



University of Catania - Department of Economics

Working Paper Series

n° 2010/05 – January 2010

Inequality of opportunity in the credit market

by

Giuseppe Coco

Giuseppe Pignataro

Inequality of Opportunity in the Credit Market

Giuseppe Coco* and Giuseppe Pignataro†
University of Bari and University of Bologna

January 19, 2010

Abstract

Credit market imperfections can prevent the poor from making profitable investments. Under asymmetric information observable features, such as wealth and collateral, play an important role in determining who gets credit, in violation of the *Equality of Opportunity* principle. We define equality of opportunity as the equal possibility of getting credit for a given aversion to effort. We first establish that, due to larger cross subsidization in high collateral classes of borrowers, richer individuals are more likely to get credit for a given aversion to effort. Our second result is that *Inequality of Opportunity* is associated with an inefficient allocation of resources among classes of borrowers. The marginal borrower in classes that post more collateral exerts less effort in equilibrium (and therefore produces lower aggregate surplus) than the marginal borrower in lower collateral classes. This suggests that public credit policies should be targeted at poorer classes of would be borrowers both for equity and efficiency reasons, which rarely occurs in practice.

Keywords: equality of opportunity, credit, moral hazard, cross-subsidization, collateral

JEL classification: D63, D8, H8

*Corresponding author: Department of Economics, University of Bari, Bari, Italy. E-mail: g.coco@dse.uniba.it

†Corresponding author: Department of Economics, University of Bologna, Bologna, Italy. E-mail: giuseppe.pignataro@unibo.it

1 Introduction

Credit market imperfections can prevent the poor from making profitable investments. Asymmetric information is crucial to understand the role of individual wealth in relation to credit availability. Lack of collateral may obviously represent a barrier. Under full information each project is funded only upon the evaluation of the borrowers' effort. Under asymmetric information instead observable features such as wealth and collateral play an important role in determining who gets credit. Collateral is important also because it influences the willingness of individuals to supply effort by itself and also through lower interest rates. We focus on the relation between equality of opportunity and credit market to determine if under these circumstances the possibility of getting credit is determined by collateral.

The seminal contribution on the effect of asymmetric information in the credit market is due to Stiglitz and Weiss (1981) who demonstrate that the credit market may not clear due to the possible existence of an interest rate above which default rises so much that the profitability of the banks starts to decrease.

Bester (1985) however shows that credit rationing is not general at all, based on an enriched model. Different amounts of collateral could be devised to determine separating equilibria among different classes of borrowers, solving the asymmetric information problems. Moreover, Riley (1987) underlines that credit rationing will occur only for some classes of borrowers determined by their lowest wealth level. An increase in the number of (observable) social types progressively reduces the rationing phenomenon and only in the limit redlining may appear.

A different framework proposed by De Meza and Webb (1987) is characterized not only by the absence of rationing but also by excessive lending. They show that competitive equilibria can arise where some borrowers realize projects whose social benefits do not compensate their social costs. Cross-subsidization among safe and risky borrowers occurs due to asymmetric information. An increase in the amount of collateral delivers a reduction in cross-subsidies. On the same perspective, De Meza and Webb (1999) study how credit-market requirements may exclude low-wealth individuals, irrespective of the exogenous quality of their projects.

Following such line of analysis, we construct a model in which we analyze how a competitive equilibrium performs in terms of equality of opportunity. In particular our model is characterized by heterogeneity in the intrinsic fea-

tures of the investor, notably her aversion to effort, asymmetric information and standard moral hazard. Entrepreneurs differ also for observable wealth, which they post entirely as collateral. As in De Meza and Webb (1987, 1999) asymmetric information delivers overinvestment and cross subsidization for investor in each wealth class. However, at high levels of effort aversion richer individuals get credit while poorer ones do not. This is due to several reasons: on one side collateral mitigates the moral hazard problem and delivers more effort spent for the same level of effort aversion. Moreover richer individuals are charged lower interest rate which in turn has the same effect. Finally, and more importantly, cross subsidization occurs to a larger extent in richer borrowers' classes. For all these reasons we find that the equality of opportunity principle, defined in this context as the equal possibility of getting credit given one individual's personal features (e.g. her effort aversion) is violated. Poorer individuals have far less chance to participate in the credit market.

Our next line of inquiry concerns the relative efficiency of lending to different classes of borrowers. To this aim we investigate whether, in the resulting equilibrium, rich investors participating actually exert more or less effort. From what we know poorer individuals exert less effort, for any given effort aversion, due to the incentive effect of collateral. However richer individuals participate also for lower level of effort aversion, due to larger cross subsidization in their class. We show that for the marginal individuals participating in the credit market in different classes of wealth, effort spent is inversely correlated with wealth. This result suggests that a public policy that transfers resources and credit from rich to poor people at the margin increases output.

In the next section, after setting up our model we discuss the trade-off between collateral, repayment and effort in the benchmark case of self-financed project. Thereafter, focusing on the behavior of the marginal borrower, we observe how cross subsidization takes place in each borrower's class. The analysis goes on in section 3 demonstrating the violation of the equality of opportunity principle through the correlation between repayment and effort aversion. Section 4 is devoted to test the incentive effect evaluating the differential inefficiency of the marginal borrowers in different wealth classes due to a wrong allocation of resources from the bank. Conclusions follow in section 5.

