Jena Economic Research Papers


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## by

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www.jenecon.de

ISSN 1864-7057

The Jena Economic Research Papers is a joint publication of the Friedrich Schiller University and the Max Planck Institute of Economics, Jena, Germany. For editorial correspondence please contact markus.pasche@uni-jena.de.

Impressum:

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# Interbank Lending and the Demand for Central Bank Loans - a Simple Microfoundation 

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Revised version: February 14, 2011


#### Abstract

The paper presents a simple model of banking behavior where portfolio, liquidity, and liability management determine simultaneously the demand and supply of borrowed reserves on the interbank market. As the central bank is one player in this market due to its refinancing policy, it is able to determine the interest rate and henceforth the residual demand for central bank loans. Comparative static analysis shows how external or monetary policy shocks affect the behavior on the interbank market, the volume as well as the structure of the bank's balance sheet. It turns out that the banking firm behavior is non-linear and partially non-monotonous, indicating that the transmission of monetary measures is more complex when endogeneous banking behavior is taken into account.


Keywords: banking firm, balance sheet, interbank market, borrowed reserves, central banking, liquidity, transmission

JEL Classification: E43, E58, G21

## 1 Introduction

The lending facilities on the market for reserves are one of the most important operating targets of monetary policy. The interbank (money) market is therefore an important hinge between central bank policy and the real sector, and the behavior of banks on these markets is crucial for understanding the transmission process of monetary impulses. The microeconomic literature about banking firm behavior is well developed and should

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not be reviewed again (see e.g. Santomero (1984), van Damme (1994), Swank (1996), Freixas and Rochet (1997)). However, in most macroeconomic models the behavior of banks on credit markets, asset markets and the market for borrowed reserves is often unsatisfactorily modelled, or even missing. This holds true especially for the interbank market because interbank relations seem to play no role in an aggregated macroeconomic view of the banking sector. The recent literature on interbank market models is therefore primarily basd on multi-agent simulation studies (cf. Georg and Poschmann (2010)).

In a standard Arrow/Debreu world there are neither informational frictions nor transaction costs and hence no need for any financial intermediaries. In Keynesian macroeconomics, credit, money, and the behavior of agents in money markets under fundamental uncertainty play an important role. However, in the neoclassical synthesis - the Keynesian textbook model - bank behavior plays no role anymore, and the financial sector is reduced to a static LM curve. Also, many modern New Keynesian macroeconomic models assume that the central bank directly determines "the" interest rate without considering the conduct of monetary policy to a limited pass-through caused by financial markets (for an overview cf. Woodford (2003), Walsh (2003)). But drawing back to Stiglitz and Weiss (1981) and the related literature on information asymmetries and rationing effects in financial markets, several macro models aim to cope with this issue (overview in Dimsdale (1994), Stiglitz (2003)). Their main focus is to analyse the implication of this market imperfection for credit financed investments in the real sector. They are of limited use, however, for the analysis of interbank markets.

Models which are not related to information asymmetries differ in their assumption on whether the bank is a price taker or whether it sets the loans interest rates in a monopolistic competitive credit market, motivated either by regional aspects, or by product differentiation. They also differ in the assumption about bank's risk attitude as well as about its decision tasks: setting optimal interest rates, determining an optimal asset portfolio, determining optimal reserve endowment to face unexpected liquiditiy outflows, etc. In the following, we briefly summarize some contributions in this field:
van Loo (1980) presents a risk neutral bank which maximizes profits by setting all interest rates for loans and deposits due to a static monopolistic Cournot calculus. Additional goals like soundness are then added as a constraint to the utility function. The interdependencies of setting the deposit interest rate on the liability side and the loans interest rate on the asset side are analysed. There are neither portfolio considerations nor activities determining the scale of the balance sheet or determining the reserve management.

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In the textbook by Bofinger (2001), a risk neutral bank maximizes profits as a price taker by scaling its balance sheet with borrowed reserves. The demand for borrowed reserves is determined by the equilibrium interest rate on the credit market as well as by the refinancing cost at the central bank. There are neither portfolio considerations nor do liquidity or soundness goals play any role. Since there is one representative bank, there is of course no interbank market. In Bernanke and Blinder (1988), bank behavior is described by a portfolio calculus, where loans and bonds are risky assets, while excess reserves are the riskless part of the portfolio. Banks are risk averse and act as price takers. The scale of the bank's portfolio is determined by central bank's reserve decisions. They study the impact of bank behavior on the endogenous money multiplier. There are no liquidity considerations and no interbank lending.

The model of Nautz (1998), based on Baltensperger (1980), focuses on the reserve management by including the cost of refinancing expected outflows in terms of penalty facility rates, e.g. marginal lending facilities of the central bank. He introduces uncertainty about the future marginal lending facilities in a 2-period model and shows that announcements of the central bank affect bank's behavior by influencing the degree of uncertainty. In Agénor and El Aynaoui (2010), banks set interest rates for loans and deposits according to markup rules reflecting their risk attitude. There is no portfolio calculus on the asset side, but an active dynamic reserve management motivated by the avoidance of liquidity shortings due to deposit withdrawls, or the avoidance of costly liquidity borrowing as in the model of Nautz (1998). Borrowing from the central bank is a residual in this model since all other items in the balance sheet are endogenously determined. A similar approach can be found in Totzek (2009). In the model of Hülsewig et al. (2009), banks act on monopolistic competitive markets, setting loans interest rate optimally according to Calvo's staggered pricing approach. There is no liability management, no risk management, no liquidity management, no activities on asset or bonds markets, and reserves are exogeneously given. The outcome is that the central bank's intereset rate setting results in dampened movements of the aggregate loan interest rate.

Also in Post Keynesian macroeconomics, several approaches can be found to include microfoundations of the banking firm in order to explain typical Keynesian features of the financial system like the interdependency of liquidity provision and credit creation. In the model of Dymski (1988), banks are risk-neutral and maximize profits by setting interest rates and choosing the volume of borrowed funds to refinance their activities. Since loans have a longer maturity than other assets and liabilities and since deposits
are volatile, the author distinguishes between a planning period and an adaption period, when (eventually unperceived) changes have taken place. This intertemporal relationship is neccessary to show that decisions regarding the liability side and the credit supply side are interdependent. To some extent this approach is close to the model presented in this paper. Our model, however, exhibits strong interdependencies between different bank management tasks also in the static "timeless" version.

