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# Should Central Banks Lean Against Changes in Asset Prices? *

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**Abstract**

How should monetary policy be conducted in the presence of endogenous feedback loops between asset prices, firms' financial health, and economic activity? We reconsider this question in the context of the financial accelerator model and show that, when the level of natural output is inefficient, the optimal monetary policy under commitment leans considerably against movements in asset prices and risk premia. We demonstrate that an endogenous feedback loop is crucial for this result and that price stability is otherwise quasi-optimal absent this feature. We also show that the optimal policy can be closely approximated and implemented using a speed-limit rule that places a substantial weight on the growth of financial variables.

*The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Swiss National Bank, the Federal Reserve Bank of San Francisco or any other person associated with the Federal Reserve System.

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# 1 Introduction

How should monetary policy be conducted in the presence of endogenous feedback loops between asset prices, firms' financial health, and economic activity? In a series of papers, Bernanke and Gertler [1999, 2001] argued that aggressive inflation-targeting rules perform best in the presence of large movements in asset prices that then affect credit-constrained firms.[1] Central banks should thus react to movements in asset prices to the extent that they affect the forecast for inflation over the medium run. Yet, the recent financial crisis has to some extent weakened this policy prescription. Indeed, many policymakers have acknowledged that the financial turmoil has refined their views on the role of asset prices in the conduct of monetary policy.[2] Consistent with this introspection, the minutes of the November 2009 Federal Open Market Committee meeting, for instance, revealed that some Committee members were concerned that keeping the federal funds rate too low for too long could lead to excessive risk-taking in financial markets.

In this paper, we reconsider the role of asset prices and financial variables in general in the implementation of monetary policy using the financial accelerator model of Bernanke, Gertler, and Gilchrist [BGG, 1999]. In this framework, entrepreneurs need external sources of funding to finance investment and their level of net worth affects their cost of capital. By directly affecting entrepreneurs' net worth, swings in asset prices affect the cost of credit financing and tend to amplify movements in investment. In contrast to the work of Bernanke and Gertler [1999, 2001] and Gilchrist and Leahy [2002], who focused on simple interest rate rules, we emphasize the design of optimal monetary policy under commitment and study its impact on asset prices and risk premia (external finance premium, in the words of BGG). We show that the optimal policy deviates substantially from perfect inflation targeting – the canonical New Keynesian benchmark – and instead leans considerably against asset prices and/or changes in risk premia.

In this economy, the feedback loop that connects asset prices, net worth, risk premia, leverage, investment and leads back to asset prices – the so-called financial accelerator – is crucial to understanding the optimal design of monetary policy. As the tightness of the financial

[1] Cecchetti et al. [2000] present an opposing view.

[2] See, for instance, Yellen [2009], Stern [2009], and Hoenig [2010].

constraint varies with the state of the business cycle (a central feature of the financial accelerator), so does the degree of inefficiency of natural output, i.e., the level of output that ensures perfect price stability. In this context, price stability is suboptimal. The central bank leans against swings in asset prices and excessive changes in risk premia because doing so reduces inefficient fluctuations in output. Hence, optimal monetary policy trades off volatility in the inflation rate for a more efficient allocation of production. The trade-off is particularly acute when the economy is hit by financial shocks. We experiment with two types of shocks that have been emphasized in the literature as potential drivers of the business cycle: a net worth shock and a risk shock (see Christiano et al. [2003, 2010] and Gilchrist and Leahy [2002]).

The presence of an endogenous feedback loop between asset prices and economic fluctuations – through net worth and leverage – is crucial for optimal policy to lean against asset price movements and changes in risk premia. Absent large endogenous movements in asset prices (when the capital stock is not costly to adjust, for instance), we show that the optimal policy closely follows the standard prescription to stabilize prices. Only when financial fluctuations are significant enough to lead to substantial inefficiencies in the equilibrium allocations does optimal policy mitigate movements in asset prices and other financial variables. This result points to the importance of incorporating capital accumulation in the model to generate potentially large endogenous movements in asset prices and net worth and to consider economies in which the natural and efficient allocations differ markedly.

In terms of implementability, we show that in practice the optimal policy can be closely approximated by simple speed-limit interest rate rules (in the spirit of Walsh [2003]) that place a considerable weight on the growth of financial variables in response to real or financial shocks. These rules have the advantage of relying solely on observables, as only the growth rates of the variables in the rule need to be known. They do not require undue knowledge about the efficient levels of variables in the rule, which are typically difficult to assess, particularly so in the case of financial variables.[3] We consider a rule that includes the change in the external finance premium, but our results are similar if we instead include alternative measures such as the

[3] Interest rate rules including a change in asset prices have also been studied by Gilchrist and Saito [2008] in a model with imperfect information, and by Tetlow [2006] under model uncertainty.

changes in equity prices or net worth. In the midst of the financial crisis, many policymakers also advanced the idea of looking at credit growth as a possible indicator of financial excess (see, e.g., Mishkin [2008]). Including this variable in our simple speed-limit rule allows a close approximation of the allocation under the optimal policy as well. We find that the optimized interest rate rule (the one that best approximates the allocation under the optimal policy) in response to either real or financial shocks is one with a weight on the growth of financial variables that is substantially higher than that on inflation, with no weight on output growth.

Overall, this result differs substantially from the typical findings in the literature that the optimal weight on financial variables in optimized rules is either zero or orders of magnitude smaller than the weight on inflation. We trace back the difference in results to the fact that movements in asset prices in our economy imply large and inefficient movements in natural output that the policymaker can mitigate by leaning heavily on the growth rate of financial variables.

By modeling endogenous feedback loops through capital accumulation, our analysis of optimal policy complements the recent contributions of Curdia and Woodford [2009] and particularly those of De Fiore and Tristani [2009] and Carlstrom, Fuerst, and Paustian [2009]. De Fiore and Tristani [2009] study the optimal monetary policy in a model with costly state verification and price rigidity, but one in which there are no endogenous movements in net worth, partly reflecting the absence of capital accumulation. They derive the loss function of the central bank, which they show to depend on the volatility of the nominal interest rate and credit spreads, in addition to the volatility of inflation and that of the output gap. They find that following a financial shock, interest rates should be aggresively lowered but price stability remains nearly optimal otherwise.

In contrast, Carlstrom, Fuerst, and Paustian [2009] derive the optimal monetary policy in an environment in which firms' labor hiring is constrained by their net worth, as in the model of Kiyotaki and Moore [1997]. They show that the central bank's loss function is partly a function of the tightness of the credit constraint, which they interpret as a risk premium. Although their model abstracts from capital accumulation, it captures endogenous movements in net worth through movements in share prices of monopolistic sticky-price firms. They show that stabilizing inflation is nearly optimal in their framework, even if the credit constraint is quite severe, because the weight on inflation volatility in the central bank's loss function dwarfs that on the variability of the risk premium.

Our paper also relates to Faia and Monacelli [2007], who study the design of optimized interest rate rules in response to technology and government spending shocks in a model with agency costs adapted from the work of Carlstrom and Fuerst [1997]. Faia and Monacelli find that the optimal rule remains focused on stabilizing inflation. We complement their analysis by emphasizing the design of optimal policies under financial shocks.

The remainder of this paper is organized as follows. We briefly present the main building blocks of our model in the next section. We then describe the model's calibration before presenting our main results, emphasizing the importance of endogenous fluctuations in asset prices. We then discuss a simple and implementable speed-limit rule that closely approximates the optimal policy and conduct a robustness analysis. The last section concludes.

# 2 The model

Our analysis draws on BGG's seminal work and on the more recent contributions of Christiano, Motto, and Rostagno (CMR, 2003, 2010). The economy consists of four sectors. The first two sectors produce intermediate and final goods, respectively, while the third produces physical capital and the fourth provides loans to investors and can be interpreted as a pseudo-banking sector. Banks in our framework are risk averse and hold a perfectly diversified portfolio of entrepreneurial loans. The economy is populated by households composed of consumers and workers and by entrepreneurs. The latter own the capital stock and provide capital services to intermediate goods producers. Entrepreneurs finance their purchases of capital both with internal funds (own net worth) and external funds (bank loans). Entrepreneurs face idiosyncratic productivity shocks and are subject to bankruptcy if their project fails. The banking sector receives deposits from households (which are considered riskless and are thus remunerated as such) and make loans to entrepreneurs. A key mechanism of the model is that the premium over the risk-free rate – the so-called external finance premium – that entrepreneurs must pay to borrow is related to their leverage. The more "skin in the game" the entrepreneurs have, the smaller is the moral hazard problem and the premium.

In the following, we describe the main building blocks of the model, which features a traditional New Keynesian model augmented by a financial accelerator following BGG and CMR. More details can be found in the appendix and in CMR.

## 2.1 Main building blocks

### 2.1.1 Capital producers

As in CMR, there is a large number of identical capital producers operating under perfect competition who, at time $t$, produce new physical capital $k_{t+1}$ to be used in $t+1$, using the following production function:

$$k_{t+1} = i_t - \frac{\varphi}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 + (1-\delta)\,k_t,$$

where $i_t$ denotes investment, $\delta$ is the depreciation rate $0 \leq \delta \leq 1$ and $\varphi$ is a capital adjustment cost. The new capital stock is sold at price $Q_t$ and the old capital stock is purchased at price $\widetilde{Q}_t$ on the capital market. Therefore, profits are given by

$$\Pi_t^k = Q_t k_{t+1} - P_t i_t - \widetilde{Q}_t k_t.$$

Maximizing profit subject to the production constraint leads to the following two first-order conditions:

$$Q_t\left(1 - \varphi\left(\frac{i_t}{k_t} - \delta\right)\right) = P_t \tag{1}$$

$$\widetilde{Q}_t = Q_t\left[1 - \delta + \frac{\varphi}{2}\left(\left(\frac{i_t}{k_t}\right)^2 - \delta^2\right)\right]. \tag{2}$$

### 2.1.2 Entrepreneurs

There is a large number of heterogenous entrepreneurs, indexed by $j$, who buy new capital stock $k_{j,t+1}$ at price $Q_t$ from the capital producers and transform it into capital services $x_{j,t+1}$

according to the linear technology:

$$x_{j,t+1} = \omega_j k_{j,t+1}, \tag{3}$$

where $\omega_j$ denotes the productivity of the transformation technology which is entrepreneur-specific. The random variable $\omega$ is drawn from a cumulative distribution denoted by $F_t(\omega^\star) = P(\omega_j \leqslant \omega^\star)$ with mean $E(\omega) = 1$. Entrepreneurial investment is risky and, as in CMR, the degree of risk is assumed to vary over time. Therefore we assume that $\log(\omega)$ is normally distributed with mean $\mu_{\omega,t}$ and standard deviation $\sigma_{\omega,t}$. The standard deviation $\sigma_{\omega,t}$ is the realization of a mean-preserving stochastic process referred to below as a "risk shock," which follows an AR(1) process with autoregressive coefficient $\rho_{\sigma_\omega}$ and innovations, $\varepsilon_{\sigma_\omega}$, assumed to be normally distributed with mean zero and standard deviation $\sigma_{\varepsilon_{\sigma_\omega}}$.

Each entrepreneur draws its type $\omega_j$ after capital $k_{j,t+1}$ has been purchased. To purchase capital, each entrepreneur can either use her net worth $N_{j,t+1}$ or borrow $B_{j,t+1}$ from banks at the gross rate of interest $R^b_{j,t+1}$. To ensure that entrepreneurs do not accumulate enough net worth to make the borrowing constraint nonbinding, we assume that entrepreneurs exit the economy (close business) each period with probability $1 - \gamma_t$.[4] Within each period, entrepreneurs rent their capital services to intermediate goods producers at the real price $z_{t+1}$, and at the end of the period they resell their capital stock to capital producers at price $\widetilde{Q}_t$.

Hence, entrepreneur $j$'s expected revenue from purchasing capital can be written as

$$E_t \left[ P_{t+1} z_{t+1} \omega_j k_{j,t+1} + \widetilde{Q}_{t+1} \omega_j k_{j,t+1} \right].$$

Denoting the rate of return on capital as

$$R^k_{t+1} \equiv \frac{P_{t+1} z_{t+1} + \widetilde{Q}_{t+1}}{Q_t},$$

we can rewrite an entrepreneur's expected revenue in the following way

$$E_t \left[ Q_t R^k_{t+1} \omega_j k_{j,t+1} \right]. \tag{4}$$

[4]To maintain a constant population of entrepreneurs, we assume that $1 - \gamma$ new entrepreneurs are born at the same time. These entrepreneurs finance their purchases with a transfer, $\tau^e_t$, that they receive from the government. Departing entrepreneurs consume their net worth.

