Leader, Follower
Strategic Investments with Asymmetric Spillovers

by

Jan Vandekerckhove and Raymond De Bondt

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Abstract

This paper analyzes the strategic incentives of first and second movers in sequential investment games with Stackelberg competition and price leadership on the output market. The study shows that the follower can invest more than the leader when the outgoing spillover from the leader to the follower is sufficiently high, taking into account the outgoing spillover of the follower. This result tends to apply in quantity and price settings. It is also shown that when externalities have opposite signs, the firm with the lowest outgoing spillover is investing most. However, with externalities that have the same sign, the asymmetry of spillovers determines who invests most. A beginning is made with the investigation of the robustness of the tendencies reported.

Catholic University of Leuven

Department of Managerial Economics, Strategy and Innovation

Naamsestraat 69

3000 Leuven

Belgium

Phone: +32 16 69 04

Fax: +32 16 32 67 32

Jan.Vandekerckhove@econ.kuleuven.be

Raymond.DeBondt@econ.kuleuven.be

1 Introduction

Firms tend to be frequently involved in strategic investments in their attempt to achieve or maintain sustainable competitive advantages. Strategic investments may take many forms, such as expenditures to increase business and technological knowledge accumulation, advertising or service outlays to develop or maintain goodwill in the market, and investments directed at modifying product characteristics, production processes or features of the internal organization and/or the external institutional environment.

A number of the main characteristics of these investments are fairly well understood and are helpful in inspiring competitive analysis. Strategic investments, for example, tend to change the parameters of the market rivalry outcomes, they may hurt or benefit rivals and firms may have an incentive to temper or exaggerate efforts for strategic reasons. Some investments may involve special features. Asymmetric information between parties involved, knowledge spillovers and cooperation between some or all of the players involved, for example, will influence innovative strategic efforts in research and development.

In many cases strategic investments take place in an ongoing dynamic process. In some stages of this process some players will be leading and others will be following. The followers can observe earlier results and choices of leaders and the leaders may be able to anticipate how followers will react. It is well known that this role playing affects the incentives and outcomes in quantity competition. Mergers that seek market power, with all suppliers making simultaneous output decisions, tend to reduce profits of the member firms, unless the merger involves almost all suppliers. But when the merged firm can irreversibly commit to output choices and act as market leaders, the picture for profitability is much better (Daughety 1990). All of this suggests that role playing is also likely to affect the incentives for and results of strategic efforts in oligopoly.

Earlier contributions, however, tend to focus on firms choosing simultaneously their strategic investments, with some possible symmetric spillovers (De Bondt 1997). But in reality, firms may be engaged in first mover efforts, while others imitate, catch up or even leapfrog on pioneering moves (Halmenschlager 2004). Spillovers between leaders and followers will tend to be asymmetric, and may carry direct negative and positive effects and may influence role playing (Amir, Amir and Jin 2000).

In markets for video games, for example, a leader and follower may have different standards. A leader investing in an innovation promoting his standard, may lower the demand of the follower. Investments by the follower in an innovation compatible with his own standard may reduce the demand of the leader. In the software industry, on the other hand, the innovator may benefit from research of the follower and the follower may benefit from the efforts of the innovator (Crampes and Langinier 2003).

The intention here is to clarify the effects of spillovers on the strategic investments in leader follower settings. A first objective is to see how leader and follower efforts compare under various spillover scenarios. Some efforts suggest larger efforts by leading players but followers may also be involved in larger efforts, because of accumulated knowledge (Doraszelski 2003) or role playing (Amir et al. 2000, Khanna and Iansiti 1997). A second objective is to see how the leader and follower efforts are influenced by spillovers and to see how, if at all, effects of spillovers with simultaneous choices have to be modified.

2 The Model

In this section, the basic models of this paper are explained. First, the model with output competition is expounded, followed by the model with price competition.

2.1 The Model with Output Competition

The focus is on an oligopoly market consisting of n firms competing non-cooperatively on the output market with differentiated or homogenous products. Of these n firms, k firms behave as leaders while the remaining $n-k$ firms are followers, both in the investment and in the output stage. The leading and the following firms are not necessarily symmetric. Either the ex ante unit costs or the market size can be different because of, for example, accumulated knowledge or goodwill. The inverse demand functions of the leaders (denoted by L) and the followers (denoted by F) are given by the following equations:

$$p_i^L = a^L - \sum_{i=1}^{k} q_i^L - \sigma \sum_{j=k+1}^{n} q_j^F \tag{1}$$

$$p_j^F = a^F - \sigma \sum_{i=1}^{k} q_i^L - \sum_{j=k+1}^{n} q_j^F \tag{2}$$

- a^L and a^F are the initial market size of the k leaders and the $n-k$ followers,
- $\sum_{i=1}^{k} q_i^L$ and $\sum_{j=k+1}^{n} q_j^F$ the total output of the leaders and the followers and
- σ a parameter denoting the degree of product differentiation ($0 < \sigma \leq 1$).

