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Department of Economics
University of Munich

Volkswirtschaftliche Fakultät
Ludwig-Maximilians-Universität München

Trade Liberalization and Welfare Inequality: a Demand-Based Approach *

Alexander Tarasov†
University of Munich

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Abstract

There is strong evidence that different income groups consume different bundles of goods. This evidence suggests that trade liberalization can affect welfare inequality within a country via changes in the relative prices of goods consumed by different income groups (the price effect). In this paper, I develop a framework that enables us to explore the role of the price effect in determining welfare inequality. There are two core elements in the model. First, I assume that heterogenous in income consumers share identical but nonhomothetic preferences. Secondly, I consider a monopolistic competition environment that leads to variable markups affected by trade and trade costs. I find that trade liberalization does affect the prices of different goods differently and, as a result, can benefit some income classes more than others. In particular, I show that the relative welfare of the rich with respect to that of the poor has a hump shape as a function of trade costs.

Keywords: nonhomothetic preferences, income distribution, monopolistic competition.

JEL classification: F12

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†Seminar for International Economics, Department of Economics, University of Munich, Ludwigstr. 28, 80539, Munich, Germany. E-mail: alexander.tarasov@lrz.uni-muenchen.de.

1 Introduction

It is well known that different income classes consume different bundles of goods. This evidence suggests that trade liberalization can affect welfare inequality within a country through at least two effects. First, trade liberalization can lead to changes in income distribution in a country and, thereby, affect the income inequality (*the income effect*). Secondly, trade liberalization can have a different impact on prices of different goods, affecting welfare inequality through changes in the relative prices of goods consumed by different income groups (*the price effect*). While the income effect is intensively explored in the trade literature (see Goldberg and Pavcnik (2007)), the price effect is not paid much attention.

In this paper, I construct a general equilibrium model of trade between symmetric countries that enables us to examine the role of the price effect in determining welfare inequality. The core element of the model is nonhomothetic consumer preferences.[1] Indeed, trade models with homothetic preferences are not appropriate for studying the impact of trade liberalization on welfare inequality through the price effect, as irrespective of their income, consumers purchase identical bundles of goods. In contrast, in the present model, nonhomotheticity of preferences leads to that some goods (luxuries) are available only to the rich. Another key element is a monopolistic competition environment. Imperfect competition induces variable markups and, therefore, allows us to explore the effects of trade liberalization on prices set by firms. In particular, I find that trade liberalization does affect the prices of different goods (necessities and luxuries) differently and, as a result, can benefit some income classes more than others.

The key assumption about consumer preferences is that goods are indivisible and consumers purchase at most one unit of each good (see Murphy et al. (1989) and Matsuyama (2000)). This implies that, given the prices, goods are arranged so that consumers can be considered as moving down a certain list in choosing what to buy. For instance, in developing countries, consumers first buy food, then clothing, then move up the chain of durables from kerosene stoves to refrigerators, to cars. Furthermore, consumers with higher income buy the same bundle of goods as poorer consumers plus some others.[2]

[1] There is strong empirical evidence that consumer preferences are nonhomothetic (see for example Deaton and Muellbauer (1980) and Hunter and Markusen (1988)).

[2] This structure of consumer preferences has enough flexibility to be applied as to the whole economy as to a certain industry where goods differ in quality. On the one hand, each good can be interpreted as a distinct good

I assume that each good is produced by a distinct firm and goods differ according to the valuations consumers attach to them.[3] Depending on the valuations placed on their goods, firms decide whether to serve both domestic and foreign markets, to serve only the domestic market, or not to produce at all. I limit the analysis in the paper to a two-class society (the rich and the poor).[4] Then, given the preferences, firms serving a certain market face a trade-off between selling to the both income classes at a lower price and selling only to the rich at a higher price. Specifically, firms with sufficiently high valuations find it profitable to sell to all consumers, while firms with low valuations decide to sell only to the rich. Hence, available goods in each market are divided into two groups: the necessities include goods that are consumed by both income classes, while the luxuries include goods that are consumed by the rich only.

Since the income distribution in the model is exogenous, I focus only on the price effect and do not explore the impact of trade liberalization on income distribution. I find that the reduction in trade costs affects the prices of necessities and luxuries differently and, therefore, changes welfare inequality within a country via the price effect. In particular, I show that the relative welfare of the rich with respect to that of the poor has a hump shape as a function of trade costs. If trade costs are sufficiently low, then further trade liberalization benefits the poor more, while if trades costs are high enough, then the rich gain more from the reduction in trade costs.

To understand better the intuition behind these findings, consider separately two submarkets: one for the necessities and one for the luxury goods. Since the rich consume the same bundle of goods as the poor plus the luxuries, the relative welfare in the model is determined by the relative prices of the luxuries with respect to those of the necessities. If trade costs are sufficiently low, then exporting firms find it profitable to serve both income classes in a foreign market: exporting firms with high valuations of their goods serve all consumers, while exporting firms with lower

sold in the market. On the other hand, we might think that firms sell not distinct goods but some characteristics of a good produced in a certain industry. For instance, consider a car industry. Each good can be treated as some characteristic of a car. The poor purchase main characteristics associated with a car, while the rich buy the same characteristics as the poor plus some additional luxury characteristics. That is, both groups of consumers buy the same good but of different quality.

[3]By the valuation of a good, I mean the utility delivered to consumers from the consumption of one unit of this good.

[4]Income heterogeneity in the model is introduced by assuming that consumers differ according to the efficiency units of labor they are endowed with. That is, the income distribution is exogenous and shaped by the relative income of the rich and the fraction of the rich. Hence, I focus only on the price effect and do not explore the impact of trade liberalization on income distribution.

valuations serve only the rich. In this case, a rise in trade costs leads to that some exporting firms exit from both foreign submarkets.[5] This reduces the intensity of competition in the submarkets and, therefore, drives up the prices. However, since exporting firms that exit from the submarket for the necessities do not stop exporting, but enter the submarket for the luxury goods (increasing the intensity of competition in this submarket), the prices of the luxuries rise by less than those of the necessities. This in turn implies that the rich lose relatively less from a rise in trade costs than the poor do. I find that, depending on the parameters of the model, the rich can even gain from higher trade costs. In contrast, if trade costs are high enough, then exporting firms find it profitable to serve only the rich. Then, a rise in trade costs does not have a direct impact on the poor and, as a result, the rich lose relatively more.

This paper is closely related to Fajgelbaum et al. (2009), who develop a general equilibrium model with nonhomothetic preferences for studying trade in vertically differentiated products. Their framework also implies that trade liberalization can affect welfare of different income groups differently. However, the mechanism developed in their paper is based on the home market effect (à la Krugman (1980)), while the present paper provides another, possibly complimentary, view, which is based on the price effect. Ramezzana (2000) and Foellmi et al. (2007) use the similar preference structure in a monopolistic competition framework to examine how similarities in per capita incomes affect trade volumes between countries. In these papers, consumers are assumed identical within a country and the impact of trade on relative welfare is not explored. Mitra and Trindade (2005) also consider a model of monopolistic competition with nonhomothetic preferences. However, they focus on the income effect of trade liberalization rather than on the price effect.

The present paper also complements a broad strand of literature that explores the role of supply-side factors in determining trade patterns. Markusen (1986) extends the Krugman type model of trade with monopolistic competition and differences in endowments by adding nonhomothetic demand. He examines the role of per capita income in interindustry and intra-industry trade. Flam and Helpman (1987), Stokey (1991), and Matsuyama (2000) develop a Ricardian model of North-South trade with nonhomothetic preferences. They examine the

[5] Some exporting firms that served all consumers start selling only to the rich, whereas some firms that served only the rich stop exporting at all.

impact of technological progress, population growth, and redistribution policy on the patterns of specialization and welfare. Stibora and Vaal (2005) extend the model in Matsuyama (2000) by studying the effects of trade liberalization. They show that the South loses in terms of trade from unilateral trade liberalization, while the North may gain by liberalizing its trade. Fieler (2009) modifies a Ricardian framework à la Eaton and Kortum (2002) by introducing nonhomothetic preferences and technology distribution across sectors. This modification allows her to separate the effects of per capita income and population size on trade volumes.

The rest of the paper is organized as follows. Section 2 introduces the basic concepts for the closed economy case of the model. Section 3 extends the analysis to the open economy case and explores the effects of trade liberalization on prices, market structure, and consumer welfare. Section 4 concludes.

2 Closed Economy

The structure of the closed economy version of the model is adopted from Tarasov (2009).

2.1 Consumption

In the model, all consumers have identical preferences that are represented by the following utility function:

$$U = \int_{\omega \in \Omega} b(\omega) x(\omega) d\omega,$$

where Ω is the set of available goods in the economy, $b(\omega)$ is the valuation of good ω, and $x(\omega) \in \{0, 1\}$ is the consumption of good ω. Note that goods are indivisible and consumers can purchase at most one unit of each good. To find the optimal consumption bundle, consumer i maximizes

$$U_i = \int_{\omega \in \Omega} b(\omega) x_i(\omega) d\omega \tag{1}$$

subject to her budget constraint

$$\int_{\omega \in \Omega} p(\omega) x_i(\omega) d\omega \leq I_i, \tag{2}$$

where I_i is the income of consumer i and $p(\omega)$ is the price of good ω. This maximization problem implies that

$$x_i(\omega) = 1 \iff \frac{b(\omega)}{p(\omega)} \geq Q_i, \tag{3}$$

where Q_i is the Lagrange multiplier associated with the maximization problem and represents the marginal utility of income of consumer i. In words, consumer i purchases good ω if and only if the valuation to price ratio $\frac{b(\omega)}{p(\omega)}$ of this good is sufficiently high.

2.2 Production

The only factor of production in the economy is labor. There is free entry into the market. Each good ω is produced by a distinct firm. To enter the market, firms have to pay costs f_e that are sunk. If a firm incurs the costs of entry, it obtains a draw b of the valuation of its good from the common distribution $G(b)$ with the support on $[0, B]$. I assume that $G'(b) = g(b)$ exists. This captures the idea that before entry, firms do not know how well they will end up doing due to uncertainty in valuations of their products. Such differences among goods generate ex-post heterogeneity across firms. Depending on the valuation drawn, firms choose whether to exit from the market or to stay. Firms that decide to stay engage in price competition with other firms. I assume that marginal cost of production is identical for all firms and is equal to c, i.e., it takes c units of labor (which are paid a wage of unity) to produce a unit of any good.