To finance capital purchases, the entrepreneur can either use her net worth $N_{j,t+1}$ or enter a contract with a bank to borrow $B_{j,t+1}$ at gross rate $R^b_{j,t+1}$, such that

$$Q_t k_{j,t+1} = N_{j,t+1} + B_{j,t+1}.$$

The debt contract then specifies the loan amount $B_{j,t+1}$ and the nominal gross rate $R^b_{j,t+1}$. If the entrepreneur's project is a success he pays back the loan and interest ($R^b_{j,t+1}B_{j,t+1}$) to the bank. If the project fails (because the project's productivity, $\omega_j$, turns out to be too low) the bank pays a proportional cost $\mu$ to monitor the entrepreneur and confiscates the remaining assets. Thus, there exists a cutoff value $\omega^\star_{j,t+1}$ defined as

$$R^b_{j,t+1}B_{j,t+1} = \omega^\star_{j,t+1} Q_t R^k_{t+1} k_{j,t+1} \tag{5}$$

below which the entrepreneur declares bankruptcy.

The expected profit of an entrepreneur is therefore given by

$$E_t\Bigg[ \underbrace{\int_0^{\omega^\star_{j,t+1}} \omega Q_t R^k_{t+1} k_{j,t+1} \mathrm{d}F(\omega)}_{\substack{\text{Revenues} \\ \text{(Failing entrepreneurs)}}} + \underbrace{\int_{\omega^\star_{j,t+1}}^{\infty} \omega Q_t R^k_{t+1} k_{j,t+1} \mathrm{d}F(\omega)}_{\substack{\text{Revenues} \\ \text{(Successful entrepreneurs)}}}$$
$$- \underbrace{\int_0^{\omega^\star_{j,t+1}} \omega Q_t R^k_{t+1} k_{j,t+1} \mathrm{d}F(\omega)}_{\substack{\text{Expenses} \\ \text{(Failing entrepreneurs)}}} - \underbrace{\int_{\omega^\star_{j,t+1}}^{\infty} R^b_{j,t+1} B_{j,t+1} \mathrm{d}F(\omega)}_{\substack{\text{Expenses} \\ \text{(Successful entrepreneurs)}}} \Bigg]$$

which simplifies to

$$E_t\left[\int_{\omega^\star_{j,t+1}}^{\infty} (\omega Q_t R^k_{t+1} k_{j,t+1} - R^b_{j,t+1} B_{j,t+1}) \mathrm{d}F(\omega)\right].$$

Using the definition of the cutoff value $\omega^\star_{j,t+1}$, and making use of the fact that $k_{j,t+1}$ is decided in period $t$, the expected profit function rewrites as

$$E_t\left[\int_{\omega^\star_{j,t+1}}^{\infty} (\omega - \omega^\star_{j,t+1}) R^k_{t+1} \mathrm{d}F(\omega)\right] Q_t k_{j,t+1}, \tag{6}$$

which defines the entrepreneur's objective function.

#### 2.1.3 Banks

Since the bank receives $R^b_{j,t+1}B_{j,t+1}$ if an entrepreneur's productivity is higher than the cutoff value $\omega^\star_{j,t+1}$ and otherwise seizes all the entrepreneur's assets $\omega_j Q_t R^k_{t+1} k_{j,t+1}$ if the project fails (after having paid a proportional monitoring cost, $\mu$), the bank's revenue corresponds to:

$$\int_{\omega^\star_{j,t+1}}^{\infty} R^b_{j,t+1}B_{j,t+1}\mathrm{d}F(\omega) + (1-\mu)\int_0^{\omega^\star_{j,t+1}} \omega Q_t R^k_{t+1} k_{j,t+1}\mathrm{d}F(\omega)$$

which can be rewritten as

$$(1 - F(\omega^\star_{j,t+1}))R^b_{j,t+1}B_{j,t+1} + (1-\mu)\int_0^{\omega^\star_{j,t+1}} \omega Q_t R^k_{t+1} k_{j,t+1}\mathrm{d}F(\omega).$$

Banks are assumed to be perfectly competitive and riskless. They finance themselves via household deposits $B_{j,t+1}$ that earn the nominal (risk-free) gross interest rate $R_t$, where $R_t$ is not contingent on shocks realized in $t+1$. Thus, the zero profit condition implies

$$(1 - F(\omega^\star_{j,t+1}))R^b_{j,t+1}B_{j,t+1} + (1-\mu)\int_0^{\omega^\star_{j,t+1}} \omega Q_t R^k_{t+1} k_{j,t+1}\mathrm{d}F(\omega) = R_t B_{j,t+1}, \tag{7}$$

which also corresponds to the bank's participation constraint.

#### 2.1.4 Aggregated net worth

The contract between a bank and an entrepreneur specifies a level of loans, $B_{j,t+1}$, and a gross interest rate, $R^b_{j,t+1}$, that maximizes the expected profit of the entrepreneur in equation (6) subject to the bank's participation constraint in equation (7), or identically a level of capital $k_{j,t+1}$ and a cutoff point $\omega^\star_{j,t+1}$. Clearly, the amount of loans and therefore the level of investment will depend on entrepreneurs' net worth $N_{j,t+1}$ (the new state variable associated with asymmetric information). Aggregating over all entrepreneurs, it can be shown (see Appendix and BGG for details) that entrepreneurial net worth evolves according to

$$N_{t+1} = \gamma_t \Xi_t + \tau^e_t,$$

where $\Xi_t$ is the aggregate profit flow to entrepreneurs, and $\tau^e_t$ is the government transfer to "newly born" entrepreneurs. Following Gilchrist and Leahy [2002], we assume the entrepreneurs' exogenous survival probability, $\gamma_t$, to be stochastic, representing a shock to net worth.

The shock follows an AR(1) process with autoregressive coefficient $\rho_\gamma$ and innovations, $\varepsilon_\gamma$, assumed to be normally distributed with mean zero and standard deviation $\sigma_\gamma$.

#### 2.1.5 Intermediate goods producers

Intermediate goods producers are monopolistically competitive. They produce an intermediate good by means of capital and labor according to a constant return to scale production function:

$$y_t(i) = a_t k_t(i)^\alpha h_t(i)^{1-\alpha} \qquad \text{with } \alpha \in (0,1), \tag{8}$$

where $k_t(i)$ and $h_t(i)$ respectively denote the physical capital and the labor input used by firm $i$ to produce $y_t(i)$, and where $a_t$ represents total factor productivity, which is assumed to follow an AR(1) process with autoregressive coefficient $\rho_a$ and where the innovation, $\varepsilon_a$, is normally distributed with mean zero and standard deviation $\sigma_a$.

The firm determines its production plan to minimize its total cost :

$$\min_{\left\{h_{t(i)}, k_t(i)\right\}} P_t w_t h_{t(i)} + P_t z_t k_t(i)$$

subject to production (8). As is standard in the New Keynesian literature, we assume that firms set their prices according to Calvo's staggering scheme and define $\tau$ as the probability that a particular firm is able to reset its price next period.

#### 2.1.6 Final goods producers

Final goods are produced by competitive retailers that assemble intermediate goods according to a CES aggregator

$$y_t = \left(\int_0^1 y_t(i)^{\frac{1}{\lambda}} di\right)^\lambda \qquad \text{with } \lambda \in [1, \infty),$$

where $\lambda$ stands for the elasticity of substitution between intermediate goods and determines the intermediate good producers' market power (or the steady-state markup of price $P$ over their marginal costs).

#### 2.1.7 Households

The representative household maximizes the discounted value of its lifetime utility that features preferences over consumption and labor:

$$\mathcal{U}_t = E_0 \sum_{j=0}^{\infty} \beta^j \left( \frac{1}{1-\sigma_c} c_{t+j}^{1-\sigma_c} - \frac{\nu_h}{1+\sigma_h} h_{t+j}^{1+\sigma_h} \right),$$

subject to the budget constraint

$$P_t C_t = P_t w_t h_t + \Pi_t + R_{t-1} B_t - B_{t+1},$$

where $\Pi_t$ represents the dividends earned on the profits of intermediate goods firms, $0 < \beta < 1$ represents the rate of time preference, and $\nu_h$ is a normalizing constant.

The household is also subject to the time endowment constraint

$$l_t + h_t = 1,$$

where $l_t$ denotes the proportion of total time dedicated to leisure.

### 2.2 Calibration

In our benchmark calibration, we assume that the coefficient of relative risk aversion, $\sigma_c$, is 1.5 and we set $\sigma_h$ to 1, implying a unit Frish elasticity of labor supply. We choose $\beta$=0.99, leading to an annual real interest rate of 4 percent.

Because of monopolistic competition, the steady state of the model is distorted, leading to a gross markup of price over marginal costs, $\lambda$, equal to 1.1. We assume a capital share, $\alpha$, of 36 percent in the Cobb-Douglas production function, and a quarterly depreciation rate of 2.5 percent per quarter. We set the Calvo parameter, $\theta$, to 0.75, so that, on average, firms expect to be able to change their price once a year. Those parameter values also imply that the economy's steady-state capital-output ratio, $\frac{k}{y}$, and investment-output ratio, $\frac{i}{y}$, are in line with estimations provided by CMR (2010) for the euro area $(8.70; 0.21)$ and the United States $(6.98; 0.25)$. The equity to debt ratio, $\frac{N}{(K-N)}$, implied by our calibration is closer to the lower end of estimates reported by CMR (2010) for the euro area and the United States.

The financial intermediation block draws heavily on values assumed by CMR (2010) for the United States and the euro area. The share of entrepreneurs who survive from one quarter to the next, $\gamma$, is set to its U.S. value of 97.62. We set the bank's monitoring cost as a percent of final output, $\mu$, to 40 percent, which lies between the U.S. estimate of 34 percent and that of 53 percent in the euro area. The percent of businesses going bankrupt per quarter, $F(\omega^{\star})$, is assumed to be 17.4 percent, between U.S. and euro area estimates ([0.34; 0.53] for $\mu$ and [0.26; 0.15] for $F(\omega^{\star})$). These calibration choices lead to an annual external finance premium $(\frac{R^k}{R})$ of 2.2 percent in equilibrium.

For our simulation exercises in which we consider the three shocks simultaneously, we set the parameters ($\rho_a$, $\rho_\gamma$, $\rho_{\sigma_\omega}$ and $\sigma_{\varepsilon_a}$, $\sigma_{\varepsilon_\gamma}$, $\sigma_{\varepsilon_{\sigma_\omega}}$) of the AR(1) processes to correspond to the mode of the CMR (2010) Bayesian estimates for the United States. However, for simplicity and ease of exposition, we will first present our main results emphasizing the optimal monetary policy response to single shocks. In this case, we set the standard deviations for each innovation to 1 percent and set $\rho_a$=$\rho_{\sigma_\omega} = 0.9$ and $\rho_\gamma = 0.5$, in line with the CMR (2010) estimation results.

# 3 The results

In this section, we compute the optimal monetary policy under commitment following the timeless perspective approach of Woodford [2003]. We first do so assuming a distorted steady state, that is, we derive the optimal policy assuming the presence of steady state markups that are not offset by lump-sum subsidies (see, for instance, Benigno and Woodford [2005]). We study the responses to productivity and financial shocks and contrast the optimal precommitment policy to the traditional New Keynesian optimal benchmark, which consists of ensuring perfect price stability. We show that the financial friction introduces a policy trade-off that is especially acute in response to financial shocks. We then highlight the importance of endogenous fluctuations in net worth and the role of asset prices by trimming down capital adjustment costs so that asset prices barely move in response to shocks. In this case, a policy that perfectly stabilizes prices is close to optimal. To study whether asset prices, interest rate spreads, or leverage ratio targeting could play a role in the conduct of monetary policy, we simulate the economy under different interest rate rules to which we append these alternative indicators and verify whether any of them can get us close to the optimal allocation. Finally, we conclude by conducting various robustness exercises on different dimensions of the model. In particular, we show that the large consensus around a policy of perfect price stabilization, even in the presence of a financial accelerator (see, for instance, Faia and Monacelli [2007]), is due to the assumption that policymakers can compensate workers for the monopolistic competition distortion in a lump-sum fashion.

## 3.1 Optimal monetary policy

Consider first the impact of a positive productivity shock when monetary policy either is conducted optimally or targets price stability.[5] In the financial accelerator model, higher productivity leads to higher asset prices, which increases entrepreneurs' net worth and results in higher investment. In turn, the increase in investment and output stimulates asset prices, which then feed back into higher net worth and investment. Figure 1 shows that optimal policy tends to dampen the response of the economy compared to a policy of perfect price stability (PPS), the canonical New Keynesian benchmark. On impact the rise of asset prices under the optimal policy is almost half of that under PPS, which in turn dampens the responses of net worth, leverage, and the risk premium by similar magnitudes. At the same time, optimal policy allows some fluctuations in the inflation rate as policymakers trade off inflation stabilization for output stabilization.

