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# Dating U.S. Business Cycles with Macro Factors

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# Dating U.S. Business Cycles with Macro Factors

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## Abstract

A probit model is used to show that latent common factors estimated by principal components from a large number of macroeconomic time series have important predictive power for NBER recession dates. A pseudo out-of-sample forecasting exercise shows that predicted recession probabilities consistently rise during subsequently declared NBER recession dates. The latent variable in the factor-augmented probit model is interpreted as an index of real business conditions which can be used to assess the strength of an expansion or the depth of a recession.

*Keywords:* Business Cycle, Forecasting, Factors, Probit Model, Bayesian Methods.

*JEL Codes:* E32, E37, C01, C22, C25.

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# 1 Introduction

Is the U.S. economy in recession? This was one of the central questions in the business and policy communities during the year 2008. While the consensus among analysts was that the economy was in fact in recession, most business cycle indicators failed to signal the downturn.<sup>1</sup> This question was answered in December 2008 when the Business Cycle Dating Committee of the NBER determined that a peak in economic activity (beginning of a recession) occurred in the U.S. economy in December 2007. The year 2009 brought forth several related questions: Is the U.S. economy still in recession? How deep is the current recession? Is it a depression? What is the shape of the recession? V-, U-, L-shaped? Answering these questions in real time (or shortly after) is not an easy task since business conditions are not observable, and NBER announcements come out long after the fact.<sup>2</sup>

With these questions in the background, this paper uses a factor-augmented probit model to show that latent common factors estimated by principal components from a large number of macroeconomic time series have important predictive power for NBER recession dates. The main driving force of this result is a factor that loads heavily on measures of real output and employment, a ‘real’ factor. This result is in line with recent empirical research using factor models which has found that a few factors extracted from a large number of series can be useful in many forecasting exercises; see, e.g., Stock and Watson (2002a,b, 2006), Ludvigson and Ng (2009a,b), and Giannone et al. (2008).

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<sup>1</sup> For example, Krugman (2008) writes: “Suddenly, the economic consensus seems to be that the implosion of the housing market will indeed push the U.S. economy into a recession, and that it’s quite possible that we’re already in one”. Leamer (2008), on the other hand, concludes that: “[The recession-dating] algorithm indicates that the data through June 2008 do not yet exceed the recession threshold, and will do so only if things get much worse”.

<sup>2</sup>The NBER has taken between 6 to 20 months to announce peaks and troughs.

While recession probabilities have traditionally been generated using Markov switching models as in Hamilton (1989), Chauvet (1998), Chauvet and Hamilton (2006), and Chauvet and Piger (2008), the use of binary class models to predict NBER recession dates is not new.<sup>3</sup> For example, Estrella and Mishkin (1998), Dueker (1997), Chauvet and Potter (2002, 2005), Kauppi and Saikkonen (2008), and Katayama (2009) examine the usefulness of several economic and financial variables, e.g. the interest rate spread, as predictors of future U.S. recessions. The approach I take is closer to Chauvet and Potter (2010) who consider the performance of four monthly coincident macroeconomic variables as predictors of current (rather than future) business conditions. Instead of relying on a small number of observed variables, in this paper I consider the information contained in a large number of macroeconomic time series. In addition, this paper focuses on the out-of-sample performance of the probit models which was not analyzed in Chauvet and Potter (2010).

The main results of this paper can be summarized as follows. First, the factor-augmented probit model proposed here fits NBER recession dates significantly better than a probit model based on the four monthly coincident macroeconomic variables traditionally considered in the literature. Second, the latent variable in the probit models is interpreted as an index of business conditions which can be used to assess the strength of an expansion or the depth of a recession; see, e.g., Dueker (2005). The (standardized) latent variable from the factor-augmented probit model almost perfectly overlaps with the index of real business conditions constructed by Aruoba et al. (2009) that is regularly updated by the Federal Reserve Bank of Philadelphia. Third, a pseudo out-of-sample forecasting exercise shows that predicted recession probabilities from the factor-augmented probit model consistently rise during subsequently declared

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<sup>3</sup> A nice review of the different approaches to dating business cycle turning points is provided by Hamilton (2010).

NBER recession dates. On the other hand, the probit model based on the four monthly coincident macroeconomic variables exhibits a poor performance, generating probabilities that are low and volatile during NBER recession dates. In addition, probit models that incorporate an autoregressive term exhibit the worst out-of-sample performance, generating probabilities that are very low during NBER recession dates and yielding significantly delayed recession calls. As a result, dynamic probit models appear to offer no out-of-sample improvements over traditional probit models. In sum, among the models considered here, the factor-augmented probit model generates the sequence of class predictions that better approximates subsequently declared NBER recession dates.

This paper is organized as follows. Section 2 presents the factor-augmented probit models and discusses its estimation using Bayesian methods. Section 3 presents preliminary results using single-regressor traditional probit models. Section 4 presents in-sample estimation results and out-of-sample forecast results in the form of posterior means. An evaluation of the out-of-sample forecasts is also presented in this section. Section 5 concludes.

## **2 The Econometric Model**

This section presents the econometric framework. First, I present the factor-augmented probit models and discuss the use of principal components to estimate latent common factors from a large number of macroeconomic time series. Subsequently, I discuss the estimation of the probit models using Gibbs sampling.

## 2.1 Factor-Augmented Probit Models

Define a latent variable  $y_t^*$ , which represents the state of the economy as measured by the Business Cycle Dating Committee of the NBER, such that

$$y_t^* = \alpha + \beta' x_t + \epsilon_t, \quad (1)$$

where  $x_t$  is a vector of exogenous predictors,  $(\alpha, \beta')$  are regression coefficients, and  $\epsilon_t|x_t \sim i.i.d. N(0, 1)$ .<sup>4</sup> We do not observe  $y_t^*$  but rather  $y_t$ , which represents the observable recession indicator according to the following rule

$$y_t = \begin{cases} 1 & \text{if } y_t^* \geq 0 \\ 0 & \text{if } y_t^* < 0 \end{cases}, \quad (2)$$

where  $y_t$  is 1 if the observation corresponds to a recession and 0 otherwise. In the case of the traditional probit model, the conditional probability of recession is

$$p_t = P(y_t = 1|x_t) = P(y_t^* \geq 0|x_t) = \Phi(\alpha + \beta' x_t), \quad (3)$$

where  $\Phi(\cdot)$  is the distribution function of the standard normal.

Chauvet and Potter (2010) analyze the performance of four coincident macroeconomic variables (industrial production, sales, personal income, and employment) as predictors of  $y_t$ . Instead of relying on a small number of observable variables, I consider the information contained in a large number of macroeconomic time series. As in Stock and Watson (2002a,b, 2006) and Ludvigson and Ng (2009a,b), among others, consider the case where we observe a  $T \times N$  panel of macroeconomic data, where  $N$  is large, and possibly larger than  $T$ . I want to estimate (1), where  $x_t$  denotes the  $N \times 1$  vector of panel observations at time  $t$ . One way of dealing with the possible degrees of freedom problem is by summarizing the information in the panel using a small number

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<sup>4</sup> Note that since  $y_t^*$  is not observable, if  $\epsilon_t|x_t \sim i.i.d. N(0, \sigma^2)$  is assumed, the regression coefficients  $(\alpha, \beta')$  and  $\sigma$  are not separately identified. As a result, it is standard to normalize  $\sigma$  to 1.

of common factors. Assume  $x_{it}$ ,  $i = 1, \dots, N$ ,  $t = 1, \dots, T$ , has a factor structure of the form

$$x_{it} = \lambda_i' f_t + e_{it}, \quad (4)$$

where  $f_t$  is a  $r \times 1$  vector of latent factors,  $\lambda_i$  is a  $r \times 1$  vector of latent factor loadings, and  $e_{it}$  is the idiosyncratic error. Since  $r \ll N$ , an important dimension reduction can be obtained by considering the factor-augmented regression

$$y_t^* = \alpha + \delta' F_t + \epsilon_t, \quad (5)$$

where  $F_t \subseteq f_t$ . Note that  $F_t$  does not have to include all elements of  $f_t$ , only those that are relevant for predicting  $y_t^*$ .

