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THE OPTIMAL NUMBER OF CONTRACTS IN CROSS- OR DELTA-HEDGES

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Abstract. When hedging in futures markets, the hedge instruments typically fail to match the exposed asset or portfolio by expiration date and/or by underlying asset. The theoretical variance-minimizing hedge is given by the slope coefficient of the conditional (forward-looking) regression of the spot price that one is exposed to on the futures price used as a hedge. We explore the hedging performance of simple rules of thumb and of unconditional regressions on past data, focusing on the effect of the choice of observation frequency, sample period, percentage vs. dollar returns, and lead/lag effects. Our findings are the following: (a) the effects of varying the observation frequency, sample period, etc, are much larger than the effects of using GARCH instead of OLS. (b) Regardless of sample size and estimation technique, the exposure is best estimated using percentage returns rather than (dollar) first differences. (c) In the case of delta hedges, and also of cross-hedges among closely related currencies, regressions are systematically beaten by naive rules of thumb. (d) This relatively poor performance of regression-based hedges is not just due to errors in data. (e) The optimal estimation technique depends on the situation. For cross-hedges involving two European currencies, high-frequency OLS estimates is flawed by EMS-induced leads and lags among exchange rate changes, and the best regressions are those using monthly data from longish sample periods. For delta-hedges the dominant source of estimation problems seems to be a time-varying relationship between the regression variables, and the best regressions use daily data from short sample periods.

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The Optimal Number of Contracts in Cross- or Delta-Hedges

Abstract. When hedging in futures markets, the hedge instruments typically fail to match the exposed asset or portfolio by expiration date and/or by underlying asset. The theoretical variance-minimizing hedge is given by the slope coefficient of the conditional (forward-looking) regression of the spot price that one is exposed to on the futures price used as a hedge. We explore the hedging performance of simple rules of thumb and of unconditional regressions on past data, focusing on the effect of the choice of observation frequency, sample period, percentage vs. dollar returns, and lead/lag effects. Our findings are the following: (a) the effects of varying the observation frequency, sample period, etc, are much larger than the effects of using GARCH instead of OLS. (b) Regardless of sample size and estimation technique, the exposure is best estimated using percentage returns rather than (dollar) first differences. (c) In the case of delta hedges, and also of cross-hedges among closely related currencies, regressions are systematically beaten by naive rules of thumb. (d) This relatively poor performance of regression-based hedges is not just due to errors in data. (e) The optimal estimation technique depends on the situation. For cross-hedges involving two European currencies, high-frequency OLS estimates is flawed by EMS-induced leads and lags among exchange rate changes, and the best regressions are those using monthly data from longish sample periods. For delta-hedges the dominant source of estimation problems seems to be a time-varying relationship between the regression variables, and the best regressions use daily data from short sample periods.

I. Introduction

Relative to a tailor-made hedge in the forward currency market, a hedge in the currency futures market is almost invariably imperfect. First, the contract size being fixed, it is difficult to exactly match the position to be hedged. More importantly, also the expiration dates that are available in the futures markets rarely coincide with those for the currency flows that they are meant to hedge. Similarly, the menu of underlying exchange rates is typically limited, so that there may be no contract available for the desired currency. Similar matching problems arise when an interest exposure is hedged in the market for Eurocurrency or T-bill futures rather than using a tailor-made forward rate agreement. Because of its low transactions cost and the availability of a secondary market, a hedger may nevertheless prefer the futures markets over their over-the-counter forward counterparts. And when the exposure to be hedged originates from stock market risk or commodity price risk, tailored forward contracts typically are entirely absent, so that an imperfect hedge in a futures market is the only feasible option. With respect to the problem of fixed contract sizes, the hedger in the futures market has little choice but to round the ideal number of contracts to the nearest integer. Still, the question arises as to how to set this ideal number of contracts—the hedge ratio—taking into account the maturity mismatch and the imperfect correlation between the portfolio that is to be hedged and the asset that underlies the futures contract.

The usual approach is to select the number of futures contracts that minimizes the variance of the hedged position. The corresponding optimal hedge ratio is given by the slope coefficient of a regression between the future spot rate that one is exposed to and the futures price that is being used as a hedge. Like the market-model β in the Capital Asset Pricing Model (CAPM), the required regression coefficient is a conditional coefficient, and should therefore be extracted from the conditional joint distribution of future spot and futures prices. In practice, CAPM betas are often estimated unconditionally from past data, and a similar procedure is often applied for hedge ratios, typically using first-differenced spot and futures data from the past as regression inputs.[1] Such an approach inevitably produces errors. As Stoll and Whaley (1993) note, a first type of problems has to do with data imperfections. For one thing, the spot and futures prices used in the regressions are often not fully synchronized because of reporting lags, infrequent trading (at least in some markets), or differential adjustment speeds reflecting cross-market differences in liquidity or transaction costs. In addition, futures prices contain bid-ask noise. Lastly, futures data have ever-changing maturities whereas the hedger is interested in the joint distribution of a spot value and a futures price for a single, known time to maturity. The familiar effect of all these errors-in-the-regressor is that the estimated slope coefficient is biased towards zero. In addition, the relation between the variables of interest may not be constant over time—that is, unconditional estimates from the past may be very different from conditional, forward-looking parameters. Kroner and Sultan (1993), for example, illustrate how in a delta-hedge the use of a bivariate GARCH error-correction model (ECM) allows one to reduce the variance of the hedged cash flow by about 6% in-sample, and 4.5%

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[1] See, for instance, Ederington, 1979; Grammatikos and Saunders, 1983; Hill and Schneeweis, 1982; Stoll and Whaley, 1990; Stoll and Whaley, 1993, Chapter 4.

out-of-sample, relative to regression on first differences. Lastly, even with error-free data and a constant joint distribution there still is estimation error because any real-world sample is finite.

Like Kroner and Sultan, we compare the out-of-sample performances of various estimation techniques and of naive rules of thumb, and the market we select for our performance race is the currency market. However, we focus on the impact of errors in variables and the (related) issue of optimal observation frequencies in the regressions, as in Stoll and Whaley (1993). In a nutshell, choosing a high observation frequency offers the advantage of a larger sample without having to go back far into the past; but the cost is that the errors-in-variables bias becomes more acute: the higher the observation frequency, the smaller the signal (the change in true futures price) relative to the noise (like bid-ask bounce or imperfect synchronization in the data). We extend Stoll and Whaley's work in the following ways. First, we consider not just OLS regressions on first-differenced data, but also the Scholes-Williams (SW) instrumental-variable estimator (which takes care of poor synchronization and other lead-lag patterns), and we experiment also with regressions between percentage changes rather than first differences in spot and futures prices. Second, we attempt to isolate problems of the errors-in-the-regressor type from problems associated with inevitable estimation noise or changes in the relationship between spot and futures prices. Specifically, we eliminate the impact of regressor errors by using noise-free currency forward prices—computed from midpoint spot and interest rate data—instead of actual currency futures prices. Relative to Kroner and Sultan (1993), the innovations in our work are as follows. First, we consider cross hedges, delta hedges, and cross-and-delta hedges rather than just delta hedges,[2] and the horizon is three months rather than one week. Second, we consider more than one naive rule, and our naive rules take into account the information in the current spot or forward interest rates. Lastly, our focus is on errors in variables rather than on time-varying distributions; we show that the impact of choosing SW rather than OLS or of selecting an

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[2]In a delta hedge, the expiration date of the futures contract does not match the hedging horizon. In a cross-hedge, the currency underlying the futures contract differs from the currency in which the exposure is expressed.

observation frequency and a sample period are, most of the time, more important than the improvements they achieve with a GARCH-ECM model.[3]

The remainder of the paper is structured as follows. Section II briefly reviews the problem and its theoretical solution. In Section III we set out the tests. Section IV describes the data and presents the results. The conclusions are summarized in Section V.

