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Should More Risk-Averse Agents Exert More Effort?

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Abstract

Consider an agent facing a risky distribution of losses who can change this distribution by exerting some effort. Should he exert more effort when he becomes more risk-averse? For instance, should we expect more risk-averse drivers to drive more cautiously? In this article, we give sufficient conditions under which the answer is positive, using results presented in Jewitt (1989). We first extend the standard models of self-insurance and self-protection and show that the comparative statics depends only on the effect of effort on the net loss. We then present conditions for the continuous case with applications.

Key words: self-insurance, self-protection, comparative statics

1. Introduction

Consider an agent who faces the risk of a monetary loss and who can choose the level of a preventing activity (hereafter "effort") so as to modify this distribution. One can think for instance of an incompletely insured car owner who may drive more or less cautiously. The study of the effect of increased risk-aversion on the level of effort is generally based on the classical distinction proposed by Ehrlich and Becker [1972] between self-insurance and self-protection. Self-insurance designates an effort aiming at loss reduction, for a given probability of loss; while self-protection applies to an effort aimed at reducing the probability of a given loss. It is now well-known (see Dionne and Eeckhoudt [1985] and Briys and Schlesinger [1990] that while more risk-averse agents always choose a higher level of self-insurance, it is not necessarily the case for self-protection.

In this article, we revisit this distinction using general single-crossing conditions presented in Jewitt [1989]. In Section 2, we first propose a more general model that encompasses both self-insurance and self-protection, and significantly extends previous results. Effort is shown to increase with risk-aversion if conditional on the occurrence of a loss, effort is a desirable good, irrespective of the impact of effort on the probability of loss. This condition in fact seems to be the main difference between self-insurance and self-protection. Indeed, if it does not hold, then increasing risk-aversion *a priori* leads to ambiguous effects on the level of effort. However, we are also able to prove in Section 3 that self-protection increases with risk-aversion if and only if the initial probability of loss is low enough (see Dionne, Eeckhoudt, and Godfroid [1998] for a related result).

The results above apply in a model where conditional on effort, the level of losses is deterministic. In Section 4, we examine the general case in which the distribution of wealth depends on effort. We there exhibit a simple sufficient condition for effort to increase with risk-aversion. This condition appears to be satisfied in the most usual models, as shown by illustrative examples. We show in particular that the comparative statics of coinsurance and of franchise contracts is independent of the premium function. We conclude by providing a simple proof of the link between the single-crossing condition and the comparative statics.

2. Self-insurance

Consider an agent with an increasing Von-Neuman-Morgenstern utility function U. This agent faces a risk of loss (or accident) and can engage in effort, chosen from an interval $[0, \bar{e}]$. For a level e of effort, his wealth is $w = W - c(e)$ with probability $(1 - p(e))$ and $w = W - d(e)$ with probability $p(e)$. The function $c(e)$ can be thought of as the cost of effort. The difference $l(e) = d(e) - c(e)$ is the loss, and W is the initial wealth. We assume that effort is costly and that the loss is positive:

C0: *$c(e)$ is nondecreasing, and $l(e) > 0$.*

Thus the expected utility of the agent is

$$p(e)U(W - d(e)) + (1 - p(e))U(W - c(e)).$$

Note that this model encompasses the traditional models of both self-insurance (for which $p(e) \equiv p$, c is increasing, and d is decreasing)[1] and self-protection (p is decreasing, c is increasing, and $d(e) = c(e) + l$ with $l > 0$).

We are interested in whether a more risk-averse agent, whose utility function V is an increasing concave transformation of U, will choose a higher level of effort. Denote $F(w, e)$ the cdf of wealth given effort. Jewitt [1989] gives the following minimal sufficient condition for this monotone comparative statics property to hold:[2]

Single-crossing condition: *For all $e_1 < e_2$, $F(w, e_1) - F(w, e_2)$ changes sign at most once, from nonnegative to nonpositive, when w increases.*

Jewitt's condition relies on earlier results by Hammond [1974] and Diamond and Stiglitz [1974]. We give a simple, self-contained proof for the sufficiency of this condition in the last section.