The optimal policy trade-off illustrated in Figure 1 means that there is an externality associated with the financial accelerator mechanism: The financial friction (monitoring cost) gives rise to an overreaction of investment and output with respect to the efficient allocation that optimal policy takes into account. Figure 2 compares the efficient responses to those under either PPS or the optimal policy following a positive productivity shock.[6] As in Figure

[5] Note that because we assume a nonefficient steady state, the natural allocation ensuring price stability is not necessarily efficient.

[6] The efficient response is defined as the response of the economy assuming no financial friction, a lump sum tax-financed subsidy that offsets the monopolistic competition distortion in the steady state, and perfectly flexible prices.

1, under the optimal policy the rise in net worth and equity is more muted than under PPS. Fewer investment projects are financed which aligns the response of the economy more closely with the efficient allocation where financial frictions are absent.

Although the difference between optimal and perfect inflation-targeting policies remains relatively small following a productivity shock, the departure from price stability becomes much more pronounced when the economy is hit by financial shocks. Because financial shocks interfere directly in the financial intermediation process, they have a more direct bearing on asset prices, net worth, and finance premia than productivity shocks. We focus here on two different shocks. The first is a "risk" shock, i.e., an innovation to the cross-sectional distribution of entrepreneurs' individual productivity (a $\sigma_\omega$-shock). A drop in $\sigma_\omega$ is to be interpreted as a decrease in the perception of market risk, which affects risk premia directly (see Christiano et al. [2010]).[7] As in the case of a productivity shock, optimal policy leans against movements in asset prices and net worth, but the difference between optimal policy and perfect inflation targeting is much starker. As Figure 3 shows, a 1 percent decrease in $\sigma_\omega$ leads to a persistent increase in output and investment when policy is directed exclusively at price stability (PPS). In contrast, the output hike is only temporary under the optimal monetary policy. The stance of policy is more restrictive – as reflected in the sudden tightening of real interest rates – dampening equity prices and avoiding the swelling of net worth that occurs under PPS. Overall, by leaning against the jump in asset prices, optimal monetary policy avoids the investment spree that characterizes the solution under PPS, but requires a temporary drop in inflation.

Interestingly, since optimal policy prevents an increase in entrepreneurs' net worth, leveraging is larger than under PPS despite lower investment.[8] Optimal policy limits the amplitude of investment fluctuations that arise out of the financial accelerator's externality by leaning against asset prices. However, as banks are perfectly safe in the model (risk is perfectly diversified), there is no concern about the amount of leverage in the economy.

The second financial shock that we consider is an innovation to net worth (a $\gamma$-shock),

[7] The analysis is almost isomorphic for shocks to net worth (or $\gamma$-shocks) that are central to the analysis in Gilchrist and Leahy [2002]. However, as explained in Christiano et al. [2010], these shocks have counterfactual implications for credit growth and are therefore less likely to be primary drivers of the business cycle.

[8] Under our calibration, optimal policy actually engineers a *decrease* in net worth.

which is central to the analysis in Gilchrist and Leahy [2002]. This shock can be interpreted as a nonfundamental increase in asset prices, which, like a negative $\sigma_\omega$-shock, implies a decrease in moral hazard and a boost in asset prices, investment, and output. Figure 4 shows that, again, optimal policy tends to dampen movements in asset prices, net worth and finance premia compared to perfect price stability, which leads to more output stability in the medium run. The efficient economy (no monopolistic competition and no price or financial frictions) is not affected by financial shocks: output, consumption, and labor are constant. Because of the policy trade-off mentioned above, however, optimal monetary policy reaches a middle ground between perfect price stability and constant output.

## 3.2 Role of asset prices

How important are movements in asset prices in determining the optimal monetary response? To isolate the effect of fluctuations in asset prices via the financial accelerator, we trim down the capital adjustment cost in equation (1). Mechanically, when investment is not costly, asset prices (Tobin's Q) do not need to increase by much to induce the required level of investment; in our example, asset prices remain almost fixed. Figure 5 shows the response of the economy to positive productivity, $\sigma_\omega$-, and $\gamma$-shocks under the optimal policy (solid line) and PPS (dashed line) when the asset prices transmission channel is shut off. Comparing Figures 1 and 5 demonstrates that endogenous fluctuations in asset prices and their impact on net worth are first order in determining the optimal monetary response to shocks. When asset prices do not move (as in Figure 5) optimal monetary policy is virtually identical to a policy targeting price stability, as the wedge between natural and efficient output remains almost constant.

This result again highlights the importance of the "multiplier effects" of asset prices on investment, i.e., the feedback loop through which an increase in asset prices boosts investment via a rise in net worth and a corresponding drop in the external finance premium. This dynamic effect pushes the economy to deviate substantially from the efficient allocation and price stability is not the best solution in terms of welfare.

### 3.3 Implementing the optimal policy

One problem with welfare-based optimal policies is that they rely on unobservables such as the efficient level of output or various shadow prices, which, in practice, makes them difficult to implement. A straightforward alternative is to rely on simple but suboptimal rules that are functions of observables only. Previous literature (see Walsh [2003]) has shown that the optimal precommitment monetary policy rule can be approximated by a simple inertial policy rule – or speed-limit rule – in the New Keynesian context. In particular, just as optimal policies with commitment, a speed-limit rule that lean on the *change* in the output gap can introduce inertia into output and inflation that would otherwise be absent with other types of simple rules. To take into account the extra friction due to the financial accelerator, we also append commonly accepted measures of financial excesses and postulate the following general specification:

$$\widehat{r_t} = g_\pi \widehat{\pi_{y,t}} + g_y \left(\widehat{y_t} - \widehat{y_{t-1}}\right) + g_f \left(\widehat{fe_t} - \widehat{fe_{t-1}}\right). \qquad (9)$$

where variables with a hat denote deviations from steady state and $\widehat{fe_t}$ is our indicator of financial excess. We consider a rule in which we set this indicator to equal the external finance premium. However, our results are robust to alternative measures such as equity prices, net worth, or loans because they are all interrelated via the optimal debt contract between banks and entrepreneurs. The general rule relies on a reasonable information set, as only the growth rates of the variables are required. Using equation (9), we search for the simple rule that best matches the optimal precommitment plan. To do so, we rely on the following distance minimization algorithm defined over the $n$ impulse response functions of $m$ variables of interest to the policymakers. The algorithm searches the space of parameters $g_\pi$, $g_y$, and $g_f$ that minimize the distance criterion:

$$\arg\min_{\vartheta} \left(IRF_{SR}\left(\vartheta\right) - IRF_O\right)' \left(IRF_{SR}\left(\vartheta\right) - IRF_O\right),$$

where $IRF_{SR}(\vartheta)$ is an $mn \times 1$ vector of impulses under the postulated simple interest rate rule, and $IRF_O$ is its counterpart under the optimal plan.[9] The algorithm matches the responses of 12 variables ($y_t$, $c_t$, $h_t$, $i_t$, $\pi_t$, $R_t$, $RR_t = \frac{R_t}{\pi_t}$, $B_t$, $spread_t \equiv \frac{R_t^k}{R_t}$, $LR_t \equiv \frac{B_t}{N_t}$, $Q_t$, $N_t$) over a 30 quarters period using constrained versions of the simple rule (9)where $\vartheta = (g_\pi, g_y, g_f)'$.

Figures 6, 7, and 8 show impulse responses to a productivity shock, $a_t$, a risk shock, $\sigma_\omega$, and a net worth shock, $\gamma_t$, respectively, under the optimal plan, the optimized speed-limit rule, and a traditional Taylor rule. Table 2 reports the value of the parameters of the optimized rule for the different exercises.[10] The optimized rule leans strongly against inflation (because of the price friction) and the change in risk premium (because of the financial friction), but does not react to the change in output ($g_y = 0$). Moreover, the financial friction is quantitatively important and leads the monetary authority to place a larger weight on financial excesses than on movements in inflation. Figures 6, 7, and 8 show that our speed-limit rule reproduces very closely the allocation generated under the optimal policy and particularly leads to inertial, hump-shaped, movements in many variables. In contrast, the standard Taylor rule often misses on the level of the variables' responses and typically doesn't generate hump-shaped movements.

The importance of leaning against financial excesses is highlighted in Figure 9 by showing the allocations in response to each shock when the weight on the growth of the financial indicator in the speed-limit rule is reduced compared to that in the optimized rule. The figure shows that a larger weight on the change in $fe_t$ helps create inertial responses, particularly for investment and output. When the policymaker leans less against changes in the risk premium, investment responds too much too quickly, which leads to suboptimal movements in consumption (not shown) and output.

As a final exercise, we also searched for the parameters of the speed-limit rule that best match the optimal plan when the three shocks are considered simultaneously. The result, shown

[9] Another possibility is to search within a predetermined space of simple interest rate rules for the one that minimizes the central bank loss function (see, e.g., Söderlind, 1999, and Dennis, 2004). However because different combinations of output gaps and inflation variability could in principle produce the same welfare loss, we rely on a more stringent exercise consisting of matching impulse response functions.

[10] Note also that similar results would be obtained with rules that lean against other financial variables such as equity prices, net worth, or loans because they are all interrelated via the optimal debt contract between banks and entrepreneurs.

in the last row of Table 2, is qualitatively similar to the individual shock exercises presented in Figures 6 to 8.

Finally, note that our results are qualitatively similar if instead we find the optimized rule by minimizing the distance between welfare under the optimal policy and welfare under the rule. In fact, using our procedure, the level of welfare under the optimized rule ends up matching that under the optimal policy. In contrast, a policy of perfect price stability implies a welfare cost of 0.05 percent of steady-state consumption when the economy is subject to all three shocks.

## 3.4 Robustness analysis

### 3.4.1 Subsidized steady-state

Most authors routinely assume that the governement is able to levy a lump-sum tax and transfer the proceeds to workers in the form of an employment subsidy that compensates them for the welfare loss associated with the steady-state monopolistic competition distortion. This assumption makes the equilibrium under flexible prices efficient and, under certain conditions (see Woodford [2003]), optimal monetary policy delivers the flexible price allocation (or constant markups and prices). Although this assumption is innocuous in a canonical New Keynesian model where the only other distortion is price stickiness (since the wedge between efficient and natural output is constant), it can have important consequences when the economy is simultaneously subject to another real friction, like the countercyclical credit market imperfections inherent to the financial accelerator model.

In general, the flexible price allocation does not maximize household welfare when there is a nontrivial real friction and when the steady-state markup is non-zero.[11] Figure 10 shows the gap between the efficient and natural responses of output to productivity and financial shocks. It appears that the wedge between natural and efficient output is not constant under flexible prices, which opens up the door to welfare-improving monetary policy under sticky prices. In this case, the optimal policy trades some output variability against movements in inflation.

[11] See Adao, Correia, and Teles (2003) for a formal analysis in the context of a monetary model with cash-in-advance constraints and firms that set prices one period in advance.

As shown in Galí, Gertler, and Lopez-Salido [2007], however, the welfare loss from output fluctuations is increasing in the amount of steady-state distortion. In the case of a subsidized steady state, the welfare loss associated with inefficient output variations is dwarfed by the welfare cost of inflation, and price stability may remain the welfare-maximizing policy. Figure 11 shows that, indeed, the optimal policy response to productivity, $\sigma_\omega$-, and $\boldsymbol{\gamma}-$shocks is to aim at perfect price stability when a subsidy is available to offset the steady-state monopolistic competition distortion.

As in Carlstrom, Fuerst, and Paustian [2009] and Faia and Monacelli [2007], who also assume a nondistorted steady state, we find that welfare is mostly affected by inflation variability and that consequently policymakers follow a policy of quasi price stability.

### 3.4.2 Inflation protected deposits

Christiano et al. [2010] emphasize the importance of the so-called "debt-deflation effect" for the transmission of real and financial shocks in a model with financial frictions. Under optimal monetary policy, however, assuming that household bank deposits are protected against surprise inflation (like in BGG) or not (like in CMR) is almost irrelevant in terms of welfare. Figure 12 shows the response of the economy to a risk shock under optimal policy in both instances. The main difference concerns the entrepreneur's leverage ratio. Because the shock is slightly inflationary, it tends to boost net worth, equity prices, and loans in the case of a CMR contract.

# 4 Conclusion

The financial crisis has forced policymakers and academics to revisit to role of financial variables in the conduct of monetary policy. Our work is in this vein and shows that in the seminal framework of Bernanke, Gertler, and Gilchrist [1999] there is a strong impetus for the optimal monetary policy to lean against movements in asset prices in response to either real or financial shocks. Our result hinges on the presence of two reasonable, though often overlooked, conditions. First, the natural allocation must differ from the efficient one, with a wedge between the two that varies along the business cycle. This naturally occurs in the model when we realistically abstract from the presence of employment subsidies to monopolistic firms. Second, we show that the presence of an endogenous feedback loop between asset prices and economic fluctuations is also crucial for our results. Absent that feedback, a policy of strict inflation targeting is the optimal prescription for monetary policy.