Each firm can commit resources to strategic investments, x^L and x^F for respectively the leaders and the followers, that attempt to maintain or improve its competitive position. These investments can take many forms and result in either lower unit costs or in higher market size.

They moreover carry spillovers to the other firms in the industry. These spillovers can be positive or negative. Positive spillovers occur, for example, when knowledge resulting from cost-reducing R&D spills over to rival firms. Negative spillovers can be a consequence of negative technological and negative demand externalities. Four different spillovers are looked at. These are:

- the spillovers between the leaders (δ),
- the spillovers betweens the followers (ε),
- the spillovers between the group of leaders and the group of followers (β_{LF}) and
- the spillovers from the group of followers to the group of leaders (β_{FL}).

These spillovers do not need to be equal to each other. All spillovers are assumed to be exogenous.

Firm's efforts, for example, may take the form of cost-reducing R&D investments which result in lower ex post unit costs. Incoming positive (negative) spillovers reinforce (weaken) this cost-reducing effect and ex post unit cost of the leaders and the followers ($c^L_{expost,i}$ and $c^F_{expost,j}$) can than be written in the following way:

$$c^L_{expost,i} = c^L - (1-\delta)x^L_i - \delta \sum_{i=1}^{k} x^L_i - \beta_{FL} \sum_{j=k+1}^{n} x^F_j \tag{3}$$

$$c^F_{expost,j} = c^F - (1-\varepsilon)x^F_j - \varepsilon \sum_{j=k+1}^{n} x^F_j - \beta_{LF} \sum_{i=1}^{k} x^L_i \tag{4}$$

All firms face the same cost function $g(x)$ that reflects diminishing returns.

$$g(x) = \frac{\tau}{2} x^2 \tag{5}$$

with τ a given parameter.

Ex post, profits of leaders and followers will be respectively

$$\pi^L_i = (P^L_i - c^L_{expost,i})q^L_i - g(x^L_i) \tag{6}$$

$$\pi^F_j = (P^F_j - c^F_{expost,j})q^F_j - g(x^F_j) \tag{7}$$

The profit functions are the same when investments take the form of demand enhancing efforts.

2.2 The Model with Price Competition

Firms may also compete with prices. Then, the demand functions can be calculated from equations (1) and (2) and are given by the following equations:

$$q^L = \frac{1}{k}(A^L - Bp^L + Cp^F) \tag{8}$$

$$q^F = \frac{1}{n-k}(A^F - Bp^F + Cp^L) \tag{9}$$

with $A^L = \frac{a^L - \sigma a^F}{1-\sigma^2}$, $A^F = \frac{a^F - \sigma a^L}{1-\sigma^2}$, $B = \frac{1}{1-\sigma^2}$, $C = \frac{\sigma}{1-\sigma^2}$, $i = 1...k$ and $j = k+1...n$.

Again, each firm invests in strategic investments intending to improve its competitive position. These investments can carry spillovers to rival players. The same four spillovers as above are introduced. When thinking of cost-reducing R&D investments, ex post unit cost functions can be written as above (see (3) and (4)).

Moreover, the cost function of the efforts is given by equation (5).

As a result, the profit functions of the leaders and the followers are given by the following equations:

$$\pi_i^L = (P_i^L - c_{expost,i}^L)q_i^L - g(x_i^L) \tag{10}$$

$$\pi_j^F = (P_j^F - c_{expost,j}^F)q_j^F - g(x_j^F) \tag{11}$$

These profit functions are the same whether investments are cost-reducing R&D investments or demand enhancing efforts.

2.3 Leading, Following Scenarios

The purpose is to analyze incentives for strategic investments in a sequential game. With more than one leader and one follower, each firm makes its decision independently from the other firms and is playing the same role. So leaders and followers make decisions simultaneously with other leaders and followers.

There are four scenarios that can be analyzed:

- the early entrance game,
- the late entrance game,
- the sequential investment game with simultaneous competition and
- the simultaneous investment game with simultaneous competition.

1) Early Entrance	Sequential Investments Leader and Follower	Sequential Market Entrance
	x_i^L / x_j^F	q_i^L (p_i^L) / q_j^F (p_j^F)
2) Late Entrance	Investment Decision and Market Entrance Leader	Investment Decision and Market Entrance Follower
	x_i^L / q_i^L (p_i^L)	x_j^F / q_j^F (p_j^F)

Table 1: Sequential Output (Price) Choices.

