE 2**]

So, the efficiency function proposed is:

$$\Psi_t = \Psi(Cg_t/G_t) = \Psi(\Phi_t) \; . \tag{13}$$

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[8]This assumption is standard in the literature, and it captures the fact that labor's share of output in many observed economies has remained relatively constant over time.

[9]Specifying Kg_{t-1} as a per capita quantity can be viewed as the incorporation of an implicit congestion effect associated with the number of firms, which is one single representative firm here (See Barro and Sala-i-Martín (1992)).

An efficiency index between 0 and 1 (where 1 denotes maximum efficiency), has the advantage that public capital is not overvalued with respect to private capital (whose effective value always coincides with the nominal value). The functional form proposed is:

$$\Psi_t = \Psi(\Phi_t) = \left(\frac{e^{(\Phi_t)} - 1}{e - 1} \right)^{\lambda} , \tag{14}$$

where $\lambda \geq 0$ is a parameter that makes possible modelling two types of public consumption:

- If $\lambda = 0$: $\Psi_t = 1$. This would be the standard case in which effective infrastructures coincide with nominal infrastructures; this would be the usual model with non productive public consumption. In this case, a reallocation of public spending in favor of consumption just crowds out public investment, leading to welfare losses.

- If $\lambda > 0$: $\Psi_t \to 0$ when $\Phi_t \to 0$ and $\Psi_t \to 1$ when $\Phi_t \to 1$. In this case, a reallocation of public spending towards consumption ($\Delta\Phi_t > 0$) induces two opposite effects on aggregate production: i) a negative effect created by the decline in nominal infrastructures accumulation, the 'crowding-out effect' of productive consumption (see equation 8); and simultaneously, ii) a positive effect created by the more efficient use of the available nominal infrastructure stock, the 'efficiency effect' of productive consumption (see equation 12). Therefore, under this assumption, the net effect of the reallocation of public spending on aggregate production is undetermined in advance.

2.3.1 Interpreting the effective infrastructures function

Taking (12) and (14) together, infrastructures in efficiency units are:

$$E_t = E(\underset{+}{\Phi_t}, \underset{+}{Kg_{t-1}}) = \left(\frac{e^{(\Phi_t)} - 1}{e - 1} \right)^{\lambda} Kg_{t-1} , \tag{15}$$

that can be interpreted as the production technology for a public good, in which both public consumption and public investment are necessary. This formulation implies that productive public consumption may act as a partial substitute for public capital: a reallocation of public spending in favor of productive consumption rises the efficiency index, but it also lowers the stock of infrastructures; however, it does not necessarily reduce the efficient infrastructure stock, and could even increase it.

From (11) and (15), the aggregate production function is:

$$Y_t = N_t^{1-\alpha} \ K_{t-1}^{\alpha} \ (\Psi(\Phi_t) \ Kg_{t-1})^{\alpha_g} , \tag{16}$$

that verifies the standard conditions of cuasi-concavity, positive and decreasing inputs marginal productivity under α, $\alpha^g \in (0,1)$.

Note that

$$MP_{Kg} = \frac{\partial Y}{\partial Kg} = \underset{+}{f(\Phi_t)} \ \Rightarrow \frac{\partial MP_{Kg}}{\partial \Phi_t} > 0,$$

capturing the efficiency effect of productive public consumption.

This production function (16) can be interpreted as the reduced form of a two-stage productive process. In the first stage, a public good E_t is obtained by using technology (15). In the second stage, the resulting outcome is incorporated into the aggregate production technology (11).

The Ψ_t function can be seen as a transformation of Aschauer's efficiency index[10]. However, using Aschauer's function would imply that $\Psi_t \geq 1$ whenever $\Phi_t \geq 0$, an undesirable property in our framework, and our normalization avoids this inconvenience.

On the other hand, our model resembles the one in Devarajan et al (1996), because both include public consumption and public investment in the aggregate production function[11]. However, our model differs from Devarajan et al in two fundamental respects: i) we formulate an efficiency index, where the role of productive public consumption is incorporated; and ii) Devarajan's optimal composition of public expenditure depends on the output elasticity of public capital, whose value is rather uncertain because a wide range of empirical estimates exists; on the contrary, the optimal composition of public expenditure here is not related to such elasticity, as will be shown later.

Note that public consumption has not been included in the consumer's utility function (as opposed to Barro (1990), Lee (1992), Lau (1995), Turnovsky (1996), Ambler and Paquet (1996), Lansing (1998) or Judd (1999), among others). Althought the welfare effects of several public consumption components is clear (education, health), the purpose of using this formulation is to isolate the welfare effects of a reallocation of public spending towards productive consumption, as a consequence of the productive channel proposed in the paper.

3   Calibration

Parameters are calibrated to replicate some empirical characteristics of the US economy. Data are annual for the period 1952-2001. They have been obtained from the National Income and Product Accounts and the 'Fixed reproducible tangible wealth' data from the Bureau of Economic Analysis (US Department of Commerce).

To calibrate the composition of public expenditure included in the infrastructures efficiency index, we have collected those components of the NIPA's public consumption

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[10]Aschauer (2000) formulates the infrastructures efficiency index as:

$$\theta = \frac{E}{Kg} = e^{(\epsilon \, Eff)}$$

where Eff is an standarized version of the efficiency index in Hulten (1996), built from a set of physical indicators of efficiency, and ϵ is estimated parameter for the countries in the sample. Parameter ϵ is equivalent to the λ parameter in our model, and the empirical Aschauer's indicator Eff reminds the composition ratio Φ.

[11]In their Cobb-Douglas version, aggregate output in Devarajan et al (1996) is given by:

$$y = k^{\alpha} \, g_1^{\beta} \, g_2^{\gamma}$$

where k is the private capital stock, g_1 is public investment and g_2 is current consumption.

and investment that previous literature has related to the aggregate production process through a variety of channels[12]:

i) By enhancing human capital accumulation, like Mankiw, Romer and Weil (1992) or Barro (1989, 1991), or by increasing labor supply efficiency, like Glomm and Ravikumar (1997). The NIPA's consumption and investment components that could be assimilated to these functions are those allocated to 'health' and 'education' functions;

ii) By acting as productive factors complementary to private factors. This would be the case for productive infrastructures in strict sense, according to definitions in Aschauer (1989) or Easterly and Rebelo (1993). The NIPA's consumption and investment components that could be included in this function are those allocated to the 'economic affairs' function[13];

iii) By protecting property rights, according to Barro and Sala-i-Martin (1995) or Hall and Jones (1996, 1998). The NIPA's consumption and investment components that correspond to this function are those allocated to 'general public service' and 'public order and safety' functions.

Therefore, calibrated productive public consumption is the amount of NIPA's public consumption allocated to 'education', 'health', 'economic affairs', 'general public service' and 'public order and safety'. Calibrated investment is the amount of NIPA's public investment allocated to the enumerated functions. The addition of productive consumption plus investment yields the amount of public expenditures. The composition of public expenditure is characterized by the percentage of productive consumption over public expenditures.

