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Exchange Rate Volatility and Trade : A General Equilibrium Analysis

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Abstract

In this paper, we use insights from the literature on financial options to analyze the effect of exchange rate volatility on the volume of trade between countries. In contrast to existing work, this analysis is carried out in a model where the exchange rate is determined endogenously, and the volatility of the exchange rate depends on the volatility of the amounts available for consumption of the traded and non-traded goods. Our main result is to show that, in a one-good world, and contrary to the popular conjecture, an increase in exchange rate volatility is associated with an increase in the volume of trade. If a non-traded good is added, the above remains true when the source of exchange rate volatility is the uncertainty in the traded goods sector. However, when the source of exchange rate volatility is the non-traded goods sector then the volume of trade may decline. Thus, our model offers at least a partial explanation for the results of empirical studies that find only a weak relation between exchange rate volatility and trade. A policy implication of the model is that the volatility of the real exchange rate can be reduced, and welfare increased, in two ways: by reducing the volatility of fundamentals and by reducing the barriers to trade. However, while a reduction in trade barriers is associated with an increase in trade, a reduction in the volatility of fundamentals leads to a reduction in trade. Thus, more trade does not always mean a higher welfare.

Key words: Exchange risk, trade volume, option pricing, transactions costs, non-traded goods

JEL Classification: F31, F32

Our objective in this article is to analyze the relation between exchange rate volatility and the volume of international trade in developed economies, and to study how this relation is affected by the degree of openness of an economy. The analysis is carried out in a general equilibrium model where the volume of trade, the exchange rate, and the interest rates are all determined endogenously. The commodity markets are assumed to be partially segmented, but—given the focus of the study on developed economies—financial markets are perfectly integrated.

Our work is motivated by the ongoing debate on the advantages of trading blocs that decrease barriers to trade, as a complement or a substitute for fixed-rate exchange regimes. Understanding the relation between exchange rate volatility and commodity trade is fundamental for the setting of exchange rate and tariff policy, and the importance of these issues can be gauged by the attention being given to exchange rate arrangements envisaged by members of the European Monetary System, and to trade agreements such as the North American Free Trade Agreement (NAFTA) between the U.S., Canada and Mexico.[1]

However, there is no consensus as to how exchange rate risk is associated with the volume of trade. A popular conjecture is that an increase in exchange rate volatility leads to a reduction in the level of international trade, and a number of partial-equilibrium models support this view (see Côté (1994) for a survey of existing work). A typical argument in this literature is that higher exchange risk lowers the risk-adjusted expected revenue from exports, and therefore reduces the incentives to trade. In contrast, another strand of the literature argues that, when firms are allowed to optimally respond to exchange rate changes, the revenue per unit of an exportable good (Sercu, 1992) or the entire cashflow from exporting (Franke (1991) and Sercu and van Hulle (1992)) are convex functions of the exchange rate.

[1]Other trade coalitions include APEC, which consists of a trans-Pacific collection of countries; Caricom, which includes most of the Caribbean islands and Belize and Guyana, and is being extended to the Association of Caribbean Countries, which will include also the mainland countries around the rim; the Central American Common Market, linking Guatemala, Honduras, El Salvador, Nicaragua and Costa Rica; the Andean Group, which consists of Venezuela, Columbia, Ecuador, Peru and Bolivia; and, Mercosur, which is a group consisting of Brazil, Argentina, Paraguay and Uruguay.

From this it follows that expected unit revenue or the expected cashflow increases when the volatility of the exchange rate increases, which then acts as a stimulant to trade rather than a deterrent. This lack of consensus on the theoretical side is matched by a lack of agreement on the empirical front.[2] Most empirical work fails to find a strong negative relation between exchange rate volatility and trade, and some empirical studies find that an increase in volatility may be associated with an increase in international trade (see, for example, Asseery and Peel (1991)).

The result that exchange rate uncertainty always lowers the risk-adjusted expected revenue from exporting is derived in a partial equilibrium setting: most of this literature assumes that exchange rate uncertainty is the sole source of risk, and either ignores the availability of non-linear hedges (options, and portfolios of options) or takes the prices of the hedge instruments (or some of the determinants of these prices) as given. A similar criticism can be made regarding the results based on the convexity of revenues: this view takes the demand functions or the cashflow function as given, and therefore ignores the issue of how changes in the economy that have lead to a higher exchange risk affect the demand or cashflow function.[3]

To address these criticisms, we need a general-equilibrium approach as adopted in, for instance, the (neo-)classical trade models. However, these models assume that all commodity markets are perfect; that is, the drawback of the neoclassical approach is that Commodity Price Parity (CPP) is assumed to hold at all times and for all goods, implying that there is no real exchange rate risk.[4] Accordingly, our objective is to develop a model that

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[2]See Côté (1994) for a summary of the results of empirical studies.

[3]See, for example the papers by de Grauwe (1988), Sercu (1992), Sercu and van Hulle (1992)), and Viaene and de Vries (1992).

[4]In a perfect-markets setting, PPP-deviations can still arise because of international differences in commodity preferences. However, such PPP-deviations are economically less interesting. In addition, Engel (1993) shows that, as a source of PPP deviations, violations of Commodity Price Parity are far more important than differences in commodity preferences.

has the internal consistency of the general-equilibrium models of trade, but where commodity markets are sufficiently segmented to allow for deviations from CPP and meaningful changes in the real exchange rate. In our analysis, we model the segmentation of commodity markets by allowing for both non-traded goods (as in, for example, Stockman (1980) and Stulz (1987)) and for costs for transferring commodities across countries (as in Dumas (1992) and Sercu, Uppal and Van Hulle (1995)). Financial markets, on the other hand, are assumed to be complete and perfectly integrated in our model. Thus, consumers can make cross-border financial investments—reflecting the fact that international capital markets are far less subject to restrictions than commodity markets. Likewise, firms can make optimal hedging decisions, and the prices of the hedge instruments are determined in a general-equilibrium framework.

Our major results in this general-equilibrium model are the following. First, in an economy with only one (imperfectly tradable) good, an increase in exchange rate volatility must be caused by a higher riskiness of the outputs of that good; and this higher output risk, in turn, is associated with an increase in the expected volume of trade in goods. That is, while an increase in exchange rate uncertainty is associated with a lower welfare level, the optimal response of agents to the increase in uncertainty nevertheless is to trade more, not less (as conventional wisdom holds). Thus, the convexity argument from the partial-equilibrium literature is qualitatively correct in a *one*-good economy. Second, we show that this conclusion is quite sensitive to the one-good assumption. If we introduce a non-traded good, the conclusions from the one-good model remain valid only if the source of increased exchange risk is an increased volatility in the tradable good's own output. However, when there is a higher output risk in the *non*-traded goods sector, this risk may either increase or decrease expected trade in the other good, and may also either decrease or increase the variance of the exchange rate. Thus, in an economy with one non-traded and one traded good, there is no longer a clear association between exchange rate volatility and the expected level of trade—unless the source of the increased risk is just the output of the tradable goods sector. And even that last conclusion needs to be qualified as soon as a second tradable good is introduced: With two tradable goods, a higher risk in one of the tradable goods sectors still boosts expected trade within that same sector, but now generally has ambiguous effects on trade in the other tradable good and on exchange risk. Thus, the convexity argument, as invoked in partial-equilibrium models, very much depends on the *ceteris paribus* assumptions about the demand function or the cash-flow function; as soon as one allows for changes elsewhere in the economy, the cash-flow functions can shift in a way that obscures, or even reverses, the conclusions from the partial analysis.

Our model also has the following policy implications. Given the construction of the model, volatility of the real exchange rates can be reduced in two ways: (a) by reducing the volatility of fundamentals and (b) by reducing the barriers to trade. Both measures lead to an increase in welfare but have opposite effects on trade: a reduction in trade barriers leads to an increase in trade, while a reduction in the volatility of fundamentals leads to a drop in trade. Thus, to draw conclusions about welfare, it is not sufficient to focus on just the volume of trade.

The rest of the paper is organized as follows. In Section I, we describe the general economy that we use in our analysis. In Section II, we consider a world with one good, and show that in this setting the relation between exchange rate volatility and the volume of international trade is positive. In Section III, we add a non-traded good, and show how the volatility of the non-traded good output may affect the conclusions of the one-good model. Section IV examines the effects of adding a second tradable good. We present our conclusions in Section V. The major results of each section are collected in propositions, the proofs of which are presented in Appendix A and B.

I. The Economy

In this section, we present a model of two countries (k = 1, 2) that have perfectly integrated financial markets but segmented commodity markets. That is, capital markets are assumed to be complete and frictionless (implying that asset prices are equal across countries, after conversion into the same reference currency), but it is costly to trade goods internationally. In what follows, we describe the endowment process and the preferences of consumers, and derive the implications of the model for the volume of trade and the real exchange rate.

In every period, each country has a stochastic endowment of up to three non-storable goods with different degrees of tradability. We index the three goods in ascending order of tradability: the first commodity (j = 1) is entirely non-traded, the second good (j = 2) is homogenous across countries and can be traded at a cost, and the third good (j = 3), also homogenous across countries, that is perfectly tradable. The introduction of a perfectly traded good, besides the imperfectly tradable one, allows us to assess the interactions between the volumes of trade of the two tradable goods.[5]

The endowment in country k at time t of each of these goods is denoted by $q_{kj}(t)$. These stochastic endowments may be given exogenously, as in Lucas' (1982) exchange economy, or could be the result of endogenous investment decisions, as in Dumas' (1992) production economy. If, in the production economy, the goods act simultaneously as capital goods and consumption goods, the "output" of good j in country k has to be interpreted as output net of reinvestment.[6]

The home and foreign country (k = 1 and 2, respectively) are assumed to be populated by a large and equal number of infinitely-lived consumers with identical, multiplicative commodity preferences over the three goods, and identical, constant relative risk aversion:

[5] We introduce this third good also because, in the absence of such a perfectly tradable good, there can be protracted periods where there is absolutely no trade. We have also worked with a three-country model that has two imperfectly traded goods besides a non-traded one, without obtaining any new insights. These results are discussed briefly in the concluding section.

[6] The process for net output, in a production economy, is likely to be more complicated than in an exchange economy—see, for instance, Dumas (1992)—but our conclusions do not depend on a particular stochastic process for the net outputs.

$$U_k[c_k(t)] = \frac{1}{1-\eta}\left(\Pi_{j=1}^{3} c_{kj}(t)^{\varepsilon_j}\right)^{1-\eta}, \ \varepsilon_j > 0, \ \Sigma_{j=1}^{3}\varepsilon_j = 1, \ 0 \leq \eta \leq \infty, \eta \neq 1,^{7} \tag{1}$$

where $c_{kj}(t)$ denote the consumption of good j in country k.

The factors that distinguish one country from another, and lead to trade between them, are the following. First, the initial wealth of the two countries may be different. Second, the outputs of the three goods generally differ across countries. By definition, in the non-tradable goods sector it is not possible to reduce such imbalances by shipping these goods (the cost for trading good 1, τ_1, is equal to infinity). While it is possible to export the imperfectly tradable good from one country to another, any such transfer is costly. This transaction cost is modeled, following Dumas (1992), as a waste of resources: if one unit is shipped, only $1/(1+\tau_2)$ units actually arrive ($\infty > \tau_2 > 0$). The transaction cost implies that, within a certain region, it will be optimal not to trade even when the price of the tradable good at home is different from that abroad. Thus, the different outputs generally imply international deviations from Commodity Price Parity for both the non-traded good and the imperfectly tradable good.

Given that individuals are identical within each country, the model can be expressed in terms of two representative consumers, one for each country. Rather than considering decentralized decision-making, we look at the problem from a central planner's perspective. Given our assumption of complete and frictionless financial markets, the decentralized solution is identical to that of the central planner, but analyzing the central planner's problem allows us to identify the optimal policies for consumption and trade in a relatively straightforward way.[8] In the decentralized solution, the initial relative wealth of the two

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[7]The special case where $\eta = 1$ is represented by the log utility function. This utility specification yields the same first-order conditions as the ones obtained by setting η equal to unity in the case for the utility function in (1), and the same expressions for trade and the real exchange rate. Thus, the implications for the log utility function are similar to the ones we derive for the case $\eta \neq 1$.

[8]For example, Uppal (1993) sho ws how the solution to the central planner's problem obtained in Dumas (1992) can be derived in a decentralized setting.

countries would determine how the claims on future consumption are distributed between them. In the equivalent central planner's problem, given in (2), differences in initial wealth are reflected by different weights in the central planner's objective function; these weights then determine the distribution of consumption across the two countries.

Let $x_{kj}(t)$ denote the amount of good j exported from country k (measured before transactions costs) at time t. The central planner's objective is to choose the sequence $\{x_{kj}(t)\}$ so as to maximize the weighted aggregate utility, where the weights are denoted by θ_k:

$$\underset{\{xkj(t)\}}{\text{Max}} \; \theta_1 \, \mathrm{E}\left\{ \sum_{t=0}^{\infty} \delta^{-t} \, U_1[c_1(t)] \right\} + \theta_2 \, \mathrm{E}\left\{ \sum_{t=0}^{\infty} \delta^{-t} \, U_2[c_2(t)] \right\}, \tag{2}$$

subject to:

$$c_{1j}(t) = q_{1j}(t) - x_{1j}(t) + \frac{x_{2j}(t)}{1+\tau_j}, \quad j=1, 2, 3\,, \tag{3a}$$

$$c_{2j}(t) = q_{2j}(t) - x_{2j}(t) + \frac{x_{1j}(t)}{1+\tau_j}, \quad j=1, 2, 3, \tag{3b}$$

$$x_{kj}(t) \geq 0, \;\; j=1, 2, 3 \text{ and } k=1, 2\,, \tag{3c}$$

and where $\tau_1 = \infty$, $0 < \tau_2 < \infty$, $\tau_3 = 0$, δ is the subjective discount factor, and $U(c_k(t))$, $k = \{1, 2\}$ is as defined in (1).

The central planner's decision rules for consumption and trade in the general case are summarized in Propositions 1.1 and 1.2, and the equilibrium real exchange rate is derived in Proposition 1.3. The proofs are given in the appendix, and the implications are discussed in Sections II to IV. Thus, in this section we just provide a qualitative introduction to the propositions.

The logic behind the optimal consumption policies is as follows. If all the goods were perfectly tradable, then it would be optimal for the central planner to equate the θ-weighted marginal utility of consumption for each good across the two countries. This is, in fact, what the central planner does for the perfectly tradable good (good 3). However, for a good that is costly to trade, it is not optimal to equate, across countries, the weighted marginal utility from consuming this good. Thus, for good 2 which has a strictly positive shipping cost, the first-order conditions imply that there will be a no-trade zone within which international imbalances in the weighted marginal utility of consuming this good will be left uncorrected—notably when the cost of shipping outweighs the utility gained by reducing the international imbalance in the consumption of this good. Similarly, even when this good is actually transferred across countries, shipments will still be restricted to the level where the cost of shipping the last unit has become equal to the incremental gain in aggregate weighted utility; that is, such shipments will still fall short of equating the weighted marginal utilities from consuming that good. Finally, since the cost for shipping the non-traded good (good 1) is infinite, no attempt is made to balance internationally the marginal utility from consuming this good.

