SEASONAL FLUCTUATIONS AND DYNAMIC EQUILIBRIUM MODELS OF EXCHANGE RATE[#].

Juan Ángel Jiménez Martín[a (*)] Rafael Flores de Frutos[b]

[a] Dpto. de Fundamentos de Análisis Económico II, Universidad Complutense, Somosaguas, 28223 Spain

[b] Dpto. de Fundamentos de Análisis Económico II, Universidad Complutense, Somosaguas, 28223 Spain

Abstract

Most dynamic equilibrium models of exchange rate are not able to generate monthly time series with the typical properties of actual exchange rate. If the exogenous endowments in an equilibrium exchange rate model contain seasonal variations, then the exchange rate will as well. In this paper, we show how in this framework, seasonal preferences can help to remove seasonality of the exchange rate simulated time series.

Keywords: exchange rate, equilibrium model, seasonality

JEL classification: F31, F37, G15

* Corresponding author. Tel.: +34 91 394 23 55. Fax: +34 91 394 2613;

e_mail: juanangel@ccee.ucm.es

We would like to acknowledge the financial support of the Ministerio de Educación, Spain, trough project BEC2003-03965

1.- Introduction

The model of the representative agent has become a fundamental tool for explaining the behavior of the exchange rate. Lucas (1978, 1982) Helpman and Razin (1979, 1982) Stockman (1980, 1983, 1987) Svensson (1985), Hodrick (1989) or Grilli and Roubini (1992) are classical references.

This literature has also contributed to the financial theory development. Lucas (1982) or Svensson (1985) are core references in the research on models for assets valuation in foreign currencies [see Baskhi and Chen, 1997 or Cao, 2001] or on determinants of risk premiums in the foreign exchange market [see Hodrick, 1989, Singleton, 1990, Kaminsky and Peruga, 1990, Engel, 1992a and 1992b, Dutton, 1993, Bekaert, 1994 and Hu, 1997].

Jiménez and Flores (2004) have shown that standard versions of dynamic equilibrium models of exchange rate are unable to generate monthly time series with similar properties to those observed in actual exchange rate. Seasonality, present in outputs and money stocks, is also present in most simulated time series of the exchange rate.

This feature is inherent to many economic time series, but it is rarely considered to be a key issue in model-building. Many econometricians consider seasonality as a noise, which must be removed. The use of seasonally adjusted data in empirical work has been so extended as criticised, see Wallis (1974), Sims (1993) and Hansen and Sargent (1993).

Miron (1986) proposed an alternative view for dealing with seasonality. For this author, seasonality is a feature of agents' behaviour that should be captured by the theoretical model. Optimisation-based structural models should reflect: (i) the decision-making process of a representative agent facing seasonal fluctuations, and (ii) that seasonal fluctuations are in fact an important source of information about the dynamic propagation mechanism.

Seasonal preferences have been mainly used to explain the presence of seasonal fluctuations in important economic variables, but never to explain the lack of them. This paper shows how modifying Grilli and Roubini (1992) model, by including seasonal preferences, as in Miron (1986), it is possible to generate time series for some exchange rate with similar properties to their actual counterparts, that is : (i) No seasonal fluctuations and (ii) degree of integration equal to 1.

The same order of integration between actual and simulated exchange rate time series leads to the study of cointegration. This property has been found in some cases, but only for the British pound / US dollar exchange rate the cointegration vector had the right sign.

The paper remains as follows. Section II presents a two countries cash in advance model of exchange rate whit a specific utility function that allows seasonal shocks. In addition, the equilibrium exchange rate as a function of money supply, production, asset returns, and consumer's preferences is derived. In section III, model parameters are estimated by applying the GMM to stochastic Euler equations and monthly time series of nominal exchange rate for several currencies are simulated. Concluding remarks appear in section IV.

II.- An exchange rate model whit seasonal shock in preferences

In this section, we utilize a version of the two-country cash in advance model proposed by Grilli and Roubini (1992), GR, to derive the equilibrium exchange rate. We first generalize previous papers by allowing seasonal shocks to preferences. We then study its implications for the relationship between exchange rate and asset returns, fundamental variables, and preferences.

Seasonal shifts in preferences

Representative agents from both countries have preferences described by identical infinite-horizon expected utility functions given by:

$$E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}U\left(C_{is}\right)\right] \quad 0<\beta<1 \tag{1}$$

Where, E_t denotes the mathematical expectation conditional on information known at the beginning of period t, β is a constant discount factor. Function U is assumed to be bounded, continuously differentiable, increasing in both arguments, and strictly concave. In this case, C_{it} is the consumption services received in each period t.

C_{it} depends on the stock of consumption goods owned by the consumer and on a set of exogenous variables:

$$C_{it} = \mathrm{F}(\mathrm{c}_{\mathrm{it}}^{D}, \mathrm{c}_{\mathrm{it}}^{F}, \mathrm{X}_{\mathrm{t}}) \tag{2}$$

Where c_{it}^{j} is the stock of consumption good produced in country j (j =D, F) and consumed by agent from country i (i=D, F) and X_t is a $sx1$ vector of variables that affect the flow of consumption services obtained from any level of c_{it}^{j}. The function $F(.)$ can be thought as the production function that consumers use to transform consumption goods into consumption services, following Lancaster (1966) and Becker and Stigler (1977). X_t variables are then interpreted as shocks to consumers' preferences. In this case, the variables included in X_t are seasonal dummies for month 2, 3 …, and 12.

In order to parameterize the model for estimation, it is necessary to specify functional form for U and F. We assume, as Miron (1986), that utility function, $U(.)$, is additively separable from the two consumption services and with constant intertemporal substitution elasticity:

$$U\left(C_{it}\right) = \frac{1}{1-\gamma^{D}}\left(\mathrm{c}_{\mathrm{it}}^{\mathrm{s}^{D}}\right)^{1-\gamma^{D}} + \frac{1}{1-\gamma^{F}}\left(\mathrm{c}_{\mathrm{it}}^{\mathrm{s}^{F}}\right)^{1-\gamma^{F}} \tag{3}$$

Where $c_{it}^{S^{j}}$ is the consumption services flow received in period t by agent from country i for consumption of good produced in country j, in season S; γ^{j} (j=D, F) is the intertemporal elasticity of substitution in consumption of the consumption service of the good produced in country j.

The production function, $F(.)$, that describes the flow of consumption services that the consumer receives, is a function of the stock of the consumption good, c_{it}^{j}, and the prevailing seasonal state μ_{st}:

$$c_{it}^{s^j} = \exp\left(\theta_s^j \ \mu_{st}\right) \ c_{it}^j, \ \text{para i}, \ j = D, F; \ \ s = 1, 2, \ldots 12 \tag{4}$$

$$\mu_{st} = \begin{cases} 1 & \text{si } t = s + \tau * 12 \qquad \tau = 0, 1, 2, \ \ldots \ \text{int}\, eger \ < \ T/12 \\ 0 & \qquad\qquad\qquad\qquad\quad else \end{cases}$$

Where θ_S^j is the preference seasonal in season S

Equilibrium exchange rate

The *GR* model is a simple variant of the cash in advance model proposed by Lucas (1982). The *GR* model posits that enough money must be set aside in advance to cover later purchases of both bonds and goods. In the particular model proposed here, the consumer has the choice of investing in stocks, rather than investing in nominal bonds. There are two countries, Domestic (D) and Foreign (F), and each country has a firm. Firms are assumed to be able to sell claims of their future outputs. Domestic (Foreign) claims entitle the owner to a proportionate share in the future stream of dividends. This framework permits to discuss the links between movements in the exchange rate and stock returns. For details, see Appendix 2.

The first-order conditions for the consumer's optimal consumption plan are analogous to those of the *GR* model. Some first order conditions follow from the assumption that the consumer has access to a capital market in which he can freely trade money and equities. In this framework, the equilibrium exchange rate clears the asset market:

$$S_t = \frac{Q_t^D}{Q_t^F} \frac{E_t\left(\dfrac{U_F\left[c_{Dt+1}^D, c_{Dt+1}^F\right]}{P_{t+1}^F} \left[d_t^F + Q_{t+1}^F\right] \right)}{E_t\left(\dfrac{U_D\left[c_{Dt+1}^D, c_{Dt+1}^F\right]}{P_{t+1}^D} \left[d_t^D + Q_{t+1}^D\right] \right)} \tag{5}$$

Where S_t is nominal spot exchange rate expressed as the domestic price for foreign currency, Q^j_t is the price of equities of firm *j* in units of currency *j* (*j*=D, F), d^j_t are dividends per equity of firm *j* (*j*=D, F), and P_t^j is the domestic and foreign currency prices for good *j*.