2 The model

2.1 The Projects

Consider a project with capital requirement K . It yields a fixed gross return Y with probability $p(e)$ or zero revenue with probability $1 - p(e)$, where e is the amount of effort. Returns to effort are positive and diminishing as usual, i.e. $p'(e) > 0$ and $p''(e) < 0$. In more general terms, we can express the distribution of returns $F(e)$ in terms of the random probability of success $p(e)$, such that greater effort reflects a continuous distribution of returns which stochastically dominates any distribution with lower effort. Each borrower must raise an outside finance.

2.2 The Borrowers

We consider an economy with a finite number of would be borrowers each endowed with a project described above. Agents are risk-neutral and are characterized by different levels of effort aversion, which is the responsibility variable in the opportunity egalitarian context. The effort cost for a borrower i , with effort aversion μ_s is $\mu_s e$. At the same time, each agent belongs to a certain class of wealth, j , that they put up entirely as collateral c_j ¹, where $c_j < K \forall j$. The borrower's aversion to effort and her actual effort choice are assumed to be private information and are independently distributed from c_j , while we will not make any explicit assumption about the distribution of the effort aversion parameter μ_s . Let $X_j = (1 + r_j)K$ be the total repayment where r_j is the interest rate required by the bank for class of observable collateral. The borrowers' expected utility when the project is funded is given by:

$$U_i = p(e)(Y - X_j) - (1 - p(e))c_j - \mu_s e \quad (1)$$

¹They cannot choose the preferable amount of wealth to realize their project. Therefore, in this analysis wealth and collateral assume the same meaning.

2.3 The lenders

The lenders do not know the effort characteristics of borrowers. They know the population distribution of effort aversion, whatever it is, observe the wealth of each borrower, which they post as collateral, and therefore charge different interest rates to different classes of borrowers according to their wealth. We assume zero risk-free interest rate. For a single borrower, the representative bank's profit in a competitive market is:

$$\pi = p(e)X_j + (1 - p(e))c_j - K = 0 \quad (2)$$

2.4 First best

It can be interesting to note that if individual's investment is realized entirely as self-financed project, the effort level will be chosen with:

$$\max_e p(e)Y - \mu_s e - K \quad (3)$$

Therefore, the optimum choice e^* follows from the FOC:

$$p'(e^*) = \frac{\mu_s}{Y} \quad (4)$$

This represents our first-best level of effort (benchmark case).

2.5 Comparative statics

Now we analyze the possibility to receive a loan from a bank in order to invest in the same project. In a context of hidden action, we assume that is not verifiable by the banks, hence it is not contractible. Moreover, there is limited liability, i.e., if projects returns are less than the repayment obligations, the borrowers bear no responsibility to pay out of pocket. The effort choice of a borrower follows from:

$$\max_e p(e)(Y - X_j) - (1 - p(e))c_j - \mu_s e \quad (5)$$

Therefore, the optimal choice $\tilde{e}(Y, X, c)$ is described by the following FOC:

$$p'(\tilde{e}) = \frac{\mu_s}{Y + c_j - X_j} \quad (6)$$

From straightforward comparative statics it follows:

$$\frac{d\tilde{e}}{dY} > 0; \quad \frac{d\tilde{e}}{dc} > 0; \quad \frac{d\tilde{e}}{dX} < 0 \quad (7)$$

Proof. See the appendix ■

We can see that $\tilde{e}(Y, X, c)$ is increasing in c and decreasing in X . Reasonably, if the borrower works harder, the probability of success increases and the risk of default decreases. A higher repayment negatively affects the borrower's return in case of success, but not in the case of failure, thus reducing incentives to apply more effort. On the other side, a higher amount of collateral reflects higher penalty in case of failure providing incentives to put more effort.

Looking at the representative bank's net profit, we observe that:

$$\pi = p(e)X_j + (1 - p(e))c_j - K = 0 \quad (8)$$

In equilibrium due to competitive pressure the banks are constrained to zero profit on each observable class of borrower and therefore also on the aggregate pool. The banks then maximize the borrower's utility subject to incentive compatibility curve (eq. 6). Given $p''(e) < 0$ and comparing (4) and (6), we point out that $\tilde{e} < e^*$. Equations (6) and (8) jointly determine the amount of effort and consequently the probability of success into the project. Moving along the incentive curve, the amount of repayment is decreasing. If the borrowers put higher effort in the project, the risk of default is reduced and the amount of repayment X_j must be lower to keep the net profits of banks at the competitive level. As a consequence, a decrease in X_j raises the incentive to work hard. Due to competitive market, the highest possible level

of effort is generated even if this is less than the first-best case. This implies that the source of the inefficiency is due simply by the incentive distortion in limited liability, i.e., no capital losses beyond the collateral posted.