Since the common factors are not observed, we must replace  $f_t$  with an estimate  $\hat{f}_t$ . Stock and Watson (2002a) show that, when  $N, T \rightarrow \infty$ ,  $f_t$  can be consistently estimated by principal components analysis. Bai and Ng (2006) provide the framework for inference in the linear factor-augmented regression model and show that estimated factors can be used instead of the true factors in this model; see, also, Stock and Watson (2002b, 2006) and Ludvigson and Ng (2009a,b). Similar results for non-linear models, including the probit model, are provided in Bai and Ng (2008). Finally, the number of latent common factors,  $r$ , to be estimated by principal components analysis can be determined using model selection criteria as in Bai and Ng (2002).

A common strategy in this literature consists in including an autoregressive term in (5) in order to capture dependence in the latent variable such that

$$y_t^* = \alpha + \delta' F_t + \theta y_{t-1}^* + \epsilon_t, \quad (6)$$

where  $|\theta| < 1$ . This model is similar to the models considered in Dueker (1999) and Chauvet and Potter (2005, 2010). As in the case of the traditional probit model, the

conditional probability of recession is given by

$$p_t = P(y_t = 1|x_t, y_{t-1}^*) = P(y_t^* \geq 0|x_t, y_{t-1}^*) = \Phi(\alpha + \delta' F_t + \theta y_{t-1}^*). \quad (7)$$

The final regression for a factor-augmented autoregressive probit model is then

$$y_t^* = \gamma' z_t + \theta y_{t-1}^* + \epsilon_t, \quad (8)$$

where  $\gamma = (\alpha, \delta)'$  and  $z_t = (1, \hat{F}_t)'$ .

## 2.2 Model Estimation

I estimate the models in two steps. First, I estimate the latent common factors by principal components analysis, as explained above, and then I estimate the probit models using the estimated factors as predictors. Maximum likelihood estimation of dynamic probit models can be quite difficult. The problem is the evaluation of the likelihood function which requires numerical evaluation of a  $T$ -variate normal distribution (see Eichengreen et al., 1985). Bayesian methods, on the other hand, can greatly simplify the problem. The approach I take consists on using data augmentation via Gibbs sampling, allowing me to treat  $y_t^*$  as observed data. This strategy turns the probit model into a standard linear regression model. The implementation of the Gibbs sampler for the traditional probit model follows Koop (2003) and is not discussed here. The implementation of the Gibbs sampler for the autoregressive probit model is similar to that of Dueker (1999) and Chauvet and Potter (2005, 2010) and is discussed in the appendix.



### 3 Data and Preliminary Results

The sample period is 1961:1 – 2010:12 and the recession indicator,  $y_t$ , is coded according to the business cycle turning points of the NBER:  $y_t$  is 1 if the observation corresponds to a recession and 0 otherwise. Common factors are estimated from a balanced panel of 102 monthly U.S. macroeconomic time series spanning the period 1960:1 – 2010:12. The data set is similar to the one used in Stock and Watson (2002b, 2006) and Ludvigson and Ng (2009a,b). The series include a wide range of macroeconomic variables in the broadly defined categories: output and income; employment, hours, and unemployment; inventories, sales, and orders; housing and consumption; international trade; prices and wages; money and credit; interest rates and interest rates spreads; stock market indicators and exchange rates. The data in  $x_t$  were transformed in order to ensure stationarity and standardized prior to estimation.<sup>5</sup>

As in Ludvigson and Ng (2009a,b), eight static common factors are estimated by principal components analysis. The first factor accounts for the largest amount of total variation in the panel, the second factor accounts for the largest variation in the panel that was not accounted for by the first factor, and so on.<sup>6</sup> Since factors that are important for explaining the total variation in the panel data  $x_{it}$  need not be relevant for modeling  $y_t$ , the first question is then which estimated factors have predictive power for  $y_t$ . To address this question, I estimate eight single-regressor traditional probit models with  $y_t^* = \alpha + \delta \hat{f}_{it} + \epsilon_t$  for  $i = 1, \dots, 8$  and  $t = 1, \dots, T$  by maximum likelihood. Note that the normalization imposed for identification purposes implies that estimated factors are mutually orthogonal. Table 1 reports parameter estimates, McFadden’s pseudo- $R^2$ , the value of the log likelihood, and the likelihood ratio (LR) test statistic

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<sup>5</sup>A complete description of the series and transformations is given in the appendix.

<sup>6</sup>“Total variation” is the sum of the variances of the variables in the panel  $x$ .

for the hypothesis that  $\delta = 0$  with its associated probability value. Although several factors appear to be significant (p-value  $< 0.1$ ), the estimated first factor not only explains most of the variation in the panel  $x$ , but also has the largest (in-sample) predictive power for  $y_t$  with pseudo- $R^2 = 0.544$ . The other significant factors exhibit very low values of pseudo- $R^2$ .

[ TABLE 1 ABOUT HERE ]

While economic interpretation of the individual factors is difficult because of identification issues, it is sometimes possible to interpret the factors by measuring on which series in the panel they load heavily. Results in Ludvigson and Ng (2009a, Figure 1) show that the first factor loads heavily on real variables such as employment, production, capacity utilization, and manufacturing orders. Figure 1 presents the estimated first factor along with the (standardized) index of capacity utilization. The series are similar, with major troughs corresponding closely to NBER recession dates (shaded areas). As concluded in Stock and Watson (2002b) and Ludvigson and Ng (2009a,b), the first factor appears to be an index of real economic activity. Figure 2 presents the probability of recession estimated from the traditional probit model using the first factor as predictor. Recession probabilities consistently rise during NBER recession dates and the model signals recessions with high probability values. The model, however, shows probabilities that are relatively volatile during recessions and exhibits several false positives during expansions.

[ FIGURE 1 ABOUT HERE ]

[ FIGURE 2 ABOUT HERE ]

## 4 Empirical Results

The results in the previous section suggest that a probit model with the first estimated factor as predictor is a good starting point. Two factor models are then considered: (1) a traditional probit model with the first factor as predictor; (2) an autoregressive probit model with the first factor as predictor. The equation to be estimated is  $y_t^* = \alpha + \delta_1 \hat{f}_{1t} + \theta y_{t-1}^* + \epsilon_t$ , with  $\theta = 0$  in the first case. I will refer to these models as factor probit (FP) and autoregressive factor probit (AFP) respectively. Additionally, I consider the predictive power of the four main monthly coincident indicators considered in Chauvet and Potter (2010) among others.<sup>7</sup> Two additional models are then considered: (3) a traditional probit model with the four coincident indicators as predictors; (4) an autoregressive probit model with the four coincident indicators as predictors. The equation to be estimated takes the form  $y_t^* = \alpha + \sum_{i=1}^4 \delta_i x_{it} + \theta y_{t-1}^* + \epsilon_t$ , with  $\theta = 0$  in case (3). I will refer to these models as coincident probit (CP) and autoregressive coincident probit (ACP) respectively.

The next section presents in-sample results where the common factors  $\hat{f}_t$  at each date  $t$  are estimated using the full sample of time series information, and where it is assumed that the entire series of NBER dates is known. To provide a more accurate evaluation of the models, section 4.2 presents out-of-sample results from a pseudo real time exercise. In this case, the factors are estimated recursively, each period using data only up to time  $t$ . Furthermore, since NBER dates are not known for some time, I assume that at time  $t$  the forecaster does not know whether the true state of the economy has changed over the last twelve months such that  $y_{t-i} = y_{t-12}$  for  $i = 0, 1, \dots, 11$ . Further details are given below.