3) Sequential Investment	Sequential Investments Leader and Follower	Simultaneous Market Competition
	x_i^L / x_j^F	q_i^L (p_i^L) and q_j^F (p_j^F)
4) Simultaneous Investment	Simultaneous Investments Leader and Follower	Simultaneous Market Competition
	x_i^L and x_j^F	q_i^L (p_i^L) and q_j^F (p_j^F)

Table 2: Simultaneous Output (Price) Choices.

In an early entrance setting (see table 1), each of the leading firms decides on its strategic investment x_i^L, knowing that each of the $n-k$ followers will observe the efforts of the leaders. After observing the investments of the followers x_j^F, each of the leaders will commit to an output (price) level q_i^L (p_i^L), anticipating the subsequent output (price) choices of each of the followers q_j^F (p_j^F). Followers decide on their strategic investment x_j^F , observing the efforts of the leaders and anticipating the subsequent Stackelberg type output (price leadership type) competition.

In the late entrance game (see table 1), each leader first decides on its investment and a subsequent output (price) level, anticipating later entry by followers. Followers will observe investment and output (price) decisions of the leaders, and choose their best response of strategic investments followed by output (price) choices.

To perform a check on robustness of tendencies, the above two scenarios will be compared with two settings that involve simultaneous choices of output (prices), see table 2. Model 4 is the usual setting in most of the strategic investment literature (see De Bondt (1997)). Model 3 has been looked at as well (see Amir et al. (2000)).

The focus of this paper is on the early entrance scenario. Checks will be reported on the robustness of the obtained tendencies across scenarios.

The purpose is to compare strategic investments of leaders and followers. The role of (asymmetric) knowledge spillovers, product differentiation and the number of players will be analyzed. Furthermore, the impact of asymmetric ex ante knowledge can be clarified. First, the case with one leader and one follower is analyzed. In another paragraph, the robustness of the found tendencies for the more general n firm oligopoly is looked at. Computations and simulations for these settings tend to be complicated and are performed with the Maple program.

3 Investment incentives of the leader and the follower in a duopoly

In this section, the general tendencies of strategic investments are analyzed in a duopoly, with one leader and one follower. The main focus is on the comparison of the investment levels of both players in the early entrance game with both output (in the first subsection) and price competition (in the next subsection). Moreover, the results apply to the symmetric case, thus with equal ex ante demand sizes ($a^L = a^F$) and equal ex ante unit costs ($c^L = c^F$).

3.1 Early entrance game with Stackelberg competition

First, the investment incentives are analyzed for the early entrance game with Stackelberg competition. After that, the same analysis is executed for the early entrance game with price leadership. For both, the role of the spillovers is analyzed and investments are compared in different scenarios.

3.1.1 The role of asymmetric spillovers with Stackelberg competition

Numerous numerical simulations indicate that the asymmetry of spillovers is playing an important role in the comparison of the investment incentives of the leader and the follower. This role of the asymmetry of spillovers is formulated in proposition 1.

Proposition 1 *In the early entrance game with Stackelberg competition, there exists, for each value of β_{FL} and each value of σ, a β^e_{LF}, which is called the equalizer value, so that given these values of β_{FL} and σ:*

- *if $\beta_{LF} < \beta^e_{LF}$ then $x^L > x^F$,*
- *if $\beta_{LF} = \beta^e_{LF}$ then $x^L = x^F$ and*
- *if $\beta_{LF} > \beta^e_{LF}$ then $x^L < x^F$.*

The level of this equalizer spillover β^e_{LF} is always higher than β_{FL}.

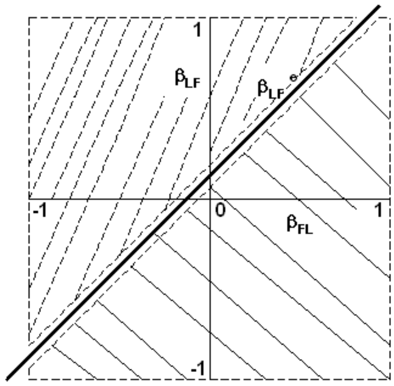

Figure 1: Equalizer value, β_{LF}^{e}, in the early entrance game with Stackelberg competition as a function of β_{FL} for a given σ.

In figure 1, this equalizer value β_{LF}^{e} is plotted in function of β_{FL} for a given value of σ. The curve of β_{LF}^{e} roughly divides the two dimensional spillover space in two sections, namely a region with rather high β_{LF} and rather low β_{FL} (area shaded with the dotted lines on figure 1) and another region with rather low β_{LF} and rather high β_{FL} (area shaded with the full lines on figure 1). Spillover combinations above the β_{LF}^{e} curve yield higher investments of the follower, while those below the β_{LF}^{e} curve result in higher investment incentives of the leader.