Further overviews about banking behavior which could be cosidered for macroeconomic theoretical foundations can be found in Santomero (1984), van Damme (1994), Freixas and Rochet (1997), and Swank (1996). The present paper contributes to this literature by combining several aspects which drive the utility of the bank, and deriving its behavior regarding optimal portfolio decisions, optimal scaling of the balance sheet by demanding financial sources, and optimal reserve management to face deposit volatility. Therefore, it provides a rationale for the behavior on the credit market and on the market for other risky assets as well as on the interbank market for borrowed reserves which is strongly influenced by central bank policy measures. While the behavior on credit and other asset markets has been studied in Georg and Pasche (2008), the focus of the present paper is on the market for borrowed reserves. The recent financial crisis emphasized the importance of interbank loans which play an important role on both, the asset and the liability side of the bank's balance sheet. As an important feature of the crisis, we observed a sharp reduction in interbank lending and a drastic increase in holding excess reserves (cf. Keiser and McAndrews (2009), von Hagen (2009)).

The interconnection of banks via loans is also addressed in the literature on financial contagion (see Georg and Poschmann (2010)). Hence, a microfoundation of the supply and demand behavior on the market for borrowed reserves may also contribute to the contagion literature. To keep the analysis simple, we confine to a static approach with perfectly competitive markets and we neglect the problem of information asymmetries. In the following, we first describe the main management tasks on the asset and the liability side, emphasizing their interdependency (chapter 2). We then set up a model where decisions about asset structure, refinancing operations and reserve management are consistently derived by a calculus (chapter 3). It turns out, that the resulting behavioral functions exhibit some nonlinearities and partially non-monotonies which require a numerical analysis. Finally, we analyse the impact of different kinds of exogenous shocks as well as of monetary policy measures on the behavioral functions in a comparative-static way (chapter 4). Chapter 5 concludes.

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## 2 The bank's management tasks

It is too simple to represent the banking firm behavior with a profit maximization problem. The utility function of a (risk averse) banking firm should consist of different components like risk, return, soundness, and illiquidity avoidance, which reflect different management tasks. In our model, these tasks are represented as follows :

- Balancing risk and return: The optimal structure of the asset side of the bank's balance sheet is determined by a portfolio calculus according to a given degree of risk aversion. By adopting the portfolio approach, we have the problem that expected returns and risk are taken as given. However, depending on product differentiation and specializing on monitoring or screening technologies, banks may charge different interest rates for loans of differentiated types, and therefore act as monopolistic price setting firms. The decision problem then becomes more complex if one considers that the expected return of one asset in the portfolio calculus (loans) depends on the bank's own decision. We neglect this problem by assuming that loans are more or less perfect substitutes, and that we have one credit market interest rate in the model which renders banks as price takers.
- Liability management: The bank attracts financial resources in order to expand the asset side. This is called scaling the balance sheet or determining its optimal volume. Since we consider banks to be price takers, they cannot attract more deposits by raising the deposit interest rate, which is assumed to be given and set to zero for the sake of simplicity. Henceforth, it must take the deposit volume as given. A long run equilibrium level of the asset volume will be refinanced by long run refinancing instruments like bonds or equity capital. We do not model the calculus for issuing bonds or shares. Instead, we assume that long run refinancing instruments are on their optimal level, but short run fluctuations e.g. of deposits and reserves require a short run adaptation to the optimal portfolio volume by changing demand for loans on the interbank market.
- Liquidity management: Deposits and hence reserves are volatile. If unexpected withdrawals occur, the deposit outflows may exceed reserves which leads to an (il)liquidity problem. In this case the bank has to borrow short run reserves at high penalty interest rates, otherwise it could be subject to bankrun effects and finally insolvency. In the literature it is common to model this by balancing the costs of avoiding illiquidity with the opportunity costs of holding reserves instead
of a risky profitable portfolio. We follow this approach and integrate the liquidity consideration into the portfolio calculus.
- Solvency management: If losses from the risky portfolio exceed the bank's capital, the bank is insolvent. This risk is controlled by regulating the volume of risky assets compared to the bank capital. Shareholders are interested in a large return on their capital but also in a low risk of insolvency. Both goals are in conflict since an expansion of the portfolio which is financed by debt enhances both, the bank capital return due to the leverage effect, and also the risk of insolvency. Furthermore, the relation between risky assets and banking capital is also constrained by external regulations like Basel III. We integrate this solvency consideration into the calculus of the liability management.

Bank firms have different decision variables to address these management tasks. The difficulty is that these tasks are interdependent. Note, for example, that excess reserves have a double function in this framework: They are the riskless part of the portfolio (e.g. like in the approach of Bernanke and Blinder (1988)), and they are also held to avoid expected liquidity shortings (e.g. like in the approaches of Agénor and El Aynaoui (2010) or Nautz (1998)). The demand for borrowed reserves is also related to two management tasks. One aim is to keep the scale of the portfolio on its optimal level. Therefore, they are a refinancing instrument. But since deposits and reserves are volatile, the need to hold enough liquidity arises. The latter drives the money market demand. However, this is not in contrast to the implication of the liability management task to keep the optimal portfolio scale.

On the asset side, we will not consider loans to firms and households and bonds as specific risky assets. We subsume them under the term "risky investments" and assume that the internal structure of these investments is determined optimally (for details see Georg and Pasche (2008)). We are interested in the combination of these investments with risky interbank loans. The risk-return properties of the asset "provided interbank loans" will differ from other (non-bank) investments. Banks will typically have a lower default rate because they have access to short run standing facilities from the central bank. Furthermore, the banking sector is regulated to a larger extent than private firms. Thus, interbank loans could be considered as an asset with lower risk than other investments. Since these loans typically have a lower expected return than investments (otherwise investments would be a strictly dominated alternative), it is rational to mix both assets in a risky portfolio, and to mix the risky portfolio with risk-free excess reserves. At the
same time, this portfolio is refinanced (partially) with interbank loans on the liability side ${ }^{2}$. This is non-trivial since the ability to provide interbank loans may depend on the ability of the bank to refinance itself on the same market. The interdependency requires some additional consistency requirements for an equilibrium analysis. The main point is that the consistency of simultaneous supply and demand plans depends on the activity of the central bank. We show that by determining the interest rate for main refinancing operations, the central bank also determines the excess demand in the money market.

## 3 A Model of Interbank Market and Demand for Central Bank Loans

### 3.1 An Introductory Two-Bank Example

There are several reasons why bank A might borrow reserves to bank B. One reason might be that interbank lending helps compensating short-run liquidity needs or to faciliate window-dressing. Such activities would not explain the important quantitative role of interbank lending between European banks where interbank exposures are roughly about $20-30 \%$ of the balance sheet. Another reason might be that banks are to some extent specialized and do not hold identical asset portfolios. Interbank lending would enable one bank to participate in other's portfolios, i.e. to enhance diversification effects. Both reasons point imply that interbank lending fosters the efficient allocation of liquidity. The following model shows that this diversification effect might also take place in case of banks with the same asset structure as long as the characteristic of an interbank loan makes it attractive to mix it with other risky investments. As argued before, interbank loans are considered to have a lower risk than other investments.