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<sup>7</sup> These variables include: industrial production, real manufacturing sales, real personal income less transfer payments, and employment. A data set of these variables was generously provided by Jeremy Piger.

## 4.1 In-Sample Results

To estimate the probit models, the Gibbs sampler was run with 25,000 iterations. After discarding the first 5,000 draws (burn-in period), posterior means are computed using a thinning factor of 20, i.e. computed from every 20th draw. As a result, the subsequent analysis is based on the means of these 1000 draws. Table 2 (panel A) reports the posterior mean and standard deviation of the models' parameters. The factor probit models show parameter posterior distributions that are concentrated away from zero and the first factor is clearly important. Bayes factors are the main tool of Bayesian model selection but with improper priors, Bayes factors are not well defined. As a consequence, I compute standard frequentist goodness of fit statistics using the posterior means (table 2, panel B). These statistics can be directly compared with the maximum likelihood estimates reported in Table 1. The results show a pseudo- $R^2$  of 0.544 for the FP model which can be compared to 0.366 of the CP model. As a result, the first factor ( $\hat{f}_{1t}$ ) exhibits more predictive power for  $y_t$  than the four monthly coincident indicators traditionally considered in the literature. The inclusion of the autoregressive term yields large improvements in pseudo- $R^2$  in both cases. The inclusion of additional factors in the factor models, on the other hand, does not yield important improvements (results not reported).

[ TABLE 2 ABOUT HERE ]

The latent variable in the probit model can be interpreted as an index of business cycle conditions that can be used to assess the strength of an expansion or the depth of a recession. Figure 3 plots the standardized negative posterior mean latent variable from the FP model for the full sample. By construction, the index takes negative values during recessions and perfectly matches NBER dates. The index suggests that

the 2007–09 recession was almost as deep as the 1973–75, and 1980 recessions and relatively deeper than the other recessions in the sample. Similarly, Aruoba et al. (2009) propose an index of business conditions –the Aruoba-Diebold-Scotti (ADS) business conditions index– that is designed to track real business conditions at high frequency and is regularly updated by the Federal Reserve Bank of Philadelphia. The figure shows that the latent variable from the FP model almost perfectly overlaps with the ADS index, showing an important degree of correlation (0.89).<sup>8</sup>

[ FIGURE 3 ABOUT HERE ]

Figure 4 plots the posterior mean probabilities of recession estimated from the four probit models. The FP model produces probabilities that consistently rise during NBER recession dates and signals recessions with high probability. While the model shows probabilities that are relatively volatile during recessions and exhibits some false positives during expansions, the FP model fits NBER recession dates significantly better than the CP model. Comparing the estimated recession probabilities from these models with the ones from the autoregressive models can be useful to understand the effect of including the autoregressive term in the regression. The autoregressive probit models generate recession probabilities that are smooth and eliminate, for the most part, false alarms. The inclusion of additional factors in the factor probit models improves the fit by generating recession probabilities that are marginally closer to 1 during recessions and closer to 0 during expansions (results not reported).

[ FIGURE 4 ABOUT HERE ]

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<sup>8</sup> The ADS index series was taken from a spreadsheet of vintages of ADS business conditions indices available for download at the Federal Reserve Bank of Philadelphia’s website. Since the index is constructed at a daily frequency, the observation corresponding to the first day of a given month was assigned to the previous month (e.g., the value of the ADS index on 1/1/2011 was assigned to December 2010). The spreadsheet is available at:  
<http://www.philadelphiafed.org/research-and-data/real-time-center/business-conditions-index/>

## 4.2 Out-of-Sample Results

To provide a more realistic assessment of the probit models, I evaluate their predictive performance in a pseudo out-of-sample forecasting exercise. This exercise requires that we make some assumptions about what was known at each time  $t$ . First, the factors are estimated recursively, each period using data only up to time  $t$ . This requires assuming that all series in the panel were available up to time  $t$  at time  $t$ .<sup>9</sup> Second, since recent NBER dates are not known, I assume that the forecaster does not know whether the true state of the economy has changed over the last twelve months. This implies that, at time  $t$ , each model is estimated assuming that  $y_{t-i} = y_{t-12}$  for  $i = 0, 1, \dots, 11$ . As a result, the sign condition on  $y_t^*$  is not imposed on these last twelve observations when generating draws of the latent variable in the Gibbs sampler. Since end-of-sample recession probabilities for time  $t$  at time  $t$  ( $\hat{p}_{t,t}$ ) are generated without making use of  $y_t$ , these are in fact out-of-sample recession probabilities.

I use the hold-out sample period 1988:1 – 2010:12 to generate the end-of-sample forecasts  $\hat{p}_{t,t}$ . The models are estimated recursively, expanding the estimation window by one observation each month. At each time  $t$ , the Gibbs sampler was run 6,000 iterations and, after discarding the first 1,000 draws to allow the sampler to converge, results are computed using a thinning factor of 10. Figure 5 presents end-of-sample posterior mean probabilities of recession from the four probit models. The FP model generates recession probabilities that consistently rise during subsequently declared NBER recession dates and exhibits few false positives. On the other hand, the CP model based on the four coincident indicators shows low and volatile probabilities during recessions and also more noise during expansions. Finally, the autoregressive probit

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<sup>9</sup> This is not likely since some series are only available after a few weeks or months. Giannone et al. (2008), however, develop a formal framework for forecasting in real time using a large number of series released with different lags that could be used here.

models exhibit the worst performance, generating probabilities that are smooth but very low during NBER recession dates and yielding significantly delayed recession calls. As a result, the autoregressive probit models fail to identify the 1990 and 2001 recessions with high probabilities and only identify the 2007 recession with an important lag.

[ FIGURE 5 ABOUT HERE ]

Figure 6 presents the full paths of recession probabilities from which the end-of-sample probabilities are obtained (tentacle plot). Each probability path is estimated without making use of the last twelve values of  $y_t$ ; i.e., without imposing the sign condition on the last twelve observations. In the case of the FP and CP models, the probability paths do not exhibit much variation as more data is incorporated and, as a result, in- and out-of-sample probabilities do not differ significantly. The results for the autoregressive probit models, on the other hand, are quite different. In this case, the paths exhibit important changes as additional observations are added to the sample and this issue is particularly evident during recession dates. As a result, the sign condition plays an important role in the case of the autoregressive models and significantly affects the end-of-sample results, generating delayed recession calls.

