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Deep Habits in the New Keynesian Phillips Curve*

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Abstract

We derive and estimate a New Keynesian Phillips curve (NKPC) in a model where consumers are assumed to have deep habits. Habits are deep in the sense that they apply to individual consumption goods instead of aggregate consumption. This alters the NKPC in a fundamental manner as it introduces expected and contemporaneous consumption growth as well as the expected marginal value of future demand as additional driving forces for inflation dynamics. We construct the driving process in the deep habits NKPC by using the model's optimality conditions to impute time series for unobservable variables. The resulting series is considerably more volatile than unit labor cost. GMM estimation of the NKPC shows an improved fit and a much lower degree of indexation than in the standard NKPC. Our analysis also reveals that the crucial parameters for the performance of the deep habit NKPC are the habit parameter and the substitution elasticity between differentiated products. The results are broadly robust to alternative specifications.

JEL Classification: E31; E32.

Key Words: Phillips curve; GMM; marginal costs; deep habits.

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1 Introduction

The New Keynesian Phillips curve (NKPC) is the center piece of modern macroeconomic models that are used for monetary policy analysis. It is derived from the optimal price-setting problem of a monopolistically competitive firm that operates in an environment where firms face downward-sloping demand curves. The NKPC, in contrast to earlier accelerationist Phillips curves, is explicitly forward-looking and imposes theoretical restrictions on the comovement of its components. Specifically, theory identifies marginal cost as the main driver of inflation dynamics. However, the NKPC faced the early criticism that marginal cost is not observable to the empirical researcher and that the stochastic properties of various proxies do not line up with the properties of the inflation process they claim to explain.

In a seminal contribution, Galí and Gertler (1999) show that the performance of the NKPC can be improved by introducing backward-looking price-setting or indexation to the effect that lagged inflation enters the structural specification. Moreover, they demonstrate that marginal cost is well proxied by unit labor cost under the assumption of perfectly competitive factor markets. Their results established a benchmark in the literature, namely that inflation dynamics are explained both by intrinsic factors, such as inflation indexation in price-setting, and by external driving forces, such as marginal cost movements. Much of the follow-up research confirmed their initial findings and established their robustness under the chosen modeling environment (for instance, Galí et al., 2005).

This paper follows in the steps of more recent research that modifies the environment in which firms operate. We introduce deep habits in the preferences of the consumer and derive the corresponding NKPC. Habit formation is deep in the sense that it extends to each individual good of the consumption bundle available to consumers, and not only to the consumption composite. This seemingly simple modification, however, has far-reaching consequences. It implies a downward-sloping demand function that depends on the lagged level of the consumer's purchases. Since firms take this demand function as a constraint in their optimal price-setting problem, the time dependence carries over to the NKPC and results in the introduction of future, current, and lagged consumption in this relationship. We thus show that deep habits fundamentally affect the analytical form and interpretation of the driving process and the interaction of marginal cost with inflation.

We estimate the NKPC under deep habits using generalized methods of moments (GMM) techniques. More specifically, our empirical approach is a mixture of calibration and structural estimation. In our benchmark specification, we combine the additional explanatory variables introduced by the deep habits environment with marginal cost into a single driving process. We then impute this unobservable series using data on consumption and real unit labor cost. The weights on the various elements are functions of the model's structural parameters, which we calibrate. This procedure allows us to compare the driving process of the standard NKPC, namely marginal cost, with the one implied by deep habits. We show that the latter is considerably more volatile than real unit labor cost. This observation is reflected in the NKPC estimates for the coefficients on the driving process and the weight on the intrinsic contribution to inflation dynamics, which is much smaller than in the standard specification. Moreover, the fit of the deep habits NKPC is much improved over the standard NKPC per typical specification measures in the GMM literature.

The representation of the driving process under deep habits involves expectations of both observable variables, such as consumption, and unobservables, specifically the marginal value of demand. In order to back out processes for these variables we pursue a parametric approach in that we use the optimality conditions of the model to link the marginal value of demand to observable consumption. In the deep habits environment, this involves an expectational difference equation which we solve forward to express the marginal value of demand as the present value of future consumption growth and marginal cost. We then pursue alternatively a univariate and a multivariate approach to produce forecasts for the latter variables, which are used to construct a synthetic series for these unobservables. Our approach is therefore of a partial equilibrium and limited information nature in that we do not use all potential information available within the general equilibrium context of the full theoretical model.

We also study an alternative representation of the deep habits NKPC that uses analytical representations for the unobserved and expectational terms in the driving process. This yields a more reduced-form representation in terms of marginal cost and current consumption growth, which we can estimate directly. The coefficients in this representation are functions of the structural parameters. This representation then allows us to identify the crucial elements for the improved performance of the deep habits model. Not surprisingly, the size of the deep habits parameter is the central element. We show that performance notably improves for values above 0.6. Our GMM-estimate of this parameter is 0.85, which is identical to previous estimates in the literature (see Ravn et al., 2010). The second important element is the degree of substitutability of differentiated products in the consumer's preferences, which is inversely related to the firm's markup. We estimate this to be consistent with a markup of 74%, which may be considered high. A robustness analysis shows, however, that the performance of the deep habits NKPC is still very satisfactory for more typical markups of 10%.

The modeling environment in our paper draws from, and contributes to, an emerging literature on deep habits, starting with the original contribution by Ravn et al. (2006). Their key insight was that deep habits impart additional internal propagation on a model, on top of what an external habit formation would produce, while at the same time being arguably more plausible. We extend this insight to the specification of the NKPC by showing that at a purely empirical level deep habits add additional regressors to the empirical inflation equation. Moreover, Ravn et al. (2006) show that deep habits give rise to countercyclical markups, which translates into procyclical marginal cost in the context of the NKPC. We utilize this insight in the construction of a modified driving process. Ravn et al. (2010) build on their earlier paper and extend it to a New Keynesian framework. They estimate the model using impulse-response function matching for identified monetary policy shocks. Our estimates of the key structural parameters using a GMM approach are virtually identical to theirs, which suggests robustness of the derived insights.

Our paper also touches upon a host of other research on the NKPC that is concerned with modifying the notion of the driving process. In the wake of Galí and Gertler (1999), researchers at first assessed the robustness of their conclusions with respect to proxies for marginal costs, the instrument set used in the estimation, and the empirical approach being taken.[1] Our paper follows the spirit of this research in that we do not modify or specifically address the nature of marginal cost. We thus continue to proxy marginal cost with unit labor cost, which is justified under the assumption of competitive factor markets and a standard neoclassical production structure. More recent contributions, however, deviate from this assumption. The key insight is that modifications to the production structure change the nature of the driving process in the NKPC either by introducing additional elements, that is, regressors, or by altering the responsiveness of inflation to marginal cost movements

For instance, this literature introduces search and matching frictions in the labor market (e.g. Krause et al., 2008), or incorporates models with finished goods inventories that highlight the distinction between marginal costs of production and marginal costs of sales (e.g. Lubik and Teo, 2011). In both approaches, modification of the production side does not improve the performance of the NKPC. The reason is that the driving process in

[1] Galí et al. (2005) is an example of the former, Kurmann (2007) an example of the latter. Nason and Smith (2008) give a comprehensive survey of the developments in this literature.

either specification relies solely on present-value relationships of the cost of, respectively, maintaining long-run employment relationships and of keeping an inventory. By their very nature, present discounted values tend to smooth out the movements imparted by volatile driving processes. In the two cases this virtually negates the effect of additional regressors. This effect is present in our paper, too, but it is compensated by the presence of current consumption growth, as we show in the reduced-form representation of the NKPC.

Guerrieri et al. (2010) provide a bridge between these different aspects of the literature. In an otherwise standard model of a small open economy, they introduce both variable demand elasticities on the preference side as well as a production structure that relies on intermediate inputs from domestic and foreign sources. They derive an NKPC, whose driving process is a function of marginal cost, relative prices and exogenous variations in the aggregate elasticity of final products, and find that the estimated NKPC provides a better fit than a standard specification, mainly due to the addition of the relative price term. Our paper similarly improves the fit of the NKPC via changes to the demand structure in the form of persistence-inducing deep habits.

We now proceed as follows. In the next section, we present our theoretical model and show how the introduction of deep habits modifies the specification of the NKPC. In Section 3, we discuss our empirical approach, the data being used, and the various specifications of the deep habits NKPC that we estimate. Section 4 presents the GMM estimation results for a standard NKPC and our deep habits model. In Section 5, we conduct various robustness checks and consider alternative specifications. The final section concludes.

2 The Model

Our theoretical model is based on Ravn et al. (2010). It describes a New Keynesian monetary economy with utility-maximizing households and profit-maximizing firms. Households consume a bundle of consumption goods, each of which individually is subject to habit formation. Firms are monopolistically competitive and hire labor from the households as input in the production process. They set their prices subject to a downward-sloping demand schedule, which is derived from household preferences, and they are subject to quadratic costs of adjusting nominal prices. It is the latter element that gives rise to an NKPC. In what follows, we describe the full model of the economy, but focus mainly on the relationships that are needed to derive the NKPC.

2.1 Households

The household sector is described by the decisions of a representative household. Its intertemporal utility function is given by:

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} x_t^{1-\sigma} - h_t \right]. \tag{1}$$

$0 < \beta < 1$ is the discount factor, and $\sigma > 0$ is the coefficient of relative risk aversion. h_t is the amount of labor supplied by the household to the firm sector. The core of the deep habits formulation is captured by the term x_t. It is a sub-utility function that depends on a CES aggregate of consumption goods i, c_{it}, relative to the previous period's consumption of goods i:

$$x_t = \left[\int_0^1 (c_{it} - \theta c_{it-1})^{1-1/\varepsilon} \, di \right]^{1/(1-1/\varepsilon)}. \tag{2}$$

$\varepsilon > 1$ is the substitution elasticity between the differentiated goods, while $0 \leq \theta < 1$ is the deep habits parameter. In contrast to habits at the aggregate level of consumption, deep habits apply to each individual consumption good and are thus deeply embedded into the utility function.

Solving for the household's optimality conditions is slightly more involved than in the standard case. We note that we can rewrite the household's problem as an expenditure minimization problem:

$$\min_{c_{it}} X_t = \int_0^1 P_{it} c_{it} di,$$

subject to the definition of the habit stock (2). The first-order condition implies the following demand function:

$$c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} x_t + \theta c_{it-1}, \tag{3}$$

where

$$P_t = \left[\int_0^1 P_{it}^{1-\varepsilon} di \right]^{1/(1-\varepsilon)} \tag{4}$$

is the associated aggregate price index. The demand function has the typical feature that it is downward-sloping in its own relative price P_{it}/P_t. In addition, deep habits render consumption demand persistent through lagged consumption c_{it-1}. It is specifically this feature which changes the nature of the driving process in the NKPC.

We can now describe the household's intertemporal utility optimization problem. The budget constraint of the representative household is given by:

$$P_t x_t + \varkappa_t + B_t = R_{t-1} B_{t-1} + W_t h_t + \Phi_t, \tag{5}$$

where

$$\varkappa_t = \theta \int_0^1 P_{it} c_{it-1} di$$

is the current consumption expenditure required to maintain the habit consumption level from the previous period. W_t is the nominal wage payment for labor services h_t. Φ_t is the residual profit accruing to the household from its ownership of the firms. Finally, we assume that households have access to a risk-free one period nominal government bond B_t that pays a gross interest rate of R_t.

The household maximizes the utility function (1) by choosing sequences of x_t, h_t, and B_t. The resulting first-order conditions are:

$$x_t^{\sigma} = \frac{W_t}{P_t}, \tag{6}$$

$$x_t^{-\sigma} = \beta R_t E_t \frac{P_t}{P_{t+1}} x_{t+1}^{-\sigma} \tag{7}$$

The first equation is the standard labor-leisure trade-off a household faces, where the variable that determines the overall level of economic activity is the habit stock x_t instead of the usual consumption aggregate. Similarly, the second optimality condition is a standard Euler-equation for intertemporal smoothing with x_t instead of consumption. This relationship also defines a stochastic discount factor which firms use to evaluate future profit streams.