Consider the case of two banks $A$ and $B$ with the following simplified balance sheets:

$$
\begin{aligned}
E+I & =(1-r) D+C \quad\left(=V_{A}=V_{B}=V\right) \\
\Rightarrow \quad\left(1-\lambda_{R}\right) V+\lambda_{R} V & =V
\end{aligned}
$$

with $E$ as excess reserves which are assumed to have zero risk and a low interest rate $\rho_{E}$, and $I$ as a risky portfolio with expected return $\rho_{I}$ and risk $\sigma_{I}^{2}$. We have deposits $D$

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where the required reserves $r D$ (with $r$ as the required reserve rate) are subtracted from both sides of the balance sheet because it is assumed that the bank is not free in their portfolio structure decision to hold $r D$. Furthermore we have $C$ as the "capital" of the bank, including net worth and different types of long-run refinancing instruments which are considered to be fixed in our analysis. We call $V$ the volume (or scale) of the portfolio. The analysis starts with no interbank loans. Then $\lambda_{R}$ is the optimally chosen share of the portfolio invested into risky assets $I$ due to the portfolio calculus.

Now consider that bank $A$ is able to provide an interbank loan $K$ to bank $B$. We denote the expected return from $K$ by $\rho_{K}$ and its risk is given by $\sigma_{K}^{2}$. Ceteris paribus, this implies a restructuring of the asset side:

$$
\begin{array}{r}
E_{A}+I_{A}+K=V_{A} \\
\left(1-\lambda_{R}^{A}\right) V_{A}+\lambda_{R}^{A} \lambda_{I}^{A} V_{A}+\lambda_{R}^{A}\left(1-\lambda_{I}^{A}\right) V_{A}=V_{A}
\end{array}
$$

The optimal structure is now given by an adjusted $\lambda_{R}^{A}$, while the risky part of the portfolio consists of the share $\lambda_{I}^{A}$ of non-bank assets, and $\left(1-\lambda_{I}^{A}\right)$ of interbank loans.

On the other side, bank $B$ has extended its portfolio volume by $K$ :

$$
\begin{aligned}
E_{B}+I_{B} & =V_{B}+K \\
\left(1-\lambda_{R}^{B}\right)\left(V_{B}+K\right)+\lambda_{R}^{B}\left(V_{B}+K\right) & =V_{B}+K
\end{aligned}
$$

where the optimal portfolio structure $\lambda_{R}^{B}$ is not affected. In the aggregated balance sheet of the banking sector, $K$ could be cancelled out as it is done in almost all macro models, but obviously $K$ has an impact on the single bank's behavior and should therefore be taken into consideration.

This interbank loan may be profitable for both banks if the ( $\rho_{K}, \sigma_{K}^{2}$ )-profile is not dominated by the $\left(\rho_{I}, \sigma_{I}^{2}\right)$-profile of the non-bank asset. As argued above, it could be assumed that due to regulation and standing facilities the probabiliy of debt failure is low relative to non-bank assets. Hence we have $\sigma_{K}^{2}<\sigma_{I}^{2}$. Then it is useful to diversificate risk by including $K$. But also for bank $B$ it might be profitable to demand for an interbank loan as long as the expected return of the portfolio exceeds the marginal refinancing cost $\rho_{K} \cdot{ }^{3}$ But due to the symmetry of both banks, the interbank loans contract could have been made just in the opposite direction: bank $B$ borrows reserves to bank $A$. Moreover, it is

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not only possible but also reasonable that we have simultaneously borrowing and lending activities. In the latter case it is convenient to study the behavior of a representative bank which holds $K$ as an asset and also extends its portfolio by interbank lending. Since we follow the traditional approach of modelling a representative agent, and since aggregated bank statistics shows these exposures on both sides of the balance sheet, we have to assume that the bank manages provided and demanded reserves at the same time.

### 3.2 Behavior of a Representative Bank on the Interbank Market

### 3.2.1 Preliminaries

As the example in the previous section shows, the bank decides about the optimal portfolio structure and the optimal portfolio volume. It is well-known that only utility functions with constant relative risk aversion (CRRA) allow to separate the decision about structure and volume, i.e. that the optimal portfolio shares $\lambda_{R}$ are independend from $V$. Unfortunately, CRRA functions lead to analytically extensive expressions for the optimal volume and have some non-intuitive properties which seem not to be empirically reasonable, i.e. the effect that the portfolio scale will be reduced instead of expanded in case of an increased attractiveness of the portfolio (see Georg and Pasche (2008)). We therefore assume that structural decisions on the asset side and volume decisions on the liability side are done by internally specialized departments of the bank. According to Krainer (2009), there exist theoretical arguments (based on agency theory) as well as empirical evidence which justify this assumption. He argues that structural decisions about the asset side are made by a risk-averse portfolio manager, while decisions about the portfolio scale are taken by a risk-neutral liability manager. We will follow this approach.

As discussed before, excess reserves have a double function as a riskless part of the portfolio and as a liquidity device to avoid or alleviate the problem of liquidity shortings in case of large deposit withdrawals. The latter motive, however, interferes with the portfolio decision which is based only on risk and return: The optimal share of $E$ from the portfolio calculus may be suboptimal for an optimal liquidity management, and vice versa. We solve this problem by a common assumption about the expected outflow of reserves which allows to formulate the costs of avoiding illiquidity per unit of the portfolio. The expected cost of liquidity shortage could then be integrated into the portfolio calculus.

### 3.2.2 Calculating the expected refinancing requirements due to deposit outflows

Deposits are a stochastic variable. Adopting the approach of Agénor and El Aynaoui (2010) we assume that with probability $\Phi$ there is a liquidity shock where a fraction $\delta$ of the deposits $D$ is withdrawn ${ }^{4}$. A withdrawal of deposits also reduces the required reserves $r D$, so there is an expected liquidity shortage if $\delta(1-r) D$ exceeds $E$. The expected value of refinancing requirements is therefore given by $\Phi(\delta(1-r) D-E)$. The expected cost of liquidity which has to be borrowed in the short run for marginal lending rates ("penalty rates") is given by

$$
\begin{equation*}
\rho_{p} \cdot(\delta(1-r) D-E) \Phi \tag{1}
\end{equation*}
$$

We follow Totzek (2009), among others, in that the penalty interest rate is a markup on the money market interest rate $\rho_{K}$ which is determined by the central bank: $\rho_{p}=(1+\zeta) \rho_{K}$. In case of the Euro system, the penalty rate is the marginal lending facility of the ECB. Equation (1) could also be expressed in terms of one portfolio unit by dividing by $V$ :

$$
\begin{equation*}
\rho_{p} \cdot\left(\delta(1-r) \lambda_{D}-\left(1-\lambda_{R}\right)\right) \Phi \tag{2}
\end{equation*}
$$

where $\lambda_{D}$ is the deposit fraction in the balance sheet (liability side), and $\left(1-\lambda_{R}\right)$ is the fraction of excess reserves in the balance sheet (asset side). This allows to easily integrate the costs of avoiding liquidity shortages into the portfolio calculus.