Some empirical research has offered evidence supporting the assumption of a positive effect on growth from the three types of public consumption enumerated above. First, Devarajan et al (1996) find that total public spending allocated to education, health and transport is productive for those countries with a large ratio of current public consumption over total spending. They explain this result as an evidence that these economic functions are productive for those governments that use public capital in an efficient manner. Kneller et al (1999) find productive effects for the total public spending on education, health, transport and communications and general services (administration, justice, defense, etc.), once the negative effects of their financing through taxes is taken into account. Pérez (2001), by using bivariate Autorregresive Vectors with annual american data (1952-1996), finds a positive relationship between the rate of GDP-growth and the rate of growth for the three types of public consumption detailed in points i)-iii).

Private and public capital stocks have been obtained from the 'Fixed reproducible tangible wealth' data from the BEA (yearend estimates in 'chained (1996) dollars').

Private capital includes non residential fixed assets, in per capita terms. On the other hand, nominal public capital (or infrastructures) includes government owned equipment and nonresidential structures, excluding military components, in per capita terms. This

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[12]See Steven Lin (1994, page 83) for a similar list of the economic functions by which government activity could act on the aggregate production through its interaction with the private sector. Current public consumption in Devarajan et al (1996, page 323) also includes a similar group of economic functions to the ones enumerated here.

[13]Which includes: General economic and labor affairs, Agriculture, Energy, Natural resources, Transportation, Postal service.

same definition is used, for example, by Cassou and Lansing (1998). Private and public capital are net of depreciation.

Depreciation rates for private and public capital are calibrated from the law of motion for each capital stock. The resulting depreciation rates for private capital (δ_k) and public capital (δ_g) are, respectively, 0.1 and 0.04. The output elasticity for private capital (α) is calibrated as the percentage of GNP that remunerates private capital (following Cooley and Prescott (1995)[14]), obtaining a level of 0.34, which is standard in the literature. By emboding the calibrated levels of α and the private capital to output ratio ($K/Y = 2.1$) into the household's intertemporal first order condition (in the stationary steady state), the discount parameter is obtained (β=0.98).

There exists a wide range of empirical estimates for the output elasticity of public capital (α_g). Estimates go from near zero (Ratner (1983), Aaron (1990) or Tatom (1991)) to near 0.4 (Aschauer (1989), Holtz-Eakin (1988), Munell (1990)), with some intermediate levels (Finn (1993) estimates a value of 0.16). Hence, we have repeated the normative analysis for different values of α_g: 0.32, 0.16 y 0.08, covering more or less the whole range. It will be shown that the public spending optimal composition results are not affected by the elasticity parameter, supporting the robustness of the normative conclusions.

Because the sum of elasticities for the cumulative factors is lower than unity ($\alpha + \alpha_g < 1$), whatever the calibration for α_g, the model does not exhibit endogenous growth. Anyway, this is not a limitation here because the analysis is centered in the study of ratios: Cg/G, Kg/K and E/K.

The tax rate is calibrated as the ratio of total public tax revenues (personal taxes, indirect business taxes, taxes on business profits and contributions to social insurance) to GNP for the US economy. This definition seems appropiate to capture the only tax defined in the model, which affects agent's total rents, those obtained from labor and capital renting. The resulting tax rate (τ) is 0.28.

The public spending composition (Cg_t/G_t ratio) path, displays two clearly differenced sample levels (see figure 3).

[INSERT FIGURE 3]

The mean value for the first sample, from 1952 to 1968, is 0.74, while for the second sample, from 1975 to 2001, is 0.83. Parameter λ will be calibrated in order to be consistent with the observed values for the composition ratio, assuming an optimizing government behaviour.

The ratio of public spending to GDP (γ) is calibrated so that the nominal infrastructures to private capital ratio (Kg/K) replicates the mean level observed for the US economy in the period 1952 to 1968 (Kg/K=0.69), before the public spending reallocation took place. The resulting value for γ is 0.216.

Finally, the persistence parameter for the public spending composition ratio is also calibrated. It will be used to characterize the transition dynamics of the model, as well as to obtain the dynamic golden rule for the composition of public spending. The persistence parameter (ρ^Φ) is computed from the first order autorregresive parameter for the ciclical

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[14]The computation of α is included in a technical appendix that is available from the author upon request.

component for the US composition ratio, obtained from the original series by using the Hodrick-Prescott filter. The calibrated value for this persistence parameter is 0.6 (with annual data).

We have performed a sensitivity analysis for $\alpha_g, \rho^\Phi, \sigma, \alpha, \tau, \gamma$.

4   Static golden rule

4.1   Competitive Equilibrium

Let $\Pi = \{\gamma, \tau, \{\Phi_t\}_{t=0}^\infty\}$ be an exogenously given public policy. A competitive equilibrium, given Π, is defined as the set of allocations $\{C_t, N_t, K_t\}_{t=0}^\infty$, together with the set of prices $\{w_t, r_t\}_{t=0}^\infty$ guaranteing:

i) $\{C_t, N_t, K_t\}_{t=0}^\infty$ solve the household's optimization problem, given $\{\tau, \{w_t, r_t, TR_t\}_{t=0}^\infty\}$.

ii) $\{N_t, K_{t-1}\}_{t=0}^\infty$ solve the firm's optimization problem, given $\{w_t, r_t\}_{t=0}^\infty$.

iii) The government budget constraint and, iv) the aggregate resources aggregate constraint, are both satisfied each period:

$$
\begin{aligned}
TR_t &= T_t - G_t \,, \\
Y_t &= C_t + I_t + G_t \,.
\end{aligned}
$$

where all the variables are in aggregate per capita terms.

From the firm's and household's first order conditions, together with the aggregate resources constraint and the goverment budget constraint the economy stationary steady state is obtained, whose output level is (see technical appendix at the end):

$$
Y = \chi \left[\Psi(\Phi) \left(1 - \Phi\right) \right]^{\frac{\alpha_g}{1-\alpha-\alpha_g}} \,, \tag{17}
$$

where $\chi = \left[\frac{\alpha(1-\tau)}{\frac{1}{\beta} - (1-\delta_k)} \right]^{\frac{\alpha}{1-\alpha-\alpha_g}} \left[\frac{\gamma}{\delta_g} \right]^{\frac{\alpha_g}{1-\alpha-\alpha_g}}$, and the variables without the t index denote steady state levels.

4.2   Static Golden Rule

Definition 1 *The static public spending composition Golden Rule is the level of the composition ratio that maximizes the steady state level of private consumption.*

Proposition 1 *The static public spending composition Golden Rule depends only on the λ parameter, in a monotonically increasing manner.*

Proof.   The static golden rule is obtained as the solution to the problem:

$$
\underset{\{\Phi\}}{Max}\ C = \eta Y = \eta\chi \left[\Psi(\Phi) \left(1 - \Phi\right) \right]^{\frac{\alpha_g}{1-\alpha-\alpha_g}} \,,
$$

where $\eta = 1 - \gamma - \frac{\alpha\beta\delta_k(1-\tau)}{1-\beta(1-\delta_k)}$.

The first order condition for that problem is:

$$\frac{\partial Y_t}{\partial \Phi_t} = 0 : \ \Omega \left[\Psi(\Phi)\,(1-\Phi)\right]^{\frac{\alpha_g}{1-\alpha-\alpha_g}-1} \left[(1-\Phi)\Psi'(\Phi) - \Psi(\Phi)\right] = 0 \ ,$$

where $\Omega = \eta\chi\frac{\alpha_g}{1-\alpha-\alpha_g}$ and $\Psi'(\Phi) = \frac{\partial\Psi(\Phi)}{\partial\Phi}$.