2.2 Firms

The firm sector is composed of a continuum of monopolistically competitive firms that face a downward-sloping demand schedule for their product. Demand for firm $i's$ output is given by equation (3), where we associate each good i with a specific firm. Firms choose an optimal price, but they are subject to quadratic costs of price adjustment. We assume that firms have access to a linear production technology that uses labor as its only input.[2] Production is subject to aggregate productivity disturbances A_t. We write the production function as $y_{it} = A_t h_{it}$. Each firm hires labor input from the representative household for a competitive wage W_t.

The firm has to solve an intertemporal profit maximization problem:

$$\max_{\{P_{it}, c_{it}, h_{it}\}} E_0 \sum_{t=0}^{\infty} q_t \left[P_{it} c_{it} - W_t h_{it} - \frac{\zeta}{2} \left(\frac{P_{it}}{P_{it-1}} - \overline{\pi}_t \right)^2 P_t c_t \right], \tag{8}$$

[2] The assumption of constant returns to scale is immaterial for the NKPC as it is independent of the curvature of the production function. Derivations and results for alternative production functions are available from the authors upon request.

by choosing sequences of prices P_{it}, output c_{it}, and labor input h_{it}, subject to the demand function (3) and the production function. The discount factor $q_t = \beta^t x_t^{-\sigma}/P_t$ reflects household's ownership of the firm. $\zeta > 0$ is the price adjustment cost parameter. We assume that firms only incur this cost when the chosen price path deviates from the weighted inflation rate $\overline{\pi}_t = \pi^{1-\eta}(\pi_{t-1})^{\eta}$, where π is steady state inflation and $0 \leq \eta \leq 1$ is the degree of indexation in the targeted inflation rate.

The first order conditions of firm i are given by:

$$\frac{W_t}{P_t} = \lambda_t^y A_t, \tag{9}$$

$$\lambda_t^c + \lambda_t^y - \frac{P_{it}}{P_t} = \theta E_t \frac{q_{t+1}}{q_t}\frac{P_{t+1}}{P_t}\lambda_{t+1}^c, \tag{10}$$

$$\zeta \frac{P_t}{P_{it-1}}\left(\frac{P_{it}}{P_{it-1}} - \overline{\pi}_t\right) c_t + \varepsilon \frac{P_t}{P_{it}}\left(c_{it} - \theta c_{it-1}\right)\lambda_t^c - c_{it} = E_t \frac{q_{t+1}}{q_t}\zeta \frac{P_{it+1}}{P_{it}}\left(\frac{P_{it+1}}{P_{it}} - \overline{\pi}_{t+1}\right)\frac{P_{t+1}}{P_{it}} c_{t+1}. \tag{11}$$

The first condition equates the real wage to the marginal product of the worker, which is simply productivity with linear production . λ_t^y is the Lagrange-multiplier on the production function. It can be interpreted as the real marginal cost. To see this, denote the real wage $w_t = W_t/P_t$. Total cost of production is $w_t h_{it} = w_t\left(\frac{y_{it}}{A_t}\right)$. Take derivative with respect to y_{it}, to get $mc_t = w_t/A_t = \lambda_t^y$, where the last equality follows from (9). This expression can now be used to eliminate λ_t^y from the optimality condition (10), which becomes an expectational difference equation in λ_t^c, with the driving variable being marginal cost.

The second first-order condition connects marginal cost λ_t^y with λ_t^c, the multiplier on the demand function (3). λ_t^c can be interpreted as the marginal value of demand. In the absence of deep habits (when $\theta = 0$), it equals relative prices minus marginal cost. Under deep habits, however, the persistence of demand for individual goods affects firms' demand for labor intertemporally. Finally, the third first-order condition captures the optimal price-setting problem of the firm. We now derive the NKPC from this equation.

2.3 Deriving the NKPC

The first step is to impose a symmetric equilibrium. That is, we assume in line with the literature that all firms behave identically and are ex-post homogeneous. This amounts to erasing the firm-specific subscripts i, which simplifies the above expressions considerably. We now define the aggregate, consumption-based (gross) inflation rate $\pi_t = P_t/P_{t-1}$. Substituting in the stochastic discount factor q_t results in the following expression:

$$\varepsilon \lambda_t^c x_t + \zeta \pi_t \left(\pi_t - \bar{\pi}_t\right) c_t = c_t + \beta \zeta E_t \left(\frac{x_{t+1}}{x_t}\right)^{-\sigma} \pi_{t+1}\left(\pi_{t+1} - \overline{\pi}_{t+1}\right) c_{t+1}. \tag{12}$$

This is an expectational difference equation in cross-products of inflation π_t and consumption c_t. We identify as driving forces the terms involving x_t and the marginal value of demand λ_t^c. Our goal is now to re-write this expression in terms of marginal cost and potentially other variables.

As is common in the literature, we consider a linearized version in terms of deviations from the steady state. Denote the (log-) deviation of a variable z_t from its steady state z as $\widetilde{z}_t = \log z_t - \log z$. We can now linearize (12) around its steady state, whereby we note that the resulting relationship is independent of the steady-state inflation rate up to first order since firms face price adjustment cost only to the extent that their prices deviate from the aggregate price path $\bar{\pi}_t$. The expression for marginal value of demand λ_t^c can also be linearized around its steady state. We substitute these into the linearized form of (12) and collect terms.[3]

The NKPC in the model with deep habits is:

$$\widetilde{\pi}_t = \frac{\beta}{1+\beta\eta} E_t \widetilde{\pi}_{t+1} + \frac{\eta}{1+\beta\eta} \widetilde{\pi}_{t-1} + \psi_1 \widetilde{mc}_t + \psi_2 E_t \Delta \widetilde{c}_{t+1} - \psi_3 \Delta \widetilde{c}_t - \psi_4 E_t \widetilde{\lambda}^c_{t+1}, \quad (13)$$

where the coefficients are given by:

$$\psi_1 = \frac{\varepsilon(1-\theta)-(1-\beta\theta)}{\zeta(1+\beta\eta)}, \quad \psi_2 = \frac{\sigma\beta\frac{\theta}{1-\theta}}{\zeta(1+\beta\eta)}, \quad \psi_3 = \frac{(1+\sigma\beta\theta)\frac{\theta}{1-\theta}}{\zeta(1+\beta\eta)}, \quad \psi_4 = \frac{\beta\theta}{\zeta(1+\beta\eta)}.$$

It is straightforward to verify that ψ_2, ψ_3 and ψ_4 are strictly non-negative. ψ_1 is positive if $\varepsilon > (1-\beta\theta)/(1-\theta)$. We impose this condition henceforth. The critical value $(1-\beta\theta)/(1-\theta)$ increases monotonically with θ.[4] For $\theta = 0$, the critical value equals 1. For $\theta \in (0, 0.99)$, the critical value is below 2 for $\beta = 0.99$. Figure 1 shows how $(1-\beta\theta)/(1-\theta)$ changes as θ varies between 0 and 0.99 with β set at 0.99.

There are a few observations to make. First, in the absence of deep habits, when $\theta = 0$, it can be easily verified that $\psi_2 = \psi_3 = \psi_4 = 0$. The relationship thus reduces to the standard NKPC derived in Galí and Gertler (1999):

$$\widetilde{\pi}_t = \frac{\beta}{1+\beta\eta} E_t \widetilde{\pi}_{t+1} + \frac{\eta}{1+\beta\eta} \widetilde{\pi}_{t-1} + \frac{\varepsilon - 1}{\zeta(1+\beta\eta)} \widetilde{mc}_t, \quad (14)$$

so that he specification with deep habits cleanly nests the standard specification. Second, the introduction of deep habits affects the conditional responsiveness of inflation to marginal cost. It is straightforward to show that $\psi_1 < \kappa \equiv \frac{\varepsilon-1}{\zeta(1+\beta\eta)}$, the standard NKPC-coefficient. Ceteris paribus inflation under the deep habits formulation is less reliant on marginal cost as driving process.

[3] Details of the derivation are given in the Appendix.

[4] It is easy to show that the derivative of $(1-\beta\theta)/(1-\theta)$ with respect to θ is positive.

The third observation is that deep habits add additional terms to the NKPC, above and beyond marginal cost, to wit, expected and current consumption growth and the expected marginal value of future demand. They stem from the fact that firms have to consider the effect of current pricing decisions on future demand through its feedback via the persistence of demand. Contemporaneous consumption growth $\Delta\widetilde{c}_t$ engenders future consumption growth via deep habits formation, which firms encourage by lowering their prices. Higher expected marginal value of future demand $E_t\widetilde{\lambda}^c_{t+1}$ reduces current inflation since it creates an incentive for firms to lower their prices in order to capture future market share (Ravn et al., 2010). In contrast, higher expected future consumption growth raises current inflation as firms do not have to lower their prices to generate an increase in future demand.

We find it useful for comparison with the standard NKPC to rewrite equation (13) in a slightly different way by factoring out a common coefficient and by grouping the relevant terms together:

$$\widetilde{\pi}_t = \frac{\beta}{1+\beta\eta}E_t\widetilde{\pi}_{t+1} + \frac{\eta}{1+\beta\eta}\widetilde{\pi}_{t-1} + \kappa\left[\tilde{\psi}_1\widetilde{mc}_t + \tilde{\psi}_2E_t\Delta\widetilde{c}_{t+1} - \tilde{\psi}_3\Delta\widetilde{c}_t - \tilde{\psi}_4E_t\widetilde{\lambda}^c_{t+1}\right]. \quad (15)$$

We refer to $\kappa = \frac{\varepsilon-1}{\zeta(1+\beta\eta)} > 0$ as the NKPC-coefficient. The expressions for the response-coefficients are $\tilde{\psi}_i \equiv \kappa^{-1}\psi_i$, $i = 1, ..., 4$ which preserves the sign restrictions that we impose on the original coefficients.

Factoring out the coefficient κ thus allows us to define the driving process $\widetilde{d}_t$ as:

$$\widetilde{d}_t = \tilde{\psi}_1\widetilde{mc}_t + \tilde{\psi}_2E_t\Delta\widetilde{c}_{t+1} - \tilde{\psi}_3\Delta\widetilde{c}_t - \tilde{\psi}_4E_t\widetilde{\lambda}^c_{t+1}. \quad (16)$$

Note that if $\theta = 0$, we have $\widetilde{d}_t = \widetilde{mc}_t$, and the specification reduces to the standard NKPC. We treat (15) as our benchmark specification for the NKPC under deep habits. Summarizing the additional regressors in terms of a driving process allows us to compare it directly to the driving process in the standard NKPC, namely marginal cost. The remainder of our paper is concerned with computing this driving process. The key challenge is to determine the behavior of the unobserved term $E_t\widetilde{\lambda}^c_{t+1}$. Once we derive a series for $\widetilde{d}_t$ we can then assess the performance of the NKPC under deep habits in a limited information setting.

3 Inflation Dynamics and Deep Habits: A Limited Information Approach

We now proceed to a formal empirical analysis of the NKPC under deep habits. We pursue a limited information approach in that we do not use all the information available in the full general equilibrium model that embeds the NKPC. To be more precise, we do not impose the cross-equation and cross-coefficient restrictions on the comovement of the endogenous variables that the full model would prescribe. We thus treat the NKPC as a moment condition which we estimate with a generalized methods of moments (GMM) approach. We begin with a short description of the data and our estimation method. We then describe how to deal with unobservable variables in the formulation of the driving process by backing them out of intertemporal optimality conditions. We apply two methods: First, our benchmark specification, which treats observable, but exogenous processes as separate univariate processes, and, second, a VAR-based method.