II. The Problem

In the problem we consider there is one unit of asset j, whose value at time T_1 is uncertain and needs to be hedged. For instance, at time T_1 there may be a cash inflow of one NLG, which needs to be converted into USD (the hedger's home currency) with minimal risk. A futures contract is available for a 'related' asset or exchange rate i—for instance, the DEM against the USD—with an expiration date T_2 ($\geq T_1$). The size of the futures contract is one unit of the underlying i (for instance, one DEM). Contracts are assumed to be infinitely divisible; that is, one can buy or sell any fraction of the unit contract. Only one type of futures contracts is being used as a hedge.

Denote the number of futures contracts sold by $\beta_{t,T1}$ (where t is the current time), the stochastic time-T_1 spot value of asset j by $\tilde{S}_{j,T1}$, and the time-t futures rate for asset i and expiration date T_2 by $\tilde{f}_{i,T1,T2}$. Ignoring the (small) effect of marking to market, the total cash flow generated by the futures contracts between times t and T_1 is then equal to $-\beta_{t,T1}\,(\tilde{f}_{i,T1,T2} - f_{i,t,T2})$. Thus, the value of the hedged cash flow is

$$\text{hedged cash flow} = \tilde{S}_{j,T1} - \beta_{t,T1}\,(\tilde{f}_{i,T1,T2} - f_{i,t,T2})\,. \tag{II.1}$$

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[3]As argued below, it is also not obvious how one should set up an ECM for cross-hedges.

The usual rule is to choose $\beta_{t,T1}$ such that, conditionally on time-t information, the variance of this hedged cash flow is minimized.[4] Adding t-subscripts to the variance and covariance operators to stress the conditional nature of these distribution parameters and using the fact that, at time t, $f_{i,t,T2}$ is known, we can formulate this problem as

$$\min_{\beta_{t,T1}} \; var_t(\tilde{S}_{j,T1}) - 2\,\beta_{t,T1}\, cov_t(\tilde{f}_{i,T1,T2}, \tilde{S}_{j,T1}) + \beta_{t,T1}{}^2\, var_t(\tilde{f}_{i,T1,T2})\,. \tag{II.2}$$

The familiar solution is

$$\beta_{t,T1} = \frac{cov_t(\tilde{S}_{j,T1}, \tilde{f}_{i,T1,T2})}{var_t(\tilde{f}_{i,T1,T2})}\,. \tag{II.3}$$

This expression for $\beta_{t,T1}$ coincides with the population slope coefficient in the linear decomposition ("regression") of the relation between the future spot and futures rates:

$$\alpha_{t,T1}, \beta_{t,T1}\!: \; \tilde{S}_{j,T1} = \alpha_{t,T1} + \beta_{t,T1}\, \tilde{f}_{i,T1,T2} + \tilde{\varepsilon}_{t,T1} \;\; \text{s.t.} \;\; E_t(\tilde{\varepsilon}_{t,T1}) = 0 = cov_t(\tilde{\varepsilon}_{t,T1}, \tilde{f}_{i,T1,T2}) \tag{II.4}$$

As the joint distribution of the future prices, $\tilde{S}_{j,T1}$ and $\tilde{f}_{i,T1,T2}$, is unknown, it has become common practice to estimate $\beta_{t,T1}$ from a regression on (suitably differenced) past data. In doing so, the issues are (a) what estimator is to be used, taking into account the statistical properties of the data series; (b) what differencing interval is to be chosen; and (c) whether one should consider simple first differences or percentage changes. A more fundamental question is whether simple rules of thumb may not provide useful alternatives, or complements, to regression-based estimators. The practical answers to these questions, as adopted in this paper, are described in the next section.

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[4]The same result can be obtained if (a) the decision maker has a mean-variance utility function with a (non-tradable) foreign-currency position as the sole source of risk and (b) the exchange rate is a martingale. See for instance Stoll and Whaley (1993) or Kroner and Sultan (1993).

III. Regression-Based vs. Naive Rules for Currency Hedging: Test Design

The tests are carried out in currency markets. This choice is motivated by the following consideration: The problems of poor synchronization, bid-ask noise, and variability in the basis are avoided if one uses not the actual futures quotes, but theoretical futures prices computed from spot prices and net convenience yields for the exact maturity needed for the hedging problem at hand.[5] In the case of a stock market hedge, as in Stoll and Whaley (1993), one component of the net convenience yield—the *ex ante* dividend yield—is unobservable, so that no noise-free shadow futures prices can be computed. In currency markets, however, the net convenience yields are observable from "swap" forward quotes or can be computed from interbank interest rates. Thus, we can use noise-free data in the regressions. This has two advantages. First, by ruling out errors in variables, the interpretation of differences between various regression-based results becomes easier. Second, in the race between regression-based and naive hedging rules, the dice are no longer loaded against the former; thus, if notwithstanding the noise-free data the naive rules still do better than the regression-based hedging ratios, then we can safely conclude that (a) the regression-based hedges suffer from more fundamental problems than just noise in the regressor, and (b) when using other data, naive rules must be even more recommendable.

The availability of a theoretical forward rate for a currency hedge offers two additional boons beside providing noise-free input data for the regressions. First, as explained in Section III.C, below, it allows us to formulate additional, and somewhat more subtle, rules of thumb than the naive model employed by Kroner and Sultan. Second, forward rates can be computed for any exchange rate with unrestricted money markets. Therefore, the analysis is not confined to currency pairs for which a futures contract is actually traded in the US. This allows us to go

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[5]To be true, one can compute only theoretical *forward* prices, but these are virtually indistinguishable from theoretical futures prices.

beyond pure field tests and set up something of a laboratory experiment; for instance, one can obtain a wide sample of closely related currencies—for example, the BEF-NLG pair, where an intra-Benelux agreement has limited exchange rate movements to an even narrower band than the EMS band—which can then be compared with currency pairs that are less closely related.

A. Computation of the Theoretical Forward Prices

To estimate the forward-looking regression $\tilde{S}_{j,T1} = \alpha_{t,T1} + \beta_{t,T1}\, \tilde{f}_{i,T1,T2} + \tilde{\varepsilon}_{t,T}$ from past data, we first construct a data series that is clean from the errors-in-variables that plague the tests presented in Section II. We consider a hedging horizon, T_1–t, of three months, and we now specify that the remaining life of the hedge, $\Delta T \equiv T_2 - T_1$, is equal to zero (for a cross hedge) or one quarters (for a delta or cross-and-delta hedge).[6] Thus, for every date we can compute forward prices with a constant time to maturity of three or six months. This eliminates the change in the life of futures prices as one source of errors-in-variables bias in the regression; and if swap forward quotes are used, or if forward rates are computed from spot exchange rates and interest rates or swap rates, then also synchronization of the observations is no longer a problem. Lastly, if midpoint data are used, also bid-ask noise is avoided.