Cash in advance spending constraints are assumed in the good market, therefore equilibrium prices of the two goods depend on domestic and foreign money supply.

$$P_t^D = N_t^D \; / \; Y_t^D \tag{6}$$

$$P_t^F = N_t^F \; / \; Y_t^F \tag{7}$$

Where, N_t^j is the amount of money of country j for transactions in the goods market at time t.

Following earlier papers, these constraints are assumed to be binding. Pooling equilibrium[1] is assumed, so each representative agent consumes half of endowment of each country.

Thus given the utility function in (3), and the domestic and foreign goods nominal prices in (6)-(7), equation (5) becomes:

$$S_t = \frac{E_t \left(\frac{1}{2}\right)^{-\gamma^F} \frac{\left(\exp\left[\sum_{s=2}^{12} \theta_S^F \; \mu_{s,\,t+1}\right] Y_{t+1}^F\right)^{1-\gamma^F}}{N_{t+1}^F} \frac{\left[d_t^F + Q_{t+1}^F\right]}{Q_t^F}}{E_t \left(\frac{1}{2}\right)^{-\gamma^D} \frac{\left(\exp\left[\sum_{s=2}^{12} \theta_s^D \; \mu_{s,\,t+1}\right] Y_{t+1}^D\right)^{1-\gamma^D}}{N_{t+1}^D} \frac{\left[d_t^D + Q_{t+1}^D\right]}{Q_t^D}} \tag{8}$$

The spot exchange rate solution is a function of money and goods endowments, the rate of return on the equities between dates t an $t+1$, and the seasonal shifts in preferences.

III. **Estimation and Results**

In this section, we present the procedure for evaluating the exchange rate equilibrium model under seasonal shifts in preferences. It starts by estimating the parameters of agents' utility function. Then, using (8), a monthly exchange rate time series is simulated. Finally, using standard time series tools (Box-Jenkins, 1970), actual and simulated rates are compared.

[1] One equilibrium solution is the perfectly pooled equilibrium of Lucas (1982).

III.1. **Parameters estimation procedure**

This section describes how the vector of parameter θ_{13*1} (discount factor, intertemporal elasticity of substitution, and eleven coefficients on the seasonal dummies) is estimated for each country. The parameters are estimated by Generalized Method of Moments (GMM). Hansen and Singleton (1982) show how to apply GMM to Consumption-Based Capital Asset Pricing Model. The estimation strategy is to use a theoretical economic model to generate a family of orthogonality conditions. Rational expectations hypothesis suggests orthogonality conditions that can be used in the GMM framework.

First order conditions from theoretical economic model imply the following stochastic intertemporal Euler equation of good *j* (=D, F)[2]:

$$\beta \, E_t \frac{U_i\left[c^{s^D}_{Dt+1}, c^{s^F}_{Dt+1}\right] / P^j_{t+1}}{U_i\left[c^{s^D}_{Dt}, c^{s^F}_{Dt}\right] / P^j_t} \frac{\left[d^j_t + Q^j_{t+1}\right]}{Q^j_t} = 1 \tag{9}$$

When (3) is substituted in (9):

$$E_t \, \beta \left(\exp\left[\sum_{s=2}^{12} \theta^j_s \left(\mu_{s,t+1} - \mu_{s,t}\right) \right] \right)^{1-\gamma^j} \left(\frac{C^j_{t+1}}{C^j_t}\right)^{-\gamma^j} \left(\frac{P^j_t}{P^j_{t+1}}\right)\left(\frac{d^j_t + Q^j_{t+1}}{Q^j_t}\right) - 1 = 0 \tag{10}$$

The above expression represents the rate at which purchases of good *j* must grow relative to the growth of prices and asset return. Nominal price of good *j* is given by cash in advance constraint, (6)-(7). Assuming pooling equilibrium, (10) becomes:

$$E_t \left\{ \beta \left(\exp\left[\sum_{s=1}^{11} \theta^j_s \left(\mu_{s,t+1} - \mu_{s,t}\right) \right] \right)^{1-\gamma^j} \left(\frac{Y^j_{t+1}}{Y^j_t}\right)^{1-\gamma^j} \left(\frac{M^j_t}{M^j_{t+1}}\right)\left(\frac{d^j_t + Q^j_{t+1}}{Q^j_t}\right) - 1 \right\} = 0 \tag{11}$$

Equation (11) represents a set of orthogonality conditions which seasonal taste parameters, factor discount and intertemporal elasticity of substitution parameters are estimated from.

III.2. **Empirical results**

Table 1 shows the GMM estimation of equation (11). Outputs are approximated by the corresponding monthly industrial production indexes (IPI), monetary aggregates by the corresponding M2 and asset returns are generated by taking a first difference on the natural logarithm of the equity price index. Appendix 1 describes the data and their stochastic properties.

In Table 1, $\theta_s^{country}$ is the seasonal preference in *s* month relative to January, for each country. Seasonal taste parameters are statistically significant in most cases. The fact that seasonal preference shocks are significantly different from zero means that inclusion of these shocks is necessary in order to explain the joint behaviour of consumption and asset returns.

The estimates of γ^j are similar to those found in other studies, ranging from 0.5 to 2.75.[3] In order to test the validity of overidentifying restrictions, the J-statistics are also displayed. The null hypothesis (overidentifying restrictions are satisfied) is not rejected at 5% significance level, in any case.

Table 1: GMM estimation of utility function parameters[(a)]

	θ_2[(c)(d)]	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}	γ^j	J_Sta[(e) (f)]
$\theta_s^{GM\ (b)}$	0.021* (0.02)	-0.073* (0.03)	-0.027 (0.03)	-0.007 (0.03)	-0.064* (0.03)	0.002 (0.03)	0.086* (0.04)	-0.086* (0.03)	-0.063 (0.04)	-0.159* (0.02)	-0.146* (0.04)	1.281 (0.11)	28.41 (0.98)
θ_s^{SP}	-0.036 (0.04)	-0.054 (0.06)	-0.098 (0.08)	-0.245* (0.11)	-0.282* (0.12)	-0.163 (0.10)	0.600* (0.08)	0.122* (0.07)	0.012* (0.06)	0.029 (0.06)	0.169* (0.05)	0.814 (0.06)	28.07 (0.86)
θ_s^{JP}	-0.069* (0.02)	-0.181* (0.02)	-0.086* (0.03)	-0.089* (0.03)	-0.129* (0.03)	-0.147* (0.03)	-0.073 (0.04)	-0.159* (0.03)	-0.123* (0.03)	-0.105* (0.03)	-0.093* (0.02)	0.417 (0.19)	32,73 (0.58)
θ_S^{UK}	-0.102 (0.08)	-0.356* (0.17)	-0.212 (0.16)	-0.299 (0.21)	-0.448 (0.30)	-0.200 (0.19)	-0.172 (0.21)	-0.224 (0.17)	-0.263 (0.17)	-0.311 (0.16)	-0.167 (0.12)	1.135 (0.08)	21,78 (0.47)

[2] See expression (A15) in Appendix 2.

[3] Kydland and Prescott (1982), in their study of aggregate fluctuations, found that they needed a value between one and two to mimic the observed relative variability of consumption and investment. Altug (1983) estimates the parameter to near cero. Kehoe (1983), studying the response of small countries balance of trade to terms of trade shocks, obtained estimates near one. Hildreth and Knowles (1986), in their study of the behaviour of farmers also obtain estimates between one and two. Mehra and Prescott (1985) present evidence for restricting the value of relative risk aversion to be a maximum of ten, though without specifying a concrete value. Eichenbaum et. al. (1984), Mankiw et al. (1985) and Hansen and Singleton (1982) report values of γ between cero and one 1, Mankiw (1985) reports values between 2 and 4.

θ_S^{US}	-0.023* (0.003)	-0.018* (0.002)	-0.019* (0.003)	-0.001 (0.002)	-0.046* (0.004)	-0.005 (0.003)	-0.045* (0.004)	-0.057* (0.004)	-0.044* (0.004)	-0.030* (0.003)	-0.017* (0.003)	2.735 (0.036)	33,33 (0.98)

Notes:

(a) Instruments are: a constant term, lagged production growth rates, lagged monetary aggregates growth rates, and lagged rates of return.
(b) Germany (GM), Spain (SP), Japan (JP), United Kingdom (UK), United States (US)
(c) Estimated standard errors in parentheses
(d) * Denotes significantly different from zero at the 95% confidence level
(e) J-statistic, for testing the validity of overidentifying restrictions. Under the null hypothesis, the overidentifying restrictions are satisfied, the J-statistic (i.e, the minimized value of the objective function) times the number of observations is asymptotically χ_q^2, with degrees of freedom equal to the number of overidentifying restrictions
(f) P values represented in parentheses

Figures 1-5 in Appendix 3, show residual graphs, autocorrelation function (ACF) and partial autocorrelation function (PACF) associated to equation (11). Residuals are white noise.