Two empirically relevant issues are not addressed in this model: (i) We assumed that required reserves must be held by the bank. In fact, the bank has to meet the reserve requirements only in the mean of a certain period. This is controlled at certain cutoff days which induces some demand peaks on the reserve market immediately before these days. We assume that these effects are negligible. (ii) The existence of deposit insurances makes it less probable that a bank falls into liquidity shortage, and they reduce the probability of bank runs (see Diamond and Dybvig (2000) on this issue).

### 3.2.3 Determining the optimal asset structure

The asset structure is determined by a risk-averse portfolio manager who follows a conventional portfolio calculus. The structure of the risky part of the portfolio is independent from the portfolio volume. Let $\rho_{I}, \sigma_{I}^{2}$ be the expected return and risk of investments $I$, and $\rho_{K}, \sigma_{K}^{2}$ the expected return and risk of interbank loans $K$. Furthermore, cov is the

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covariance between both returns. It is reasonable to assume that this covariance is positive: When banks face a negative shock on their asset $I$, the bank's ability to pay back the interbank loan will also be affected negatively. Furthermore, let $\rho_{E}$ be the return of the excess reserves as the riskless part of the portfolio. Expected return and variance of the risky portfolio is then given by

$$
\begin{align*}
\mu_{R} & =\lambda_{I} \rho_{I}+\left(1-\lambda_{I}\right) \rho_{K}  \tag{3}\\
\sigma_{R}^{2} & =\lambda_{I}^{2} \sigma_{I}^{2}+\left(1-\lambda_{I}\right)^{2} \sigma_{K}^{2}+\lambda_{I}\left(1-\lambda_{I}\right) c o v \tag{4}
\end{align*}
$$

The fraction $\lambda_{I}$ is determined in a way that the risky portfolio is the tangential point to the Capital Allocation Line (CAL):

$$
\mu_{P}=\rho_{E}+\underbrace{\left(\frac{\mu_{R}-\rho_{E}}{\sigma_{R}}\right)}_{b} \sigma_{P}
$$

where $\mu_{P}, \sigma_{P}$ are the expected return and the standard deviation of the total portfolio. Maximzing $b$ with respect to $\lambda_{I}$ gives the optimal risky portfolio structure:

$$
\begin{equation*}
\lambda_{I}=\frac{\left(\rho_{I}-\rho_{E}\right) \sigma_{K}^{2}-\left(\rho_{K}-\rho_{E}\right) \operatorname{cov}}{\left(\rho_{I}-\rho_{E}\right) \sigma_{K}^{2}+\left(\rho_{K}-\rho_{E}\right) \sigma_{I}^{2}-\left(\rho_{I}+\rho_{K}-2 \rho_{E}\right) \operatorname{cov}} \tag{5}
\end{equation*}
$$

To determine the optimal share of the excess reserves as the riskless part of the portfolio, we use a quadratic utility function in order to keep the analysis as simple as possible. We assume that excess reserve's interest rate $\rho_{E}$ is set by the central bank in the same manner as the marginal lending facility, but with a negative markup on the money market interest rate: $\rho_{E}=(1-\zeta) \rho_{K}$. The central bank is therefore able to determine $\rho_{K}$ (this is shown in the next section) and the spread $2 \zeta$ which determines the deposit and the marginal lending rate.

The standard portfolio calculus is now extended in order to account for the expected costs of liquidity shortings as expressed by (2). We assume that balancing risk and return with a given degree of risk aversion $\theta$ on the one hand, and avoiding the expected cost of liquidity shortages on the other hand are separable. We furthermore account only for the expected cost of liquidity shortages, not its variance in order to keep analysis simple. Then the extended calculus for one portfolio unit reads as

$$
\begin{aligned}
\max _{\lambda_{R}} u & =\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}-\frac{1}{2} \theta \lambda_{R}^{2} \sigma_{R}^{2}-\rho_{p}\left(\delta(1-r) \lambda_{D}-\left(1-\lambda_{R}\right)\right) \Phi \\
& =\underbrace{\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right)(1-\zeta) \rho_{K}}_{\text {return }} \underbrace{-\frac{1}{2} \theta \lambda_{R}^{2} \sigma_{R}^{2}}_{\text {risk }} \underbrace{-(1+\zeta) \rho_{K}\left(\delta(1-r) \lambda_{D}-\left(1-\lambda_{R}\right)\right) \Phi}_{\text {liquidity shorting }}
\end{aligned}
$$

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First order condition gives

$$
\begin{equation*}
\lambda_{R}=\frac{\mu_{R}-(1-\zeta) \rho_{K}-(1+\zeta) \rho_{K} \Phi}{\theta \sigma_{R}^{2}} \tag{6}
\end{equation*}
$$

The last term in the numerator makes the difference to the standard solution without liquidity considerations. Intuively, a higher penalty rate makes it more attractive to care for sufficient liquidity in expense of investing into a risky portfolio. But in general, the dependency of $\lambda_{R}$ on $\rho_{K}$ is ambigous, recalling that an increasing $\rho_{K}$ also increases $\mu_{R}$ and reduces $\sigma_{R}^{2}$ (via $\lambda_{I}$ ), cf. equations (3) and (4).

Now all structural decisions, based on risk, return, and liquidity are completely determined: $\lambda_{R} \lambda_{I}, \lambda_{R}\left(1-\lambda_{I}\right),\left(1-\lambda_{R}\right)$ are the shares of risky investments, interbank loans supply, and excess reserves, respectively. It has to be noted that all shares should be truncated to the $[0,1]$ interval.

### 3.2.4 Determining the optimal portfolio volume

The risk-neutral liability manager's task is to attract financial funds in order to scale the portfolio volume to its optimal level. Since we take deposits as well as long-run refinancing instruments like bank capital as given, the decision variable is the amount of interbank loans $K$. As mentioned before, we implicitly assume that long-run refinancing instruments have been adjusted to an optimal level, but short-run fluctuations of deposits and reserves require an adaptation of $K$. Furthermore, long-run refinancing instruments and $K$ are imperfect substitutes which justifies a certain mix of both instruments.

We assume that the utility function of the liability manager is simply the expected net profit from holding the portfolio: $\pi(K)=\mu_{R} \lambda_{R} V+\rho_{E}\left(1-\lambda_{R}\right) V-\rho_{K} K$. As long as $\mu_{R}>$ $\rho_{K}$ holds true, the liability manager would expand the portfolio volume and henceforth the $I / C$ ratio to infinity. Thus, the choice of an optimal $K$ is always a border solution due to external constraints like Basel III requirements for the $I / C$ ratio, or rationing effects on the interbank market. But empirically seen, many banks hold more capital than required by regulation, indicating an interior solution of the liability management problem. One of the main problems of a liability manager is that expanding the portfolio by debt and hence enhancing the $I / C$ ratio implies a trade off between increased returns to capital and the probability of bank distress (see Diamond and Rajan (2000) on this issue).