In practice, we have chosen to compute forward rates from interest rates rather than from 3- and 6-month swap rates, for the following reason. In our story, the hedge is liquidated on the expiration date, T_1. For this reason we want T_1 to be a working day, a condition that is

[6]The use of a three-month horizon has the drawback that there is overlap in the month-by-month hedging errors, but is dictated by data availability. Datastream provides one-, three-, and six-month interest rates, which allows us to analyze a problem of hedging a three-month exposure using a six-month hedge but (because of the absence of two-month Euro-rates) not the problem of hedging a one-month exposure using a two-month hedge. Other data series consulted by us provided much shorter time series and were hard to splice into the Datastream exchange rate files.

not always met for the expiration day of standard 90- or 180-day market quotes.[7] Thus, starting from every working day t, we first go to the date three months ("90 days") later; and if this tentative T_1-date is not a working day, we define T_1 as the first working day after that. T_2 is defined similarly. Delivery then takes place on the second working day after this date T_2.

To compute the t-to-T_1 forward exchange rate, we next need to consider the replicating deposits (or loans) made at time t. Such a deposit earns interest from the second working day following day t and until the calendar day before the delivery date. We therefore compute the number of interest-earning days between these dates as a fraction of a year, either from the number of calendar days and a 365-day year (the interbank convention for the GBP and the BEF), or using the 30-days-per-month, 360-days-per-year rule applicable for other currencies. We compute the return on the deposit or loan by multiplying the time to maturity, $T_1 - t$, by the three-month interest rate; that is, following interbank practice, we ignore the fact that $T_1 - t$ may be one or two days off the three-month mark. Our three-month forward rate then follows. For the six-month rate the procedure is analogous, except that we start from date T_1 rather than t. As mentioned before, we use midpoint rates so as to eliminate bid-ask noise.

B Estimation of Backward-Looking Hedge ratios

We first consider a number of regression-based estimators. In estimating the forward-looking regression $\tilde{S}_{j,T1} = \alpha_{t,T1} + \beta_{t,T1}\, \tilde{f}_{i,T1,T2} + \tilde{\varepsilon}_{t,T}$ from past data, one generally starts from the conceptual linear decomposition of the relationship between these variables: for every date τ there always exist conditional parameters α_τ and β_τ such that

$$\alpha_\tau, \beta_\tau : S_{j,\tau} = \alpha_\tau + \beta_\tau F_{i,\tau} + \varepsilon_\tau \quad \text{s.t. } \operatorname{cov}_{\tau-1}(\varepsilon_\tau, f_{i,\tau}) = 0 = E_{\tau-1}(\varepsilon_\tau) . \qquad \text{(III.1)}$$

[7] The delivery day is, of course, always a working day, but this is not always true for the expiration day. For instance, a 90-day contract taken out on February 25, 1997 (a Tuesday) expires on April 25, 1997 (a Sunday). The delivery day would then be April 27 (a Tuesday), but on Sunday April 25 itself we cannot trade.

The simplest operationalization of (III.1) is obtained by assuming that β_τ is an intertemporal constant and that α_τ is, at most, linear in time. Differencing (III.1) so as to eliminate problems of non-stationarity, and setting $\alpha_\tau - \alpha_{\tau-1} = \alpha$, we then obtain the regression equation that is standard in this literature,

$$[S_{j,\tau} - S_{j,\tau-1}] = \alpha + \beta\,[F_{i,\tau} - F_{i,\tau-1}] + \varepsilon'_\tau . \tag{III.2}$$

One potential problem with (III.2) is that, if the level of the variables changes substantially through time, there may be heteroscedasticity in the variables. In fact, the accepted view in capital market studies is that percentage changes are closer to I.I.D. than dollar price changes. Thus, one could also consider a regression that relates percentage changes,

$$[\frac{S_{j,\tau}}{S_{j,\tau-1}} - 1] = a + b\,[\frac{F_{i,\tau}}{F_{i,\tau-1}} - 1] + \varepsilon''_\tau . \tag{III.3}$$

In (III.3), the coefficient b has the dimension of an elasticity, while β's dimension is that of a partial derivative; thus, we now compute $\hat{\beta}$ as

$$\hat{\beta} = \hat{b}\,\frac{S_{j,t}}{F_{i,t}}, \tag{III.4}$$

where t refers to the last day in the estimation sample.

Another problem, which may affect both (III.2) and (III.3), is that, especially at high observation frequencies, problems like imperfect synchronization between spot and futures data may become relatively important. True, with our data there cannot be any (spurious) lead-lag relationships due to imperfect time stamping; but for intra-European currency pairs the exchange rate mechanism (ERM), or managed floating, may very well introduce (non-spurious) cross-correlations among changes in two exchange rates. The reason is as follows: when there is an ERM band or an informal target zone linking two currencies, exchange rate changes relative to the USD must be either perfectly identical (which we know is not the case), or they must follow each other's movements within a relatively short time span—thus creating a lead-lag relation akin to the one caused by poorly synchronized data. Whatever the cause of the cross-correlations, OLS estimators that consider only contemporaneous returns will underestimate the link between the two currencies as soon as the hedging horizon exceeds the observation period. Accordingly, we also experiment with the Scholes-Williams (1977) (SW) instrumental variable estimator, which is designed to pick up lagged responses between the regressor and the regressand:

$$\text{SW estimator} = \frac{\hat{\text{cov}}(RS_\tau, IV_\tau)}{\hat{\text{cov}}(Rf_\tau, IV_\tau)}. \tag{III.5}$$

In (III.5), RS_τ and Rf_τ stand for either the first-differenced spot and futures price, respectively, as in (III.2), or the percentage changes in spot and futures prices, respectively, as in (III.3); and IV_τ, the Scholes-Williams instrumental variable, is defined as $Rf_{\tau-1} + Rf_\tau + Rf_{\tau+1}$.[8]

We estimate (III.2) and (III.3) using OLS and SW using various sampling frequencies and periods. To streamline the programming of the regressions, we used either all London working days ("daily"), or every fifth working day ("weekly"), or every tenth working day ("biweekly"), or twentieth working day ("monthly"). For daily and weekly sampling, we use two years of data. As a two-year interval leaves rather few observations for regressions with biweekly and especially monthly sampling, we also show results from four-year samples for the estimation of bi-weekly and monthly regressions.

Our data base eliminates errors in the regressor as a source of bias, the focus of this article and also the prime problem discussed in Stoll and Whaley (1993). In contrast, Kroner and Sultan (1993) stress over-differencing of the data and GARCH-effects as potential shortcomings in standard regression tests. While their results are positive and interesting, in the case of cross- and cross-and-delta hedges there are practical problems in implementing a GARCH error-correction model. Specifically, while there is little a priori doubt that spot and

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[8]See Apte, Kane, and Sercu (1994) for a theoretical justification and application of the Scholes-Williams estimator to lead-lag situations other than those caused by thin trading. As in Apte, Kane and Sercu (1994), or Fowler and Rorke (1983) one could extend the lead-lag window to more than one period (one day, here), but tests in Sercu and Wu (1997) reveal that there are no significant cross-correlations beyond the one-day interval.

forward rates for one given currency (as in a delta hedge) are cointegrated, for a cross-hedge or a cross-and-delta hedge the existence of a cointegration relation between non-related currencies is not clear at all; and for EMS pairs, the relation imposed by the exchange rate mechanism is not constant over time, being subject to "trend breaks" (realignments) that are, ex ante, difficult to predict. Thus, it is not clear how an ECM for cross hedges should be constructed and estimated. In addition, over a three-month horizon GARCH-effects are likely to be less important than over a one-week interval. For these reasons, our regression-based hedge ratios are confined to standard estimation techniques.

