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# Coase Theorem and Exchangeable Rights in Non-Cooperative Games

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## Abstract

*In this paper, we consider the Coase theorem in a non cooperative game framework. In particular, we explore the robustness of the Coase theorem with respect to the final distribution of alienable property rights which constitutes, as far as we know, a less cultivated field of research. In our framework, in order to reach efficiency, agents have to stipulate binding contracts. In the analysis, we distinguish between permanent and temporary contracts showing the different implication of the two kinds of contracts with respect to the final attribution of individual rights. More precisely, we show that, with temporary binding contracts and under particular assumptions, the final attribution of individual rights does not converge.*

Keywords: Coase theorem, binding contracts, side payments.

JEL classification codes: C7, D6.

## 1 Introduction

The Coase Theorem<sup>1</sup>, as it is well known, asserts that in absence of transaction costs, the final outcome of bargaining (i.e., the resource allocation among individuals) does not depend on the initial distribution of rights or liability rules<sup>2</sup>.

As noted about a decade ago by Usher (1998), efficiency may be achieved not only with a well defined assignment of initial property rights (or liability rules), but also in situations in which property rights are insecure and/or ill-defined.

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<sup>1</sup>See Coase (1960, 1988). It was Stigler (1966, p.113) the first scholar introducing the label "Coase Theorem".

<sup>2</sup>For a standard definition of the Coase theorem see, among others, Mueller (2003, p. 28) and Myles (1995, p. 319).

This aspect of the Coase theorem is full analyzed - among others - in Schmitz (2001), and more recently by Robson-Skaperdas (2008)<sup>3</sup>.

In this work, we explore the robustness of the Coase theorem with respect to the final distribution of property rights which constitutes, as far as we know, a less cultivated field of research.

The analysis is made in a non-cooperative game setting. Indeed, we follow an important branch of literature<sup>4</sup> according to which individual rights may be represented in a non-cooperative game. That is, we associate to each assignment of individual rights a set of permissible strategies that individuals can play. More precisely, in our framework, each assignment of property rights is associated to a non-cooperative game in pure strategies.

For the sake of simplicity, we assume the existence of two different right regimes to which correspond two different non-cooperative games<sup>5</sup>.

Under this *two-games* framework, we explore the possibility that individual rights (or liability rules) are alienable. In other terms, we consider situations in which the assignment of property rights is exchangeable, by assuming the possibility of switching from a certain game to another one.

We will see that in our framework, in order to reach efficiency, agents have to stipulate binding contracts. In the analysis we distinguish between permanent and temporary contracts showing (section 4) the difference of the two kinds of contracts with respect to the final attribution of individual rights. More precisely, we will show that, with temporary contracts and under particular assumptions, the right regime switch may be endless. Section 4 also shows the necessary conditions on individual preferences in order to obtain such result.

An other result of the paper (section 4) is that our findings are robust with respect to the introduction of side payments.

The rest of the paper is organized as follows: In section 2, we briefly describe the way in which rights are introduced in economic analysis. In section 3, we set the economic environment. In section 4, we present the results of the paper. Finally, section 5 concludes.

## 2 Rights in economic theory

As it is well known, individual rights play an important role in the Coase theorem. As far as we know, the first author who introduced rights in economics - at least in a formal way - was Amartya Sen with his famous liberal paradox (Sen, 1970). Sen's notion of rights was articulated by attributing to each individual the decisiveness over one pair of social states different among them for a feature concerning the individual himself. This idea is quite simple and intuitive: being these alternatives different only for personal characteristics which refers to the single individual, the individual himself is entitled to fix the social ranking of

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<sup>3</sup>See also Schiff (1995) for an informal definition of uncertain property rights.

<sup>4</sup>See section 2.