III.3.- Comparing observed and simulated exchange rates

Expression (8) describes the equilibrium exchange rate as a non linear function of outputs, monetary aggregates and asset returns. From this expression, by using the estimates of utility function parameters of Table 1, the corresponding IPI, M2 and the asset returns, monthly time series of nominal exchange rates for several currencies are generated: German mark (DEM/USD), Japanese yen (JPY/USD), Spanish peseta (ESP/USD), and British pound (GBP/USD) relative to the US dollar. Also Japanese yen (JPY/DEM), Spanish Peseta (ESP/DEM), and British pound (GBP/DEM) relative to the German mark.

The currency is calculated as the value of the second country's currency. For example, (GBP/USD) is the number of British pounds needed to purchase a US dollar, in this case UK is the domestic country and US is the foreign.

Table 2 reports a variety of descriptive statistics of simulated exchange rate, (*SimExRa*) and observed exchange rate (*ObsExRa*) over the sample period 1990:01-1998:04. Mean (M), standard deviation (Std), skewness (Skw), kurtosis (Kt) and the order of integration (d). Then, the stochastic process of the *SimExRa* time series is analysed and compared with the stochastic processes characterizing the *ObsExRa*. Table 2 shows time series analysis results, diagnostic checks are developed to detect model inadequacy. Descriptive statistics of the residuals from estimated models are reported:

mean ($\bar{a}_t$) and estimated mean standard error ($\hat{\sigma}_{\bar{a}_t}$), estimated standard errors ($\hat{\sigma}_{a_t}$) and Ljung-Box Q-statistics at lag 12 to test for serial correlation (Q(12))

Table 2: Summary of ARIMA[4] models fitted to the *SimExRa* and the *ObsExRa*

	M	Std.	Skw	Kt	∇^d	$\bar{a}_t / (\hat{\sigma}_{\bar{a}_t})$	$\hat{\sigma}_{a_t}$	Q(12)	ARIMA MODELS[(a) (b)]
***Obs* DEM/USD**	$1.6*10^{0}$	$1.2*10^{-1}$	-0.03	2.1	∇^1	$1.0*10^{-3}$ $(4.6*10^{-3})$	$4.6*10^{-2}$	9.7	$Y_t = 0.20\ \xi_t^{S3/91} + N_t \quad \nabla N_t = a_t$ (0.05)
***Sim* DEM/USD**	$2.1*10^{-4}$	$1.9*10^{-5}$	-0.70	2.3	∇^1	$-4.3*10^{-7}$ $(4.7*10^{-7})$	$4.7*10^{-6}$	12.6	$(1-0.16B+0.32B^2)\ \nabla Y_t = a_t$ (0.10) (0.10)
***Obs* ESP/USD**	$1.2*10^{2}$	$1.6*10$	0.07	2.0	∇^1	$2.5*10^{-1}$ $(3.4*10^{-1})$	$0.3*10$	0.9	$Y_t = (7.5 + 10.2\ B)\ \xi_t^{S9/92} + 12.7\ \xi_t^{I7/93} + N_t \quad \nabla N_t = a_t$ (3.5) (3.5) (2.5)
***Sim* ESP/USD**	$6.8*10^{-4}$	$8.1*10^{-5}$	-0.19	2.2	∇^1	$-2.1*10^{-6}$ $2.6*10^{-6}$	$2.6*10^{-5}$	18.4	$(1+0.52B+0.15B^2+0.23B^4)\ \nabla Y_t = a_t$ (0.10) (0.11) (0.10)
***Obs* GBP/USD**	$6.1*10^{-1}$	$4.6*10^{-2}$	-0.61	2.6	∇^1	$-1.9*10^{-3}$ $(1.5*10^{-3})$	$1.5*10^{-2}$	6.6	$Y_t = (0.08 + 0.08B)\ \xi_t^{S1092} + N_t \quad \nabla N_t = a_t$ (0.02) (0.02)
***Sim* GBP/USD**	$4.6*10^{-5}$	$4.4*10^{-6}$	-0.75	2.8	∇^1	$-9.2*10^{-8}$ $(7.3*10^{-8})$	$7.3*10^{-7}$	10.9	$(1-0.37B+0.25B^2+0.18B^4)\ \nabla Y_t = a_t$ (0.10) (0.10) (0.10)
***Obs* JPY/USD**	$1.2*10^{2}$	$1.7*10$	0.16	2.5	∇^1	$-1.0*10^{-2}$ $(3.6*10^{-1})$	3.6	22.4	$Y_t = -10.5\ \xi_t^{S5/97} + N_t \quad \nabla N_t = a_t$ (3.59)
***Sim* JPY/USD**	$2.1*10^{-6}$	$3.5*10^{-7}$	-0.47	1.9	∇^1	$-1.08*10^{-8}$ $(6.8*10^{-9})$	$6.8*10^{-8}$	18.7	$(1+0.35B)\ \nabla Y_t = a_t$ (0.09)
***Obs* ESP/DEM**	$7.6*10$	$1.0*10$	-0.43	1.4	∇^1	$5.2*10^{-2}$ $(8.1*10^{-2})$	$8.1*10^{-1}$	12.6	$Y_t = 5.37\ \xi_t^{S9/92} + (6.26-3.0B+6.45B^2)\ \xi_t^{S5/93} + 3.59\ \xi_t^{I3/95} + N_t$ (0.83) (0.83) (0.83) (0.83) (0.58) $\nabla N_t = a_t$
***Sim* ESP/DEM**	$0.3*10$	$1.6*10^{-1}$	0.53	2.6	∇^1	$1.7*10^{-3}$ $1.2*10^{-2}$	$1.2*10^{-2}$	14.5	$(1+0.64B+0.27B^2+0.29B^3)\ \nabla Y_t = a_t$ (0.10) (0.11) (0.10)
***Obs* GBP/DEM**	$3.8*10^{-1}$	$4.0*10^{-2}$	0.22	1.8	∇^1	$4.9*10^{-4}$ $(1.3*10^{-3})$	$1.3*10^{2}$	6.55	$(1+0.25B)\ \nabla Y_t = a_t$ (0.09)
***Sim* GBP/DEM**	$2.2*10^{-1}$	$6.6*10^{-3}$	-0.10	3.5	∇^1	$-1.0*10^{-4}$ $(3.6*10^{-4})$	$3.6*10^{-4}$	10	$(1-0.21B+0.41B^2)\ \nabla Y_t = a_t$ (0.10) (0.10)
***Obs* JPY/DEM**	$7.4*10$	$0.9*10$	0.47	2.2	∇^1	$-1.1*10^{-1}$ $(2.4*10^{-1})$	$2.4*10^{0}$	8.5	$\nabla Y_t = a_t$
***Sim* JPY/DEM**	$9.7*10^{-3}$	$1.0*10^{-3}$	0.33	1.9	∇^1	$3.6*10^{-5}$ $(3.1*10^{-5})$	$3.1*10^{-4}$	17.3	$(1+0.34B+0.22B^2)\ \nabla Y_t = a_t$ (0.10) (0.11) (0.10)

Notes:

(a) Estimated standard errors in parentheses

(b) $\xi_t^{I"T"} = \begin{cases} 1 & t = T \\ 0 & t \neq T \end{cases}; \qquad \xi_t^{S"T"} = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases};$

The stochastic processes for all currencies (observed and simulated) are modelled in first differences. Outliers in *ObsExRa* are analysed to conclude that the random walk process is a valid representation for six of the seven analysed cases. The first difference of time series GBP/DEM behaves as a first order autorregresive process. The analysis of *SimExRa,* suggests that one seasonal difference is not necessary. Figures 6-12 in Appendix 4 plot monthly data from 1990:01 to 1998:04 for both simulated and observed exchange rates.

---

[4] Autoregressive Integrated Moving Average (ARIMA) Model

Both, *ObsExRa* and *SimExRa* time series follow integrated processes of order 1 [I(1)], we will test for cointegration. In this framework, cointegration means that the economic model replicates the long-run evolution of actual exchange rate. Additionally, if the OLS estimate of β in (12) is positive, then the economic model also replicates the appreciation or depreciation process in the observed time series. To test for cointegration, the following model for the exchange rate data, from t=1990:01 trough 1998:04, is estimated by OLS:

$$ObsExRa_t = \beta_0 + \beta_1 SimExRa_t + u_t^{ExRa} \tag{12}$$

The standard unit root test on the estimated residuals is carried out, as well as the augmented Dickey-Fuller t test (ADF). If u_t^{ExRa} is I(0), regression (12) implies that the variables *ObsExRa* and *SimExRa* will be cointegrated with cointegrating vector (1, -β_1). Table 3 reports test results for cointegration

Table 3. Testing for cointegration among *ObsExRa* and *SimExRa*.