In the paper, I limit the analysis to a framework with two types of consumers indexed by L and H. A consumer of type $i \in \{L, H\}$ is endowed with I_i units of labor where $I_H > I_L$. The fraction of consumers with income I_H in the aggregate mass N of consumers is given by α_H. Then, the total labor supply in the economy is equal to $N(\alpha_H I_H + (1 - \alpha_H) I_L)$. I assume that each consumer owns a balanced portfolio of shares of all firms producing the goods. Note that due to free entry, the total firm profits are equal to zero in the equilibrium. This implies that the value of any balanced portfolio is equal to zero. Hence, the total income of consumer i is equal to her labor income I_i.

Using (3), the budget constraint in (2) can be rewritten as follows:

$$\int_{\omega: \frac{b(\omega)}{p(\omega)} \geq Q_i} p(\omega) d\omega = I_i.$$

It is straightforward to see that given the prices and the valuations, the left hand side of the equation is decreasing in Q_i. This suggests that the marginal utility of income is lower for richer consumers, i.e., $Q_H < Q_L$. Hence, the preferences considered in the paper imply that rich consumers purchase the same goods as the poor plus some others. That is, available in the economy goods can be divided into two groups: the necessities include goods that are purchased by all consumers; the luxuries includes goods that are purchased only by the rich. As a result, the demand for good ω is given by

$$D(p(\omega)) = \begin{cases} N, & \text{if } \frac{b(\omega)}{p(\omega)} \geq Q_L, \\ \alpha_H N, & \text{if } Q_L > \frac{b(\omega)}{p(\omega)} \geq Q_H, \\ 0, & \text{if } \frac{b(\omega)}{p(\omega)} < Q_L. \end{cases} \tag{4}$$

Taking Q_L and Q_H as given, firms maximize their profits

$$\pi(\omega) = (p(\omega) - c)D(p(\omega)). \tag{5}$$

. The following proposition holds.

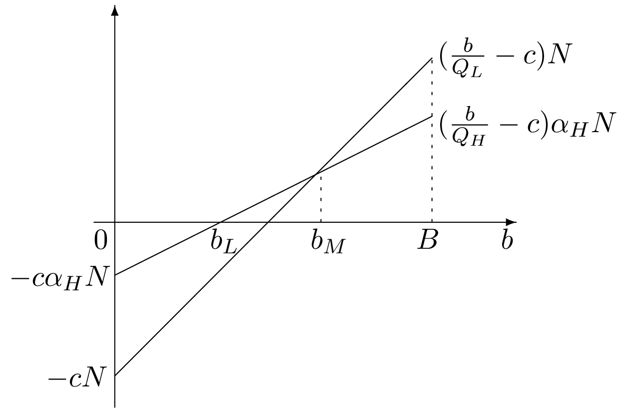
Proposition 1 *Goods from the same group have the same valuation to price ratio in the equilibrium.*

Proof. Suppose the opposite is true. Then, there exists some group, in which there are at least two goods with different $\frac{b(\omega)}{p(\omega)}$ ratios in the equilibrium. Since both goods belong to the same group, the firm producing the good with higher $\frac{b(\omega)}{p(\omega)}$ can raise its $p(\omega)$ without affecting the demand. This in turn would increase its profits contradicting the equilibrium concept. ■

A direct implication of *Proposition 1* is that if good ω is purchased by all consumers in the equilibrium, then its price is equal to $\frac{b(\omega)}{Q_L}$. Indeed, a lower price would not affect demand for the good and, thereby, would reduce the profits, while a higher price would exclude the poor from purchasing ω. Similarly, if good ω belongs to the luxury goods, then it price is given by $\frac{b(\omega)}{Q_H}$. Hence, if a firm with valuation $b(\omega)$ serves all consumers, its profits are given by

$$(p(\omega) - c)\, N = \left(\frac{b(\omega)}{Q_L} - c \right) N,$$

Figure 1: Profit Functions



while if the firm serves only the rich, its profits are given by

$$(p(\omega) - c)\alpha_H N = \left(\frac{b(\omega)}{Q_H} - c\right)\alpha_H N.$$

In other words, to maximize their profits, firms choose between selling to more people at a lower price and selling to fewer of them, but at a higher price.

In the equilibrium, the price of good ω depends only on $b(\omega)$. Therefore, hereafter I omit the notation of ω and consider prices as a function of b. Let us denote b_M as the solution of the equation

$$\left(\frac{b}{Q_L} - c\right) N = \left(\frac{b}{Q_H} - c\right)\alpha_H N. \tag{6}$$

Then,

$$\begin{aligned} \left(\frac{b}{Q_L} - c\right) N &\geq \left(\frac{b}{Q_H} - c\right)\alpha_H N, \quad \text{if } b \geq b_M, \\ \left(\frac{b}{Q_L} - c\right) N &< \left(\frac{b}{Q_H} - c\right)\alpha_H N, \quad \text{otherwise.} \end{aligned}$$

Thus, if a firm draws $b \geq b_M$, then it is more profitable for the firm to serve both types of consumers. Otherwise, the firm serves only the rich or exits. Firms with valuation b_M are indifferent between selling to all consumers or only to the rich (see *Figure 1*). In *Figure 1*, b_L is the exit cutoff such that firms with valuations $b < b_L$ exit from the market because of negative potential profits.

2.3 The Equilibrium

Let us denote M_e as the mass of firms entering the market. One can think of M_e as that there are $M_e g(b)$ different firms with a certain valuation b. In the equilibrium, two conditions should be satisfied. First, due to free entry, the expected profits of firms have to be equal to zero. Second, the goods market clears.

Definition 1 *The equilibrium in the model is defined by* $\left\{b_L,\ b_M,\ M_e,\ \{p(b)\}_{b \geq b_L},\ \{Q_i\}_{i\in\{L,H\}}\right\}$ *such that*

1) Consumers solve the utility maximization problem resulting in (3)*.*

2) By setting the prices, firms maximize their profits.

3) The expected profits of firms are equal to zero.

4) The goods market clears.

Further, I derive the equations that are sufficient to describe the equilibrium in the model. Remember that firms with valuation b_L have zero profits. This implies that $Q_H = \frac{b_L}{c}$. Using this expression for Q_H and the equation (6), we can find Q_L as a function of b_L and b_M. Namely, the following lemma holds.

Lemma 1 *In the equilibrium,*

$$p(b) = \begin{cases} \frac{b}{Q_L} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right), & \text{if } b \geq b_M, \\ \frac{b}{Q_H} = \frac{cb}{b_L}, & \text{if } b \in [b_L, b_M), \end{cases}$$

$$\pi(b) = \begin{cases} \left(b\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - 1\right) cN, & \text{if } b \geq b_M, \\ \left(\frac{b}{b_L} - 1\right) c\alpha_H N, & \text{if } b \in [b_L, b_M). \end{cases}$$

Due to free entry, the ex-ante profits of the firms are equal to zero in the equilibrium. This means that

$$\int_0^B \pi(t)dG(t) = f_e.$$

Using *Lemma 1* and taking into account that firms with $b < b_L$ exit, the last equation is equivalent to

$$\frac{f_e}{cN} + 1 = \alpha_H H(b_L) + (1 - \alpha_H) H(b_M), \tag{7}$$

where $H(x) = G(x) + \frac{\int_x^B tdG(t)}{x}$.

The goods market clearing condition implies that for any $i \in \{L, H\}$,

$$\int_{\omega \in \Omega} p(\omega) x_i(\omega) d\omega = I_i.$$

Using the findings in *Lemma 1*, it is straightforward to see that

$$I_L = cM_e \left(\frac{\alpha_H}{b_L} + \frac{(1 - \alpha_H)}{b_M} \right) \int_{b_M}^{B} tdG(t), \tag{8}$$

$$I_H - I_L = \frac{cM_e}{b_L} \int_{b_L}^{b_M} tdG(t). \tag{9}$$

Therefore, dividing the second line by the first one, we obtain

$$\frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{B} tdG(t)} = \left(\frac{I_H}{I_L} - 1 \right) \left(\alpha_H + \frac{b_L (1 - \alpha_H)}{b_M} \right). \tag{10}$$

Hence, given the parameters I_H, I_L, α_H, f_e, c, N, and the distribution of draws $G(\cdot)$, we can find the endogenous variables b_M and b_L from the following system of equations:[6]

$$\begin{cases} \frac{\int_{b_L}^{b_M} tdG(t)}{\int_{b_M}^{B} tdG(t)} = \left(\frac{I_H}{I_L} - 1 \right) \left(\alpha_H + \frac{b_L (1 - \alpha_H)}{b_M} \right), \\ \\ \frac{f_e}{cN} + 1 = \alpha_H H(b_L) + (1 - \alpha_H) H(b_M). \end{cases} \tag{11}$$

Note that if we know b_L and b_M, we can find the equilibrium value of Q_L and Q_H using *Lemma 1.* Furthermore, the mass of entrants into the industry producing the differentiated good can be found from equation (8) or (9).

[6] The existence and uniqueness of the equilibrium are proved in Tarasov (2007).

3 Open Economy

This section focuses on the open economy extension of the model described above. In particular, I develop a model of trade between two symmetric countries. The notation in this section is the same as in the previous one.

3.1 Production and Exporting

In the model, trade costs take the Samuelson's iceberg form and equal to τ. To simplify the analysis, I assume that there are no fixed costs of trade. Since the countries are symmetric, it is sufficient to describe the equilibrium conditions only for one country. As before, I assume that there are two types of consumers. That is, given the preferences, goods are divided into two groups: the necessities and luxuries. The presence of trade costs implies that some firms find it profitable to serve only the domestic market, as exporting would lead to negative profits. Hence, a firm has three options: to exit, to serve only the domestic market, or to serve both domestic and foreign markets. In the paper, I consider *pricing-to-market.* I assume that the markets are segmented and firms are able to price discriminate between domestic and foreign markets. Furthermore, it is not possible for any third party to buy a good in one country and then to resell it in the other to arbitrage price differences.