Apart from the trivial solution ($\Phi = 1$, that would yield a zero value for all the endogenous variables in the stationary steady state), the optimal composition (Φ^*) is obtained:

$$\left[(1-\Phi^*)\Psi'(\Phi^*) - \Psi(\Phi^*)\right] = 0 \ ,$$

or equivalently,

$$\Psi(\Phi^*) \left[\lambda(1-\Phi^*)\frac{e^{\Phi^*}}{e^{\Phi^*}-1} - 1\right] = 0 \ . \tag{18}$$

From which we get an implicit function for the composition ratio:

$$\lambda = \Theta(\underset{+}{\Phi^*}) \ , \tag{19}$$

where $\Theta(\Phi^*) = \left(1 - \frac{1}{e^{\Phi^*}}\right)\left(\frac{1}{1-\Phi^*}\right)$. $\blacksquare$

The second order condition for a maximun is[15]

$$\left\{(1-\Phi^*)\Psi''(\Phi^*) - 2\,\Psi'(\Phi^*)\right\} < 0 \rightarrow \left\{\Phi^* + e^{\Phi^*}\left[\lambda(1-\Phi^*) - 2\right]\right\} < -1 \ , \tag{20}$$

where $\Psi''(\Phi^*) = \frac{\partial^2\Psi(\Phi)}{\partial\Phi^2}$. This condition is always verified for the zeros of equation (19).

The positive relationship between the optimal composition ratio (Φ^*) and λ parameter is displayed in the following figure:

[INSERT FIGURE 4]

Characterization of the optimal solution is readily obtained from the analysis of the bracketed terms in (18). The first term represents the efficiency effect that a reallocation of public spending towards productive consumption induces on the nominal infrastructures (because they are used in a more efficient way), which depends positively on λ. The second term represents the crowding-out effect that such reallocation induces on nominal infrastructures (because fewer resources are allocated to the accumulation of physical infrastructures), and it is equal to one. The balance between the efficiency and the crowding-out effects determines the optimal composition of public spending. The larger λ, the larger the relative weight of the efficiency effect with respect to the crowding-out effect, and hence, the larger the optimal percentage of public spending allocated to productive consumption.

---

[15] $\frac{\partial^2 Y}{\partial\Phi^2} = \Omega\left(\frac{\alpha_g}{1-\alpha-\alpha_g} - 1\right)\left[\Psi(\Phi)\,(1-\Phi)\right]^{\frac{\alpha_g}{1-\alpha-\alpha_g}-2} \left[(1-\Phi)\Psi'(\Phi) - \Psi(\Phi)\right]^2 + \ldots$

$\Omega\left[\Psi(\Phi)\,(1-\Phi)\right]^{\frac{\alpha_g}{1-\alpha-\alpha_g}-1} \left[(1-\Phi)\Psi''(\Phi) - 2\,\Psi'(\Phi)\right] < 0$

Because the first term is zero in the optimum, the second order condition is given by equation (20).

In particular, for the case $\lambda = 0$, that is, for the standard model of non-productive public consumption, the normative results obtained here resembles the one from the previous literature: because the only effect on infrastructures of a reallocation of government spending towards consumption is the crowding-out effect, the optimal ruling is to allocate the whole spending to investment (many authors have found that the growth-maximizing ratio of public non-productive consumption to output is zero, for example Barro, 1990, or Judd, 1999).

Now it is possible to calibrate the λ parameter from the observed composition ratio in the US economy, by assuming that such composition is the result from an optimizing government behaviour within a competitive private sector framework. First, taking as optimal the mean composition for 1952 to 1968 (0.74), λ level is aproximately 2.0, but taking as optimal the mean composition for 1975 to 2001 (0.83), λ level changes to 3.3. Therefore, the analysis of real data must be done for an interval of λ between 2 and 3.3.

The planner solution is also computed and it is included in the technical appendix at the end of the paper. The main conclusion is that, like the competitive solution, the planner rule is also increasing in the λ parameter and, furthermore, it does not depend on the output elasticity of public capital (α_g).

Because the value of α_g is subject to great uncertainty, this property looks advantageous with respect to Devarajan et al (1996): the optimal composition in their model depends positively on the output elasticity of public consumption and negatively on the output elasticity of public investment[16].

5 Transitional dynamics

The question now is, which are the effects on the endogenous variables of a change in the composition of productive public spending in favor of consumption?. This has been the observed process for the US economy since 1968 (see Figure 3 again).

The observed process in the composition of public spending is simplified in the following way. We assume that the composition ratio follows an autorregresive process, whose mean value represents the objective level established by government. The experiment consists of assuming that the objective level changed after 1968.

The assumed process for the composition ratio is given by:

$$\ln \Phi_t = (1 - \rho^\Phi) \ln \bar{\Phi} + \rho^\Phi \ln \Phi_{t-1} \ ,$$

where ρ^Φ is the persistence parameter and $\bar{\Phi}$ is the objective level for the composition established by government.

We assume that the objective level until 1968 was 0.74 ($\bar{\Phi}^0$) rising thereafter to 0.83 ($\bar{\Phi}^1$). Note that this experiment does not increase the financing needs of the public sector,

---

[16]The optimal composition in these authors' model, in its Cobb-Douglas version of footnote 9, is given by the relative level of the output elasticity of public investment (g_1) and public consumption (g_2):

$$\frac{g_2}{g_1 + g_2} = \frac{\gamma}{\beta + \gamma}$$

which are given by the percentage of total spending over output (γ), that has not changed. Consequently, changing the tax rate on income (τ) is not necessary. The modelled path for the composition ratio is displayed in figure 5a.

[INSERT FIGURE 5a]

This reallocation of public spending induces the transition towards a new steady state for the endogenous variables.[17]

In particular, we are interested on the transition path for the nominal and effective infrastructures, and also on the induced effect on output and consumption of the change in the target for the composition of public expenditures.

The experiment requires a level for the λ parameter. From the previous section results, a value of $\lambda = 3.3$ is chosen, because we assume that the observed path for the composition ratio is the result of an optimizing behaviour of the government. We also assume an output elasticity for public capital of 0.16.

The effects of the public spending reallocation appear in figures 5b and 5c.

[INSERT FIGURES 5b 5c]

The public spending reallocation induces:

i) A lower accumulation of nominal infrastructures because of the crowding-out effect of productive consumption.

ii) A rise of the infrastructures efficiency index that counterbalances the nominal infrastructures decline. This is the efficiency effect of productive consumption.

iii) As a consequence, the effective infrastructures to private capital ratio rises after the public spending reallocation and, at the end of the sample, is even larger than at the begining. Under the proposed theoretical framework the lower accumulation of nominal infrastructures has been compensated by a better use of them after the policy change.

iv) The observed reallocation of public spending leads to an increase of output and consumption that achieves a welfare gain of 3.9 percentage points of output. This gain represents the percentage in which initial private consumption should be increased every year so that the discounted lifetime utility would coincide with the one achieved as a result of the policy reform. The computation of welfare gain is carried out as usual[18].