3.1 Data and Empirical Approach

We extract quarterly data from the FRED database at the Federal Reserve Bank of St. Louis. Our sample period ranges from 1955:1 to 2011:2, but we also consider a sub-sample from 1984:1 onwards, which covers the Great Moderation during which the behavior of many macroeconomic time series changed. Output and consumption are constructed by dividing real GDP in chained dollars (GDPC96 in FRED) and real consumption in chained dollar (PCECC96) by the civilian non-institutional population aged 16 and over (CNP16OV). The GDP implicit price deflator is our measure of P_t (GDPDEF). Real unit labor cost is constructed by dividing nominal unit labor cost of the nonfarm business sector (ULCNFB) by the price deflator. We remove a linear trend from GDP and consumption. We also use compensation per hour in the nonfarm business sector (COMPNFB) as a measure of the nominal wage to construct wage inflation, which we then use as an instrument in the GMM estimation.

Table 1 reports some moments of the data series. Over the full sample period, GDP is more volatile than real unit labor cost, which has been used in the literature as a proxy for marginal cost. Concentrating on the sub-sample from 1984 on, we find, however, that the volatility of GDP drops, while that of the marginal cost proxy increases, to the effect that the latter becomes now more volatile than the former. Inflation and marginal cost are mildly positively correlated, both for the full sample and for the sub-sample. This pattern is almost a requirement for the validity of the NKPC as the logic of the relationship suggests that increases in marginal cost should drive up inflation. We also note that the correlation pattern between GDP and real unit labor costs is negative for the full sample but quite positive for the period of the Great Moderation. Finally, we report second moments for consumption growth as it appears in the driving process of the deep habits specification. It comoves negatively with inflation over both the full sample and sub-sample. Since current consumption growth affects inflation negatively per equation (16), this suggests that deep habits have a role to play in explaining inflation dynamics.

Let z_t denote a vector of variables observed at time t. The NKPC then defines a set of orthogonality conditions: $E_t\left[\widetilde{\pi}_t - \gamma_f \widetilde{\pi}_{t+1} - \gamma_b \widetilde{\pi}_{t-1} - \kappa \widetilde{d}_t\right] * \; z_t = 0$. Given these conditions, we can estimate the model using a GMM approach. To aid comparison with the recent literature, we use the same set of instruments as Galí et al. (2005). Specifically, we use 4 lags of inflation and 2 lags of the regressors and wage inflation as instruments. The weighting matrix is computed from the estimated heteroskedasticity- and autocorrelation-adjusted (HAC) covariance matrix, where the number of lags in the HAC estimation is chosen based on the criterion in Andrews (1991). We consider two empirical specifications: first, a reduced-form version which estimates only the coefficients in the moment conditions. The focus here is on the relative importance of the backward and forward inflation terms, and thus the degree of intrinsic price dynamics, and on the NKPC-coefficient κ, which captures the strength of the transmission mechanism between the real and nominal side and indicates the presumed degree of price stickiness. The second version attempts to estimate the underlying structural parameters of the model embedded in the reduced-form coefficients.

3.2 Computing the Driving Process (I): Baseline

The baseline specification we intend to estimate is:

$$\widetilde{\pi}_t = \gamma_f E_t \widetilde{\pi}_{t+1} + \gamma_b \widetilde{\pi}_{t-1} + \kappa \widetilde{d}_t, \tag{17}$$

where the driving process $\widetilde{d}_t$ is given by equation (16). The advantage of this specification is that the coefficient estimates are immediately comparable to those from the standard NKPC. The only difference is that allowing for deep habits affects the nature of the driving process, which is no longer marginal costs alone, but a composite of marginal cost, expected and current consumption and the expected marginal value of future demand. Once we have constructed a times series for $\widetilde{d}_t$, estimating this specification is straightforward.

However, this approach presents a few challenges. First, marginal cost is unobservable to the econometrician. We proxy $\widetilde{mc}_t$ with real unit labor cost in line with most of the NKPC literature. Second, while we can compute current consumption growth straight from the data, *expected* consumption growth is unobservable. This can be obtained in several ways. One possibility is to proxy expected consumption using survey data from sources such as the Survey of Consumer Finances. This approach has numerous drawbacks, such as the potential inconsistency of forecast horizons and forecast object between the model and the survey respondents. Second, we can try to back out expected consumption from other equilibrium conditions. However, as it turns out, all possible relationships involve unobservable variables, specifically, Lagrange multipliers that also would have to be proxied. We therefore choose to specify a parametric model for consumption, which we use to compute conditional expectations. In this section, we specify univariate processes for the observable variables, while the subsequent section assumes a multi-variate relationship that allows for richer interactions.

The third challenge to computing the driving process $\widetilde{d}_t$ is the presence of the term $E_t\widetilde{\lambda}^c_{t+1}$, which involves the conditional expectation of an unobservable quantity, namely the marginal value of future demand. We follow the approach of Lubik and Teo (2011) and use an intertemporal equilibrium condition to relate the unobservable expected marginal value of future demand to observables. We find such a relationship in the firm's first-order condition (10). Using the relationship $mc_t = \lambda^y_t$, and linearizing (10) around the steady state yields:[5]

$$\widetilde{\lambda}^c_t = \beta\theta E_t\widetilde{\lambda}^c_{t+1} - \beta\theta E_t\left[\frac{\sigma}{1-\theta}\left(\Delta\widetilde{c}_{t+1} - \theta\Delta\widetilde{c}_t\right)\right] - \left[\varepsilon\left(1-\theta\right) - \left(1-\theta\beta\right)\right]\widetilde{mc}_t. \qquad (18)$$

This is an expectational difference equation in $\widetilde{\lambda}^c_t$ which can be solved forward.[6] We find that:

$$\begin{aligned}\widetilde{\lambda}^c_t &= \sum_{j=0}^{\infty}(\beta\theta)^j E_t\left\{\left[-\beta\frac{\sigma\theta}{1-\theta}\left(\Delta\widetilde{c}_{t+j+1} - \theta\Delta\widetilde{c}_{t+j}\right)\right] - \left[\varepsilon\left(1-\theta\right) - \left(1-\beta\theta\right)\right]\widetilde{mc}_{t+j}\right\} \\ &= \sum_{j=0}^{\infty}(\beta\theta)^j E_t\left\{\left[-\beta\frac{\sigma\theta}{1-\theta}\left(1-\beta\theta^2\right)\Delta\widetilde{c}_{t+j+1}\right] - \left[\varepsilon\left(1-\theta\right) - \left(1-\beta\theta\right)\right]\widetilde{mc}_{t+j}\right\} \\ &\quad +\beta\frac{\sigma\theta^2}{1-\theta}\Delta\widetilde{c}_t. \qquad (19)\end{aligned}$$

The last equality follows from collecting terms in consumption growth. The marginal value of demand can now be expressed as a function of observable $\Delta\widetilde{c}_t$ and as the present discounted value of future consumption growth and marginal cost. Proxying the latter by real unit labor cost, we can now back out $\widetilde{\lambda}^c_t$ from a parametric model for the two series, $\Delta\widetilde{c}_t$ and $\widetilde{mc}_t$.

As a first pass, we assume that both variables follow AR(1)-processes:

$$\widetilde{mc}_t = \rho_{mc}\widetilde{mc}_{t-1} + \varepsilon_{mc,t}, \qquad (20)$$

[5] The derivation is shown in the Appendix.

[6] Since $0 < \beta\theta < 1$, the equation has a unique solution.

$$\Delta\widetilde{c}_t = \rho_{\Delta c}\Delta\widetilde{c}_{t-1} + \varepsilon_{\Delta c,t}, \tag{21}$$

where $|\rho_{mc}, \rho_{\Delta c}| < 1$ and $\varepsilon_{mc,t}$ and $\varepsilon_{\Delta c,t}$ are *i.i.d.* random variables with zero mean. Noting that $E_t\widetilde{mc}_{t+j} = \rho_{mc}^j\widetilde{mc}_t$, we can substitute this into the above expression and solve out the infinite discounted sum. This results in the following expression:

$$\widetilde{\lambda}_t^c = \left[\frac{\sigma\beta\theta^2}{1-\theta} - \frac{1}{1-\beta\theta\rho_{\Delta c}}\frac{\sigma\beta\theta}{1-\theta}\left(1-\beta\theta^2\right)\rho_{\Delta c}\right]\Delta\widetilde{c}_t - \frac{\varepsilon\left(1-\theta\right)-\left(1-\beta\theta\right)}{1-\beta\theta\rho_{mc}}\widetilde{mc}_t, \tag{22}$$

which is a weighted average of consumption growth and marginal cost. Since the latter can be proxied by unit labor cost, this expression allows us to impute a time series for the unobservable $\widetilde{\lambda}_t^c = \psi_{\Delta c}\Delta\widetilde{c}_t - \psi_{mc}\widetilde{mc}_t$.

The conditional one period-ahead forecast can then be computed by iterating forward one more time:

$$E_t\widetilde{\lambda}_{t+1}^c = \psi_{\Delta c}\rho_{\Delta c}\Delta\widetilde{c}_t - \psi_{mc}\rho_{mc}\widetilde{mc}_t. \tag{23}$$

The sign of the coefficients depends on the size of the deep habits parameter. ψ_{mc} is positive since we impose $\varepsilon > (1-\beta\theta)/(1-\theta)$. Furthermore, it is easy to show that $\psi_{\Delta c} > 0$ if $\theta > \rho_{\Delta c}$. Whether movements in $E_t\widetilde{\lambda}_{t+1}$ reinforce or dampen movements of the other variables in the driving process thus depends on the size of the habit parameter and whether consumption growth and marginal cost comove positively or negatively.[7] Before we can make further empirical progress, however, our final step assigns numerical values to the structural parameters. In our benchmark exercise, we calibrate all parameters required to impute the unobservable series since we focus on the impact of changes in the driving process only. In a robustness exercise below, we show how to use the previous expression to compute a reduced-form representation of the NKPC, which allows us to estimate some of these parameters.

The calibrated parameter values are detailed in Table 2. We base our calibration on the estimates in Ravn et al. (2010), which is to the best of our knowledge the first empirical study of a deep habits model. We fix the discount factor $\beta = 0.99$ to be consistent with an annual real interest rate of 4%. We set the coefficient of relative risk aversion to $\sigma = 1$, which implies log utility. As an alternative, we consider a value of $\sigma = 3$, which implies much more risk averse households. We follow again Ravn et al. (2010) in choosing the habit parameter $\theta = 0.85$. The substitution elasticity between differentiated products ε, which can be interpreted as a demand elasticity is set at $\varepsilon = 2.48$, based on the empirical estimates in Ravn et al. (2010). In standard models without deep habits this parameter is

[7] Recall that the coefficient on the expected marginal value of future demand in (15), $\tilde{\psi}_4$ is strictly positive.

usually fixed at $\varepsilon = 11$ to imply a markup of 10% over marginal cost. In our model with deep habits, the steady state markup is given by $\frac{\varepsilon(1-\theta)}{\varepsilon(1-\theta)-(1-\theta\beta)}$.[8]

Figure 2 plots the steady state markup as ε varies for the standard model without deep habits and our deep habits model. β and θ are fixed at 0.99 and 0.85, respectively. The markup is slightly higher for a given value of ε in our deep habits model compared to the standard model without deep habits. In our deep habits model, $\varepsilon = 11$ implies a steady state markup of 10.6% instead of 10%. The difference arises because for a given value of ε the presence of deep habits makes demand less elastic, giving firms incentive to charge a higher markup. Our benchmark $\varepsilon = 2.48$ imposes a steady state markup of 74.2%, which may seem excessive. We discuss this assumption further in the robustness section, where we also investigate alternative values. Finally, we fit separate AR(1)-processes to consumption growth and real unit labor costs. This results in estimates of $\rho_{mc} = 0.98$ and $\rho_{\Delta c} = 0.31$, which satisfies the restriction $\theta > \rho_{\Delta c}$ for the coefficient $\psi_{\Delta c}$ to be positive.