The definition of expected net profit given above does not include the case of insolvency. A bank is insolvent if the losses from the portfolio exceed the bank capital. Let $R$ be the

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realized (eventually negative) return rate from the risky portfolio, and $\mu_{R}=E[R]$. The probability of insolvency is then given by

$$
\begin{align*}
\Psi(K) & =\operatorname{Prob}\left\{R \lambda_{R} V+\rho_{K}\left(1-\lambda_{R}\right) V<-C\right\} \\
& =\operatorname{Prob}\left\{R<\frac{-C-\rho_{K}\left(1-\lambda_{R}\right) V}{\lambda_{R} V}\right\} \\
& =F\left(\frac{-C-\rho_{K}\left(1-\lambda_{R}\right) V}{\lambda_{R} V}\right) \tag{7}
\end{align*}
$$

with $F(\cdot)$ as the cumulative distribution function of $R$. We assume that in case of insolvency, shareholders lose their complete capital $C$. Including this insolvency probability, the expected net profit is now defined as

$$
\begin{equation*}
\pi(K)=(1-\Psi(K))\left[\mu_{R} \lambda_{R} V+\rho_{E}\left(1-\lambda_{R}\right) V-\rho_{K} K\right]-\Psi(K) C \tag{8}
\end{equation*}
$$

Enhancing the portfolio volume by additional loans $K$ will therefore increase both, the returns from the portfolio and the risk of insolvency:

$$
\begin{equation*}
\Psi^{\prime}(K)=\frac{d F}{d R} \cdot \frac{C}{\lambda_{R} V^{2}}>0 \tag{9}
\end{equation*}
$$

which implies a trade-off. In order to avoid too complicated analytical expressions, we make the convenient assumption that $\Psi(K)$ has a constant Arrow-Pratt measure by assuming

$$
\frac{1-\Psi(K)}{\Psi^{\prime}(K)}=\gamma \quad \Rightarrow \quad-\frac{\Psi^{\prime \prime}(K)}{\Psi^{\prime}(K)}=1 / \gamma
$$

The marginal impact of an additional portfolio unit on the insolvency probability is a linear decreasing function of the insolvency probability. Inserting $V=(1-r) D+K+C$ into (8), the liability manager maximizes $\pi$ with respect to $K$. First order condition gives (see appendix)

$$
\begin{equation*}
K^{d}=\max \left\{\gamma-\frac{\left(\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}\right)((1-r) D+C)+C}{\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}-\rho_{K}}, 0\right\} \tag{10}
\end{equation*}
$$

with $\lambda_{R}$ according to (6). As we will see later, these demand plans are always realized when the central bank serves the excess demand on the money market. Since $\rho_{K}$ also affects $\mu_{R}, \sigma_{R}^{2}, \lambda_{R}$, the dependency of $K^{d}$ on $\rho_{K}$ is non-linear and non-monotonous (depending on the parametrization): An increasing $\rho_{K}$ makes the risky portfolio more attractive and induces an expansion, while the refinancing of this expansion becomes more expensive. Moreover, the penalty rate as well as the deposit rate increase, so that the portfolio is restructured in favor or in expense of reserves.

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With the optimal structural variables $\lambda_{I}$ and $\lambda_{R}$, and the refinancing demand volume $K^{d}$ (and henceforth $V$ ), all behavioral functions in this model are determined. Suppressing the extensive analytical expressions, we have:

$$
\begin{aligned}
I\left(\rho_{K}, \cdot\right) & =\lambda_{R} \lambda_{I}\left((1-r) D+C+K^{d}\right) \\
K^{s}\left(\rho_{K}, \cdot\right) & =\lambda_{R}\left(1-\lambda_{I}\right)\left((1-r) D+C+K^{d}\right) \\
E\left(\rho_{K}, \cdot\right) & =\left(1-\lambda_{R}\right)\left((1-r) D+C+K^{d}\right)
\end{aligned}
$$

where $K^{s}$ are the interbank loans held as an asset, i.e. the (private) supply side of the interbank market. Note, that the behavior which drives investment (e.g. loans to firms and households, bonds), the supply of interbank credits, and the holding of reserves also depend non-linearly and eventually non-monotonously on $\rho_{K}$, although the underlying calculus is quite simple. As mentioned before, this results from the complex interdependencies of portfolio, liquidity, and liability considerations. Since the structural variables $\lambda_{R}, \lambda_{I}$ are truncated to the $[0,1]$ interval, it is also likely that the behavioral functions may have kinks. The effect of $\rho_{K}$, as the central bank's operating target, on $I, K, E$ depends on the parametrization, i.e. the underlying data generating economic process.

### 3.2.5 Consistency Condition and Derivation of Central Bank Loans Demand

The motives to provide $K^{s}$ and to demand $K^{d}$ are different, and for a single bank supply and demand for interbank loans will differ for a given $\rho_{K}$. The consistency of plans, however, is a matter of the aggregated sector. In a closed bank sector there is an equilibrium when $K^{d}\left(\rho_{K}\right)=K^{s}\left(\rho_{K}\right)$. For $\rho_{K}=0$ it is unattractive to hold $K$ as an asset but more attractive to expand the portfolio, so that $K^{d}(0)>K^{s}(0)$. If $\rho_{K}$ increases to a critical value where it becomes completely unattractive to refinance the portfolio by interbank loans, we have $K^{s}>K^{d}=0$. From this fact and the continuity of $K^{d}, K^{s}$ in $\rho_{K}$, we can conclude that there is (at least) one $\rho_{K}^{*}$ which equilibrates supply and demand: $K^{s}\left(\rho_{K}^{*}\right)=K^{d}\left(\rho_{K}^{*}\right)$. The interbank loans market is, however, a financial market which should be characterized by information asymmetries. Henceforth, it is reasonable to assume non-market clearing interest rates and rationing effects (see Freixas and Jorge (2008)). With the assumption that banks have more or less similar management policies and that they demand interbank loans primarly for short-run liquidity reasons and not to finance projects with private information about the project type, we regard these agency problems to be negligible.