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[17]The transition path for the control variable and the remains of endogenous variables are obtained by Sims' (1990) procedure to compute a numerical solution to a dynamic general equilibrium model. Stability conditions (characterizing the convergence subspace towards steady state, and guaranteing that transversality conditions are fulfilled) are given by the left eigenvectors corresponding to the unstable eigenvalues of the transition matrix in the linear approximation to the model economy.

[18]Computation of welfare gain includes the following steps: i) computing the discounted lifetime utility corresponding to the public spending reallocation: $\widehat{U} = \sum_{t=0}^{\infty} \beta^t \ U\left(C_t(\Phi_{t\in(1952,1968)} = 0.74; \Phi_{t\in(1969,\infty)} = 0.83)\right)$; ii) computing the constant level of private consumption that would achieve the level of discounted utility obtained in (i): $\widehat{C} = [(1-\beta)(1-\sigma)\widehat{U} + 1]^{1/(1-\sigma)}$; iii) computing the percent increment in inicial steady state private consumption $(C_{SS,0}(\Phi_{t\in(1952,\infty)} = 0.74))$ necessary to match $\widehat{C}$, and expressing it as a percentage of initial steady state output $(Y_{SS,0}(\Phi_{t\in(1952,\infty)} = 0.74))$, that is: $\frac{\widehat{C}-C_{SS,0}}{Y_{SS,0}} \cdot 100$.

15

Next we repeat the experiment assuming that the composition ratio at the begining of the sample is the optimal one, which leads to a calibrated λ of 2.0. The effects of the government spending reallocation are qualitatively similar, although the welfare gain is lower: around 1 percentage point of aggregate output. As a conclusion, even if the optimal composition was the one prevailing at the begining of the sample, the short-run effects of the public spending reallocation induce a significant welfare gain, through the rise in effective infrastructures.

6   Dynamic golden rule

It has been shown that the transition effects of a public spending composition reform are not negligible. Hence, it seems sensible to assume that a government should be interested in implementing the reform that maximizes the discounted utility of households along the transition. Consequently, a new Golden Rule definition is proposed now.

Definition 2 *Dynamic Golden Rule is defined as the public spending composition that maximizes the agent's discounted utility along the transition from the initial situation to the steady state corresponding to the new composition ratio.*

Computation of the dynamic golden rule has been made by partially following Cooley and Hansen (1992):

i) Given a level for the λ parameter, and starting at the initial composition ratio ($\bar{\Phi}^0 = 0.74$) the government changes the objective level for that ratio to $\bar{\Phi}^1$.

ii) The transition from the initial composition towards the final one is performed according to the calibrated persistence parameter for the US.

iii) The transition path for the state and control variables is computed from the competitive equilibrium conditions.

iv) The achieved welfare gain from the reform is then computed.

This experiment is repeated for every possible new objective level of the composition ratio; that is, it is repeated for a grid of $\bar{\Phi}^1 \in (0, 1)$. The optimal level of the composition ratio is the one that leads to the largest welfare gain as a result of the reform, given the level of λ.

In the experiment, there is an implicit assumption that the government will actually implement the composition path $\left\{ \bar{\Phi}^0, \left\{ \bar{\Phi}^1 \right\}_{t=1}^{\infty} \right\}$, once the optimal final ratio has been chosen.

The results for a wide range of λ parameter values are displayed in the figure below, together with the static golden rule. The experiment has assumed the output elasticity parameter for infrastructures (α_g) to be 0.16.

[**INSERT FIGURE 6**]

The main implications from the figure are:

i) The dynamic golden rule level of the composition ratio is larger than the static one, for any positive level of the λ parameter. The reason is that the dynamic rule achieves better short-run results on welfare, lower losses or larger gains, althought the long-run

results on welfare are worse than for the static rules, lower gains or larger losses. There exist short run losses whenever the initial ratio is larger than the final one, and there exist long-run gains whenever the final ratio is nearer to the static golden rule than the initial ratio.

ii) Similarities between the static and dynamic rules allow us to interpret the dynamic rule as the intertemporal balance between the efficiency and the crowding-out effects of productive consumption on the nominal infrastructures. Because the positive efficiency effect of productive consumption is concentrated in the short-run and the dynamic rule confers a larger weight to short-run effects, a larger ratio of consumption to total spending is chosen when transitional dynamics are taken into account.

Considering the dynamic rule, which should be the calibrated λ for the US economy?. If we take as optimal the composition ratio at the end of the sample, λ would be aproximately 2.05. Ignoring the transitional dynamics of the economy would (wrongly) suggest a much lower composition ratio, of around 0.74.

The dynamic golden rule has been obtained for different values of the most significant parameters $(\alpha_g, \bar{\Phi}^0, \rho^\Phi, \sigma, \gamma, \tau)$ and a summary of these results are available in table 1.

[**INSERT TABLE 1**]

The main conclusion of the sensitivity analysis is that the dynamic golden rule is hardly altered by changes in any of these parameters. In particular, the output elasticity of public capital (α_g parameter) is irrelevant when characterizing the optimal composition ratio corresponding to a given λ. This result is similar to the one obtained for the static golden rule. As it was said before, this is a crucial advantage of the model because of the large uncertainty about the real level of this parameter (α_g). Furthermore, the initial level for the composition ratio ($\bar{\Phi}^0$) and the persistence parameter (ρ^Φ) also have a negligible relevance, supporting the robustness of the results.

6.1   A look at US data

Next, we compare the observed series paths for US data and the model. We assume the same public spending reallocation used in the transitional dynamics section: that is, until 1968 the objective level of the composition ratio is 0.74 ($\bar{\Phi}^0$) increasing to 0.83 ($\bar{\Phi}^1$) thereafter. Experiments are carried out for the two λ values calibrated for the last part of the data sample under the static and the dynamic golden rule definitions (2.05 and 3.3). Three levels for α_g have also been considered. The analysis is focused on characterizing the transition path for both the ratio of nominal infrastructures (Kg) over private capital (K) and the ratio of effective infrastructures (E) over private capital, assuming the efficiency index proposed in the paper. See figures 7a, 7b, 7c, 7d

For $\lambda = 3.3$ and $\alpha_g = 0.32$, the Kg/K ratio (figure 7a) observed in the US economy has been at the level or slightly above the model ratio for most of the period after 1968, with the exception of the last years (1998 and after). On the other hand, assuming $\alpha_g = 0.16$, there are some periods of time in which the Kg/K ratio has been a little lower than the model ratio (periods 1978-1991 and 1996 onwards); however, there is not evidence in any of the two cases that the available public capital is too low in relative terms to

private capital. Only when the assumed α_g is 0.08, the observed path for the ratio is systematically below the model ratio, although the distance is not very large.

The model Kg/K path changes with α_g because this parameter alters the convergence speed of the ratio: even though the final steady state is the same in the three cases (see equation 21 in the technical appendix), the speed of convergence to such ratio is larger for higher α_g.

The conclusions for the analysis of the E/K ratios (figure 7b) for different α_g are similar to the ones for Kg/K; all of the model ratios suggest that the observed ratio is too low for the last part of the sample (after the mid-nineties). The reason is that in the last years the decline of the Kg/K ratio adds to the decline of the efficiency index (as a consequence of a lower composition ratio, see figures 1 and 3 again).

The experiment is repeated for $\lambda = 2.05$ and the three levels of α_g already mentioned (figures 7c and 7d), leading to cualitatively similar conclusions.