[ FIGURE 6 ABOUT HERE ]

A formal evaluation of the end-of-sample recession probabilities requires the selection of a loss function that reflects the preferences of the forecaster. In the case of recession indicators, the loss is greater in the case of missed signals and, hence, an asymmetric loss function may be appropriate. The cost-weighted misclassification loss function assumes that the two types of misclassifications (false positives and false neg-

atives) involve differing costs while assuming that the sum of costs add to 1 (see, e.g., Buja et al., 2005). The loss function is given by

$$ML = \frac{1}{N} \sum_{t=1}^N ((1 - q)y_t(1 - \hat{y}_{t,t}) + q(1 - y_t)\hat{y}_{t,t}), \quad (9)$$

where  $N$  is the number of end-of-sample forecasts,  $\hat{y}_{t,t}$  is the predicted class,  $q$  is the cost of a false positive, and  $(1 - q)$  is the cost of a false negative. The loss is 0 if the predicted classification is perfect and takes positive values otherwise. In order to compute the loss we need to select a classification rule that translates the end-of-sample recession probabilities into class predictions. A simple rule is given by

$$\hat{y}_{t,t} = \begin{cases} 1 & \text{if } \hat{p}_{t,t} \geq c \\ 0 & \text{otherwise} \end{cases}, \quad (10)$$

for some  $c$  to be chosen by the forecaster, with  $0 < c < 1$ . The usual choice is  $c = 0.5$  (see, e.g., Chauvet and Potter, 2010). To compute the misclassification loss (9) we need to specify the relative cost of false positives and false negatives. Since the cost is greater in the case of a missed signal, I specify  $q = 1/3$  and  $(1 - q) = 2/3$ ; i.e., the cost of a false negative is twice the cost of a false positive. The choice of  $q$ , although arbitrary, is not important for the results. Table 2 (panel C) presents the misclassification loss for  $c = 0.5$ . Recession probabilities from the FP model generate the sequence of class predictions that better approximate subsequently declared NBER recession dates. Since the predictive performance of the models is different for expansion and recession periods, Table 2 (panel C) also provides the loss for these sub-periods. The FP model exhibits a much lower loss during recessions at the cost of a larger loss during expansions due to some false positives. The other models miss most recession signals and, as a result, exhibit a much larger loss during recessions.



## 5 Conclusion

This paper shows that latent common factors estimated by principal components from a large number of macroeconomic time series have important predictive power for NBER recession dates and can be used to assess current business conditions. The main driving force of the results is a factor that loads heavily on measures of real output and employment. The latent variable in the probit model is interpreted as an index of real business conditions and the index from the factor-augmented probit model is highly correlated with the index extracted by Aruoba et al. (2009). End-of-sample predicted recession probabilities consistently rise during subsequently declared NBER recession dates and the model exhibits good performance as a dating algorithm.

The model I consider can be extended in a number of ways. First, it can be extended to allow for non-linear dynamics. Expansions and recessions may be probabilistically different regimes and a Markov switching dynamic probit model (as in Dueker, 1999) may be more adequate. Second, the model can be used to evaluate the predictive power of the macro factors for future (rather than current) business conditions. In particular, it is of interest to evaluate which factors are relevant at different horizons. These extensions are topics for future research.

## A Autoregressive Probit Model Estimation

The regression equation for the factor-augmented autoregressive probit model is

$$y_t^* = \gamma' z_t + \theta y_{t-1}^* + \epsilon_t, \quad (\text{A.1})$$

where  $\gamma = (\alpha, \delta)'$  and  $z_t = (1, \hat{F}_t)'$ , and the likelihood function for the model is

$$L(y|z, \gamma, \theta, y_0) = \prod_{t=1}^T [\Phi(\gamma' z_t + \theta y_{t-1}^*)]^{y_t} [1 - \Phi(\gamma' z_t + \theta y_{t-1}^*)]^{1-y_t}. \quad (\text{A.2})$$

The implementation of the Gibbs sampler is similar to that of Dueker (1999) and Chauvet and Potter (2005, 2010). After generating initial values of the latent variable  $y_t^*$ , the sampler proceeds as follows: (i) generate draws of the latent variable  $y_t^*$  conditional on  $(\gamma', \theta)$  and the observed data; (ii) generate draws of  $\gamma'$  conditional on  $(y_t^*, \theta)$  and the observed data; (iii) generate draws of  $\theta$  conditional on  $(y_t^*, \gamma')$  and the observed data. Prior and posterior distributions are discussed next.

### A.1 Generating Draws of the Latent Variable

Initial values of the latent variable,  $y_t^{*(0)}$  for  $t = 1, \dots, T$ , are drawn from  $f(y_t^{*(0)} | y_{t-1}^{*(0)}, y_t)$  with  $y_0^{*(0)} = 0$ . Conditional on  $y_{t-1}^*$  and  $y_t$ ,  $y_t^*$  has a truncated normal distribution where  $y_t^* \geq 0$  if  $y_t = 1$  and  $y_t^* < 0$  if  $y_t = 0$ . The truncation imposes a sign condition on  $y_t^*$  based on the observed value  $y_t$ . Then, potential values of  $y_t^{*(0)}$  are drawn from  $y_t^{*(0)} \sim N(\gamma' z_t + \theta y_{t-1}^{*(0)}, 1)$ . Draws are discarded if the sign condition is not satisfied.

Obtaining subsequent draws of the latent variable  $y_t^*$  conditional on the parameters and the observed data requires the derivation of the conditional distribution  $y_t^* | y_{t-1}^*, y_{t+1}^*$ . Since the vector  $(y_{t+1}^*, y_t^*, y_{t-1}^*)$  has a joint normal distribution, the conditional distribution  $y_t^* | y_{t-1}^*, y_{t+1}^*$  is also normal. Starting with (A.1) and substituting

backwards for lagged  $y^*$ 's on the right side, the following results can be derived:

$$y_t^* = \sum_{s=0}^{t-1} \theta^s \gamma' z_{t-s} + \sum_{s=0}^{t-1} \theta^s \epsilon_{t-s},$$

$$E(y_t^*) = A_t = \sum_{s=0}^{t-1} \theta^s \gamma' z_{t-s} = \gamma' z_t + \theta A_{t-1},$$

$$\text{Var}(y_t^*) = B_t = \sum_{s=0}^{t-1} \theta^{2s} = 1 + \theta^2 B_{t-1},$$

$$\text{Cov}(y_t^*, y_{t-1}^*) = \theta B_{t-1}.$$

The joint distribution of the vector  $(y_{t+1}^*, y_t^*, y_{t-1}^*)$  is then

$$\begin{bmatrix} y_{t+1}^* \\ y_t^* \\ y_{t-1}^* \end{bmatrix} \sim N \left( \begin{bmatrix} A_{t+1} \\ A_t \\ A_{t-1} \end{bmatrix}, \begin{bmatrix} B_{t+1} & \theta B_t & \theta^2 B_{t-1} \\ & B_t & \theta B_{t-1} \\ & & B_{t-1} \end{bmatrix} \right).$$

Using standard results for the multivariate normal distribution,  $y_t^* | y_{t+1}^*, y_{t-1}^* \sim N(\tilde{\mu}_t, \tilde{\Sigma}_t)$

for  $t = 2, \dots, T-1$ , with truncation such that  $y_t^* \geq 0$  if  $y_t = 1$  and  $y_t^* < 0$  if  $y_t = 0$  and

$$\tilde{\mu}_t = A_t + \theta \begin{pmatrix} B_t \\ B_{t-1} \end{pmatrix}' \begin{pmatrix} B_{t+1} & \theta^2 B_{t-1} \\ & B_{t-1} \end{pmatrix}^{-1} \begin{pmatrix} y_{t+1}^* - A_{t+1} \\ y_{t-1}^* - A_{t-1} \end{pmatrix},$$

$$\tilde{\Sigma}_t = B_t - \theta^2 \begin{pmatrix} B_t \\ B_{t-1} \end{pmatrix}' \begin{pmatrix} B_{t+1} & \theta^2 B_{t-1} \\ & B_{t-1} \end{pmatrix}^{-1} \begin{pmatrix} B_t \\ B_{t-1} \end{pmatrix}.$$

Finally, assuming  $y_0^* = 0$ ,  $y_1^* | y_2^* \sim N(\tilde{\mu}_1, \tilde{\Sigma}_1)$ , with truncation such that  $y_1^* \geq 0$  if  $y_1 = 1$  and  $y_1^* < 0$  if  $y_1 = 0$  and

$$\tilde{\mu}_1 = A_1 + \theta B_1 B_2^{-1} (y_2^* - A_2) = A_1 + \frac{\theta}{1 + \theta^2} (y_2^* - A_2),$$

$$\tilde{\Sigma}_1 = B_1 - \theta^2 B_1 B_2^{-1} B_1 = 1 - \frac{\theta^2}{1 + \theta^2}.$$