In the absence of spillovers, the leader has higher investment incentives than the follower due to its first mover advantage in the market competition. In figure 1, this is represented by the origin lying under the β_{LF}^{e} curve. The presence of spillovers, however, can change those incentives to invest in strategic investments. If the spillover from the leader to the follower is high compared to the spillover from the follower to the leader, the investment incentives are reversed, so the follower invests more than the leader.

It can be verified that the introduction of (sequential) strategic investments can have an impact on the typical Stackelberg obtained tendencies without investments. For highly asymmetric spillover combinations with $\beta_{LF} > \beta_{FL}$, the leader can lose his first mover advantage when $\sigma \in [0, \sigma^{*}[$ (with σ^{*} dependent on β_{LF} and β_{FL} and $\pi^{L} = \pi^{F}$ when $\sigma = \sigma^{*}$), resulting in a higher profit for the follower than for the leader.

3.1.2 Comparison investment incentives in four different scenarios

Externalities describe the impact of strategic investments, made by one firm, on the profits of the rival firms and are either positive or negative.[1] Although the sign and magnitude of the externalities are not equal to the spillovers, both concepts are clearly related to each other.

The sign and magnitude of the externalities on the leader and on the follower are given, respectively, by the sign of the following expressions (12 and 13):

$$\beta_{FL} - \frac{\sigma}{2} \tag{12}$$

$$\beta_{LF} - \frac{2\sigma}{4-\sigma^2} \tag{13}$$

The sign and magnitude of the externalities are thus determined by the outgoing spillover of the rival and the degree of product differentiation. Contrary to the traditional two stage models with simultaneous strategic investments, simultaneous output decisions and symmetric spillovers, the externalities in the early entrance game can be asymmetric (i.e. one externality is positive while the other is negative), due to the possibility of asymmetric spillovers.

There are four possible scenarios, dependent on the sign of the externalities, namely

- $\eta_{FL} > 0$ and $\eta_{LF} > 0$ when β_{FL} is high and β_{LF} is high,
- $\eta_{FL} < 0$ and $\eta_{LF} < 0$ when β_{FL} is low and is β_{LF} is low,
- $\eta_{FL} > 0$ and $\eta_{LF} < 0$ when β_{FL} is high and is β_{LF} is low and
- $\eta_{FL} < 0$ and $\eta_{LF} > 0$ when β_{FL} is low and is β_{LF} is high

with η_{FL} (η_{LF}) representing the externality on the leader (follower) [2].

Numerical simulations lead to the following proposition, which is also represented in figure 2.

Proposition 2 *In the early entrance game with Stackelberg competition, the leader (follower) commits, in general, most resources on strategic investments when the externality on the leader is positive (negative) and the externality on the follower is negative (positive). However, when the sign of the externalities is the same, the comparison of strategic investment levels does not depend on the sign of the externalities but on the asymmetry of spillover levels.*

[1] The externalities of the followers' investments on the profits of the leaders are given, for the general oligopoly, by $\frac{\partial \pi_i^L}{\partial x_j^F}$ with $i = 1, ..., k$ and $j = k+1, ..., n$.
The externalities of the leaders' investments on the profits of the followers are given, for the general oligopoly, by $\frac{\partial \pi_j^F}{\partial x_i^L}$ with $i = 1, ..., k$ and $j = k+1, ..., n$.

[2] β_{FL} is high when term (12) is positive (negative) and β_{LF} is high (low) when term (13) is positive (negative)

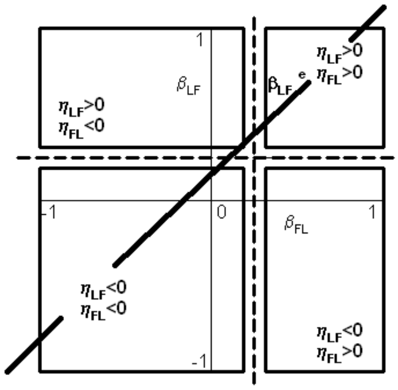

Figure 2: Comparison of strategic investments of the leader and the follower related to externalities.

For example, when $\eta_{FL} < 0$ and $\eta_{LF} > 0$, which can be seen as the traditional imitation case, the leader will be reluctant to invest as its investment positively influences the profits of the follower. Contrary, benefiting from the investments made by the leader, the follower has high incentives to invest. The result is that the follower invests more than the leader, consistent with the fact that the innovation (imitation) creates a positive (negative) externality on the follower's profit (leader's profit).