Now consider that the central bank is an additional player on the interbank market by

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offering its main refinancing instruments. In this model, we regard such central bank loans as perfect substitutes to interbank loans. In reality, however, both loan types may differ due to collateral requirements, different maturities, and simply by the timing of tender procedures. Since we treat loans as perfect substitutes, there is only one interest rate $\rho_{K}$. The equilibrium condition is now modified to $K^{d}\left(\rho_{K}\right)=K^{s}\left(\rho_{K}\right)+K_{c}$ where $K_{c}$ is the central bank loans supply. The central bank is now able to set an interest rate $\rho_{K}$ which induces an excess (or residual) demand $K^{d}\left(\rho_{K}\right)-K^{s}\left(\rho_{K}\right)=K_{c}>0$. It is assumed that this excess demand is satisfied by central bank loans, i.e. there is full allotment and no rationing effects. We obtain a continuum of ( $K_{c}, \rho_{K}$ ) combinations which represent an equilibrium:

$$
\begin{equation*}
K_{c}\left(\rho_{K}\right)=K^{d}\left(\rho_{K}\right)-K^{s}\left(\rho_{K}\right) \tag{11}
\end{equation*}
$$

This allows the central bank to set the interest rate $\rho_{K}$ as an operating target and therefore to induce a residual demand for borrowed reserves $K_{c}\left(\rho_{K}\right)$. The latter is hence microfounded by portfolio, liquidity and liability management decisions of a representative bank. If the central bank follows an interest rate rule, it must respond to shifts in $K^{d}$ and $K^{s}$ by accomodating the excess demand for borrowed reserves. The money base is therefore endogenously determined (cf. Georg and Pasche (2008)).

The previously described way of supplying the banking sector with borrowed liquidity is more close to the practice of the ECB rather than the Fed. If there is an excess demand for reserves and the central bank aims to keep the money market interest rate on the current level, it is also possible to supply liquidity by purchasing bonds which are a part of $I$ in our model. In this case we have a structural change of the asset side in favor of reserves $(-\Delta I=+\Delta E)$, but no extension of the balance sheet's scale. This way of expanding the monetary base by purchasing bonds is more typical for the practice of the Fed. Purchasing bonds and paying with reserves immediately affects the reserve demand $K^{d}$ on the money market so that the targeted interest rate equals the equilibrium rate. In equilibrium, this operation is only possible if the bank is willing to sell bonds to the Fed, i.e. to restructure its asset side. This implies higher bonds prices and a lower interest rate $\rho_{I}$. Since the purpose of this paper is an outline of a model of bank behavior especially on the interbank market and to derive the central bank loans demand, we will not study the implications of these different central bank policy strategies.

There are two further consistency conditions which must hold true in a closed model of the financial sector: (a) A bank will demand additional central bank loans $\Delta K_{c}$ in order to refinance its asset side. For a single bank there is no need that the additional loans are

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held only as additional liquidity reserves since e.g. they could be used to finance investments. For the aggregated banking sector, however, $\Delta K_{c}=\Delta E$ holds true as a matter of accounting record. This is because the central bank's balance sheet reads $S+K_{c}=r D+E$ (with $S$ as securities) and $D$ is assumed to be given. A model of the complete financial sector would require that equilibrium interest rates adjust so that for the representative bank $\Delta K_{c}=\Delta E$ holds true in terms of planned values. (b) Observe, that $I$ includes loans to firms and households (besides bonds and other securities). Additional loans, however, create additional deposits by accounting record: $\Delta I=\Delta D$. These additional deposits are typically not held by the borrower at the bank which has provided the loan, but neccessarily at some bank in the banking sector. Therefore, a single bank's microeconomic calculus does not care about these both consistency conditions, but they are neccessary when plugging the model of bank behavior into a macro model of the aggregated financial sector.

### 3.2.6 The transmission of $\rho_{K}$ to $\rho_{I}$

Up to now, our model consists of several exogenously given parameters, i.e. the return of risky investments $\rho_{I}$. These investments are mainly loans to non-banks like firms and households, and bonds. A change of the refinancing conditions $\rho_{K}$ affects the desired supply of financial ressources $I\left(\rho_{I}, \rho_{K}, \cdot\right)$. We will now replace the assumption of an exogenously given $\rho_{I}$. We have to consider that a shift of $I\left(\rho_{I}, \rho_{K}, \cdot\right)$, together with a given downward sloping demand curve for financing investments $I^{d}\left(\rho_{I}, \cdot\right)$ will induce a shift of the equilibrium interest rate $\rho_{I}^{*}$ which is now an endogeneous variable. We consider $\rho_{I}^{*}$ as the market clearing interest rate, neglecting rationing effects which are typical, however, for financial makets with information asymmetries. Since this is the interest rate which explains the level of investments, it is the most important link between the monetary and the real sphere, and henceforth of the transmission process of monetary policy. To account for this effect, we assume a simple linear decreasing demand function

$$
I^{d}\left(\rho_{I}\right)=a-b \rho_{I}
$$

With the condition $I\left(\rho_{I}^{*}, \rho_{K}\right)=\lambda_{R} \lambda_{I}\left((1-r) D+C+K^{d}\right)=a-b \rho_{I}^{*}=I^{d}\left(\rho_{I}^{*}\right)$ we have the equilibrium interest rate $\rho_{I}^{*}\left(\rho_{K}\right)$ as a positive implicit function of $\rho_{K}$. Since the analytical expression of $I\left(\rho_{I}, \rho_{K}\right)$ is very complicated, figure 1 shows a graphical derivation of $\rho_{I}^{*}\left(\rho_{K}\right)$ (parameter values are given in the next section). A given value of $\rho_{K}$ parametrizes $I\left(\rho_{I}, \rho_{K}\right)$ in the upper left graphic which gives - together with the demand $I^{d}$ - the equilibrium interest rate $\rho_{I}^{*}$. The latter parametrizes the corresponding investment function in

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the upper right graphic. Hence, we have constructed one pair of interest rates $\left(\rho_{K}, \rho_{I}^{*}\left(\rho_{K}\right)\right)$ which is depicted in the lower right quadrant. It turns out that the relationship $\rho_{I}^{*}\left(\rho_{K}\right)$ could be approximated fairly good by a linear function. The lower left quadrant is simply the $45^{\circ}$-line.


Figure 1: The impact of $\rho_{K}$ on $\rho_{I}$ via the market for $I$

### 3.3 A Numerical Example

Even if the basic calculus of the banking firm is simple, the resulting optimal structural and volume decisions are not. The resulting functions could be analytically derived but are too complicated to conduct an analytical comparative-static analysis. For a numerical example, we consider $a=2000, b=15000$ for the linear investment demand. The other parameters are given by $\sigma_{I}=0.12, \sigma_{K}=0.08, \operatorname{cov}=0, \zeta=0.5, \Phi=0.05, \delta=0.1, r=$ $0.02, \gamma=3000$. The risk aversion parameter is $\theta=8$. On the liability side we have $D=800$ and $C=20$. It is also possible to choose $D$ and $C$ on a complete different scale, but then also $\gamma$ should be adapted to these values.