Let us denote $\pi_D(b)$ and $\pi_F(b)$ as the profits of a firm with valuation b from selling at home and abroad, respectively. Then, the total profits of a firm with b are given by

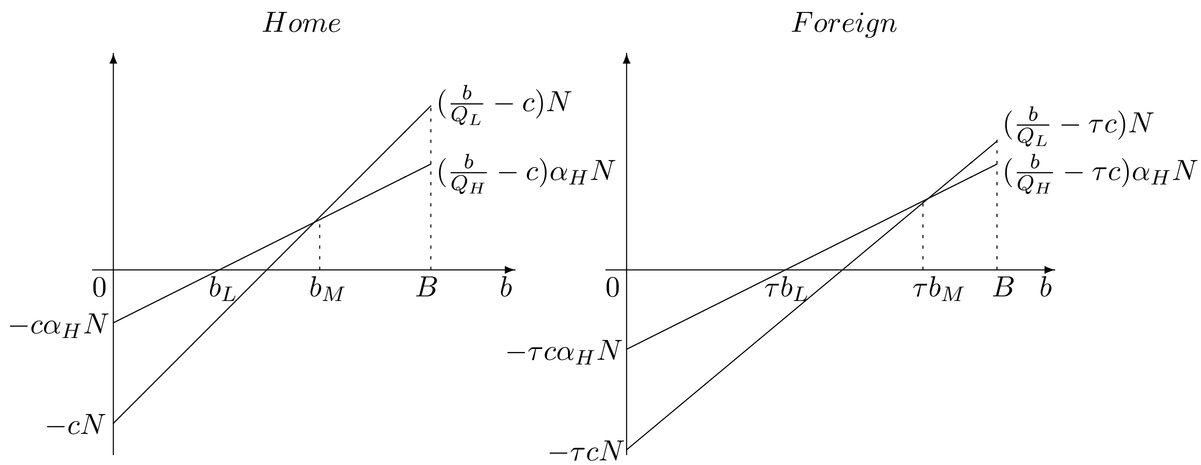
$$\pi(b) = \begin{cases} 0, & \text{if the firm exits,} \\ \pi_D(b), & \text{if the firm serves only the domestic market,} \\ \pi_D(b) + \pi_F(b), & \text{if the firm serves both the markets.} \end{cases} \tag{12}$$

By analogy with the results in the previous section, firms with valuations $b \in [b_M, B]$ serve all consumers at home, while firms with $b \in [b_L, b_M)$ serve only the rich. Therefore, the profits from selling at home are given by

$$\pi_D(b) = \begin{cases} \left(\frac{b}{Q_L} - c\right) N = \left(b\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - 1\right) cN, & \text{if } b \geq b_M, \\ \\ \left(\frac{b}{Q_H} - c\right) \alpha_H N = \left(\frac{b}{b_L} - 1\right) c\alpha_H N, & \text{if } b \in [b_L, b_M). \end{cases} \tag{13}$$

Similarly, as the countries are symmetric, it is straightforward to show (see *Figure 2*) that

Figure 2: Profit Functions: Open Economy

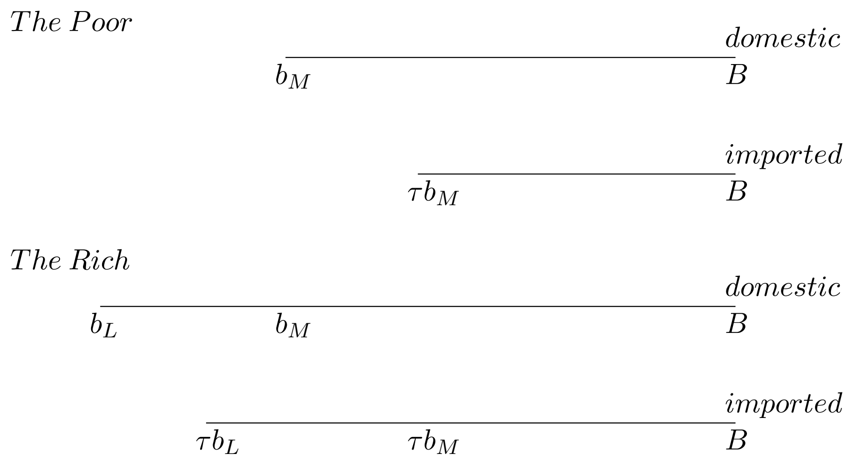


$$\pi_F(b) = \begin{cases} \left(\frac{b}{Q_L} - \tau c\right) N = \left(b\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) - \tau \right) cN, & \text{if } \ b \geq \tau b_M, \\ \\ \left(\frac{b}{Q_H} - \tau c\right) \alpha_H N = \left(\frac{b}{b_L} - \tau\right) c\alpha_H L, & \text{if } \ b \in [\tau b_L, \tau b_M). \end{cases} \tag{14}$$

Thus, firms with $b < b_L$ exit, firms with $b \in [b_L, \tau b_L)$ serve only the domestic market, while firms with $b \geq \tau b_L$ serve both domestic and foreign markets. In addition, as illustrated in *Figure 3*, domestic goods with valuations $b \in [b_M, B]$ and imported goods with $b \in [\tau b_M, B]$ are purchased by all consumers and, thereby, belong to the necessities, while domestic goods with $b \in [b_L, b_M)$ and imported goods with $b \in [\tau b_L, \tau b_M)$ belong to the luxury goods.

Note that due to transport costs, there are goods that are available to consumers of type i at home but not available to consumers of the same type abroad. In particular, goods with valuations $b \in [b_M, \tau b_M)$ are sold to all consumers at home, but exported only to the rich in a foreign country. Hence, the model provides an explanation why some imported goods are available to the rich and not available to the poor. Moreover, as it can be seen, if transport costs τ are sufficiently high ($\tau b_M \geq B$ in the equilibrium), then imported goods are so expensive that only the rich can afford purchasing them.

Figure 3: Consumption



3.1.1 Prices and Arbitrage Opportunities

Let us denote $p_D(b)$ and $p_F(b)$ as the prices of goods with valuation b sold at home and exported, respectively. Then,

$$p_D(b) = \begin{cases} \frac{b}{Q_L} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right), & \text{if } \ b \geq b_M, \\ \\ \frac{b}{Q_H} = \frac{cb}{b_L}, & \text{if } \ b \in [b_L, b_M), \end{cases} \tag{15}$$

$$p_F(b) = \begin{cases} \frac{b}{Q_L} = cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right), & \text{if } \ b \geq \tau b_M, \\ \\ \frac{b}{Q_H} = \frac{cb}{b_L}, & \text{if } \ b \in [\tau b_L, \tau b_M). \end{cases} \tag{16}$$

Hence, the prices of goods with sufficiently high and low valuations are the same at home and abroad, i.e., $p_D(b) = p_F(b)$, implying that the f.o.b. export prices of those goods (given by $\frac{p_F(b)}{\tau}$) are strictly less than the prices in the domestic market.[7] This is reminiscent of reciprocal dumping in Melitz and Ottawiano (2008).

Note that the assumption about the infeasibility of arbitrage is a necessary ingredient of the model. In particular, for goods with $b \in [b_M, \tau b_M)$, $p_D(b)$ and $p_F(b)$ are different with $p_F(b) > p_D(b)$ and, therefore, it can be profitable for a third party to ship those goods from one country to the other to arbitrage the price difference. Namely, the absence of arbitrage

[7] In the model, the prices are not directly affected by the transport costs. The impact of τ on the equilibrium prices goes through the effects on b_L and b_M only.

opportunities is equivalent to

$$\tau p_F(b) \geq p_D(b) \geq \frac{p_F(b)}{\tau}. \tag{17}$$

In our case, inequality (17) holds for goods with $b \in [\tau b_L, b_M) \cup [\tau b_M, B]$ and does not necessarily hold for goods with $b \in [b_M, \tau b_M)$. Specifically, for any $b \in [b_M, \tau b_M)$,

$$\frac{p_D(b)}{p_F(b)} = \alpha_H + \frac{b_L(1-\alpha_H)}{b_M}.$$

Hence, the no-arbitrage condition means that

$$\alpha_H + \frac{b_L(1-\alpha_H)}{b_M} \geq \frac{1}{\tau} \iff \frac{b_L}{b_M} \geq \frac{1-\alpha_H\tau}{(1-\alpha_H)\,\tau}. \tag{18}$$

Later in the paper, I show that the ratio $\frac{b_L}{b_M}$ is increasing in τ in the equilibrium. As $\frac{1-\alpha_H\tau}{(1-\alpha_H)\tau}$ is decreasing in τ, this implies that there exists τ^* such that for any $\tau \geq \tau^*$, inequality (18) holds. Hence, arbitrage opportunities are ruled out in the equilibrium if and only if the transport costs are sufficiently high.[8]

3.2 The Equilibrium

As before, the equilibrium is characterized by the free entry and the goods market clearing conditions. The free entry condition means that in the equilibrium, the ex-ante profits of firms are equal to zero. That is,

$$f_e = \int_0^B \pi(t)dG(t),$$

where the function $\pi(t)$ is given by (12). Using the expressions for $\pi_D(b)$ and $\pi_F(b)$ (see (13) and (14)), the last equation can be rewritten as follows:

$$\frac{f_e}{cN} + 1 + \tau = \alpha_H\left(H(b_L) + \tau H(\tau b_L)\right) + (1-\alpha_H)\left(H(b_M) + \tau H(\tau b_M)\right),$$

where $H(x) = G(x) + \frac{\int_x^B tdG(t)}{x}$.

[8] Notice that τ^* lies in the interval $\left(1, \frac{1}{\alpha_H}\right)$.