Intuitively, the effect of effort must be to decrease risk (in the sense of first-order stochastic dominance) in the lower tail of the distribution of wealth at the cost of an increase in risk in the upper tail. Applying this to our context leads to:

Proposition 1: *Assume that C0 holds and that $d(e)$ is nonincreasing. If $e_1 < e_2$ and U prefers e_2 to e_1, then any V more risk-averse than U prefers e_2 to e_1.*

Proof. For a level of effort e, the cdf of wealth is 0 if $w < W - d(e)$, $p(e)$ if $W - d(e) < w < W - c(e)$, and 1 if $w > W - c(e)$.

For two levels $e_1 < e_2$, the assumptions on c and d imply that $-d(e_1) \leq -d(e_2) \leq -c(e_2) \leq -c(e_1)$, so that

$$F(w, e_1) - F(w, e_2) = \begin{cases} p(e_1) \geq 0 & \text{if } W - d(e_1) < w < W - d(e_2) \\ p(e_1) - p(e_2) & \text{if } W - d(e_2) < w < W - c(e_2) \\ p(e_1) - 1 \leq 0 & \text{if } W - c(e_2) < w < W - c(e_1) \end{cases}$$

and 0 elsewhere. Thus the property quoted in the text is verified, whatever the sign of $(p(e_1) - p(e_2))$. □

As a consequence, a more risk-averse agent must choose a higher level of effort, the reason for this being simply that a higher effort increases the worst outcome. Indeed our result is independent of whether the probability increases or decreases with e; neither does it require that the agent be risk-averse. Thus this proposition significantly extends previous results in the literature on self-insurance, which is the special case in which $p(e)$ is constant.

Another generalization of earlier results on self-insurance obtains when assuming that the probability of loss is constant ($p(e) = p \in (0, 1)$). Let us add risks on final wealth, so that the expected utility of the agent is

$$pE[U(w_1) \mid e] + (1 - p)E[U(w_0) \mid e],$$

where w_0 and w_1 are stochastic variables whose distribution may depend on e (indeed they generalize the previous terms c and d).

Proposition 2: *If for all* e, $\Pr\{w_1 < w_0 \mid e\} = 1$, *and for all* $e_1 < e_2$, $w_0 \mid e_1$ *(resp.* $w_1 \mid e_2$*) dominates* $w_0 \mid e_2$ *(resp.* $w_1 \mid e_1$*) in the sense of first-order stochastic dominance, then a more risk-averse agent chooses a higher level of effort.*

Proof. For $i = 0, 1$, denote $G_i(w, e)$ the cdf of $w_i \mid e$, which is positive on the interval $(a_i(e), b_i(e))$. For $e_1 < e_2$, $G_0(w \mid e_1) \leq G_0(w \mid e_2)$ and $G_1(w \mid e_1) \geq G_1(w \mid e_2)$. Therefore,

$$b_1(e_1) \leq b_1(e_2) \leq a_0(e_2) \leq a_0(e_1).$$

For a level e, the cdf $F(w, e)$ of wealth is $pG_1(w \mid e)$ if $w < a_0(e)$, and $p + (1 - p)G_0(w \mid e)$ otherwise, so that $F(w, e_1) - F(w, e_2)$ is equal to

$$\begin{cases} p(G_1(w, e_1) - G_1(w, e_2)) \geq 0 & \text{if } w < a_0(e_2) \\ -(1 - p)G_0(w, e_2) \leq 0 & \text{if } a_0(e_2) < w < a_0(e_1) \\ (1 - p)(G_0(w, e_1) - G_0(w, e_2)) \leq 0 & \text{if } a_0(e_1) < w \end{cases}$$

and the single-crossing condition is verified. □

The assumption on supports carries the idea that the occurrence of an accident reduces the agent's revenue. Assuming moreover that p is a constant allows to avoid any other assumption on the distributions w_0 and w_1 and yields this simple result.

To summarize, this section offers a new characterization of self-insurance: self-insurance obtains when effort is desirable conditional on having an accident and undesirable conditional on not having an accident. Under self-insurance, a more risk-averse agent chooses a higher level of effort.

3. Self-protection

Let us now turn to the case where $d(.)$ is increasing—so that conditional on having an accident, one would like to reduce effort. One important result due to Meyer (theorem 4 in Meyer [1975]) states that given $e_1 < e_2$, it is possible to find an increasing U such that, if U prefers e_2 to e_1, then so does any V more risk-averse than U. Thus a monotone comparative static result would hold, provided attention is restrained to agents that are risk-averse enough. However, this result vanishes when applied to our case because we consider that effort is a continuous variable: therefore, for a given e_2 chosen by U, we have to discard an infinite number of candidates $e_1 < e_2$ to ensure that V will indeed choose an effort not lower than e_2. The distinction between ranking two levels of effort (as discussed by Meyer) and finding the optimal effort (as in this article) is illustrated by the following result.