We also show that in practice the optimal monetary policy can be well approximated by a speed-limit interest rate rule that places a large weight on deviations of inflation from a target and on the growth rate of financial variables. We have emphasized changes in the risk premia in our speed-limit rule. However, as has been suggested by many policymakers during the crisis, leaning against the growth rate of credit would also closely approximate the optimal policy and improve on a policy of strict inflation targeting.

In our analysis, we abstracted from the presence of financial constraints on banks and other financial institutions, which clearly played an important role in the financial turmoil of the past three years. We conjecture that qualitatively our results would also hold in a model with those features (see Gertler and Karadi [2009], Gertler and Kiyotaki [2010], or Brunnermeier and Sannikov [2011] for models with constraints on financial institutions). Nevertheless, the extent to which the optimal policy leans against changes in financial variables or which one is a better guide for monetary policy may very well depend on whether or not financial institutions also face financial constraints. We intend to pursue this avenue in future research.

## Appendix I: The contract (not for publication)

The contract specifies a level of loans, $B_{j,t+1}$ and a gross interest rate $R^b_{j,t+1}$ that maximizes the expected profit of the entrepreneur subject to the participation constraint of the financial intermediary (the bank), or identically a level of capital and a cutoff point.

To simplify matters, it is convenient to operate certain substitutions.[12] Let us first introduce:

$$\Gamma(\omega) = \omega(1 - F(\omega)) + G(\omega) \text{ and } G(\omega) = \int_0^\omega x\mathrm{d}F(x)$$

which satisfy

$$\Gamma'(\omega) = 1 - F(\omega)$$
$$G'(\omega) = \omega F'(\omega).$$

The expected profit of the entrepreneur is given by equation (6), which rewrites as

$$E_t\left[\int_{\omega^\star_{j,t+1}}^\infty \omega\mathrm{d}F(\omega)R^k_{t+1} - \omega^\star_{j,t+1}\int_{\omega^\star_{j,t+1}}^\infty \mathrm{d}F(\omega)R^k_{t+1}\right]Q_t k_{j,t+1}.$$

Note that since $E(\omega) = 1$, we have

$$1 = \int_0^\infty \omega\mathrm{d}F(\omega) = \int_0^{\omega^\star_{j,t+1}} \omega\mathrm{d}F(\omega) + \int_{\omega^\star_{j,t+1}}^\infty \omega\mathrm{d}F(\omega)$$

such that

$$\int_{\omega^\star_{j,t+1}}^\infty \omega\mathrm{d}F(\omega) = 1 - \int_0^{\omega^\star_{j,t+1}} \omega\mathrm{d}F(\omega) = 1 - G(\omega^\star_{j,t+1}).$$

Hence, the expected profit function rewrites

$$E_t\left[[1 - G(\omega^\star_{j,t+1}) - \omega^\star_{j,t+1}(1 - F(\omega^\star_{j,t+1}))]R^k_{t+1}\right]Q_t k_{j,t+1}$$

which can be compactly written as

$$E_t\left[\left[1 - \Gamma(\omega^\star_{j,t+1})\right] R^k_{t+1}\right]Q_t k_{j,t+1}.$$

[12] Note that we consider the steady-state contract only. As $\sigma_{\omega,t}$ takes the form of a "risk shock" in certain simulations presented, the equations must be updated accordingly.

Considering now the participation constraint (7) and using the definition of the cutoff we can write:

$$\left[(1-F(\omega^{\star}_{j,t+1}))\omega^{\star}_{j,t+1}+(1-\mu)\int_{0}^{\omega^{\star}_{j,t+1}}\omega \mathrm{d}F(\omega)\right]Q_tR^k_{t+1}k_{j,t+1}=R_tB_{j,t+1}$$

or identically

$$\left[(\Gamma(\omega^{\star}_{j,t+1}))-\mu G(\omega^{\star}_{j,t+1})\right]Q_tR^k_{t+1}k_{j,t+1}=R_tB_{j,t+1}.$$

Then using the fact that $Q_tk_{j,t+1}=N_{j,t+1}+B_{j,t+1}$ we rewrite the equation above as

$$\left[(\Gamma(\omega^{\star}_{j,t+1}))-\mu G(\omega^{\star}_{j,t+1})\right]Q_tR^k_{t+1}k_{j,t+1}=R_t(Q_tk_{j,t+1}-N_{j,t+1}).$$

The CSV problem therefore amounts to finding a cutoff point $\omega^{\star}_{j,t+1}$ and a level of capital $k_{j,t+1}$ that solves

$$\max_{\{\omega^{\star}_{j,t+1},k_{j,t+1}\}} E_t\left[\left[1-\Gamma(\omega^{\star}_{j,t+1})\right]R^k_{t+1}\right]Q_tk_{j,t+1}$$
$$\left[(\Gamma(\omega^{\star}_{j,t+1}))-\mu G(\omega^{\star}_{j,t+1})\right]Q_tR^k_{t+1}k_{j,t+1}=R_t(Q_tk_{j,t+1}-N_{j,t+1}).$$

Denoting by $\Psi_{t+1}$ the Lagrange multiplier associated with the constraint, and remembering that $\omega^{\star}_{j,t+1}$ is indexed by each possible $R^k_{t+1}$, the set of first-order conditions is given by

$$E_t\left[\left[1-\Gamma(\omega^{\star}_{j,t+1})\right]R^k_{t+1}+\Psi_{t+1}\left(\left[(\Gamma(\omega^{\star}_{j,t+1}))-\mu G(\omega^{\star}_{j,t+1})\right]R^k_{t+1}-R_t\right)\right]=0$$
$$-\Gamma'^{\star}_{j,t+1})+\Psi_{t+1}\left(\Gamma'^{\star}_{j,t+1}))-\mu G'^{\star}_{j,t+1}\right)=0$$
$$\Psi_{t+1}\left[\left(\left[(\Gamma(\omega^{\star}_{j,t+1}))-\mu G(\omega^{\star}_{j,t+1})\right]Q_tR^k_{t+1}k_{j,t+1}-R_t(Q_tk_{j,t+1}-N_{j,t+1})\right)\right]=0$$

Restricting ourselves to interior solutions, we have $\Psi_{t+1}>0$ and the system becomes

$$E_t\left[\left[1-\Gamma(\omega^{\star}_{j,t+1})\right]R^k_{t+1}+\frac{\Gamma'^{\star}_{j,t+1}}{\Gamma'^{\star}_{j,t+1}-\mu G'^{\star}_{j,t+1}}\left(\left[\Gamma(\omega^{\star}_{j,t+1})-\mu G(\omega^{\star}_{j,t+1})\right]R^k_{t+1}-R_t\right)\right]=0$$
$$\left[(\Gamma(\omega^{\star}_{j,t+1}))-\mu G(\omega^{\star}_{j,t+1})\right]Q_tR^k_{t+1}k_{j,t+1}-R_t(Q_tk_{j,t+1}-N_{j,t+1})=0$$

Recalling that $\Gamma'^{\star}_{j,t+1} = 1 - F(\omega^{\star}_{j,t+1})$, $G(\omega^{\star}_{j,t+1}) = \omega^{\star}_{j,t+1} F'^{\star}_{j,t+1}$, and defining $h(\omega^{\star}_{j,t+1}) = F'^{\star}_{j,t+1}/(1 - F(\omega^{\star}_{j,t+1}))$, we can reduce the system to

$$E_t\left[\left[1-\Gamma(\omega^{\star}_{j,t+1})\right]R^k_{t+1} + \frac{\left[(\Gamma(\omega^{\star}_{j,t+1})) - \mu G(\omega^{\star}_{j,t+1})\right]R^k_{t+1} - R_t}{1-\mu\omega^{\star}_{j,t+1}h(\omega^{\star}_{j,t+1})}\right] = 0$$

$$\left[(\Gamma(\omega^{\star}_{j,t+1})) - \mu G(\omega^{\star}_{j,t+1})\right]Q_t R^k_{t+1} k_{j,t+1} - R_t(Q_t k_{j,t+1} - N_{j,t+1}) = 0.$$

It is clear from the first equation that $\omega^{\star}_{j,t+1}$ only depends on $R^k_{t+1}$ and $R_t$. Therefore, we have $\omega^{\star}_{j,t+1} = \omega^{\star}_{t+1}$ for all $j$, such that

$$E_t\left[\left[1-\Gamma(\omega^{\star}_{t+1})\right]R^k_{t+1} + \frac{\left[(\Gamma(\omega^{\star}_{t+1})) - \mu G(\omega^{\star}_{t+1})\right]R^k_{t+1} - R_t}{1-\mu\omega^{\star}_{t+1}h(\omega^{\star}_{t+1})}\right] = 0 \tag{10}$$

$$\left[(\Gamma(\omega^{\star}_{t+1})) - \mu G(\omega^{\star}_{t+1})\right]Q_t R^k_{t+1} k_{j,t+1} - R_t(Q_t k_{j,t+1} - N_{j,t+1}) = 0. \tag{11}$$

# Appendix II: Aggregation (not for publication)

This section discusses the evolution of aggregate net worth, denoted by $N_{t+1}$. At this stage it is useful to introduce the distribution of net worth, $\Upsilon(N)$, such that

$$N_{t+1} = \int_0^{\infty} N d\Upsilon(N).$$

Recalling that the capital stock held by an individual $j$ is a function of individual net worth, it is clear that aggregate capital $k_{t+1}$ is given by

$$k_{t+1} = \int_0^{\infty} k_{t+1}(N) d\Upsilon(N).$$

Noting that equation (11) is linear in both $k_{j,t+1}$ and $N_{j,t+1}$, we have, aggregating over individuals

$$E_t\left[\left[1-\Gamma(\omega^{\star}_{t+1})\right]R^k_{t+1} + \frac{\left[(\Gamma(\omega^{\star}_{t+1})) - \mu G(\omega^{\star}_{t+1})\right]R^k_{t+1} - R_t}{1-\mu\omega^{\star}_{t+1}h(\omega^{\star}_{t+1})}\right] = 0 \tag{12}$$

$$\left[(\Gamma(\omega^{\star}_{t+1})) - \mu G(\omega^{\star}_{t+1})\right]Q_t R^k_{t+1} k_{t+1} - R_t(Q_t k_{t+1} - N_{t+1}) = 0. \tag{13}$$

We now turn our attention to the law of motion of aggregate net worth. Let us denote by $\Xi_{j,t}$ the average actual profit of an individual entrepreneur in period $t$,

$$\Xi_{j,t} = [1-\Gamma(\omega^{\star}_t)]R^k_t Q_{t-1} k_{j,t}.$$

Note that this individual profit is linear in $k_{j,t}$, such that aggregate profit flow is given by

$$\int_0^\infty \Xi_t(N)d\Upsilon(N) = [1-\Gamma(\omega_t^\star)]R_t^k Q_{t-1} \int_0^\infty k_t(N)d\Upsilon(N)$$

and

$$\Xi_t = [1-\Gamma(\omega_t^\star)]R_t^k Q_{t-1} k_t.$$

Since entrepreneurs are randomly drawn for survival with probability $\gamma_t$, and newly born entrepreneurs receive a transfer $\tau_t^e$ from the government, aggregate net worth evolves as

$$N_{t+1} = \gamma_t \Xi_t + \tau_t^e$$

or

$$N_{t+1} = \gamma_t[1-\Gamma(\omega_t^\star)]R_t^k Q_{t-1} k_t + \tau_t^e. \tag{14}$$

The $(1-\gamma_t)$ entrepreneurs selected to close their business consume a constant share $\chi$ of their profit, the remaining being kept by the government to finance transfers to newly born entrepreneurs. Hence

$$P_t c_t^e = (1-\gamma_t)\chi[1-\Gamma(\omega_t^\star)]R_t^k Q_{t-1} k_t. \tag{15}$$

## Appendix III: Risk (not for publication)

In this Appendix we come back to the determination of $\omega^\star$. Recall that we use a log–normal distribution for $\omega$, such that

$$F(\omega^\star) = \frac{1}{\sigma_\omega\sqrt{2\pi}}\int_0^{\log(\omega^\star)} e^{-\frac{1}{2}\left(\frac{x-\mu_\omega}{\sigma_\omega}\right)^2}\mathrm{d}x.$$

Making the change of variable $y = \frac{x-\mu_\omega}{\sigma_\omega}$, we have

$$F(\omega^\star) = \frac{1}{\sqrt{2\pi}}\int_0^{\frac{\log(\omega^\star)-\mu_\omega}{\sigma_\omega}} e^{-\frac{y}{2}}\mathrm{d}y = \Phi\left(\frac{\log(\omega^\star)-\mu_\omega}{\sigma_\omega}\right)$$

where $\Phi(\cdot)$ denotes the cdf of the normal distribution.