The consumption behavior described above has the following implications for trade: good 1 is never traded, and good 3 is always traded so that the marginal utility from its consumption is balanced across countries. Trade in good 2 occurs only when the ratio of the two marginal utilities from its consumption exceeds a particular bound. Thus, it is possible to divide the state space into three critical regions indexed by (superscript) i, where $i = \{0, 1, 2\}$. The three states depend on trade in good 2: in state 0, there is no trade in the imperfectly tradable good (good 2); in state 1, country 1 exports good 2; and, in state 2, country 2 exports good 2. It will be convenient to express our results in terms of these three states. Many of our results also depend on consumption ratios rather than levels; thus, we will use $\kappa_j^i(t) \equiv [c_{2j}(t)/c_{1j}(t)]$ to denote the ratio, at time t and in state i, of the optimal consumptions of good j.

Proposition 1.1: *In each of the states i, the optimal consumption ratios across countries for the three goods are :*

$$\kappa_1(t) \equiv \frac{c_{21}(t)}{c_{11}(t)} = \frac{q_{21}(t)}{q_{11}(t)} \quad \text{in all states } i, \tag{4}$$

$$\kappa_2^i(t) \equiv \frac{c_{22}(t)}{c_{12}(t)} = \begin{cases} \dfrac{q_{22}(t)}{q_{12}(t)} & \text{if } i=0 \\ (1+\tau_2)^{-\alpha}\left(\theta_2/\theta_1\right)^{\alpha}\left(\kappa_1(t)\right)^{\beta} & \text{if } i=1 \\ (1+\tau_2)^{+\alpha}\left(\theta_2/\theta_1\right)^{\alpha}\left(\kappa_1(t)\right)^{\beta} & \text{if } i=2 \end{cases} \tag{5}$$

$$\kappa_3^i(t) \equiv \frac{c_{23}(t)}{c_{13}(t)} = \begin{cases} \left[(\theta_2/\theta_1)\left(\kappa_1(t)\right)^{\varepsilon_1(1-\eta)}\left(\kappa_2^0(t)\right)^{\varepsilon_2(1-\eta)}\right]^{\frac{1}{1-\varepsilon_3(1-\eta)}} & \text{if } i=0 \\ (1+\tau)\,\kappa_2^1(t) & \text{if } i=1 \\ \dfrac{1}{1+\tau}\,\kappa_2^2(t) & \text{if } i=2 \end{cases} \tag{6}$$

where

$$i = \begin{cases} 1 \ \text{if } \dfrac{q_{22}(t)}{q_{12}(t)} < (1+\tau_2)^{-\alpha}\left(\theta_2/\theta_1\right)^{\alpha}\left(\kappa_1(t)\right)^{\beta} & \text{[country 1 is exporting good 2];} \\ 2 \ \text{if } \dfrac{q_{22}(t)}{q_{12}(t)} > (1+\tau_2)^{+\alpha}\left(\theta_2/\theta_1\right)^{\alpha}\left(\kappa_1(t)\right)^{\beta} & \text{[country 2 is exporting good 2];} \\ 0 \ \text{otherwise} & \text{[good 2 is not traded]} \end{cases} \tag{7}$$

$$\alpha = \frac{1-\varepsilon_3(1-\eta)}{1-(1-\varepsilon_1)(1-\eta)} > 0; \ \textit{and} \ \beta = \frac{\varepsilon_1(1-\eta)}{1-(1-\varepsilon_1)(1-\eta)} \gtrless 0\,. \tag{8}$$

The optimal export policies, presented in Proposition 1.2, follow from the consumption behavior described above. Equation (9) states that the first good, being non-tradable, is never exported. Equation (10) gives the optimal amount of good 2 that should be exported from country 1 (in state 1), and (11) gives the optimal exports of the same good from country 2 (in state 2). Note that the conditions in the max operators in (10) and (11) corresponds to the conditions for trade to occur, given in (7). Equations (12) and (13) describe the volume of good three that will be exported in state i from country 1 and country 2, respectively.

Proposition 1.2: *The optimal levels of trade for the three goods are given by:*

For good 1: $\quad x_{11}(t) = x_{21}(t) = 0\,;$ (9)

For good 2:

$$x_{12}(t) = \frac{1}{1/[\kappa_2^1(t)\,(1+\tau_2)] + 1}\,\mathrm{Max}\left(q_{12}(t) - \frac{1}{\kappa_2^1(t)}\,q_{22}(t),\, 0\right), \tag{10}$$

$$x_{22}(t) = \frac{1}{\kappa_2^2(t)/(1+\tau_2) + 1}\,\mathrm{Max}\big(q_{22}(t) - \kappa_2^2(t)\,q_{12}(t),\, 0\big)\,; \tag{11}$$

For good 3:

$$x_{13}(t) = \frac{1}{1 + \kappa_3^i(t)}\,\mathrm{Max}\big(\kappa_3^i(t)\,q_{13}(t) - q_{23}(t)\,,\, 0\big)\,, \tag{12}$$

$$x_{23}(t) = \frac{1}{1 + \kappa_3^i(t)}\,\mathrm{Max}\big(q_{23}(t) - \kappa_3^i(t)\,q_{13}(t)\ ,\, 0\big)\,. \tag{13}$$

Lastly we derive the real exchange rate. The intuition underlying this derivation is the following. The real exchange rate in a one-good economy is given by the ratio of weighted marginal utility of consumption abroad to that at home. In our multi-good economy with homothetic utility functions, it is possible to define one composite consumption good per country, and the real exchange rate can be shown to be the ratio of the θ-weighted marginal utilities of consuming each composite good. In the case of multiplicative utility functions, these marginal utilities of composite consumption can then be decomposed into the marginal utilities of consuming each separate good. As, for good 3, the θ-weighted marginal utilities are perfectly equalized, we end up with a real exchange rate that depends only on the consumption ratios for goods 2 and 3:

Proposition 1.3: *The real exchange rate can be expressed as:*

$$S(t) = \left[\left(\frac{\theta_2}{\theta_1}\right)^{1-\varepsilon_3} \big(\kappa_1(t)\big)^{-\varepsilon_1\,\eta}\, \big(\kappa_2^i(t)\big)^{-\varepsilon_2\,\eta}\right]^{\frac{1}{1-\varepsilon_3(1-\eta)}} \tag{14}$$

From Propositions 1.1, 1.2 and 1.3, we see that it is possible to express explicitly the real exchange rate and the volume of trade as functions of the state variables, the outputs of the goods. In the rest of the paper, we will examine how an increase in the volatility in one of the state variables affects the expected volume of trade and the volatility of the exchange rate. To facilitate the exposition, we will start by considering a specialized version of the general economy described above—an economy with only the imperfectly tradable good. We will consider the effect of introducing a non-traded good in Section III and that of the perfectly-tradable good in Section IV.

II. Trade and Exchange Rate Risk in a Single-Good Model

In this section, we consider a special case of the model described in the previous section—one in which there is only a single good—the imperfectly-tradable good (good 2). In what follows, we first establish the relation between expected trade and output uncertainty, and then consider the effect of output risk on the volatility of the exchange rate. We conclude this section with a discussion of the generality of these results.

II.A. The Effect of Output Risk on Expected Trade

To obtain the model with only the imperfectly-tradable good, we set $\varepsilon_2 = 1$ and $\varepsilon_1 = \varepsilon_3 = 0$ in Proposition 1.1. The implication is that the consumption ratios when trade is positive, κ_2^1 and κ_2^2, become constants:

$$\kappa_2^1 = \left(\frac{\theta_2}{\theta_1}\,\frac{1}{1+\tau_2}\right)^{1/\eta} \text{ and } \kappa_2^2 = \left(\frac{\theta_2}{\theta_1}\,(1+\tau_2)\right)^{1/\eta}. \tag{15}$$

Thus, the optimal trade quantities, from Proposition 1.2, are:

$$x_{12}(t) = \frac{1}{1/[\kappa_2^1\,(1+\tau_2)] + 1}\,\text{Max}\left(q_{12}(t) - \frac{1}{\kappa_2^1}\,q_{22}(t), 0\right), \tag{16}$$

and

$$x_{22}(t) = \frac{1}{\kappa_2^2/(1+\tau_2) + 1}\,\text{Max}\big(q_{22}(t) - \kappa_2^2\,q_{12}(t), 0\big). \tag{17}$$

To see how the expected volume of trade is affected by the uncertainty of the underlying future outputs, we need to define what "higher uncertainty" means. As we do not want to restrict the analysis to normally or lognormally distributed variables, the variance of the future net output is not a sufficiently general measure of uncertainty. Instead, we define the "riskiness" of the realization of a variable at a future date, T, as in Rothschild and Stiglitz (1970): a random variable $y_1(T)$ is said to be more risky than another variable $y_2(T)$ if $y_1(T)$ can be decomposed into $y_1(T) = y_2(T) + \varepsilon$, with $E[\varepsilon \mid y_2(T)] = 0$.[9]

We first apply this concept to a relative measure of trade, viz. the ratios of time-T exports to local output. From (16) and (17), these ratios are given by

$$\frac{x_{12}(T)}{q_{12}(T)} = \frac{1}{1/(1+\tau_2) + \kappa_2^1} \operatorname{Max}\left(\kappa_2^1 - \frac{q_{22}(T)}{q_{12}(T)}, 0\right), \tag{18}$$

$$\frac{x_{22}(T)}{q_{22}(T)} = \frac{1}{1/(1+\tau_2) + 1/\kappa_2^2} \operatorname{Max}\left(\frac{1}{\kappa_2^2} - \frac{q_{12}(T)}{q_{22}(T)}, 0\right). \tag{19}$$

Noting that the functions in (18) and (19) are convex in the ratio of relative outputs, Jensen's Inequality immediately implies that the expected ratio of time-T exports to time-T output increases when the riskiness of relative output increases. This conclusion can be related to analogous results from the literature on financial options. The volume of domestic exports as a fraction of local output, given in (18), is similar to the payoff of a European-style put option, with (known) contract size $[1/(1+\tau_2) + \kappa_2^1]^{-1}$ and (known) strike price κ_2^1, written on a process $[q_{22}(T)/q_{12}(T)]$. Since this payoff is convex in $[q_{22}(T)/q_{12}(T)]$, the expected payoff is an increasing function of the riskiness of $[q_{22}(T)/q_{12}(T)]$.[10]

Similar results can be obtained for the ratio of net *imports* over local output:

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[9]Thus, a higher riskiness also implies a higher variance. However, the reverse is not necessarily true: with non-normal or non-lognormal distributions, for instance, a higher variance need not imply a higher riskiness. Specifically, a higher Rothschild-Stiglitz riskiness affects the entire distribution and, therefore, rules out changes in the variance that are due only to a change in an extreme tail of the distribution.

[10]Merton (1973, Theorem 8) shows that, independent of the investors attitudes towards risk, the market value of an option is a positive function of the riskiness of the underlying process. The expected value is the market value in the special case of risk-neutrality and a zero interest rate. Thus, Merton's theorem about market values also applies to expected values.

$$\frac{x_{12}(T)/(1+\tau_2)}{q_{22}(T)} = \frac{1}{1/[(1+\tau_2)\kappa_2^1] + 1} \, \mathrm{Max}\left(\frac{q_{12}(T)}{q_{22}(T)} - \frac{1}{\kappa_2^1}, 0\right), \tag{20}$$

$$\frac{x_{22}(T)/(1+\tau_2)}{q_{12}(T)} = \frac{1}{\kappa_2^2/(1+\tau_2) + 1} \, \mathrm{Max}\left(\frac{q_{22}(T)}{q_{12}(T)} - \kappa_2^2, 0\right). \tag{21}$$

These functions are convex in relative output and are similar to payoffs from a European-style call option. We conclude that, in a comparative statics sense, an increased riskiness of the relative output leads to a higher expected ratio of exports and imports relative to local output.

From equations (16) and (17), we can make similar statements about the expected *level* of trade rather than trade expressed as a fraction of local output. The volume of exports is convex in both $q_{12}(T)$ and $q_{22}(T)$, and corresponds to the payoff from a European-style option to exchange two risky payoffs, $q_{22}(T)$ and $q_{12}(T)$.[11] From the convexity of these payoffs it follows that, conditional on one of the outputs, the expected volume of trade is a positive function of the riskiness of the other output. This, in turn, implies that also the unconditional expected volume of trade is positive in the riskiness of either output.[12] We summarize these results as follows.

Proposition 2.1: *In a model with a single good that is tradable at a finite cost, the expected volume of trade, and the expected ratio of trade to local output, are positive functions of the riskiness of the output of the good abroad or at home.*

To get an exact expression for expected trade, and to illustrate these distribution-free results in closed-form, one needs to make specific assumptions about the endowment

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[11]See Margrabe (1978) for an analysis of such options.

[12]The above results relate to the expected level of exports of either country 1 or 2, but immediately carry over to imports and to the total level of a country's international trade. This is because net domestic imports, after transactions costs, are obtained by simply dividing the above gross foreign exports by $(1+\tau_2)$. Thus, expected trade for one country, being the sum of its expected exports and expected imports, is a positive function of the risk of relative output.

process. For instance, let the endowment processes for the tradable good be given by log random walks, in continuous or discrete time, with constant drifts and variances:[13]

$$\ln q_{k2}(T) \sim \mathrm{N}\left(\ln q_{k2}(t) + [\mu_{k2} - \tfrac{1}{2}\sigma_{k2}{}^2]\,(T-t),\ \ \sigma_{k2}\sqrt{T-t}\right),\ k=\{1,2\} \tag{22}$$

where the correlation between the output of the tradable goods at home and abroad is denoted by ρ_2, which is assumed to be constant and less than unity. Given that the volume of domestic [foreign] exports (16) [(17)] is similar to the payoff of an option to exchange two risky assets at the rate κ_2^2 [$1/\kappa_2^1$] and the underlying distribution is jointly lognormal, the expression for the expected trade volume is similar to the solution by Margrabe (1978) for the value of an option to exchange two risky assets with log-normally distributed future prices. Thus, we obtain the following corollary to Proposition 2.1.

Corollary 2.1: *In a one-good model with lognormal outputs, the conditional expectation at time t, of foreign exports at a later date T, is a positive function of the variance of* $[q_{22}(T)/q_{12}(T)]$, *and therefore a positive function of the variance of either* $q_{22}(T)$ or $q_{12}(T)$. *This expectation is given by*

$$\mathrm{E}_t(x_{22}(T)) = \frac{\mathrm{E}_t[q_{22}(T)]\,N(d_1) - \kappa_2^2\,\mathrm{E}_t[q_{12}(T)]\,N(d_2)}{\kappa_2^2/(1+\tau_2) + 1}, \tag{23}$$

where $$\mathrm{E}_t(q_{k2}(T)) = q_{k2}(t)\exp\{\mu_{k2}\,(T-t)\},\ \ k=\{1,2\} \tag{24}$$

$$\phi^2 = \sigma_{12}{}^2 - 2\,\rho_2\,\sigma_{12}\,\sigma_{22} + \sigma_{22}{}^2,\ \textit{the p.a. variance of}\ \ln\frac{q_{22}(T)}{q_{12}(T)} \tag{25}$$

$$d_1 = \frac{\ln\dfrac{\mathrm{E}_t[q_{22}(T)]}{\mathrm{E}_t[q_{12}(T)]} + \dfrac{1}{2}\,\phi^2(T-t) - \ln\kappa_2^2}{\phi\sqrt{T-t}},$$

$$d_2 = \frac{\ln\dfrac{\mathrm{E}_t[q_{22}(T)]}{\mathrm{E}_t[q_{12}(T)]} - \dfrac{1}{2}\,\phi^2\,(T-t) - \ln\kappa_2^2}{\phi\sqrt{(T-t)}},$$

[13]With a lognormal process, an increased variance also means an increased riskiness.

$N(d)$ = *the probability that* $z \leq d$, z *a unit normal random variable.*

In the next section, we find that a higher level of output riskiness also leads to a higher exchange rate volatility, implying that, in a one-good model, there is a positive association between expected trade and exchange rate volatility.