We focus our attention on the behavior of the marginal borrower. For any class of wealth, the marginal borrower is defined as the individual who is indifferent to exit or remain active in the credit market. Due to the first order stochastic dominance, we also know that the marginal borrower is the individual with the highest aversion to effort for any class of wealth. Under asymmetric information, this implies that her repayment X_j will be below that of the full information case due to cross-subsidization. Therefore, as in DeMeza and Webb (1987, 1999, 2000), for each observable class of borrowers overlending occurs.

Remark 1: *For any borrower's' class of wealth, the more averse to effort the borrower is, the lower the amount of effort chosen*

$$\frac{d\tilde{e}}{d\mu} = \frac{1}{(Y - X + c)p''(\tilde{e})} < 0 \quad (9)$$

Remark 1 implies that individuals with a greater aversion to effort also display a higher probability of default. Since the marginal individuals capture the lowest share of project expected returns, their choice of effort is farthest from the socially efficient value showed above. However, in such framework, credit rationing is impossible given that individuals with the highest aversion to effort are the first to exit from the market as the interest rates rise. Further, a representative borrower will undertake a project if and only if:

$$\bar{U}(\tilde{e}, \mu, X) \geq 0 \quad (10)$$

From (10) a borrower enters into the credit market applying for funds if and only if:

$$p(\tilde{e}) \geq \frac{\mu_s \tilde{e} + c_j}{Y - X_j + c_j} \quad (11)$$

Looking at the marginal case, from (11), there is a cut-off probability of success below which loans are not asked by borrowers. Let us define

this value with equality as $p_M(\tilde{e})$, while the average probability of success into the project is denoted as $\bar{p}(\tilde{e})$, such that $\bar{p}(\tilde{e}) > p_M(\tilde{e})$. This implies that $\bar{p}(\tilde{e})X_j > p_M(\tilde{e})X_j$ where $\bar{p}(\tilde{e})X_j$ can be denoted as the representative bank's average payment on each demand of K . Given that the utility from the project of the marginal borrower is zero, the marginal borrower is indifferent to entry but gives rise to an expected loss of $[\bar{p}(\tilde{e}) - p_M(\tilde{e})] X_j$ to the bank. We can state that:

Remark 2: *The expected value of the marginal borrower's project is negative in equilibrium*

Further, let us define the marginal set as the set of the marginal individuals, one for each class of wealth, whose utilities are zero at the equilibrium X and c . Define the aversion to effort parameter along the marginal set as the marginal aversion to effort. It is a function of the equilibrium collateral c and repayment X for each class s . Then, differentiating the utility function of the marginal borrower (hence with the maximum value function of utility set at 0) with respect to collateral and aversion to effort, it follows that:

$$\frac{d\mu}{dc} \Big|_{U_i(X,c)=0} = \frac{p'(\tilde{e}) \left(\frac{c+\mu\tilde{e}}{\mu} \right)}{\frac{c}{\mu^2} p'(\tilde{e})(Y - X + c) + \tilde{e}} > 0 \quad (12)$$

Proof. See the Appendix ■

Formula (12) shows that along the marginal set, a higher wealth is accompanied by higher aversion to effort, i.e., richer marginal individuals are more averse to effort in the project. The intuition for the result is that in a context of hidden information, individuals are evaluated just on the basis of their collateral independently by the aversion to effort they have. Moreover richer individuals receive loans at lower interest rate and, as a consequence, spend more effort other things equal. This implies that for a richer class of borrowers, cross-subsidization is wider than in other classes.

3 Inequality of opportunity

In the previous section, we have showed that as in Bester (1985) and De Meza and Webb (1987), an increase in the amount of wealth has only positive

effects for both borrower's utility and lender's profit function. No other effect is taken into account. However, Stiglitz and Weiss (1992) show that an increase in the amount of collateral may have quite different impacts with respect to Bester's environment. Under certain conditions, notably decreasing risk aversion, richer borrowers are those who are more willing to undertake riskier projects². As a consequence the representative bank may be forced to increase the interest rate in response to an increase in wealth. It follows that an ex-post moral hazard question must be analyzed. Higher interest rates affect individuals in two ways. On one side, there is a reduction in the share of low-risk borrowers (negative selection effect), while, on the other side, borrowers are motivated to use riskier techniques (positive incentive effects).

Relative to Stiglitz and Weiss (1992), although we do not consider decreasing absolute risk aversion, our setting is complicated by the independent role of the aversion to effort parameter. From the borrower's point of view, an increase in the repayment X_j brings forth a reduction in terms of utility. From (12), we also know that an increase in the amount of collateral is positively correlated to the aversion to effort in the marginal set. Moreover, given that for each class of wealth, the marginal borrower has the highest aversion to effort, the higher the class of wealth, the higher the aversion to effort accompanied to it. Now, additional information about the link between the marginal aversion to effort and the amount of repayment can be developed.

Remark 3: *In the marginal set, other things being equal, the lower the amount of repayment X assigned, the more averse to effort the borrower is*

$$\frac{d\mu}{dX} \Big|_{U_i(X,c)=0} = \frac{-p'(\tilde{e}) \left(\frac{c+\mu\tilde{e}}{\mu} \right)}{\frac{c}{\mu^2} p'(\tilde{e})(Y - X + c) + \tilde{e}} < 0 \quad (13)$$

Proof. See the Appendix ■

Expression (13) implies that, as expected, the individuals with the highest aversion to effort in the marginal set (i.e. the richest) are those who pay

²They study the case where individuals have the same opportunity sets with decreasing absolute risk aversion. Instead, Wette (1983) shows that in the dynamic of incentive effects no assumption of risk aversion is required if opportunity sets differ across borrowers.

the lowest interest rate. This is a logical consequence of eq. (12) as richer borrowers, posting more collateral, receive a contract with a lower repayment.