Although in this second case the model Kg/K and E/K ratios differ more markedly from the observed ratio, the observed ratio is not significantly below the model ratio, except for the last years.

On the other hand, under this theoretical framework, welfare gains achieved by the public spending reallocation are far from being negligible. Under $\lambda = 3.3$, welfare gains obtained from the government spending reallocation range from 1.9 percentage points of GDP, under $\alpha_g = 0.08$, to 10.5 percentage points of GDP, under $\alpha_g = 0.32$. For $\lambda = 2.05$, welfare gains are placed between 0.5 and 2.5 percentage points respectively.

The major conclusion of the experiment is that, whatever the chosen λ value, the similarity between the model ratios and the observed ratios (Kg/K and E/K) is large. Therefore, under this interpretation, there is not evidence that available public capital has been significantly low relative to private capital, with the exception of the last years. This suggests that the observed paths in the US could also be viewed as the convergence towards an optimal composition of productive public spending. A lower stock of infrastructures has been accumulated, but it has been used more efficiently. Only for the last years of the sample (1996 and onwards) a growing divergence of the observed from the optimal ratio is found.

7   Conclusions

This paper links the works of Hulten (1996) and Aschauer (2000), on the one hand, and Devarajan *et al.* (1996) on the other, by analyzing: i) the role played in the aggregate production process by the efficiency with which infrastructures are used; ii) the role played in the efficient use of infrastructures by the public consumption expenditure allocated to the operation and maintenance of the infrastructure stock.

The aim of this paper is to obtain the optimal allocation of public spending between investment, i.e., expenditures that accumulate physical infrastructures, and productive consumption, i.e., expenditures for operation and maintenance of infrastructures.

The main contributions of the paper are:

i) Extending the neoclassical growth model with productive public capital by including an infrastructure efficiency index, which is assumed to depend on a public choice variable,

in particular, the share of public spending allocated to productive public consumption.

ii) Specifying a golden rule for the allocation of public expenditure between productive consumption and investment. Two definitions of golden rule are proposed: the first is obtained by maximizing the steady state welfare, and the second is characterized by maximizing the welfare along the transition from the initial situation to the final steady state. Both of them can be interpreted as the balance between the efficiency and the crowding-out effects that a reallocation of public spending towards productive consumption induces on the nominal infrastructures: they are used in a more efficient way, but also fewer resources are allocated to the accumulation of physical infrastructures. We also show that a larger ratio of consumption to total spending is chosen when transitional dynamics are taken into account.

iii) We conclude, under this framework, that the observed paths for the ratio of nominal infrastructures to private capital and for the efficiency index in the US economy during the last fifty years have been close to optimal. There is not evidence that nominal public capital has been significantly low relative to private capital, with the exception of the last years. This suggests that the observed paths in the US could also be viewed as the convergence towards an optimal composition of productive public spending. A lower stock of infrastructures has been accumulated, but it has been used more efficiently. Only for the last years of the sample (1996 and onwards) a growing divergence of the observed from the optimal ratio is found.

8   References

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9 Figures and Tables

UNITED STATES ECONOMY:
'PUBLIC CAPITAL EFFICIENCY' AND
PUBLIC TO PRIVATE CAPITAL RATIO

Figure 1

Ratios of Government Consumption (Cg) over Government Investment
(Ig) and over Government Exhaustive Expenditure (G)
Standardized series
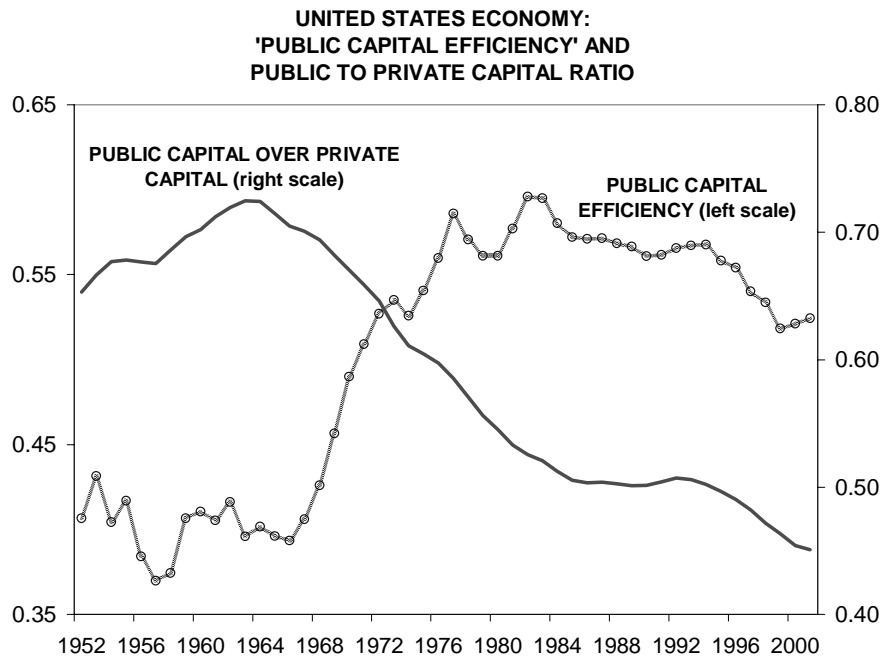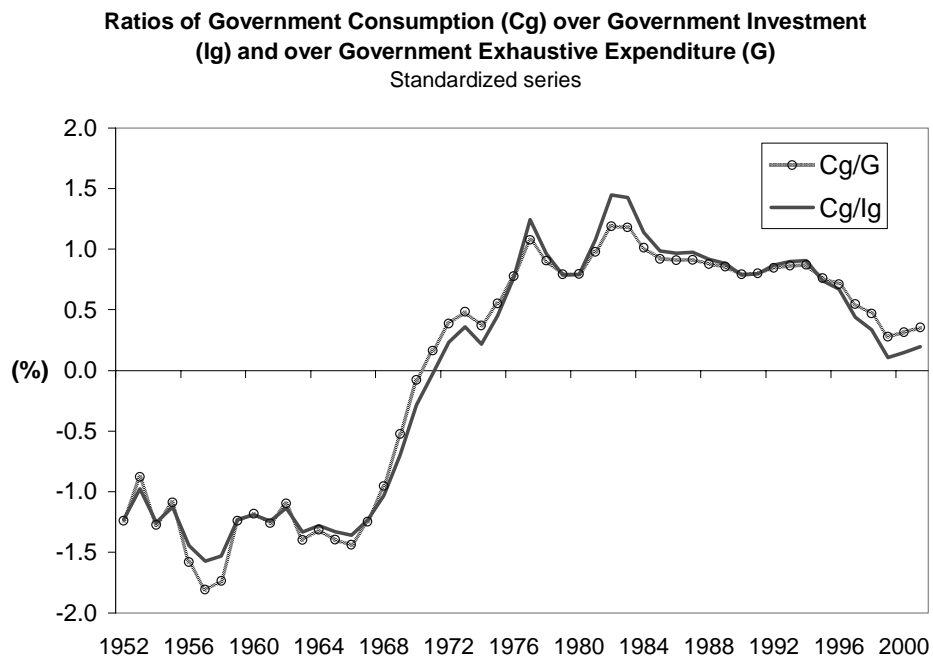
Figure 2