II.B. The Effect of Output Risk on the Variance of the Real Exchange Rate

With $\varepsilon_1 = \varepsilon_3 = 0$ in the general expression for the exchange rate in (14), the real exchange rate in the one-good case is:

$$S(t) = \frac{\theta_2}{\theta_1}\left(\kappa_2^i\right)^{-\eta} \tag{26}$$

As is standard in the literature (because of its symmetry), we consider the log of the exchange rate. Using (5), we can then expand (26) to:

$$\ln S(t) = \begin{cases} \ln\dfrac{\theta_2}{\theta_1} - \eta \ln \kappa_2^1 & \text{if } \dfrac{q_{22}(t)}{q_{12}(t)} \leq \kappa_2^1; \\[2ex] \ln\dfrac{\theta_2}{\theta_1} - \eta \ln \dfrac{q_{22}(t)}{q_{12}(t)} & \text{if } \kappa_2^1 \leq \dfrac{q_{22}(t)}{q_{12}(t)} \leq \kappa_2^2; \\[2ex] \ln\dfrac{\theta_2}{\theta_1} - \eta \ln \kappa_2^2(t) & \text{if } \dfrac{q_{22}(t)}{q_{12}(t)} \geq \kappa_2^2. \end{cases} \tag{27}$$

Thus, the log real exchange rate is a linear transformation of a truncated variate, $\ln[q_{22}(t)/q_{12}(t)]$, where the values of the log output ratio that fall outside the no-trade region are replaced by the constants $\ln\kappa_2^1$ and $\ln \kappa_2^2$. We show, in Appendix B, that the an increase in riskiness in the output of either country leads to an increase in the volatility of the exchange rate under the following conditions.

Proposition 2.2: *Three alternative conditions sufficient for* $\mathrm{var}_t(\ln S(T))$ *to be a positive function of the riskiness of relative output are:*

(a) *The underlying variable,* $\ln[q_{22}(T)/q_{12}(T)]$, *is normally distrbuted;*

(b) *The distribution of* $\ln[q_{22}(T)/q_{12}(T)]$ *is unimodal and the mode lies within the no-trade region.*

(c) *The density function is concave over the no-trade region.*

Proposition 2.2 can be interpreted as follows. Suppose first that the no-trade zone is around the middle of the distribution of relative output. From (15), we know that an increase in the variance of the relative output does not affect κ_2^1 and κ_2^2, that is, the bounds of this no-trade domain. With a unimodal and symmetric distribution (such as the normal distribution), the effect of an increase in uncertainty then is to lower the probability that future relative output will be in the no-trade zone; that is, the probability of trade increases. But non-zero trade also implies that $S(T)$ is at one of its bounds (see (27)). The implication is that, when risk increases, more and more of the probability mass of $\ln S(T)$ is moved from the middle of the distribution towards the bounds, which increases the variance of the log exchange rate.

From Proposition 2.2, we see that one case where the variance of the log of the exchange rate may decrease with an increase in the riskiness of relative output is when, in the no-trade domain, the density function is substantially more convex than the normal distribution. One example that has a sufficiently strong local convexity is the mixture of uniform distribitions (the "step" distribution): if the discountinuity in the density is situated within the no-trade zone and very close to one of the bounds of that zone, the variance of the truncated variable will actually decrease when the riskiness of the underlying goes up.[14] However, most economists would consider this example far-fetched. We numerically verified the properties of the Pareto-Lévy sum-stable and the Student's distributions with a low number of degrees of freedom. These distributions are less unappealing than the step distribution, and, being very fat-tailed and peaked-at-the-center, appeared to be *a priori* candidates for a sufficienly strong convexity at the beginning of the tails. However, no negative association between the riskiness of the underlying and the variance of the truncated

[14]This is shown formally in the appendix, in the concluding part of the proof to Proposition 2.2.

variable was observed—not even for the limiting case of the Cauchy distribution, which is so fat-tailed that even the first moment does not exist. Thus, a negative link between $\text{var}_t(\ln S(T))$ and the riskiness of $\ln[q_{22}(T)/q_{12}(T)]$ is unlikely, except possibly for extremely artificial distributions.

II.C. Concluding Comments Relating to the Analysis of the One-Good Case

We summarize the results for the one-good case as follows.

Proposition 2.3: *In a model with only one good that is tradable at a cost, there is a positive association between exchange rate volatility and the expected volume of trade.*

The existence of a positive shipping cost is crucial for our final conclusion. With $\tau_2 = 0$, in a single-good world the real exchange rate would always equal unity and its variance would therefore be zero regardless of the riskiness of relative output. Thus, as in a neo-classical free-trade model, there would be no association between the average volume of trade and real exchange rate volatility. In contrast, the effect of output volatility on the expected volume of trade would be stronger rather than weaker when $\tau_2 = 0$.[15] Thus, from a policy perspective, a reduction in the barriers to trade ($\tau_2 \rightarrow 0$), leads to both a decline in real exchange rate volatility and an increase in expected trade.

Our conclusions still hold when a non-traded good is added to the model, as long as the output process for this good is non-stochastic. To see this, note from Propositions 1.1 and 1.2 that the critical output ratios for good 2, κ_2^1 and κ_2^2, would then also become functions of the relative output of the non-traded good (good 1). However, in the deterministic case the effect of a non-unit value for $q_{21}(T)/q_{11}(T)$ is no different from the effect of unequal θ's.

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[15]Observe that if τ_2 is zero, the zone of no shipping would shrink to a single ray with slope $(\theta_2/\theta_1)^{1/\eta}$, which would mark the (smooth) transition from exports to imports. That is, when we set τ_2 equal to zero, the volume of trade becomes similar to the payoff from a straddle rather than the payoff from a vertical combination. Thus, with $\tau_2 = 0$ the convexity would be even more marked than in the case $\tau_2 > 0$, and expected trade would be more sensitive to changes in riskiness in the same way as a straddle is more sensitive to changes in volatility than a vertical combination.

Specifically, κ_2^1 and κ_2^2 would still be non-stochastic and all our earlier inferences would, therefore, continue to hold. The assumption of risk-free outputs in the non-traded goods sector is not necessarily unreasonable: empirically the output of, say, services is much less volatile than the output of industrial goods and, crucially, is also less related to changes in the real exchange rate. However, the non-tradable goods sector in most countries is also large, so that it remains useful to explore the effect of risk in this sector on the expected level of trade. This is done in Section III.

Note that our conclusions with respect to trade of good 2 hold even when a perfectly tradable good—good 3 with $\tau_3 = 0$—is added. To see this, note from Propositions 1.2 and 1.3 that, neither trade in good 2 nor $S(t)$ is affected by $q_{23}(t)/q_{13}(t)$; thus, the results about trade for good 2 are qualitatively unaffected by the addition of good 3.[16] However, the reverse is not true: from equations (12) and 13), trade in good 3 itself (and, therefore, also total trade) *is* affected by the output of good 2. Thus, to make any statement about total trade, we also need to know whether the effect of increased risk in the output of good 1, on trade of good 2, is not undone by an opposite effect on trade of good 3. This analysis is considered in Section IV.

It is important to note that although the unconditional relation between exchange rate volatility and trade in the single-good model is positive, this effect need not be obvious in small-sample data. First, sample parameters (such as average trade and ex post exchange rate volatility) are noisy estimates of the population parameters used in our analysis. Second, when trade is already intense and can be expected to remain so in the future, the effect of increased risk on the variance of the real exchange rate and on the expected volume of trade is small. With respect to expected trade, this is because strict convexities occur only at the points κ_2^1 or κ_2^2. Thus, if it is unlikely that these critical values will be reached, the effect of the convexity on the expected level of trade is small. With respect to the real exchange rate,

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[16]The intuition for this is that it is always optimal to balance internationally the marginal utility from consuming the third good. This, with our assumption of a multiplicative utility function, implies that the ratio of outputs of the third good drops out of the expressions for the exchange rate, trade, and the consumption of goods 1 and 2.

when the probability of trade is very high, the effect of riskiness on the variance of the real exchange rate is small because the exchange rate then, almost surely, is at one of its bounds. Thus, under those circumstances exchange rate variability is already small and is hardly affected by variations in output risk.

Lastly, the relation between trade and exchange rate volatility implied by the general equilibrium model is non-linear. Thus, the linear regression model frequently used by empirical studies to estimate the relation between trade and exchange rate volatility is misspecified.

III. A Model with Stochastic Output of Non-Traded Goods

In this section, we consider whether the results derived in Section 2 for a single-good economy generalize to an economy with two goods—a non-traded good and an imperfectly traded good—both with stochastic output. Our analysis of the relation between exchange rate volatility and trade is divided into two parts: we first study the case where the source of the increase in risk is the sector producing the imperfectly tradable good and then the case where the source of the increase in risk is the non-traded goods sector. Our main results in this section are that: (a) The conclusion of the previous section is still valid: when the increase in exchange rate volatility comes from the sectors producing the *imperfectly-tradable* goods, then exchange rate volatility is positively related to the volume of trade; (b) However, an increase in exchange rate uncertainty arising from increased variability in the *non-traded* goods sectors, may be associated with either an increase or a decrease in expected trade, and the effect on the variability of the exchange rate is also ambiguous.

The presence of a non-traded good (good 1) with stochastic output affects the bounds on the relative consumption of good 2 in two ways: one, the location of the region of no-trade is affected and two, the slope of the trade function is affected.[17] Or, using options terminology, the volume of trade now becomes similar to the payoff from a portfolio of options where both the strike prices and the contract sizes are functions of two stochastic variables, the outputs in sector 1. In this section, we examine how the conclusions of the previous section are affected by this randomness in $\kappa_2^1(t)$ and $\kappa_2^2(t)$.

III.A. Effect of Increased Risk in the Imperfectly-Tradable Goods Sector

As we noted, in the presence of a non-traded goods sector with a stochastic output, $\kappa_2^1(t)$ and $\kappa_2^2(t)$ may vary over time. From this it follows that *conditional* on the levels of these non-traded outputs, the relation between trade and the volatility of outputs of good 2 is similar to that in the one-good case. Since this is true for all possible levels of the outputs of the non-traded goods, it is also true unconditionally. Thus, an increase in risk arising from the non-traded goods sector leads to an increase in the volume of trade.

Similarly, the effect of an increase in risk in the imperfectly-tradable goods sector on exchange rate volatility is positive—as it was in the previous section. This is because, *conditional* on relative output in the non-traded goods sector, an increase in riskiness in sector 2 has the same effect as in a one-good model: the variance of the log real exchange rate increases with the riskiness of sector 2.

Proposition 3.1: *In an economy with a non-traded good and a good that is tradable at a cost, both the expected level of trade and* $\mathrm{var}_t(\ln S(T))$ *are positive functions of the riskiness of output in the imperfectly-tradable good sector. Thus, when the source of increased risk is the tradable goods sector, expected trade and the variance of the real exchange rate are positively related.*

[17]The location of the bounds now depends not only on relative wealth, the shipping cost, and risk aversion (as in the one-good case), but also on the relative output of good 1 and on the openness of the economy (as given by ε_2, the proportion of expenditure of the tradable good in total spending).

The results of Sections II and III.A are similar to the conclusions of the partial-equilibrium model of Sercu (1991): a higher exchange risk is associated with a higher probability that the deviation from CPP will be sufficient to allow trade. However, Sercu's analysis takes the demand curve as given. Thus, in Sections III.B and IV we analyze how the demand for good 2 is affected by the changing endowments of other goods, and how this affects trade.

III.B. Effect of Increased Risk in the Non-Traded Goods Sector

To identify whether increased risk in the non-traded goods sector on average boosts trade or not, we need to find out whether trade in good 2 is convex or concave in the relative outputs of the non-tradable good. This can be done by differentiating the expression for exports and imports with respect to the relative outputs of the non-tradable good. We find that the sign of this derivative cannot be determined unambiguously: exports are convex or concave depending on the magnitude of η relative to 1, and the size of κ_1.[18] The presence of both convex and concave sections in the function for net trade implies that, when the riskiness of the relative output in sector 1 increases, the expected value of trade may either increase or decrease, depending on the location and dispersion of the probability distribution.

Proposition 3.2: *In a model with a non-traded good besides the imperfectly tradable good, an increase in the riskiness in the non-traded goods sector may be associated either with an increase or a decrease in expected trade.*

We next consider the relation between risk in the non-traded goods sector and exchange rate volatility. From (14), when $\varepsilon_3 = 0$, the log of the real exchange rate is:

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[18]The effects of a higher output of good 1 are, first, an increase in real wealth in at least one country, and second, a change of the relative price of goods 1 and 2 in at least one country. Thus, a higher output of good 1 in one country could either increase or decrease demand for good 2, depending on whether the income effect dominates the substitution effect or vice versa. The watershed case is the log utility function (η=1, that is, $U(c_1,c_2) = \ln c_1 + \ln c_2$), where demand for good 2 is independent of consumption of good 1.

$$\ln S(t) = \ln\left(\frac{\theta_2}{\theta_1}\right) - \varepsilon_1\eta \ln\big(\kappa_1(t)\big) - \varepsilon_2\eta \ln\big(\kappa_2^j(t)\big). \tag{28}$$

A higher riskiness of $\kappa_1(T)$ now has three effects. First, it directly increases the variance of the second term in (28). Second, a higher riskiness in $\kappa_1(T)$ affects the covariance between the last two terms. Lastly, it randomizes the bounds on $\kappa_2^j(t)$. It turns out that the sign of the net effect cannot be determined unambiguously.

Proposition 3.3: *In a model with a non-traded good besides the imperfectly tradable good, an increase in the riskiness in the non-traded goods sector may be associated either with an increase or a decrease in* $\mathrm{var}_t(\ln S(T))$.

III.C. Conclusions from the Model with a Non-Traded and a Tradable Good

We find that, for given riskiness of the non-traded goods sector, when the change of risk is confined to the tradable-goods sector, our earlier conclusions still obtain: trade and exchange rate volatility are positively related. However, when the source of the extra risk is the *non*-traded goods sector, this may no longer be true; the outcome depends on whether the conditional joint distribution of the outputs implies a predominantly convex relation between relative output in sector 1 and trade in good 2, or a predominantly concave one. We also find that a higher riskiness in sector 1 may affect the variance of the exchange rate positively or negatively, depending on the value of the risk aversion coefficient and the openness of the economy. Thus, the addition of a non-traded good with stochastic output obscures the positive effect of sector-2 risk on both the expected level of trade and exchange rate volatility.

To summarize: the addition of the non-traded good allows us to identify one reason why the results from the simple one-good model may be misleading. In the next section, we show how interactions between the trade volumes of two goods, *both* tradable, provide similar sources of concavities and strict convexities, thus reinforcing the conclusions of this section.

IV. The Effect of Adding a Second Traded Good

In this section, we consider the general model with all three goods: one that cannot be traded at all, a second that is tradable at a cost, and a third that can be traded costlessly. Below, we discuss the effects of increased risk in a particular sector, starting with sector 3 itself.

IV.A. The Effects of Risk in the Perfectly Traded Goods Sector

From equations (12) and (13), we see that for the perfectly tradable good, the absolute amount of trade is a piecewise linear (V-shaped) function of the endowments of that good. That is, because good 3 is always traded, the no-trade zone observed for good 2 is absent for this good, but the pattern is otherwise similar to what holds for good 2. It follows that, like for sector 2, expected trade of good 3 is a positive function of the riskiness of the output in its own sector.