In a context of perfect information, individuals with a higher aversion to effort do not receive credit, independently by the class of wealth they belong. Instead, with asymmetric information, in equilibrium, there are pooling interest rates for each wealth class, such that richer individuals with high aversion to effort may stay in the market. Since interest rates are higher for poorer individuals, selection among poorer individuals is more severe due to a lower cross-subsidization.

Such pooling equilibria perfectly characterize a violation of the equality of opportunity principle. Aversion to effort being equal, richer individuals with higher wealth receive a loan, while poorer individuals with a lower amount of wealth may drop out of the market. This occurs due to two reasons. On one side, the richer individuals post more collateral, are charged lower interest rates and for both reasons exert more effort other things equal. On the other side, more cross subsidization occurs in the richer class (because of the reason above) and therefore more negative-surplus projects are realized in this class than in the others. For both these reasons, the aversion to effort of the marginal borrower in a rich class is bound to be larger than in lower wealth classes.

However, that is not all. We can go further in our analysis. An increase in the level of wealth of richer types induces a decrease in the interest rate, more entry of even more effort adverse rich types and more cross subsidization.

Proposition 1: *In the marginal set, due to incentive effect, an increase in the wealth of borrowers leads to the entry of individuals with higher aversion to effort.*

Proof. Bester (1985) and De Meza and Webb (1987) show that an increase in the wealth of a borrower brings to a reduction in the amount of repayment required. Cross-subsidization is reduced pushing worse entrepreneurs out of the market. Here, instead, due to the presence of incentive effects (SW, 1992), an increase in collateral implies more effort into the project from participating borrowers and a reduction of the repayment such that some marginal borrowers can enter. Entry has a countervailing (negative) effect on the repayment but the incentive effect necessarily prevails- as entry is only triggered by lower interest rates. ■

To sum up, there is discrimination among classes of wealth. An increase in inequality in the form of a higher wealth for rich classes implies a lower repayment that in turn causes richer individuals with higher aversion to effort to enter into the market further discriminating low-wealth individuals with a lower aversion to effort.

4 Inefficiency due to incentive effect

The analysis of the marginal borrower focuses on the individuals' choice to enter or not in the credit market. It is useful in our context because it allows the analysis of the conditions under which the marginal poor individuals' effort level is higher than the marginal rich individuals' one. The combination of personal wealth and individual aversion to effort assumes a crucial role in determining who becomes a borrower. As demonstrated above, due to more collateral, lower interest rates and more cross subsidization, richer individuals characterized by high aversion to effort may decide to enter into the credit market affecting the composition of the bank's lending portfolio. An efficiency question must then be faced. Competition will force the banks to offer pooled-contracts dependent on collateral yielding zero expected profit in each wealth class. To make the problem manageable we added the hypothesis that the lowest aversion to effort in each class is 0. This entails necessarily that the best borrower in the class never defaults. We can express the profit function in terms of probability distributions as follows:

$$\text{Bank's profit} \quad \pi = \int_{p_M(\tilde{e})}^1 [p(\tilde{e})X_j + (1 - p(\tilde{e}))c_j - K] dp(\tilde{e}) = 0 \quad (14)$$

where $p_M(\tilde{e})$ is defined as the probability of success into the project of the marginal borrower for any class of wealth c_j . Given the choice about participation in the credit market, the marginal borrower's utility must be zero:

$$\text{Marginal borrower's utility} \quad U_i = p(\tilde{e})(Y - X_j + c_j) - c_j - \mu_s \tilde{e} = 0 \quad (15)$$

Combining conditions (14)-(15) and the standard optimal choice of effort (6), a study about the possible behaviour of the marginal borrower for any wealth level c_j in the marginal set can be developed. Therefore, starting by the bank's net return function (14), we can derive the probability of success of the marginal borrower $p_M(\tilde{e})$ in terms of monetary measures as:

$$p_M(\tilde{e}) = \frac{2K - c_j - X_j}{X_j - c_j} \quad (16)$$

Proof. See the Appendix ■

Formula (16) provides a clear link between the choice to remain active in the market and the amount of collateral owned by each individual. It's a function of the capital K required for the realization of the project and the class of wealth c_j which the individuals belong to. Substituting (16) into the utility function of a borrower (15), we can write that:

$$U_i = \left(\frac{2K - c_j - X_j}{X_j - c_j} \right) (Y - X_j + c_j) - c_j - \mu_s \tilde{e} = 0 \quad (17)$$

In particular, from effort choice (6):

$$U_i = \left(\frac{2K - c_j - X_j}{X_j - c_j} \right) (Y - X_j + c_j) - c_j - p'(\tilde{e})(Y - X_j + c_j)\tilde{e} = 0 \quad (18)$$