<sup>5</sup>For example, while in the first game agent  $i$  has the right to pollute, in the other game the other agent ( $-i$ ) has the right to avoid pollution.

these two alternatives. This first approach was based on the existence of some social choice rule which was asked to satisfy individual rights, the Pareto principle and an universal domain assumption on individual preferences. Rights are a sort of constraint that the social choice rule should satisfy. Sen (1970) shows that such social choice rule does not exist. Always by referring to a social choice rule, a somewhat more articulated notion of rights was subsequently introduced by Gibbard in an influential paper (Gibbard, 1974)<sup>6</sup>.

Another, and more recent, stream of literature interprets rights in terms of game forms. The basic idea, in this case, is that rights attribute to the individual the entitlement of choosing within a set of permissible strategies. Once rights are established, and consequently the sets of individual admissible strategies, agents exercise their rights by playing the non-cooperative game in pure strategies<sup>7</sup>. In this paper, in order to analyze the Coase theorem, we will use the game form representation of rights<sup>8</sup>. We will assume that by choosing his strategy, the agent is able to fix a 'feature' of the social state and thus he is able to influence the final outcome<sup>9</sup>.

As it is well-known, in the Coase theorem an important ingredient is the initial assignment of rights or liability rules (e.g. the right to pollute, to make noise, etc.). In our work, to each initial assignment of rights it corresponds a non-cooperative game with different strategies. For example, to the right to pollute for a firm it corresponds a set of available strategies (say game *A*) that are different in the event the firm has no right to pollute (say game *B*). At the same time, the other firm damaged from pollution has different strategies according to the initial assignment of pollution right (game *A* and *B*).

Furthermore, we explore the possibility that rights are alienable. In other terms, we consider situations in which the assignment of individual rights (liability rules) are exchangeable. The agents may 'bargain' among them by switching from a certain assignment of individual rights to another, that is by switching from a game to another<sup>10</sup>.

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<sup>6</sup>This literature is partially surveyed, among others, by Sen himself (1976), Suzumura (1983) and Wriglesworth (1985).

<sup>7</sup>This is, roughly speaking, the essence of game form articulation of individual rights. See, among others Gaertner, Pattanaik and Suzumura (1992). Sen (1992) [contains] a rejoinder to the game form articulation of individual rights.

<sup>8</sup>In other terms, within a particular game, we assume that agents may exercise their rights by choosing a strategy among their set of available strategies.

<sup>9</sup>We remind the reader that this assumption is not generally true in the realm of game form articulation of individual rights. A different notion refers to concept of 'effectivity function' (see, among others, Peleg 1998).

<sup>10</sup>Since we analyze the Coase theorem, a standard assumption we make is that efficiency is the implicit goal of individuals. If, on the contrary, rights concern purely private matters, such as in Sen's Liberal Paradox, then it is questionable that social efficiency represents still a desideratum. See, e.g., Sen (1983, 1995); Suzumura (1996).

### 3 The economic environment

Let  $N = \{1, 2\}$  be the set of economic actors<sup>11</sup> with generic element  $i = 1, 2$ . Each actor has a set of strategies that depend on the liability or responsibility rules (*i.e.*, the right to emit pollution, to make noise, and so on). For simplicity we consider two different attributions of individual rights, that we label by  $h \in \{A, B\}$ . Each right attribution offers different possibilities of actions to economic agents: in particular, we suppose that it is possible to associate to each right attribution rule a set of strategies to the individuals. We associate to the generic right regime  $h$  the set of strategies  $\Sigma^h = \{\Sigma_1^h, \Sigma_2^h\}$ , where, for simplicity,  $\Sigma_i^h = \{\sigma_{i1}^h, \sigma_{i2}^h\}$ .

Given the initial attribution of rights (and thus of strategies), agents play a non-cooperative game in pure strategies.

Furthermore, we introduce a payoff function  $u$  which maps the combination of individual strategies  $\Sigma_1^h \times \Sigma_2^h$  into the set of individual payoffs

$$u : \Sigma_1^h \times \Sigma_2^h \rightarrow \Pi = \{\pi_1, \pi_2\}$$

The payoff of each individual is therefore a function of the emerging couple of strategy in the game,

$$\pi_i = \pi_i(\sigma_{is}^h, \sigma_{-it}^h)$$

where  $s$  is the generic strategy played by agent  $i$  in game (right regime)  $h$ , while  $t$  is the generic strategy played by the other agent, say  $-i$ , in the same game.