Proposition 3: *Let $p(e), c(e)$, and $d(e)$ be continuously differentiable, with $c(e)$ and $d(e)$ increasing. Assume U is increasing and continuously differentiable and that the agent characterized by U strictly prefers effort $e_0 \in]0, \bar{e}[$ to any other effort, with $0 < p(e_0) < 1$.*

Then there exists more risk-averse utility functions V_1 and V_2 such that V_1 (resp. V_2) chooses a higher (resp. lower) level of activity than U.

Proof. Let $0 < \lambda < 1$, $0 < \alpha < \inf\{1 - p(e_0), p(e_0)\}$.

Choose $e_2 < e_0$ close enough to e_0 so that

$$\max_{p(e) \leq \alpha} E(U \mid e) < E(U \mid e_2) \text{ and} \tag{1}$$

$$E(U \mid e_0) - E(U \mid e_2) < \lambda\alpha[U(W - d(e_2)) - U(W - d(e_0))] \tag{2}$$

and define the more risk-averse utility function $V_2(x) = (1 + \lambda)U(x) - \lambda \max\{U(x), U(W - d(e_2))\}$. For $e \geq e_0$ (assuming $c(e) < d(e_2)$ for the comparison between e_2 and e to be nontrivial),

$$E(V_2 \mid e) - E(V_2 \mid e_2) = [E(U \mid e) - E(U \mid e_2)] \\ - \lambda p(e)[U(W - d(e_2)) - U(W - d(e))].$$

This is strictly negative for $p(e) \leq \alpha$ because of (1) and $d(e_2) < d(e)$. It is strictly negative for $p(e) \geq \alpha$ because of (2), $E(U \mid e) \leq E(U \mid e_0)$ and $d(e) \geq d(e_0)$. Therefore, V_2 prefers $e_2 < e_0$ to any $e \geq e_0$.

The reverse case obtains by choosing $e_1 > e_0$ such that

$$\max_{1-p(e)\leq\alpha} E(U \mid e) < E(U \mid e_1) \text{ and}$$
$$E(U \mid e_0) - E(U \mid e_1) < \lambda\alpha[U(W - c(e_0)) - U(W - c(e_1))]$$

and the utility function $V_1(x) = (1-\lambda)U(x) + \lambda \inf\{U(x), U(W - c(e_1))\}$. Then a similar argument using

$$E(V_1 \mid e) - E(V_1 \mid e_1) = [E(U \mid e) - E(U \mid e_1)] - \lambda(1 - p(e)) \times [U(W - c(e)) - U(W - c(e_1))]$$

shows that V_1 prefers e_1 to any $e \leq e_0$. □

The intuition is given graphically in Briys and Schlesinger [1990]. When $d(.)$ is increasing, a higher level of effort would lead an agent to obtain a lower wealth when he has an accident. But the marginal utility of wealth in case of an accident can be made arbitrarily higher for V than for U, so that reducing effort may be optimal.

The ambiguity is illustrated by the following example. A driver must go through a cross-road without any visibility, with the risk of a collision. Going faster through the cross-road reduces the time of exposure and therefore the probability of collision but increases the severity of the damage in case of accident. It is not clear whether a more risk-averse driver prefers to reduce or to increase the speed. This illustrative example has the same structure that the standard model of self-protection presented in Ehrlich and Becker [1972]. Indeed, there have been difficulties in providing comparative statics results on risk aversion for self-protection (see Dionne and Eeckhoudt [1985] and Briys-Schlesinger [1990]). To address this issue, we restrict attention to risk-averse agents and assume that

C1: *U is concave*, $c(e)$ *and* $d(e)$ *are increasing convex*, $d(e) > c(e)$, $p(e)$ *is decreasing convex and* $p''(e)p(e) \geq 2(p'(e))^2$.

Condition $C1$ ensures that self-protection can be desirable and also that the maximization program determining its level is concave.[3]

For the problem to be meaningful, we also assume that the optimal level of effort for U is interior (this can be ensured through appropriate Inada-like conditions):

C2: *The level of effort for U is strictly between* 0 *and* $\bar{e}$.