Further, a negative correlation between collateral and effort can be derived. It follows that:

Proposition 2: *Due to incentive effects, the richer the marginal borrower, the lower her own effort level into the project*

$$\frac{d\tilde{e}}{dc} = \frac{\frac{1}{p'(\tilde{e})} \left[\frac{2Y(K-X)}{(X-c)^2} \right] - \tilde{e}}{\frac{p''(\tilde{e})}{p'(\tilde{e})^2} \left[\frac{-(2K-X)(Y+X-c)-Yc}{(X-c)} \right] + (Y - X + c)} < 0 \quad (19)$$

Proof. See the Appendix ■

The fact that, in the marginal set, effort actually spent is negatively correlated with collateral has far reaching consequences. Indeed, it means that marginal richer individuals not only are more averse to effort, causing a violation of equality of opportunity, but also exert less effort than marginal poorer individuals. The traditional literature about the credit market suggests that an increase in the amount of collateral brings forth an increase in the amount of effort for all classes of borrowers. Here, instead, we show that in the marginal set, the effort levels of richer individuals are lower than those of the poorer ones. An inefficiency question due to the wrong allocation of credit arises. Although more wealth motivates better individual participating in the credit market before and after the increase in wealth, entry of some new types occurs as well. The entrants are certainly characterized by higher aversion to effort. However our last result suggests also that in equilibrium they actually exert a lower effort. Hence while for the infra marginal individuals more wealth can only imply more effort, the entry of new marginal participant worsens the pool and may decrease average effort spent in each class.

Some interesting consequences follow.

Credit allocation is not only unequal but also inefficient. Particularly, in our model, two sources of inefficiency are now revealed. The first traditional inefficiency belongs to the overlending phenomenon class due to cross-subsidization as in De Meza and Webb (1987). More interestingly the second source of inefficiency is derived from a wrong credit allocation among classes of wealth i.e. individuals who receive funds from the bank may also be those who put less effort into the realization of the project. In the second sense, inequality and inefficiency are clearly intertwined. The two problems can be addressed jointly through a government action aimed at changing the composition of loans rather than the overall amount of credit. Since richer individuals exert less effort (in the margin) a redistributive policy towards poorer ones might increase the surplus in the system.

5 Conclusion

We have explored the relationship between equality of opportunity and efficiency in the credit market. Building on leading models of asymmetric information (both ex-ante and ex-post) in the credit market, our model allows for heterogeneity of would-be entrepreneurs both in wealth and preferences over effort aversion. Equality of opportunity is evaluated relative to effort aversion, which is obviously also the unobservable variable. The wealth of different individuals, on the contrary, is observable and entirely posted as collateral.

In this context we find two important results. On one side we demonstrate that, due to effects linked to collateral both direct and indirect (notably greater effort and cross subsidization), richer individuals participate more in the credit market even when relatively more averse to effort. This is characterized as a violation of the equality of opportunity principle. An important caveat in this result is that more participation for the richer results also from more effort due to the own participation in the project and the consequent lower interest rates, which by themselves mitigate the moral hazard problem.

However we also find that marginal richer borrowers exert less effort than poorer ones in equilibrium, notwithstanding these counterbalancing effects. This result has far reaching consequences for public policies. In particular it strongly suggests that the allocation of credit can be made more efficient by transferring resources from richer to poorer borrowers. More in general it suggests that public programs are more likely to produce results if targeted at lower wealth individuals. This is at odds with some evidence about the way existing public policies are devised and implemented particularly in Italy. On the contrary it is coherent with the growing interest for programs of micro-credit in poor countries.

An interesting joint consequence of Proposition 1 and Proposition 2 is that an increasing inequality determines entry in the high wealth segment of the credit market and exit from the low wealth segment. In our setting this implies a lower aggregate surplus, as marginal richer borrowers spend less effort than marginal poor borrowers. More in general we think that the link between inequality dynamics and credit market performance is the most promising argument for further research suggested by our results.

A The Appendix

A.1 Comparative statics

A) Proof of (7):

From (6), straightforward comparative statics are given by:

$$p''(\tilde{e})d\tilde{e} = -\frac{\mu dY}{(Y + c - X)^2} \quad (7,a)$$

$$\frac{d\tilde{e}}{dY} = -\frac{\mu}{p''(\tilde{e})(Y + c - X)^2}$$
$$\frac{d\tilde{e}}{dY} = -\frac{p'(\tilde{e})}{p''(\tilde{e})(Y + c - X)} > 0$$

$$p''(\tilde{e})d\tilde{e} = -\frac{\mu dc}{(Y + c - X)^2} \quad (7,b)$$

$$\frac{d\tilde{e}}{dc} = -\frac{\mu}{p''(\tilde{e})(Y + c - X)^2}$$
$$\frac{d\tilde{e}}{dc} = -\frac{p'(\tilde{e})}{p''(\tilde{e})(Y + c - X)} > 0$$

$$p''(\tilde{e})d\tilde{e} = \frac{\mu dX}{(Y + c - X)^2} \quad (7,c)$$

$$\frac{d\tilde{e}}{dX} = \frac{\mu}{p''(\tilde{e})(Y + c - X)^2}$$
$$\frac{d\tilde{e}}{dX} = \frac{p'(\tilde{e})}{p''(\tilde{e})(Y + c - X)} < 0$$