An example of the traditional imitation game can be found in the emerging satellite radio market in North America (Besanko and Braeutigam 2005). The only two players in this market are XM Satellite Radio and Sirius Satellite Radio. XM entered the new market in September 2001, almost one year before Sirus. In order to inform the public about its service and satellites radios in general, XM has executed advertising campaigns, which had a positive impact on the demand size of XM but also on the demand size of Sirius. Thus, these campaigns carry a positive spillover from XM to Sirius. Sirius, on the other hand, entered the satellite radio market nearly a year After XM. Although Sirius also did some advertising efforts, the spillover from Sirius to XM is negative as Sirius' technology is incompatible with XM's technology, lowering the demand size of XM. As negative spillovers always lead to negative externalities (see 12), XM is hurt by the investments of Sirius. The positive spillover from the XM to Sirius are probably high, resulting in a positive externality on Sirius.

On the other hand, it is also possible that leaders benefit from efforts by the follower but that the follower may be hurt by the leader's efforts. Than the leader will invest more than the follower.

When both firms benefit from each other investments, both firm's outgoing spillovers are rather high. The comparison of the investment levels of the leader and the follower is not dependent on

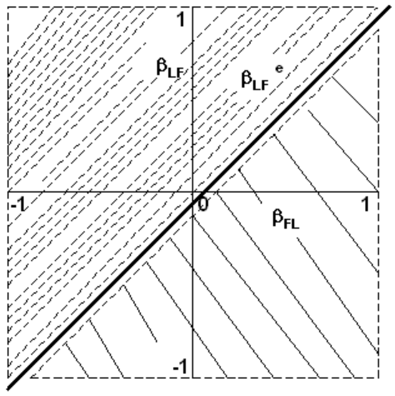

Figure 3: Equalizer value, β^e_{LF}, in the early entrance game with price leadership as a function of β_{FL} for a given σ.

the sign of the externalities, but, as is shown in firgure 2, is now determined by the asymmetry of the spillovers, as stated in proposition 1. The same applies for the case where both the leader and the follower are hurt by the strategic efforts of each other.

3.2 Early entrance game with price leadership

The following analysis concerns the comparison of strategic investments of the leader and the follower in the early entrance game with price leadership.

3.2.1 The role of asymmetric spillovers with price leadership

Asymmetric spillovers do also play a crucial role in the comparison of strategic efforts of the leader and the follower when firms compete with their price. The result of the simulations is formulated in proposition 3.

Proposition 3 *In the early entrance game with price leadership, there exists, for each value of β_{FL} and each value of σ, a β^e_{LF}, which is called the equalizer value, so that given these values of β_{FL} and σ:*

- *if $\beta_{LF} < \beta^e_{LF}$ then $x^L > x^F$,*
- *if $\beta_{LF} = \beta^e_{LF}$ then $x^L = x^F$ and*
- *if $\beta_{LF} > \beta^e_{LF}$ then $x^L < x^F$.*

The general tendency with respect to the comparison of investments of the leader and the follower in the early entrance game with price leadership is thus more or less the same as with Stackelberg leadership. However, the values for the equalizer spillover, β^e_{LF}, differ compared to Stackelberg competition.

Proposition 3 indicates that the spillover asymmetry is crucial for the comparison of investment levels of the leader and the follower. This can also be seen in figure 3. Asymmetric spillover combinations with rather high β_{LF} and rather low β_{FL} result in higher investment of the follower, which is represented by the part above the curve of the equalizer value (area shaded with dotted lines). Reverse spillover combinations lead to $x^L > x^F$. Consequently, the equalizer spillover value β^e_{LF} roughly divides the spillover space in two regions, namely one region with higher investments of the leader (area shaded with full lines) and another region where the follower invests most (area shaded with dotted lines above the β^e_{LF} curve).

Without strategic investments, the price follower has a second mover advantage. Observing the price of the leader, the follower sets a lower price by which he captures a higher market share than the leader. When there are no spillovers, the follower invests more than the leader. However, asymmetric spillovers with $\beta_{FL} > \beta_{LF}$, can change this relation so that the leader invest most.

In a price leadership setting without strategic investments, the follower sets a slightly lower price than the observed price of the leader, by which the follower is able to capture a higher market share and to realize a higher profit. In other words, the follower enjoys a second mover advantage. However, the introduction of (sequential) strategic investments can alter this general characteristic. When spillovers are highly asymmetric with $\beta_{FL} > \beta_{LF}$, it is possible that the leader gets the highest profit. The second mover advantage is than transformed into a first mover advantage when $\sigma \in [0, \sigma^o[$, with σ^o dependent on β_{LF} and β_{FL} and $\pi^L = \pi^F$ when $\sigma = \sigma^o$.