Figure 3 depicts the constructed driving process and marginal cost. The former has a standard deviation of 5.68%, which is higher than marginal cost. The correlation of the two series is 0.57. This confirms that the introduction of the additional elements into the NKPC via deep habits renders the driving process more volatile. This is also reflected in the less than perfect comovement, since some elements of $\widetilde{d}_t$ enter the driving process with negative signs, as the previous discussion has shown. Nevertheless, what we cannot distinguish at this stage is whether the changed properties of the driving process are simply due to the increased number of regressors or to the changed responsiveness of the coefficients. We attempt to disentangle this further below.

3.3 Computing the Driving Process (II): A VAR Approach

In the previous section we used independent AR(1) processes for marginal cost and consumption growth as predictors for the behavior of $E_t\widetilde{\lambda}^c_{t+1}$. In order to capture potential additional information in the data, we alternatively pursue a VAR-based approach. Consider a generic data vector v_t, which contains consumption growth, marginal cost and other variables that we judge useful for forecasting. Assume that v_t is described by a VAR: $v_t = Av_{t-1} + \epsilon_t$.[9] The conditional forecast is then given by $E_t v_{t+j} = A^j v_t$, $j \geq 1, \forall t$. Denote the extraction vector for some element a_t of the vector v_t as ι_a, so that, for instance, $E_t(\Delta\widetilde{c}_{t+j}) = \iota_{\Delta c}A^j v_t$ and $E_t(\widetilde{mc}_{t+j}) = \iota_{mc}A^j v_t$. Estimating the VAR and the coefficient

[8] See the Appendix for a derivation.

[9] The first-order specification is without loss of generality since any higher-order VAR can be written in first-order companion form. We discuss this specification for expositional expediency.

matrix $\widehat{A}$ therefore allows us to construct a time series for conditional expectations of the variables of interest.

We can use these expressions in the equation for the expected marginal value of future demand. Following the same steps as above, this yields:

$$\begin{aligned} E_t\widetilde{\lambda}_{t+1}^{c} &= \left\{\left[-\frac{\sigma\beta\theta}{1-\theta}\left(\left(1-\beta\theta^2\right)\iota_{\Delta c}\widehat{A}^2\right)\right]-\left[\varepsilon\left(1-\theta\right)-\left(1-\beta\theta\right)\right]\iota_{mc}\widehat{A}\right\}\left[I-\beta\theta\widehat{A}\right]^{-1}v_t \\ &+\frac{\sigma\beta\theta^2}{1-\theta}\iota_{\Delta c}\widehat{A}v_t. \end{aligned} \tag{24}$$

Given this expression, observed consumption growth $\Delta\widetilde{c}_t$, expected consumption growth $E_t\Delta\widetilde{c}_{t+1} = \iota_{\Delta c}\widehat{A}v_t$, and our proxy for marginal cost $\widetilde{mc}_t$, we can now construct an imputed time series for the driving process $\widetilde{d}_t$ from equation (16). As before, we impose our benchmark calibration, that is, $\theta = 0.85$, $\varepsilon = 2.48$, and $\sigma = 1$. We estimate a VAR(4) in consumption growth, real unit labor cost and output growth to construct expectations.

The constructed driving process from the deep habits specification is depicted in Figure 4, together with the marginal cost proxy. The standard deviation of $\widetilde{d}_t^{VAR}$ is 4.82%, whereas that of real unit labor cost is 3.44%. The correlation of the two series is 0.45. Both numbers are lower than the corresponding values from the baseline specification with independent AR(1) processes for expected marginal cost and consumption growth.[10] Nevertheless, the imputed driving process exhibits substantial volatility. Figure 5 depicts the imputed series for $E_t\widetilde{\lambda}_{t+1}^{c}$ against marginal cost. The contemporaneous correlation of both series is -0.94, while the standard deviation of the expected marginal value of future demand term is 2.27%. Since the latter term enters the driving process (16) with a negative sign, and its own coefficient is positive, the negative correlation thus imparts positive comovement with marginal cost and reinforces its contribution to the driving process.

4 Estimating the NKPC

We now provide formal estimates of the NKPC using a GMM approach. Our benchmark specification relies on the use of constructed driving processes. We first estimate a standard NKPC, where we use real unit labor cost as a proxy for marginal cost. We then estimate the corresponding NKPC, where the driving process is imputed from the first-order conditions of a deep habits model, using the two methods described in Section 3.

[10] This is reminiscent of the finding in Lubik and Teo (2011), where the use of a VAR-based imputation process tends to smooth out the present discounted value much more than simple univariate processes.

4.1 The Standard NKPC

In order to provide a benchmark for our deep habits specification, we first estimate both unrestricted and restricted versions of the standard NKPC with a proxy for marginal cost as in the original model of Galí and Gertler (1999). Specifically, we estimate the following standard NKPC specification:

$$\widetilde{\pi}_t = \gamma_f E_t \widetilde{\pi}_{t+1} + \gamma_b \widetilde{\pi}_{t-1} + \kappa \widetilde{mc}_t. \tag{25}$$

The GMM-estimation results for the standard NKPC are reported in Table 3. The estimates are quite similar to those found in the literature and statistically significant throughout. In the fully unrestricted specification, the coefficient γ_f on expected inflation is 0.79 while the coefficient γ_b on lagged inflation is 1/5, which is consistent with the findings of Galí and Gertler (1999) and subsequent work. The coefficient on marginal cost $\kappa = 0.004$. The J-test for overidentifying restrictions does not reject the specification, as evidenced by a high p-value. When γ_b is restricted to zero, γ_f is estimated to be 0.989, while the estimate of κ increases by 50% to 0.006. At the same time, the p-value for the J-test increases, which suggests that the specification without indexation is preferred.

Next, we estimate the structural parameters of the NKPC. From (14), the coefficient on expected inflation $\gamma_f = \beta/(1+\beta\eta)$, while the coefficient on past inflation $\gamma_b = \eta/(1+\beta\eta)$. Note that when $\eta = 0$ the specification reduces to the purely forward looking NKPC. The slope coefficient $\kappa = (\varepsilon - 1)/[\zeta(1+\beta\eta)]$. We impose $\beta = 0.99$ on the estimation, which is consistent with the implied value from the restricted NKPC estimation. We also note the parameters in the coefficient $\kappa' = (\varepsilon - 1)/\zeta$ are not separately identifiable in this specification as the coefficient simply scales the marginal cost term and appears nowhere else. We therefore only report estimates for κ' and the indexation parameter η. The results are in the last line of Table 3. We note that the high p-value of the J-statistic suggests that the cross-coefficient restrictions are informative in the estimation. The estimate $\eta = 0.27$ corresponds to an implied backward-coefficient $\gamma_b = 0.21$, which is consistent with the reduced-form estimate.

4.2 The NKPC with Deep Habits

We now estimate the NKPC specification with deep habits:

$$\widetilde{\pi}_t = \gamma_f E_t \widetilde{\pi}_{t+1} + \gamma_b \widetilde{\pi}_{t-1} + \kappa \widetilde{d}_t, \tag{26}$$

where the driving process $\widetilde{d}_t$ is either imputed using independent AR-processes for the observables or from a VAR-based approach. We note again that the specification of the NKPC is such that we only vary the term $\widetilde{d}_t$. Estimates for the forward-looking coefficient γ_f, the back-ward-looking coefficient γ_b and the NKPC-coefficient κ thus allow us to make direct comparisons between the models. We also report results from the structural specification:

$$\widetilde{\pi}_t = \frac{\beta}{1+\beta\eta} E_t \widetilde{\pi}_{t+1} + \frac{\eta}{1+\beta\eta} \widetilde{\pi}_{t-1} + \frac{\varepsilon - 1}{\zeta (1+\beta\eta)} \widetilde{d}_t. \tag{27}$$

Not all parameters in this specification are identifiable, however. We therefore focus on the indexation parameter η and the price adjustment cost parameter ζ, and fix the remaining parameters. Specifically, we set $\beta = 0.99$ and $\varepsilon = 2.48$, following the estimates reported in Ravn et al. (2010). The calibrated parameter values that go into the imputed driving process $\widetilde{d}_t$ are as reported in the previous section. We will consider alternative calibrations in our robustness analysis.

Table 4 contains the GMM estimates for the deep habits NKPC when the marginal cost and consumption growth processes are assumed to be independent AR(1)-processes. Compared to the standard NKPC, two observations stand out. First, the degree of indexation and thus the weight on the lagged inflation term is much lower for the deep habits specification. In the unrestricted version, $\gamma_b = 0.10$ is barely half as big as in the standard NKPC. Restricting γ_b to zero results in an estimate for $\gamma_f = 0.984$, which is identical to the implied value for the discount factor $\beta = 0.99$ in the restricted specification. Second, the p-value for the J-test rises substantially from the unrestricted to the restricted specification. The respective p-values are also much higher than the corresponding values for the standard NKPC. When interpreted as a specification test, this suggests that the deep habits NKPC captures inflation dynamics exceedingly well with only a minor degree of intrinsic inflation persistence. This is also reflected in the structural estimates. The fraction of price-indexing firms is estimated at a highly significant $\eta = 0.11$, which translates into a backward-looking coefficient of $\gamma_b = 0.099$. Moreover the p-value of the J-statistic is almost one, which suggests an excellent fit, based on the information content in the cross-coefficient restrictions. Finally, the estimated NKPC-coefficients κ are statistically significant and almost twice as large as those for the standard NKPC.

We now vary the specification for the driving process and use $\widetilde{d}_t^{VAR}$ which has been constructed from the VAR-based forecasts. The estimation results are reported in Table 5. Conceptually, the estimates do not differ from those based on AR-forecasts. In fact, the fit of the model is improved in the case of the unrestricted specification where the coefficient on lagged inflation comes in at a statistically insignificant $\gamma_b = 0.034$. The estimates for the restricted and structural specification are essentially unchanged from before. This suggests that the performance of the deep habits NKPC does not rest solely on the specification of the forecasting model for variables that are extraneous to the inflation dynamics equation.[11] The more important aspect is the fact that introducing deep habits imparts additional regressors into the driving process. We will take up this issue again in the robustness section.

The conclusion we can draw from our benchmark analysis is straightforward. Deep habits dramatically improve the performance of the NKPC in describing inflation dynamics. We demonstrated in the previous section that the implied driving process generated from our model is more volatile than marginal cost. We find that this improves the fit of the model in terms of a standard J-type specification test, but it also much reduces, even negates, the role of indexation in price-setting for explaining inflation dynamics. This device was introduced by Galí and Gertler (1999) in order to better capture inflation persistence through an intrinsic, that is, built-in, mechanism. We show that this role is played by the process for consumption growth in the driving process above and beyond indexation in price-setting.

5 Robustness

We assess the robustness of our conclusion in three directions. First, we consider the role that calibration plays in generating the desired stochastic properties of the driving process. In the second exercise, we look at an alternative specification for the driving process. Instead of first imputing a time series for the driving process, which is then used as a single regressor, we use the theoretical model restrictions to derive an alternative representation for a second set of observable driving forces. This allows us to decompose the effects on the NKPC into two underlying forces, namely marginal cost and consumption growth. Specifically, we use the imputed representations for expected consumption growth and the expected marginal value of future demand, substitute them into the driving process, and thus generate a reduced-form in marginal cost and consumption growth. We then use these as independent regressors in the NKPC. Finally, we also look at the performance of the model for a sub-sample of our full data set that considers only the period of the Great Moderation from 1984 on.

[11] This stands in contrast to the results in Lubik and Teo (2011), where the specification of the forecasting model matters.