Based on these results, subsequent draws of the latent variable,  $y_t^{*(i)}$  for  $t = 1, \dots, T$ , are taken from  $f(y_t^{*(i)} | y_{t-1}^{*(i-1)}, y_{t+1}^{*(i)}, y_t)$  for  $t = 1, \dots, T - 1$  and  $f(y_t^{*(i)} | y_{t-1}^{*(i-1)}, y_t)$  for  $t = T$  where  $i$  denotes the  $i$ th cycle of the Gibbs sampler. As in Chauvet and Potter (2005, 2010), I start drawing a value of  $y_T^*$  conditional on a value of  $y_{T-1}^*$  and  $y_T$  from  $y_T^{*(i)} \sim N(\gamma' z_T + \theta y_{T-1}^{*(i-1)}, 1)$ , with truncation such that  $y_T^{*(i)} \geq 0$  if  $y_T = 1$  and  $y_T^{*(i)} < 0$  if  $y_T = 0$ . With this value of  $y_T^*$ , I generate draws of  $y_t^*$  for  $t = 1, \dots, T - 1$  backwards using the results described above. Potential draws of  $y_t^*$  are discarded if the sign condition is not satisfied.

## A.2 Prior and Posterior for $\gamma$

Following Albert and Chib (1993) and Dueker (1999), I use a flat non-informative prior for  $\gamma$ . Initial values for  $\gamma$  in the first cycle of the Gibbs sampler are the least squares estimates from a regression on the observed variable  $y_t$  without autoregressive terms. Let  $W_t^\gamma = y_t^* - \theta y_{t-1}^*$ , then draws of  $\gamma$  are generated from the multivariate normal distribution  $\gamma | y^*, \theta, y \sim N(\hat{\gamma}, (z'z)^{-1})$  where  $\hat{\gamma} = (z'z)^{-1} z' W^\gamma$ .

## A.3 Prior and Posterior for $\theta$

Similarly, I use a flat non-informative prior for the autoregressive parameter  $\theta$ . The initial value of  $\theta$  to start the Gibbs sampler is set at 0.5. Let  $W_t^\theta = y_t^* - \gamma' z_t$  and  $W_t^y = y_{t-1}^*$ , with  $W_1^y = 0$ . Then, potential draws of  $\theta$  are generated from  $\theta | y^*, \gamma, y \sim N(\hat{\theta}, (W^{y'W^y})^{-1})$  where  $\hat{\theta} = (W^{y'W^y})^{-1} W^{y'W^\theta}$ . Draws are discarded if the stationarity condition  $|\theta| < 1$  is not satisfied.

## A.4 Recession Probabilities

Conditional recession probabilities are generated at each draw of the Gibbs sampler such that

$$p_t^{(i)} = \Phi\left(\gamma^{(i)'} z_t + \theta^{(i)} y_{t-1}^{*(i)}\right), \quad (\text{A.3})$$

where  $i$  denotes the  $i$ th cycle of the Gibbs sampler. The posterior mean probability of recession is given by

$$\hat{p}_t = \frac{1}{I} \sum_{i=1}^I p_t^{(i)}, \quad (\text{A.4})$$

where  $I$  denotes the total number of draws.

## B Data Appendix

This appendix lists the 102 time series included in the balanced panel. The table lists the short name of each series, the transformation applied, and a brief data description. All series are from FRED – St. Louis Fed –, unless the source is listed as ECON (Econmagic), GFD (Global Financial Data), or AC (author’s calculation). The transformation codes are: 1 = no transformation; 2 = first difference; 3 = second difference; 4 = logarithm; 5 = first difference of logarithms; 6 = second difference of logarithms.

	<b>Short Name</b>	<b>Trans.</b>	<b>Description</b>
1	PI	5	Personal Income (Bil. Chain 2005 \$)
2	PILT	5	Personal Income Less Transfer Payments (AC)
3	CONS	5	Real Consumption (Bil. Chain 2005 \$)
4	IP	5	Industrial Production Index - Total Index
5	IPP	5	Industrial Production Index - Products, Total (ECON)
6	IPF	5	Industrial Production Index - Final Products
7	IPCG	5	Industrial Production Index - Consumer Goods
8	IPDCG	5	Industrial Production Index - Durable Consumer Goods
9	IPNDCG	5	Industrial Production Index - Nondurable Consumer Goods
10	IPBE	5	Industrial Production Index - Business Equipment
11	IPM	5	Industrial Production Index - Materials
12	IPDM	5	Industrial Production Index - Durable Goods Materials
13	IPNDM	5	Industrial Production Index - Nondurable Goods Materials
14	IPMAN	5	Industrial Production Index - Manufacturing
15	NAPMPI	1	Napm Production Index (%)
16	MCUMFN	2	Capacity Utilization
17	CLFT	5	Civilian Labor Force: Employed, Total (Thous.,sa)
18	CLFNAI	5	Civilian Labor Force: Employed, Nonagric. Industries (Thous.,sa) (ECON)
19	U: all	2	Unemployment Rate: All Workers, 16 Years & Over (% ,sa)
20	U: duration	2	Unempl. By Duration: Average Duration In Weeks (sa)
21	U <5 wks	5	Unempl. By Duration: Persons Unempl. Less Than 5 Wks (Thous.,sa)
22	U 5-14 wks	5	Unempl. By Duration: Persons Unempl. 5 To 14 Wks (Thous.,sa)
23	U 15+ wks	5	Unempl. By Duration: Persons Unempl. 15 Wks + (Thous.,sa)
24	U 15-26 wks	5	Unempl. By Duration: Persons Unempl. 15 To 26 Wks (Thous.,sa)
25	U 27+ wks	5	Unempl. By Duration: Persons Unempl. 27 Wks + (Thous,sa)
26	UI claims	5	Average Weekly Initial Claims, Unempl. Insurance
27	Emp: total	5	Employees On Nonfarm Payrolls: Total Private
28	Emp: gds prod	5	Employees On Nonfarm Payrolls - Goods-Producing
29	Emp: mining	5	Employees On Nonfarm Payrolls - Mining
30	Emp: const	5	Employees On Nonfarm Payrolls - Construction
31	Emp: mfg	5	Employees On Nonfarm Payrolls - Manufacturing
32	Emp: dble gds	5	Employees On Nonfarm Payrolls - Durable Goods
33	Emp: nondbles	5	Employees On Nonfarm Payrolls - Nondurable Goods
34	Emp: serv	5	Employees On Nonfarm Payrolls - Service-Providing
35	Emp: TTU	5	Employees On Nonfarm Payrolls - Trade, Transportation, And Utilities
36	Emp: wholesale	5	Employees On Nonfarm Payrolls - Wholesale Trade