ExRa	β_0 [(a)]	β_1	D-F [(b) (c)]				
			1 LAG [(d)]	2 LAG	3 LAG.	4 LAG	5 LAG
DEM/USD	0.2*10 (0.1*10)	$-1.5*10^3$ ($0.6*10^3$)	-2.54	-2.26	-2.52	-2.26	-2.26
ESP/USD	$17*10^2$ ($0.08*10^2$)	$-9.3*10^4$ ($1.1*10^4$)	-1.97	-2.36	-2.16	-2.20	-2.57
GBP/USD	$2.0*10^{-1}$ ($0.4*10^{-1}$)	$6.6*10^3$ ($0.8*10^3$)	-3.41	-3.86	-3.97	-3.80	-3.94
JPY/USD	8.4*10 (1.0*10)	$1.6*10^7$ ($0.5*10^7$)	-1.45	-1.86	-2.12	-1.68	-1.65
ESP/DEM	$2.0*10^2$ ($0.2*10^2$)	-3.9*10 (0.5*10)	-2.64	-2.54	-2.23	-2.75	-2.86
GBP/DEM	$8.4*10^{-1}$ ($1.2*10^{-1}$)	-0.2*10 (0.6*10)	-1.43	-1.04	-1.16	-0.91	-1.04
JPY/DEM	1.3*10 (0.6*10)	$6.3*10^3$ ($0.6*10^3$)	-2.04	-1.89	-2.04	-1.87	-1.94

Notes:

a) Estimated standard errors in parentheses
b) Augmented Dickey-Fuller *t* test
c) Critical values are taken from MacKinnon (1990): -3.50 (1 %), -2.89 (5 %), -2.58 (10 %)
d) Report number of lags . They indicate the lag length of the autoregresive parameters in the Dickey-Fuller Tests. Regression includes a constant.

Test results are mixed:

1.- At the 5% critical value, the ADF *t* statistic in Table 3 suggests that estimated residuals $\hat{u}_t^{GBP/USD}$ are I(0) and, with $\hat{\beta}'s$ positive. Thus, the economic model seems to be able to replicate the long run evolution and depreciation of actual data. Figure 7 in Appendix 4 shows *ObsExRa* and *SimExRa* times series.

2.- ADF test reported in Table 3, for ESP/DEM exchange rate, suggests at 10 % significance level, that observed and simulated time series appear to be cointegrated, but $\hat{\beta}_1 < 0$, i.e. the economic model forecasts an appreciation when real data shows depreciation. ESP/DEM exchange rate shows outliers in 10/92 and 5/93 due to European Monetary System crisis. However, an intervention analysis reveals that cointegration tests were not distorted.

3.- For the remaining currencies the null hypothesis of no cointegration is accepted at the 10% significance level. In these cases, the economic model is able to remove seasonality from the exchange rate, but not able to replicate other important features related to long run evolution.

IV. **Conclusions**

Standard dynamic equilibrium models of exchange rate generate equilibrium pricing functions relating exchange rate to real production, monetary aggregates, and asset returns. Those equilibrium pricing functions allow for the transmission of statistical properties from production, money and asset returns to the exchange rate. This becomes a problem when seasonality is the transmitted property. Either seasonal variables are not determinants of the exchange rate or agents take these fluctuations into account when deciding their behaviours.

This paper generalises standard dynamic equilibrium models by allowing for seasonal shocks in preferences. This new feature makes the theoretical model consistent with observed stylised facts in the mentioned variables.

In contrast to prior studies, the theoretical model is tested directly with seasonal unadjusted data. Empirical results suggest that the model is able to reproduce for some currencies the stochastic process of the actual exchange rate. For instance, in the case of GBP/USD, the model captures the long-term patterns and the depreciation found in the data.

The results in this paper are modest but allow a more optimistic view of exchange rates equilibrium models.

Acknowledgements

We would like to acknowledge the financial support of the Ministerio de Educación, Spain, trough project BEC2003-03965. We are grateful to A. Aznar, M. Gracia, J. M. González-Páramo, J. Del Hoyo and A. Novales for helpful comments.

References

Altug, S., 1983. "Gestion lags and the business cycle: an empirical analysis", *Carnegie-Mellon Working Paper* , presented in The Econometric Society meeting, Stanford University, Carnegie-Mellon University, Pittsburgh, PA.

Becker, G., Stigler, G., 1977. "De Gustibus Non Est Disputandum", *American Economic Review,* 67, 76-90.

Box, G.E.P., Jenkins, G.M., 1970. *Time Series Analysis, Forecasting and Control*, ed. Holden-Day, San Francisco.

Box, G. E. P., Tiao, G. C., 1975. "Intervention Analysis with applications to economic and environmental problems", *Journal of American Statistical Association,* 70, 70-79.

Eichenbaum, M. S., Hansen L. P., Singleton, K. J., 1984. "A time series analysis of representative agent models of consumption and leisure choice under uncertainty", mimeo, Pittsburgh, Carnegie-Mellon University.

Grilli, V., Roubini, N., 1992. "Liquidity and exchange rates", *Journal of International Economics,* 33, 339-352.

Hansen, L. P., Sargent, T. J., 1993. "Seasonality and approximation error in rational expectations models", *Journal of Econometrics,* 55, 21-55.

Hansen, L. P., Singleton, K. J., 1982. "Generalized instrumental variables estimation of nonlinear rational expectations models", *Econometrica*, 50 (5), 1269-1286.

Hildreth, C., Knowles, G. J., 1986. "Farmers' Utility Functions", in *Bayesian Inference and Decision Techniques: Essays in Honor of Bruno de Finetti. Studies in Bayesian Econometrics and Statistics Series*, 6, Amsterdam and Oxford: North-Holland; distributed in the U.S. and Canada by Elsevier Science, New York, in Goel, P. and Zellner, A. (ed.), 291-317.

Jiménez, J. A., Flores, R., 2004. "The Fit of Dynamic Equilibrium Models of Exchange Rate", in evaluation. http://www.ucm.es/info/ecocuan/jajm

Kehoe, T., 1983. "Dynamics of the current account: theoretical and empirical analysis", *Working Paper*, Harvard University, Cambridge, MA.

Kydland, F. E., Prescott, E. C., 1982. "Time to Build and Aggregate Fluctuations", *Econometrica*, 50, 1345-1370.

Lancaster, K. J., 1966. “A new approach to consumer theory”, *Journal of Political Economic*, 74, 132-157.

Lucas, R. E. JR, 1982. “Interest rates and currency prices in a two-country world”, *Journal of Monetary Economics*, 10, 335-359.

Lucas, R. E. JR, 1990. “Liquidity and interest rates”, *Journal of Economic Theory,* 50, 237-264.

MacKinnon, J., 1990. "Critical Values for Cointegration Tests", University of California at San Diego, Economics Working Paper Series 90-4, Department of Economics, UC San Diego.

Mankiw, N. G., 1985. “Consumer durables and the real interest rate”, *Review Economics and Statistic*, 67, 353-362.

Mankiw, N. G., Rotemberg, J. J., Summers, L. H., 1985. “Intertemporal substitution in macroeconomics”, *Quarterly Journal Economic*, 100, 225-251.

Mehra, R., Prescott, E. C., 1985. “The Equity premium: a puzzle”, *Journal of Monetary Economics,* 15, 145-161.

Miron, A. J., 1986. “Seasonal fluctuations and the life cycle-permanent income model of consumption”, *Journal of Political Economy,* 94, 1258-1279.

Sims, C. A., 1993. “Rational expectations modelling with seasonally adjusted data”, *Journal of Econometrics,* 55, 9-19.

Wallis, K. F., 1974. “Seasonal Adjustment and the relation between variables”, *Journal of the American Statistical Association,* 69, 13-32.

Appendix 1.- The data and stochastic properties

Monthly seasonally–unadjusted data from 1986:01 to 1998:04 are used for five countries: Germany (GM), Spain (SP), Japan (JP) United Kingdom (UK), and United States (US). Monetary aggregate, M2, is taken from EcoWin. Industrial Production Index (IPI) is used as a proxy for income, and is compiled from OCDE. The exchange rates of German Mark (DEM), Japanese yen (JPY), Spanish Peseta (ESP), and British Pound (GBP) relative to US dollar are taken from OCDE.

Asset return data are generated by taking the first difference of the natural logarithm of equity price index: DAX-XETRA (DAX) for GM, the General Index of the Madrid Stock Exchange (IGBM) is sufficiently representative of the Spanish stock exchange, for JP Nikkei-225 index (NIKKEI) is used, the FT-100 (FT) for UK, and Dow-Jones (DJ) for US, (December 1994=100). Stock index data are taken from Financial Times, London. Table 4 below reports time series analysis. Previously to the simulation we start by checking for the presence of extreme values. We performed intervention analysis [Box and Tiao, 1975]. Time series analysis of data indicates that these series do not display mean-reversion and hence, they are I(1) process. IPI and M2 series show very regular seasonal patterns. The random walk process is consistent with the data generating process of the exchange rate, and stock index. All stock index show extreme values in 1987 October crash.