Figure 2 shows the dependency of $\mu_{R}, 1-\lambda_{I}, \lambda_{R}$, as well as the behavioral functions determining $I, E$ and $K^{s}, K^{d}$ on $\rho_{K}$. As long as we have $\sigma_{K}<\sigma_{I}$ and a zero covariance, an increase of $1-\lambda_{I}$ leads to a monotone decrease of $\sigma_{R}$ which is therefore not depicted. For each $\rho_{K}$ we have computed the equilibrium interest rate $\rho_{I}^{*}$ on the market for investment
funds, as depicted in the center quadrant ("transmission"). The demand for central bank loans $K_{c}$ is shown in the right mid figure. The volume and composition of the asset side is depicted on the lower right side. The regions betwween the three curves depict the excess reserves $E$, investments $I$, and interbank loans $K^{s}$ (from bottom to top). Note, that all graphs have no meaning for values $\rho_{K}>\rho_{K}^{*}$ because the central bank is able to set $\rho_{K}<\rho_{K}^{*}$ to induce a positive residual demand. Otherwise the interbank market would equilibrate without intervention of the central bank. For these values of $\rho_{K}$ the functions are depicted as dotted lines to indicate that they do not represent equilibrium behavior. Note that the comparative-static analysis does not include feedbacks from the other agents, i.e. households and firms.










Figure 2: Effects of $\rho_{K}$ on bank behavior

Starting with extreme low values, with increasing $\rho_{K}$ the interbank loans become more attractive relative to non-bank investments which has an unambigous effect on $1-\lambda_{I}$. The expected return from the risky portfolio $\mu_{R}$ will initially decrease because it is the variance reducing effect which makes interbank loans more attractive so that the bank is willing to restructure the portfolio in favor of $K$ although $K$ has a lower return than $I$. The same reasoning holds true for the increase of $\lambda_{R}$. The share of the risky portfolio will approach to 1 , implying that there is no systematic excess reserve holding anymore.

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While the demand $K^{d}$ is decreasing in $\rho_{K}$, the supply function $K^{s}$ will initially be sloped upwards due to the increase of the attractiveness of the risky portfolio. The attractiveness is also fosetered by an increasing share $\lambda_{R}$. But then the effect of a decreasing $K^{d}$ and henceforth $V$ overcompensates the former effect, and $K^{s}$ becomes downward sloping. As a result, we have a decreasing demand curve for central bank loans $K_{c}$. The lower right quadrant summarizes all effects: The total scale of the portfolio decreases with $\rho_{K}$ and we have a substantial change in its structure: With an increasing $\rho_{K}$ we have a shift from risk-free to risky assets, and a shift from investments to interbank loans.

## 4 Effects of exogenous shocks

In the following, we take the same parameter set as in figure 2 and change one variable to study the comparative-static effects on bank behavior. In all figures the solid lines represent the situation before the parameter change, and the dotted lines show the effect of the shock. For a proper interpretation it has to be underlined that all lines represent the equilibrium behavior of the bank only up to the equilibrium interest rate $\rho_{K}^{*}$ where $K^{s}$ and $K^{d}$ intersect (the upper right quadrant). On the right side of the equilibrium point $\left(\rho_{K}>\rho_{K}^{*}\right)$ all depicted lines could be computed, but they are meaningless since these notional plans could not be realized.

We first analyse the effect of a changing soundness of banking firms subjectively perceived by the banks. A lower soundness means that banks expect a higher probability of debt failure, expressed in a larger volatility of returns from interbank credits $\sigma_{K}^{2}$. If banks trust less to each other, the attractiveness of holding interbank loans as an asset will decline. As can seen in figure 3 , an increased $\sigma_{K}$ from 0.08 to 0.11 dampens the supply of loans $K^{s}$ significantly, but not the demand $K^{d}$ which implies a shift of the central bank loans demand $K_{c}$. Intuitively, the share $1-\lambda_{I}$ as well as the share $\lambda_{R}$ are reduced since interbank loans became less attractive. But both effects have a countervailing impact on the absolute supply of investment funds: The resulting investment supply is virtually unaffected by this shock. The reduction of $K^{s}$ corresponds to an increase of the risk-free excess reserves. Since the liability manager is assumed to be risk-neutral, the shock has no impact on the total portfolio scale but only on the portfolio structure as can been seen in the lower right quadrant. Also the transmission of $\rho_{K}$ to $\rho_{I}$ (center quadrant) remains unaffected.

Now consider a risk shock on the market for investments: The standard deviation $\sigma_{I}$


Figure 3: Effects of changing $\sigma_{K}$
increases from 0.12 to 0.14 . Figure 4 shows that we have reversed effects compred to the case of a $\sigma_{K}$ shock: There is no effect on interbank market activities. The reason is that the reduced attractiveness of investments reduces $\lambda_{R}$ and increases $1-\lambda_{I}$. The resulting effect on interbank loan supply is therefore zero. The total portfolio volume is unaffected by the shock, but we have a restructured portfolio in favor of excess reserves. Due to the reduced supply of financial funds $I$, we have larger equilibrium interest rates $\rho_{I}^{*}$ which implies a shift of the transmission relation in the center quadrant.

If we combine both negative shocks $\left(\sigma_{K}, \sigma_{I}\right)$ as they have been typical for the 20072009 financial crisis, and assuming an accompanying monetary policy which reduces $\rho_{K}$, the bank behavior in this model replicates the stylized facts: Interbank borrowing is significantly reduced, central bank borrowing is expanded, excess reserves are massively increased, and the effect on investments is small and ambigous since the reduced $\rho_{K}$ compensates the reduction of the supply $I$.

Now consider the case of a deposit volatility change (see figure 5). This could be represented by a higher probability of a liquidity shock $\Phi$. We assume an increase of $\Phi$ from 0.05 to 0.15 . Intuitively, we observe a slight reduction of $\lambda_{R}$ and an expansion of excess reserves at the expense of investments.


Figure 4: Effects of changing $\sigma_{I}$

Finally, we study the impact of a risk aversion decrease as it may take place in boom phases (see figure 6 ). We assume a shift of $\theta$ from 8 to 6 . As the decision about the portfolio scale depends on a risk-neutral expected profit function, the risk aversion parameter has no direct effect on $K^{d}$ and the portfolio volume. The very small changes are explained by indirect effects of an increased $\lambda_{R}$. The structural effects are straightforward: We observe an increase of $\lambda_{R}$ which results in reduction of excess reserves in favor of the risky assets $I$ and $K^{s}$. An almost unchanged $K^{d}$ and an increased $K^{s}$ implies a reduction of central bank loans.

The central bank is not only able to determine its operating target $\rho_{K}$. Independently from this target, it could also change the marginal lending and the deposit facility. In our model, this is done by adapting the spread $2 \zeta$ ( $\zeta$ moves from 0.5 to 0.7 ). Figure 7 shows the results. Since the penalty rate in case of liquidity shorting is larger, excess reserves become more attractive. On the other hand, the deposit facility rate is lower which makes excess reserve holding less attractive. In our numerical example the latter effect dominates. The portfolio is slightly restructured in favor of both risky assets.