To verify whether this conclusion holds not just for trade in good 3 but also for overall trade, we need to verify that the higher output risk in sector 3 does not induce lower trade in good 2. This is easily established: from equations (10) and (11), trade in good 2 is unaffected by the output or consumption of good 3. Therefore, a higher output risk in sector 3 boosts trade in good 3 with no offsetting effect on trade in good 2, implying that overall trade increases.

However, a higher riskiness in the output of good 3 does *not* lead to an increased variability in the exchange rate. The absence of such a relation follows immediately from Proposition 1.3: output of the perfectly tradable good has no direct effect on the real exchange rate, nor does it have an indirect effect through relative consumption of the other goods (see equations (4) and (5)). We summarize as follows:

Proposition 4.1: *Increases in output risk of a perfectly traded good boost expected trade but not the variability of the exchange rate. Thus, such changes in risk cannot be a source of* *positive or negative association between the expected level of trade and exchange rate volatility.*

IV.B. The Effects of Output Risk in the Non-traded Goods Sector

We already know that the addition of a perfectly tradable good does not affect the real exchange rate (Proposition 1.3) nor the expected level of trade in sector 2 (Proposition 1.2). Thus, our conclusions from Section III, regarding the effects of the riskiness of the non-traded goods sector, still hold when a perfectly traded good is added: the variability of the output in the non-traded goods sector has ambiguous effects on both the volatility of the exchange rate and the expected level of trade in good 2. All we need to verify is whether the effect of output risk in sector 1 on trade in good 3 reinforces, or offsets, the effect of trade in sector 2.

From the discussion in Section III.B. we already know that trade in both good 2 and 3 can be concave or convex functions of the relative output in the non-tradable good sector. Thus, increased risk in sector 1 may be associated with either an increase or a decrease in trade for good 3, depending on whether the distribution of $\kappa_1(T)$ is mostly in the concave section of the function or in the convex section. Nor can we hope that concavities for trade in good 2 are always more than offset by convexities in good 3 and *vice versa*. Indeed, from the expressions for $\kappa_2^i(t)$ and $\kappa_3^i(t)$ in Proposition 1.2, we see that the effect of $q_{21}(t)/q_{21}(t)$ on $\kappa_2^i(t)$ and $\kappa_3^i(t)$ is similar: when the downward-sloping part of trade of good 2 is convex (concave), the downward-sloping part of trade of good 3 is convex (concave) too, and similarly for the positive-sloped sections. Thus, concavities for one good are not generally offset by convexities for the other good. As a result, an increase in risk in sector 1 may lead to a reduction of expected trade in each of the traded goods. This confirms the conclusions from the two-good case discussed in Section III.B.

Proposition 4.2: *An increase in the riskiness in the non-traded goods sector has ambiguous effects on total trade and on the volatility of the exchange rate.*

IV.C. The Effects of Output Risk in the Imperfectly Tradable Goods Sector

Lastly, we investigate how our conclusions from Section II are affected by the addition of the third good. First, both the exchange rate and trade in good 2 are unaffected by the introduction of a perfectly tradable good; thus, output risk in sector 2 still boosts the variability of the exchange rate as well as the expected level of trade in good 2, as in Section II. All we need to add is an analysis of how output risk in sector 2 affects trade in good 3.

The arguments used in Section III.B can be repeated to show that trade in good 3, as a function of relative consumption of another good, is V-shaped with one convex and one concave leg. When this other good is the non-traded one (good 1, as in Section III), relative consumption of good 1 is identical to its relative output, thus inducing a concave/convex V-shaped relation between trade of good 3 and relative output of good 1. But when the other good is the (imperfectly tradable) good 2, relative consumption of good 2 becomes a piecewise linear function of its output rather than being identical to it (Proposition 1.1). Thus, trade in good 3 as a function of relative consumption of another good, has not only both convex and concave sections, but also abrupt changes in the slopes—notably when the relative output in sector 2 reaches the critical values that determine whether good 2 is traded or not. Thus, the concave (convex) sections are now interrupted by local convexities (concavities). Thus, it is *a fortiori* difficult to predict the effect of increased risk in sector 2 on trade in good 3.

Proposition 4.3: *When a perfectly tradable good is introduced, a higher risk in the relative output of sector* 2 *still increases exchange risk and expected trade in good* 2, *but has an uncertain effect on trade in good* 3. *Thus, the effects on total trade are uncertain, implying that the relation between exchange rate volatility and total trade is ambiguous.*

V. Conclusions

In this paper, we study the relation between the volume of trade and exchange rate volatility in a general equilibrium, two-country economy with three goods: one that cannot be traded, a second that is tradable at a cost and a third that can be traded costlessly. While commodity markets are assumed to be segmented, we assume that financial markets are complete and perfect. Our main results are the following. If there is only one (imperfectly-tradable) good, a change in output risk affects the expected volume of trade and exchange rate volatility in the same direction, thus creating a positive association between the two. We show, however, that the inferences from a one-good economy can be misleading: if the source of volatility in the exchange rate is the non-traded goods sector, the relation between exchange rate volatility and trade is no longer unambiguously positive. Given that the magnitude of the non-traded goods sector in developed economies is large, our model is consistent with the findings of empirical studies that the relation between exchange rate volatility and trade is weak. These conclusions are reinforced when a perfectly-tradable good is added: a higher risk in the non-traded goods sector has ambiguous effects on expected trade in both the imperfectly-tradable good and the perfectly-tradable one; likewise, a higher risk in the imperfectly-tradable goods sector may boost or reduce expected trade in the perfectly-tradable goods sector.

The general-equilibrium analysis sheds light on two types of conclusions that are obtained from partial-equilibrium models. The first result obtained using a partial-equilibrium approach is that an increase in exchange risk lowers the risk-adjusted expected revenue from exports and, therefore, reduces the incentives to trade. This view ignores the existence of hedges against exchange risk, or takes the price of these hedges (or some of the determinants of these prices) as given. In contrast, our general-equilibrium approach allows us to detect the outcome of decentralized decision making in a complete market where traders take optimal hedging decisions and where the prices of these hedge instruments are set in equilibrium. The second set of results obtained by partial equilibrium models is that a higher exchange risk is always associated with a higher probability that deviations from Commodity Price Parity will be sufficiently large to generate trade. This view takes the demand curve as given, and ignores the issue about the source of the higher exchange risk. We have shown that the shifts in the demand curves caused by changing endowments of other goods are likely to obscure, or possibly even dominate, the effects that would be observed in a one-good or partial-equilibrium model.

From a policymaker's perspective, our model implies that the volatility of the real exchange rate can be reduced by (a) reducing the volatility of fundamentals and (b) by reducing the barriers to trade. However, these two measure may have different effects on trade: while a reduction in trade barriers is associated with an increase in the volume of trade, a reduction in the volatility of fundamentals may be associated with a reduction in trade. That is, even though both measures enhance welfare, they may have opposite effects on the endogenous variable, trade. This means that, in a model with risk, that one cannot simply look at expected trade to draw conclusions about welfare.

Appendix A: Derivation of the Amount of Trade and the Real Exchange Rate

This appendix contains the proofs of the propositions that relate to the amount of trade and the exchange rate. The proofs for Propositions 2.2 and 3.3, which are more related to statistics than economics, are given in Appendix B.

Proof of Proposition 1.1

Given that the utility function in (2) is time-separable, and the constraints in (3) apply period-by-period, we can rewrite the intertemporal problem of the central planner as a static optimization program. Thus, the central planner's problem at time t is:

$$\underset{x_{kj}(t)}{\text{Max}}\ \theta_1 \left(\prod_{j=1}^{3} c_{1j}(t)^{\varepsilon_j} \right)^{1-\eta} + \theta_1 \left(\prod_{j=1}^{3} c_{2j}(t)^{\varepsilon_j} \right)^{1-\eta} \tag{A1}$$

subject to the constraints in (3). Letting $\Lambda(t)$ denote the Lagrangian function and $\lambda_{kj}(t)$ the Lagrangian multipliers on the consumption of good j in country k, we get the following first-order conditions:

$$0 = \frac{\partial \Lambda(t)}{\partial c_{kj}(t)} = (1-\eta)\, \theta_k\, \varepsilon_j \left(\prod_{j=1}^{3} c_{1j}(t)^{\varepsilon_j} \right)^{1-\eta} \frac{1}{c_{kj}} - \lambda_{kj},\ j\text{=1, 2, 3 and } k\text{=1, 2};$$

$$0 = x_{kj}(t)\, \frac{\partial \Lambda(t)}{\partial x_{kj}(t)},\ j\text{=1, 2, 3 and } k\text{=1, 2};$$

$$0 \geq \frac{\partial \Lambda(t)}{\partial x_{kj}(t)} = -\lambda_{1j}(t) + \frac{\lambda_{2j}(t)}{1+\tau_j} \Rightarrow \frac{\lambda_{1j}(t)}{\lambda_{2j}(t)} \geq \frac{1}{1+\tau},\ j\text{=1, 2, 3};$$

$$0 \geq \frac{\partial \Lambda(t)}{\partial x_{kj}(t)} = -\lambda_{2j}(t) + \frac{\lambda_{1j}(t)}{1+\tau_j} \Rightarrow \frac{\lambda_{2j}(t)}{\lambda_{1j}(t)} \geq \frac{1}{1+\tau},\ j\text{=1, 2, 3}.$$

Substituting the appropriate value for τ_j $\{j = 1, 2, 3\}$, we infer that for the non-tradable good ($j = 1$, $\tau_1 = \infty$) it is optimal not to correct any difference in the ratio of marginal utility of consumption of this good across countries (that is, this good is never traded). Thus:

$$\frac{c_{21}(t)}{c_{11}(t)} = \frac{q_{21}(t)}{q_{11}(t)} \equiv \kappa_1, \tag{A2}$$

which is the expression in (4). In contrast, because $\tau_3 = 0$, the first-order conditions imply that it is optimal to trade good 3 so as to equalize the marginal utilities of consumption of this good across countries:

$$1 = \frac{\partial U_2(t)/\partial c_{23}(t)}{\partial U_1(t)/\partial c_{13}(t)} = \frac{\theta_2}{\theta_1}\left(\prod_{j=1}^{3}\left(\frac{c_{2j}(t)}{c_{1j}(t)}\right)^{\varepsilon_j}\right)^{1-\eta}\left(\frac{c_{23}(t)}{c_{13}(t)}\right)^{-1}. \tag{A3}$$

This implies the following optimal consumption ratio for good 3:

$$\frac{c_{23}(t)}{c_{13}(t)} = \left[\frac{\theta_2}{\theta_1}\left(\frac{c_{21}(t)}{c_{11}(t)}\right)^{\varepsilon_1(1-\eta)}\left(\frac{c_{22}(t)}{c_{12}(t)}\right)^{\varepsilon_2(1-\eta)}\right]^{\frac{1}{1-\varepsilon_3(1-\eta)}}. \tag{A4}$$

Lastly, for good 2 that can be traded only at a cost ($0 < \tau_2 < \infty$), the first order conditions yield the following bounds on relative marginal utility:

$$\frac{1}{1+\tau_2} \leq \frac{\partial U_2(t)/\partial c_{22}(t)}{\partial U_1(t)/\partial c_{12}(t)} = \frac{\theta_2}{\theta_1}\left(\prod_{j=1}^{3}\left(\frac{c_{2j}(t)}{c_{1j}(t)}\right)^{\varepsilon_j}\right)^{1-\eta}\left(\frac{c_{22}(t)}{c_{12}(t)}\right)^{-1} \leq 1+\tau_2, \tag{A5}$$

implying that it is optimal to trade this good only when these bounds are violated in the absence of trade. After substituting (A2) and (A4) into (A5) and rearranging, we obtain the following bounds on the relative consumption of good 2:

$$\kappa_2^1(t) \leq \frac{c_{22}(t)}{c_{12}(t)} \leq \kappa_2^2(t), \tag{A6}$$

where $\kappa_2^1(t)$ and $\kappa_2^2(t)$ are as defined in (5). To obtain the bounds for good 3 expressed in terms of output ratios, as in (6), substitute into (A4) the appropriate expression for $c_{22}(t)$ and $c_{12}(t)$ in (5).

Proof of Proposition 1.2

From (A2), it is never optimal to trade good one; this yields the result in (9). To obtain equations (10)-(13), note that the relevant state space can be divided into three distinct regions: a) where good 2 is not traded (state $i = 0$); b) where good 2 is exported by country 1 (state $i = 1$); and c) where good 2 is exported by country 2 (state $i = 2$).

a. No trade in good 2. In the absence of trade in sector 2, we have $\frac{c_{22}(t)}{c_{12}(t)} = \frac{q_{22}(t)}{q_{12}(t)}$, implying that

$$x_{12}(t) = x_{22}(t) = 0. \tag{A6}$$

To compute the exports for the third good, note that the consumption ratio for the perfectly tradable good in this state is given by $\kappa_3^0(t)$, defined in (6). The volume of trade for good 3 can be identified from the sharing rule $c_{23}(t) = \kappa_3^0(t)\, c_{13}(t)$ and the market clearing condition $c_{13}(t) = q_{13}(t) - x_{13}(t) + x_{23}(t)$ and $c_{23}(t) = q_{23}(t) - x_{23}(t) + x_{13}(t)$. The solution is:

$$x_{13}(t) - x_{23}(t) = \frac{\kappa_3^0(t)\, q_{13}(t) - q_{23}(t)}{1 + \kappa_3^0(t)}. \tag{A7}$$

Note that $x_{13}(t)$ and $x_{23}(t)$ are constrained to be positive, which implies that if $\kappa_3^0(t)\, q_{13}(t) - q_{23}(t) > 0$, then $x_{23}(t)$ is zero in (A7); otherwise, $x_{13}(t)$ is zero.

b. Exports of good 2 from country 1. From (5), in state $i = 1$, country 1 must be exporting an amount $x_{12}(t)$ of good 2 such that $c_{22}(t)/c_{12}(t)) = (1+\tau_2)^{-\alpha} \left(\theta_2/\theta_1\right)^{\alpha} \left(\kappa_1(t)\right)^{\beta}$. The amount of good 2 being exported from country 1 can be identified from the sharing rule $c_{22}(t) = \kappa_2^1(t)\, c_{12}(t)$ (with $\kappa_2^1(t)$ defined in (5)) and the market clearing condition $c_{12}(t) = q_{12}(t) - x_{12}(t)$ and $c_{22}(t) = q_{22}(t) + x_{12}(t)/(1 + \tau_2)$. The solution is

$$x_{12}(t) = \frac{q_{12}(t) - q_{22}(t)/\kappa_2^1(t)}{1/[\kappa_2^1(t)(1+\tau_2)] + 1}, \tag{A8}$$

which is positive since we are considering states where $q_{12}(t)\ \kappa_2^1(t) > q_{22}(t)$, implying that $x_{22}(t) = 0$. The optimal consumption ratio for good 3 in state 1 is $\kappa_3^1(t)$, given in (6). By a similar argument as before, we identify the amount of trade in the perfectly tradable good:

$$x_{13}(t) - x_{23}(t) = \frac{\kappa_3^1(t)\, q_{13}(t) - q_{23}(t)}{1 + \kappa_3^1(t)}. \tag{A9}$$

where $x_{13}(t)$ and $x_{23}(t)$ can be determined by imposing the non-negativity constraint on the export quantities.

c. Exports of good 2 from country 2. In this state, country 2 must be exporting an amount $x_{22}(t)$ such that $c_{22}(t)/c_{12}(t) = (1+\tau_2)^{+\alpha}\ \left(\theta_2/\theta_1\right)^{\alpha}\ \left(\kappa_1(t)\right)^{\beta}$. Imposing the market clearing condition, the volume of trade in good 2 is:

$$x_{22}(t) = \frac{q_{22}(t) - \kappa_2^2(t)\, q_{12}(t)}{\kappa_2^2(t)/(1+\tau_2) + 1}, \tag{A10}$$

which is positive since we are considering states where $q_{22}(t)\ \kappa_2^2(t) > q_{12}(t)$ implying that $x_{12}(t) = 0$. Finally, the optimal consumption ratio for good 3 is given by $\kappa_3^2(t)$, and using the same arguments as above, allows us to obtain:

$$x_{13}(t) - x_{23}(t) = \frac{\kappa_3^2(t)\, q_{23}(t) - q_{13}(t)}{1 + \kappa_3^2(t)} \tag{A11}$$

where $x_{13}(t)$ and $x_{23}(t)$ can be identified by imposing the non-negativity constrain on the export quantities. Collecting the results in (A6) and (A8) gives (10), and the results in (A6) and (A10) give (11). Collecting the results in (A7), (A9) and (A11) yields (12) and (13).