In our framework, a *bargain* is defined as a decision among individuals which "exchanges" (transfers) their rights or liability rules. This decision will affect the individual set of strategies (the environment) and therefore their way to operate.

In our setting, an exchange of individual rights is a "switch" from right regime  $A$  (resp.  $B$ ) to  $B$  (resp.  $A$ ). In the following we will call the result of the bargain process a *switch right regime* (SRR).

We have now the basic ingredients that we will use to show our results.

### 4 Results

The results we prove in this section are based on six assumptions and two definitions that we present and discuss:

- **Assumption 1.** Given a right attribution, agents have the possibility to switch to the other one.
- **Assumption 2.** There are no transaction costs.
- **Assumption 3.** There are no side payments.

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<sup>11</sup>Dixit and Olson (2000) show the non-validity of the Coase theorem when the number of actors is sufficiently high, because of free-riding behavior.

- **Assumption 4.** In absence of binding contracts, given a right attribution  $h \in \{A, B\}$ , each agent acts according to individual rationality, that is by choosing (whether exists) his/her dominant strategy.
- **Assumption 5.** The game may be played for an indefinite number of times. Agents discount future payoffs at rate  $\delta > 0$ .

Assumption 1 relies on the literature interpreting individual right as a non-cooperative game (see section 2). Accordingly, a change of individual rights implies a change of the game played.

Assumption 2 is standard in the Coase theorem literature. The issue whether the presence of transaction costs affects the robustness of Coase theorem was analyzed, among others, by Anderlini and Felli (2001, 2006).

Assumption 3 is made for the sake of simplicity. Nevertheless, in sub-section 4.2 we relax assumption 3 and analyze situations in which side payments are allowed.

Assumption 4 and 5 are quite standard in the literature.

About binding contracts<sup>12</sup>, we analyze two cases according to the duration of the contract. In particular, we distinguish between *permanent* and *temporary* binding contracts.

- **Definition 1.** With a *permanent binding contract* agents may switch right regime and are obliged to play forever the strategy they have stipulated.
- **Definition 2.** With a *temporary binding contract* agents may switch right regime and are obliged to play the strategy they have stipulated for a finite number of moves.
- **Assumption 6.** *Temporary binding contract* obliges agents only for a single move and can be stipulated more than one time.

As far as we know, in the literature, *temporary binding contract* is a less cultivated field: The issue of contract duration has been mainly analyzed, in contract theory, with reference to *relationship-specific investment* (Joskow, 1987).

Regarding the robustness of the Coase theorem, in this paper when we say that it does not hold, we refer to the fact that agents are no able to attain the most efficient outcome.

We are now ready to state the main result of the paper.

### Proposition 1

Given  $N = \{1, 2\}$  and given two different attributions of individual rights,  $A$  and  $B$ , there exists a payoff function  $u^* : \Sigma_1^h \times \Sigma_2^h \rightarrow \Pi^* = \{\pi_1^*, \pi_2^*\}$ , such that:

- (i). If there are no *binding contracts* the Coase theorem does not hold.
- (ii). If there are *permanent binding contracts* the Coase theorem holds.

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<sup>12</sup>In the cooperative games framework, the role of binding contracts is analyzed by Bernholz (1997, 1999).

(iii). If there are *temporary binding contracts* the Coase theorem holds. But there is no convergent final allocation of property rights in the sense that agents have an incentive to continuously switch from one right attribution to the other.

**Proof:**

Suppose the payoffs the players receive in  $A$  and  $B$  are the following:

$A$	$\sigma_{21}^A$	$\sigma_{22}^A$
$\sigma_{11}^A$	(4, 2)	(5, 1)
$\sigma_{12}^A$	(1, 8)	(3, 7)

Table 1: Matrix payoffs in right regime  $A$ .