Likewise,

$$G(\omega^\star) = \frac{1}{\sigma_\omega\sqrt{2\pi}}\int_0^{\log(\omega^\star)} e^x e^{-\frac{1}{2}\left(\frac{x-\mu_\omega}{\sigma_\omega}\right)^2}\mathrm{d}x.$$

Making a first change of variable $y = \frac{x-\mu_\omega}{\sigma_\omega}$, we have

$$G(\omega^\star) = \frac{e^{\mu_\omega}}{\sqrt{2\pi}}\int_0^{\frac{\log(\omega^\star)-\mu_\omega}{\sigma_\omega}} e^{-\frac{1}{2}(y^2-2\sigma_\omega y)}\mathrm{d}y.$$

Adding and substracting $\sigma_\omega/2$ in the exponential under the integral, we have

$$G(\omega^\star) = \frac{e^{\mu_\omega+\frac{\sigma_\omega^2}{2}}}{\sqrt{2\pi}}\int_0^{\frac{\log(\omega^\star)-\mu_\omega}{\sigma_\omega}} e^{-\frac{1}{2}(y-\sigma_\omega)^2}\mathrm{d}y.$$

Making a last change of variable $z = y - \sigma_\omega$, this rewrites as

$$G(\omega^\star) = \frac{e^{\mu_\omega+\frac{\sigma_\omega^2}{2}}}{\sqrt{2\pi}}\int_0^{\frac{\log(\omega^\star)-\mu_\omega}{\sigma_\omega}-\sigma_\omega} e^{-\frac{z^2}{2}}\mathrm{d}z = e^{\mu_\omega+\frac{\sigma_\omega^2}{2}}\Phi\left(\frac{\log(\omega^\star)-\mu_\omega}{\sigma_\omega}-\sigma_\omega\right).$$

Since $E(\omega)=1$, $G(\omega^\star)$ reduces to

$$G(\omega^\star) = \Phi\left(\frac{\log(\omega^\star)-\mu_\omega}{\sigma_\omega}-\sigma_\omega\right).$$

Note finally that $E(\omega)=1$ imposes $\mu_\omega = -\frac{\sigma_\omega^2}{2}$. Therefore, in the case of a risk shock, $\mu_{\omega,t} = -\frac{\sigma_{\omega,t}^2}{2}$.

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**Table 1. Calibration**

| | | |
|---|---|---|
| **Households and production** | | |
| Discount rate | $\beta$ | 0.990 |
| Frisch elasticity | $\sigma_h$ | 1.000 |
| Intertemporal elasticity of substitution | $\sigma_c$ | 1.500 |
| Depreciation rate on capital | $\delta$ | 0.025 |
| Capital share | $\alpha$ | 0.361 |
| Steady-state gross markup | $\lambda$ | 1.100 |
| Calvo parameter | $\theta$ | 0.750 |
| **Financial intermediation** | | |
| Entrepreneur's survival probability | $\gamma$ | 97.620 |
| Monitoring cost | $\mu$ | 0.4 |
| Percent of bankrupt business p/quarter | $F(\omega^{\star})$ | 0.174 |
| Rental rate on capital (gross p/quarter) | $R^k$ | 1.015 |
| Risk premium in annual percentage terms | $\frac{R^k}{R}$ | 2.200 |
| **Steady-state great ratios** | | |
| Capital-output ratio | $\frac{k}{y}$ | 8.071 |
| Investment-output ratio | $\frac{i}{y}$ | 0.202 |
| Equity-debt ratio | $\frac{N}{(K-N)}$ | 1.030 |
| **Shock processes: Benchmark** | | |
| AR(1) coefficients | $\rho_a; \rho_\gamma; \rho_{\sigma_\omega}$ | $0.9; 0.5; 0.9$ |
| Standard deviations | $\sigma_{\varepsilon_a}; \sigma_{\varepsilon_\gamma}; \sigma_{\varepsilon_{\sigma_\omega}}$ | $0.01; 0.01; 0.01$ |
| **Shock processes: Christiano et al. (2010), US** | | |
| AR(1) coefficients | $\rho_a; \rho_\gamma; \rho_{\sigma_\omega}$ | $0.883; 0.529; 0.821$ |
| Standard deviations | $\sigma_{\varepsilon_a}; \sigma_{\varepsilon_\gamma}; \sigma_{\varepsilon_{\sigma_\omega}}$ | $0.005; 0.005; 0.05$ |

**Table 2. Optimized Interest-Rate Rule**

| | Parameters | | |
|---|---|---|---|
| **Shocks** | $g_\pi$ | $g_y$ | $g_f$ |
| TFP | 4.800 | 0 | 21.087 |
| Risk | 4.275 | 0 | 10.078 |
| Net Worth | 4.552 | 0 | 10.765 |
| All shocks | 4.500 | 0 | 11.651 |

# Figures

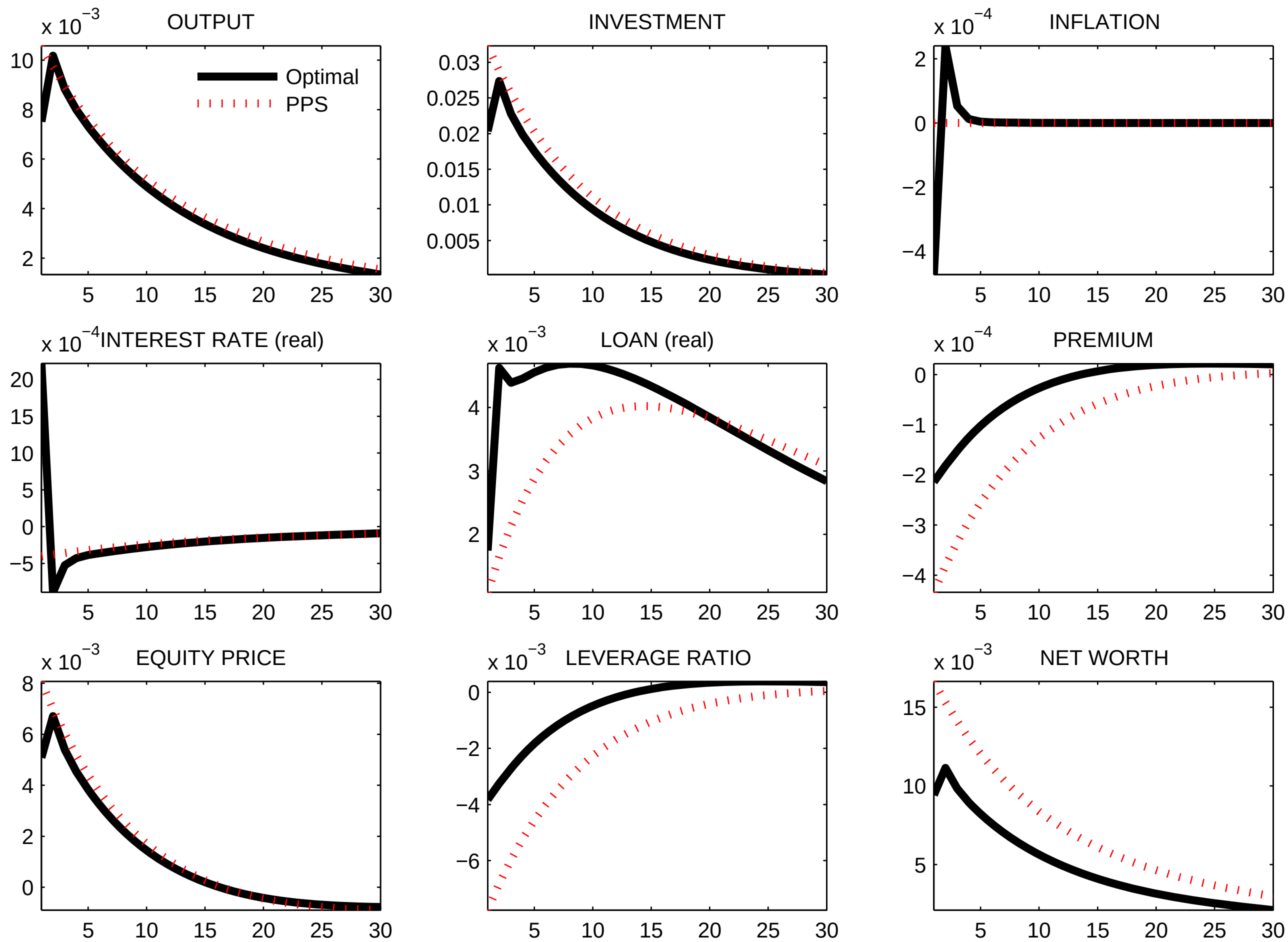


Figure 1: Responses to a Productivity Shock Under the Optimal Policy (Optimal) and Perfect Price Stability (PPS).

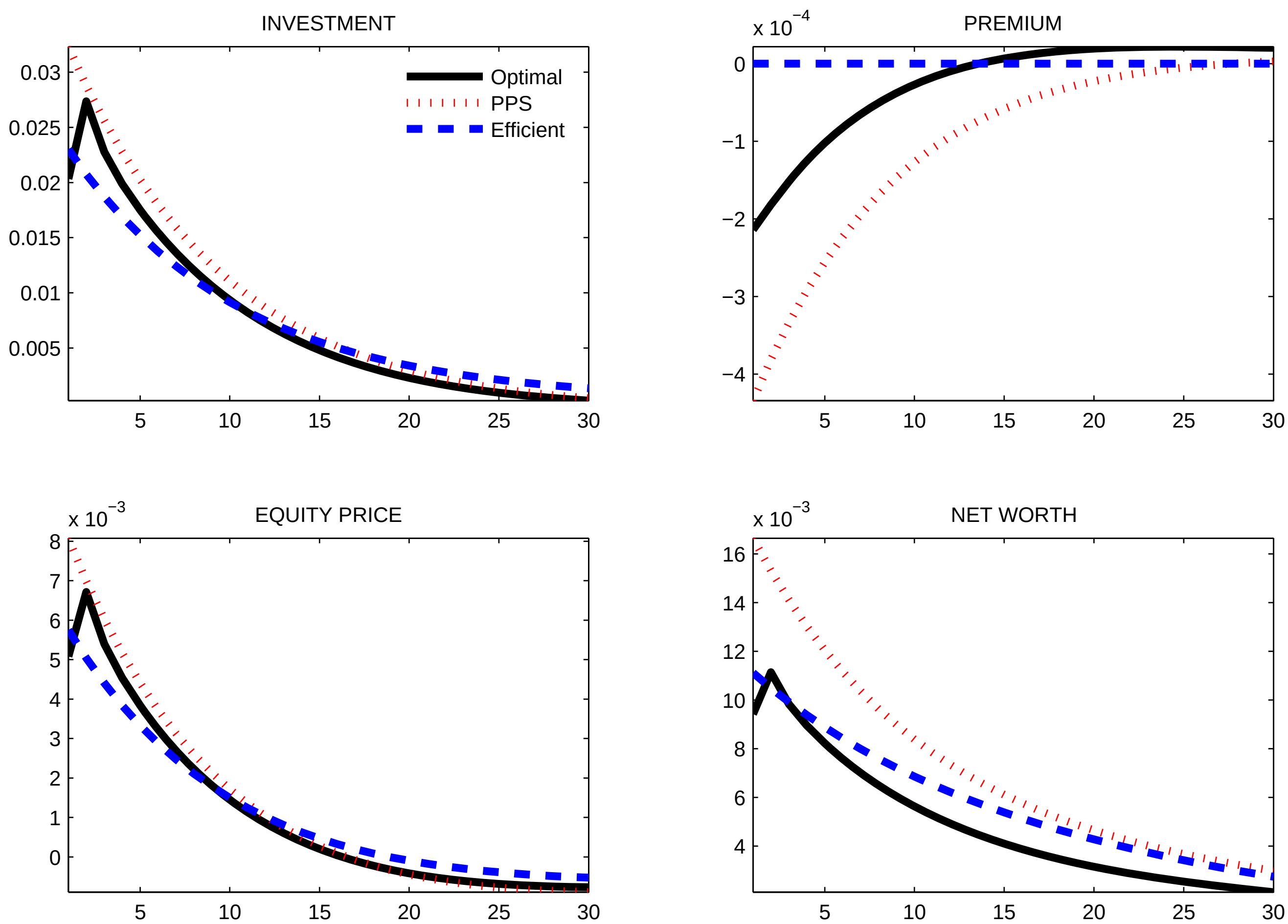


Figure 2: Responses to a Productivity Shock Under the Optimal Policy (Optimal), Perfect Price Stability (PPS) and the Efficient Allocation (Efficient)