Table .4. Summary of ARIMA models fitted to real data[(a)]

Variables	$\nabla^d \nabla_s$	ARIMA (R) (p, d, q)	ARIMA (S) (P, D, Q)	$\bar{a}$ $(\sigma_{\bar{a}})$	$\hat{\sigma}_a$	Q(12)	Outliers[(b)]	ARIMA MODELS [(c) (d)]
IPIGM	$\nabla\nabla_{12}$	(2,1,0)	(2,1,0)	0.10 (0.15)	1.85	12.41	4/90,6/91,8/91*,11/91, 10/92,12/92,1/93,7/97	$(1+0.57 B +0.21 B^2)(1+0.58 B^{12} +0.36 B^{24}) \nabla\nabla_{12}Y_t = a_t$ (0.08) (0.08) (0.08) (0.08)
IPIJP	$\nabla\nabla_{12}$	(0,1,2)	(3,1,0)	0.24 (0.21)	2.43	12.65		$(1+0.32B^{12} +0.23 B^{24}+0.44 B^{36}) \nabla\nabla_{12} Y_t = (1-0.58B + 0.29 B^2) a_t$ (0.08) (0.09) (0.09) (0.08) (0.08)
IPISP	$\nabla\nabla_{12}$	(2,1,0)	(3,1,1)	0.24 (0.21)	2.43	12.65	7/87,3/88,10/91,10/92*, 12/93,4/97	$Y_t = - 6.97 >_t^{SS} + N_t$ (0.71) $(1+0.73 B +0.34 B^2)(1-0.23B^{12} +0.30 B^{24}+0.21 B^{24}) \nabla\nabla_{12}N_t = (1-0.84 B^{12}) a_t$ (0.08) (0.08) (0.08) (0.07) (0.07) (0.03)
IPIUK	∇_{12}	(3,0,0)	(3,1,1)	-0.006 (0.128)	1.46	19.05	9/88,9/90,4/91,5/92,1/93, 10/93, 2/96	$Y_t = -2.193\ \xi_t^{SS} + N_t$; (0.472) $(1+0.002 B - 0.296 B^2 - 0.583 B^3) (1- 0.245 B^{12} + 0.343 B^{24} + 0.184 B^{36}) [\nabla_{12} N_t - 1.684] = (1-0.852 B^{12}) a_t$ (0.072) (0.067) (0.071) (0.091) (0.078) (0.076) (0.172) (0.029)
	$\nabla\nabla_{12}$	(2,1,0)	(3,1,1)	0.162 (0.131)	1.6	19.36	9/88,9/90,4/91,5/92, 2/96 11/97.	$Y_t = -2.105\ \xi_t^{SS} + N_t$; (0.481) $(1+0.962 B +0.623 B^2) (1- 0.221 B^{12} + 0.303 B^{24} + 0.187 B^{36}) \nabla\nabla_{12} N_t = (1-0.826 B^{12}) a_t$ (0.068) (0.068) (0.094) (0.078) (0.076) (0.039)
IPIUSA	$\nabla\nabla_{12}$	(3,1,0)	(1,1,0)	0.045 (0.050)	0.58	9.82	7/89, 11/90*, 12/90, 4/95, 2/96.	$Y_t = -0.675\ \xi_t^{SS} + N_t$; (0.134) $(1+0.045 B - 0.131 B^2 - 0.239 B^3) \nabla\nabla_{12} N_t = (1-0.532 B^{12}) a_t$ (0.083) (0.084) (0.084) (0.073)
	$\nabla\nabla_{12}$	(3,1,0)	(3,1,0)	0.047 (0.049)	0.57	5.83	2/87, 7/89, 11/90*, 12/90 2/96.	$Y_t = -0.711\ \xi_t^{SS} + N_t$; (0.116) $(1+0.051 B - 0.182 B^2 - 0.217 B^3)(1- 0.386 B^{12} + 0.360 B^{24} + 0.257 B^{36}) \nabla\nabla_{12}N_t = a$ (0.084) (0.084) (0.085) (0.084) (0.084) (0.087)
M2GM	$\nabla\nabla_{12}$	(3,1,0)	(3,1,0)	0.12 (0.43)	5.06	9.66	3/98,9/90,12/90,12/91, 1/92,12/92,11/94,12/96	$Y_t = 82.56 >_t^{S6/90} + 26.40 >_t^{S12/93} + 14.16 >_t^{E1296} + N_t$ (4.11) (4.37) (3.09) $(1-0.006 B - 0.12 B^2 - 0.23 B^3) (1+0.38 B^{12}+ 0.27 B^{24} + 0.34 B^{36}\nabla\nabla_{12}N_t = a_t$ (0.09) (0.08) (0.08) (0.09) (0.08) (0.08)
	$\nabla\nabla_{12}$	(3,1,0)	(3,1,0)	0.11 (0.47)	5.43	8.82	9/90,12/91,1/92,7/94, 12/96*	$Y_t = 82.66 >_t^{S6/90} + 31.91 >_t^{S12/93} + N_t$ (4.27) (4.48) $(1+0.04 B - 0.11 B^2 - 0.20 B^3) (1+0.35 B^{12}+ 0.27 B^{24} + 0.42 B^{36}\nabla\nabla_{12}N_t = a_t$ (0.08) (0.08) (0.08) (0.08) (0.09) (0.09)
	$\nabla\nabla_{12}$	(3,1,0)	(2,1,0)	0.26 (0.50)	5.81	9.08	12/90,12/91,1292,3/93, 12/93*,12/94,12/96*	$Y_t = 85.03 >_t^{S6/90} + N_t$ (4.86) $(1+0.08 B - 0.08 B^2 - 0.32 B^3) (1+0.70 B^{12}+ 0.29 B^{24}\nabla\nabla_{12}N_t = a_t$ (0.08) (0.08) (0.08) (0.08) (0.10)
M2JAP	$\nabla\nabla_{12}$	(2,1,0)	(2,1,0)	-0.07 (0.15)	1.73	14.17	11/90,2/91,9/92,4/93, 11/95	$Y_t = 4.12 >_t^{S4/90} - 4.48 >_t^{S11/90} + N_t$ (1.33) (1.29) $(1-0.35 B - 0.15 B^2) (1+0.31 B^{12}+ 0.35 B^{24}) \nabla\nabla_{12} N_t = a_t$ (0.08) (0.09) (0.08) (0.08)
	$\nabla\nabla_{12}$	(2,1,0)	(3,1,1)	0.07 (0.16)	1.85	17.05	4/90*,11/90*,2/91,11/95	$(1-0.29 B - 0.22 B^2) (1+0.39 B^{12}+ 0.37 B^{24})\nabla\nabla_{12}Y_t = a_t$ (0.08) (0.09) (0.08) (0.08)
M2SP	$\nabla\nabla_{12}$	(2,1,0)	(0,1,1)	8.46 (13.07)	151.85	8.67	8/89, 4/90, 12/90, 2/92, 10/92, 7/97	$(1-0..25B-0.35B^2) \nabla\nabla_{12}Y_t = (1+0.43B^{12}) a_t$ (0.08) (0.08) (0.080
M2USA	$\nabla\nabla_{12}$	(1,1,0)	(0,1,1)	0.012 (0.773)	8.98	10.85	5/93, 5/94, 6/95, 3/96, 8/97.	$(1-0.561B)\nabla\nabla_{12} Y_t = (1-0.480B^{12}) a_t$ (0.072) (0.080)
	$\nabla\nabla_{12}$	(1,1,0)	(-1,1,0)	0.109 (0.779)	9.05	8.78	5/87, 2/93, 5/93, 5/94 6/95, 3/96, 4/96,8/97	$(1-0.558B) (1-0.480B^{12}\nabla\nabla_{12}Y_t = a_t$ (0.072) (0.078)