$B$	$\sigma_{21}^B$	$\sigma_{22}^B$
$\sigma_{11}^B$	(2, 4)	(8, 1)
$\sigma_{12}^B$	(1, 5)	(7, 3)

Table 2: Matrix payoffs in right regime  $B$ .

It can be easily checked that in  $A$  agent 1's dominant strategy is  $\sigma_{11}^A$  and that agent 2's dominant strategy is  $\sigma_{21}^A$  and that in  $B$ , 1's dominant strategy is  $\sigma_{11}^B$  and that 2's dominant strategy is  $\sigma_{21}^B$ . Therefore, in game  $A$  there is one Nash equilibrium in pure strategies  $(\sigma_{11}^A, \sigma_{21}^A)$  corresponding to the payoff (4, 2); in the symmetric game  $B$  there is one Nash equilibrium in pure strategies  $(\sigma_{11}^B, \sigma_{21}^B)$  corresponding to the payoff (2, 4).

Let's start, without loss of generality, with the regime  $A$ . Now, it can be seen that there is a combination of strategies in  $B$ ,  $(\sigma_{12}^B, \sigma_{22}^B)$  with an associated payoff higher for everyone (7, 3).

According to Assumption 1, agents may switch to game  $B$  without any transaction cost (Assumption 2). However, if there are no binding contracts, given Assumption 4 agent 1 knows that after the switch, agent 2 will play his/her dominant strategy  $\sigma_{21}^B$ . Therefore, 1's payoff will be 2 which is lower than his/her actual payoff (4). Further, given Assumption 3, agents are not able to reach the outcome (3,7), since there are no side payments. This prove part (i) of Proposition 1.

If there are *permanent* binding contracts, according to Assumption 1, agents may switch to game  $B$  without any transaction cost (Assumption 2). Further, thanks to binding contracts they are able to oblige themselves in playing forever the strategies  $(\sigma_{12}^B, \sigma_{22}^B)$  which assure a Pareto improvement. This prove part (ii) of Proposition 1.

If there are *temporary* binding contracts, by Assumptions 1 and 2, agents may switch to game  $B$  without any transaction cost. If they stipulate a *temporary* binding contract they will earn initially a payoff (7, 3). Then, after this single move, by Assumption 4, they will end up in the Nash equilibrium of game  $B$  with a new payoff (2,4). However this situation is harmful for agent 1 who



would lose 5. Therefore in order to reduce such loss, agent 1 will propose a new *temporary* binding contract in order to switch in game  $A$  in  $(\sigma_{12}^A, \sigma_{22}^A)$  which assures to each player a higher payoff (7, 3) with respect to the Nash equilibrium in  $B$ . However we note that  $(\sigma_{12}^A, \sigma_{22}^A)$  is not a Nash equilibrium and, therefore, according to our previous reasoning they will switch to  $(\sigma_{12}^B, \sigma_{22}^B)$ , and so on. We note also that is rational for each agent to prefer this “continuous switch” to the initial Nash equilibrium. Indeed, without stipulating contracts, agent 1 would earn the following discounted payoff

$$4 + \frac{4}{1 + \delta} + \frac{4}{(1 + \delta)^2} + \frac{4}{(1 + \delta)^3} + \dots$$

which is lower (for every  $\delta > 0$ ) than the discounted payoff agent 1 would earn with *temporary* binding contracts:

$$7 + \frac{3}{1 + \delta} + \frac{7}{(1 + \delta)^2} + \frac{3}{(1 + \delta)^3} + \dots$$

It can be easily checked that the same holds also for agent 2. This proves the second part of (iii).