C. Naive Forward-Looking Estimators for Currency Hedges

In the empirical application, the regression-estimated hedge ratios put forward in the preceding section are competing against simple strategies that require no statistical analysis of past data. Thus, while the naive estimators may very well be biased, they nevertheless have zero estimation error. To understand the logic of these naive hedge ratios, consider a situation where the US investor's currency-j inflow, occurring at time T_1, is hedged using a currency-i future that expires at T_2 ($\geq T_1$). For instance, a 3-month NLG inflow is hedged using a 6-month DEM futures contract. The conditional regression relation that is to be estimated is

$$\tilde{S}_{j,T1} = \alpha_{t,T1} + \beta_{t,T1} \; \tilde{f}_{i,T1,T2} + \tilde{\varepsilon}_{t,T1,T2}. \tag{III.6}$$

To obtain a simple forward-looking estimator of $\beta_{t,T}$, we consider two elementary no-arbitrage conditions. First, forward rates satisfy Interest Rate Parity. Thus, our first no-arbitrage relation is

$$\tilde{f}_{i,T1,T2} = \tilde{S}_{i,T1} \; \frac{1+\tilde{r}_{T1,T2}}{1+\tilde{r}^{i}_{T1,T2}} \; . \tag{III.7}$$

where $r_{T1,T2}$ is the effective rate of return, without any annualization, on a risk-free investment between times T_1 and T_2 in the domestic currency (the USD), and $r^{i}_{T1,T2}$ is the effective return on the currency-i (DEM) risk-free investment. Rearranging, we obtain the following relation between the spot value of the hedge currency and its futures price:

$$\tilde{S}_{i,T1} = \frac{1+\tilde{r}^{i}_{T1,T2}}{1+\tilde{r}_{T1,T2}}\,\tilde{f}_{i,T1,T2}\,. \tag{III.8}$$

Note that the futures price on the right hand side is the regressor in (III.6). The spot rate on the left hand side of (III.8) is not yet the regressand in (III.6), except in the case of a pure delta hedge. We can, however, make a link with the regressand by invoking a second arbitrage relationship, triangular arbitrage:

$$\tilde{S}_{j,T} = \tilde{S}^{i}_{j,T}\,\tilde{S}_{i,T}\,, \tag{III.9}$$

where $\tilde{S}^{i}_{j,T}$ is the cross-rate (the value of the exposure currency, j, in units of the hedge currency, i). For example, when the currency to be hedged (j) is the NLG and the hedge currency (i) is the DEM, the relevant cross-rate is the time-T value, in DEM, of one NLG. Combining (III.8) and (III.9), we obtain the following no-arbitrage condition,

$$\tilde{S}_{j,T1} = \tilde{S}^{i}_{j,T1}\,\frac{1+\tilde{r}^{i}_{T1,T2}}{1+\tilde{r}_{T1,T2}}\,\tilde{f}_{i,T1,T2}\,. \tag{III.10}$$

The variables on the right- and left-hand sides of (III.10) now correspond to the ones appearing in regression (III.6). We see that if the time-T_1 cross rate and the interest rates were known, then there would be no need to estimate $\beta_{t,T1}$; in fact, the exposure would be *a priori* equal to

$$\beta_{t,T1} = S^{i}_{j,T1}\,\frac{1+r^{i}_{T1,T2}}{1+r_{T1,T2}} \qquad \text{(certainty model)}\,. \tag{III.11}$$

In practice, the future cross-rate and interest rates are, of course, unknown, but we can experiment with simple predictors. For example, the unbiased expectations (UE) hypothesis suggests $E_t(\tilde{S}^{i}_{j,T}) = F^{i}_{j,t,T}$, where $F^{i}_{j,t,T}$ is the forward cross rate. Alternatively, if spot rates are random walks (RW), then $E_t(\tilde{S}^{i}_{j,T})$ equals $S^{i}_{j,t}$, the current cross rate. Thus, our alternative price-based estimators for the future spot rate in (III.11) are

$$\text{UE:} \qquad \hat{S}^{i}_{jT1} = F^{i}_{j,t,T}, \tag{III.12}$$

and

RW: $$\hat{S}^{i}_{jT1} = S^{i}_{j,t}. \qquad \text{(III.13)}$$

This already provides two naive estimators for $\beta_{t,T1}$ in a cross-hedge problem (where $T_1 = T_2$, that is, where no future interest rates need to be predicted.) Analogously, as alternative predictors for the future interest rates we use either the current relative return ratio for the same time to maturity (T_2–T_1)—the no-change or random-walk (RW) forecast:

RW: $$\frac{1+\hat{r}^{i}_{T1,T2}}{1+\hat{r}_{T1,T2}} = \frac{1+r^{i}_{t,t+T2-T1}}{1+r_{t,t+T2-T1}}, \qquad \text{(III.14)}$$

or the current forward interest rates—the unbiased-expectations (UE) forecast:

UE: $$\frac{1+\hat{r}^{i}_{T1,T2}}{1+\hat{r}_{T1,T2}} = \frac{\dfrac{1+r^{i}_{t,T2}}{1+r^{i}_{t,T1}}}{\dfrac{1+r_{t,T2}}{1+r_{t,T1}}}. \qquad \text{(III.15)}$$

Expressions (III.14) and (III.15) provide our alternative naive estimators for the exposure in a delta hedge (where j = i, that is, where no future cross rate needs to be predicted.) Note, in passing, that these naive hedge ratios do take into account the information in the current spot or forward interest rates. As such, they are somewhat more sophisticated than the naive rule adopted in Kroner and Sultan (1993), who match the sizes of the spot and forward positions (that is, they set $\beta = 1$).

Lastly, for a cross-and-delta hedge we use the following four combinations of the random-walk (RW) and unbiased expectations (UE) estimators:

UE/RW: $$\hat{\beta}_{t,T1} = F^{i}_{j,t,T1}\,\frac{1+r^{i}_{t,t+T2-T1}}{1+r_{t,t+T2-T1}}. \qquad \text{(III.16)}$$

UE/UE: $$\hat{\beta}_{t,T1} = F^{i}_{j,t,T1}\,\frac{\dfrac{1+r^{i}_{t,T2}}{1+r^{i}_{t,T1}}}{\dfrac{1+r_{t,T2}}{1+r_{t,T1}}}, \qquad \text{(III.17)}$$

RW/RW: $$\hat{\beta}_{t,T1} = S^{i}_{j,t}\,\frac{1+r^{i}_{t,t+T2-T1}}{1+r_{t,t+T2-T1}}, \qquad \text{(III.18)}$$

$$\text{RW/UE:} \qquad \hat{\beta}_{t,T1} = S^{i}_{j,t} \frac{\dfrac{1+r^{i}_{t,T2}}{1+r^{i}_{t,T1}}}{\dfrac{1+r_{t,T2}}{1+r_{t,T1}}} . \tag{III.19}$$

D. Performance Evaluation Criterion

We make two alternative assumptions regarding the assumed size of the forex cash flow contracted at t for delivery at T_1. Under the first approach, the size of each foreign-currency inflow is set such that, at the hedging date, its spot value corresponds to one USD; that is, the number of foreign currency units one is exposed to at time t is assumed to be equal to $1/S_{j,t}$. The alternative procedure is that each cash flow is one unit of foreign exchange, regardless of this currency's USD value at the hedging date. We discuss the pros and cons of either approach after we have set out the evaluation procedures.