3.2.2 Comparison investment incentives in four different scenarios

Strategic investments of the leader and the follower are now compared with each other in four different scenarios, based on the sign of the externalities. Although the demand functions are derived from the equations (1) and (2), the terms determining the sign and magnitude of the externality on respectively the leader (η_{FL}) and on the follower (η_{LF}) are not the same as in the early entrance game with Stackelberg competition but are given by:

$$\beta_{FL} - \frac{\sigma}{2 - \sigma^2} \tag{14}$$

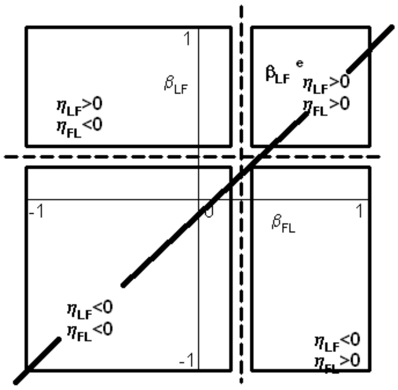

Figure 4: Comparison of strategic investments of the leader and the follower related to externalities.

$$\beta_{LF} - \frac{\sigma(2-\sigma^2)}{4-3\sigma^2} \tag{15}$$

Obviously, the sign and magnitude of the externalities is again determined by the outgoing spillover of the rival and the degree of product differentiation. Asymmetric spillovers result in the possibility of having asymmetric externalities (different sign). The same four scenarios as in paragraph 3.1.2. are looked at. The general tendency is represented in figure 4 and can be formulated in proposition 4.

Proposition 4 *In the early entrance game with price leadership, the leader (follower) commits in general most resources on strategic investments when the externality on the leader is positive (negative) and the externality on the follower is negative (positive). However, when the sign of the externalities is the same, the comparison of strategic investment levels does not depend on the sign externalities but on the spillover levels.*

As proposition 4 is similar to proposition 2, the tendencies of the comparison of strategic investments in the different scenarios in the early entrance game with price leadership are more ore less the same as with Stackelberg leadership.

4 Impact of sequential competition

In this section, the robustness of the tendencies formulated in proposition 1 and 3 is looked at by analyzing model 3 and model 4 (see table 2). First, the tendencies of the early entrance game with Stackelberg competition are compared with two models with Cournot competition. Second, a similar analysis is performed for the models with price competition (price leadership versus Bertrand competition).

4.1 Simultaneous Output Choices

Next to Stackelberg leadership, firms can also compete with quantities à la Cournot, thus choosing quantities simultaneously. In model 3, one firm is taking the lead in the investment stages (without loss of generality, firm 1 is the leader). However, on the output market, both firms compete simultaneously.

The comparison of the investment incentives of the leader and the follower in model 3, which has been analyzed by Amir et al. (2000), can be described by using proposition 5.

Proposition 5 *In the simultaneous output choice game with sequential investments, there exists, for each value of β_{FL} and each value of σ, a β^e_{LF}, which is called the equalizer value, so that given these values of β_{FL} and σ:*

- *if $\beta_{LF} < \beta^e_{LF}$ then $x^L > x^F$,*
- *if $\beta_{LF} = \beta^e_{LF}$ then $x^L = x^F$ and*
- *if $\beta_{LF} > \beta^e_{LF}$ then $x^L < x^F$.*

The level of this equalizer spillover β^e_{LF} is always higher than β_{FL}.

This proposition is similar to proposition 1 concerning the equalizer value β^e_{LF} with Stackelberg competition. However, the level of this equalizer spillover is always lower with simultaneous output choices than with Stackelberg competition. This indicates the impact of the competition mode on the comparison of investment incentives of the leader and the follower.

Comparing the investment incentives of the leader (follower) in model 1 and model 3, the incentives of the Stackelberg leader are not necessarily higher than the incentives of the investment leader with Cournot competition[3]. The incentives of the follower, on the contrary, are always lower with sequential (model 1) than with simultaneous output choices (model 3).

Model 4 is the basic two stage game, as have been introduced by D'Aspremont and Jacquemin (1988), but more generalized for different degrees of product differentiation (De Bondt and Veugelers 1991) and asymmetric spillovers (Attalah 2005, De Bondt and Henriques 1995). In this model, one firm invests more than the other firm if its outgoing spillover is the lowest. The asymmetry of spillovers determines completely the comparison of investment incentives of the leader and the follower as the equalizer value of β_{LF} in model 4 is equal to β_{FL}.

Based on the findings in this paragraph and paragraph 3.1.1, it is possible to conclude that the tendencies of the comparison of strategic investments of the leader and the follower are more or

[3]Stackelberg leader's investment incentives are more sensitive to the outgoing spillover β_{LF} than the incentives of the Cournot leader ($|\frac{\partial x^L}{\partial \beta_{LF}}|$ is higher with Stackelberg competition than with Cournot competition).

less the same in the different scenarios (model 1, model 3 and model 4). However, the value of the equalizer spillover β^e_{LF} is the highest in the early entrance game and the lowest in the two stage game (model 4).