Proof of Proposition 1.3

Let $Z(t)$ denote the nominal exchange rate. Then, the relative price of a good across countries is given by the marginal rate of substitution, in the central planner's utility function, of consumption of that good abroad versus at home:

$$\frac{Z(t)\, p_{2j}(t)}{p_{1j}(t)} = \frac{\theta_2\, \partial U_2(t)/\partial c_{2j}(t)}{\theta_1 \partial U_1(t)/\partial c_{1j}(t)} \tag{A12}$$

We can relate the right-hand-side to the indirect utility function, $V(m_k(t), p_k(t))$, defined as

$$V(m_k(t), p_k(t)) \equiv \underset{c_{kj}(t)}{\text{Max}} \ \{U_k(c_k(t)) - \Lambda_k(t)\, [\sum_{j=1}^{N} c_{kj}(t)\, p_{kj}(t) - m_k(t)]\}, \tag{A13}$$

where $m_k(t)$ is nominal spending, $p_k(t)$ is the vector of prices (of goods j, $\{j=1, 2, 3\}$), and $\Lambda_k(t) = \partial V(m_k(t), p_k(t))/\partial m_k(t)$ is the marginal indirect utility of nominal spending in country k. The first order condition of this optimization problem is $\partial U_k(c_k(t))/\partial c_{kj}(t) = \Lambda_k(t)\ p_{kj}(t)$. Substituting this condition into (A12) and rearranging, we obtain

$$Z(t) = \frac{\dfrac{\theta_2 \partial U_2(c_2(t)) \,/\, \partial c_{2j}(t)}{p_{2j}(t)}}{\dfrac{\theta_1 \partial U_1(c_1(t)) \,/\, \partial c_{1j}(t)}{p_{1j}(t)}} = \frac{\dfrac{\theta_2 \partial V(m_2(t), p_2(t))}{\partial m_2(t)}}{\dfrac{\theta_1 \partial V(m_1(t), p_1(t))}{\partial m_1(t)}} . \tag{A14}$$

Thus, the nominal exchange rate is the ratio of the marginal indirect utilities of nominal spending abroad versus at home.

If the utility function is homothetic, $U_k(c_k(t))$ can be written as $\Phi[u_k(c_k(t))]$, where $u_k(c_k(t))$ is a linear homogenous function of the consumption amounts of the individual goods and Φ is an order-preserving transformation. Thus, if $U_k(c_k(t))$ is maximized subject to the budget constraint, also $u_k(c_k(t))$ must be at its maximum subject to the budget constraint. Denote the solution to that last problem as $v_k(m_k(t), p_k(t))$:

$$v(m_k(t), p_k(t)) \equiv \underset{c_{kj}(t)}{\text{Max}} \ \{v_k(c_k(t)) - \lambda_k(t)\, [\sum_{j=1}^{N} c_{kj}(t)\, p_{kj}(t) - m_k(t)] . \tag{A15}$$

It is well known—see, for instance, Samuelson and Swamy (1974))—that the multiplier $\lambda_k(t)$ in (A15) is of the form $\lambda_k(t) = 1/\Pi_k(p_k(t))$, with Π a linear homogenous function of the prices $p_{kj}(t)$ and independent of the nominal consumption budget, $m_k(t)$. In addition, the indirect utility functions are related as follows:

$$V(m_k(t), p_k(t)) = \Phi[v_k(t)] = \Phi[m_k(t)/\Pi_k(p_k(t)], \tag{A16}$$

As $\Pi_k(p_k(t))$ is linear homogenous in the prices of individual goods, it can be interpreted as the price level, and $v_k(t) = m_k(t)/\Pi_k(p_k(t)$ can be interpreted as the indicator of (total) real consumption. Using the chain-rule to differentiate (A16), we can re-express the marginal utilities of nominal spending, that appear on the right hand side of (A14), to the marginal utility of real total consumption, $d\Phi(v_k(t))/dv_k(t)$, divided by the price level. Substituting this into (A14), yields:

$$Z(t) = \frac{\theta_2}{\theta_1} \frac{d\Phi_2(t)/dv_2(t)}{d\Phi_1(t)/dv_1(t)} \frac{\Pi_1(t)}{\Pi_2(t)} \tag{A17}$$

For our utility function, real consumption is given by $c_k(t) \equiv \left(\prod_{j=1}^{3} (c_{kj}(t))^{\varepsilon_j} \right)$. It follows that

$$S(t) \equiv Z(t)\frac{\Pi_2(t)}{\Pi_1(t)} = \frac{\theta_2}{\theta_1} \left(\prod_{j=1}^{3} \left(\frac{c_{2j}(t)}{c_{1j}(t)} \right)^{\varepsilon_j} \right)^{-\eta}. \tag{A18}$$

Upon substituting the optimum consumption ratio for each good, given in equations (4), (5) and (6), we obtain (14).

Proof of Proposition 2.1

The results follow from the convexity of the functions (16), (17), (18) and (19) in $q_{12}(T)$ and $q_{22}(T)$.

Proof of Corollary 2.1

We consider foreign exports $x_{22}(T)$ as given in (17), and rewrite Max[$q_{22}(T) - \kappa_2^2\, q_{21}(T)$, 0] as $q_{22}(T) - \kappa_2^2\, q_{21}(T)$ times an indicator function:

$$x_{22}(T) = \frac{[q_{22}(T) - \kappa_2^2\, q_{21}(T)]\; I(\frac{q_{22}(T)}{q_{21}(T)})}{\kappa_2^2/(1+\tau_2) + 1},$$

$$\text{where } I(\frac{q_{22}(T)}{q_{21}(T)}) = \begin{cases} 1 & \text{if } \frac{q_{22}(T)}{q_{21}(T)} > \kappa_2^2(t) \\ 0 & \text{otherwise} \end{cases}.$$

Thus, the expectation to be evaluated can be written as

$$\mathrm{E}_t(x_{22}(T)) = \frac{\mathrm{E}_t\left(q_{22}(T)\, I(\frac{q_{22}(T)}{q_{21}(T)})\right) - \kappa_2^2\, \mathrm{E}_t\left(q_{21}(T)\, I(\frac{q_{22}(T)}{q_{21}(T)})\right)}{\kappa_2^2/(1+\tau_2) + 1}. \tag{A19}$$

To solve the expectations in the above expression, we use the following result:

Lemma: *Let X and Y (where Y may be a vector) be joint lognormal with means of the log-transforms denoted by m_x and m_y, variances of the log-transforms denoted by v_x and v_y, and covariance between the log-transforms denoted by c_{xy}. Let $f(Y)$ be a function of Y. Then, provided the expectation exists,*

$$\mathrm{E}(X f(Y)\,;\, m_x, m_y, v_x, v_y, c_{xy}) = \mathrm{E}(X\,;\; m_x, v_x)\;\; \mathrm{E}(f(Y)\,;\, m_y + c_{xy}, v_y). \tag{A20}$$

That is, in $\mathrm{E}(f(Y)\,;\, m_y + c_{xy}, v_y)$ *the mean(s) of lnY has (have) been shifted by adding the covariance of* ln*Y with* ln*X*.

Proof: Please see Beckers and Sercu (1987).

We apply Lemma 1 to each term in the expectation of equation (A19), choosing the corresponding tradable good output as the X-variable, and the indicator $I(\frac{q_{22}(T)}{q_{21}(T)})$ as the function $f(Y)$. Noting that the expectation of this indicator function is a probability, the expected volume of foreign exports can be written as the difference of the two expected values, each of them multiplied by a cumulative normal probability, $E_t\{I(\frac{q22(T)}{q21(T)})\} = N(d)$, evaluated on the basis of an appropriately shifted distribution function:

$$E_t\left(q22(T)\, I(\frac{q22(T)}{q21(T)})\right) - \kappa_2^2\, E_t\left(q21(T)\, I(\frac{q22(T)}{q21(T)})\right) = E_t[q22(T)]\, N(d_1) - \kappa_2^2\, E_t[q21(T)]\, N(d_2)$$

where $N(d)$ is the cumulative standard normal probability (prob ($z \leq d$)). To obtain the argument for the (shifted) normal probability function, we rewrite the shifted mean in the first expectation on the left-hand side of the above expression as follows:

$$E_t\left(\ln \frac{q22(T)}{q22(T)}\right) + \mathrm{cov}_t\left(\ln q22(T)\, ,\, \ln \frac{q22(T)}{q22(T)}\right)$$

$$= [\ln q22(t) + (\mu_{22} - \tfrac{1}{2}\sigma_{22}{}^2)\,(T-t)] - [\ln q21(t) + (\mu_{21} - \tfrac{1}{2}\sigma_{21}{}^2)\,(T-t)]$$
$$+ [\sigma_{22}{}^2 - \rho_2\, \sigma_{21}\, \sigma_{22}]\,(T-t)$$

$$= [\ln q22(t) + \mu_{22}\,(T-t)] - [\ln q21(t) + \mu_{21}\,(T-t)] + \tfrac{1}{2}[\sigma_{21}{}^2 - 2\,\rho_2\, \sigma_{21}\, \sigma_{22} + \sigma_{22}{}^2]\,(T-t)$$

$$= \ln \frac{E_t[q22(t)]}{E_t[q21(t)]} + \frac{1}{2}\phi^2\,(T-t)$$

where $\phi^2 \equiv \sigma_{21}{}^2 - 2\,\rho_2\, \sigma_{12}\, \sigma_{22} + \sigma_{22}{}^2$ is the variance of the log output ratio. Thus, the probability associated with $E_t(q22(t))$ can be worked out as

$$E_t\left(I(\frac{q22(T)}{q21(T)})\, ;\ \ln \frac{E_t[q22(t)]}{E_t[q21(t)]} + \frac{1}{2}\phi^2(T-t)\, ,\ \phi^2(T-t)\right)$$

$$= \mathrm{Prob}\left(\ln\frac{q22(T)}{q21(T)} > \ln\kappa_2^2\, ;\ \ln \frac{E_t[q22(t)]}{E_t[q21(t)]} + \frac{1}{2}\phi^2(t_1 - t)\, ,\ \phi^2(t_1 - t)\right)$$

$$= N(d_1)\, , \qquad \text{(A21)}$$

where $d_1 = \dfrac{\ln \frac{E_t[q22(t)]}{E_t[q21(t)]} + \frac{1}{2}\phi^2(t_1 - t) - \ln \kappa_2^2}{\phi\, \sqrt{t_1 - t}}$.

Analogously, the shifted mean in the second expectation in equation (A19) can be rewritten as

$$E_t\left(\ln\frac{q_{22}(T)}{q_{22}(T)}\right)+\mathrm{cov}_t\left(\ln q_{21}(T)\,,\,\ln\frac{q_{22}(T)}{q_{22}(T)}\right)=\ln\frac{E_t[q_{22}(t)]}{E_t[q_{21}(t)]}+\frac{1}{2}\phi^2(T-t)\,, \tag{A22}$$

implying that the associated probability is

$$E_t\left(I\left(\frac{q_{22}(T)}{q_{21}(T)}\right);\ln\frac{E_t[q_{22}(t)]}{E_t[q_{21}(t)]}-\frac{1}{2}\phi^2(t_1-t)\,,\ \phi^2(t_1-t)\right)$$

$$=\mathrm{Prob}\left(\frac{q_{22}(T)}{q_{21}(T)}>\kappa_2^2\ ;\ \ln\frac{q_{22}(t)}{q_{21}(t)}-\frac{1}{2}\phi^2(t_1-t)\,,\ \phi^2(t_1-t)\right)$$

$$= N(d_2)\,, \tag{A23}$$

where $d_2=\dfrac{\ln\dfrac{E_t[q_{22}(t)]}{E_t[q_{21}(t)]}-\dfrac{1}{2}\phi^2(t_1-t)-\ln\kappa_2^2}{\phi\sqrt{t_1-t}}$.

Using the Lemma, and (A21) and (A23), we obtain

$$\frac{E_t\left(q_{22}(T)-\kappa_2^2\,q_{21}(T)]\,I\left(\frac{q_{22}(T)}{q_{21}(T)}\right)\right)}{\kappa_2^2/(1+\tau_2)+1}=\frac{E_t[q_{22}(T)]\,N(d_1)-\kappa_2^2\,E_t[q_{21}(T)]\,N(d_2)}{\kappa_2^2/(1+\tau_2)+1}, \tag{A24}$$

which is equation (25). We can rewrite the numerator of the right hand side of (A24) as

$$E_t[q_{22}(T)]\,N(d_1)-\kappa_2^2\,E_t[q_{21}(T)]\,N(d_2)=E_t[q_{21}(T)]\times\left(\frac{E_t[q_{22}(T)]}{E_t[q_{21}(T)]}N(d_1)-\kappa_2^2\,N(d_2)\right).$$

The part in the curly brackets is formally identical to the valuation formula of Black and Scholes (1973) and Merton (1973) for a call option on an asset with current price $\dfrac{E_t[q_{22}(T)]}{E_t[q_{21}(T)]}$, strike price κ_2^2, a zero interest rate and variance ϕ^2 p.a. Because option prices increase, *ceteris paribus*, when the variance increases, the expression in square brackets is a positive function of ϕ^2.

Proof of Proposition 2.3

This follows from Propositions 2.1 and 2.2.

Proof of Proposition 3.1

Conditional on the output in the non-traded goods sector, the relation between riskiness of output in the imperfectly traded goods sector and trade is positive, as established in Proposition 2.1. Similarly, conditional on the output in the non-traded goods sector, the relation between riskiness of output in the imperfectly traded goods sector and the volatility of the exchange rate is also positive, as established in Proposition 2.2. Given that these results are true for all levels of output in the non-traded goods sector, they are also true unconditionally.