In order to prove the first part of (iii), i.e. the fact that the Coase theorem holds with *temporary* binding contracts, we note that with *temporary* binding contracts agents will shift “continuously” from a payoff (3, 7) to a payoff (7, 3) which are, both of them, efficient payoffs. ||

## 4.1 Individual preferences

Probably the first reaction to Proposition 1 is to say that it is not generally true: more precisely, with *temporary* binding contracts we have a ‘continuous’ switch of property rights because of the particular values of individual payoffs. This consideration is obviously correct: our result is not generally true but holds only in some circumstances. In particular we showed in the proof of the proposition that the “trick” by which we obtained this switch depends on the fact that there is an outcome in game  $A$  such that everyone prefers it to the Nash equilibrium of game  $B$  and, symmetrically, there exists an outcome in  $B$  such that every agent prefers it to the Nash equilibrium in game  $A$ . We can show that a necessary condition on individual preferences in order to have a no stable allocation of final rights under temporary binding contracts is that there are at least *two externalities/spillovers one for each game and one for each agent*. In other terms, it has to exist an agent  $i$  having an externality/spillover in right regime  $A$  and the other agent ( $-i$ ) having an externality/spillover in right regime  $B$ .

In order to show this result, we state the following definition:

**Definition 3.** We say that an agent  $i$  has no externalities/spillovers in game  $h$  if

$$\pi_i(\sigma_{ir}^h, \sigma_{-it}^h) = \pi_i(\sigma_{ir}^h, \sigma_{-iv}^h) \quad \forall \sigma_{-it}^h, \sigma_{-iv}^h \in \Sigma_{-i}^h, \sigma_{-it}^h \neq \sigma_{-iv}^h; \quad \forall \sigma_{ir}^h \in \Sigma_i^h$$

This implies that for agent  $i$  there are no externalities if his payoff is not altered by the decision of the other agent, and therefore,  $i$  may neglect the decisions of the other agent.

**Proposition 2**

Given  $N = \{1, 2\}$  and given two different attributions of individual rights,  $A$  and  $B$ , under assumptions A1-A6, a necessary condition in order to have a no stable allocation of final rights in presence of temporary binding contracts is that player  $i$  has an externality/spillover in game  $A$  and player  $(-i)$  has an externality/spillover in game  $B$ .

**Proof:**

For notational convenience, the Nash equilibrium in game  $h$  will be denoted by  $NE_h$ . In order to have a no convergent allocation of final rights, it is necessary that there is a combination of strategies in game  $A$ , that we denote  $(a)$  with a little abuse of notation<sup>13</sup>, such that:

$$\pi_l(a) > \pi_l(NE_B) \quad \forall l \in N \tag{1}$$

and a combination of strategies in  $B$ , say  $(b)$ , such that:

$$\pi_l(b) > \pi_l(NE_A) \quad \forall l \in N \tag{2}$$

Now assume, by contradiction, that  $i$  has no externalities. The absence of externalities for  $i$  implies that there is no combination of strategies which assures to him a higher payoff than the Nash equilibrium, i.e.  $\pi_i(NE_B) \geq \pi_i(b)$  and  $\pi_i(NE_A) \geq \pi_i(a)$ . Combining with equation (2), it results:  $\pi_i(NE_B) \geq \pi_i(b) > \pi_i(NE_A) \geq \pi_i(a)$  which contradicts equation (1). Therefore  $i$  has at least one externality (the same arguments also holds for  $-i$ ).

In order to show that both externalities have to lie in different games, suppose, by contradiction, that they both are in the same game, without loss of generality, game  $B$ . Now, if in game  $A$  there are no externalities, then we have  $\pi_l(NE_A) \geq \pi_l(a) \quad \forall l \in N$ , which implies, by equations (1) and (2),  $\pi_l(b) > \pi_l(NE_A) \geq \pi_l(a) > \pi_l(NE_B) \quad \forall l \in N$ . However in this case we haven't got an endless switch, since individuals will always have interest to choose  $b$ . Indeed, assume that for some reason they are in game  $B$ . They have no incentive to exchange their rights and to switch to game  $A$ , since  $b$  is the best result for all. On the other hand, if they are in game  $A$ , they will have an incentive to switch in  $B$  and, through a binding contract, to choose  $b$ . ||

In order to have a better understanding of Proposition 2, we present the following numerical example.