The goods market clearing condition implies that

$$\begin{cases} I_L = M_e \left(\int_{b_M}^{B} p_D(t) dG(t) + \int_{\tau b_M}^{B} p_F(t) dG(t) \right), \\ \\ I_H - I_L = M_e \left(\int_{b_L}^{b_M} p_D(t) dG(t) + \int_{\tau b_L}^{\tau b_M} p_F(t) dG(t) \right). \end{cases} \tag{19}$$

Using the expressions for the domestic and export prices derived in the previous section and dividing the second line by the first one, we obtain

$$\frac{\int_{b_L}^{b_M} t dG(t) + \int_{\tau b_L}^{\tau b_M} t dG(t)}{\int_{b_M}^{B} t dG(t) + \int_{\tau b_M}^{B} t dG(t)} = \left(\frac{I_H}{I_L} - 1 \right) \left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M} \right).$$

Hence, by analogy with the closed economy case, the equilibrium values of b_M and b_L can be found from the following system of equations:

$$\begin{cases} \frac{\int_{b_L}^{b_M} t dG(t) + \int_{\tau b_L}^{\tau b_M} t dG(t)}{\int_{b_M}^{B} t dG(t) + \int_{\tau b_M}^{B} t dG(t)} = \left(\frac{I_H}{I_L} - 1 \right) \left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M} \right), \\ \\ \frac{f_e}{cN} + 1 + \tau = \alpha_H \left(H(b_L) + \tau H(\tau b_L) \right) + (1-\alpha_H) \left(H(b_M) + \tau H(\tau b_M) \right). \end{cases} \tag{20}$$

The existence and the uniqueness of the equilibrium can be proved in the same manner as in the closed economy case (see Tarasov (2009)).

3.3 Consumer Welfare

Before analyzing comparative statics of the equilibrium, I focus on consumer welfare. Recall that welfare of consumer i is given by

$$U_i = \int_{\omega \in \Omega} b(\omega) x_i(\omega) d\omega.$$

Thus, welfare of consumers with income I_L is equal to

$$U_L = M_e \left(\int_{b_M}^{B} t dG(t) + \int_{\tau b_M}^{B} t dG(t) \right).$$

Meanwhile, the marker clearing conditions in (19) imply that

$$M_e = \frac{I_L}{\int_{b_M}^{B} p_D(t) dG(t) + \int_{\tau b_M}^{B} p_F(t) dG(t)}.$$

Therefore, using the expressions for the prices, we obtain that

$$U_L = I_L Q_L.$$

Welfare of the poor naturally rises with an increase in either their income or the valuation to price ratio of goods they consume.

Similarly, welfare of the rich is given by

$$U_H = I_L Q_L + (I_H - I_L) Q_H.$$

As the rich consume the same bundle of goods as the poor plus some others, welfare of the rich is equal to welfare of the poor plus additional welfare from the consumption of the luxury goods, which is in turn equal to income spent on those goods multiplied by their valuation to price ratio.

The findings above suggest that relative welfare of the rich with respect to the poor is given by

$$\frac{U_H}{U_L} = 1 + \left(\frac{I_H}{I_L} - 1\right) \frac{Q_H}{Q_L}.$$

Note that all changes in the relative welfare are due to two effects: the price and income effects. The price effect is determined by changes in $\frac{Q_H}{Q_L}$, while the income effect is determined by changes in $\frac{I_H}{I_L}$.

3.4 Trade Liberalization and Relative Welfare

This section focuses on the effects of changes in transport costs on the relative welfare. To simplify the analysis and to avoid some ambiguity in the results, I assume that the aggregate utility from the consumption of goods with a certain valuation b given by $M_e b g(b)$ does not decrease too fast in b. Specifically, I limit the analysis to the case when the distribution of draws $G(b)$ is such that $b^2 g(b)$ is increasing and convex in b.[9] This assumption also guarantees that the probability of getting higher values of b does not decrease too fast with b.

[9] For instance, the family of power distributions with $G(b) = \left(\frac{b}{B}\right)^k$, $k > 0$, satisfies this assumption. The convexity of $b^2 g(b)$ is rather a technical condition, which substantially simplifies some proofs.

Recall that the relative welfare is given by

$$\frac{U_H}{U_L} = 1 + \left(\frac{I_H}{I_L} - 1\right)\frac{Q_H}{Q_L}. \tag{21}$$

To understand better the intuition behind the effects of τ on the relative welfare, I separately consider two submarkets: the submarket for the necessities and the submarket for the luxuries. First, I consider the effects of higher trade costs on the prices of the necessities. A rise in τ leads to that some exporting firms exit from the submarket for the necessities and start selling only to the rich (i.e., τb_M rises). This reduces the intensity of competition among firms that serve all consumers and, therefore, drives up the prices of the necessities. Because of higher prices of the necessities, some domestic firms that served only the rich consumers find it profitable start selling to all consumers. This implies that the domestic cutoff b_M decreases.

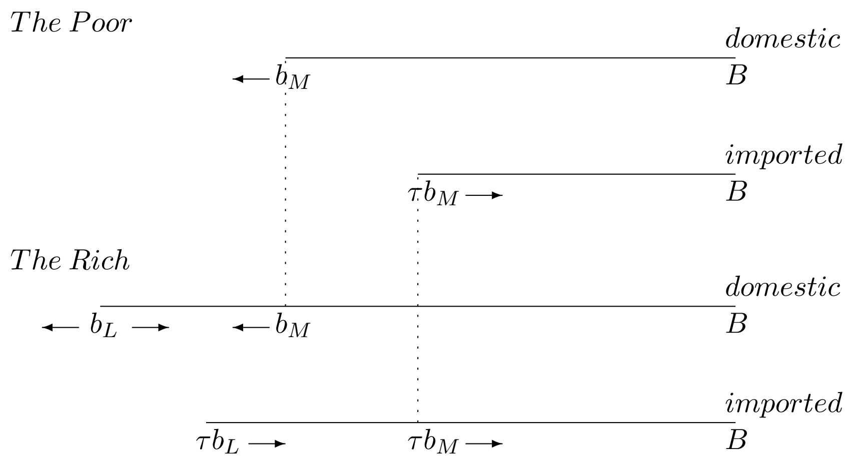
Notice that we should also take into account changes in the mass of entrants M_e and their effects on the cutoffs and the prices. In general, the impact of τ on M_e is unclear. On the one hand, a rise in τ reduces the profits from exporting. On the other hand, higher τ can raise the profits from selling domestically due to lower competition. The overall effect on the expected profits and, therefore, on M_e is ambiguous. However, I find that the results claimed in the previous paragraph hold irrespective of changes in M_e. Hence, the following lemma holds.

Lemma 2 *Higher transport costs raise the exporting cutoff τb_M, decrease the domestic cutoff b_M, and lead to higher prices of the necessities.*

Proof. In the Appendix. ■

Similarly, higher transport costs imply that some exporting firms exit from the submarket for the luxuries (in fact, those firms stop exporting at all), implying that the exporting cutoff τb_L rises. In addition, as it was discussed above, some domestic firms find it more profitable to serve all consumers (b_M decreases). Both effects reduce the intensity of competition in the submarket, resulting in higher prices of the luxury goods and, thereby, decreasing the exit cutoff b_L. However, there is an additional effect working in the opposite direction. Remember that a rise in τ results in higher τb_M (see *Lemma 2*). That is, some exporting firms that served all consumers before start serving only the rich. This creates more competition in the submarket for the luxuries and, therefore, negatively affects the prices. Hence, we observe two opposite

Figure 4: The Impact of τ on Consumption



effects of changes in τ on the prices of the luxury goods.

I find that, in general, the overall impact is unclear. For instance, in the extreme case when the fraction of the rich is close to zero and the income difference between the rich and the poor is sufficiently high (there is a tiny minority of very rich consumers), the rich can even gain from higher transport costs because of lower prices of the luxuries. In other words, in very unequal societies trade liberalization can even harm the rich. The following lemma summarizes the findings above.

Lemma 3 *Higher transport costs raise the exporting cutoff τb_L and have an ambiguous impact on the exit cutoff b_L and, thereby, on the prices of the luxury goods. However, in very unequal economies, where α_H is close to zero and $\frac{I_H}{I_L}$ is sufficiently high, a rise in τ can reduce the prices of the luxuries and benefits the rich.*

Proof. In the Appendix. ■

Figure 4 illustrates the results formulated in *Lemmas 1* and *2*. As it can be seen from the lemmas, the poor always gain from trade liberalization, while the impact on the rich is unclear in general. Hence, we might expect that the reduction in transport costs benefit the poor more than the rich. Indeed, I show that for any parameters in the model, the ratio $\frac{Q_H}{Q_L}$ is increasing in τ. In words, higher transport costs increase the relative prices of the necessities with respect to those of the luxuries. This is because exporting firms that exit from the submarket for the necessities in fact enter the submarket for the luxury goods inducing tougher competition. The following proposition holds.

Proposition 2 *The poor gain more from a decrease in τ than the rich do.*

Proof. In the Appendix. ■

It should be emphasized that the results above are based on two key features of the model: non-homothetic preferences and monopolistic competition. Nonhomotheticity of preferences implies that different groups of consumers purchase different bundles of goods. While monopolistic competition allows firms to choose what group of consumers to serve and what prices to set. Note that in traditional literature with homothetic preferences, bilateral trade liberalization has the same or no impact on prices set by firms, implying that trade liberalization is beneficial for all consumers. While in the present model, it is not necessarily the case. In very unequal economies, the rich consumers can even loose from trade liberalization due to higher prices of the luxury goods.