	<b>Short Name</b>	<b>Trans.</b>	<b>Description</b>
37	Emp: retail	5	Employees On Nonfarm Payrolls - Retail Trade
38	Emp: fin	5	Employees On Nonfarm Payrolls - Financial Activities
39	Emp: govt	5	Employees On Nonfarm Payrolls - Government
40	Avg hrs	2	Avg Weekly Hrs, Private Nonfarm Payrolls - Goods-Producing
41	Overtime	1	Avg Weekly Hrs, Private Nonfarm Payrolls - Mfg Overtime Hours
42	Avg hrs mfg	1	Average Weekly Hours, Mfg. (Hours)
43	NAPM emp	1	NAPM Employment Index (%)
44	Starts: nonfarm	4	Housing Starts: Total (Thous.,saar)
45	Starts: NE	4	Housing Starts: Northeast (Thous.U.,sa)
46	Starts: MW	4	Housing Starts: Midwest(Thous.U.,sa)
47	Starts: S	4	Housing Starts: South (Thous.U.,sa)
48	Starts: W	4	Housing Starts: West (Thous.U.,sa)
49	BP: total	4	Housing Authorized: Total New Priv Housing Units (Thous.,saar)
50	NAPM new ords	1	NAPM New Orders Index (%)
51	NAPM vend del	1	NAPM Vendor Deliveries Index (%)
52	NAPM invent	1	NAPM Inventories Index (%)
53	M1	6	Money Stock: M1 (Bil \$,sa)
54	M2	6	Money Stock: M2 (Bil \$,sa)
55	MB	6	Monetary Base, Adj For Reserve Requirement Changes (Mil \$,sa)
56	Rsrv tot	3	Depository Inst Reserves: Total, Adj For Reserve Req Chgs (Mil \$,sa)
57	Rsrv nonbor	3	Depository Inst Reserves: Nonborrowed, Adj Res Req Chgs (Mil \$,sa)
58	Cons credit	6	Consumer Credit Outstanding - Nonrevolving
59	S&P 500	5	S&P's Common Stock Price Index: Composite (1941-43=10) (GFD)
60	S&P indst	5	S&P's Common Stock Price Index: Industrials (1941-43=10) (GFD)
61	S&P div yield	5	S&P's Composite Common Stock: Dividend Yield (% per annum) (GFD)
62	S&P PE ratio	5	S&P's Composite Common Stock: Price-Earnings Ratio (%) (GFD)
63	Fed Funds	2	Interest Rate: Federal Funds (Effective) (% per annum)
64	Comm paper	2	Commercial Paper Rate
65	3-m T-bill	2	Interest Rate: U.S.Treasury Bills, Sec Mkt, 3-Mo. (% per annum)
66	6-m T-bill	2	Interest Rate: U.S.Treasury Bills, Sec Mkt, 6-Mo. (% per annum)
67	1-y T-bond	2	Interest Rate: U.S.Treasury Const Maturities, 1-Yr. (% per annum)
68	5-y T-bond	2	Interest Rate: U.S.Treasury Const Maturities, 5-Yr. (% per annum)
69	10-y T-bond	2	Interest Rate: U.S.Treasury Const Maturities, 10-Yr. (% per annum)
70	AAA bond	2	Bond Yield: Moody's AAA Corporate (% per annum) (GFD)
71	BAA bond	2	Bond Yield: Moody's BAA Corporate (% per annum) (GFD)
72	CP spread	1	Comm paper – Fed Funds (AC)

	<b>Short Name</b>	<b>Trans.</b>	<b>Description</b>
73	3-m spread	1	3-m T-bill – Fed Funds (AC)
74	6-m spread	1	6-m T-bill – Fed Funds (AC)
75	1-y spread	1	1-y T-bond – Fed Funds (AC)
76	5-y spread	1	5-y T-bond – Fed Funds (AC)
77	10-y spread	1	10-y T-bond – Fed Funds (AC)
78	AAA spread	1	AAA bond – Fed Funds (AC)
79	BAA spread	1	BAA bond – Fed Funds (AC)
80	Ex rate: index	5	Exchange Rate Index (Index No.) (GFD)
81	Ex rate: Swit	5	Foreign Exchange Rate: Switzerland (Swiss Franc per U.S.\$)
82	Ex rate: Jap	5	Foreign Exchange Rate: Japan (Yen per U.S.\$)
83	Ex rate: U.K.	5	Foreign Exchange Rate: United Kingdom (Cents per Pound)
84	Ex rate: Can	5	Foreign Exchange Rate: Canada (Canadian\$ per U.S.\$)
85	PPI: fin gds	6	Producer Price Index: Finished Goods (82=100,sa)
86	PPI: cons gds	6	Producer Price Index: Finished Consumer Goods (82=100,sa)
87	PPI: int mat	6	Producer Price Index: Intermed. Mat. Supplies & Components (82=100,sa)
88	PPI: crude mat	6	Producer Price Index: Crude Materials (82=100,sa)
89	Spot Mrk Price	6	Spot market price index: all commodities (GFD)
90	CPI-U: all	6	Cpi-U: All Items (82-84=100,sa)
91	CPI-U: app	6	Cpi-U: Apparel & Upkeep (82-84=100,sa)
92	CPI-U: transp	6	Cpi-U: Transportation (82-84=100,sa)
93	CPI-U: med	6	Cpi-U: Medical Care (82-84=100,sa)
94	CPI-U: comm	6	Cpi-U: Commodities (82-84=100,sa) (ECON)
95	CPI-U: dbles	6	Cpi-U: Durables (82-84=100,sa) (ECON)
96	CPI-U: serv	6	Cpi-U: Services (82-84=100,sa) (ECON)
97	CPI-U: ex food	6	Cpi-U: All Items Less Food (82-84=100,sa)
98	CPI-U: ex shelter	6	Cpi-U: All Items Less Shelter (82-84=100,sa) (ECON)
99	CPI-U: ex med	6	Cpi-U: All Items Less Medical Care (82-84=100,sa) (ECON)
100	PCE defl	6	PCE, Implicit Price Deflator: PCE (1987=100)
101	AHE: const	6	Avg Hourly Earnings - Construction
102	AHE: mfg	6	Avg Hourly Earnings - Manufacturing



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Table 1: Single-Factor Probit Models for  $y_t$ 

Regressor	$\hat{f}_{1t}$	$\hat{f}_{2t}$	$\hat{f}_{3t}$	$\hat{f}_{4t}$	$\hat{f}_{5t}$	$\hat{f}_{6t}$	$\hat{f}_{7t}$	$\hat{f}_{8t}$
$\alpha$	-1.674 (0.121)	-1.039 (0.063)	-1.029 (0.063)	-1.031 (0.063)	-1.055 (0.064)	-1.023 (0.062)	-1.027 (0.062)	-1.026 (0.062)
$\delta$	-1.660 (0.157)	0.187 (0.057)	0.121 (0.055)	0.139 (0.054)	0.248 (0.061)	0.024 (0.056)	-0.088 (0.062)	-0.108 (0.059)
$R^2$	0.544	0.021	0.009	0.013	0.033	0.000	0.004	0.007
$\ln \hat{L}$	-117.190	-251.679	-254.671	-253.811	-248.537	-256.980	-256.073	-255.347
$LR$	279.757	10.779	4.795	6.516	17.063	0.178	1.990	3.444
p-value	0.000	0.001	0.029	0.011	0.000	0.673	0.158	0.063

Note: Probit models with  $y_t^* = \alpha + \delta \hat{f}_{it} + \epsilon_t$  for  $i = 1, \dots, 8$  and  $t = 1, \dots, T$  are estimated by maximum likelihood. Top panel reports parameter estimates and standard errors (in parentheses).  $R^2 = 1 - \ln \hat{L} / \ln L_0$  is McFadden's pseudo- $R^2$ , where  $\ln \hat{L}$  is the value of the log likelihood function evaluated at the estimated parameter values and  $\ln L_0$  is the log likelihood computed only with a constant term.  $LR = -2(\ln \hat{L} - \ln L_0)$  is the likelihood ratio test statistic and p-value is the associated probability value.