Proof of Proposition 3.2

To prove the claim that the volume of trade may *not* be increasing in the volatility of the non-traded good (good 1), it is sufficient to show that exports may be non-convex in the ratio of the output of good 1 across the two countries, $\kappa_1(t) \equiv q_{21}(t)/q_{11}(t)$. That is, we show that, for some parameter values,

$$\frac{\partial^2 x_{12}(t)}{\partial[\kappa_1(t)]^2} < 0.$$

From (10), we can re-write the expression for exports in the case where they are positive as:

$$x_{12}(t) = \frac{(1+\tau_2)\,[\kappa_2^1(t)\, q_{21}(t) - q_{22}(t)]}{\kappa_2^1(t)\,(1+\tau_2) + 1}. \tag{A25}$$

Thus, the volume of exports depends on the output of good 1 only through the effect on $\kappa_2^1(t)$. From (5),

$$\frac{\partial\, \kappa_2^1(t)}{\partial\, \kappa_1(t)} = \beta\, \frac{\kappa_2^1(t)}{\kappa_1(t)}, \tag{A26}$$

$$\frac{\partial^2\, \kappa_2^1(t)}{\partial\, [\kappa_1(t)]^2} = \beta\,(\beta - 1)\frac{\kappa_2^1(t)}{[\kappa_1(t)]^2}. \tag{A27}$$

where $\beta = \dfrac{\varepsilon_1(1-\eta)}{1-(1-\varepsilon_1)(1-\eta)}$. Then, differentiating (A25), and simplifying the resulting expression gives:

$$\frac{\partial x_{12}(t)}{\partial \kappa_1(t)} = \frac{\partial \kappa_2^{\frac{1}{2}}(t)}{\partial \kappa_1(t)} \times \frac{(1+\tau_2)\,[q_{21}(t) + (1+\tau_2)\,q_{22}(t)]}{[\kappa_2^{\frac{1}{2}}(t)\,(1+\tau_2) + 1]^2}\,.$$

Differentiating one more time, and collecting terms we get:

$$\frac{\partial^2 x_{12}(t)}{\partial\,[\kappa_1(t)]^2} = \frac{(1+\tau_2)\,[q_{21}(t) + (1+\tau_2)\,q_{22}(t)]}{[\kappa_2^{\frac{1}{2}}(t)\,(1+\tau_2) + 1]^3}$$

$$\times\left\{ [1 + (1+\tau_2)\,\kappa_2^{\frac{1}{2}}]\,\frac{\partial^2 \kappa_2^{\frac{1}{2}}(t)}{\partial\,[\kappa_1(t)]^2} - 2\,(1+\tau_2)\left(\frac{\partial \kappa_2^{\frac{1}{2}}(t)}{\partial \kappa_1(t)}\right)^2 \right\}. \quad \text{(A28)}$$

Thus, the sign of (A28) depends on the expression in curly brackets. To show that this may be negative, substitute from (5), (A26) and (A27), and simplify to get:

$$\frac{\partial^2 x_{12}(t)}{\partial\,[\kappa_1(t)]^2} = (1+\tau_2)\,[q_{21}(t) + (1+\tau_2)\,q_{22}(t)]\,(\kappa_1(t))^{\beta-2} \times \beta$$

$$\times \left\{ (\beta - 1) - (\beta + 1)\,(1+\tau_2)^2\,[q_{21}(t) + (1+\tau_2)\,q_{22}(t)]\,[\kappa_1(t)]^{\beta} \right\} \qquad \text{(A29)}$$

where $\beta = \dfrac{\varepsilon_1(1-\eta)}{1-(1-\varepsilon_1)(1-\eta)}$. Consider the case where $0 < \eta < 1$, and thus, $0 < \beta < 1$. The sign of (A29) then depends on the terms in the curly brackets (on the second line)—but the first term in curly brackets is negative and the second one is positive. Thus, the difference of the two terms in curly brackets is always negative for the case $0 < \eta < 1$.

Proof of Proposition 4.1

The direct relation between trade in good 3 and riskiness of output in sector 3 follows from the convexity of the expressions for exports and imports in (12) and (13). The fact that overall trade also increases follows from (10) and (11), where we see that trade in the imperfectly-tradable good is not affected by the output in sector 3. Finally, the fact that there is no effect of riskiness in sector 3 on the exchange rate can be seen from the expression for the exchange rate in (14), which is independent of output in sector 3.

Proof of Proposition 4.2

Given that a perfectly tradable good does not affect either trade or the exchange rate (as explained in the proof for Proposition 4.1), the effect of an increase in riskiness in sector 2 on trade in good 2 and exchange rate volatility can be deduced from the results in Propositions 3.2 and 3.3. To show that the effect of an increase in riskiness in sector 2 on trade in good 3 is ambiguous, use the same approach as that in the proof for Proposition 3.2.

Proof of Proposition 4.3

The fact that trade in good 3 is not always an increasing function of the riskiness of output in sector 2 can be established using the same arguments as in the proof for Proposition 3.2. That a higher risk in the relative output of sector 2 still increases exchange risk and expected trade in good 2, follows from the arguments used in the proofs for Propositions 2.1 and 2.2.

Appendix B: The Variance of a Truncated Variable

Proof of Proposition 2.2

Let $x(T)$ be a random variable and let $y(T) = Y(x(T), U, L)$ be equal to $x(T)$ truncated at U and L, as follows:[19]

$$y(T) = Y(x(T), U, L) \equiv \begin{cases} L, & x(T) < L\,; \\ x(T), & L \leq x(T) \leq U\,; \\ U\,, & x(T) > U\,. \end{cases} \tag{B1}$$

This is the model for the real exchange rate in the one-good case, where $x(T) = \ln[q_{22}(T)/q_{12}(T)]$, $L = \ln\kappa_2^1$, and $U = \ln\kappa_2^2$. We need to compare the variance of the truncated variable $y(T) = Y(x(T), U, L)$ to the variance of $y'(T) = Y(x'(T), U, L)$ where $x'(T) = x(T) + e(T)$, $\mathrm{E}(e(T) \mid x(T)) = 0$. Thus, $x'(T)$ is more risky than $x(T)$ in the sense of Rothschild and Stiglitz(1970). Initially, we restrict $e(T)$ to be a binomial variable,

$$e(T) = \begin{cases} +\delta, & \text{prob } 0.5 \\ -\delta, & \text{prob } 0.5 \end{cases}\,. \tag{B2}$$

Note that the results obtained from the binomial case are quite general. For example, by repeating our analysis for a new increase in risk, $x''(T) = x'(T) \pm \delta$, and taking limits for many such increases in risk, the conclusions also hold for normally distributed $e(T)$. Moreover, subsequent changes in riskiness need not always be of the same step size, δ. Thus, the conclusions also hold for mixtures of normals or any other distribution that can be generated by a binomial model. Finally, $x(T)$ itself may have any distribution and may be a transformation (for example, a log) of another variable.

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[19]Although $x(t)$ may have infinite variance, $y(t)$ must have a finite variance because of the truncation.

To investigate the effect of a perturbation of $x(T)$ on var($y(T)$), we write var($y(T)$) as $E(y(T)^2) - [E(y(T))]^2$. Consider the limit of the change in $E[f(y'(T))]$ for $\delta \to 0$,

$$\lim_{\delta \to 0} \frac{E[f(y'(T))] - E[f(y(T))]}{\delta^2} \quad . \tag{B3}$$

This limit tells us how a small increase in riskiness affects the expectation of some function of $y(T)$. Note also that, if $x(T)$ has a finite (conditional) variance, (B2) implies that var($x'(T)$) – var($x(T)$) = var($e(T)$) = δ^2. Thus, if $x(T)$ (and, by implication, $x'(T)$) has finite variance, we can interpret this limit as the derivative of the expectation of a function of $y(T)$ with respect to the variance of $x(T)$, with the proviso that the change in this variance must result from an increase of Rothschild-Stiglitz riskiness. The function $f(y(T))$ we are interested in is the variance of the truncated variance $y(T)$. We start the proof by first stating a technical result.

Lemma 1: For $y(T)$ with any given distribution:

$$\lim_{\delta \to 0} \frac{\mathrm{var}(y'(T)) - \mathrm{var}(y(T))}{\delta^2} = \Phi(U) - \Phi(L) - U\,\phi(U) + L\,\phi(L) + E(y(T))\,[\phi(U) - \phi(L)] \tag{B4}$$

Proof: Using $\mathrm{var}(y'(T)) = E(y'(T)^2) - [E(y'(T))]^2$, we can write the left-hand-side as:

$$\lim_{\delta \to 0} \frac{\mathrm{var}(y'(T)) - \mathrm{var}(y(T))}{\delta^2} = \lim_{\delta \to 0} \frac{E(y'(T)^2) - E(y(T)^2)}{\delta^2} - 2\,E(y(T)) \lim_{\delta \to 0} \frac{E(y'(T)) - E(y(T))}{\delta^2} \tag{B5}$$

We first examine the second term on the right-hand side and then the first.

Note that the effect on $y(T)$ of the perturbation of $x(T)$ is the same as adding the error to $y(T)$ and subtracting it from the bounds U and L.

$$y'(T) \equiv Y(x(T) + e(T), L, U) = e(T) + Y(x(T), L - e(T), U - e(T)) \,. \tag{B6}$$

Thus, (B6) implies that the effect of increased riskiness on the expectation of the truncated variable equals the expected effect of the perturbation of the bounds:

$$\mathrm{E}(y'(T)) - \mathrm{E}(y(T)) = \mathrm{E}[Y(x(T), L{-}e(T), U{-}e(T)) - Y(x(T), L, U)]\,. \tag{B7}$$

If, in addition, $e(T)$ is binomial, we can further specify the effect of the randomized bounds as

$$\frac{\mathrm{E}(y'(T)) - \mathrm{E}(y(T))}{\delta^2} = \frac{\dfrac{\mathrm{E}[Y(x(T), L+\delta, U+\delta)] + \mathrm{E}[Y(x(T), L-\delta, U-\delta)]}{2} - \mathrm{E}(y(T))}{\delta^2}\,, \tag{B8}$$

which is the usual finite-difference approximation for a second derivative. Taking limits as $\delta \rightarrow 0$, and applying l'Hôpital's rule twice, we obtain the regular second derivatives:

$$\lim_{\delta \to 0} \frac{\mathrm{E}(y'(T)) - \mathrm{E}(y(T))}{\delta^2} = \frac{1}{2}\left[\frac{\partial^2\, \mathrm{E}[Y(x(T), L, U)]}{\partial\, \mathrm{L}^2} + \frac{\partial^2\, \mathrm{E}[Y(x(T), L, U)]}{\partial\, \mathrm{U}^2}\right]. \tag{B9}$$

To evaluate the second derivatives in (B9), note that the expectation of the truncated variable can be written as

$$\mathrm{E}(y(T)) = L\,\Phi(L) + \int_L^U x\, \mathrm{d}\Phi(x) + U\,[1 - \Phi(U)]\,, \tag{B10}$$

where $\Phi(x)$ is the (cumulative) distribution function, $\phi(x)$ is the density function, and $\mathrm{d}\Phi(x) = \phi(x)$. From (B10), we obtain

$$\frac{\partial\, \mathrm{E}(y(T))}{\partial L} + \frac{\partial\, \mathrm{E}(y(T))}{\partial U} = [L\,\phi(L) + \Phi(L)] + [-L\,\phi(L) + U\,\phi(U)] + [-U\,\phi(U) + 1 - \Phi(U)]$$

$$= \Phi(L) + [1 - \Phi(U)]\,, \tag{B11}$$

and, therefore, we get a reduced-form for the second part of the expression in (B10).

$$\lim_{\delta \to 0} \frac{\mathrm{E}(y'(T)) - \mathrm{E}(y(T))}{\delta^2} = \frac{1}{2}\left[\frac{\partial^2 \mathrm{E}(y(T))}{\partial\, \mathrm{L}^2} + \frac{\partial^2 \mathrm{E}(y(T))}{\partial\, \mathrm{U}^2}\right] = \frac{1}{2}\,[\phi(L) - \phi(U)]\,. \tag{B12}$$

We next identify the effect of the perturbation on $\mathrm{E}(y(T)^2)$. From (B12), we immediately obtain

$$\mathrm{E}(y'(T)^2) - \mathrm{E}(y(T)^2) = \delta^2 + 2\,\mathrm{E}[e(T)\,Y(x(T), L-e(T), U-e(T))]$$

$$+\,\mathrm{E}\Big\{\,[Y(x(T), L-e(T), U-e(T))]^2 - U\Big\}]^2)\,. \qquad \text{(B13)}$$

Again using l'Hôpital's rule, we can identify the limits for the expectations on the right hand side. For the first expectation, we have

$$\lim_{\delta\to 0}\frac{\mathrm{E}[e(T)\,Y(x(T), L-e(T), U-e(T))]}{\delta^2} = \lim_{\delta\to 0}\frac{\delta\,\mathrm{E}[Y(x(T), L-\delta, U-\delta)] + (-\delta)\,\mathrm{E}[Y(x(T), L+\delta, U+\delta)]}{2\delta^2}$$

.

Dividing numerator and denominator by δ, and adding and subtracting $\mathrm{E}[Y(x(T), L, U)]$ in the numerator, this can be rewritten as

$$\lim_{\delta\to 0}\frac{\mathrm{E}[e(T)\,Y(x(T), L-e(T), U-e(T))]}{\delta^2} = -\left[\frac{\partial\,\mathrm{E}(y(T))}{\partial L} + \frac{\partial\,\mathrm{E}(y(T))}{\partial U}\right] = [\Phi(U) - 1] - \Phi(L). \quad \text{(B14)}$$

Similarly, the limit for the second expectation on the right hand side of (B13) is

$$\lim_{\delta\to 0}\frac{\mathrm{E}\{[Y(x(T), L-e(T), U-e(T))]^2\} - \mathrm{E}\{[Y(x(T), L, U)]^2\}}{\delta^2}$$

$$= \frac{1}{2}\left[\frac{\partial^2\,\mathrm{E}\{[Y(x(T),L,U)]^2\}}{\partial\,\mathrm{L}^2} + \frac{\partial^2\,\mathrm{E}\{[Y(x(T),L,U)]^2\}}{\partial\,\mathrm{U}^2}\right]$$

$$= \Phi(L) + L\,\phi(L) + [1-\Phi(U)] - U\,\phi(U)\,. \qquad \text{(B15)}$$

Combining (B13), (B14) and (B15), we can conclude that

$$\lim_{\delta\to 0}\frac{\mathrm{E}(y'(T)^2) - \mathrm{E}(y(T)^2)}{\delta^2} = 1 + 2\,[\Phi(U) - 1 - \Phi(L)] + \Phi(L) + L\,\phi(L) + [1-\Phi(U)] - U\,\phi(U)$$

$$= \Phi(U) - \Phi(L) - U\,\phi(U) + L\,\phi(L)\,. \qquad \text{(B16)}$$

Finally, we substitute (B12) and (B16) into (B14) to get the result in (B4). n

Proof of Part 1 of Proposition 2.2: For the case of normal distributions, (B4) can be written as[20]

$$\frac{\partial \operatorname{var}(y(T))}{\partial \sigma^2} = N(u) - N(l) - U\, n(u) + L\, \phi(l) + \mathrm{E}(y(T))\, [n(u) - n(l)]\,, \qquad \text{(B17)}$$

where, $u = \dfrac{U - \mathrm{E}(x(T))}{\sigma}$; $l = \dfrac{L - \mathrm{E}(x(T))}{\sigma}$; $n(z)$ is the standard normal density; and, $N(z)$ the standard normal probability.[21] Note that the terms in (B17) can be linked to the partial mean and the partial mean square of $x(T) - \mathrm{E}(x(T))$ between L and U. Specifically,[22]

$$\int_L^U [x - \mathrm{E}(x(T))]\, \phi(x)\, dx \;=\; \int_l^u \sigma\, z\, n(z)\, dz \;=\; -\sigma\, n(z)\, |_l^u \;=\; \sigma\, [n(l) - n(u)]\,. \qquad \text{(B18)}$$

and[23]

$$\int_L^U [x - \mathrm{E}(x(T))]^2\, \phi(x)\, dx) = \int_l^u \sigma^2 z^2\, n(z)\, dz$$

$$= \sigma^2 \int_l^u z\, [z\, n(z)\, dz]$$

$$= \sigma^2\, [-z\, n(z)]\, |_l^u - \sigma^2 \int_l^u [-n(z)]\, dz$$

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[20]A direct derivation, starting from the normal distribution and not using the concept of "increased riskiness," is available on request.