This procedure works as follows. We set aside the first four years of data for the initial estimation of the regression coefficients. Thus, at the beginning of the 49th month of data we determine the hedge ratio, using either the beginning-of-the-month prices (for the naive rules) or two to four years of daily, weekly, biweekly, or monthly data (for the regression-based estimators). Let the competing estimation rules be indicated by subscripts h = 1, ... H. For each of the proposed hedge ratios $\beta_{h,t}$, the cash flow contracted and hedged in month t is then computed as $Z_t\, [\tilde{S}_{j,T1} - \beta_{h,t}(\tilde{f}_{i,T1,T2} - f_{i,t,T1})]$, where Z_t is equal to either $1/S_{j,t}$ or unity. This cash flow is usually non-stationary; and so is its conditionally stochastic component, $Z_t\, [\tilde{S}_{j,T1} - \beta_{h,t}\, \tilde{f}_{i,T1,T2}]$. To obtain a better-behaved variable, we follow standard procedure and subtract the initial spot rate; that is, we study the variable $Z_t\, \{[\tilde{S}_{j,T1} - S_{j,t}] - \beta_{h,t}[\tilde{f}_{i,T1,T2} - f_{i,t,T1}]\}$. The entire procedure is repeated for every subsequent month, each time resetting the naive hedge ratios or re-estimating the regression coefficients. For each time series of hedge ratios $\{\beta_{h,t}\}$, the N monthly hedge errors are then summarized by their mean square (MS):[9]

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[9]The rankings are not affected when the mean in subtracted, i.e. when the standard deviation is computed rather than the RMSE. We prefer the latter because the mean is (insignificantly different from zero and) not known, ex

$$MS_h = \sum_{t=1}^{N} \frac{\{Z_t\,[(\tilde{S}_{j,t+\Delta T} - S_{j,t}) - \beta_{h,t}(\tilde{f}_{i,t+\Delta T,t+n\Delta T} - f_{i,t,t+n\Delta T})]\}^2}{N} \quad . \qquad \text{(III.20)}$$

where n (in the subscript for the forward rate) is equal to unity in a pure cross hedge, and equal to two in a delta or cross-and-delta hedge.

We now briefly discuss the alternative assumptions regarding Z_t, the size of the contractual exposures. The first procedure works with a dimensionless (percentage) number, $[(\tilde{S}_{j,T1} - S_{j,t}) - \beta_{h,t}(\tilde{f}_{i,T1,T2} - f_{i,t,T1})]/S_{j,t}$, and eliminates the time-varying level of the exchange rate as a potential source of heteroscedasticity. In contrast, the dimension of the variable in the second procedure, $[(\tilde{S}_{j,T1} - S_{j,t}) - \beta_{h,t}(\tilde{f}_{i,T1,T2} - f_{i,t,T1})]$, is a number of USD per unit of foreign exchange, and its variability is partly determined by the level of the exchange rate. Percentages offer the twin advantages that the division by the initial level eliminates one source of heteroskedasticity and the measures of volatility are more comparable across currencies; in fact, percentage changes are the standard transform in studies of speculative markets. In the hedge literature, however, one often works with changes in dollar prices for both the regression estimation and the evaluation, presumably because then the regression coefficient immediately has the dimension of a number of foreign currency units. In this study, both approaches are used, and they lead to similar conclusions.

IV. Data, Results, and Discussion

We select eleven countries that have at least twelve years of daily data (1985-1996) in the Datastream data base: Belgium, Canada, Denmark, France, Germany, Italy, Japan, the Netherlands, Switzerland, the UK, and the US. Exchange rates, originally against GBP, were re-expressed into units of USD (the home currency), and forward rates were computed following the procedure outlined in the test-design section. From the total menu of 45 possible

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ante, to the trader. Nor are the rankings are affected if one relies on mean absolute deviations rather than on RMSs to rate the competing hedging rules.

pairs that could enter into a cross-hedge problem, we select three groups of three pairs each, in a way that should provide a sufficient variability in the degree of relatedness between the two members of a pair. The first group contains intimately related currency pairs that US-based traders would surely consider to be excellent candidates in a cross-hedge: NLG-BEF (where for most of the sample period an intra-Benelux agreement imposed a 1%-band around the ERM central rate), DEM-NLG (which the Nederlandse Bank unilaterally kept within a narrow band for most of the sample period), and BEF-DEM (linked indirectly through the above arrangements, and directly by unilateral intervention by the Nationale Bank van België). The second group contains a straight ERM pair (DKK-FRF), two combinations between an ERM currency and the CHF (which, until mid-1997, was widely viewed as linked to the DEM, even though Switzerland's central bank denies that it actually intervenes in exchange markets), and the ITL-GBP pair. These four pairs still show substantial common characteristics, although less so than the first group. The motivations for considering the two currency combinations in the third group, lastly, are completeness and academic curiosity rather than realism. The members of each pair in the third group are, indeed, far less connected; in fact, the only commonalities between these probably is the USD-component in the exchange rates, and it extremely doubtful whether, in reality, one would ever hedge a GBP exposure using CAD or an ITL exposure using JPY. Still, this currency selection allows us to see to what extent the degree of relatedness affects the relative performance of the naive *vs*. regression-based hedges.

To set the stage, Table 1 describes the results from naive hedges. Panel A of the table shows the root[10] MS cash flow of the exposed currency, first without hedging and then after applying each of the three naive no-change hedges. First consider the delta hedge, as studied by

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[10]In Panel A of Table 1 we use the *root* mean square for the purpose of showing risk as an absolute magnitude, because the root mean square percentage change is almost indistinquishable from the volatility of log changes (the standard measure of risk in option pricing) and squaring of the mean dollar changes would have led to numbers with inconveniently divergent orders of magnitude. In all subsequent tables, in contrast, we use the mean squares themselves, as standard in the hedging literature, and we deal with the divergent magnitudes by dividing this MS by the MS of the naive rule.

Kroner and Sultan. In all cases, the naive delta-hedge reduces the volatility by 93 percent (DKK) to almost 98 percent (JPY). The picture for cross-hedges and cross-and-delta hedges is, unsurprisingly, less homogenous across currencies: the performance of a naive hedge depends a lot on the degree of relatedness of the two pairs. For ERM pairs, applying the no-change rule of thumb reduces the risk by 82 percent (hedging BEF using NLG) to 96 percent (NLG by DEM); for non-ERM European pairs the risk-reduction ranges from 60-65 percent (the cases involving the CHF) to a lowish 33 percent (hedging ITL using GBP). For unrelated pairs, lastly, naive hedging achieves virtually no risk-reduction (CAD-GBP), or may actually backfire rather badly (ITL-JPY); recall, however, that the last two combinations are a priori not realistic for hedging purposes.

Panel B compares the MS cash flows for the naive hedges other than the no-change rule. In that panel, as in Tables 2 and 3 discussed below, all MSs are rescaled by the MS cash flow of the no-change hedging rule—RW for a cross or delta hedge, and RW/RW for a cross-and-delta hedge. All ratios in Panel B of Table 1 turn out to be extremely close to unity; that is, the choice of a particular naive rule has no meaningful impact on the MS cash flow. Thus, even though it is widely accepted that the RW model beats the UE model as an exchange rate forecaster (Froot and Thaler, 1990), for current purposes the two models are indistinguishable, and our choice of the no-change rule as the basis of comparison is not material. The more interesting question, then, is how the regression-based hedging rules fare, across sampling rules or estimation techniques and relative to the naive hedging rules.