5.1 Alternative Calibration

Our first robustness check simply looks at the implications of different parameterizations on the behavior of the driving process. We focus on three parameters, namely the coefficient of relative risk aversion σ, the demand elasticity ε, and the habit parameter θ. Table 6 contains the GMM estimation results for the imputed driving processes under different calibrations. We report only estimates for the unrestricted specification where we estimate reduced-form coefficients and where the driving processes are constructed using AR-based forecasts. Estimation results for the restricted version, the structural NKPC which estimates the model's parameters and the VAR-based driving process offer overall consistent results.

The first experiment documents the sensitivity of the model to the size of the habit parameter. When $\theta = 0.15$, the NKPC estimates are in between those of the benchmark calibration in Table 4 and the standard NKPC in Table 3. The estimates of γ_f and κ are larger than those of the standard NKPC but smaller than those of the benchmark calibration of $\theta = 0.85$. The reverse is true for the estimate of γ_b. Moreover, the standard errors are wider, and the J-test statistic has a lower p-value compared to the results of the benchmark calibration. Going to the other end of the parameter range, when $\theta = 0.95$, the estimate of γ_f is somewhat larger than that of the benchmark calibration but the J-test statistic has lower p-value. We also experimented with intermediate values of θ and find that the performance of the deep habits NKPC improves notably for values of θ above 0.6, the intuition of which we discussed in Section 2. The highest p-value is in fact attained for our benchmark calibration.

The second experiment varies σ, but keeps other parameters at their benchmark values. For $\sigma = 3$, when agents are more risk-averse, the effect is to increase the weight on the forward-looking coefficient. However, the J-test statistic has lower p-value compared to the benchmark calibration. Finally, we also consider increasing the demand elasticity ε to 11, which is the value most commonly used in the calibration literature implying a markup of 10% in standard model without deep habits. In this case, the estimates of γ_f and κ are slightly smaller and the fit of the model worsens compared to the case of the benchmark calibration. Nonetheless, even with $\varepsilon = 11$, the J-test statistic for the deep habits models still has a higher p-value than in the case of the standard NKPC. When considering joint variations of the parameters, the strongest role is played, unsurprisingly, by the habit parameter, while small values of ε improve performance. This suggest that the benchmark calibration, which has been chosen based on the empirical estimates in Ravn et al. (2010), does provide good estimates of the underlying parameters.

5.2 An Alternative Reduced-Form Specification

Our benchmark specification relies on an imputed series for the driving process, which we treat as a single regressor. However, the NKPC specification in equation (13) highlights the fact that the introduction of deep habits changes the standard NKPC in two fundamental ways. First, it affects the responsiveness of inflation to marginal cost as captured by the coefficient ψ_1, which is different from the standard NKPC coefficient. Second, it adds additional regressors to the inflation equation, namely current and expected consumption growth and the expected marginal value of future demand.

Consider the structural representation for the NKPC from equation (13), which we reproduce here for convenience:

$$\widetilde{\pi}_t = \frac{\beta}{1+\beta\eta} E_t \widetilde{\pi}_{t+1} + \frac{\eta}{1+\beta\eta} \widetilde{\pi}_{t-1} + \psi_1 \widetilde{mc}_t + \psi_2 E_t \Delta \widetilde{c}_{t+1} - \psi_3 \Delta \widetilde{c}_t - \psi_4 E_t \widetilde{\lambda}^c_{t+1}, \tag{28}$$

What prevents direct estimation of this relationship is that $E_t \widetilde{\lambda}^c_{t+1}$ is not observable to the econometrician.[12] We can, however, use the tools developed in Section 3 to provide analytical expressions for these two components of the driving process. For purposes of exposition, we focus on the univariate representation for the driving forces. These expressions depend on marginal costs and current consumption growth. Substituting them into the NKPC results in a reduced-form specification that only depends on observables. The exact derivations can be found in the Appendix.

We thus have:

$$\widetilde{\pi}_t = \gamma_f E_t \widetilde{\pi}_{t+1} + \gamma_b \widetilde{\pi}_{t-1} + \kappa_{mc} \widetilde{mc}_t - \kappa_{\Delta c} \Delta \widetilde{c}_t, \tag{29}$$

where we treat κ_{mc} and $\kappa_{\Delta c}$ as reduced-form coefficients. We also consider a representation that factors out the standard NKPC-coefficient and thus imposes the cross-coefficient restrictions implied by theory:

$$\widetilde{\pi}_t = \frac{\beta}{1+\beta\eta} E_t \widetilde{\pi}_{t+1} + \frac{\eta}{1+\beta\eta} \widetilde{\pi}_{t-1} + \frac{\varepsilon - 1}{\zeta(1+\beta\eta)} \left[\tilde{\psi}_5 \widetilde{mc}_t - \tilde{\psi}_6 \Delta \widetilde{c}_t \right]. \tag{30}$$

[12] As we discussed before, there are alternatives to our approach, one of which involves combining future inflation and consumption treating them as joint elements in the moment condition. This raises issues of normalization in the estimation, which is well known to be problematic in empirical NKPC models. Moreover, it does not solve the problem with fundamentally unobservable marginal value of demand, for which we would have to use a parametric model in any case. We therefore chose to be fully parsimonious in that we treat all non-inflation variables in the NKPC as pure elements of the driving process. A comparison of these additional alternative approaches would be a worthwhile exercise.

The coefficients on $\widetilde{mc}_t$ and $\Delta\widetilde{c}_t$ are, respectively, given by:

$$\tilde{\psi}_5 = \frac{\varepsilon(1-\theta)-(1-\beta\theta)}{\varepsilon-1}\frac{1}{1-\beta\theta\rho_{mc}},$$
$$\tilde{\psi}_6 = \frac{1}{\varepsilon-1}\frac{\theta}{1-\theta}\left[1+\frac{\sigma\beta(\theta-\rho_{\Delta c})}{1-\beta\theta\rho_{\Delta c}}\right].$$

We also show in the Appendix that both coefficients are positive for plausible calibrations. Since marginal cost and current consumption growth are negatively correlated, this implies that the latter reinforces the impact of marginal cost on inflation. Both specifications show that the main effect of deep habits is via introducing an additional regressor in the NKPC, namely consumption growth.

We can now estimate (29) without further modifications. For a preliminary assessment, Figure 6 depicts the marginal cost series and consumption growth. Clearly, the latter is less volatile (see also Table 1). Any reinforcement of marginal cost on inflation dynamics would therefore have to be generated by the relative size of the coefficient $\kappa_{\Delta c}$. This is, in fact, borne out by the estimation results in Table 7. The consumption growth coefficient in the unrestricted and the restricted specifications (where $\gamma_b = 0$) is an order of magnitude larger than κ_{mc}, which in turn is in line with the estimates from the standard NKPC. This confirms that capturing inflation dynamics is thus a simple matter of adding the correct additional regressor.[13] However, the fit of the model as measured by the J-test declines relative to both the standard NKPC as well as our benchmark specification.

We now turn to estimating the alternative specification (30), which imposes the cross-coefficient restrictions. We fix σ and ε, and estimate ζ and θ. As before, we consider a benchmark calibration that sets $\beta = 0.99$, $\sigma = 1$ and $\varepsilon = 2.48$, following the estimates reported in Ravn et al. (2010). We report the estimation results in Table 7, where we also consider variations in key parameters.

The estimates for the indexation parameter are identical across the three specifications where we vary the fixed parameters. A value of $\eta = 0.198$ corresponds to $\gamma_b = 0.166$, and is thus consistent with the reduced-form estimate. There are minor differences for the other parameters in the two specifications that set $\varepsilon = 2.48$. In that case, the value of the deep habits parameter θ is close to 0.85, which is the value estimated by Ravn et al. (2010) with a different empirical methodology and within the context of a fully-specified general equilibrium model. For the specification with $\sigma = 3$ and $\varepsilon = 11$, we note that the habit

[13] That this need not be the case is demonstrated by empirical studies of the NKPC that modify factor inputs (e.g. by introducing labor market search and matching frictions as in Krause et al., 2008) or change the structure of product markets (e.g. by introducing finished goods inventories as in Lubik and Teo, 2011).

parameter is close to one and that the price adjustment cost parameter ζ is estimated at a very high value.[14] The latter estimate reflects the fact that ε and ζ are not separately identifiable. The higher calibrated value of the elasticity parameter ε then translates into higher implied adjustment cost in order to generate the same implied NKPC-coefficient. At the same time, the increase in θ is related to the increase in ε, which also suggest identification issues regarding these two preference parameters.

We investigate this issue further by looking at how the driving process changes with respect to parameters. Figure 7 plots the marginal cost coefficient $\tilde{\psi}_5$ as well as the relative weight on consumption growth $\tilde{\psi}_6/\tilde{\psi}_5$ against the habit parameter θ over the range $[0, 1]$. The graphs are conditional on $\varepsilon = 2.48$. Analytically, the coefficient on marginal cost is not affected by σ, which only enters the weight on consumption growth (see equation 30). The bottom graph therefore contains two lines for different values of $\sigma = 1, 3$. As θ increases, the weight on marginal cost decreases. For $\theta = 0$, the weight is one and the standard NKPC obtains. $\tilde{\psi}_5$ is fairly inelastic to changes in θ until about 0.6, after which it declines rapidly. Nevertheless, the weight on marginal cost drops below 0.5 only for extremely high values of the habit parameter. The relative weight on consumption growth is the flip-side of this. For $\theta > 0.6$, $\tilde{\psi}_6/\tilde{\psi}_5$ exceeds 1 and increases exponentially afterwards. For values larger than 0.9, the weight on consumption is ten times that on marginal cost. Higher value of σ increases the relative weight on consumption growth, but the two curves are close enough to not make this the dominant effect.

The previous graphs were plotted conditional on $\varepsilon = 2.48$. We now study how the response coefficients change with variations in that parameter for a given θ. The respective graphs are in Figure 8. We fix $\theta = 0.85$, and also plot variations to σ. As before, the weight on marginal cost is not affected by σ. As ε rises over the range $[2, 11]$, the weight on marginal cost increases only slightly. At the same time, the relative weight on consumption growth decreases by an order of 10 over the range. A higher value of σ increases the relative weight on consumption growth as in Figure 7. Note that the relative weights on consumption growth are all larger than 1 in Figure 8.

This analysis shows where the improvement in fit over the standard NKPC is coming from. It is not simply the addition of another regressor, but the fact that deep habits increase the responsiveness of inflation to movements in consumption growth by an order of magnitude without a large countervailing effect from a reduced importance of marginal cost. This result is, however, predicated on two requirements. First, the demand elasticity ε has

[14] We do not report the case of $\sigma = 1$ and $\varepsilon = 11$ since the estimate of θ would exceed 1 if we did not impose the parameter restriction.

to be small enough, while the degree of deep habits has to be large enough, as is the case for the estimates of Ravn et al. (2010) and for our estimates using the alternative specification of this section. Although a high degree of habits reduces the weight on marginal costs, this is more than compensated by the increase in the relative weight of consumption growth. The second requirement is that consumption growth has to have the right statistical properties. Although it is markedly less volatile than marginal costs, this is more than compensated by the response coefficient in terms of its overall impact. However, for the amplification effect on marginal cost dynamics, consumption growth has to comove negatively with the marginal cost proxy since consumption growth shows up in the NKPC with a negative sign as per equation (29). Otherwise, deep habits would dampen the effects of marginal cost. Our final robustness check investigates such a case.

5.3 Sub-Sample Analysis

We conclude our robustness analysis by estimating the benchmark specification for a sub-sample period that starts in 1984, which covers the period of the Great Moderation. Table 1 shows that the behavior of the series we use in this paper has in fact changed. The volatility of all variables is smaller in the sub-sample than in the full sample with the exception of real unit labor cost. This need not have a dramatic effect on our estimates since the decline in consumption growth volatility is compensated by increased volatility of the marginal cost proxy. More detrimental is the change in comovement pattern between these two series; to wit, the contemporaneous correlation between mc_t and Δc_t is a positive 0.23 over the sub-sample. Since we show above that the latter enters the theoretical NKPC with a negative coefficient, this is likely to counter the effects of marginal cost.