4.2 Simultaneous Price Choices

With Bertrand competition, firms decide simultaneously on their prices. Then, investment decisions can be taken sequentially (model 3) or simultaneously (model 4).

When firms take investment decisions sequentially and competing with prices simultaneously, the general tendency of the comparison of the investments of the leader and the follower is roughly the same as with price leadership and can be summarized by formulating proposition 6.

Proposition 6 *In the simultaneous price choice game with sequential investments, there exists, for each value of β_{FL} and each value of sigma, a β^e_{LF}, which is called the equalizer value, so that given these values of β_{FL} and σ:*

- *if $\beta_{LF} < \beta^e_{LF}$ then $x^L > x^F$,*
- *if $\beta_{LF} = \beta^e_{LF}$ then $x^L = x^F$ and*
- *if $\beta_{LF} > \beta^e_{LF}$ then $x^L < x^F$.*

Simultaneous price decisions do not change the tendencies of strategic investments with sequential price choices. However, the equalizer value of β_{LF} in model is in general, slightly higher than the equalizer value of β_{LF} in the early entrance game, for given values β_{FL} and σ.

In model 4, the two firms choose simultaneously their investment levels. Just like in the two stage game with Cournot competition, a firm will invest more than its rival when its outgoing spillover is lower than the the outgoing spillover of the rival firm. In other words, the equalizer value for β_{LF} is equal to β_{FL}.

5 Conclusions

The major finding of this paper is that the comparison of the strategic investments of a leading and a following firm in a game with sequential investment and sequential output (price) decisions is mainly determined by the spillover asymmetry. These tendencies are more or less the same whether firms compete sequentially with quantities (early entrance game with Stackelberg competition) or with prices (early entrance game with price leadership).

Moreover, games with simultaneous market competition yield similar results. However, the exact value of the equalizer spillover β^e_{LF} differ among the several games.

Furthermore, with asymmetric externalities, the firm benefiting from the investment of his rival, invests most. However, when externalities are symmetric, the comparison of the incentives of the leader and the follower relies on the asymmetry of spillovers.

Future research plans with respect to this topic are the following. Firstly, the robustness of the found tendencies (proposition 1-4) must be further checked with the more general oligopoly with k leaders and $n-k$ followers. Doing so creates also the opportunity to introduce cooperation among leaders and/or followers. The role playing tendencies need to be looked at also.

Appendix

All games in this paper are solved with the technique of backward induction.

Early Entrance Game with Stackelberg Competition

Maximizing the profit of the follower in the fourth stage, yields the following equation for the output of the follower:

$$q^F = \frac{1}{2}((a-c) + (x^F + \beta_{LF}x^L) - \sigma q^L) \tag{16}$$

The leader anticipates this output level of the follower and maximizes its profit in the third stage. This leads to the following output for the leader.

$$q^L = \frac{1}{2}\frac{((2-\sigma)(a-c) + (2-\sigma\beta_{LF})x^L + (2\beta_{FL}-\sigma)x^F)}{2-\sigma^2} \tag{17}$$

Substituting equation (17) in equation (16), results in the following equation:

$$q^F = \frac{1}{4}\frac{(4-2\sigma-\sigma^2)(a-c) + ((4-\sigma^2)\beta_{LF}-2\sigma)x^L + (4-\sigma^2-2\sigma\beta_{FL})x^F}{2-\sigma^2} \tag{18}$$

It can be calculated that the profit of the leader and the follower can be rewritten as follows:

$$\pi^L = \frac{2-\sigma^2}{2}(q^L)^2 - (\tau/2)(x^L)^2 \tag{19}$$

$$\pi^F = (q^F)^2 - (\tau/2)(x^F)^2 \tag{20}$$

In the second stage, the follower observers the investment level of the leader and optimizes its own level of strategic investments:

$$x^F = \frac{2}{\tau}q^F\frac{(4-\sigma^2-2\sigma\beta_{FL})}{4(2-\sigma^2)}. \tag{21}$$

The leader anticipates the amount of resources the follower will spend and commits to a investment level in the first stage:

$$x^L = \frac{(2-\sigma\beta_{LF})}{(2\tau)}q^L \tag{22}$$

which is an expression in terms of β_{LF}, β_{FL}, σ, a, c and τ. The solutions of x^L and x^F can be obtained by solving (22), (22), (16) and (17). The software Maple is used to accomplish this complicated tasks.