Table 2: Probit Models for  $y_t$ 

	FP		AFP		CP		ACP	
<i>Panel A: In-sample parameter estimates</i>								
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
$\alpha$	-1.685	0.125	-0.587	0.136	-0.927	0.080	-0.279	0.081
$\delta_1$	-1.671	0.159	-0.482	0.144	-0.934	0.149	-0.253	0.216
$\delta_2$					-0.192	0.089	-0.649	0.209
$\delta_3$					-0.437	0.139	-0.142	0.334
$\delta_4$					-1.294	0.310	0.449	0.599
$\theta$			0.765	0.064			0.852	0.045
<i>Panel B: In-sample fit</i>								
$R^2$	0.544		0.842		0.366		0.886	
$\ln \hat{L}$	-117.195		-40.645		-162.859		-29.180	
BIC	0.412		0.167		0.596		0.161	
<i>Panel C: Out-of-sample fit</i>								
Hold-out sample	0.030		0.062		0.065		0.066	
Expansions	0.013		0.010		0.006		0.004	
Recessions	0.144		0.396		0.450		0.468	

Note: Panel A reports the parameters' posterior means and standard deviations from the four probit models for the full sample. Panel B reports goodness of fit statistics for the full sample.  $R^2 = 1 - \ln \hat{L} / \ln L_0$  is McFadden's pseudo- $R^2$ , where  $\ln \hat{L}$  is the value of the log likelihood function evaluated at the posterior means and  $\ln L_0$  is the log likelihood computed only with a constant term.  $BIC = -2(\ln \hat{L})/T + k(\ln T)/T$  is the traditional information criterion. Panel C reports the misclassification loss  $ML = \frac{1}{N} \sum_{t=1}^N ((1-q)y_t(1-\hat{y}_{t,t}) + q(1-y_t)\hat{y}_{t,t})$  where  $\hat{y}_{t,t} = 1(\hat{p}_{t,t} \geq c)$  with  $c = 0.5$  and  $q = 1/3$  for the hold-out sample period 1988:1 - 2010:12.

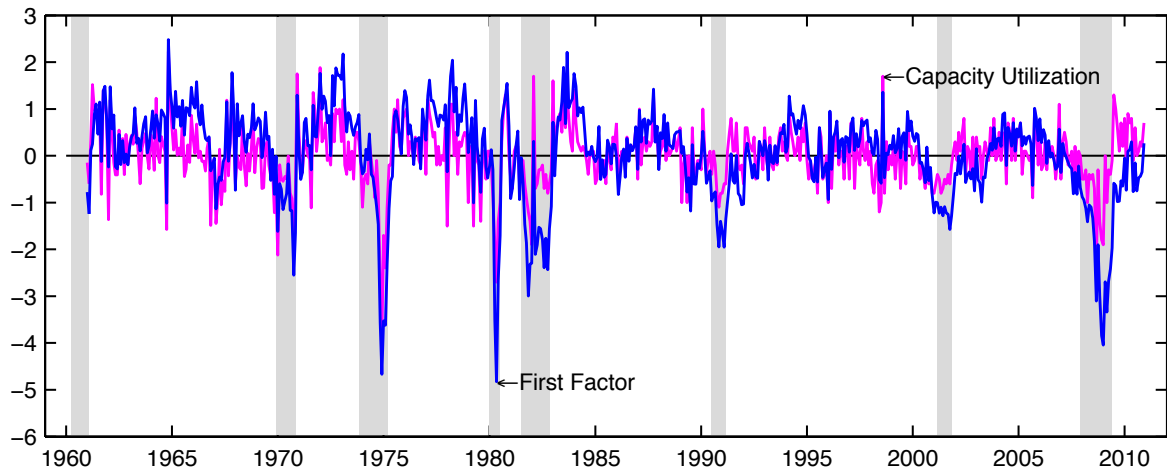


Figure 1: First Factor and Capacity Utilization. “First Factor” denotes the first estimated factor ( $\hat{f}_{1t}$ ). Standardized units are reported. Shaded areas denote NBER recession months.

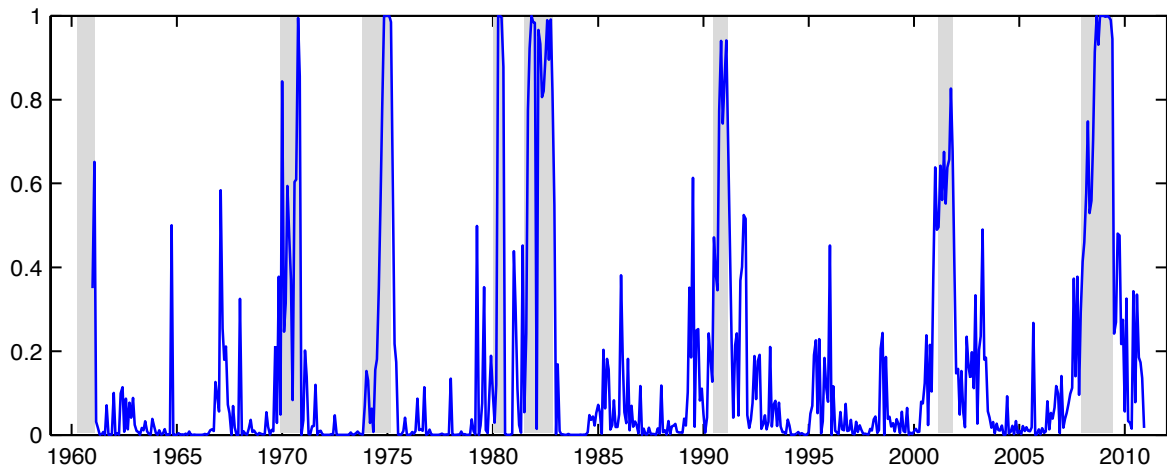


Figure 2: In-sample probabilities of recession from the single-factor probit model using the first estimated factor ( $\hat{f}_{1t}$ ) as predictor. Shaded areas denote NBER recession months.

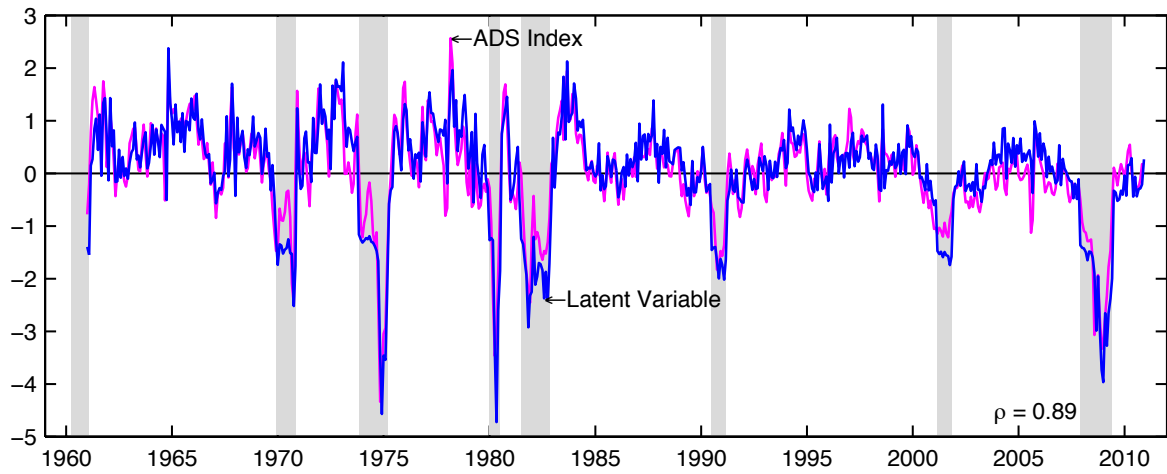


Figure 3: Standardized negative posterior mean latent variable from the FP model for the full sample and the Aruoba-Diebold-Scotti (ADS) business conditions index.  $\rho$  is the correlation between  $-y_t^*$  and the ADS index. Shaded areas denote NBER recession months.