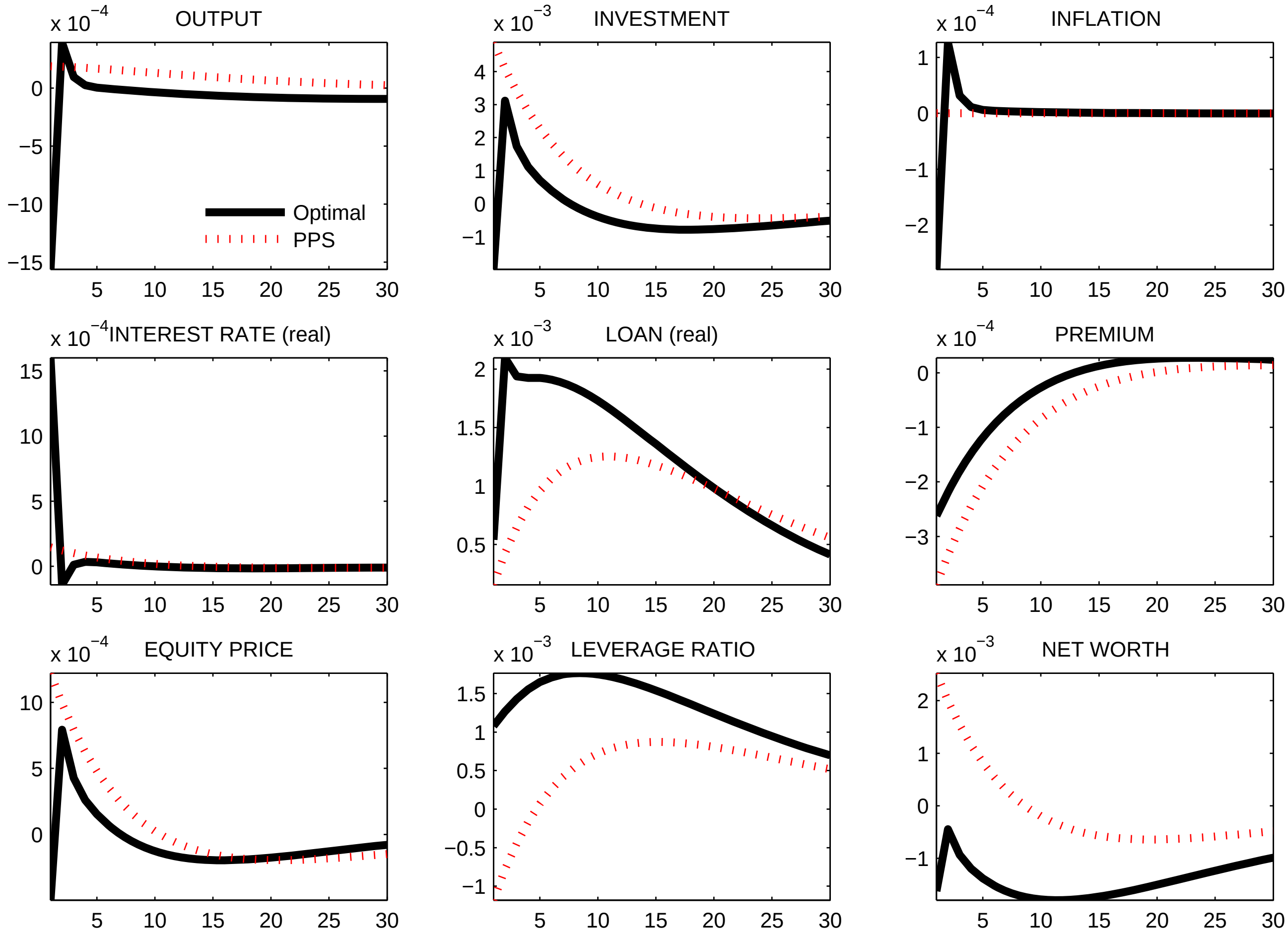


Figure 3: Responses to the Cross-Sectional Distribution of Entrepreneurs' Productivity Under the Optimal Policy (Optimal) and Perfect Price Stability (PPS)

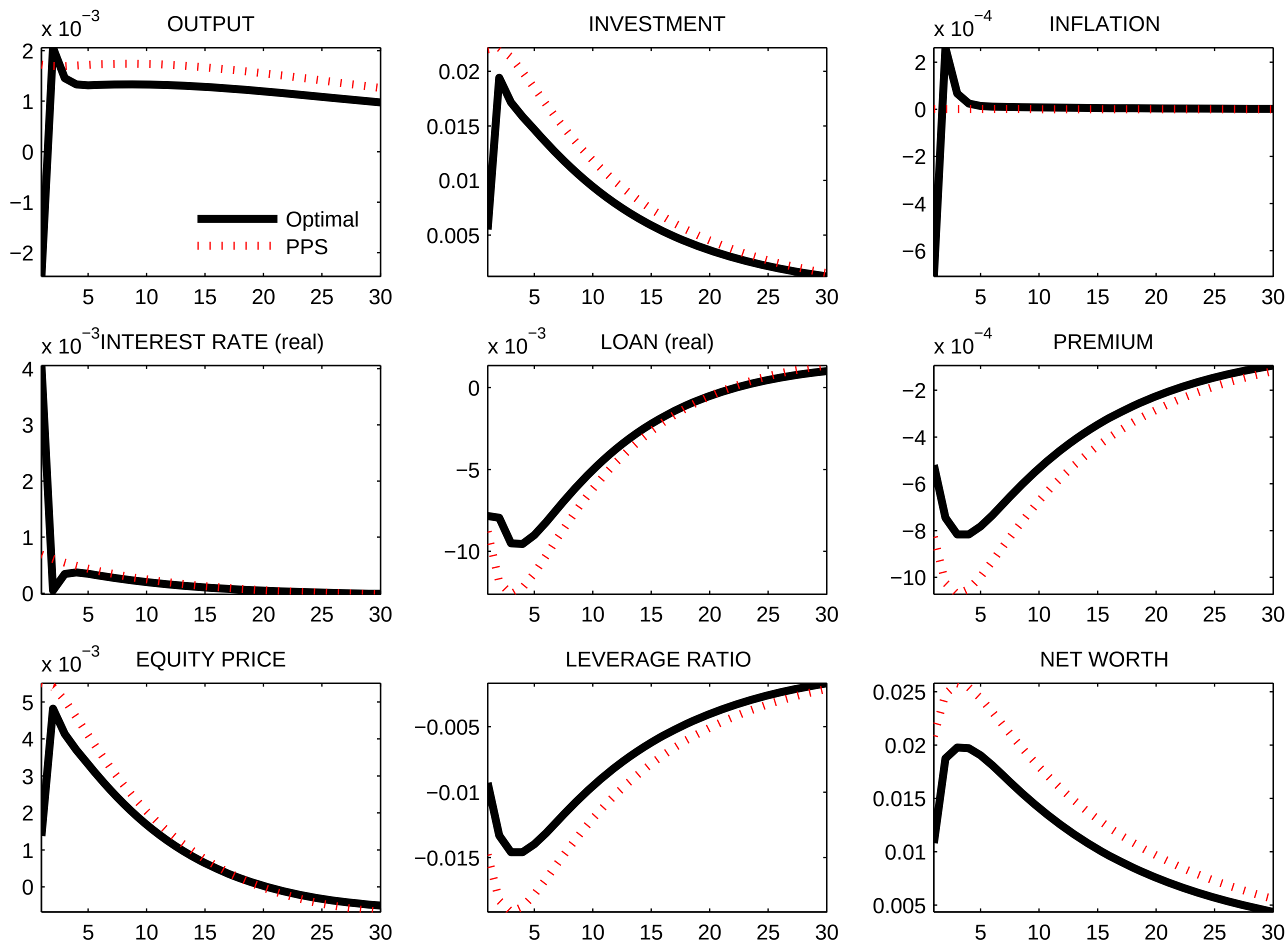


Figure 4: Responses to a Shock to Entrepreneurs' Net Worth Under the Optimal Policy (Optimal) and Perfect Price Stability (PPS)

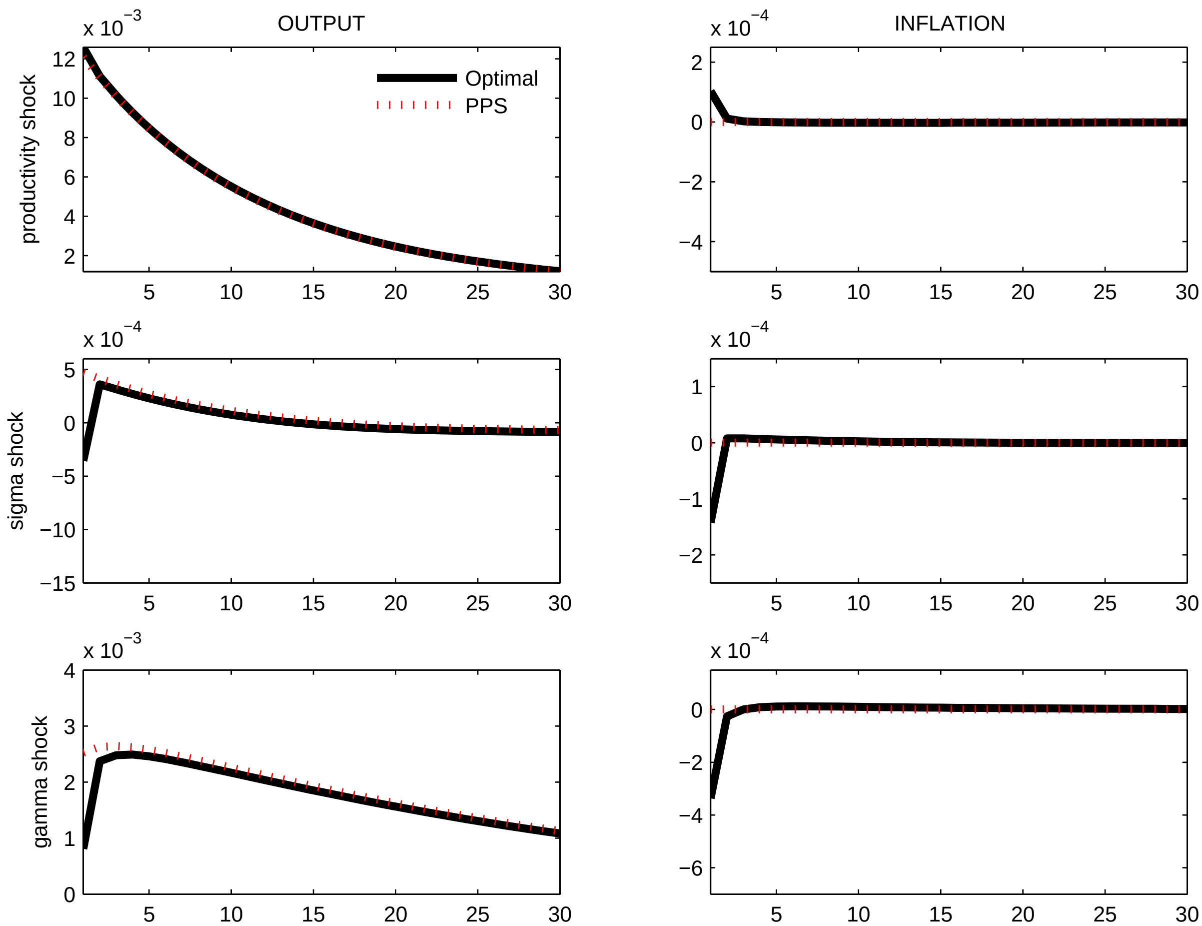


Figure 5: Responses to All Shocks Under the Optimal Policy (Optimal) and Perfect Price Stability (PPS): No Investment Adjustment Costs