[21]We immediately conclude that when u and l are many standard deviations away from zero, then (B14) approaches zero because $N(u)$, $N(l)$, $n(u)$ and $\phi(l)$ approach zero. That is, when trade is a near-certainty, the variance of the real exchange rate is hardy affected when the variance of the underlying variable changes. This conclusion also holds for any non-normal distribution as long as the density approaches zero when $x(t) \to \pm \infty$ and the mean of x is finite.

[22]Do the substitution $v = \frac{-z^2}{2}$. Then $z\, dz = -dv$, and $z\, e^{-z^2/2}\, dz = -\, e^v\, dv$. So the integral of $\{z \exp[-z^2/2]\}$ is $-e^v = -\, e^{-z^2/2}$.

[23]Integrate by parts, using $z^2\, n(z)\, dz = u\, dv$ where $u = z$ and $dv = z\, n(z)\, dz$, implying $du = dz$ and, from the previous footnote, $v = -n(z)$.

$$= \sigma^2 \left[\frac{L-\gamma}{\sigma} \, n(l) - \frac{U - \mathrm{E}(x(T))}{\sigma} \, n(u) \right] - \sigma^2 \, [N(l) - N(u)]$$

$$= \sigma \left[(L - \mathrm{E}(x(T))) \, n(l) - (U - \mathrm{E}(x(T))) \, n(u) \right]$$

$$+ \sigma^2 [N(u) - N(l)]. \qquad \text{(B19)}$$

Thus, using (B18) and (B19), (B17) can be simplified to

$$\frac{\partial \, \mathrm{var}_t(y(T))}{\partial \, \sigma^2} = \int_L^U (x - \mathrm{E}(x(T)))^2 \, \phi(x) \, \mathrm{d}x \; - \; 2 \, \mathrm{E}[y(T) - \mathrm{E}(x(T))] \int_L^U (x - \mathrm{E}(x(T))) \, \phi(x) \, \mathrm{d}x$$

$$= \int_L^U \{ (x - \mathrm{E}(x(T)))^2 - \mathrm{E}[y(T) - \mathrm{E}(x(T))] \, (x - \mathrm{E}(x(T))) \} \; \phi(x) \; \mathrm{d}x$$

$$= \int_L^U \{ (x - \mathrm{E}(x(T))) \, [x - \mathrm{E}(x(T)) - \mathrm{E}(y(T)) - \mathrm{E}(x(T))] \} \; \phi(x) \; \mathrm{d}x$$

$$= \int_L^U (x - \mathrm{E}(x(T))) \, [x - \mathrm{E}(y(T))] \; \phi(x) \; \mathrm{d}x \, . \qquad \text{(B20)}$$

The sign of this expression is the sign of

$$\int_L^U (x - \mathrm{E}(x(T))) \, [x - \mathrm{E}(y(T))] \frac{\phi(x)}{N(u) - N(b)} \, \mathrm{d}x \, , \qquad \text{(B21)}$$

where $\frac{\phi(x)}{N(u)-N(b)}$ is the density function conditional on $L \leq x(T) \leq U$. Define $\mathrm{E}_c(x(T))$ as the conditional mean of x given that $L \leq x(T) \leq U$. The integrand in (B21) can then be rewritten as

$$(x - \mathrm{E}(x(T))) \, [x - \mathrm{E}(y(T))]$$

$$= (x - \mathrm{E}_c(x(T)) + \mathrm{E}_c(x(T)) - \mathrm{E}(x(T))) \, [x - \mathrm{E}_c(x(T)) + \mathrm{E}_c(x(T)) - \mathrm{E}(y(T))]$$

$$= (x - E_C(x(T)))^2 + (x - E_C(x(T)))\,[E_C(x(T)) - E(x(T))] \tag{B22}$$

$$+ (E_C(x(T)) - E(x(T)))\,(x - E_C(x(T))) + (E_C(x(T)) - E(x(T)))\,[E_C(x(T)) - E(y(T))].$$

Substituting (B22) into (B21), and noting that the conditional expectation of $(x - E_C(x(T)))$ is zero, we obtain

$$\int_L^U \{(x - E_C(x(T)))\,[x - E(y(T))]\}\,\frac{\phi(x)}{N(u)-N(b)}\,dx$$

$$= \int_L^U \{(x - E_C(x(T)))^2\,\frac{\phi(x)}{N(u)-N(b)}\,dx \quad + \quad (E_C(x(T)) - E(x(T)))\,[E_C(x(T)) - E[y(T)]. \tag{B23}$$

The first term on the RHS of (B23), the variance of x conditional on $U \leq x \leq L$, is strictly positive. The second term is zero when $E_C(x(T)) = E(x(T))$, that is, when the no-trade domain $[L\ , U]$ is symmetric around the expected value of x. The second term becomes positive when the no-trade zone $[L\ , U]$ is more towards one of the tails of the distribution of x, rather than right at the center. To see this, note that, from (B23), $E[y(T)]$ is a weighted average of L, $E_C(x(T))$, and U, with weights $N(b)$, $N(u)–N(b)$, and $1 - N(u)$, respectively. If the no-trade zone is in the right tail of the distribution, we have $E_C(x(T)) - E(x(T)) > 0$, as $E_C(x(T))$ is to the right of $E(x(T))$. But $E_C(x(T))$ is also to the right of $E[y(T)]$: $E[y(T)]$ is a weighted average of L , $E_C(x(T))$, and U, and the weight of L, $N(b)$, is much larger than the weight of U, which is $1–N(u)$. Thus, when the no-trade zone is in the right tail of the distribution of y, then $E_C(x(T))$ is to the right of both $E(x(T))$ and $E[y(T)]$. By a similar argument, $E_C(x(T))$ is to the left of both $E(x(T))$ and $E(y(T))$ when the no-trade zone is in the left tail of the distribution of y. This means that the second term on the RHS of (B21), being a product of two factors with the same sign, is positive. Thus, (B21) is positive. Since the sign of (B23) is the sign of (B21), the variance of the exchange rate is a positive function of σ. n

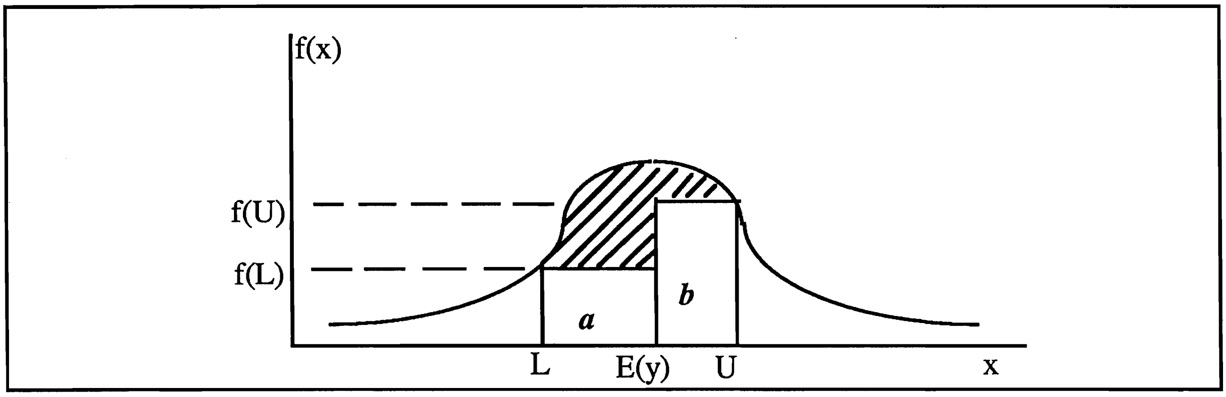
Proof of Part 2 of Proposition 2.2:

To show that for a general unimodal distribution, expression (B4) is always positive when L and U contain the mode of the distribution of $x(T)$ ($\phi(L) < \phi(E(y(T)) < \phi(U)$), rearrange (B4) as:

$$\lim_{\delta\to 0} \frac{\mathrm{var}(y'(T)) - \mathrm{var}(y(T)^2)}{\delta^2} = [\Phi(U) - \Phi(L)] - [U - E(y(T))]\,\phi(U) - [E(y(T)) - L]\,\phi(L). \quad (B24)$$

Equation (B24) describes the surface under the density function between L and U, minus the surface of the rectangles labeled ***a*** and ***b*** in Figure B.1. When $\phi(L) < \phi(E(y(T)) < \phi(U)$, the remaining surface is positive.[24] n

Figure B.1.



In (B4), the term $[\Phi(U) - \Phi(L)]$ corresponds to the surface below the distribution function between L and U. The term $[U - E(y(T))]\ \phi(U)$ corresponds to the surface of the rectangle ***b***, while $[E(y(T)) - L]\ \phi(L)$ corresponds to the surface of the rectangle ***b***. When the mode of the distribution is between U and L, the remaining area has a positive surface.

Proof of Part 3 of Proposition 2.2:

We now show that if the no-trade domain $[L, U]$ is in a concave part of the distribution, expression (B4) is always positive. Suppose that $\phi(U) > \phi(L)$—we are in the left tail of a unimodal distribution. Define

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[24]Note that is result holds whether or not there are convex sections in the distribution function within the domain $[L, U]$.

$$Z \equiv \int_L^U x \frac{\phi(x)}{\Phi(\mathrm{U}) - \Phi(\mathrm{L})} \mathrm{d}x$$

which is the mean of x conditional on there being no trade. Since we are in the left tail, $Z > \frac{U+L}{2}$, because there is more density to the right than to the left. It follows that

$$\mathrm{E}(y(T)) \; > L\,\Phi(L) + (\Phi(U) - \Phi(L))\frac{U+L}{2} + U[1 - \Phi(U)] \; = U - [\Phi(U) + \Phi(L)]\frac{U-L}{2},$$

and, therefore, from (B24),

$$\lim_{\delta \to 0} \frac{\mathrm{var}(y'(T)) - \mathrm{var}(y(T)^2)}{\delta^2}$$

$$> \Phi(U) - \Phi(L) - U\,\phi(U) + L\,\phi(L) + \left\{U - [\Phi(U) + \Phi(L)]\frac{U-L}{2}\right\}[\phi(U) - \phi(L)]$$

$$= \Phi(U) - \Phi(L) - \phi(L)\,(U-L) - [\Phi(U) + \Phi(L)]\frac{U-L}{2}[\phi(U) - \phi(L)] \qquad \text{(B25)}$$

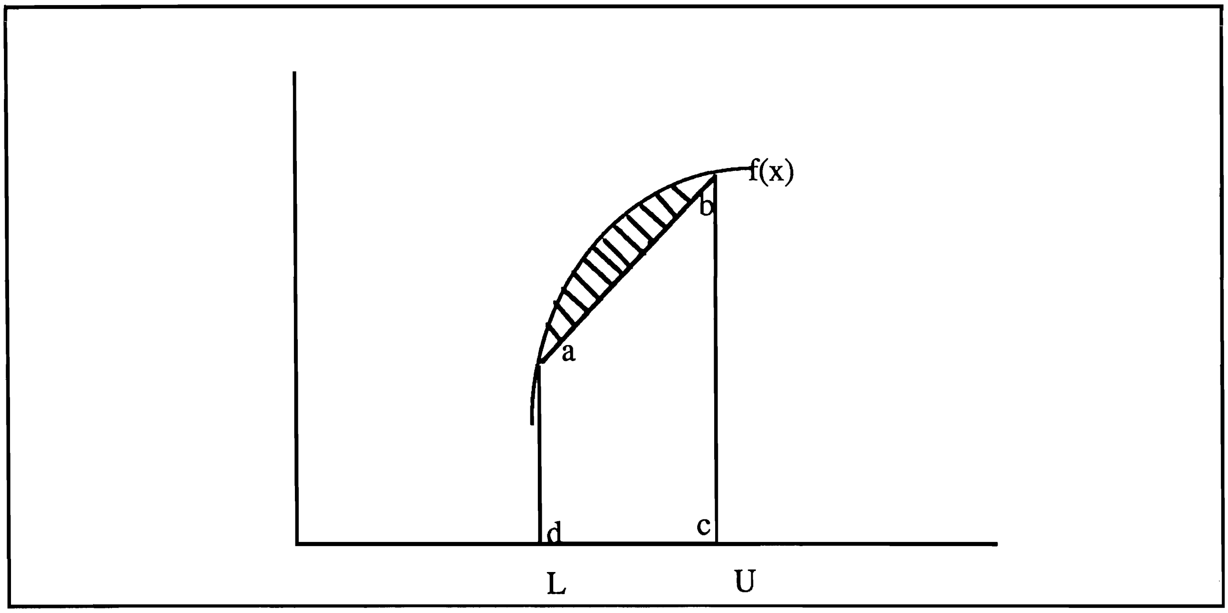
Since we are in the left tail, we have $1 - \Phi(U) > \Phi(L)$ and, therefore, $\Phi(U) + \Phi(L) < 1$. Thus,

$$\lim_{\delta \to 0} \frac{\mathrm{var}(y'(T)) - \mathrm{var}(y(T)^2)}{\delta^2} > \Phi(U) - \Phi(L) - \phi(L)\,(U-L) - \frac{U-L}{2}[\phi(U) - \phi(L)] \qquad \text{(B26)}$$

$$= \Phi(U) - \Phi(L) - \frac{U-L}{2}[\phi(U) + \phi(L)]\,. \qquad \text{(B27)}$$

This last expression corresponds to the area below $\phi(x)$ between U and L, minus the surface of the tetrahedron abcd shown in Figure B.2. If $\phi(x)$ is concave, the remaining surface is positive. Thus, for concave $\phi(x)$, the variance of $y(T)$ increases with riskiness.

Figure B.2.



Graphically, the derivative of the variance of the truncated variable w.r.t. the riskiness is at larger than the area below $\phi(x)$ between U and L, minus the surface of the tetrahedron abcd. If $\phi(x)$ is concave, the remaining surface is positive.

We lastly show that if the no-trade domain $[L, U]$ is in a sufficiently convex part of the distribution, expression (B4) may become negative. Suppose there is a strong convexity between L and U, as in Figure B.3. Then $\mathrm{E}(z(T))$ approximately equals $\phi(L)\ (U+L)/2$, and (B27) actually provides a good approximation:

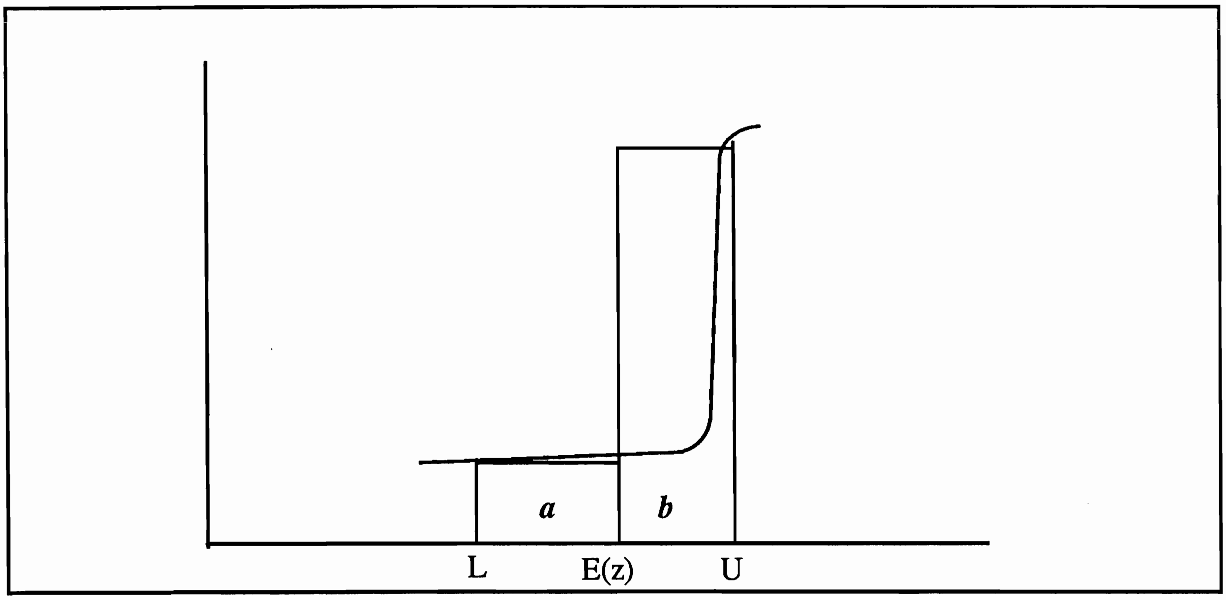
$$\lim_{\delta\to 0} \frac{\mathrm{var}(y'(T)) - \mathrm{var}(y(T)^2)}{\delta^2}$$

$$\gtrsim \Phi(U) - \Phi(L) - \phi(L)\ (U-L) - [\Phi(U) + \Phi(L)]\,\frac{U-L}{2}\,[\phi(U) - \phi(L)]\,. \qquad \text{(B28)}$$

However, with such a strong convexity, $\Phi(U) - \Phi(L)$ approximately equals $\phi(L)\ (U-L)$. Substitution of this then produces a negative value:

$$\lim_{\delta\to 0} \frac{\mathrm{var}(y'(T)) - \mathrm{var}(y(T)^2)}{\delta^2} \stackrel{\sim}{>} -[\Phi(U) + \Phi(L)]\,\frac{U-L}{2}\,[\phi(U) - \phi(L)] < 0\,. \qquad \text{(B29)}$$

Figure B.3.