B) Proof of eq. (12)

We start by the utility function for the marginal borrower i (1) for each class of wealth j and by the FOC suggested in (6) which are respectively given by:

$$U_i = p(e)(Y - X_j) - (1 - p(e))c_j - \mu_s e \quad (20)$$

$$p'(\tilde{e}) = \frac{\mu_s}{Y - X_j + c_j} \quad (21)$$

From (20) , the condition of the marginal borrower i for each class of wealth j implies that:

$$(Y - X_j + c_j) = \frac{c_j + \mu_s \tilde{e}}{p(\tilde{e})} \quad (22)$$

Substituting (22) into (21), it follows that:

$$p(\tilde{e}) = \frac{c_j + \mu_s \tilde{e}}{\mu_s} p'(\tilde{e}) \quad (23)$$

It refers to the probability of success of the marginal borrower. Therefore, the utility function of the marginal borrower i at any class of collateral j can be expressed as:

$$U_i = \frac{c_j + \mu_s \tilde{e}}{\mu_s} p'(\tilde{e})(Y - X_j + c_j) - c_j - \mu_s \tilde{e} \quad (24)$$

Looking at (24), we can differentiate the utility function U_i with respect to c, μ, X . Hence, we can establish by envelope theorem that in computing the first-order effects respectively of changes in c, μ, X on the maximum value of the utility function U_i the only effect of any consequences is the direct effect. The terms in each of the three equations respectively $d\tilde{e}/dc$, $d\tilde{e}/d\mu$ and $d\tilde{e}/dX$ all vanish by envelope theorem:

1) Derivation of $\frac{\partial U_i}{\partial c}$:

$$\frac{\partial U_i}{\partial c} = \left\{ \begin{array}{l} \left[\frac{p'(\tilde{e})(Y-X+c)}{\mu} + p'(\tilde{e}) \left(\frac{c+\mu\tilde{e}}{\mu} \right) - 1 \right] \\ + \frac{d\tilde{e}}{dc} \left[p'(\tilde{e})(Y-X+c) + \left(\frac{c+\mu\tilde{e}}{\mu} \right) p''(\tilde{e})(Y-X+c) - \mu \right] \end{array} \right\} \quad (25)$$

By envelope theorem:

$$\frac{d\tilde{e}}{dc} \left[p'(\tilde{e})(Y-X+c) + \left(\frac{c+\mu\tilde{e}}{\mu} \right) p''(\tilde{e})(Y-X+c) - \mu \right] = 0$$

Therefore eq. (25) can be rewritten as:

$$\frac{\partial U_i}{\partial c} = \left[\frac{p'(\tilde{e})(Y-X+c)}{\mu} + p'(\tilde{e}) \left(\frac{c+\mu\tilde{e}}{\mu} \right) - \frac{\mu}{\mu} \right] = \left[p'(\tilde{e}) \left(\frac{c+\mu\tilde{e}}{\mu} \right) \right] > 0 \quad (26)$$

given that $\left[\frac{p'(\tilde{e})(Y-X+c)}{\mu} - \frac{\mu}{\mu} \right] = 0$ by (6) for each class of wealth j . The same analysis can be developed for $\frac{\partial U_i}{\partial \mu}$ and $\frac{\partial U_i}{\partial X}$.

2) Derivation of $\frac{\partial U_i}{\partial \mu}$:

$$\begin{aligned} \frac{\partial U_i}{\partial \mu} &= \left[\frac{\mu\tilde{e} - c - \mu\tilde{e}}{\mu^2} p'(\tilde{e})(Y-X+c) - \tilde{e} \right] + \left[\frac{d\tilde{e}}{d\mu} p''(\tilde{e}) \left(\frac{c+\mu\tilde{e}}{\mu} \right) (Y-X+c) \right] = \\ &= -\frac{c}{\mu^2} p'(\tilde{e})(Y-X+c) - \tilde{e} < 0 \end{aligned} \quad (27)$$

3) Finally, based on the definition of the marginal set, we obtain that eq. (12) is given by:

$$\frac{d\mu}{dc} \Big|_{U_i(X,c)=0} = \frac{p'(\tilde{e}) \left(\frac{c+\mu\tilde{e}}{\mu} \right)}{\frac{c}{\mu^2} p'(\tilde{e})(Y-X+c) + \tilde{e}} > 0$$

A.2 Inequality of opportunity

3) Derivation of $\frac{\partial U_i}{\partial X}$:

$$\begin{aligned}\frac{\partial U_i}{\partial X} &= -p'(\tilde{e}) \left(\frac{c + \mu\tilde{e}}{\mu} \right) + \frac{d\tilde{e}}{dX} \left[\mu p'(\tilde{e})(Y - X + c) - \left(\frac{c + \mu\tilde{e}}{\mu} \right) p''(\tilde{e})(Y - X + c) \right] = \\ &= -p'(\tilde{e}) \left(\frac{c + \mu\tilde{e}}{\mu} \right) < 0\end{aligned}\quad (28)$$