3.4.1 When the Transportation Costs are Sufficiently High

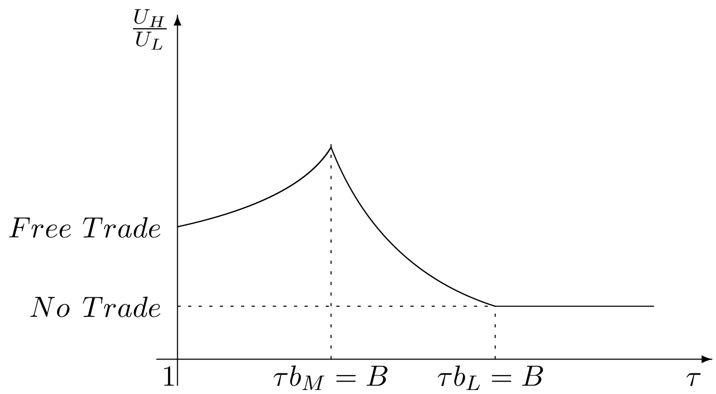
In the previous analysis, I assume that imported goods are purchased by both the rich and poor consumers. That is, the transport costs are such that $\tau b_M \leq B$ in the equilibrium. However, it is not necessarily the case. If the transport costs are so high that $\tau b_M > B$, then imported goods are purchased only by the rich. In this case, the equilibrium equations can be written as follows:

$$\begin{cases} \frac{\int_{b_L}^{b_M} tdG(t)+\int_{\tau b_L}^{B} tdG(t)}{\int_{b_M}^{B} tdG(t)} = \left(\frac{I_H}{I_L}-1\right)\left(\alpha_H+\frac{b_L(1-\alpha_H)}{b_M}\right), \\ \frac{f_e}{cN}+1+\alpha_H\tau = \alpha_H\left(H(b_L)+\tau H(\tau b_L)\right)+(1-\alpha_H)H(b_M). \end{cases} \tag{22}$$

If we consider this special case, then it is straightforward to see that trade liberalization benefits the rich more than the poor. This is explained by the fact that changes in τ do not directly affect poor consumers, as they purchase only domestic goods. Therefore, the following proposition holds.

Proposition 3 *If τ is such that $\tau b_M > B$ in the equilibrium, then the rich gain more from trade liberalization than the poor do.*

Figure 5: Relative Welfare



Proof. In the Appendix. ■

Hence, summarizing the findings in *Propositions 1* and *2*, we can see that the relative welfare has a hump shape as a function of transport costs τ. Moreover, if we assume that there are no trade costs, then the trade equilibrium is equivalent to the equilibrium in the closed economy when the mass of consumers is doubled. Meanwhile, Tarasov (2009) shows that in the closed economy, a rise in the mass of consumers benefits the rich more than the poor. Thus, we can conclude that opening a country to costless trade always benefits the rich more. However, further trade liberalization can reduce welfare inequality. *Figure 5* illustrates these findings.

3.4.2 A Numerical Example

This subsection considers a numerical example that illustrates some of the results obtained above. For certain values of the parameters, I simulate the relationship between consumer welfare and trade costs in equilibrium. Specifically, I assume that the distribution $G(x)$ is uniform with the support on $[0,1]$ and $\frac{f_e}{cN} = 1$. In addition, I assume that the rich have income three times as higher as the poor do (meaning that $\frac{I_H}{I_L} = 3$) and constitute a quarter of the total population (i.e., $\alpha_H = 0.25$). Given the assumed values of the parameters, I solve for the equilibrium values of b_L and b_M as τ is raised from 1 (free trade) to 12 (no trade).

Figure 6 shows the simulated relationship between consumer welfare and trade costs. As it can be seen, both types of consumers gain from trade liberalization. Note that the poor are slightly worse off when the economy just starts moving from the autarky to costly trade (τ falls from 9.4 to 7.8). This can be explained by the free entry effect. On the one hand, lower transport costs induce tougher competition, as domestic firms have to compete with their foreign counterparts. This positively affects the well-being of consumers in the economy. On the other hand, lower transport costs can reduce the firm's expected profits and, thereby, decrease the mass of firms entering the market (see *Figure 7*). This in turn negatively affects consumers. It appears that if the poor cannot afford to buy foreign goods (i.e., the trade costs are sufficiently high), then the latter effect can prevail over the former one and, as a result, the poor can be worse off from trade liberalization. However, further trade liberalization raises the well-being of the poor.

Figure 7 illustrates the relationship between the relative welfare and the trade costs. As it can be inferred from the figure, the relative welfare is first increasing and then decreasing as a function of τ, which is consistent with the theoretical findings obtained in the previous sections (see *Figure 5*). In particular, moving from the autarky to free trade raises the relative welfare of the rich by 9%. Furthermore, if trade liberalization does not directly affect the poor: i.e., imported goods are purchased only by the rich, then the relative welfare rises by 23%. This suggests that the impact of trade liberalization on relative welfare through the price effect can be of considerable magnitude.

Figure 6: Consumer Welfare and Trade Costs

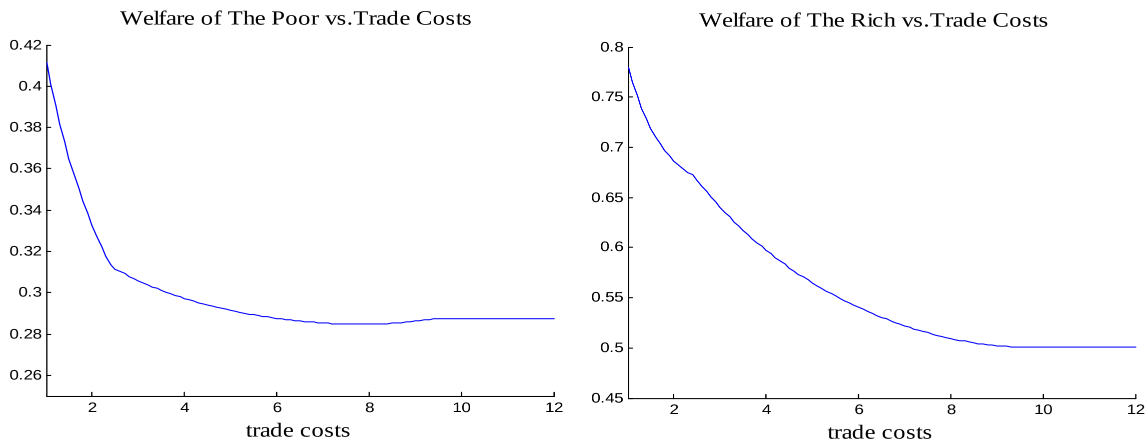
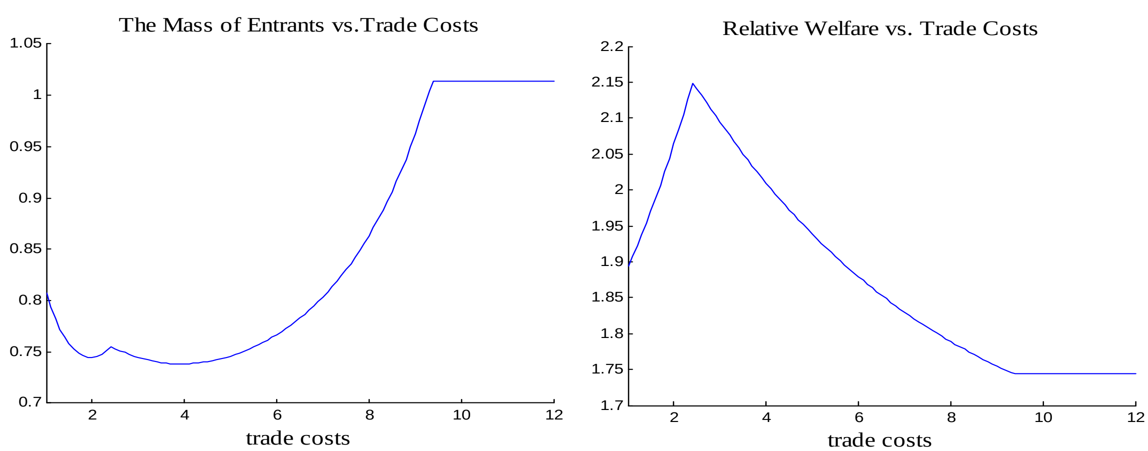


Figure 7: The Mass of Entrants, Relative Welfare, and Trade Costs



4 Concluding Remarks

In this paper, I develop a tractable framework that enables us to analyze the impact of trade and trade costs on welfare inequality through the price effect. One of the key elements of the model is nonhomothetic preferences that feature discrete choices (among horizontally differentiated goods) by heterogenous in income consumers. Such preference structure implies that consumers first buy goods that are relatively more essential in consumption and then move to less essential goods. Furthermore, the rich consumers buy the same bundle of goods (the necessities) as the poor consumers plus some others (the luxuries).

I then incorporate these preferences in the monopolistic competition model of trade à la Melitz and Ottawiano (2008). The presence of market power and nonhomothetic preferences lead to that prices set by firms are affected by trade and trade costs. Moreover, the prices of different goods (necessities and luxuries) are affected differently, implying that trade liberalization can benefit some income classes more than others. In particular, I find that if trade costs are such that imported goods are available for all consumers, then trade liberalization benefits the poor more. While if trades costs are so high that only the rich can afford to buy imported goods, then the rich gain relatively more from trade liberalization. In other words, the relative welfare of the rich has a hump shape as a function of trade costs.

The developed framework can be easily extended in at least two directions. First, it would not be difficult to consider a similar model of trade between two countries with different income distributions and to examine how this difference affects trade patterns and relative welfare. Secondly, it would be interesting to explore the case when income distribution is endogenous. This framework would allow for the both income and price effects and, therefore, could give us an idea about the relative magnitude of the effects. I leave these issues for future work.