Our concerns are only partially borne out by the estimates reported in Table 8. The standard NKPC estimates in Panel A show a shift towards a stronger weight on forward-looking behavior compared to the full sample and an overall better fit, although the structural estimates impart a value for the indexation parameter that is much higher than in the benchmark sample. Panel B shows the estimates of the deep habits NKPC using the benchmark calibration with VAR-based forecasts. The estimates do not show dramatic differences to those in Table 5. The relative fit of the sub-sample estimation is worse, but still much improved over the standard NKPC. However, standard errors of the estimates are surprisingly large, technically rendering the NKPC coefficient statistically insignificant at the 10%-level. This arguably reflects the changing pattern of the comovement between the regressors. Moreover, it may also reflect that unit labor cost may not be the best proxy for marginal cost for this sub-sample period.[15] Nevertheless, even the sub-sample analysis shows that deep habits are a central component to explaining inflation dynamics per the NKPC.

6 Conclusion

We show in this paper that deep habits in preferences are an essential element in understanding inflation dynamics. Compared to a standard version of the NKPC, a deep habits specification is an improvement in terms of fit and in terms of smaller standard errors of the estimated parameters. The estimated NKPC under deep habits also puts much less weight on lagged inflation. This suggests a lower degree of intrinsic inflation persistence, where the required propagation is derived from the properties of the imputed driving process. The impact of deep habits on the latter stems from two influences. First, the model implies additional regressors, specifically consumption growth and a marginal value of demand term. This in and of itself produces a better fit, but we also show that a large part of the improved performance is due to the altered responsiveness of inflation to the coefficients in the NKPC. Deep habits therefore preserve the standard transmission mechanism from marginal cost movements to inflation, but reinforce this through additional feedback.

The main concern about the validity of our results stems from the partial equilibrium nature of our analysis, that is, we only estimate a single equation that should naturally be seen as a part of a larger general equilibrium model. This is reflected in two aspects. First, the elements of the driving process are treated as exogenous regressors, albeit ones that still suffer from endogeneity problems. This requires the use of instrumental variables in the estimation, that may themselves be of dubious quality. Estimation of the full equilibrium model with, for instance, likelihood-based methods obviates this problem since the likelihood function and the application of the Kalman-filter automatically constructs the optimal instruments. The drawback of a systems-approach, however, is the possibility of misspecification. Resolving this issue is outside the scope of this paper. We take comfort, however, from the empirical estimates of Ravn et al. (2010), which are close to ours, despite a different empirical method that does utilize more information. The second problematic aspect lies in the way we impute the unobservable variables. We rely on present-value computations that are known to impose weak restrictions on the imputed variables. It would therefore be a useful exercise to consider alternative methods for backing out unobservables.

[15] See the discussion in Galí et al. (2005) and Nason and Smith (2008).

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Table 1. Business Cycle Statistics

Sample Period : 1955 : 1 − 2011 : 2

Variable	s.d.(%)	Cross-Correlation			
		y	π	$rulc$	Δc
GDP	3.97	1	0.15	-0.20	0.14
Inflation	0.58		1	0.24	-0.28
RULC	3.44			1	-0.09
Cons.Growth	0.71				1

Sample Period : 1984 : 1 − 2011 : 2

Variable	s.d.(%)	Cross-Correlation			
		y	π	$rulc$	Δc
GDP	3.31	1	0.16	0.42	0.28
Inflation	0.25		1	0.27	-0.11
RULC	3.71			1	0.23
Cons.Growth	0.56				1

Table 2. Benchmark Calibrated Parameter Values

Parameter	Definition	Value	Source
β	Discount Factor	0.99	Annual Real Interest Rate
σ	Risk Aversion	1	Log-utility
θ	Deep Habits	0.85	Ravn et al. (2010)
ϵ	Elasticity of Demand	2.48	Ravn et al. (2010)
$\rho_{\Delta c}$	AR(1)-coefficient	0.31	Authors' Estimates
ρ_{mc}	AR(1)-coefficient	0.98	Authors' Estimates

Table 3. GMM Estimates: Standard NKPC

Specification				
Unrestricted NKPC	γ_f	γ_b	κ	$J(7)$
	0.791	0.197	0.004	5.428
	(0.055)	(0.058)	(0.001)	(0.608)
Restricted NKPC	γ_f		κ	$J(8)$
$\gamma_b = 0$	0.989		0.006	5.813
	(0.014)		(0.002)	(0.668)
Structural NKPC	η		κ'	$J(8)$
$\beta = 0.99$	0.271		0.004	5.333
	(0.092)		(0.001)	(0.722)

Note: The numbers in parentheses are standard errors. For J-statistics, the numbers in parentheses are p-values. For the structural NKPC, $\kappa' \equiv (\varepsilon - 1)/\zeta$. The instrument set includes 4 lags of inflation and 2 lags of marginal cost, output and wage inflation. The adjusted sample period for the estimation is 1956Q3 to 2011Q2.

Table 4. GMM Estimates: Deep Habits NKPC, AR-based

Specification				
Unrestricted NKPC	γ_f	γ_b	κ	$J(7)$
	0.857	0.105	0.008	4.236
	(0.041)	(0.045)	(0.002)	(0.752)
Restricted NKPC	γ_f		κ	$J(8)$
$\gamma_b = 0$	0.984		0.010	3.133
	(0.008)		(0.001)	(0.926)
Structural NKPC	η		ζ	$J(10)$
$\beta = 0.99$, $\sigma = 1$	0.110		203.455	1.068
$\varepsilon = 2.48$, $\theta = 0.85$	(0.011)		(7.377)	(0.998)

Note: The numbers in parentheses are standard errors. For J-statistics, the numbers in parentheses are p-values. The instrument set includes 4 lags of inflation and 2 lags of constructed driving process, output and wage inflation. The adjusted sample period for the estimation is 1956Q3 to 2011Q2.

Table 5. GMM Estimates: Deep Habits NKPC, VAR-based

Specification				
Unrestricted NKPC	γ_f	γ_b	κ	$J(7)$
	0.929	0.034	0.017	2.251
	(0.028)	(0.031)	(0.002)	(0.945)
Restricted NKPC	γ_f		κ	$J(8)$
$\gamma_b = 0$	0.990		0.010	3.133
	(0.011)		(0.003)	(0.926)
Structural NKPC	η		ζ	$J(8)$
$\beta = 0.99$	0.138		334.825	1.007
	(0.004)		(10.165)	(0.998)

Note: The numbers in parentheses are standard errors. For J-statistics, the numbers in parentheses are p-values. The instrument set includes 4 lags of inflation and 2 lags of constructed driving process, output and wage inflation. The adjusted sample period for the estimation is 1956Q3 to 2011Q2.

Table 6. Robustness: Alternative Calibration

Specification	γ_f	γ_b	κ	$J(7)$
Unrestricted NKPC	0.825	0.157	0.005	6.942
$\theta = 0.15$	(0.059)	(0.059)	(0.002)	(0.543)
Unrestricted NKPC	0.902	0.064	0.004	4.615
$\theta = 0.95$	(0.051)	(0.053)	(0.001)	(0.707)
Unrestricted NKPC	0.845	0.126	0.006	4.921
$\theta = 0.60$	(0.058)	(0.066)	(0.002)	(0.670)
Unrestricted NKPC	0.884	0.071	0.008	4.565
$\sigma = 3$	(0.047)	(0.051)	(0.002)	(0.713)
Unrestricted NKPC	0.841	0.130	0.006	4.859
$\varepsilon = 11$	(0.058)	(0.066)	(0.002)	(0.677)

Note: The numbers in parentheses are standard errors. For J-statistics, the numbers in parentheses are p-values. The instrument set includes 4 lags of inflation and 2 lags of constructed driving process, output and wage inflation. The adjusted sample period for the estimation is 1956Q3 to 2011Q2.

Table 7. Robustness: Alternative Specifications

Specification					
Unrestricted NKPC	γ_f	γ_b	κ_{mc}	$\kappa_{\Delta c}$	$J(8)$
	0.825	0.157	0.005	0.069	6.942
	(0.059)	(0.059)	(0.002)	(0.030)	(0.543)
Restricted NKPC	γ_f		κ_{mc}	$\kappa_{\Delta c}$	$J(9)$
$\gamma_b = 0$	1.002		0.004	0.047	7.871
	(0.022)		(0.002)	(0.042)	(0.547)
Structural NKPC	η		ζ	θ	$J(9)$
$\beta = 0.99$	0.198		230.699	0.894	6.710
$\sigma = 1$, $\varepsilon = 2.48$	(0.084)		(62.834)	(0.038)	(0.667)
Structural NKPC	η		ζ	θ	$J(9)$
$\beta = 0.99$	0.198		254.654	0.843	6.710
$\sigma = 3$, $\varepsilon = 2.48$	(0.084)		(74.851)	(0.052)	(0.667)
Structural NKPC	η		ζ	θ	$J(9)$
$\beta = 0.99$	0.198		1228.934	0.956	6.709
$\sigma = 3$, $\varepsilon = 11$	(0.084)		306.754	(0.015)	(0.667)

Note: The numbers in parentheses are standard errors. For J-statistics, the numbers in parentheses are p-values. The instrument set includes 4 lags of inflation and 2 lags of marginal cost, consumption growth, output and wage inflation. The adjusted sample period for the estimation is 1956Q3 to 2011Q2.

Table 8. Robustness: Sub-Sample 1984:1-2011:2

Panel A: Standard NKPC

Unrestricted NKPC	γ_f	γ_b	κ	$J(7)$
	0.833	0.234	0.004	4.209
	(0.081)	(0.052)	(0.001)	(0.755)
Restricted NKPC	γ_f		κ	$J(8)$
$\gamma_b = 0$	1.146		0.005	5.051
	(0.057)		(0.002)	(0.752)
Structural NKPC	η		κ'	$J(8)$
$\beta = 0.99$	0.402		0.003	2.984
	(0.056)		(0.001)	(0.935)

Panel B: Deep Habits NKPC

Unrestricted NKPC	γ_f	γ_b	κ	$J(7)$
	0.988	0.074	0.009	3.830
	(0.196)	(0.140)	(0.012)	(0.799)
Restricted NKPC	γ_f		κ	$J(8)$
$\gamma_b = 0$	1.073		0.013	3.881
	(0.118)		(0.009)	(0.868)
Structural NKPC	η		ζ	$J(8)$
$\beta = 0.99$	0.131		139.677	4.420
	(0.159)		(128.87)	(0.817)

Note: The numbers in parentheses are standard errors. For J-statistics, the numbers in parentheses are p-values. The instrument set includes 4 lags of inflation and 2 lags of constructed driving process, output and wage inflation. The sample period for the estimation is 1984Q1 to 2011Q2.