With a very strong convexity, subtraction of the rectangles a and b from the area under the distribution function between U and L leaves a negative area. Thus, increased riskiness decreases the variance of the truncated variable.

Proof of Proposition 3.3

Define $y(T) = \dfrac{\ln[\theta_2/\theta_1] - \ln S(T)}{\eta\, \varepsilon_2}$, $z(T) = \dfrac{\varepsilon_1}{\varepsilon_2} \ln \dfrac{q_{21}(T)}{q_{11}(T)}$, and $x(T) = \ln \dfrac{q_{22}(T)}{q_{12}(T)}$. Proposition 1.3

then implies that the exchange rate can be expressed as:

$$y(T) = z(T) + \Upsilon(x(T), L(z(T)), U(z(T))) = z(T) + \begin{cases} L(z(T)) & , \ x(T) < L(z(T))\,; \\ x(T) & , \ L(z(T)) \leq x(T) \leq U(z(T))\,; \\ U(z(T)) & , \ x(T) \geq U(z(T))\,. \end{cases} \quad \text{(B30)}$$

We now want to investigate the effect on the variance of $y(T)$ when the riskiness of $z(T)$ is increased. Thus, we perturb $z(T)$ into $z^*(T) = z(T) + e^*(T)$ where $e^*(T)$ has mean zero and is independent of both $z(T)$ and $x(T)$. For $e^*(T)$ small, the effect on $U(z(T))$ and $L(z(T))$ is

$$dU(z(T)) = \frac{\partial U(z(T))}{\partial z(T)} e^*(T) = \frac{\varepsilon_2}{\varepsilon_1} \frac{\varepsilon_1 (1-\eta)}{1 - \varepsilon_2(1-\eta)} e^*(T) = \frac{\varepsilon_2 (1-\eta)}{1 - \varepsilon_2(1-\eta)} e^*(T)\,, \quad \text{(B31)}$$

$$dL(z(T)) = dU(z(T))\,. \quad \text{(B32)}$$

We immediately infer that if $\eta=1$, then $U(z(T))$ and $L(z(T))$ are unaffected. Thus, when $\eta = 1$, we have $y^*(T) = y(T) + e^*(T)$, which means that the variance of $y(T)$ increases when $z(T)$ becomes more risky. To analyze the cases where $\eta \neq 1$, we define $e(T) = -k^* e^*(T)$ where $k^* = \dfrac{\varepsilon_2 (1-\eta)}{1-\varepsilon_2(1-\eta)}$, and we let $e(T)$ take on the values $+\delta$ or $-\delta$ with equal probabilities.

Then

$$\begin{aligned} y^*(T) &\equiv z(T) + e^*(T) + \mathrm{Y}[x(T), U(z(T)) + k^* e^*(T), L(z(T)) + k^* e^*(T)] \\ &= z(T) - \left(1 + \frac{1}{k^*}\right) e(T) + [e(T) + \mathrm{Y}(x(T), L(z(T)) - e(T), U(z(T)) - e(T))] \\ &= z(T) - k\, e(T) + y'(T) . \end{aligned} \tag{B33}$$

where $k = (1 + 1/k^*) = \dfrac{1}{\varepsilon_2 (1-\eta)}$ and $y'(T) = \mathrm{Y}(x(T) + e(T), L(z(T)), U(z(T)))$, as in the preceding sections. Although $L(z(T))$ and $U(z(T))$ are now random, we can still use the results of the preceding analysis to obtain results conditional on z, and then take expectations. One immediate implication of (B33) is that $\mathrm{E}(y^*(T) - y(T)) = \mathrm{E}(y'(T) - y(T))$. Thus, from (B12),

$$\lim_{\delta \to 0} \frac{\mathrm{E}(y^*(T) - y(T))}{\delta^2} = \frac{1}{2}\, \mathrm{E}[\Phi(L(z(T))) - \Phi(U(z(T)))] . \tag{B34}$$

Noting that $z(T)$ is independent of $e(T)$ and, by implication, also from $(y'(T) - y(T))$, equation (B33) implies that

$$\lim_{\delta \to 0} \frac{\mathrm{E}[y^*(T)^2 - y(T)^2]}{\delta^2} = k^2 + \lim_{\delta \to 0} \frac{\mathrm{E}[y'(T)^2 - y(T)^2]}{\delta^2} - \lim_{\delta \to 0} \frac{2k\, \mathrm{E}[e(T)\, y'(T)]}{\delta^2} ,$$

or, using (B16) and (B14),

$$\lim_{\delta \to 0} \frac{\mathrm{E}[y^*(T)^2 - y(T)^2]}{\delta^2} = k^2 + \mathrm{E}[\Phi(U(z(T))) - \Phi(L(z(T)))$$

$$- U(z(T))\, \phi(U(z(T))) + L(z(T))\, \phi(L(z(T)))] - 2k\, \mathrm{E}[\Phi(U(z(T))) - 1 - \Phi(L(z(T)))] . \tag{B35}$$

It then follows that

$$\lim_{\delta \to 0} \frac{\mathrm{var}(y^*(T)) - \mathrm{var}(y(T))}{\delta^2} = \mathrm{E}[\Phi(U(z(T))) - \Phi(L(z(T))) - U(z(T))\,\phi(U(z(T)))$$

$$+ L(z(T))\,\phi(L(z(T)))] - \mathrm{E}(y(T))\,\mathrm{E}[\phi(L(z(T))) - \phi(U(z(T)))]$$

$$+ k^2 + 2k\,\mathrm{E}[1 - \Phi(U(z(T))) + \Phi(L(z(T)))] \tag{B36}$$

Comparing (B36) to (B28) we see that the effect of the increased riskiness of $z(T)$ equals the expected effect of an increased riskiness of $x(T)$, plus a term $k^2 + 2k$ E$[1 - \Phi(U(z(T))) + \Phi(L(z(T)))]$. When k is positive, that is, when $\eta < 1$, the additional term is always positive because $[1 - \Phi(U(z(T))) + \Phi(L(z(T)))]$, being the probability of trade, is always nonnegative. Thus, when $\eta < 1$, the relation between riskiness of output in the non-traded goods sector and the volatility of the exchange rate is positive. We have already discussed the case $\eta = 1$, and found that also in this case the effect of increasing the riskiness of $z(T)$ on the variance of y is positive. However, when $\eta > 1$, $k \equiv 1/[\varepsilon_2\,(1-\eta)]$ is negative. Then $k^2 + 2k\,[1 - \Phi(U(z(T))) + \Phi(L(z(T)))]$ is positive only if k is more negative than $-2[1 - \Phi(U(z(T))) + \Phi(L(z(T)))]$, which obviously is not always the case. Thus, in this case, the relation between riskiness in the output of the non-traded goods sector and exchange rate volatility depends on the magnitude of η and ε_2. n

References

Asseery, A. and D.A. Peel. 1991. "The Effects of Exchange Rate Volatility on Exports - Some New Estimates." *Economics Letters* 37 (October): 173-77.

Baron, David P. 1976a. "Flexible Exchange Rates, Forward Markets, and the Level of Trade." *American Economic Review* 66 (June): 253-66.

Baron, David P. 1976b. "Fluctuating Exchange Rates and the Pricing of Exports." *Economic Inquiry* 14 (September): 425-38.

Bini-Smaghi, Lorenzo. 1991. "Exchange Rate Variability and Trade: Why Is it So Difficult to Find any Empirical Relationship?" *Applied Economics* 23 (May): 927-35.

Bleaney, Michael. 1992. "Comparison of Real Exchange Rate Volatility Across Exchange Rate Systems." *Oxford Bulletin of Economics and Statistics* 54: 557-65.

Côté, Agathe. 1994. "Exchange Rate Volatility and Trade: A Survey." Working Paper, Bank of Canada, Ottawa.

Cushman, David O. 1983. "The Effects of Real Exchange Rate Risk on International Trade." *Journal of International Economics* 15 (August): 45-63.

Cushman, David O. 1986. "Has Exchange Risk Depressed International Trade? The Impact of Third-Country Exchange Risk." *Journal of International Economics* 5 (September); 361-79.

De Grauwe, Paul and Guy Verfaille. 1988. "Exchange Rate Variability, Misalignment, and the European Monetary System." In *Misalignment of Exchange Rates: Effects on Trade and Industry*, edited by Richard C. Marston, 77-100. Chicago: University of Chicago Press.

De Grauwe, Paul. 1988. "Exchange Rate Variability and the Slowdown in Growth of International Trade." *International Monetary Fund Staff Papers* 35 (March): 63-84.

De Grauwe, Paul. 1992. "The Benefits of a Common Currency." In *The Economics of Monetary Integration*, edited by Paul de Grauwe. New York: Oxford University Press.

Dellas, Harris and Ben-Zion Zilberfarb. 1993. "Real Exchange Rate Volatility and International Trade: A Reexamination of the Theory." *Southern Economic Journal* 59 (April): 641-7.

Dumas, B.(1992): Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World, *The Review of Financial Studies*, 5(3), 153-180.

Either, Wilfrid. 1973. "International Trade and the Forward Exchange Market." *American Economic Review* 63 (June): 494-503.

Engel, C. 1993. "Real Exchange Rates and Relative Prices," *Journal of Monetary Economics* 32.1: 35-50.

Farrell, Victoria S., Dean A. DeRosa and T. Ashby McCown. 1983. "*Effects of Exchange Rate Variability on International Trade and other Economic Variables: A Review of the Literature.*" Staff Studies, No. 130, U.S. Board of Governors of the Federal Reserve System.

Fenton, Paul and John Murray. 1993. "Optimum Currency Areas: A Cautionary Tale." In *The Exchange Rate and the Economy, Proceedings of a Conference Held at the Bank of Canada, 22-23 June, 1992*. Ottawa: Bank of Canada.

Franke, Gunter. 1991. "Exchange Rate Volatility and International Trading Strategy." *Journal of International Money and Finance* 10 (June) : 292-307.

Gagnon, Joseph E. 1993. "Exchange Rate Variability and the Level of International Trade." *Journal of International Economics* 34: 269-87.

Giovannini, Alberto. 1988. "Exchange Rates and Traded Goods Prices." *Journal of International Economics* 24 (February): 45-68.

Gros, Daniel. 1987. "Exchange Rate Variability and Foreign Trade in the Presence of Adjustment Costs." Working Paper no. 8704. Preliminary version. Department des sciences economiques, Universite Catholique de Louvain.

Harris, Richard G. 1993. "Exchange Rates and Hysteresis in Trade." In *The Exchange Rate and the Economy, Proceedings of a Conference Held at the Bank of Canada, 22-23 June, 1992*. Ottawa: Bank of Canada.

Hooper, Peter and Steven W. Kohlhagen. 1978. "The Effect of Exchange Rate Uncertainty on the Prices and Volume of International Trade," *Journal of International Economics* 8 (November): 483-511.

International Monetary Fund. 1984. "Exchange Rate Volatility and World Trade," Occasional Paper no. 28.

Koray, Faik and William D. Lastrapes, 1989. "Real Exchange Rate Volatility and U.S. Bilateral Trade: A VAR Approach." *Review of Economics and Statistics* 71 (November): 708-12.

Kroner, Kenneth F. and William D. Lastrapes. 1993. "The Impact of Exchange Rate Volatility on International Trade: Reduced Form Estimates using the GARCH-in-mean Model, "*Journal of International Money and Finance* 12 (June): 298-318.

Kumar, Vikram and Joseph A. Whitt. 1992. "Exchange Rate Variability and International Trade." *Economic Review*, Federal Reserve Bank of Atlanta 37 (May/June): 17-32.

Lastrapes, William D. and Faik Koray. 1990. "Exchange Rate Volatility and U.S. Multilateral Trade Flows," *Journal of Macroeconomics* 12 (Summer): 341-62.

Margrabe, W. (1978): "The Value of an Option to Exchange One Asset for Another," *Journal of Finance*, 33, 177-186.

Melitz, Jacques. 1988. "Comment on : Exchange Rate Variability, Misalignment and the European Monetary System." In *Misalignment of Exchange Rates: Effects on Trade and Industry*, edited by Richard C. Marston, 100-103. Chicago: U. of Chicago Press.

Rothschild, M. and J. E. Stiglitz (1970): Increasing Risk: I. A Definition, *Journal of Economic Theory*, 2(3), September 1970.

Samuelson, P., and S. Swamy (1974): Invariant Index Numbers and Canonical Duality: Survey and Synthesis, *American Economic Review*, 64(3), 566-593.

Savvides, Andreas. 1992. "Unanticipated Exchange Rate Variability and the Growth of International Trade." *Weltwirtschaftliches Archiv* 128:446-63.

Sercu, P. (1995): Riskiness of a Truncated Variable: The Effect of Changes in Variability of Underlying Variables, Working Paper, Katholieke Universiteit Leuven.

Sercu, P. and C. Van Hulle (1992): "Exchange Rate Volatility, International Trade, and the Value of Exporting Firms." *Journal of Banking and Finance* 16: 155-82.

Sercu, P., C. Van Hulle and R. Uppal (1995): The Exchange Rate in the Presence of Transaction Costs: Implications for Tests of Relative Purchasing Power Parity, *Journal of Finance*, 50.4, 1309-1319.

Stulz, R. (1987): An Equilibrium Model of Exchange Rate Determination and Asset Pricing with Nontraded Goods and Imperfect Information, *Journal of Political Economy*, 95, 1024-1040.

Uppal, R. (1993), A General Equilibrium Model of International Portfolio Choice, *Journal of Finance* 48(3), 529-553.

Viaene, Jean-Marie and Casper G. de Vries. 1992. "International Trade and Exchange Rate Volatility. "*European Economic Review* 36 (August): 1311-21.

Willett, Thomas D. 1986. "Exchange-Rate Volatility, International Trade, and Resource Allocation: A Perspective on Recent Research." *Journal of International Money and Finance* 5 (Supplement, March): S101-S115.