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Appendix

The algebra in the Appendix is mainly based on differentiation of implicit functions. As the intuition of this exercise is straightforward, I only present the most important details and omit unnecessary ones. To simplify the notation in the Appendix, hereafter I assume that $\int_x^y$ means $\int_x^y tdG(t)$. Before proceeding to the proofs of the lemmas and the propositions, we consider the equilibrium equations rewritten in the following way:

$$
\begin{aligned}
J_1 &\equiv \alpha_H \left(H(b_L) + \tau H(\tau b_L)\right) + (1-\alpha_H)\left(H(b_M) + \tau H(\tau b_M)\right) - \frac{f_e}{cN} - 1 - \tau = 0, && (23) \\
J_2 &\equiv I_L \left(\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}\right) - (I_H - I_L)\left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}\right)\left(\int_{b_M}^{B} + \int_{\tau b_M}^{B}\right) = 0; && (24)
\end{aligned}
$$

and establish some necessary relationships. Specifically, using the equations in (23) and (24), it is straightforward to show that[10]

$$
\begin{aligned}
\frac{\partial J_1}{\partial b_M} &= -\frac{(1-\alpha_H)}{b_M^2}\left(\int_{b_M}^{B} + \int_{\tau b_M}^{B}\right) < 0, \; \frac{\partial J_1}{\partial b_L} = -\frac{\alpha_H}{b_L^2}\left(\int_{b_L}^{B} + \int_{\tau b_L}^{B}\right) < 0, \\
\frac{\partial J_1}{\partial \tau} &= \alpha_H G(\tau b_L) + (1-\alpha_H) G(\tau b_M) - 1 < 0, \\
\frac{\partial J_2}{\partial b_M} &= I_L b_M \left(g(b_M) + \tau^2 g(\tau b_M)\right)\left(\frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}}\right) + I_L \frac{b_L(1-\alpha_H)}{b_M^2} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} > 0, \\
\frac{\partial J_2}{\partial b_L} &= -I_L b_L \left(g(b_L) + \tau^2 g(\tau b_L)\right) - I_L \frac{(1-\alpha_H)}{b_M} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} < 0, \\
\frac{\partial J_2}{\partial \tau} &= I_L \tau \left(b_M^2 g(\tau b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} - b_L^2 g(\tau b_L)\right) > 0.
\end{aligned}
$$

Finally, from (23) and (24), we have

$$
\begin{cases}
\frac{\partial J_1}{\partial b_M}\frac{\partial b_M}{\partial \tau} + \frac{\partial J_1}{\partial b_L}\frac{\partial b_L}{\partial \tau} + \frac{\partial J_1}{\partial \tau} = 0 \\
\frac{\partial J_2}{\partial b_M}\frac{\partial b_M}{\partial \tau} + \frac{\partial J_2}{\partial b_L}\frac{\partial b_L}{\partial \tau} + \frac{\partial J_2}{\partial \tau} = 0
\end{cases}.
$$

Solving for $\frac{\partial b_M}{\partial \tau}$ and $\frac{\partial b_L}{\partial \tau}$, we obtain that

$$
\begin{aligned}
\frac{\partial b_M}{\partial \tau} &= \frac{-\frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_L}}{D}, && (25) \\
\frac{\partial b_L}{\partial \tau} &= \frac{-\frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_M} + \frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_M}}{D}, && (26)
\end{aligned}
$$

where

$$
D = \frac{\partial J_1}{\partial b_M}\frac{\partial J_2}{\partial b_L} - \frac{\partial J_2}{\partial b_M}\frac{\partial J_1}{\partial b_L} > 0.
$$

Next, we proceed to the proof of *Lemma 2.*

[10] Recall that by assumption, $b^2 g(b)$ is increasing in b.

The Proof of Lemma 2

As it can be clearly seen from (25), $\frac{\partial b_M}{\partial \tau} < 0$. That is, higher τ decreases the domestic cutoff b_M. Next, I show that higher transport costs raise the exporting cutoff τb_M. We have

$$(\tau b_M)'_\tau = b_M + \tau \frac{\partial b_M}{\partial \tau}.$$

Plugging the expression for $\frac{\partial b_M}{\partial \tau}$ in (25), the derivative can be rewritten as follows:

$$(\tau b_M)'_\tau = \frac{b_M D + \tau \left(-\frac{\partial J_1}{\partial \tau} \frac{\partial J_2}{\partial b_L} + \frac{\partial J_2}{\partial \tau} \frac{\partial J_1}{\partial b_L} \right)}{D}. \tag{27}$$

Since we know that $D > 0$, one only needs to determine the sign of the numerator in the last expression. Plugging the expression for D, we obtain that the numerator is equal to

$$\frac{\partial J_2}{\partial b_L} \left(b_M \frac{\partial J_1}{\partial b_M} - \tau \frac{\partial J_1}{\partial \tau} \right) + \frac{\partial J_1}{\partial b_L} \left(\tau \frac{\partial J_2}{\partial \tau} - b_M \frac{\partial J_2}{\partial b_M} \right).$$

Using the expressions for $\frac{\partial J_i}{\partial b_L}$, $\frac{\partial J_i}{\partial b_M}$, and $\frac{\partial J_i}{\partial \tau}$ derived above, we can show that

$$\begin{aligned}
b_M \frac{\partial J_1}{\partial b_M} - \tau \frac{\partial J_1}{\partial \tau} &= -(1-\alpha_H) \left(\tau H(\tau b_M) + \frac{\int_{b_M}^B}{b_M} \right) + \tau \left(1 - \alpha_H G(\tau b_L) \right), \\
\tau \frac{\partial J_2}{\partial \tau} - b_M \frac{\partial J_2}{\partial b_M} &= -I_L \left(\tau^2 b_L^2 g(\tau b_L) + b_M^2 g(b_M) \left(\frac{\int_{b_L}^B + \int_{\tau b_L}^B}{\int_{b_M}^B + \int_{\tau b_M}^B} \right) + \frac{b_L(1-\alpha_H)}{b_M} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} \right).
\end{aligned}$$

Plugging the last expressions into the numerator and using the expressions for $\frac{\partial J_1}{\partial b_L}$ and $\frac{\partial J_2}{\partial b_L}$, we obtain that the numerator is equal to

$$\begin{aligned}
& I_L \frac{(1-\alpha_H)}{b_M} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} \left(\frac{\alpha_H \int_{b_L}^B}{b_L} + \frac{(1-\alpha_H) \int_{b_M}^B}{b_M} + \tau \left(\alpha_H H(\tau b_L) + (1-\alpha_H) H(\tau b_M) - 1 \right) \right) \\
& + I_L \tau^2 b_L g(\tau b_L) \left(\frac{\alpha_H \int_{b_L}^B}{b_L} + \frac{(1-\alpha_H) \int_{b_M}^B}{b_M} + \tau \left(\alpha_H H(\tau b_L) + (1-\alpha_H) H(\tau b_M) - 1 \right) \right) \\
& + I_L \frac{\alpha_H b_M^2 g(b_M)}{b_L^2} \frac{\left(\int_{b_L}^B + \int_{\tau b_L}^B \right)^2}{\int_{b_M}^B + \int_{\tau b_M}^B} - I_L b_L g(b_L) \left(-(1-\alpha_H) \left(\tau H(\tau b_M) + \frac{\int_{b_M}^B}{b_M} \right) + \tau \left(1 - \alpha_H G(\tau b_L) \right) \right).
\end{aligned}$$

Remember that $H(x) = G(x) + \frac{\int_x^B}{x} \geq 1$ for any $x \in [0, B]$. This means that

$$\alpha_H H(\tau b_L) + (1-\alpha_H) H(\tau b_M) - 1 > 0.$$

Furthermore, since $b_L < b_M$ and $b^2 g(b)$ is increasing in b, $b_M^2 g(b_M) > b_L^2 g(b_L)$. Finally, $\frac{\int_{b_L}^B + \int_{\tau b_L}^B}{\int_{b_M}^B + \int_{\tau b_M}^B} > 1$. Therefore, the part of the numerator given by

$$\begin{aligned}
& I_L \frac{\alpha_H b_M^2 g(b_M)}{b_L^2} \frac{\left(\int_{b_L}^B + \int_{\tau b_L}^B\right)^2}{\int_{b_M}^B + \int_{\tau b_M}^B} - I_L b_L g(b_L) \left(-(1-\alpha_H) \left(\tau H(\tau b_M) + \frac{\int_{b_M}^B}{b_M} \right) + \tau \left(1 - \alpha_H G(\tau b_L)\right) \right) \\
> \;& I_L g(b_L) b_L \left(\frac{\alpha_H \left(\int_{b_L}^B + \int_{\tau b_L}^B\right)}{b_L} + (1-\alpha_H) \left(\tau H(\tau b_M) + \frac{\int_{b_M}^B}{b_M} \right) - \tau \left(1 - \alpha_H G(\tau b_L)\right) \right) \\
= \;& I_L g(b_L) b_L \left(\frac{\alpha_H \int_{b_L}^B}{b_L} + (1-\alpha_H) \frac{\int_{b_M}^B}{b_M} + \tau \left(\alpha_H H(\tau b_L) + (1-\alpha_H) H(\tau b_M) - 1 \right) \right) > 0.
\end{aligned}$$

This implies that the numerator in (27) is positive and, thereby, $(\tau b_M)'_\tau$ is positive.

Finally, I show that the prices of the necessities given by $cb\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)$ increase with a rise in τ. Specifically, I show that $\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}$ is increasing in τ. We have

$$\begin{aligned}
\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M} \right)'_\tau &= -\frac{\alpha_H}{b_L^2} \frac{\partial b_L}{\partial \tau} - \frac{(1-\alpha_H)}{b_M^2} \frac{\partial b_M}{\partial \tau} \\
&= \frac{\frac{\partial J_1}{\partial \tau} \left(\frac{(1-\alpha_H)}{b_M^2} \frac{\partial J_2}{\partial b_L} - \frac{\alpha_H}{b_L^2} \frac{\partial J_2}{\partial b_M} \right) + \frac{\partial J_2}{\partial \tau} \left(\frac{\alpha_H}{b_L^2} \frac{\partial J_1}{\partial b_M} - \frac{(1-\alpha_H)}{b_M^2} \frac{\partial J_1}{\partial b_L} \right)}{D}.
\end{aligned}$$

Note that

$$\begin{aligned}
\frac{\alpha_H}{b_L^2} \frac{\partial J_1}{\partial b_M} - \frac{(1-\alpha_H)}{b_M^2} \frac{\partial J_1}{\partial b_L} &> 0 \text{ and} \\
\frac{(1-\alpha_H)}{b_M^2} \frac{\partial J_2}{\partial b_L} - \frac{\alpha_H}{b_L^2} \frac{\partial J_2}{\partial b_M} &< 0.
\end{aligned}$$

As $\frac{\partial J_1}{\partial \tau} < 0$ and $\frac{\partial J_2}{\partial \tau} > 0$, we can see that $\left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right)'_\tau > 0$. This finishes the proof of *Lemma 2*.