Table .4 (continued). Summary of ARIMA models fitted to real data

Variables	$\nabla^d\nabla_s$	ARIMA (R) (p, d, q)	ARIMA (S) (P, D, Q)	$\overline{a}$ $(\sigma_{\overline{a}})$	$\hat{\sigma}_a$	Q(12)	Outliers[(b)]	ARIMA MODELS
M2UK	$\nabla\nabla_{12}$	(1,1,0)	(1,1,0)	-0.017 (0.156)	1.81	9,38	6/89*, 7/96, 6/97, 9/97*	$Y_t = -21.452\ \xi_t^{S12/92} + N_t$ $(1+0.210\ B)\ \nabla\nabla_{12}N_t = (1-0.805\ B^{12})\ a_t$ (1.729) (0.084) (0.046)
	$\nabla\nabla_{12}$	(1,1,0)	(3,1,0)	-0.010 (0.157)	1.83	11,63	6/89*, 6/90, 9/92, 7/96, 6/97*, 9/97*	$Y_t = -21.238\ \xi_t^{S12/92} + N_t$ $(1+0.154\ B)(1+0.665\ B^{12} +0.436\ B^{24} +0.375\ B^{36})\ \nabla\nabla_{12}N_t = a_t$ (1.622) (0.087) (0.094) (0.108) (0.091)
	$\nabla\nabla_{12}$	(-3,1,0)	(1,1,0)	-0.026 (0.113)	1.32	12,44	11/90, 9/92, 6/93,5/94, 4/95, 3/96,7/96,8/96	$Y_t = -7.598\ \xi_t^{S6/89} - 22.457\ \xi_t^{S12/92} + 6.157\ \xi_t^{S6/97} - 6.860\ \xi_t^{I9/97} + N_t$ $(1+0.174\ B +0.229\ B^2 +0.374\ B^3)\ \nabla\nabla_{12}N_t = (1-0.648\ B^{12})\ a_t$ (1.100) (1.106) (1.265) (0.971) (0.082) (0.079) (0.081) (0.067)
	$\nabla\nabla_{12}$	(3,1,0)	(3,1,0)	-0.025 (0.113)	1.31	10,68	11/90, 9/92, 3/96, 7/96, 4/97.	$Y_t = -7.671\ \xi_t^{S6/89} - 22.414\ \xi_t^{S12/92} + 6.676\ \xi_t^{S6/97} - 6.701\ \xi_t^{I9/97} + N_t$; $(1+0.156\ B +0.254\ B^2 +0.383\ B^3)(1+0.636\ B^{12} +0.380\ B^{24} +0.280\ B^{36})\ \nabla\nabla_{12}N_t = a_t$ (1.085) (1.077) (1.255) (0.960) (0.082) (0.078) (0.084) (0.091) (0.107) (0.095)
DAX	∇	(1,1,0)	(0,0,0)	0.004 (0.37)	4.43	13.99	8/86,4/90,9/94,3/95,1/96 , 11/97,12/97,2/98,3/98*	$Y_t = -14.91 >_t^{S10/87} + (-15.44 - 16.72\ B) >_t^{S8/90} + (29.76 + 7.63\ B +19.70\ B^2) >_t^{S7/97} + N_t$ $(1-0.17B)\ [\nabla N_t - 1.56) = a_t$ (4.39) (4.44) (4.44) (3.81) (4.61) (3.78) (0.08) (0.44)
	∇	(2,1,0)	(0,0,0)	-0.00 (0.46)	5.60	8.92	9/94,7/97(5.20)*,8/97(4.20) *,10/97,12/97,3/98	$Y_t = -17.23 >_t^{S10/87} + (-16.00 - 15.12\ B) >_t^{S8/90} + N_t$ $(1+0.09B - 0.21\ B^2)\ [\nabla N_t - 1.61) = a_t$ (5.47) (5.48) (5.48) (0.08) (0.08) (0.52)
IGBM	∇	(0,1,0)	(0,0,0)	0.00 (0.52)	6.36	17.33	8/90,6/97,9/97*,3/98*, 4/98*	$Y_t = -32.72 >_t^{I1087} - 18.61 >_t^{S9/90} - 20.75 >_t^{I10/97} + 19.28 >_t^{S1/98} + N_t$ $\nabla N_t = 1.64 + a_t$ (6.38) (6.38) (4.50) (3.22) (0.53
	∇	(1,1,0)	(0,0,0)	-0.01 (0.61)	7.37	10.08	6/97,9/97,10/97*,11/97, 1/98,2/98*,3/98*	$Y_t = -32.05 >_t^{I1087} - 17.63 >_t^{E9/90} + N_t$ $(1-0.17B)\ [\nabla N_t - 2.15) = a$ (7.31) (7.40) (0.08) (0.74)
NIKKEI	∇	(0,1,0)	(0,0,0)	0.20 (0.52)	6.32	17.20	2/90,11/90,2/91,6/91, 11/91,11/93,1/94	$Y_t = -16.46 >_t^{S10/87} + (-20.91 - 20.45\ B) >_t^{I3/90} + (-15.77 - 31.22) >_t^{I8/90} + N_t$ $\nabla N_t = a_t$ (6.32) (5.17) (5.17) (5.17) (5.17)
FT ($\lambda = 0$)[(e)]	∇	(0,1,0)	(0,0,0)	$1.6*10^{-18}$ $(3.5*10^{-3})$	4.2 %	8.5	01/89,10/89,05/90,10/97	$Y_t = (-0.31 - 0.11\ B)\ \xi_t^{S1087} + N_t$; $(\nabla N_t - 0.012) = a_t$ (0.02) (0.02) (0.003)
DJ ($\lambda = 0$)	∇	(1,1,0)	(0,0,0)	$-1.5*10^{-13}$ $(3.1*10^{-3})$	3.8%	14.4	01/87,11/87,8/90, 9/90	$Y_t = -0.31\ \xi_t^{S1087} + N_t$; $(1 + 0.21\ B)(\nabla N_t - 0.014) = a_t$ (0.04) (0.08) (0.003)

Notes:

(a) ∇: *Difference Operator*; **B**: *backward shift operator;* $\nabla_s = (1-B^s)$. Descriptive statistics of the residuals from estimated models are reported: mean and estimated mean standard error $\overline{a}\ /\left(\hat{\sigma}_{\overline{a}}\right)$, estimated standard errors ($\hat{\sigma}_a$) and Ljung-Box Q-statistics at lag 12 to test for serial correlation (Q(12)).

(b) Residuals over two standard errors (* Residuals over three standard errors)

(c) Estimated standard errors in parentheses

(d) $\xi_t^{I"T"} = \begin{cases} 1 & t = T \\ 0 & t \neq T \end{cases}$; $\xi_t^{S"T"} = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases}$; $\xi_t^{SS} = \begin{cases} 1 & t = \text{month within Easter holidays} \\ 0 & \text{Re}\ st \end{cases}$

(e) Box-Cox transformation

Appendix 2: The two-country exchange rate model

In this appendix we present additional detail of two country model from section II, including the first-order conditions. There are two country, Domestic (D) and Foreign (F). Each country has a firm, each specialized in its exogenous stochastic endowment of a perishable, traded, and distinct good: Y_t^D, Y_t^F. Firms are assumed to be able to sell claims of their future outputs. Domestic (Foreign) claims entitle the owner to a proportionate share in the future stream of dividends.

Representative agents from both countries have preferences described by identical infinite-horizon expected utility functions given by:

$$Et\left[\sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}^{D}, c_{is}^{F})\right] \quad 0 < \beta < 1 \tag{A1}$$

The pattern of trading is assumed to proceed in the following way. As in Lucas (1990) we assume the convenient artifact of a three- member representative household, each of whom goes his own way during a period and the three regrouping at the period of a day to pool goods, assets, and information. One member of the household (the owner of firm) collects the endowments, which he must then sell to other households on a cash in advance. A household cannot consume any of its own endowments. Cash received from sale on date-*t* cannot be used for any purpose during period *t.* A second member of the household takes an amount N_t-Z_t of household's initial cash balances (N_t) and uses it to purchase goods from other households. Domestic (foreign) goods can be bought only with domestic (foreign) currency. The third member carries out the remaining domestic and foreign currency cash balances in the security market where domestic and foreign securities are sold and bought, and where he can buy and sell domestic and foreign currencies in the foreign exchange market. Also, domestic (foreign) assets can be bought only with domestic (foreign) cash balances. Only two securities are supposed to exist in the security market, domestic and foreign equity claims (shares in domestic and foreign future outputs) .

The transaction technology is that of the cash in advance model, extended by the assumption that the representative agent faces two liquidity constraints, on the purchase of goods and on the purchase of assets. Home and foreign cash in advance constraints for the goods market in period *t* are:

$$N_{it}^{D} = P_{t}^{D}C_{it}^{D} \Rightarrow C_{it}^{D} = \frac{N_{it}^{D}}{P_{t}^{D}} \qquad i = D,\ F, \tag{A2}$$

$$N_{it}^{F} = P_{t}^{F}C_{it}^{F} \Rightarrow C_{it}^{F} = \frac{N_{it}^{F}}{P_{t}^{F}} \qquad i = D, F, \tag{A3}$$

Where, N^{j}_{it} is the amount of money of country j (j=D, F) hold by the representative agent of country i (i=D, F) for transactions in the goods market at time t. P_t^j is the domestic and foreign currency prices for good j.