PUBLIC EXPENDITURE COMPOSITION (Cg/G)
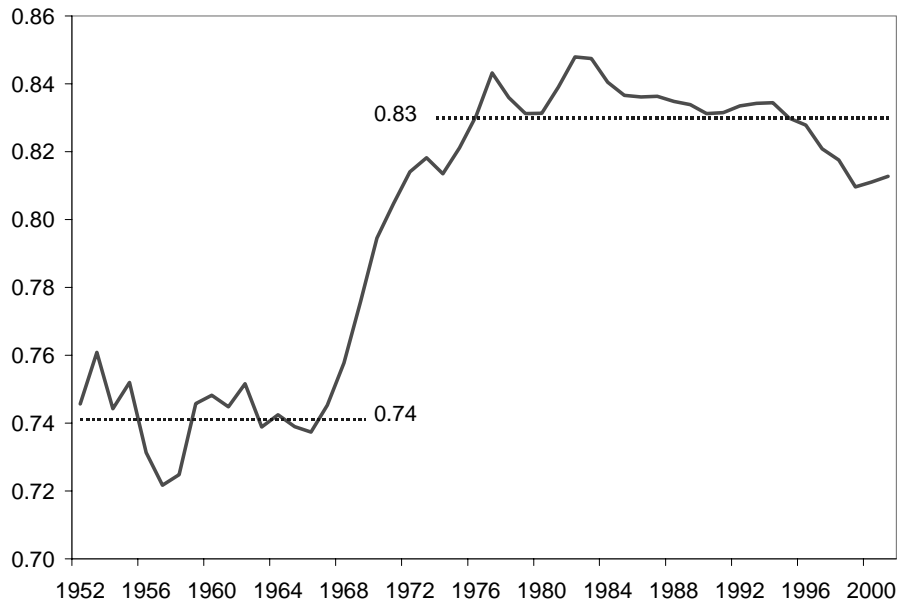
Figure 3

STATIC GOLDEN RULE
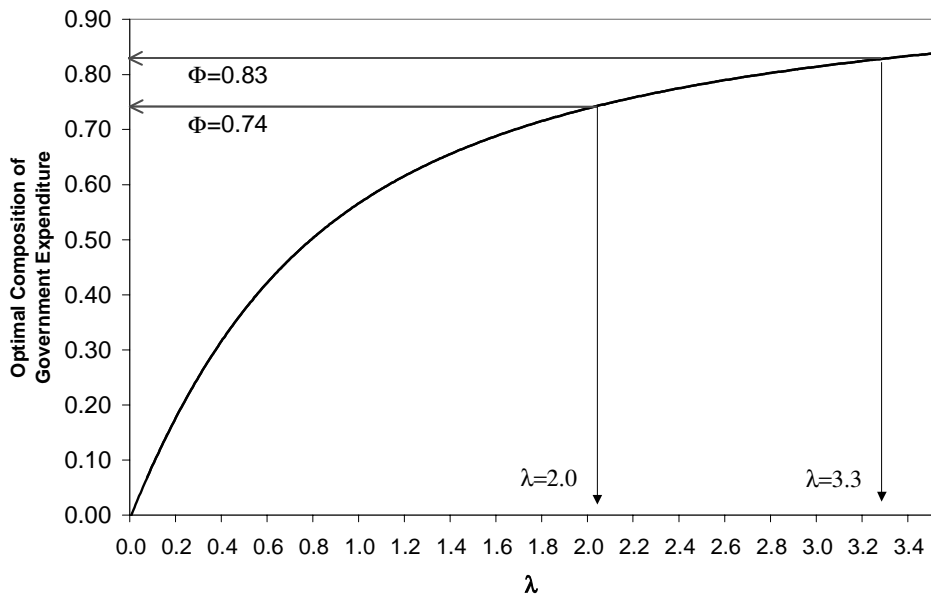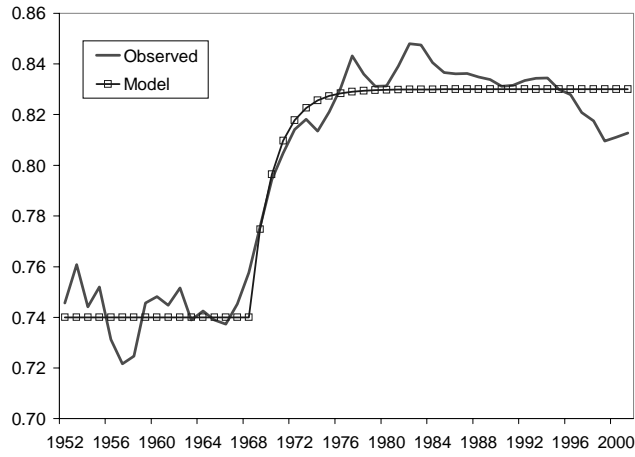
Figure 4

Observed and Model Composition Ratio (Φ)

Figure 5a

Nominal and Effective Infrastructures from model
λ= 3.3, α_g= 0.16
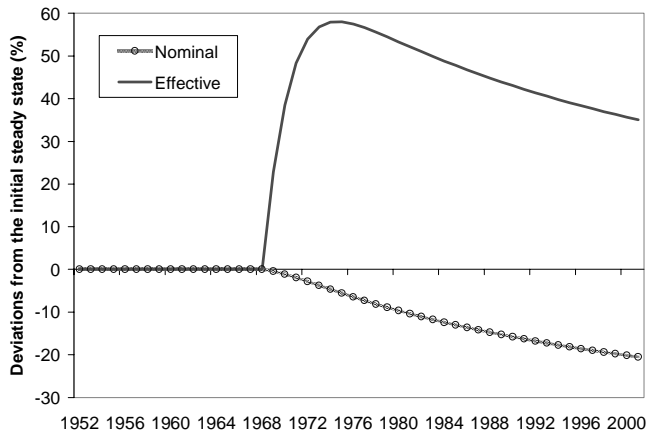
Figure 5b

Ratios of Nominal (Kg) and Effective (E) Infraestructures over Private Capital (K) from model
λ= 3.3, α_g= 0.16
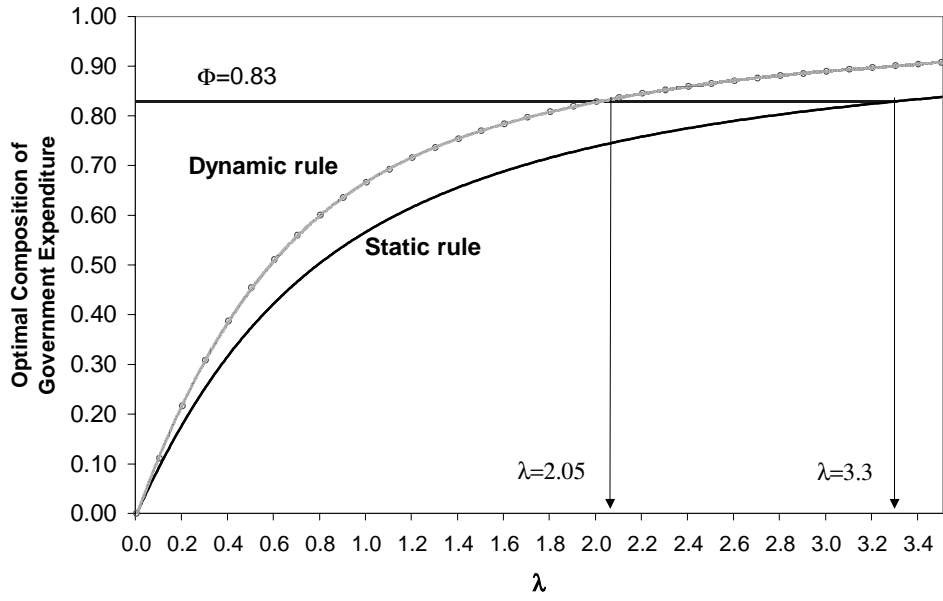