The Proof of Lemma 3

In this section, I show that τb_L is increasing in τ, while the impact of τ on b_L is unclear. We have

$$(\tau b_L)'_\tau = \frac{b_L D + \tau \left(-\frac{\partial J_2}{\partial \tau} \frac{\partial J_1}{\partial b_M} + \frac{\partial J_1}{\partial \tau} \frac{\partial J_2}{\partial b_M} \right)}{D}. \tag{28}$$

Hence, it is necessary to determine the sign of the numerator given by

$$b_L D + \tau \left(-\frac{\partial J_2}{\partial \tau} \frac{\partial J_1}{\partial b_M} + \frac{\partial J_1}{\partial \tau} \frac{\partial J_2}{\partial b_M} \right) = \frac{\partial J_1}{\partial b_M} \left(b_L \frac{\partial J_2}{\partial b_L} - \tau \frac{\partial J_2}{\partial \tau} \right) + \frac{\partial J_2}{\partial b_M} \left(\tau \frac{\partial J_1}{\partial \tau} - b_L \frac{\partial J_1}{\partial b_L} \right).$$

Plugging the expressions for all partial derivatives, we can show that the numerator equals to

$$
\begin{aligned}
& I_L \frac{b_L(1-\alpha_H)}{b_M^2} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} \left(\frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M} + \alpha_H \frac{\int_{b_L}^{B}}{b_L} + \tau\left((1-\alpha_H)H(\tau b_M) + \alpha_H H(\tau b_L) - 1\right) \right) \\
& + I_L \tau^2 b_M g(\tau b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(\frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M} + \alpha_H \frac{\int_{b_L}^{B}}{b_L} + \tau\left((1-\alpha_H)H(\tau b_M) + \alpha_H H(\tau b_L) - 1\right) \right) \\
& + \frac{I_L(1-\alpha_H)\left(\int_{b_M}^{B} + \int_{\tau b_M}^{B}\right)}{b_M^2} b_L^2 g(b_L) \\
& + I_L b_M g(b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(\alpha_H \tau H(\tau b_L) + \alpha_H \frac{\int_{b_L}^{B}}{b_L} - \tau\left(1 - (1-\alpha_H)G(\tau b_M)\right) \right).
\end{aligned}
$$

Taking into account that $(\tau b_M)^2 g(\tau b_M) \geq b_M^2 g(b_M)$, we derive

$$
\begin{aligned}
& I_L \tau^2 b_M g(\tau b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(\frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M} + \alpha_H \frac{\int_{b_L}^{B}}{b_L} + \tau\left((1-\alpha_H)H(\tau b_M) + \alpha_H H(\tau b_L) - 1\right) \right) \\
& + I_L b_M g(b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(\alpha_H \tau H(\tau b_L) + \alpha_H \frac{\int_{b_L}^{B}}{b_L} - \tau\left(1 - (1-\alpha_H)G(\tau b_M)\right) \right) \\
\geq \; & I_L b_M g(b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(\frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M} + \alpha_H \frac{\int_{b_L}^{B}}{b_L} + \tau\left((1-\alpha_H)H(\tau b_M) + \alpha_H H(\tau b_L) - 1\right) \right) \\
& + I_L b_M g(b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(\alpha_H \tau H(\tau b_L) + \alpha_H \frac{\int_{b_L}^{B}}{b_L} - \tau\left(1 - (1-\alpha_H)G(\tau b_M)\right) \right) > 0.
\end{aligned}
$$

This implies that the numerator in (26) is positive and, thereby, $(\tau b_L)'_\tau > 0$.

Next, I consider the derivative of b_L with respect to τ. Recall that

$$
\frac{\partial b_L}{\partial \tau} = \frac{-\frac{\partial J_2}{\partial \tau}\frac{\partial J_1}{\partial b_M} + \frac{\partial J_1}{\partial \tau}\frac{\partial J_2}{\partial b_M}}{D}.
$$

As $D > 0$, we only need to consider the sign of the numerator. After some simplifications, we obtain that the numerator is equal to

$$
\begin{aligned}
& I_L \frac{\tau(1-\alpha_H)}{b_M^2} \left(b_M^2 g(\tau b_M) \left(\int_{b_L}^{B} + \int_{\tau b_L}^{B} \right) - b_L^2 g(\tau b_L) \left(\int_{b_M}^{B} + \int_{\tau b_M}^{B} \right) \right) \\
& - I_L \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(1 - \alpha_H G(\tau b_L) - (1-\alpha_H)G(\tau b_M)\right) \left(b_M g(b_M) + \tau^2 b_M g(\tau b_M)\right) \\
& - I_L \frac{b_L(1-\alpha_H)}{b_M^2} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} \left(1 - \alpha_H G(\tau b_L) - (1-\alpha_H)G(\tau b_M)\right).
\end{aligned}
$$

In general, the sign of the numerator can be either positive or negative. For instance, if α_H is close to unity, then the numerator is approximately equal to

$$-I_L \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(1 - G(\tau b_L)\right) \left(b_M g(b_M) + \tau^2 b_M g(\tau b_M)\right) < 0,$$

implying that b_L is decreasing in τ. However, in very extreme cases when $\frac{I_H}{I_L}$ is sufficiently high and α_H is sufficiently low, it is possible that the sign of the numerator is positive. Specifically, Tarasov (2009) shows that all else equal, higher $\frac{I_H}{I_L}$ results in lower b_L and higher b_M. Hence, if we consider such $\frac{I_H}{I_L}$ that τb_M is close to B, the numerator would be approximately equal to

$$\begin{aligned}
& I_L \frac{\tau(1-\alpha_H)}{b_M^2} \left(b_M^2 g(\tau b_M) \left(\int_{b_L}^{B} + \int_{\tau b_L}^{B} \right) - b_L^2 g(\tau b_L) \int_{b_M}^{B} \right) \\
& -\alpha_H I_L \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B}} \left(1 - G(\tau b_L)\right) \left(b_M g(b_M) + \tau^2 b_M g(\tau b_M)\right) \\
& -\alpha_H I_L \frac{b_L(1-\alpha_H)}{b_M^2} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} \left(1 - G(\tau b_L)\right),
\end{aligned}$$

which is positive for sufficiently low α_H. This suggests that in economies with tiny minority of very rich consumers, higher transport costs can reduce the prices of the luxuries.

Finally, I show that the impact of τ on welfare of the rich is unclear in general and in some extreme cases, the rich can even be better off from higher transport costs. Recall that welfare of the rich is given by

$$U_H = I_L Q_L + (I_H - I_L) Q_H = \frac{1}{c} \left(\frac{I_L}{\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}} + (I_H - I_L)\, b_L \right).$$

Therefore, after some simplifications,

$$(U_H)'_\tau = \frac{I_L \left(\frac{\partial J_1}{\partial \tau} \left(P_2 \frac{\alpha_H}{b_L^2} \frac{\partial J_2}{\partial b_M} - \frac{(1-\alpha_H)}{b_M^2} \frac{\partial J_2}{\partial b_L} \right) + \frac{\partial J_2}{\partial \tau} \left(\frac{(1-\alpha_H)}{b_M^2} \frac{\partial J_1}{\partial b_L} - P_2 \frac{\alpha_H}{b_L^2} \frac{\partial J_1}{\partial b_M} \right) \right)}{cD \left(\frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M} \right)^2}, \tag{29}$$

where

$$P_2 = 1 + \frac{1}{\alpha_H} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \left(\alpha_H + \frac{(1-\alpha_H) b_L}{b_M} \right).$$

Plugging the expressions for $\frac{\partial J_i}{\partial \tau}$, $\frac{\partial J_i}{\partial b_M}$, and $\frac{\partial J_i}{\partial b_L}$ into (29), we can show that the sign of the numerator in

(29) is the same as the sign of the following expression:

$$
\begin{aligned}
&(\alpha_H G(\tau b_L) + (1-\alpha_H)G(\tau b_M) - 1)\left(b_M g(b_M) + \tau^2 b_M g(\tau b_M)\right) P_2 \frac{\alpha_H}{b_L^2} \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} \\
&+ (\alpha_H G(\tau b_L) + (1-\alpha_H)G(\tau b_M) - 1)\left(b_L g(b_L) + \tau^2 b_L g(\tau b_L)\right) \frac{(1-\alpha_H)}{b_M^2} \\
&+ (\alpha_H G(\tau b_L) + (1-\alpha_H)G(\tau b_M) - 1) \frac{(1-\alpha_H)}{b_M^2} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} \left(P_2 \frac{\alpha_H}{b_L} + \frac{(1-\alpha_H)}{b_M}\right) \\
&+ \left(\tau b_M^2 g(\tau b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} - \tau b_L^2 g(\tau b_L)\right) \frac{(1-\alpha_H)^2 \left(\int_{b_L}^{b_M} + \int_{\tau b_L}^{\tau b_M}\right)}{b_M^3 b_L}.
\end{aligned}
$$

As it can be seen, the sign of the last expression is unclear in general. For instance, if α_H is close to unity or the incomes of the poor and the rich are close to each other (implying that b_L is close to b_M), then the sign is negative. However, if α_H is sufficiently low and the difference between the incomes of the poor and the rich is such that τb_M is close to B, then the sign can be positive. A number of simulations I conduct for a wide range of parameters confirm these findings.