The agents who transacts in the asset market face the budget constraints given by:

$$[M_{it}^{D} - N_{it}^{D}] + S_t[M_{it}^{F} - N_{it}^{F}] = Q_t^{D}\omega_{it}^{D} + S_tQ_t^{F}\omega_{it}^{F} \quad i = D, F \tag{A4}$$

$$P_t^jY_t^j = \left(\omega_{D,t}^{j} + \omega_{F,t}^{j}\right)Q_t^j \quad j = \mathrm{D, F}, \tag{A5}$$

Where S_t is the nominal spot exchange rates expressed as the domestic price for foreign currency. M^{j}_{it} are i holdings (i=D, F) of money j (j=D, F) on date t. Q^j_t is the price of equity j in units of currency j (j=D, F). ω^{j}_{it} is the number of equities of country j (j=D, F) purchased at t by a resident of country i (i=D, F).

At the beginning of period $t+1$, the ownership of an equity entitles the owner to receive the dividend in t and to have the right to sell the equity at Q^{j}_{t+1} price. Therefore, the agent will begin $t+1$ with cash balances given by

$$M_{it+1}^{D} = d_t^{D}\omega_{it}^{D} + Q_{t+1}^{D}\omega_{it}^{D} \qquad i = D, F \tag{A6}$$

$$M_{it+1}^{F} = d_t^{F}\omega_{it}^{F} + Q_{t+1}^{F}\omega_{it}^{F} \qquad i = D, F \tag{A7}$$

Where d^j_t are dividends per equity of firm j (j=D, F).

The agent chooses $\{N^D_{Dt}, N^F_{Dt}, \omega^D_{Dt}, \omega^F_{Dt}\}^{\infty}_{t=0}$ to maximize (A1) subject to the cash in advance constraints (A2)-(A3), the budget constraint (A4) and the transition equation for state variables (A6)-(A7). The agent's decision problem motivates the Bellman equation,

$$\mathrm{V}\left[\mathrm{M}^{\mathrm{D}}_{\mathrm{Dt}}, \mathrm{M}^{\mathrm{F}}_{\mathrm{Dt}}\right] = \mathrm{Max}\ \mathrm{U}\left[\mathrm{N}^{D}_{Dt}/\mathrm{P}^{D}_{t}, \mathrm{N}^{F}_{Dt}/\mathrm{P}^{F}_{t}\right] + \beta\,\mathrm{E}_t\left\{\mathrm{V}\left[\mathrm{M}^{\mathrm{D}}_{\mathrm{Dt+1}}, \mathrm{M}^{\mathrm{F}}_{\mathrm{Dt+1}}\right]\right\} \tag{A8}$$

First order and envelope conditions associated to the problem stated in (A8) are used to characterize equilibrium behaviour, assuming that the value functions exist and are increasing, differentiable, and concave,

$$\frac{\partial V}{\partial N^{D}_{Dt}} = \frac{U_D\left[c^D_{Dt}, c^F_{Dt}\right]}{P^D_t} = \lambda_t \tag{A9}$$

$$\frac{\partial V}{\partial N^{F}_{Dt}} = \frac{U_F\left[c^D_{Dt}, c^F_{Dt}\right]}{P^F_t} = \lambda_t S_t \tag{A10}$$

$$\frac{\partial V}{\partial \omega^{D}_{Dt}} = Q^D_t \lambda_t = \beta\, E_t\left[V'_{M^D_{Dt+1}}\left(d^D_t + Q^D_{t+1}\right)\right] \tag{A11}$$

$$\frac{\partial V}{\partial \omega^{F}_{Dt}} = Q^F_t \lambda_t S_t = \beta\, E_t\left[V'_{M^F_{Dt+1}}\left(d^F_t + Q^F_{t+1}\right)\right] \tag{A12}$$

Where λ_t is the multiplier associated to the budget constraint.

The envelope conditions are:

$$V'_{M^D_{Dt+1}} = \frac{U_D\left[c^D_{Dt+1}, c^F_{Dt+1}\right]}{P^D_{t+1}} \tag{A13}$$

$$V'_{M^F_{Dt+1}} = \frac{\partial U_F\left[c^D_{Dt+1}, c^F_{Dt+1}\right]}{P^F_{t+1}} \tag{A14}$$

Substituting (A9) and (A13) into (A11) domestic firm equity price is given by:

$$Q^D_t = E_t\left\{\frac{U_D\left[c^D_{Dt+1}, c^F_{Dt+1}\right]/P^D_{t+1}}{U_D\left[c^D_{Dt}, c^F_{Dt}\right]/P^D_t}\left[d^D_t + Q^D_{t+1}\right]\right\} \tag{A15}$$

Symmetrically, substituting (A10) and (A14) into (A12) the foreign firm equity price is given by:

$$Q^F_t = \beta\, E_t\left\{\frac{U_F[c^D_{Dt+1}, c^F_{Dt+1}]/P^F_{t+1}}{U_F[c^D_{Dt}, c^F_{Dt}]/P^F_t}\left[d^F_t + Q^F_{t+1}\right]\right\} \tag{A16}$$

Finally, equilibrium exchange rate is obtained from first order conditions (A11)-(A12), and envelope conditions (A13)-(A14):

$$S_t = \frac{Q^D_t}{Q^F_t}\,\frac{E_t\left(\frac{U_F\left[c^D_{Dt+1}, c^F_{Dt+1}\right]}{P^F_{t+1}}\left[d^F_t + Q^F_{t+1}\right]\right)}{E_t\left(\frac{U_D\left[c^D_{Dt+1}, c^F_{Dt+1}\right]}{P^D_{t+1}}\left[d^D_t + Q^D_{t+1}\right]\right)} \tag{A17}$$

Appendix 3: Diagnostic analysis of GMM estimation. Residual graphs, ACF and PACF.