Figure 5c

24
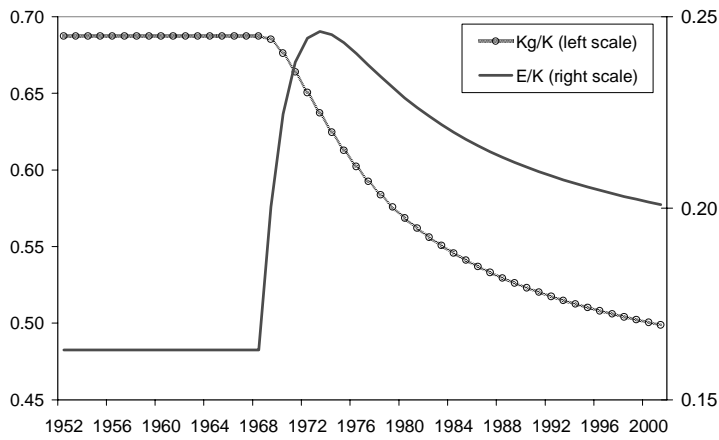
DYNAMIC VS STATIC GOLDEN RULES

Figure 6

λ	Benchmark (*)	α_g=0.08	α_g=0.32	Φ_0 = 0.5	ρ^Φ = 0	σ = 3	γ = 0.1	τ = 0.4
1	**0.667**	0.665	0.670	0.699	0.668	0.672	0.668	0.668
2	**0.829**	0.830	0.827	0.849	0.828	0.828	0.830	0.830
3	**0.890**	0.892	0.886	0.902	0.889	0.888	0.891	0.891

Table 1: Sensitivity analysis of the Dynamic Golden Rule for the Composition of Government Expenditure.

(*) Benchmark:

$\alpha_g = 0.16 \quad \tau = 0.28$

$\Phi_0 = 0.74 \quad \alpha = 0.34$

$\rho^\Phi = 0.6 \quad \beta = 0.98$

$\sigma = 1.5 \quad \delta_k = 0.1$

$\gamma = 0.216 \quad \delta_g = 0.04$

Observed and simulated Ratios of Nominal Public capital over private capital. λ = 3.3

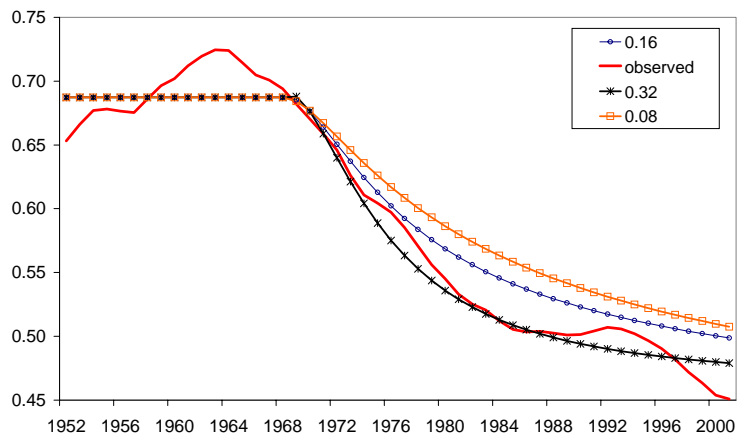Figure 7a

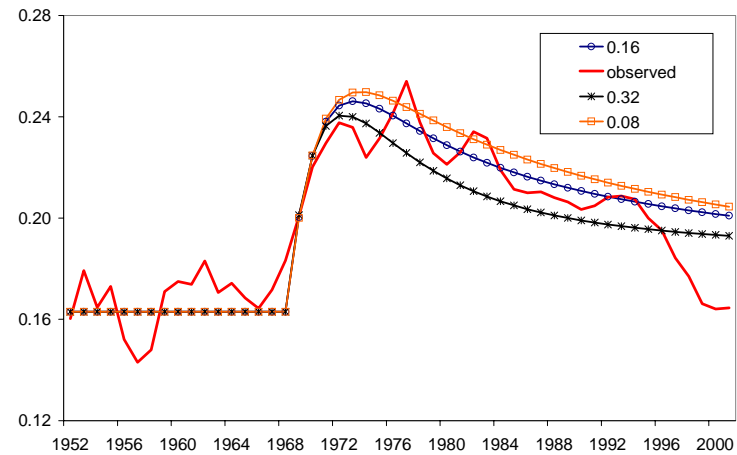**Observed and simulated Ratios of Effective public capital over private capital. λ= 3.3**

Figure 7b

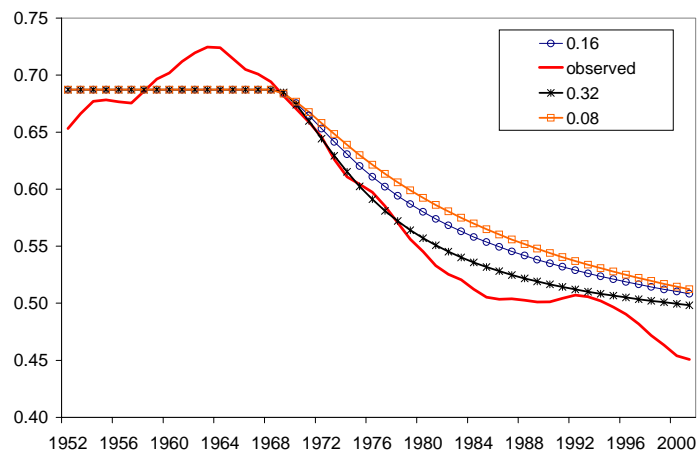**Observed and simulated Ratios of Nominal public capital over private capital. λ= 2.05**

Figure 7c

Observed and simulated Ratios os Effective public capital over private capital. λ=2.05
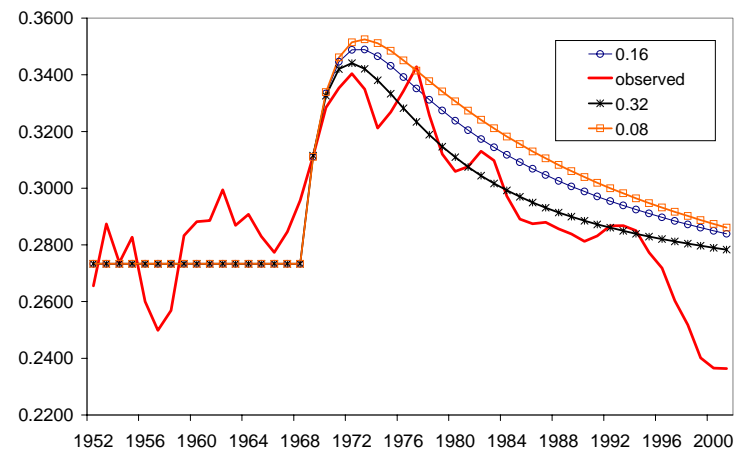