Early Entrance Game with Price Leadership

With price leadership in the competition stage, the follower observes the price of the leader and then maximizes its profit by choosing its optimal price. This price is given by:

$$p^F = \frac{(1-\sigma)a + c - \beta_{LF}x^L - x^F - \sigma p^L}{2} \tag{23}$$

The price leader, deciding on its optimal price in the third stage, anticipates (23. Optimizing its profit function leads the following equation for its price:

$$p^L = \frac{(2-\sigma^2)(a+c) - \sigma(a-c) - (2+\sigma\beta_{LF}+\sigma^2)x^L - ((2-\sigma^2)+\sigma)x^F}{2(2-\sigma^2)} \tag{24}$$

Substituting (24) into equation (23), results in the following equation for the price of the follower:

$$p^F = \frac{4(a+c) - \sigma^2(3a+c) + (\sigma^3 - 2\sigma)(a-c) - (4\beta_{LF} + 2\sigma + \sigma^2\beta_{LF} - \sigma^3)x^L - (4 + 2\beta_{FL} - \sigma^2 - \beta_{FL}\sigma^3)x^F}{4(2-\sigma^2)} \tag{25}$$

The profits of the leader and the follower can now be rewritten as follows.

$$\pi^L = \frac{2(1-\sigma^2)}{(2-\sigma^2)}(q^L)^2 - (\tau/2)(x^L)^2) \tag{26}$$

$$\pi^F = (1-\sigma^2)(q^F)^2 - \frac{\tau}{2}(x^F)^2 \tag{27}$$

with q^L and q^F given by the following expressions:

$$q^L = \frac{(2-\sigma-\sigma^2)(a-c) + (2-\sigma\beta_{LF}-\sigma^2)x^L + (2\beta_{FL} - \beta_{FL}\sigma^2 - \sigma)x^F}{4(1-\sigma^2)} \tag{28}$$

$$q^F = \frac{(4-2\sigma-3\sigma^2+\sigma^3)(a-c) + (4\beta_{LF} - 3\sigma^2\beta_{LF} + \sigma^3)x^L + (4 - 2\beta_{FL}\sigma - 3\sigma^2 + \beta_{FL}\sigma^3)x^F}{4(1-\sigma^2)(2-\sigma^2)} \tag{29}$$

In the second stage, the follower optimizes its level of strategic investments, observing the efforts of the leader:

$$x^F = \frac{q^F}{\tau}\frac{(4 - 2\beta_{FL}\sigma - 3\sigma^2 + \beta_{FL}\sigma^3)}{4(1-\sigma^2)(2-\sigma^2)}. \tag{30}$$

The leader is the first mover and decides on its strategic investments in the first stage:

$$x^L = \frac{2(1-\sigma^2)}{2-\sigma^2}\frac{q^L}{\tau}(2 - \sigma\beta_{LF} - \sigma^2). \tag{31}$$

Solving the equation (30) and (31) results in the following solutions for x^L and x^F. These solutions are given by Maple.

The investment level of the follower x^F, in function of x^L, is given by the following equation. The solutions of x^L and x^F can be obtained by solving (30), (31), (28) and (29). The software Maple is used to accomplish this complicated task

References

Amir, Madjid, Rabbah Amir, and Jim Jin (2000) 'Sequencing R&D decisions in a two-period duopoly with spillovers.' *Economic Theory* 15(2), pp. 297–317

Attalah, Gamal (2005) 'R&D cooperation with asymmetric spillovers.' *Canadian Journal of Economics* 38(3), pp. 919–936

Besanko, David, and Ronald R. Braeutigam (2005) *Microeconomics,* 2nd ed. (John Wiley & Sons)

Crampes, Claude, and Corinne Langinier (2003) 'Following the follower in the innovation game.' *Working Paper*

D'Aspremont, Claude, and Alexis Jacquemin (1988) 'Cooperative and noncooperative R&D in duopoly with spillovers.' *American Economic Review* 78(8), pp. 1133–1137

Daughety, Andrew F. (1990) 'Beneficial concentration.' *American Economic Review* 80(5), pp. 1231–1237

De Bondt, Raymond (1997) 'Spillovers and innovative activities.' *International Journal of Industrial Organization* 15(1), pp. 1–28

De Bondt, Raymond, and Irene Henriques (1995) 'Strategic investments with asymmetric spillovers.' *Canadian Journal of Economics* 28(3), pp. 656–674

De Bondt, Raymond, and Reinhilde Veugelers (1991) 'Strategic investment with spillovers.' *European Journal of Political Economy* 7, pp. 345–366

Doraszelski, Ulrich (2003) 'An R&D race with knowledge accumulation.' *RAND Journal of Economics* 34(1), pp. 20–42

Halmenschlager, Christine (2004) 'R&D-cooperating laggards versus a technological leader.' *Economics of Innovation and New Technology* 13(8), pp. 717–732

Khanna, Tarun, and Marco Iansiti (1997) 'Firm asymmetries and sequential R&D: Theory and evidence from the mainframe computer industry.' *Management Science* 43(4), pp. 405–421