Tables 2 and 3 show the results for non-naive hedges when the size of the foreign inflow, due within three months, is equal to, respectively, $1/S_{j,t}$ units of foreign exchange (Table 2) or one unit (Table 3). For ease of comparison, the results for OLS regressions using two years of data (with varying observation frequency) are presented in the central part of the table. To the left, next to the MS ratios for daily OLS regressions, we present the ratios for daily SW regressions; and to the right we add the numbers for biweekly or monthly OLS regressions obtained with four rather than with two years of data.

There are four pervasive findings. First, in this study (and unlike in Kroner and Sultan, 1993), MS ratios in excess of unity are by no means the exception. In fact, regressions seem to do systematically poorly for delta-hedges, as well as for cross hedges or cross-and-delta hedges involving strongly related currencies. Given that, in this experiment, one cannot invoke errors in data as an explanation of the less-than-impressive performance of the statistics-based hedge ratios, we conclude that the regressions must suffer from low precision and/or from some form of misspecification. We return to this issue below.

A second pervasive finding from Tables 2 and 3 is that, in this study, the choice of a sample (period length and observation frequency) or of an estimator (OLS vs. SW) has a much larger impact than has the choice of OLS vs. GARCH-ECM in Kroner and Sultan (1993) for a given observation frequency (weekly, in their case); also, the deviations from unity are larger, here, than what they observe. To a large extent this is due to the fact that most of our hedging experiments include cross-rate risk; for the delta-hedges, which are the object of Kroner and Sultan's study, the impact of the sample and estimator tend to be smaller indeed, and so do the deviations from unity. The third general pattern, related to the previous one and already apparent from Table 1, is that the results for cross-and-delta hedges are quite close to those of pure cross hedges; that is, cross-rate volatility is the dominant source of basis risk in a cross-and-delta risk, and the delta-component is rather marginal. Lastly, for a given sample and estimation technique, the results from regressions using percentage-change data are virtually always better than the ones from regressions between first differences—even in two-year samples, where the variability in the level of the exchange rates is clearly lower than in four-year samples. This finding confirms the standard view that percentage changes have better statistical properties than dollar price changes.

Closer inspection of the regression MS ratios reveals some interesting differences between the three currency groups we chose for cross (and cross-and-delta) hedges. Specifically, for highly related pairs, the regression-based cross-hedges that use two years of data have the following characteristics: (a) they do clearly worse than naive hedges; (b) the low-frequency regressions do substantially better than high-frequency regressions; and (c) for daily observations, SW resoundingly beats OLS. The last two findings imply that, at high frequencies, the ERM does induce substantial lead-lag patterns. Such leading/lagging relationships should be picked up not just by shifting from OLS to SW, but also, within OLS, by increasing the observation interval. This does, in fact, happen: even though sample sizes become smaller and smaller, weekly still OLS does better than daily, and biweekly better than weekly OLS and even SW.[11] However, when going from biweekly to monthly data, the advantage of picking up more lead/lag relations appears to be more than compensated by the concomitant loss of degrees of freedom. When, accordingly, the sample period is increased to four years, monthly sampling comes out as the winner; in fact, the results for four years of monthly data become close to the ones from the naive rules.

Thus, for highly related currencies lead-lag patterns are the prime source of problems in the high-frequency OLS regressions with two years of data, and explain why these regressions are resoundingly beaten by the naive rules. These problems can be mitigated by choosing SW, or a large sample of low-frequency data; but the result is still not worth the effort as the naive rules do at least as well. To confirm this picture, we note that none of these patterns is present in the group of unrelated currencies—the combinations that no real-world treasurer would actually select: there is no clear association between MS ratio and sample period or frequency, and SW does not improve on OLS. In the absence of an obvious mis-specification problem, the regression does about as well as the naive rule (CAD-GBP), or substantially better (ITL-JPY)[12]. The diagnosis for group 2, finally, is somewhere in between: there is some evidence of cross-correlations (as shown by the superiority of daily SW, or two years of biweekly data or four years of monthly data relative to daily—all relative to daily or weekly OLS), but the

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[11]This provides circumstantial evidence that cross-correlations may exist at horizons exceeding one day. When using daily data one could, of course, extend the SW instrument to account for higher-order lags. However, as shown in Sercu and Wu (1997), while the direct statistical evidence in favor of one-day leading/lagging is convincing, there is no direct evidence of significant higher-order cross-correlations.

[12]In fairness, recall that, in this particular case, the application of the naive rule actually increased the risk. It can easily be calculated that the regressions reduce the total variability by about 1/6.

naive rules do not systematically outperform the regression-based hedge ratios, and increasing the sample size does not help.

In contrast, for delta hedges (Panels C in Tables 2 and 3) there is no a priori reason to expect (quasi-) EMS currencies to be very different from others, nor do we see any such difference in the figures. Strikingly, even the best regression-based hedges tend to do worse than the naive rules, and in the three cases where the naive rules are actually beaten the difference remains rather small. The superior performance of the naive hedging rules that is observed here differs from the conclusion of Kroner and Sultan (1993), who find that OLS beats their naive rule in all cases but one. Nor is there any evidence of lead-lag relationships: SW is typically quite close to OLS, and the differences between the MS cash flows of these two go either way, without any clear pattern. As we found for cross-hedges that involve unrelated currencies, for delta hedges a sample of recent high-frequency data does better than low-frequency data; and increasing the sample period to four years actually worsens the results. This suggests that the main problem that plagues delta-regressions seems to be a changing relationship between the regression variables. The finding of Kroner and Sultan that the GARCH ECM does better than OLS points in the same direction.

V. Conclusions

When hedging an asset using a futures contract that has the wrong expiration data, or the wrong underlying asset, or both, the variance-minimizing hedge ratio depends on unobservable conditional (co)variances, which have to be estimated. If unconditional regression analysis of past data is used, the issues are (a) what estimator is to be used, taking into account the statistical properties of the data series; (b) what differencing interval is to be chosen; and (c) whether one should consider simple first differences or percentage changes. A more radical question is whether simple rules of thumb provide useful alternatives, or complements, to regression-based estimators.

In this paper we find that, regardless of observation frequency and estimation technique, unconditional (backward-looking) regressions are often poor proxies for the ideal regression, even to the extent that regression-based hedges are frequently beaten by simple rules of thumb. For delta hedges, this effect is rather pervasive, while for cross hedges and cross-and-delta hedges the superiority of the naive hedging rule is especially clear among closely-related currencies. As our data are free of measurement errors, this relatively poor performance of regression-based hedges cannot be due to errors in data. For cross-hedges involving two European currencies, the poor performance of high-frequency OLS estimates can be traced to EMS-induced leads and lags among exchange rate changes, while for delta-hedges the dominant source of estimation problems seems to be a time-varying relationship between the regression variables. Lastly, we find that regressions do better if they use (percentage) returns rather than dollar price changes.