The Proof of Proposition 2

In this section, I focus on the relative welfare, which is given by

$$
\frac{U_H}{U_L} = 1 + \left(\frac{I_H}{I_L} - 1\right) \frac{Q_H}{Q_L} = 1 + \left(\frac{I_H}{I_L} - 1\right) \left(\alpha_H + (1-\alpha_H)\frac{b_L}{b_M}\right).
$$

Hence, to examine the sign of $\left(\frac{U_H}{U_L}\right)'_\tau$, we need to determine the sign of

$$
\left(\frac{b_L}{b_M}\right)'_\tau = \frac{\frac{\partial b_L}{\partial \tau} b_M - \frac{\partial b_M}{\partial \tau} b_L}{b_M^2}.
$$

Algebra shows that the sign of $\left(\frac{b_L}{b_M}\right)'_\tau$ is the same as the sign of

$$
\frac{\partial J_1}{\partial \tau}\left(\frac{\partial J_2}{\partial b_M} b_M + \frac{\partial J_2}{\partial b_L} b_L\right) - \frac{\partial J_2}{\partial \tau}\left(\frac{\partial J_1}{\partial b_M} b_M + \frac{\partial J_1}{\partial b_L} b_L\right).
$$

Using the expressions for $\frac{\partial J_i}{\partial \tau}$, $\frac{\partial J_i}{\partial b_M}$, and $\frac{\partial J_i}{\partial b_L}$, we derive

$$
\begin{aligned}
\frac{\partial J_2}{\partial b_M} b_M + \frac{\partial J_2}{\partial b_L} b_L &= I_L \left(\left(b_M^2 g(b_M) + \tau^2 b_M^2 g(\tau b_M)\right) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B} + \int_{\tau b_M}^{B}} - b_L^2 g(b_L) - \tau^2 b_L^2 g(\tau b_L)\right) \\
\frac{\partial J_1}{\partial b_M} b_M + \frac{\partial J_1}{\partial b_L} b_L &= -\frac{(1-\alpha_H)\left(\int_{b_M}^{B} + \int_{\tau b_M}^{B}\right)}{b_M} - \frac{\alpha_H \left(\int_{b_L}^{B} + \int_{\tau b_L}^{B}\right)}{b_L}.
\end{aligned}
$$

Thus, we need to examine the sign of

$$\begin{aligned}
& I_L \left(\tau b_M^2 g(\tau b_M) \frac{\int_{b_L}^B + \int_{\tau b_L}^B}{\int_{b_M}^B + \int_{\tau b_M}^B} - \tau b_L^2 g(\tau b_L) \right) \tau \left(\alpha_H G(\tau b_L) + (1-\alpha_H) G(\tau b_M) - 1 \right) \\
& + I_L \left(\tau b_M^2 g(\tau b_M) \frac{\int_{b_L}^B + \int_{\tau b_L}^B}{\int_{b_M}^B + \int_{\tau b_M}^B} - \tau b_L^2 g(\tau b_L) \right) \left(\frac{(1-\alpha_H) \left(\int_{b_M}^B + \int_{\tau b_M}^B \right)}{b_M} + \frac{\alpha_H \left(\int_{b_L}^B + \int_{\tau b_L}^B \right)}{b_L} \right) \\
& - I_L \left(1 - \alpha_H G(\tau b_L) - (1-\alpha_H) G(\tau b_M) \right) \left(b_M^2 g(b_M) \frac{\int_{b_L}^B + \int_{\tau b_L}^B}{\int_{b_M}^B + \int_{\tau b_M}^B} - b_L^2 g(b_L) \right).
\end{aligned}$$

To show that the last expression is positive, it is sufficient to show that

$$\tau b_M^2 g(\tau b_M) \frac{\int_{b_L}^B + \int_{\tau b_L}^B}{\int_{b_M}^B + \int_{\tau b_M}^B} - \tau b_L^2 g(\tau b_L) > b_M^2 g(b_M) \frac{\int_{b_L}^B + \int_{\tau b_L}^B}{\int_{b_M}^B + \int_{\tau b_M}^B} - b_L^2 g(b_L)$$

and

$$\frac{(1-\alpha_H) \left(\int_{b_M}^B + \int_{\tau b_M}^B \right)}{b_M} + \frac{\alpha_H \left(\int_{b_L}^B + \int_{\tau b_L}^B \right)}{b_L} > (1+\tau) \left(1 - \alpha_H G(\tau b_L) - (1-\alpha_H) G(\tau b_M) \right).$$

The first inequality follows from the assumption that $b^2 g(b)$ is increasing and convex. The second inequality is equivalent to

$$\begin{aligned}
& \tau \left(\alpha_H H(\tau b_L) + (1-\alpha_H) H(\tau b_M) - 1 \right) + \frac{(1-\alpha_H) \int_{b_M}^B}{b_M} + \frac{\alpha_H \int_{b_L}^B}{b_L} \\
> \; & 1 - \alpha_H G(\tau b_L) - (1-\alpha_H) G(\tau b_M),
\end{aligned}$$

which is always true, as

$$\alpha_H H(\tau b_L) + (1-\alpha_H) H(\tau b_M) - 1 > 0$$

and

$$\begin{aligned}
\frac{(1-\alpha_H) \int_{b_M}^B}{b_M} + \frac{\alpha_H \int_{b_L}^B}{b_L} \;&>\; 1 - \alpha_H G(b_L) - (1-\alpha_H) G(b_M) \\
&>\; 1 - \alpha_H G(\tau b_L) - (1-\alpha_H) G(\tau b_M).
\end{aligned}$$

Hence, we show that $\frac{b_L}{b_M}$ is always increasing in τ. This finishes the proof.

The Proof of Proposition 3

In this section, I consider the equilibrium where imported goods are purchased only by the rich and show that in this case, the rich gain more from trade liberalization than the poor do. If transport costs are such that $\tau b_M \geq B$ in the equilibrium, then the equilibrium equations are given by

$$\begin{cases}
\frac{\int_{b_L}^{b_M} t dG(t) + \int_{\tau b_L}^B t dG(t)}{\int_{b_M}^B t dG(t))} = \left(\frac{I_H}{I_L} - 1 \right) \left(\alpha_H + \frac{b_L (1-\alpha_H)}{b_M} \right), \\
\frac{f_e}{cN} + 1 + \alpha_H \tau = \alpha_H \left(H(b_L) + \tau H(\tau b_L) \right) + (1-\alpha_H) H(b_M).
\end{cases}$$

As in the previous sections, I rewrite the equilibrium equations in the following way:

$$
\begin{aligned}
J_1 &\equiv \alpha_H \left(H(b_L) + \tau H(\tau b_L)\right) + (1-\alpha_H) H(b_M) - \frac{f_e}{cN} - 1 - \alpha_H \tau = 0, \\
J_2 &\equiv I_L \left(\int_{b_L}^{b_M} + \int_{\tau b_L}^{B} \right) - (I_H - I_L) \left(\alpha_H + \frac{b_L(1-\alpha_H)}{b_M} \right) \int_{b_M}^{B} = 0.
\end{aligned}
$$

By differentiating these equations, we obtain

$$
\begin{aligned}
\frac{\partial J_1}{\partial b_M} &= -\frac{(1-\alpha_H)}{b_M^2} \int_{b_M}^{B} < 0, \ \frac{\partial J_1}{\partial b_L} = -\frac{\alpha_H}{b_L^2} \left(\int_{b_L}^{B} + \int_{\tau b_L}^{B} \right) < 0, \\
\frac{\partial J_1}{\partial \tau} &= \alpha_H \left(G(\tau b_L) - 1 \right) < 0, \\
\frac{\partial J_2}{\partial b_M} &= I_L b_M g(b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B}} + I_L \frac{b_L(1-\alpha_H)}{b_M^2} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{B}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} > 0 \\
\frac{\partial J_2}{\partial b_L} &= -I_L b_L \left(g(b_L) + \tau^2 g(\tau b_L) \right) - I_L \frac{(1-\alpha_H)}{b_M} \frac{\int_{b_L}^{b_M} + \int_{\tau b_L}^{B}}{\alpha_H + \frac{b_L(1-\alpha_H)}{b_M}} < 0 \\
\frac{\partial J_2}{\partial \tau} &= -I_L \tau b_L^2 g(\tau b_L) < 0.
\end{aligned}
$$

Recall that the sign of $\left(\frac{U_H}{U_L}\right)'_\tau$ is the same as the sign

$$
\frac{\partial J_1}{\partial \tau} \left(\frac{\partial J_2}{\partial b_M} b_M + \frac{\partial J_2}{\partial b_L} b_L \right) - \frac{\partial J_2}{\partial \tau} \left(\frac{\partial J_1}{\partial b_M} b_M + \frac{\partial J_1}{\partial b_L} b_L \right). \tag{30}
$$

Using the expressions for $\frac{\partial J_i}{\partial \tau}$, $\frac{\partial J_i}{\partial b_M}$, and $\frac{\partial J_i}{\partial b_L}$ derived above, we obtain

$$
\begin{aligned}
\frac{\partial J_2}{\partial b_M} b_M + \frac{\partial J_2}{\partial b_L} b_L &= I_L \left(b_M^2 g(b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B}} - b_L^2 g(b_L) - \tau^2 b_L^2 g(\tau b_L) \right), \\
\frac{\partial J_1}{\partial b_M} b_M + \frac{\partial J_1}{\partial b_L} b_L &= -\frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M} - \frac{\alpha_H \left(\int_{b_L}^{B} + \int_{\tau b_L}^{B} \right)}{b_L}.
\end{aligned}
$$

Plugging these expressions into (30), we have

$$
\begin{aligned}
& I_L \left(b_M^2 g(b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B}} - b_L^2 g(b_L) - \tau^2 b_L^2 g(\tau b_L) \right) \alpha_H \left(G(\tau b_L) - 1 \right) \\
& - I_L \tau b_L^2 g(\tau b_L) \left(\frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M} + \frac{\alpha_H \left(\int_{b_L}^{B} + \int_{\tau b_L}^{B} \right)}{b_L} \right) \\
= \ & I_L \alpha_H \left(b_M^2 g(b_M) \frac{\int_{b_L}^{B} + \int_{\tau b_L}^{B}}{\int_{b_M}^{B}} - b_L^2 g(b_L) \right) \left(G(\tau b_L) - 1 \right) \\
& + I_L \tau b_L^2 g(\tau b_L) \left(\alpha_H \tau \left(1 - H(\tau b_L) \right) - \frac{(1-\alpha_H)\int_{b_M}^{B}}{b_M} - \frac{\alpha_H \int_{b_L}^{B}}{b_L} \right) < 0.
\end{aligned}
$$

Hence, $\left(\frac{U_H}{U_L}\right)'_\tau < 0$, implying that the rich lose more from higher transport costs. This finishes the proof.