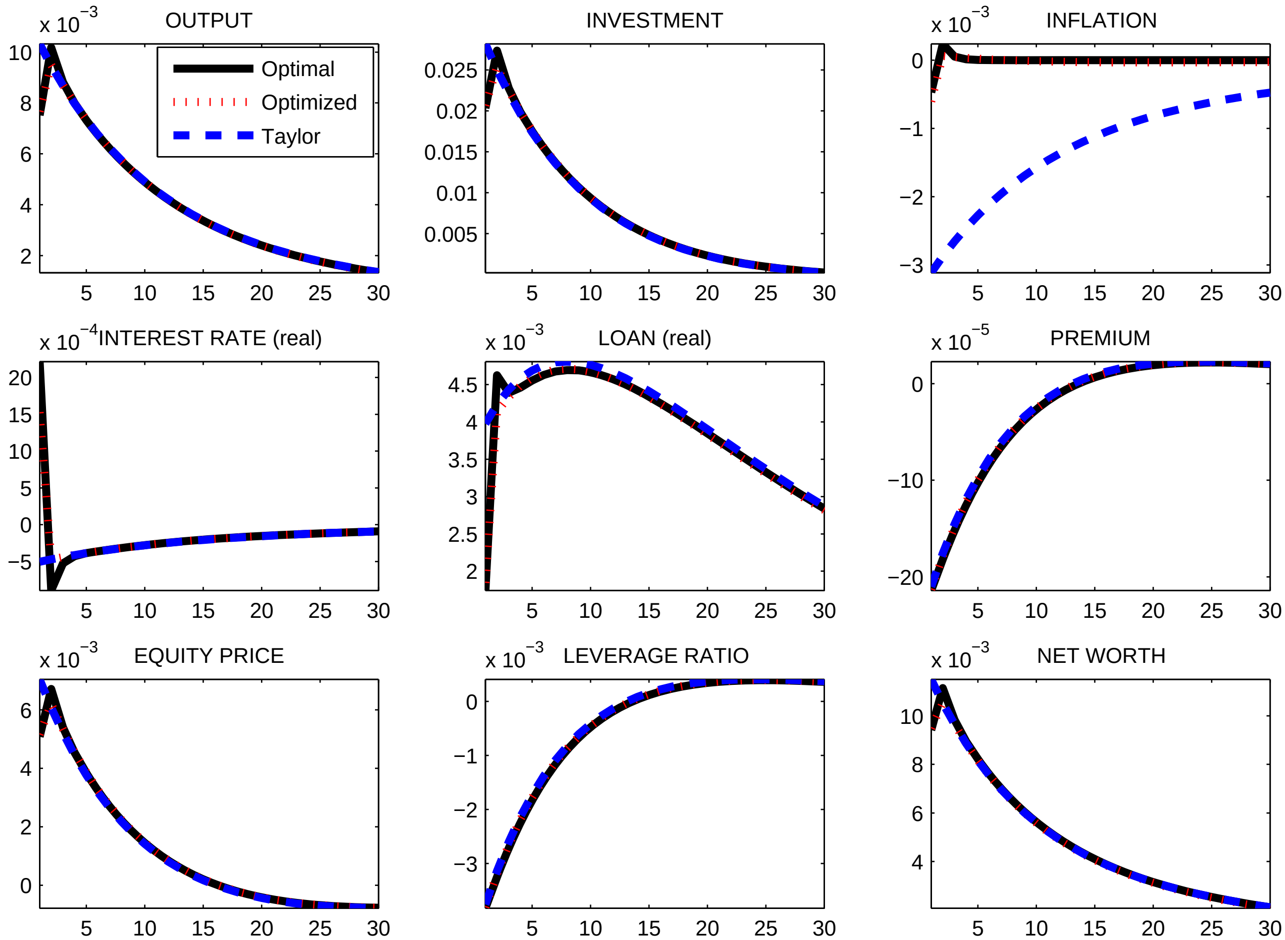


Figure 6: Optimized Speed-Limit Rule: Productivity Shock

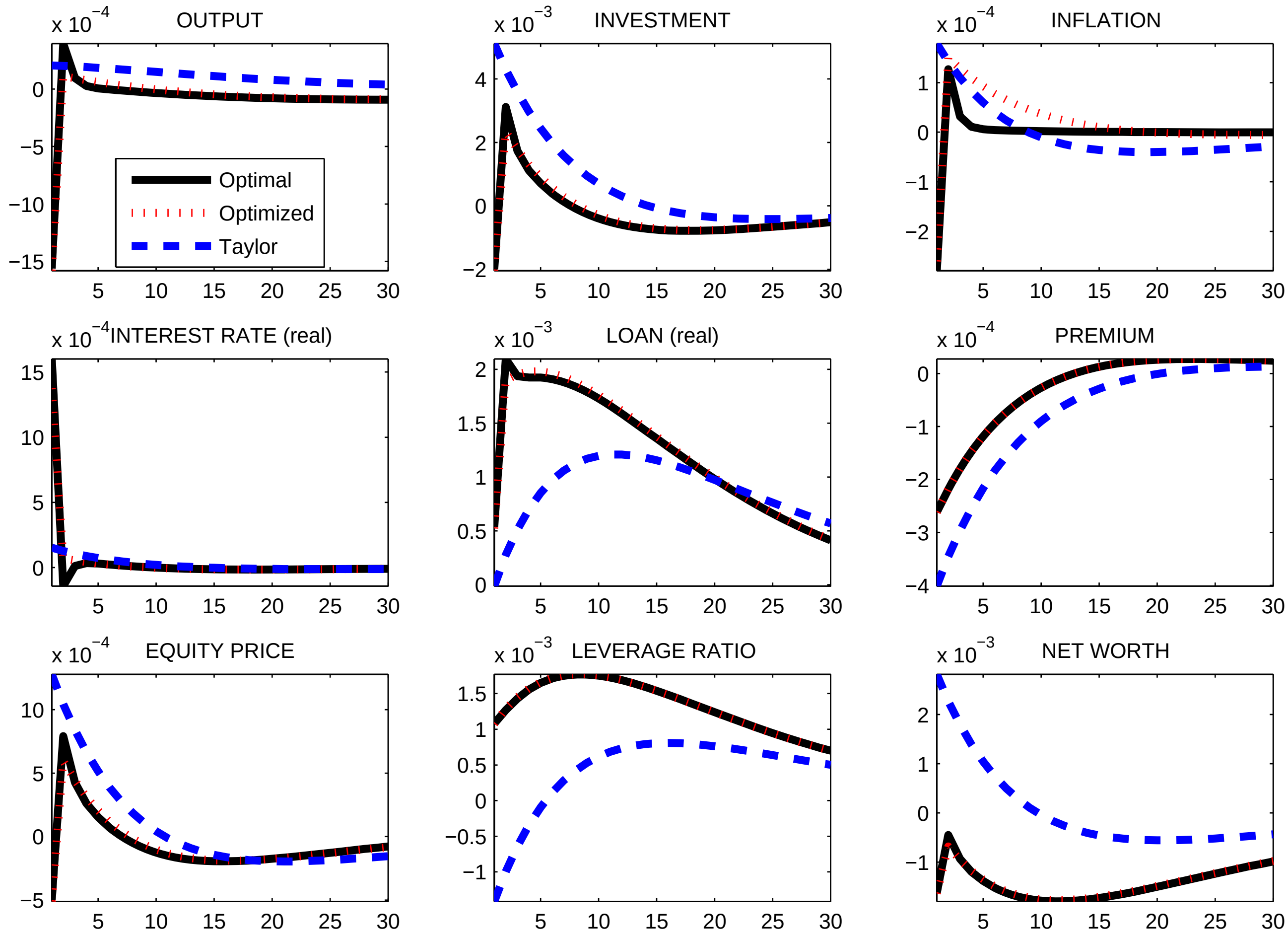


Figure 7: Optimized Speed-Limit Rule: Risk Shock

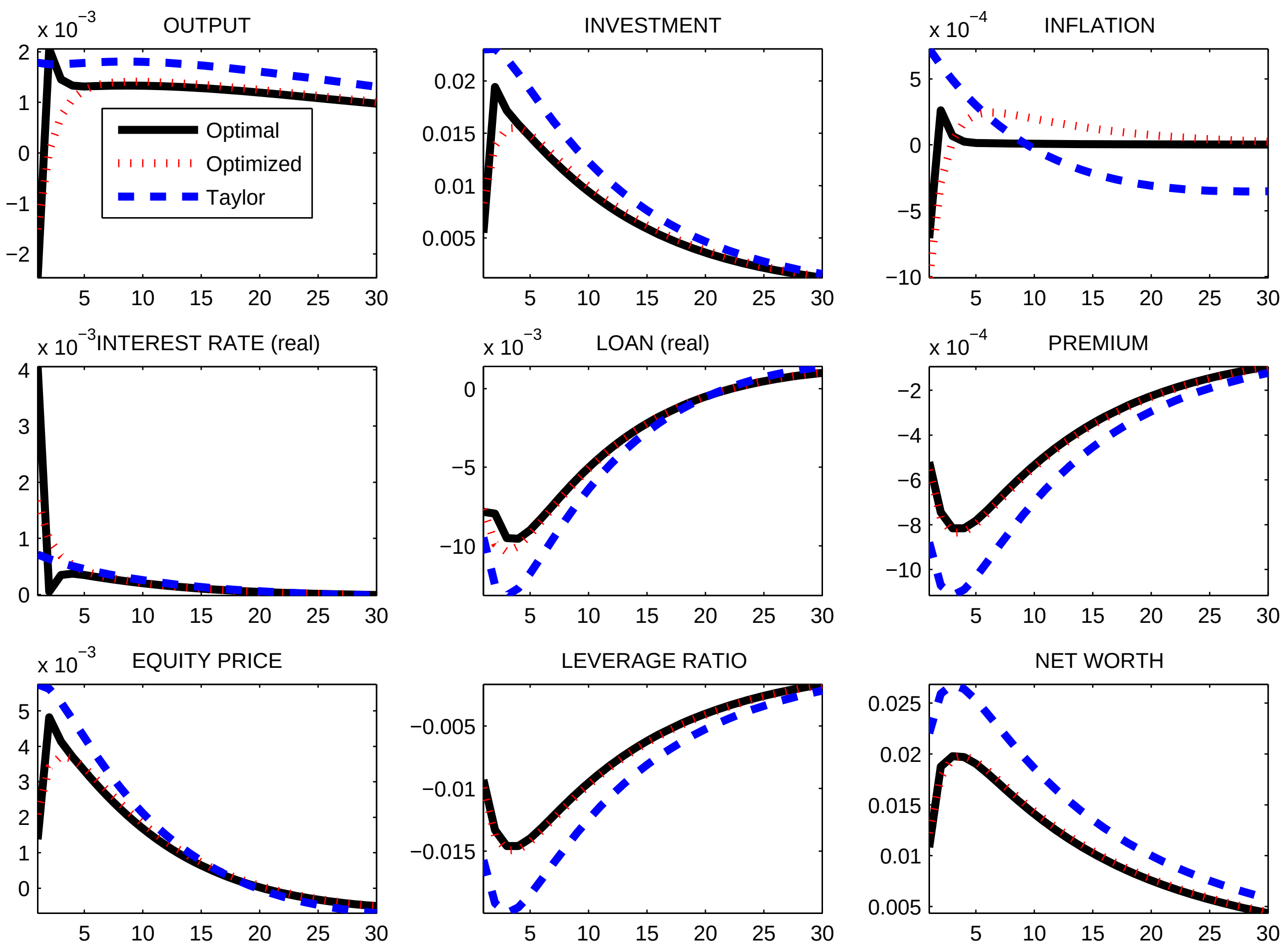


Figure 8: Optimized Speed-Limit Rule: Net Worth Shock

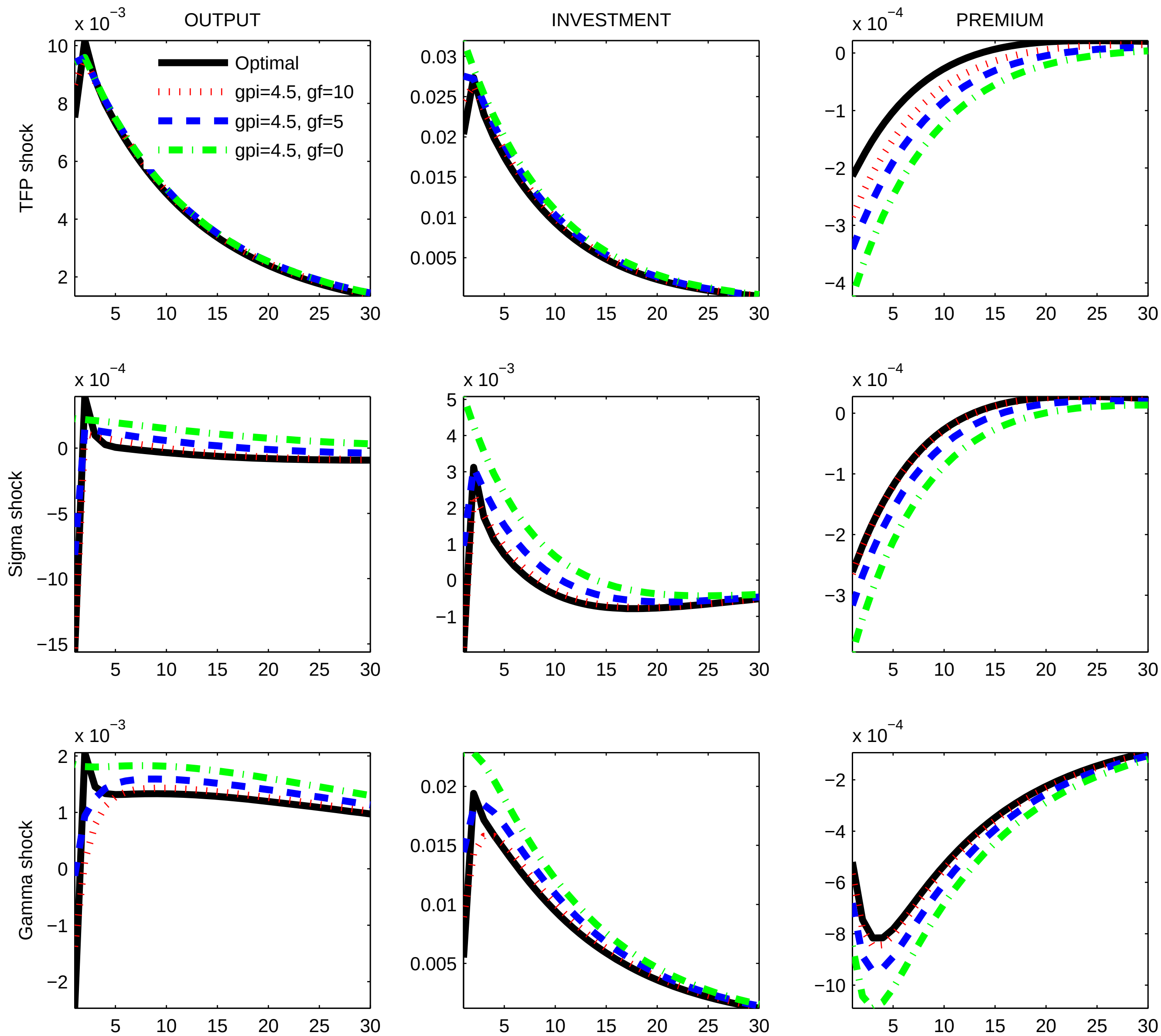


Figure 9: Varying the Weight on the Indicator of Financial Excess