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Table 1: MS cash flow from naive hedges

Panel A: RMS cash flow, E-2, unhedged or covered by a random-walk hedge

hedg-exp	Cash flow is $1/S_{j,t}$ units of foreign exchange: not hedged	cross&delta	cross	delta	Cash flow is one unit of foreign exchange: not hedged	cross&delta	cross	delta
DEM-NLG	5.95526	0.26795	0.24010	0.17021	3.32539	0.14740	0.13464	0.09266
NLG-BEF	6.03279	1.07683	1.08522	0.26349	0.18433	0.03107	0.03120	0.00771
BEF-DEM	6.01958	1.26694	1.09797	0.16317	3.78570	0.76289	0.66086	0.09943
DKK-FRF	5.71725	1.0253	0.96754	0.32070	1.05734	0.19049	0.18004	0.05878
CHF-DKK	5.97799	2.4684	2.41083	0.43770	0.97167	0.39136	0.38297	0.07002
FRF-CHF	6.52408	2.31984	2.29890	0.20071	4.76446	1.69084	1.65566	0.14245
GBP-ITL	6.41942	3.6406	3.60859	0.33057	0.00515	0.00252	0.00250	0.00026
CAD-GBP	6.04439	5.7703	5.78942	0.19197	10.86317	10.10121	10.15489	0.34342
ITL-JPY	6.18218	8.13144	8.16641	0.14255	0.05720	0.07366	0.07372	0.00116

Panel B: MS cash flow of other naive hedges, scaled by the MS cash flow of random-walk hedge

	Cash flow is $1/S_{j,t}$ units of foreign exchange					Cash flow is one unit of foreign exchange				
	cross and delta hedges			cross	delta	cross and delta hedges			cross	delta
hedg-exp	ue/rw	rw/ue	ue/ue	ue	ue	ue/rw	rw/ue	ue/ue	ue	ue
DEM-NLG	1.006	0.990	0.996	1.008	1.010	1.006	0.992	0.996	1.008	1.012
NLG-BEF	0.996	1.000	0.996	0.996	0.994	0.998	1.000	0.998	0.996	0.994
BEF-DEM	1.000	0.998	0.998	1.000	1.000	1.000	0.998	0.998	1.000	1.004
DKK-FRF	1.008	0.990	0.998	1.004	0.992	1.006	0.990	0.996	1.004	0.994
CHF-DKK	0.986	0.998	0.982	0.984	0.960	0.984	0.998	0.982	0.982	0.960
FRF-CHF	0.996	1.002	0.998	0.996	1.002	0.994	1.002	0.996	0.994	1.002
GBP-ITL	1.004	1.000	1.004	1.004	1.024	1.008	1.000	1.008	1.000	1.000
CAD-GBP	1.002	1.000	1.002	1.002	1.028	1.002	1.000	1.002	1.002	1.028
ITL-JPY	1.022	0.998	1.022	1.024	0.998	1.020	0.998	1.018	1.020	0.982

Key to Table 1: Either one unit, or $1/S_{j,t}$ units, of the currency shown in column "expo(sure)" are hedged using β futures contracts of the currency shown in column "hedg(e)". In Panel A, the MS themselves are shown, either in the absence of hedging (column "unhedged"), or when hedged by a six-month contract of the currency labeled "hedg(e)" (the cross&delta hedge), a three-month contract of the currency labeled "hedg(e)" (the cross hedge), or by a six-month contrct in the currency of the exposure (the delta-hedge). In all cases the hedge ratio is based on the no-change forecast in spot or interest rates.

In Panel B, the MS cashflow of other naive rules are shown, divided by the MS of the no-change hedging rule. The naive hedging rules are:

- "RW" (random walk): β is set assuming that exchange rates (or interest rates) will not change.
- "UE" (unbiased expectations): β is set assuming that the forward exchange rate (or interest rate) is right
- "RW/UE", "UE/RW", etc.: the first entry refers to the naive forecast used for the exchange rate, the second one to the naive forecast used for the interest rates.

The base case is for cross-and-delta hedges is RW/RW; for the other cases it is RW. All MSs are scaled by the MS of the base case.

Table 2: MS cash flow from regression-based hedges of $1/S_{j,t}$ units of foreign exchange, scaled by the MS cash flow of random-walk based hedge results

Panel A: Cross- and delta-hedges

	using two years of data										using four years of data			
	day	day	day	day	week	week	2week	2week	mnth	mnth	2week	2week	mnth	mnth
hedg-exp	SWΔ	SW%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%
DEM-NLG	1.186	1.184	2.477	2.430	1.414	1.399	1.120	1.084	1.19	1.208	1.107	1.084	1.080	1.038
NLG-BEF	1.077	1.010	1.390	1.313	1.156	1.084	1.125	1.056	1.146	1.072	1.124	1.034	1.084	1.002
BEF-DEM	1.061	1.069	1.156	1.190	1.090	1.092	1.058	1.049	1.067	1.058	1.042	1.034	1.036	1.034
DKK-FRF	0.882	0.870	0.992	0.972	0.901	0.889	0.926	0.914	0.916	0.910	0.947	0.931	0.968	0.956
CHF-DKK	0.889	0.889	0.922	0.914	0.920	0.901	0.908	0.900	0.907	0.888	0.933	0.933	0.901	0.870
FRF-CHF	1.040	1.036	1.153	1.153	1.059	1.044	0.994	0.989	1.003	0.998	1.002	0.986	1.044	1.030
GBP-ITL	0.939	0.941	0.933	0.937	0.931	0.929	0.992	1.000	0.936	0.934	1.047	1.014	1.042	1.002
CAD-GBP	1.047	1.038	1.012	1.000	1.094	1.090	1.361	1.358	1.134	1.127	1.061	1.049	1.113	1.090
ITL-JPY	0.645	0.642	0.646	0.643	0.659	0.656	0.654	0.651	0.643	0.639	0.654	0.654	0.664	0.663

Panel B: Cross-hedges

	using two years of data										using four years of data			
	day	day	day	day	week	week	2week	2week	mnth	mnth	2week	2week	mnth	mnth
hedg-exp	SWΔ	SW%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%
DEM-NLG	1.149	1.141	2.465	2.500	1.416	1.409	1.063	1.034	1.131	1.154	0.986	0.992	1.024	0.996
NLG-BEF	1.053	1.000	1.336	1.275	1.126	1.067	1.120	1.064	1.111	1.054	1.080	1.020	1.040	0.986
BEF-DEM	1.075	1.082	1.175	1.221	1.086	1.086	1.055	1.047	1.068	1.061	1.028	1.022	1.026	1.020
DKK-FRF	0.903	0.897	0.982	0.964	0.872	0.869	0.867	0.874	0.896	0.903	0.960	0.960	0.927	0.937
CHF-DKK	0.884	0.887	0.920	0.916	0.918	0.901	0.906	0.900	0.906	0.890	0.941	0.941	0.889	0.867
FRF-CHF	1.059	1.049	1.171	1.164	1.080	1.059	0.995	0.985	1.018	1.007	1.020	0.998	1.061	1.038
GBP-ITL	0.935	0.939	0.925	0.931	0.925	0.925	0.984	1.000	0.933	0.933	1.026	0.998	1.020	0.988
CAD-GBP	1.034	1.024	1.008	0.996	1.075	1.071	1.315	1.313	1.121	1.113	1.059	1.047	1.103	1.084
ITL-JPY	0.640	0.637	0.642	0.637	0.654	0.651	0.651	0.648	0.637	0.633	0.648	0.648	0.656	0.654