**Example 1.** Let's consider the following payoffs in Game  $A$  and  $B$ .

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<sup>13</sup>In other terms, in order to simplify the notation in the proof, we replace  $\pi_i(\sigma_{is}^h, \sigma_{-it}^h)$  with  $\pi_i(a)$ .

$A$	$\sigma_{21}^A$	$\sigma_{22}^A$
$\sigma_{11}^A$	(4, 2)	(4, 1)
$\sigma_{12}^A$	(3, 8)	(3, 2)

Table 3: Matrix payoffs in right regime  $A$ .

$B$	$\sigma_{21}^B$	$\sigma_{22}^B$
$\sigma_{11}^B$	(2, 4)	(8, 3)
$\sigma_{12}^B$	(1, 4)	(2, 3)

Table 4: Matrix payoffs in right regime  $B$ .

It can be easily checked that game  $A$  exhibits a Nash equilibrium with an associated payoff (4, 2) that game  $B$  exhibits a (pure strategies) Nash equilibrium with an associated payoff (2, 4), that there an outcome in  $A$  such that everyone prefers it to the Nash equilibrium in  $B$  (and symmetrically in  $B$ ), and that agent 1 in game  $A$  has not any externality and so agent 2 in game  $B$ .

## 4.2 Side payments

In this section, we relax Assumption 3 and we analyze situations in which side payments are allowed. One could argue that the introduction of side payments may be sufficient to avoid the “non-convergence” of final rights. In other words, it could be argued that with side payments, individuals would not have the necessity to switch from a right regime to another, but simply may find a proper amount of payoff transfers such that the efficient outcome (which is not a Nash equilibrium of the game without side payments) may become the new Nash equilibrium of the game. In this section, we show that the introduction of side payments cannot be considered a satisfactory solution to the problem of attribution of final rights. In other terms, in what follows Assumption 3\* replaces Assumption 3.

- **Assumption 3\***. Side-payments are allowed.

Given Assumption 3\*, we may state the following result.

**Proposition 3** Given  $N = \{1, 2\}$ , and two different attributions of individual rights,  $A$  and  $B$ , and given Assumptions 2 and 3\* then there exists a payoff function  $u^* : \Sigma_1^h \times \Sigma_2^h \rightarrow \Pi^* = \{\pi_1^*, \pi_2^*\}$ , such that the Coase theorem does not hold, i.e. individuals are not able to reach the efficient outcome.

**Proof:**

Suppose the payoffs the players receive in  $A$  and  $B$  are the following:

In order to show the result and to avoid the introduction of further notation, it is sufficient, for our purposes, to follow strictly Jackson-Wilkie (2005, example

$A$	$\sigma_{21}^A$	$\sigma_{22}^A$
$\sigma_{11}^A$	(3, 7)	(0, 8)
$\sigma_{12}^A$	(8, 1.5)	(4, 2)

Table 5: Matrix payoffs in right regime  $A$ .

$B$	$\sigma_{21}^B$	$\sigma_{22}^B$
$\sigma_{11}^B$	(7, 3)	(1.5, 8)
$\sigma_{12}^B$	(8, 0)	(2, 4)

Table 6: Matrix payoffs in right regime  $B$ .

1)<sup>14</sup>. Let's consider, without loss of generality, game  $A$  (same argumentation holds for game  $B$ ): it can be easily checked that game  $A$  has a Nash equilibrium in pure strategies with the associated payoff (4, 2). In order to reach the outcome with the greater total payoff (3, 7), agent 2 has the incentive to offer a payment of  $5 + \varepsilon$  to agent 1 contingent on the agent 1 playing  $\sigma_{11}^A$ . Now the situation becomes:

$A$	$\sigma_{21}^A$	$\sigma_{22}^A$
$\sigma_{11}^A$	( $8 + \varepsilon$ , $2 - \varepsilon$ )	( $5 + \varepsilon$ , $3 - \varepsilon$ )
$\sigma_{12}^A$	(8, 1.5)	(4, 2)

Table 7: Matrix payoffs in right regime  $A$ .