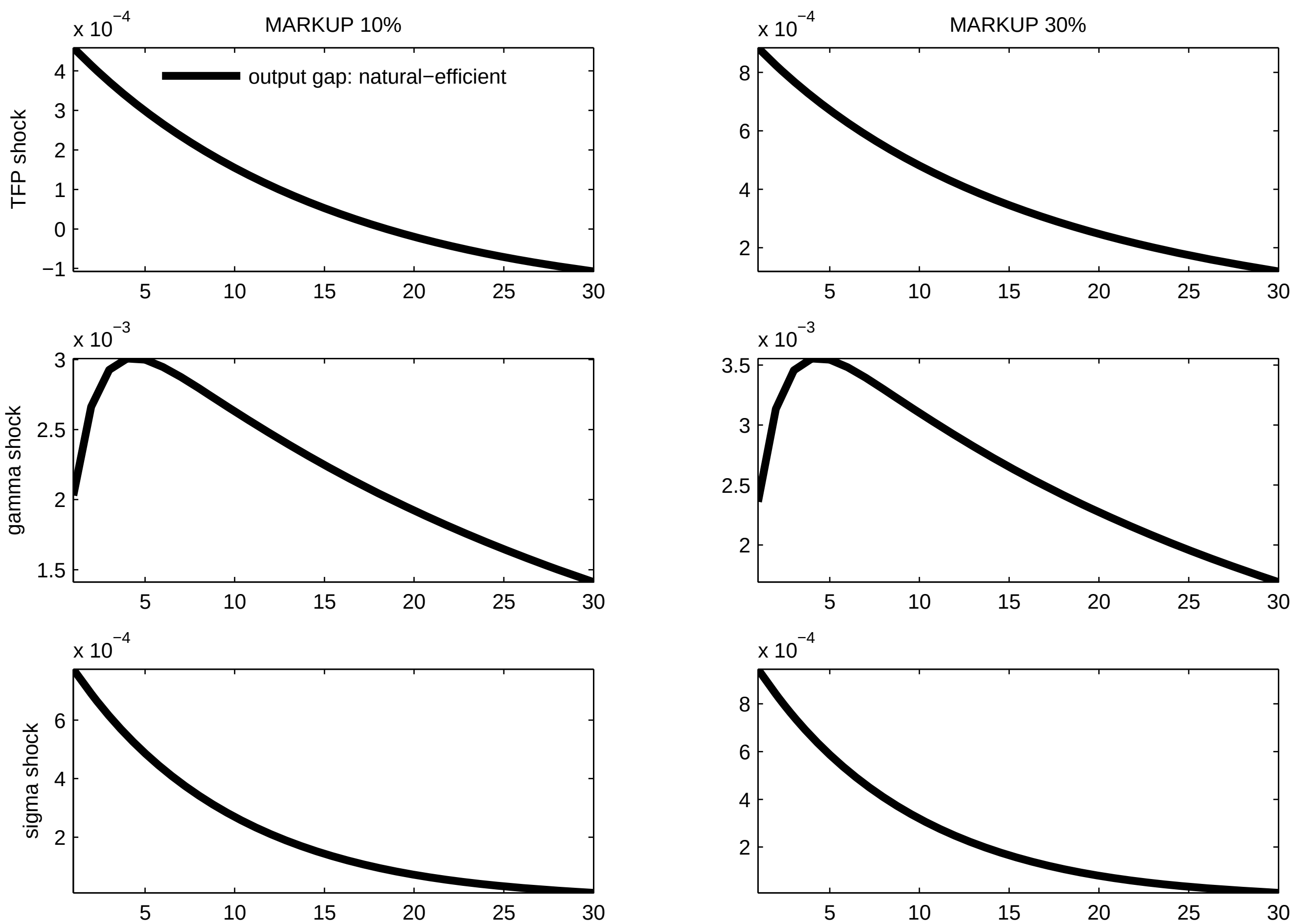


Figure 10: Output Gaps Responses to Productivity and Financial Shocks

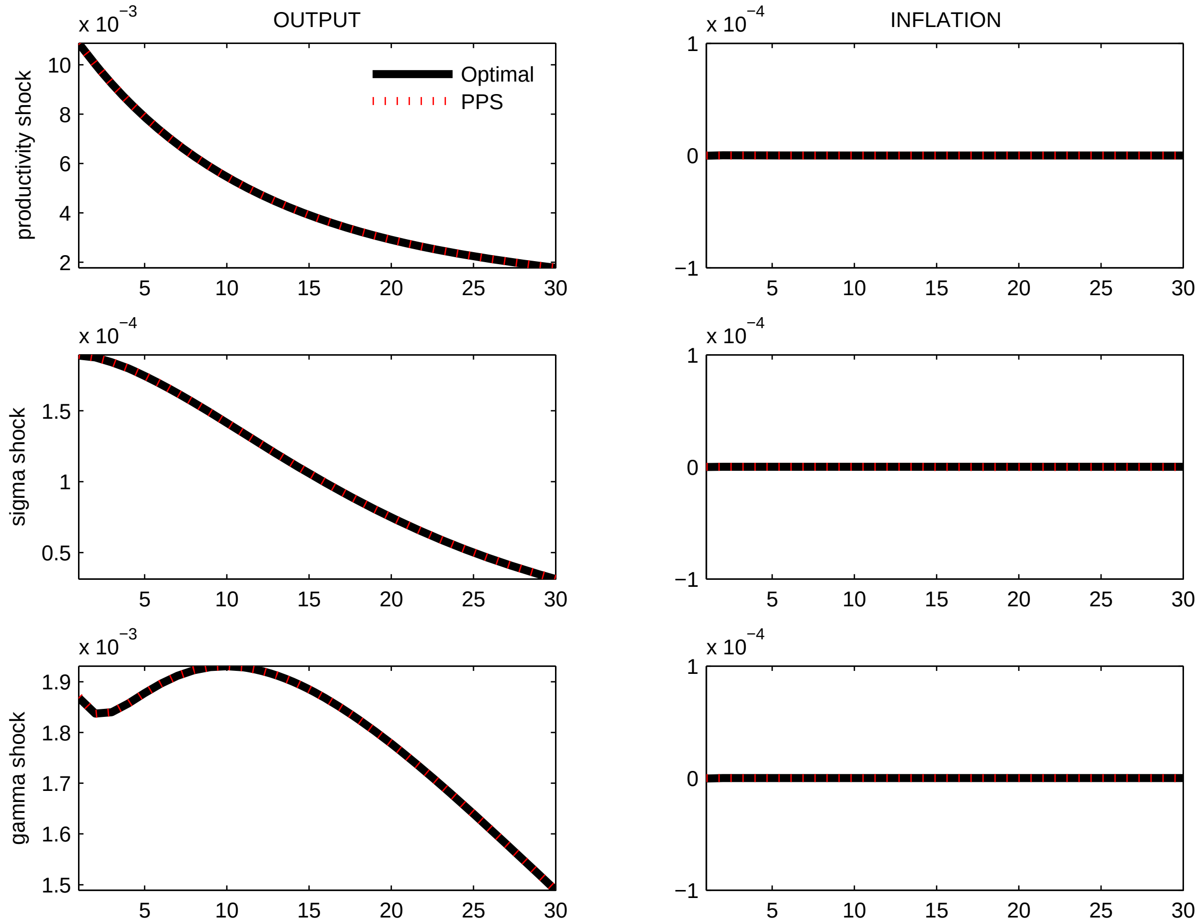


Figure 11: Responses to Productivity and Financial Shocks: Efficient Steady-State

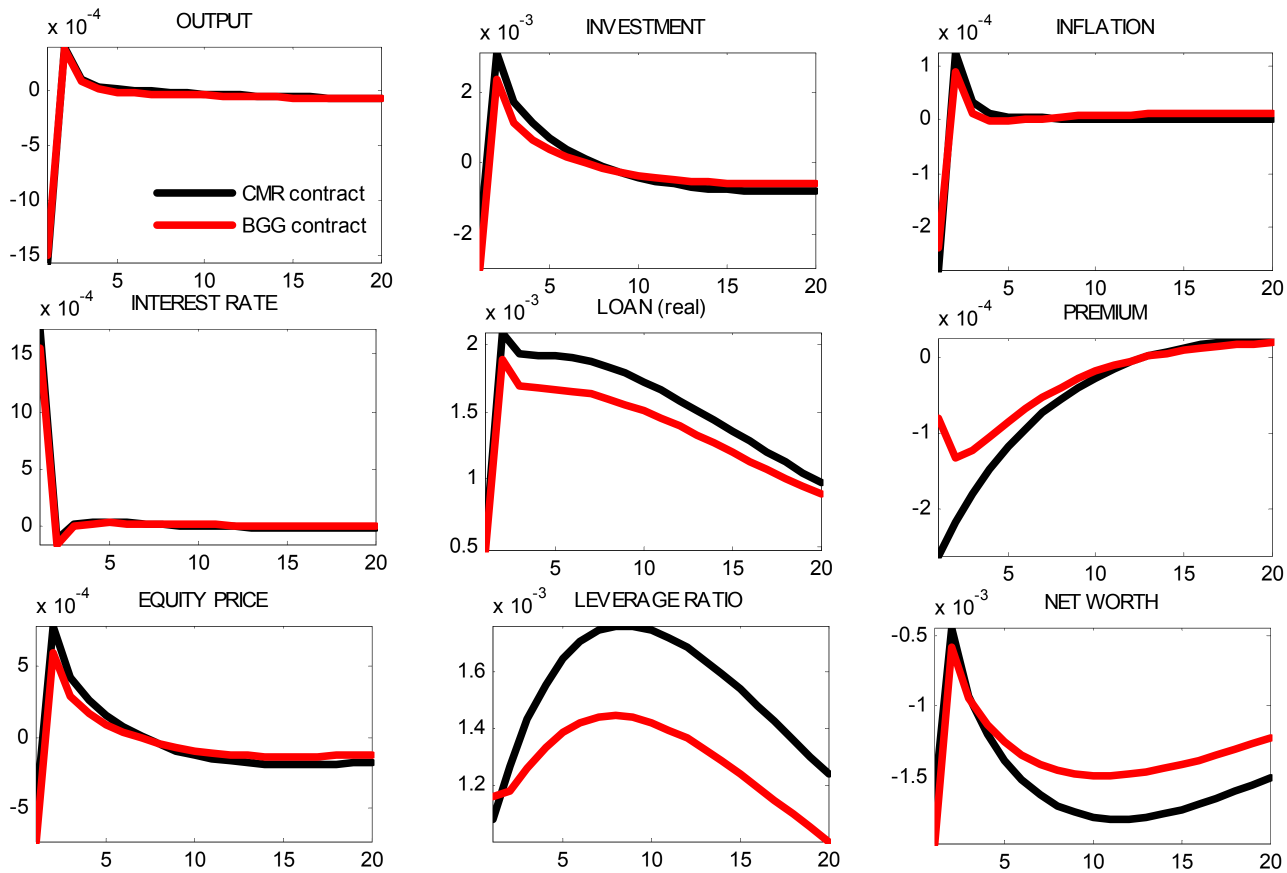


Figure 12: Responses to a Productivity Shock Under the Optimal Policy: Inflation Protected (BGG) vs Nominal (CMR) Contracts