Under these assumptions, the optimal level of self-protection e_u for U is uniquely determined by

$$-p'(e_u) = \frac{p(e_u)U'(A)d'(e_u) + (1 - p(e_u))U'(B)c'(e_u)}{U(B) - U(A)},$$

where $A = W - d(e_u)$ and $B = W - c(e_u)$.

As the maximization programs are concave, the level of self-protection is larger for V than for U if the RHS of the first-order condition evaluated at e_u is smaller for V than for U, or, with obvious notation:

$$\left.\frac{dE(V \mid e)}{de}\right|_{e_u} \geq 0.$$

A direct computation then shows that this is the case if and only if

$$p(e_u)[V'(A)\Delta U - U'(A)\Delta V] \leq (1 - p(e_u))[U'(B)\Delta V - V'(B)\Delta U]\frac{c'(e_u)}{d'(e_u)},$$

where $\Delta V = V(B) - V(A)$, $\Delta U = U(B) - U(A)$. Now if V is more concave than U,

$$\frac{V'(B)}{U'(B)} \leq \frac{\Delta V}{\Delta U} \leq \frac{V'(A)}{U'(A)}$$

as $B > A$, so that both terms in brackets are positive. Let us define p^* by

$$\frac{p^*}{1 - p^*} = \left(\frac{U'(B)\Delta V - V'(B)\Delta U}{V'(A)\Delta U - U'(A)\Delta V}\right)\frac{c'(e_u)}{d'(e_u)},$$

where $0 < p^* < 1$.

As the function $p/(1 - p)$ is increasing and maps (0, 1) into $(0, \infty)$, we obtain:

Proposition 4: *Assume $C1$ and $C2$ hold for U and V, with V more risk-averse than U. Then self-protection is higher for V than for U if and only if $p(e_u) < p^*$.*

The intuition is clear: for a low probability of loss, the more risk-averse agent exerts more effort to self-protect, as the intuition suggests. For a high probability of loss, he is mainly interested in reducing the maximal loss, which leads to less self-protection. A similar result is obtained by Dionne et al. [1998] in an independent work, in which they restrict attention to what they call "proper risk behavior."

Note however that the threshold p^* depends on both U and V, as well as e_u. We need, of course, to show that the condition is not vacuous—that there exist cases in which $p(e_u) < p^*$ and cases in which $p(e_u) > p^*$. To prove this, we note that p^* depends on the function $p(.)$ only through e_u. Assume that the initial function $p(.)$ is replaced by a new function $q(.)$ such that

$$-q'(e_u) = \frac{q(e_u)U'(A)d'(e_u) + (1 - q(e_u))U'(B)c'(e_u)}{U(B) - U(A)}.$$

Then U still chooses the same level e_u, p^* is unchanged, and V chooses a higher level of self-protection if and only if the inequality $q(e_u) < p^*$ holds. This shows that our condition has predictive content.

This result was already shown by Dionne and Eeckhoudt [1985] for the case of quadratic utility functions when $d(.) = c(.) + l, l > 0$; then the threshold p^* equals $1/2$ and is thus independent of the value of l. More generally, assume that l is small compared to wealth. Then tedious calculations show that

$$p^* \sim \frac{1}{2} - \frac{l}{12}\frac{P_v R_v - P_u R_u}{R_v - R_u},$$

where R_u and R_v denote absolute risk aversion, while P_u and P_v denote absolute prudence. We see that, whenever V is more prudent than U, p^* will be below $\frac{1}{2}$ for l close to zero.

As another example in which $d(.) = c(.) + l$, consider *CARA* utility functions with risk-aversion indices α for U and β for V. Then a direct computation shows that

$$\frac{p^*}{1 - p^*} = \frac{\beta - \alpha + \alpha e^{\beta l} - \beta e^{\alpha l}}{(\beta - \alpha)e^{\alpha l}e^{\beta l} - \beta e^{\beta l} + \alpha e^{\alpha l}},$$

which decreases from 1 to 0 as l increases from 0 to infinity. Therefore, p^* decreases with l from $1/2$ to 0. Note that e_u is always increasing with respect to l, so that $p(e_u)$ is also decreasing, and the comparison with p^* is ambiguous.