At this point, in order to avoid that agents 2 deviates, agent 1 has the interest to offer any payment of at least  $1 + \varepsilon$  to agent 2 contingent on the agent 2 playing  $\sigma_{21}^A$ . The matrix payoffs now becomes:

Now, it can be easily seen that  $(\sigma_{11}^A, \sigma_{21}^A)$  is the new Nash equilibrium in pure strategies and it also the efficient outcome.

However, we have to note that if agent 2 offers any payments of at least  $5 + \varepsilon$  to agent 1 contingent on the agent 1 playing  $\sigma_{11}^A$ , it is not interest of agent 1 to offer  $1 + \varepsilon$  to agent 2 contingent on the agent 2 playing  $\sigma_{21}^A$ . Agent 1 can do better. Indeed, suppose that agent 1 offers to agent 2 the amount  $0.5 + \varepsilon$  contingent on agent 2 plays  $\sigma_{21}^A$  and  $0.5$  contingent agent 1 himself plays  $\sigma_{11}^A$ . Therefore the new matrix payoffs becomes:

With these associated side payments, game  $A$  has a new Nash equilibrium in pure strategies  $(\sigma_{12}^A, \sigma_{21}^A)$  which assures to agent 1 a greater payoff, and which is not the most efficient outcome, since the total payoff is lower than  $(\sigma_{11}^A, \sigma_{21}^A)$ .||

The reason for which side payments cannot be considered a general solution to reach the efficient outcome is clearly identified by Jackson and Wilkie (2005, p. 546): "Players can use transfers to try to ensure that the other players internalize externalities. However, they can also use transfers to try to manipulate

<sup>14</sup>The interested readers may refer to their paper for a more general account on the question of side payments.

$A$	$\sigma_{21}^A$	$\sigma_{22}^A$
$\sigma_{11}^A$	$(7, 3)$	$(5+\varepsilon, 3-\varepsilon)$
$\sigma_{12}^A$	$(7-\varepsilon, 2.5+\varepsilon)$	$(4, 2)$

Table 8: Matrix payoffs in right regime  $A$ .

$A$	$\sigma_{21}^A$	$\sigma_{22}^A$
$\sigma_{11}^A$	$(7, 3)$	$(4.5, 3.5)$
$\sigma_{12}^A$	$(7.5-\varepsilon, 2+\varepsilon)$	$(4, 2)$

Table 9: Matrix payoffs in right regime  $A$ .

other players' behaviour more generally. Sometimes, these objectives are at odds with each other, and then it is impossible to support efficient outcomes in equilibrium.”

## 5 Conclusions

In this paper, we analyze the robustness of the Coase theorem in a non-cooperative game framework, and in cases in which it is possible to exchange the conferment of individual rights (or liability rules).

We perform the analysis by using two games each one corresponding to a different right regime. Agents can bargain their rights at zero transaction costs. An important feature is that agents decide to switch from a right regime to the other if there is, in the other right regime, a social state which constitutes a Pareto improvement. Without binding contracts, agents behave according to individual rationality, *i.e.* they always choose their dominant strategies.

We show that, in presence of temporary contracts, with at least two agents, it is possible to have an endless bargain of individual rights. This result arises from the fact that it is possible to find an utility function such that, for every game, the Nash equilibrium is Pareto dominated by an outcome of the other game.

Another result of our work is that a necessary condition in order to have an endless bargain is the existence of an agent  $i$  with an externality in the first right regime, and the other agent ( $-i$ ) with an externality in the other right regime. In other terms, it is necessary that there are two externalities/spillovers, one for each game and one for each agent.

The last result we find is that, under conditions shown in the paper, the introduction of side payments cannot avoid the endless bargain.

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