C) Proof of eq. (13):

Following the analysis proposed on point B for eq. (12) on the basis of the definition of the marginal set, eq. (13) can then be expressed as:

$$\frac{d\mu}{dX} \Big|_{U_i(X,c)=0} = \frac{-p'(\tilde{e}) \left(\frac{c + \mu\tilde{e}}{\mu} \right)}{\frac{c}{\mu^2} p'(\tilde{e})(Y - X + c) + \tilde{e}} < 0$$

A.3 Inefficiency due to incentive effect

D) Proof of eq. (16)

Starting by (14), we can compute that:

$$\begin{aligned}\pi &= \int_{p_M(\tilde{e})}^1 [(X_j - c_j) p(\tilde{e}) + (c_j - K)] dp(\tilde{e}) = \\ &= \left[(X_j - c_j) \frac{p^2(\tilde{e})}{2} + (c_j - K) p(\tilde{e}) \right]_{p_M(\tilde{e})}^1 = \\ &= \frac{(X_j - c_j)}{2} + (c_j - K) - \frac{(X_j - c_j)}{2} p_M^2(\tilde{e}) - (c_j - K) p_M(\tilde{e}) = \\ &= \frac{(X_j - c_j)}{2} (1 - p_M^2(\tilde{e})) + (c_j - K) (1 - p_M(\tilde{e})) = 0\end{aligned}$$

$$= \frac{(X_j - c_j)}{2}(1 + p_M(\tilde{e}))(1 - p_M(\tilde{e})) + (c_j - K)(1 - p_M(\tilde{e})) = 0$$

Therefore, it follows that the probability of success of the marginal borrower in terms of monetary measures is equal to:

$$p_M(\tilde{e}) = \frac{2K - c_j - X_j}{X_j - c_j}$$

E) Proof of eq. (19)

we can rewrite (18) as:

$$U_i = \frac{1}{p'(\tilde{e})} \left[(Y - X_j + c_j) \left(\frac{2K - c_j - X_j}{X_j - c_j} \right) - c_j \right] - (Y - X_j + c_j)\tilde{e} = 0 \quad (29)$$

given that the marginal borrower's utility U_i is equal to zero. It follows that:

$$U_i = \frac{1}{p'(\tilde{e})} \left[\frac{2KY - 2KX_j + 2Kc_j - X_jY + X_j^2 - X_jc_j - c_jY}{X_j - c_j} \right] - (Y - X_j + c_j)\tilde{e} = 0 \quad (30)$$

The effect of varying the level of collateral with respect to the utility function U_i is equal to:

$$\begin{aligned} \frac{\partial U_i}{\partial c} &= \frac{1}{p'(\tilde{e})} \left[\frac{(2 - X - Y)(X - c) + (2KY - 2KX + 2Kc - XY + X^2 - Xc - cY)}{(X - c)^2} \right] - \tilde{e} = \\ &= \frac{1}{p'(\tilde{e})} \left[\frac{2KX - 2Kc - X^2 + Xc - YX + 2KY - 2KX + 2Kc - YX + X^2 - Xc}{(X - c)^2} \right] - \tilde{e} = \end{aligned}$$

$$= \frac{1}{p'(\tilde{e})} \left[\frac{2Y(K-X)}{(X-c)^2} \right] - \tilde{e} < 0 \quad (31)$$

which is negative given that $X_j = (1+r_j)K$. As regards, instead, the marginal borrower i 's choice of effort satisfies:

$$\begin{aligned} \frac{\partial U_i}{\partial \tilde{e}} &= -\frac{p''(\tilde{e})}{p'(\tilde{e})^2} \left[\frac{2KY - 2KX + 2Kc - XY + X^2 - Xc - cY}{(X-c)} \right] - (Y-X+c) = \\ &= -\frac{p''(\tilde{e})}{p'(\tilde{e})^2} \left[\frac{-(2K-X)(X-c) + Y(X-K) - Y(K+c)}{(X-c)} \right] - (Y-X+c) = \\ &= -\frac{p''(\tilde{e})}{p'(\tilde{e})^2} \left[\frac{-(2K-X)(Y+X-c) - cY}{(X-c)} \right] - (Y-X+c) < 0 \quad (32) \end{aligned}$$

Summing up the first derivatives of the utility function with respect to collateral and effort, a link between collateral and effort (eq.19) can be proposed as:

$$\frac{d\tilde{e}}{dc} = \frac{\frac{1}{p'(\tilde{e})} \left[\frac{2Y(K-X)}{(X-c)^2} \right] - \tilde{e}}{\frac{p''(\tilde{e})}{p'(\tilde{e})^2} \left[\frac{-(2K-X)(Y+X-c) - cY}{(X-c)} \right] + (Y-X+c)} < 0$$

which is definitively negative.