Now consider a negative shock on the investment demand side: We have a shift of the demand function ( a changes from 3000 to 2700). The effects, as depicted in figure 8, are



$0.01 \quad 0.02 \quad 0.03 \quad 0.04 \quad 0.05 \quad 0.06$
$0.01 \quad 0.02 \quad 0.030 .040 .050 .06$







Figure 5: Effects of an increased deposit volatility
very similar to the case of an increase of $\sigma_{I}$. Investments become less attractive. Since we have also a reduced $\lambda_{R}$ and a reduced portfolio scale, the impact on $K^{d}$ and the excess reserves is ambigous: The risky part of the portfolio is slightly restructured in favor of $K^{s}$ but at the same time the portfolio volume is significantly reduced. The latter effect dominates so that $K^{s}$ decreases. The former effect dominates for the excess reserves so that we observe a very small increase of $E$. Since interbank loan demand is reduced stronger than interbank supply, the resulting central bank loans demand also decreases. Note that other parametrizations may lead to other net effects.

Since the risk-free reserves are nearly unaffected while the risky investments are reduced, the capital/investment ratio increases with a lower investment demand. This implies that the capital/investment ratio is countercyclical as it is empirically the case (Meh and Moran (2004)).

## 5 Concluding Remarks

The model derives the demand and supply behavior for interbank loans from a simple utility maximization calculus of a bank with portfolio, liquidity and liability management.










Figure 6: Effects of decreased risk aversion

The logic behind this behavior is that on the one hand banks hold interbank loans as an asset in an optimally structured portfolio, and on the other hand they refinance their portfolio by demanding borrowed reserves. Thus, the supply of loans depends on whether the refinancing demand is realized, which makes the behavior slightly complicated and implies a non-linear dependency on $\rho_{K}$ on both market sides. As the central bank also provides borrowed reserves, it is possible to determine the money market interest rate (as its operating target) and to induce a demand for central bank loans. The model allows to study the impact of monetary policy on the structure and the scale of the bank's portfolio. This is important because both issues determine the supply of financial ressouces to investors and therefore the interest rate on credit and bonds markets. The behavior of the bank is hence a complex hinge between monetary measures and the real part of the economy.

On the banking side, we have neglected several important issues which may be included into an extended analysis: (a) One important task of the bank is the management of different maturities of assets and liabilities. This would require a dynamic framework. Since investment contracts like loans have a long maturity, the interest rate could not be adapted immediately to a perceived shock. Hence, the calculus has to consider the


Figure 7: Effects of a decreased spread $2 \zeta$
possibility of future changes in variables as in the approaches of Hülsewig et al. (2009) on staggered interest rate setting or Nautz (1998) on intertemporal reserve management. (b) We have neglected information asymmetries and agency costs. A simple way to account for this issue would be to endogenize the risk as a positive function of the interest rate as in the standard literature based on Stiglitz and Weiss (1981). (c) We neglect some technical details of central bank lending and interbank lending: loans may require collaterals like specific types of bonds on the asset side, and the interest rate may be determined by the debt/equity ratio as a proxy for the individual risk of debt failure. This would impose additional constraints for both, the structural portfolio decisions and the decision about the portfolio scale. Furthermore, the central bank is able to cut the allotments of loans (rationing). (d) As pointed out in Pasche (2009), it is possible to account for fundamental uncertainty, which may play an important role especially in times of a financial crisis. Increasing fundamental uncertainty could shift the behavior in favor of holding liquidity. (e) There is no calculus determining the long-run refinancing instruments like equity capital or issued bonds which are an imperfect substitute to borrowed reserves $K^{d}$.

Although we employed a simple mechanism to determine the interest rate $\rho_{I}$ on the investment market by introducing a linear demand function and assuming market clearing,

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Figure 8: Effects of a reduced investment demand
the model is neither a complete model of financial markets, nor a full-fledged macroeconomic model. In a complete model, we would have several feedback mechanisms: While the interest rate $\rho_{I}$ and the allocated financial ressources $I$ have an impact on the real sector of the economy, i.e. on the income $y$, the real sector determines the demand for financial ressources $I^{d}$ as well as the demand for money, which is represented as deposits $D$. According to common macroeconomic theorizing, the money demand is endogenously determined by income and interest rates: $D\left(y, \rho_{I}\right)$. It has to be underlined that we do not argue on the basis of a mechanistic money multiplier. The relationship between the money base $r D+E$ (as the liability side of the central bank's balance sheet) and $D$ is determined by bank and non-bank behavior in a complex way (see e.g. Alves et al. (2008) for an attempt to study the multiplier process with actively managed interrelated banking firms).

We hope that this more rigorously microfounded and rich-structured model of banking behavior helps to improve macroeconomic models in order to understand the transmission process of monetary policy. As we have seen, the translation of $\rho_{K}$ to $\rho_{I}$ and $I$ is nontrivial due to non-linearities and non-monotonies, and it depends on several parameters. When designing an optimal monetary policy (rule) it has to be taken into account that

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the transmission channel is based on a relationship $\rho_{I}\left(\rho_{K}\right)$ which depends on changing behavior.

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## Appendix

First order condition of maximizing (8) gives

$$
\begin{aligned}
\frac{d \pi}{d K}= & -\Phi^{\prime}(K)\left[\left(\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}\right)((1-r) D+C+K)-\rho_{K} K+C\right] \\
& +(1-\Phi(K))\left[\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}-\rho_{K}\right]=0
\end{aligned}
$$

Using the assumption $(1-\Phi) / \Phi^{\prime}=\gamma$ gives

$$
\begin{aligned}
\Rightarrow \quad- & {\left[\left(\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}\right)((1-r) D+C+K)-\rho_{K} K+C\right] } \\
& +\gamma\left[\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}-\rho_{K}\right]=0
\end{aligned}
$$

Solving for K gives

$$
K^{d}=\gamma-\frac{\left(\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}\right)((1-r) D+C)+C}{\lambda_{R} \mu_{R}+\left(1-\lambda_{R}\right) \rho_{E}-\rho_{K}}
$$


[^0]:    ${ }^{1}$ markus.pasche@uni-jena.de. I thank Co-Pierre Georg, Sebastian Sienknecht, Jenny Poschmann and Severin Weingarten for helpful comments.

[^1]:    ${ }^{2}$ Of course this wouldn't make any sense in case of risk neutrality. Note that also holding deposits at another bank with a positive deposit interest rate has similar properties as providing a loan to the other bank.

[^2]:    ${ }^{3}$ To be more precise, the marginal cost are the interest rate $\rho_{K}$ payed for $K$ which is lower than the expected return $E\left[\rho_{K}\right]$ due to the small probability of debt failure. Because this probability is assumed to be small and since we should avoid additional notation, we take $\rho_{K}$ as a proxy for the expetcted return.

[^3]:    ${ }^{4}$ See Totzek (2009) for a more elaborated approach.