To conclude this section, let us consider again our model in the case when in addition to the risk of accident, the individual faces a background risk on wealth, independent from the occurrence of an accident. Then his expected utility is

$$p(e)EU(\tilde{w} - d(e)) + (1 - p(e))EU(\tilde{w} - c(e)).$$

Defines the new utility functions $\hat{U}(x) = EU(\tilde{w} + z)$ and $\hat{V}(z) = EV(\tilde{w} + z)$. Then the comparative static exercise reduces to the above problem for utility functions $\hat{U}$ and $\hat{V}$. Pratt [1988] shows that provided that either U or V has a decreasing absolute risk aversion, $\hat{V}$ is more risk averse than $\hat{U}$ whenever V is more risk averse than U. It follows that all the preceding results extends to the case of background risk and DARA utility functions.

4. The general case

Now consider the general case in which the final wealth w is a random variable with compact support. The wealth distribution is characterized by a cdf $F(w, e)$ that depends on effort e. The single-crossing condition ensures that a more risk-averse agent chooses a higher level of effort. To apply it to our framework, we define $l = W - c(e) - w$, so that l is a random loss with cdf $G(l, e)$ while $c(e)$ is interpreted as the cost of effort (it is arbitrary, introduced for clarity and needs not even be increasing). This framework clearly generalizes the two-states of nature model studied in the previous sections. A general result obtains as follows:

Proposition 5: *Assume $l = \phi(\varepsilon, e)$, where ε is a random variable whose distribution is independent of e, $\phi_\varepsilon > 0$ and $\phi_{\varepsilon e} < 0$. Then a more risk-averse agent chooses a higher level of effort.*

Proof. Under our assumptions, we can write $w = \psi(e, \varepsilon)$ with $\psi_\varepsilon < 0$ and $\psi_{e\varepsilon} > 0$. Let $H(\varepsilon)$ be the cdf of ε. For $e_2 > e_1$, we have

$$\Delta(w) \equiv F(w, e_1) - F(w, e_2) = H(\varepsilon_2) - H(\varepsilon_1),$$

where $w = \psi(e_1, \varepsilon_1) = \psi(e_2, \varepsilon_2)$. Since $\psi_{e\varepsilon} > 0$, ε_1 must decrease more slowly than ε_2 when w increases. As H is nondecreasing, Δ can thus change sign only once, from positive to negative. Therefore, the single-crossing condition applies. □

The condition $\phi_{\varepsilon e} < 0$ means that effort must reduce the loss in bad states of nature (high ε), at the cost of increasing it in good states of nature.[4] This is exactly the property captured by the definition for self-insurance at the end of Section 2. One simple and fairly usual example is provided by the case where the loss is distributed around its mean with an additive noise: $l = \mu(e) + \sigma(e)\varepsilon$. Then e increases with risk aversion if the variance decreases with e (note that the expectation $\mu(e)$ does not play any role).

This example can be applied to coinsurance: interpret $\sigma(e)$ as the uninsured share of the risk, $\mu(e)$ as the associated premium, and e as the agent's decision. Then more risk-averse agents choose to insure a higher share of the risk. Similarly, it is easily shown that in the (nondifferentiable) case in which $l = \mu(e) + \min\{\varepsilon, l(e)\}$ with $l'(e) < 0$, effort also increases with risk-aversion. This case is interesting in that self-protection reduces the maximal loss supported by the agent; thus $l(e)$ can be interpreted as the deductible in an insurance contract, and e just defines the choice of deductible: we therefore see that more risk-averse agents should choose a lower deductible. *The interesting part of these two results is that they show that the fact that more risk-averse agents buy more insurance doesn't depend on the relationship between the coverage and the premium but only on the nature of the coverage*. The design of a financial portfolio, or the choice of a production level under risk, also offer numerous examples for which Proposition 5 allows for direct conclusions.

In the case of a smooth distribution, we obtain

Proposition 6: *If the distribution of losses is characterized by a positive density $g(l, e)$ on a compact interval, self-protection increases with risk aversion if $\frac{\partial}{\partial l}(\frac{G_e}{g}) > 0$.*

Proof. Choosing ε to be uniform on [0, 1], then $G(\phi(\varepsilon, e), e) = \varepsilon$ so that $\phi_e = -\frac{G_e}{g}$, which gives the condition. □

As $g \equiv G_l$, the ratio $-\frac{G_e}{g}$ can be interpreted as the marginal rate of substitution between loss and self-protection at a constant level of the cumulative distribution of losses. In the space (l, e), the iso-cumulative curves must then be convex. This expresses a property of decreasing returns to effort. Indeed assume that the agent is ready to bear a given probability P that the loss is higher than a given level l. This defines a corresponding effort by $G(l, e(P, l)) = 1 - P$. Then our condition amounts to the concavity of $e(P, l)$ in l: if the agent is able to accept higher losses, then he can only reduce effort at a decreasing rate.