References

- [1] Besley, T. (1994), "How Do Market Failures Justify Interventions in Rural Credit Markets", *World Bank Research Observer*, 9 (1), 27-47
- [2] Bester, H. (1985), "Screening vs. Rationing in credit market with imperfect information", *American Economic Review*, 75 (4), 850-855
- [3] Black, J. and de Meza, D. (1994), "The nature of credit market failure", *Economic Letters*, 46, 243-249
- [4] Calsamiglia C. (2008), "Decentralizing Equality of Opportunity", *International Economic Review*, forthcoming
- [5] Coco, G. (1999), "Collateral, heterogeneity in risk attitude and the credit market equilibrium", *European Economic Review*, 43, 559-574
- [6] Coco, G. (2000), "On the Use of Collateral", *Journal of Economic Surveys*, 14 (2), 191-214
- [7] de Meza, D. (2002), "Overlending?", *The Economic Journal*, 112 (2), 17-31
- [8] de Meza, D. and Webb, D.C. (1987), "Too Much Investment: A Problem of Asymmetric Information", *Quarterly Journal of Economics*, 102 (2), 281-292
- [9] de Meza, D. and Webb, D.C. (1999), "Wealth, Enterprise and Credit Policy", *The Economic Journal*, 109 (455), 153-163
- [10] de Meza, D. and Webb, D.C. (2000), "Does Credit Rationing Imply Insufficient Lending?", *Journal of Public Economics*, 78 (3), 215-234
- [11] de Meza, D. and Webb, D.C. (2006), "Credit Rationing: Something's Gotta Give", *Economica*, 73, 563-578
- [12] Hoff, K. and Lyon, A., (1995), "Non-leaky Buckets: Optimal Redistributive Taxation and Agency Costs", *Journal of Public Economics*, 58 (3), 365-390
- [13] ISAE (2007), Rapporto trimestrale, november

- [14] Jimenez, G., Salas V., Saurina J. (2006), "Determinants of collateral", *Journal of Financial Economics*, 81, 255-281
- [15] Morduch, J. (1999), "Between the State and the Market: Can Informal Insurance Patch the Safety Net?", *World Bank Research Observer*, 14 (2),187-207
- [16] Okun, A.M. (1975), "*Equality and Efficiency: the Big Tradeoff*", Washington D.C.: Brookings Institution
- [17] Ravallion, M. (2002), "Targeted Transfers in Poor Countries: Revisiting the Trade-Offs and Policy Options", World Bank Policy Research Working Paper N. 3048
- [18] Riley, J., (1987), "Credit rationing: A further remark", *American Economic Review*, 77 (1), 224-227
- [19] Roemer, J.E. (1993), "A pragmatic theory of responsibility for the egalitarian planner", *Philosophy & Public Affairs*, 22, 146-166
- [20] Roemer, J.E. (1995), "Equality and Responsibility", *Boston Review*, Vol. 20 (2)
- [21] Roemer, J.E. (1998), "*Equality of Opportunity*", Harvard University Press. Cambridge, Ma.
- [22] Stiglitz, J. and Weiss, A. (1981), "Credit rationing in market with imperfect information", *American Economic Review*, 71 (3), 393-410
- [23] Stiglitz, J. and Weiss, A. (1987), "Credit rationing and collateral in recent developments in corporate finance", In J. Edwards et al. (eds), New York: Cambridge University Press, 101-35
- [24] Stiglitz, J. and Weiss, A. (1992), "Asymmetric information in credit markets and its implications for macro-economics", *Oxford Economic papers*, 44, 694-724
- [25] Stiglitz, J. (1998), "Distinguished Lecture on Economics in Government: the Private Uses of Public Interests: Incentives and Institutions", *Journal of Economic Perspectives*, 12 (2), 3-22

- [26] Wette H. (1983), "Collateral in Credit Rationing in markets with Imperfect Information", *American Economic Review*, 73, 442-5

University of Catania - Department of Economics

Corso Italia 55, Catania – Italy.

www.demq.unict.it

E-mail: demqseg@unict.it

Working Paper Series

Previous 10 papers of the series:

- 2010/04 – P. Platania, “Generalization of the financial systems of management in actuarial techniques affecting insurance of individuals”
- 2010/03 – L. Gitto, “Multiple sclerosis patients’ preferences: a preliminary study on disease awareness and perception”
- 2010/02 – A. Cristaldi, “Convergenza e costanza del sentiero di consumo ottimo intertemporale in presenza di incertezza e lasciti”
- 2010/01 – A. E. Biondo and S. Monteleone, “Return Migration in Italy: What do we Know?”
- 2009/04 – S. Angilella, A. Giarlotta and F. Lamantia “A linear implementation of PACMAN”
- 2009/03 – G. Torrisi, “A multilevel analysis on the economic impact of public infrastructure and corruption in Italian regions”
- 2009/02 – L. Bonaventura and A. Consoli, “La scelta dei criteri di priorità per il giudice penale: effetti sui carichi pendenti e sul costo sociale”
- 2009/01 – G. I. Bischi and F. Lamantia, “R&D collaboration networks in oligopoly competition with spillovers”
- 2008/03 – C. Guccio and I. Mazza, “Determinants of Regional Spending for Heritage Conservation and Valorization in Sicily: a Political-economy Approach”
- 2008/02 – C. Guccio, I. Mazza and A. Mignosa, “Analisi delle variabili determinanti la spesa per cultura e attività ricreative nei paesi OCSE”