Figure .1

GERMANY

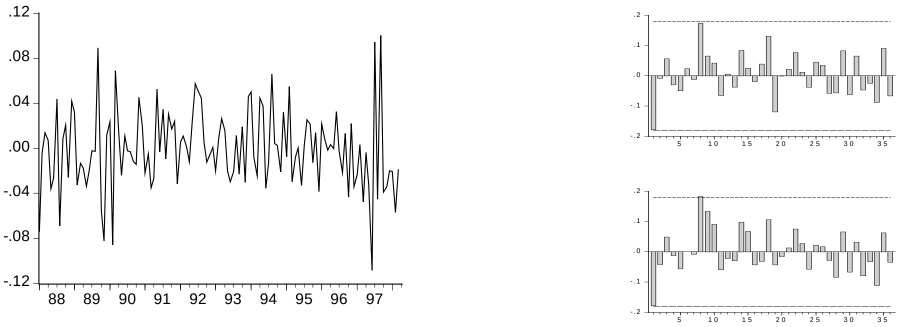


Figure 2

SPAIN

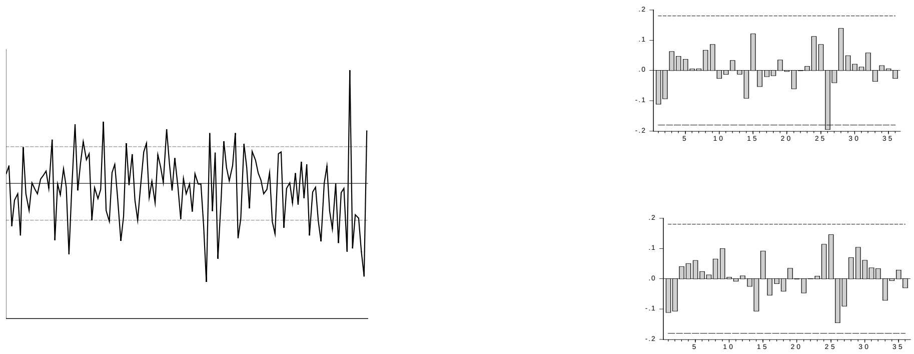


Figure.3

JAPAN

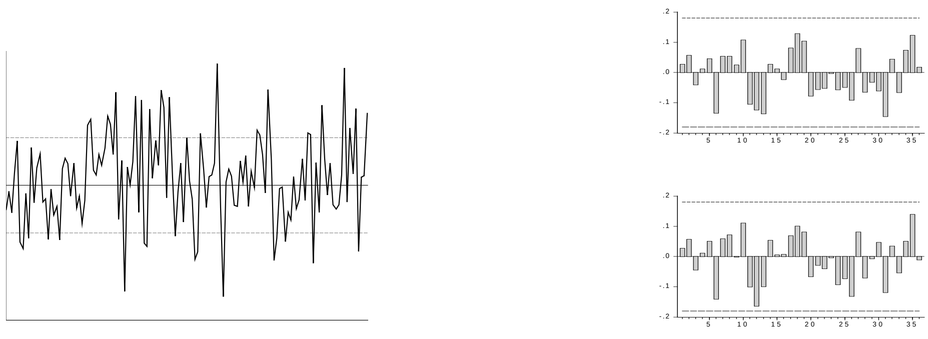


Figure 4

UNITED KINDONG

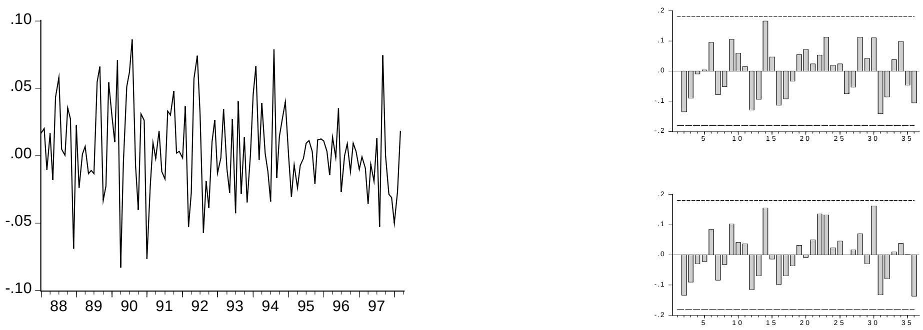
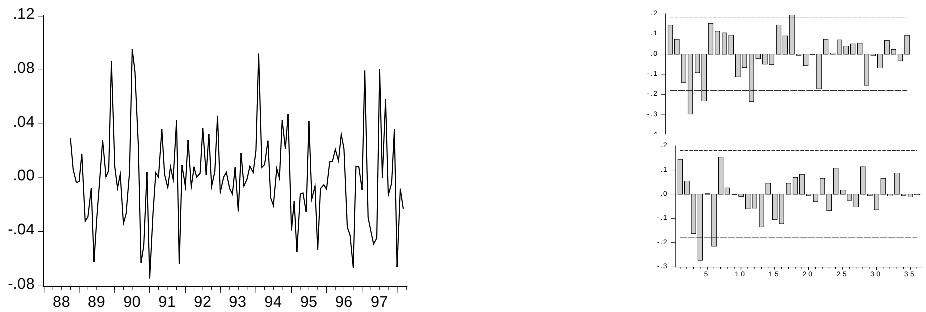


Figure 5

UNITED STATES



Appendix 4: *ObsExRa* and *SimExRa*

Figure .6

Obs DEM / USD (Left) & *Sim* DEM / USD (Right)

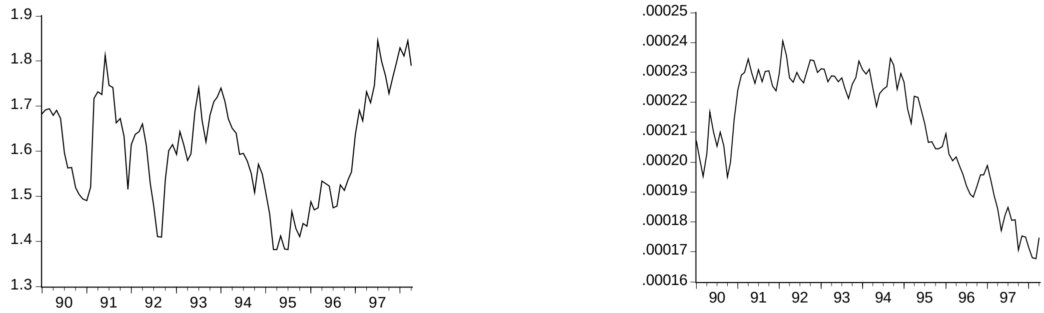


Figure 7

Obs GBP / USD (Left) & *Sim* GBP / USD (Right)

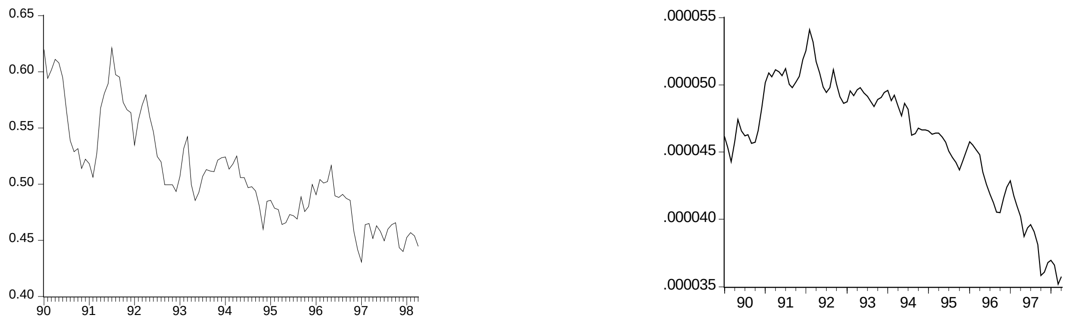


Figure.8

Obs ESP / USD (Left) & *Sim* ESP / USD (Right)

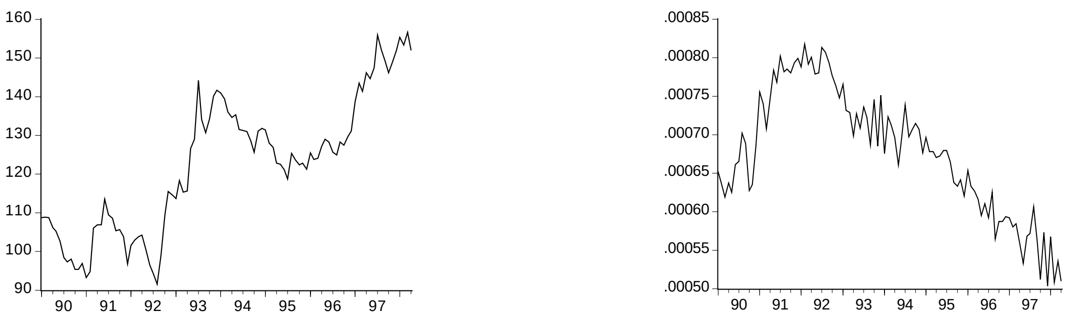


Figure 9

Obs JPY / USD (Left) & *Sim* JPY / USD (Right)

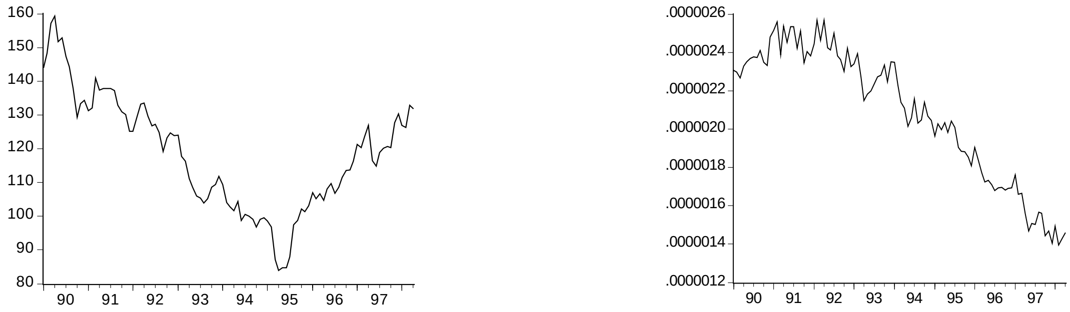


Figure 10

Obs GBP / DEM (Left) & *Sim* GBP/ DEM (Right)

Figure 11

Obs ESP / DEM (Left) & *Sim* ESP / DEM (Right)

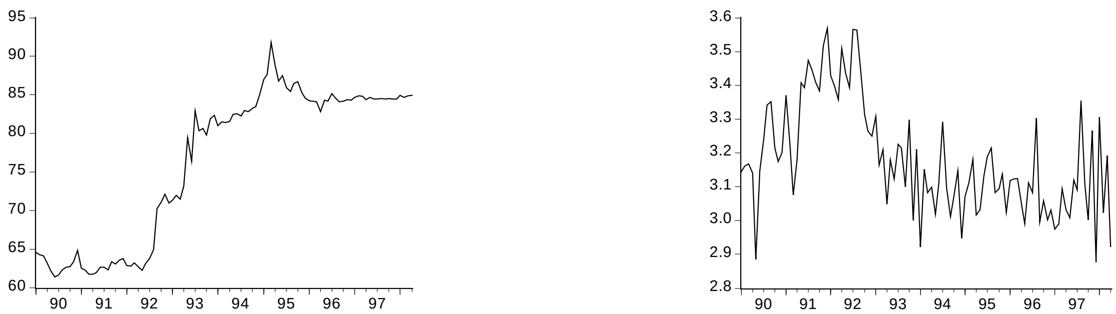


Figure.12

Obs JPY / DEM (Left) & *Sim* JPY / DEM (Right)