Panel C: Delta-hedges

	using two years of data										using four years of data			
	day	day	day	day	week	week	2week	2week	mnth	mnth	2week	2week	mnth	mnth
hedg-exp	SWΔ	SW%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%	OLSΔ	OLS%
NLG	1.173	1.014	1.175	1.012	1.201	1.026	1.297	1.072	1.281	1.077	1.357	1.036	1.464	1.119
BEF	1.111	1.032	1.117	1.032	1.145	1.073	1.190	1.087	1.199	1.100	1.179	1.082	1.151	1.059
DEM	1.332	1.107	1.320	1.094	1.350	1.115	1.376	1.109	1.366	1.113	1.488	1.080	1.605	1.173
FRF	1.032	1.010	1.038	1.024	1.055	1.036	1.067	1.072	1.062	1.056	1.008	1.024	0.988	0.998
DKK	0.861	0.920	0.925	0.964	0.933	0.988	1.013	1.074	0.949	1.011	0.922	0.922	0.943	1.020
CHF	1.175	1.053	1.158	1.051	1.162	1.042	1.151	0.995	1.147	1.033	1.252	1.047	1.306	1.077
ITL	0.986	0.882	1.040	0.925	0.941	0.878	0.925	0.925	1.02	0.918	1.098	0.947	1.105	0.956
GBP	1.132	1.018	1.134	1.040	1.166	1.057	1.204	1.032	1.337	1.183	1.631	1.383	1.721	1.421
JPY	1.182	1.044	1.177	1.057	1.212	1.055	1.395	1.180	1.25	1.085	1.277	1.069	1.383	1.134

Key to Table 2. $1/S_{j,t}$ units of the currency shown in column "exp(osure)" are hedged using β futures contracts of the currency shown in column "hedg(e)". The table show mean squares (MS) of these hedged cash flows, scaled by the MS cash flow from the corresponding random-walk-based hedging strategy. In a cross-hedge, the expiry dates of the exposure and the futures match, in a delta-hedge, the two currencies match, and in a cross-delta-hedge neither match.

The β is set using the following regression-based rules:

- OLSΔ: OLS regressions on first differences (daily to monthly)
- OLS%: OLS regressions on percentage returns (daily to monthly), with the slope rescaled into a hedge ratio using time-t rates, see (II.4).
- SWΔ: Scholes-Williams regressions on first differences (daily)
- SW%: Scholes-Williams regressions on percentage returns (daily), with the slope rescaled into a hedge ratio using time-t rates, see (II.4).

Table 3: MS cash flow from regression-based hedges of one unit of foreign exchange, scaled by the MS cash flow of random-walk based hedge results

Panel A: Cross- and delta-hedges

	using two years of data										using four years of data			
hedg-exp	day SWΔ	day SW%	day OLSΔ	day OLS%	week OLSΔ	week OLS%	2week OLSΔ	2week OLS%	mnth OLSΔ	mnth OLS%	2week OLSΔ	2week OLS%	mnth OLSΔ	mnth OLS%
DEM-NLG	1.186	1.188	2.506	2.474	1.407	1.397	1.161	1.184	1.144	1.105	1.065	1.051	1.057	1.024
NLG-BEF	1.075	1.008	1.454	1.374	1.166	1.092	1.156	1.080	1.114	1.047	1.138	1.044	1.092	1.006
BEF-DEM	1.069	1.077	1.162	1.199	1.098	1.100	1.074	1.064	1.055	1.046	1.042	1.036	1.038	1.036
DKK-FRF	0.874	0.861	0.978	0.956	0.899	0.887	0.913	0.905	0.931	0.921	0.941	0.924	0.962	0.949
CHF-DKK	0.874	0.878	0.904	0.901	0.903	0.885	0.893	0.878	0.923	0.911	0.889	0.889	0.882	0.852
FRF-CHF	1.024	1.022	1.158	1.156	1.075	1.036	0.991	0.988	1.010	1.005	1.002	0.982	1.024	1.008
GBP-ITL	0.953	0.953	0.953	0.960	0.953	0.953	0.969	0.961	0.958	0.964	1.082	1.057	1.065	1.032
CAD-GBP	1.088	1.080	1.044	1.032	1.143	0.000	1.184	1.180	1.285	1.273	1.100	1.088	1.158	1.136
ITL-JPY	0.667	0.666	0.672	0.671	0.679	0.676	0.663	0.661	0.638	0.633	0.667	0.671	0.674	0.676

Panel B: Cross-hedges

	using two years of data										using four years of data			
hedg-exp	day SWΔ	day SW%	day OLSΔ	day OLS%	week OLSΔ	week OLS%	2week OLSΔ	2week OLS%	mnth OLSΔ	mnth OLS%	2week OLSΔ	2week OLS%	mnth OLSΔ	mnth OLS%
DEM-NLG	1.169	1.153	2.468	2.506	1.440	1.423	1.133	1.148	1.063	1.044	0.978	0.974	1.026	0.990
NLG-BEF	1.057	1.002	1.409	1.341	1.145	1.084	1.130	1.072	1.102	1.047	1.094	1.034	1.051	0.994
BEF-DEM	1.082	1.090	1.186	1.239	1.094	1.094	1.077	1.068	1.050	1.043	1.032	1.024	1.030	1.024
DKK-FRF	0.904	0.897	0.980	0.958	0.874	0.869	0.897	0.902	0.870	0.878	0.964	0.960	0.933	0.939
CHF-DKK	0.874	0.880	0.906	0.904	0.903	0.889	0.895	0.882	0.919	0.910	0.901	0.901	0.870	0.848
FRF-CHF	1.038	1.030	1.177	1.169	1.121	1.047	1.003	0.994	1.016	1.004	1.018	0.992	1.038	1.014
GBP-ITL	0.937	0.945	0.937	0.945	0.937	0.945	0.953	0.953	0.957	0.965	1.049	1.024	1.032	1.008
CAD-GBP	1.067	1.057	1.034	1.022	1.117	0.000	1.164	1.159	1.250	1.239	1.096	1.084	1.145	1.126
ITL-JPY	0.666	0.664	0.671	0.669	0.677	0.674	0.662	0.659	0.629	0.624	0.666	0.669	0.671	0.672

Panel C: Delta-hedges

	using two years of data										using four years of data			
hedg-exp	day SWΔ	day SW%	day OLSΔ	day OLS%	week OLSΔ	week OLS%	2week OLSΔ	2week OLS%	mnth OLSΔ	mnth OLS%	2week OLSΔ	2week OLS%	mnth OLSΔ	mnth OLS%
NLG	1.138	0.990	1.145	0.994	1.171	1.004	1.248	1.056	1.348	1.108	1.341	1.018	1.430	1.084
BEF	1.109	1.032	1.115	1.028	1.136	1.069	1.195	1.096	1.199	1.095	1.184	1.080	1.156	1.061
DEM	1.290	1.075	1.279	1.065	1.306	1.084	1.327	1.089	1.432	1.147	1.471	1.061	1.570	1.136
FRF	1.000	0.852	1.077	0.925	0.925	0.852	1.000	0.925	1.008	0.921	1.077	0.925	1.077	0.925
DKK	0.865	0.927	0.924	0.970	0.939	1.000	0.955	1.024	1.001	1.053	0.893	0.893	0.949	1.038
CHF	1.141	1.030	1.124	1.026	1.343	1.016	1.124	1.017	1.167	1.007	1.241	1.036	1.304	1.071
ITL	1.032	1.010	1.038	1.024	1.055	1.038	1.063	1.061	1.063	1.065	1.004	1.020	0.998	1.010
GBP	1.100	0.996	1.117	1.028	1.130	0.030	1.306	1.166	1.273	1.079	1.623	1.388	1.687	1.407
JPY	1.162	1.034	1.124	1.034	1.199	1.034	1.218	1.088	1.387	1.167	1.275	1.069	1.374	1.124

Key to Table 3. The table is similar to Table 2, except that the monthly inflow now is *one* unit of the currency shown in column "expo(sure)".