5. A simple proof of the single-crossing condition

The proof is based on the following lemma, which is of more general range of applicability (a similar result appears in Meyer [1977], Theorems 1 and 2):

Lemma 7: *Consider an agent endowed with a VNM function U, nondecreasing, facing two wealth distributions with cdfs $F_1(w)$ and $F_2(w)$. Denote $\delta(w) = F_1(w) - F_2(w)$. Then the following properties are equivalent*:
(i) *If U prefers F_2 to F_1, then so does any V more risk averse than U*;
(ii) *If $\int U'(w)\delta(w)\,dw \geq 0$, then for all $w_0 : \int^{w_0} U'(w)\delta(w)\,dw \geq 0$.*

Proof. Suppose (i) holds and $\int U'(w)\delta(w)\,dw \geq 0$. Integrating by parts, one gets $\int U(w)dF_1(w) \leq \int U(w)dF_2(w)$, so that U prefers F_2 to F_1. For a given w_0, denote $V(w) = \min(U(w), U(w_0))$. V is more risk-averse than U, so that from (i) we get $\int V'(w)\delta(w)dw \geq 0$, or equivalently $\int^{w_0} U'(w)\delta(w)\,dw \geq 0$. This shows (ii).

Reciprocally, suppose that U prefers F_2 to F_1. Then $\int U'(w)\delta(w)\,dw \geq 0$. Suppose that (ii) holds. If V is more risk-averse than U, then there exists a concave nondecreasing function k such that $V = k \circ U$, so that

$$\begin{aligned}\int V'(w)\delta(w)\,dw &= \int k'(U)U'(w)\delta(w)\,dw \\ &= k'(U(\bar{w}))\int U'(s)\delta(w)\,dw \\ &\quad - \int \left\{ k''(U)U'(w) \int^{w} U'(s)\delta(s)\,ds \right\} dw,\end{aligned}$$

where $\bar{w}$ is an upper bound of the support of the wealth for both distributions. This is nonnegative, from (ii) and the concavity of k. Therefore, V prefers F_2 to F_1, and (i) holds. □

We can now prove the sufficiency of single-crossing condition. Denote by e_u the optimal choice of effort for U, assuming it is unique. For $e < e_u$, suppose that $\delta(w) = F(w, e) - F(w, e_u)$ is nonnegative, then nonpositive. Define $u(w) = \int^{w} U'(s)\delta(s)ds$. We know that $u(-\infty) = 0$, and that u is nondecreasing, then nonincreasing, when w increases. Moreover, $u(+\infty) \geq 0$ since U prefers e_u to e. Therefore u is nonnegative everywhere. This proves (ii) in Lemma 7, which is equivalent to (i). From (i) any more risk-averse V prefers e_u to all $e < e_u$, so that his optimal choice of effort cannot be smaller than e_u.

If U has multiple optimal choices, take e_u to be the largest optimal choice for U. Then any optimal choice for V that is not an optimal choice for U is larger than e_u.[5]

The fact that the condition is a minimal condition, follows from the fact that if $\delta(w)$ changes sign from negative to positive, it is possible to find a nonnegative function $U'(w)$ such that (ii) is violated.

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Notes

1. This last condition is necessary for the existence of an interior optimal level of effort.
2. See also Athey [1997].
3. The proof of concavity is omitted: simply write the second-order condition and use the first-order condition.
4. A related but stronger condition, referred to as a simple risk-reducing deterministic transformation, appears in Dionne and Gollier [1992] and Meyer [1992], where it is used for different purposes.
5. We only assume that V is more risk averse, so that $U'(w) - V'(w)$ may cancel on some range. It is thus possible that an optimal choice for V coincides with a smaller optimal choice of U, then e_u is also optimal for V and $\delta(w)$ evaluated between the two optimal choices cancels on the range where V' differs from U'.

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