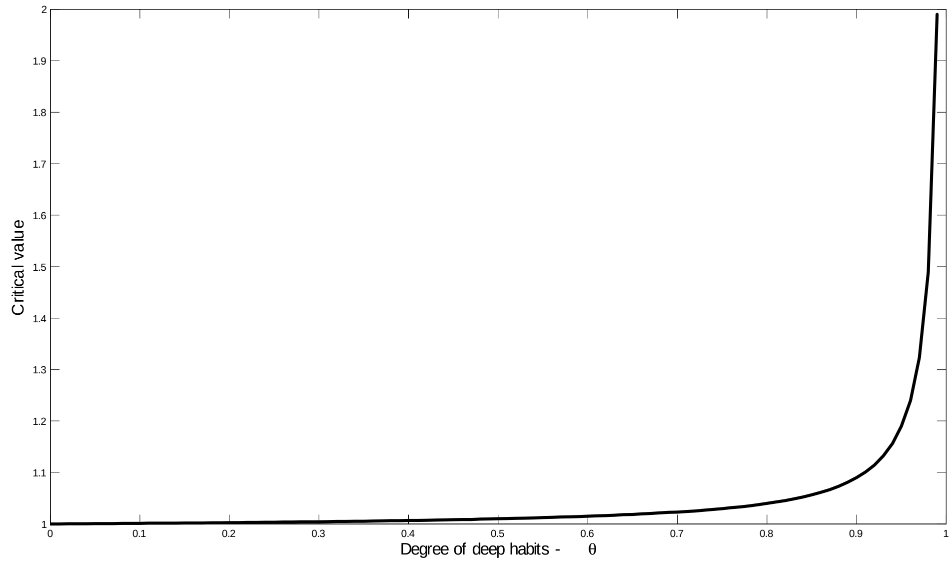

Figure 1: Critical value of ε for ψ_1 to be positive

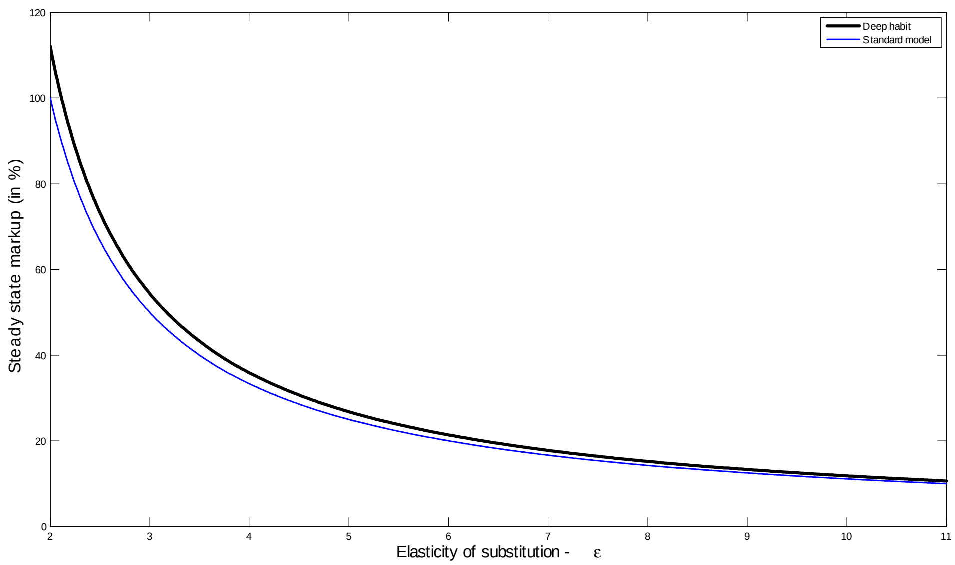

Figure 2: Steady-state markup as ε varies

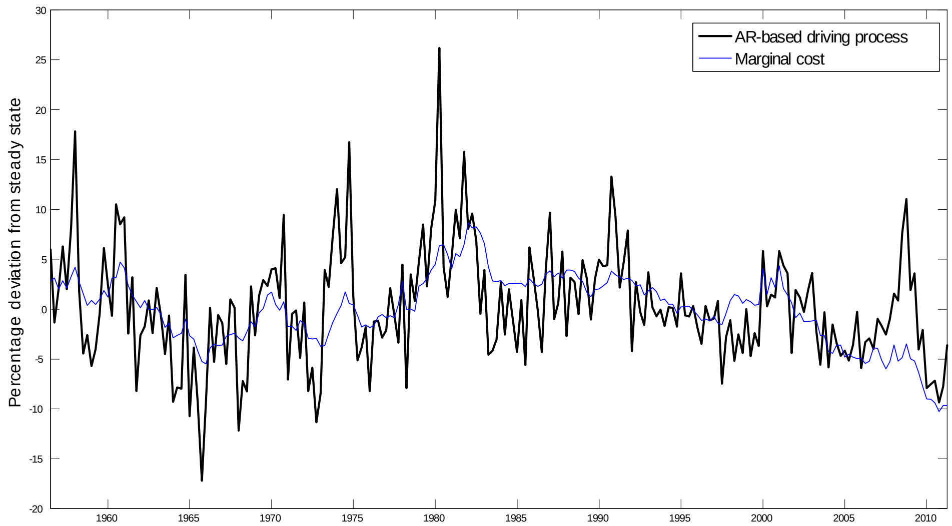

Figure 3: AR-based constructed driving process and marginal cost

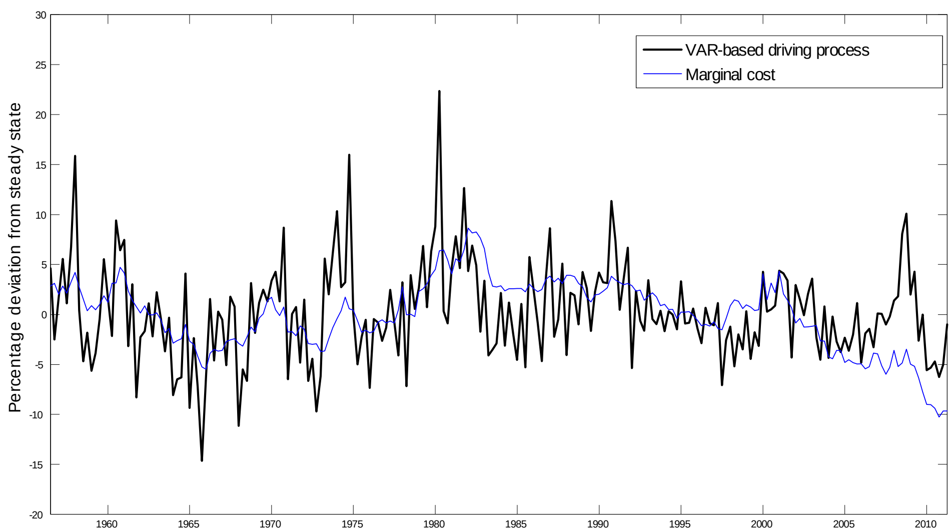

Figure 4: VAR-based constructed driving process and marginal cost

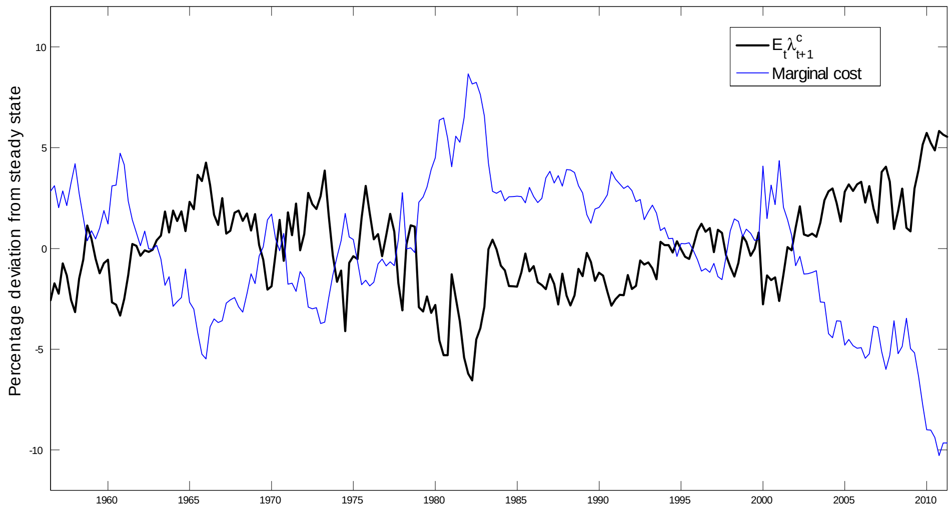

Figure 5: Expected marginal value of future demand and marginal cost

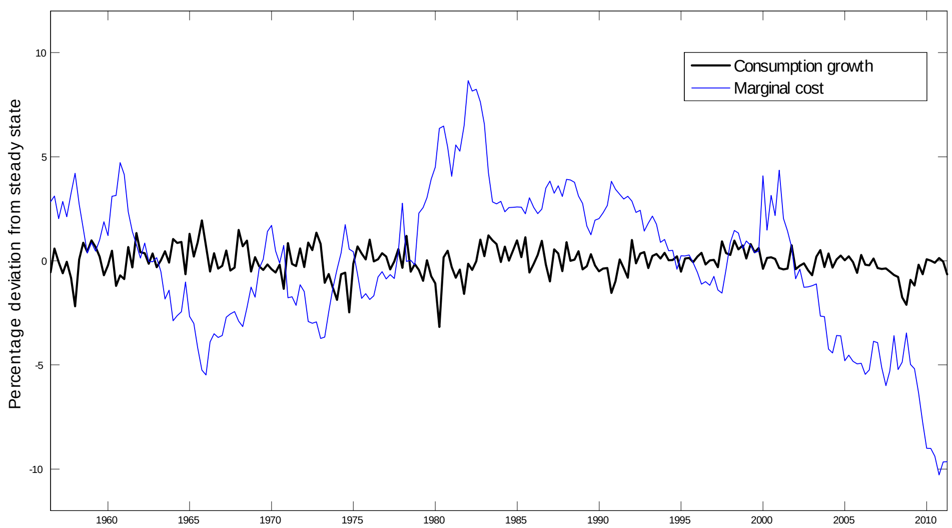

Figure 6: Consumption growth and marginal cost

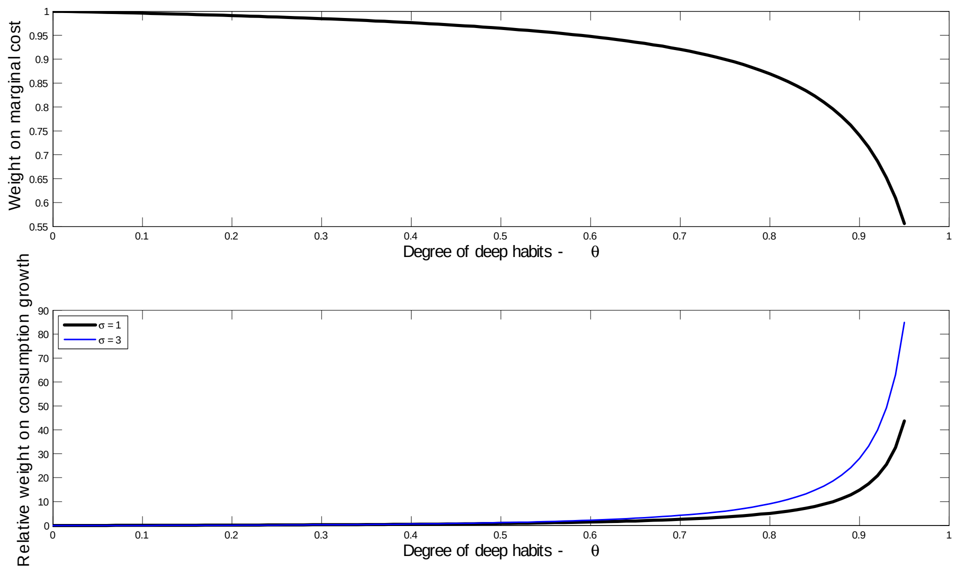

Figure 7: Weights on marginal cost and consumption growth as θ varies

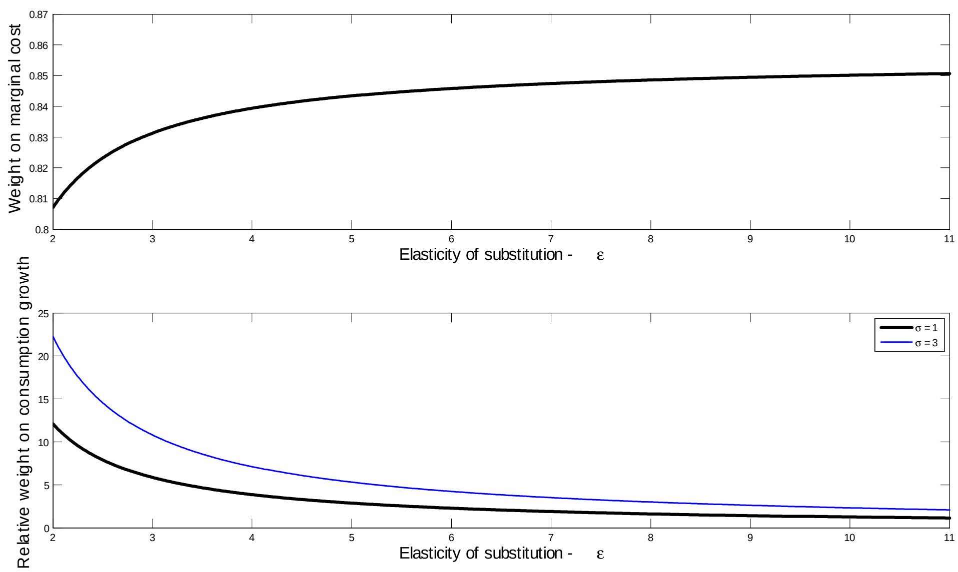

Figure 8: Weights on marginal cost and consumption growth as ε varies

A Appendix: Analytical Derivations

A.1 Derivation of Equation (18) and the Deep Habits NKPC

Substituting the definitions of the habit stock $x_t = c_t - \theta c_{t-1}$ and the weighted inflation rate $\overline{\pi}_t = \pi^{1-\eta} (\pi_{t-1})^{\eta}$ into equation (12), we have:

$$\varepsilon \lambda_t^c (c_t - \theta c_{t-1}) + \zeta \pi_t \left(\pi_t - \pi^{1-\eta} (\pi_{t-1})^{\eta} \right) c_t = c_t + \beta \zeta E_t \left(\frac{x_{t+1}}{x_t} \right)^{-\sigma} \pi_{t+1} \left(\pi_{t+1} - \pi^{1-\eta} (\pi_t)^{\eta} \right) c_{t+1}. \tag{A1}$$

Log-linearization of the equation above gives:

$$\varepsilon \lambda^c c (1-\theta) \left[\tilde{\lambda}_t^c + \frac{1}{1-\theta} \tilde{c}_t - \frac{\theta}{1-\theta} \tilde{c}_{t-1} \right] + \zeta c (\tilde{\pi}_t - \eta \tilde{\pi}_{t-1}) = c \tilde{c}_t + \beta \zeta c E_t (\tilde{\pi}_{t+1} - \eta \tilde{\pi}_t). \tag{A2}$$

Using the steady-state relation:

$$\varepsilon \lambda^c (1-\theta) = 1, \tag{A2b}$$

which is obtained from equation (A1), we can simplify equation (A2) as:

$$\tilde{\pi}_t = \frac{\eta}{1+\beta\eta} \tilde{\pi}_{t-1} + \frac{\beta}{1+\beta\eta} E_t \tilde{\pi}_{t+1} - \frac{1}{(1+\beta\eta)\zeta} \left[\tilde{\lambda}_t^c + \frac{\theta}{1-\theta} (\tilde{c}_t - \tilde{c}_{t-1}) \right]. \tag{A3}$$

Imposing symmetry and substituting the discount factor q_t, $x_t = c_t - \theta c_{t-1}$, and $\lambda_t^y = mc_t$ into equation (10), we have:

$$mc_t + \lambda_t^c = 1 + \theta \beta E_t \left(\frac{c_{t+1} - \theta c_t}{c_t - \theta c_{t-1}} \right)^{-\sigma} \lambda_{t+1}^c. \tag{A4}$$

Log-linearization of the equation above gives:

$$mc \widetilde{mc}_t + \lambda^c \tilde{\lambda}_t^c = \theta \beta \lambda^c E_t \left[-\frac{\sigma}{1-\theta} (\tilde{c}_{t+1} - \theta \tilde{c}_t - \tilde{c}_t + \theta \tilde{c}_{t-1}) + \tilde{\lambda}_{t+1}^c \right]. \tag{A5}$$

Using the steady-state relation:

$$mc = 1 - (1 - \beta\theta) \lambda^c, \tag{A5b}$$

which is obtained from equation (A4) and (A2b), we can simplify equation (A5) as:

$$\tilde{\lambda}_t^c = \beta \theta E_t \tilde{\lambda}_{t+1}^c - \beta \theta E_t \left[\frac{\sigma}{1-\theta} (\Delta \tilde{c}_{t+1} - \theta \Delta \tilde{c}_t) \right] - (\varepsilon (1-\theta) - (1-\theta\beta)) \widetilde{mc}_t, \tag{A6}$$

which is equation (18) in the main text. Substituting equation (A6) into equation (A3) and rearranging, we then obtain equation (13) in the main text.

A.2 Derivation of Equation (22)

Substituting (20) and (21) into (19), we get:

$$\begin{aligned}
\widetilde{\lambda}_t^c &= \sum_{j=0}^{\infty} (\beta\theta)^j E_t \left\{ \left[-\beta \frac{\sigma\theta}{1-\theta} \left(1-\beta\theta^2\right) \Delta\widetilde{c}_{t+j+1} \right] - \left[\varepsilon(1-\theta) - (1-\beta\theta)\right] \widetilde{mc}_{t+j} \right\} \\
&\quad + \beta \frac{\sigma\theta^2}{1-\theta} \Delta\widetilde{c}_t. \\
&= -\sum_{j=0}^{\infty} (\beta\theta\rho_{\Delta c})^j \left[\beta \frac{\sigma\theta}{1-\theta} \left(1-\beta\theta^2\right) \rho_{\Delta c} \Delta\tilde{c}_t \right] \\