Figure 7d

10 Appendix

10.1 Competitive equilibrium

The firm's objective function is:

$$\underset{\{N_t, K_{t-1}\}}{Max} \; Y_t - w_t N_t - r_t K_{t-1}$$

and the first order conditions are (obtained from the objective function maximization with respect to the decision variables, taking as given the input prices because the firm is competitive in the input markets):

$$w_t = (1-\alpha) \frac{Y_t}{N_t} = w(N_t, K_{t-1}, Kg_{t-1}, \Phi_t) \,,$$

$$r_t = \alpha \frac{Y_t}{K_{t-1}} = r(N_t, K_{t-1}, Kg_{t-1}, \Phi_t) \,.$$

The consumer lagrangean function is:

$$\pounds = \sum_{t=0}^{\infty} \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \mu_t \, \beta^t \, [K_t - (w_t N_t + r_t K_{t-1})(1-\tau) + TR_t \dots \right. \\ \left. -C_t + (1-\delta_k) K_{t-1}] \right\}$$

and the first order conditions (by substituing the firm's first order conditions):

$$\frac{\partial \pounds}{\partial N_t} = 0 : \; N_t = 1 \,,$$

$$\frac{\partial \pounds}{\partial C_t} = 0 : \; \mu_t = C_t^{-\sigma} \,,$$

$$\frac{\partial \pounds}{\partial K_t} = 0 : \; \frac{\mu_t}{\mu_{t+1}} = \beta \left\{ 1 - \delta_k + (1-\tau) \frac{\alpha Y_{t+1}}{K_t} \right\} \,.$$

where μ_t is the budget constraint Lagrange multiplier.

Solution must also verify the aggregate resources and the goverment budget constraints:

$$(1-\gamma) Y_t = C_t + K_t - (1-\delta_k) K_{t-1} \,,$$

$$Kg_t = (1-\delta_g) \, Kg_{t-1} + (1-\Phi_t) \, \gamma \, Y_t \,,$$

$$TR_t = (\tau - \gamma) Y_t \,,$$

$$Y_t = N_t^{1-\alpha} \, K_{t-1}^{\alpha} \left(\left(\frac{e^{(\Phi_t)} - 1}{e - 1} \right)^{\lambda} Kg_{t-1} \right)^{\alpha_g} \,.$$

And also the transversality condition:

$$\lim_{t \to \infty} E_0 \, \beta^t \, \mu_t \, K_t = 0 \,,$$

which guarantees that the first order conditions for optimality are sufficient.

From this equations system, the economy steady state can be obtained, whose production level is:

$$Y = \chi \left[\Psi(\Phi) \, (1 - \Phi) \right]^{\frac{\alpha_g}{1 - \alpha - \alpha_g}} \, ,$$

where $\chi = \left[\frac{\alpha(1-\tau)}{\frac{1}{\beta} - (1 - \delta_k)} \right]^{\frac{\alpha}{1 - \alpha - \alpha_g}} \left[\frac{\gamma}{\delta_g} \right]^{\frac{\alpha_g}{1 - \alpha - \alpha_g}} .$

The expression for the infrastructures to public capital ratio is given by:

$$\frac{Kg}{K} = \left[\gamma \frac{1 - \Phi}{\delta_g} \right] \left[\frac{\alpha \, (1 - \tau)}{\frac{1}{\beta} - (1 - \delta_k)} \right]^{-1} \tag{21}$$

10.2 Planner Equilibrium

The planner solution is given by a set of allocations
$\{C_t, N_t, K_t, \Phi_t, Kg_t\}_{t=0}^{\infty}$, that solves the following optimization problem:

$$\underset{\{C_t, N_t, K_t, \Phi_t, Kg_t\}_{t=0}^{\infty}}{Max} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1 - \sigma} \right) \right\}$$

$$s.t. \quad :$$
$$(1 - \gamma)Y_t = C_t + K_t - (1 - \delta_k)K_{t-1} \, ,$$
$$Kg_t = (1 - \delta_g) \, Kg_{t-1} + (1 - \Phi_t) \, \gamma \, Y_t \, ,$$
$$Y_t = N_t^{1-\alpha} \, K_{t-1}^{\alpha} \left(\left(\frac{e^{(\Phi_t)} - 1}{e - 1} \right)^{\lambda} Kg_{t-1} \right)^{\alpha_g} \, .$$

By combining equations, the lagrangean is:

$$\pounds = \sum_{t=0}^{\infty} \left\{ \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \mu_t \, \beta^t \left[N_t^{1-\alpha} \, K_{t-1}^{\alpha} \, \left(\Psi(\Phi_t) \, Kg_{t-1} \right)^{\alpha_g} - C_t \, ... \right. \right.$$
$$\left. \left. -K_t + (1 - \delta_k)K_{t-1} - \frac{1}{1 - \Phi_t} \left[Kg_t - (1 - \delta_g) \, Kg_{t-1} \right] \right] \right\}$$

and the first order conditions are:

$$\frac{\partial \pounds}{\partial N_t} = 0 : \ N_t = 1 \, ,$$

$$\frac{\partial \pounds}{\partial C_t} = 0 : \ \mu_t = C_t^{-\sigma} \, ,$$

$$\frac{\partial \pounds}{\partial K_t} = 0 : \ \frac{\mu_t}{\mu_{t+1}} = \beta \left\{ 1 - \delta_k + \frac{\alpha Y_{t+1}}{K_t} \right\} \, ,$$

$$\frac{\partial \pounds}{\partial Kg_t} = 0 : \ \frac{\mu_t}{\mu_{t+1}} = \beta(1 - \Phi_t) \left\{ \frac{1 - \delta_g}{1 - \Phi_{t+1}} + \frac{\alpha_g Y_{t+1}}{Kg_t} \right\} \, ,$$

$$\frac{\partial \pounds}{\partial \Phi_t} = 0 : \ \alpha_g Y_t \frac{\Psi'(\Phi_t)}{\Psi(\Phi_t)} = \frac{1}{(1 - \Phi_t)^2} \left[Kg_t - (1 - \delta_g) \, Kg_{t-1} \right] \, .$$

Proposition 2 *The static planner Golden Rule is larger than the competitive equilibrium Golden Rule.*

Proof. By combining the first order conditions of the optimization problem and solving the stationary steady state, a new implicit function of the optimal composition ratio is obtained:

$$\lambda = \Delta(\Phi^P) = \varphi \, \Theta(\Phi^P) \,, \tag{22}$$

where P-index denotes the planner solution, $\varphi = \frac{\delta_g}{\frac{1}{\beta} - 1 + \delta_g} < 1$ and $\Theta(\Phi) = \left(1 - \frac{1}{e^\Phi}\right)\left(\frac{1}{1-\Phi}\right)$ (see (19)). ∎

Consequently, the optimal planner composition is larger than the competitive optimal composition for a given λ. That is, the planner would allocate a larger percentage of public exhaustive spending to productive consumption:

$$\Phi^P > \Phi^* \ , \ \forall \lambda.$$