&\quad - \sum_{j=0}^{\infty} (\beta\theta\rho_{mc})^j \left(\varepsilon(1-\theta) - (1-\theta\beta)\right) \widetilde{mc}_t + \beta \frac{\sigma\theta^2}{1-\theta} \Delta\tilde{c}_t \\
&= \psi_{\Delta c} \Delta\tilde{c}_t - \psi_{mc} \widetilde{mc}_t, \qquad \text{(A7)}
\end{aligned}$$

where:

$$\begin{aligned}
\psi_{\Delta c} &= \beta \frac{\sigma\theta^2}{1-\theta} - \frac{1}{1-\beta\theta\rho_{\Delta c}} \beta \frac{\sigma\theta}{1-\theta} \left(1-\beta\theta^2\right) \rho_{\Delta c} \\
&= \beta \frac{\sigma\theta}{1-\theta} \left[\theta - \frac{1-\beta\theta^2}{1-\beta\theta\rho_{\Delta c}} \rho_{\Delta c} \right], \qquad \text{(A8)}
\end{aligned}$$

$$\psi_{mc} = \frac{\varepsilon(1-\theta) - (1-\beta\theta)}{1-\theta\beta\rho_{mc}}. \qquad \text{(A9)}$$

Since $\beta \in (0,1)$ and $|\rho_{mc}| \in [0,1)$, the condition for $\psi_{mc} > 0$ is $\varepsilon > (1-\beta\theta)/(1-\theta)$, which is the same condition as for $\psi_1 > 0$. It is then easy to verify that a sufficient condition for $\psi_{\Delta c} \geq 0$ is $\theta > \rho_{\Delta c}$.

A.3 Steady-State Markup

Combining equations (A2b) and (A5b), we obtain:

$$mc = \frac{\varepsilon(1-\theta) - (1-\theta\beta)}{\varepsilon(1-\theta)}. \qquad \text{(A10)}$$

The steady-state markup is the inverse of steady-state real marginal cost. We therefore have:

$$\text{Steady-state markup} = \frac{\varepsilon(1-\theta)}{\varepsilon(1-\theta) - (1-\theta\beta)}. \qquad \text{(A11)}$$

A.4 Derivation of Equations (29) and (30)

Substituting (23) and (21) into (13), we have:

$$\begin{aligned}
\widetilde{\pi}_t &= \frac{\beta}{1+\beta\eta}E_t\widetilde{\pi}_{t+1} + \frac{\eta}{1+\beta\eta}\widetilde{\pi}_{t-1} + \psi_1\widetilde{mc}_t + \psi_2\rho_{\Delta c}\Delta\widetilde{c}_t - ... \\
&\quad -\psi_3\Delta\widetilde{c}_t - \psi_4\left(\psi_{\Delta c}\rho_{\Delta c}\Delta\widetilde{c}_t - \psi_{mc}\rho_{mc}\widetilde{mc}_t\right) \\
&= \frac{\beta}{1+\beta\eta}E_t\widetilde{\pi}_{t+1} + \frac{\eta}{1+\beta\eta}\widetilde{\pi}_{t-1} + \left(\psi_1 + \psi_4\psi_{mc}\rho_{mc}\right)\widetilde{mc}_t \\
&\quad - \left(\psi_3 + \psi_4\psi_{\Delta c}\rho_{\Delta c} - \psi_2\rho_{\Delta c}\right)\Delta\widetilde{c}_t,
\end{aligned} \tag{A12}$$

which is in the form of equation (29) in the main text. Factoring out $\frac{\varepsilon-1}{\zeta(1+\beta\eta)}$, we can write equation (A12) as equation (30), where:

$$\begin{aligned}
\tilde{\psi}_5 &= \tilde{\psi}_1 + \tilde{\psi}_4\psi_{mc}\rho_{mc} \\
&= \frac{\varepsilon(1-\theta) - (1-\beta\theta)}{\varepsilon-1} + \frac{\beta\theta}{\varepsilon-1}\frac{\varepsilon(1-\theta)-(1-\theta\beta)}{1-\beta\theta\rho_{mc}}\rho_{mc} \\
&= \frac{\varepsilon(1-\theta)-(1-\theta\beta)}{\varepsilon-1}\frac{1}{1-\beta\theta\rho_{mc}}.
\end{aligned}$$

$$\begin{aligned}
\tilde{\psi}_6 &= \tilde{\psi}_3 + \tilde{\psi}_4\psi_{\Delta c} - \tilde{\psi}_2\rho_{\Delta c} \\
&= \frac{1+\sigma\beta\theta}{\varepsilon-1}\frac{\theta}{1-\theta} + \frac{\beta\theta}{\varepsilon-1}\frac{\beta\sigma\theta}{1-\theta}\left(\theta - \frac{1-\beta\theta^2}{1-\beta\theta\rho_{\Delta c}}\rho_{\Delta c}\right)\rho_{\Delta c} - \frac{\sigma\beta\theta}{1-\theta}\frac{1}{\varepsilon-1}\rho_{\Delta c} \\
&= \frac{1}{\varepsilon-1}\frac{\theta}{1-\theta}\left[1+\sigma\beta\theta+\sigma\beta^2\theta\left(\theta - \frac{1-\beta\theta^2}{1-\beta\theta\rho_{\Delta c}}\rho_{\Delta c}\right)\rho_{\Delta c} - \sigma\beta\rho_{\Delta c}\right] \\
&= \frac{1}{\varepsilon-1}\frac{\theta}{1-\theta}\left[1+\frac{\sigma\beta\left(\theta-\rho_{\Delta c}\right)}{1-\beta\theta\rho_{\Delta c}}\right].
\end{aligned}$$

$\tilde{\psi}_5$ is positive if $\varepsilon > (1-\beta\theta)/(1-\theta)$, which we have imposed. A sufficient condition for $\tilde{\psi}_6$ to be non-negative is $\theta > \rho_{\Delta c}$, which is the same condition for $\psi_{\Delta c} \geq 0$.