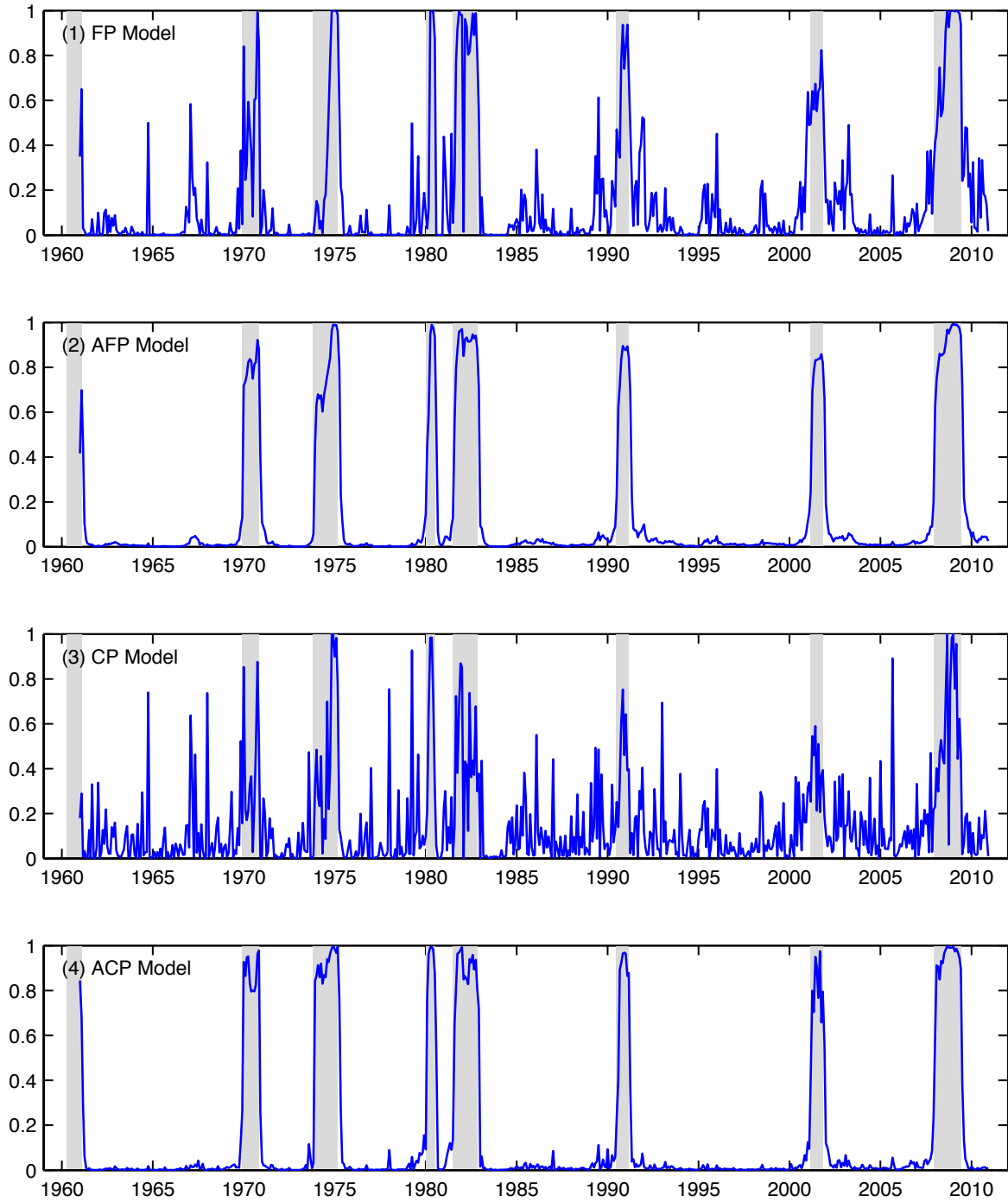


Figure 4: In-sample posterior mean probabilities of recession ( $\hat{p}_t$ ) from the four probit models. Shaded areas denote NBER recession months.



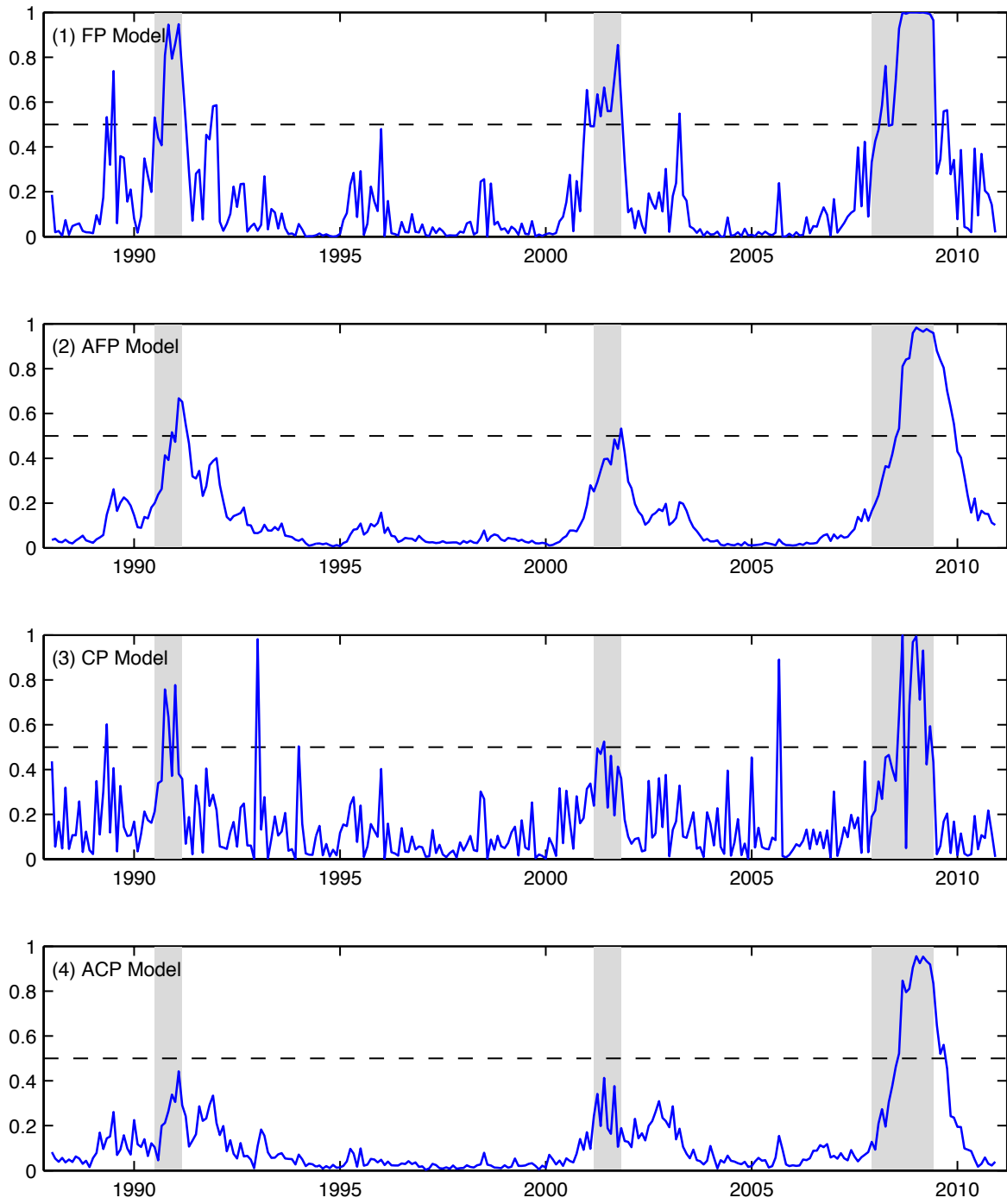


Figure 5: End-of-sample posterior mean probabilities of recession ( $\hat{p}_{t,t}$ ) from the four probit models for the hold-out sample period 1988:1 – 2010:12. Shaded areas denote NBER recession months.

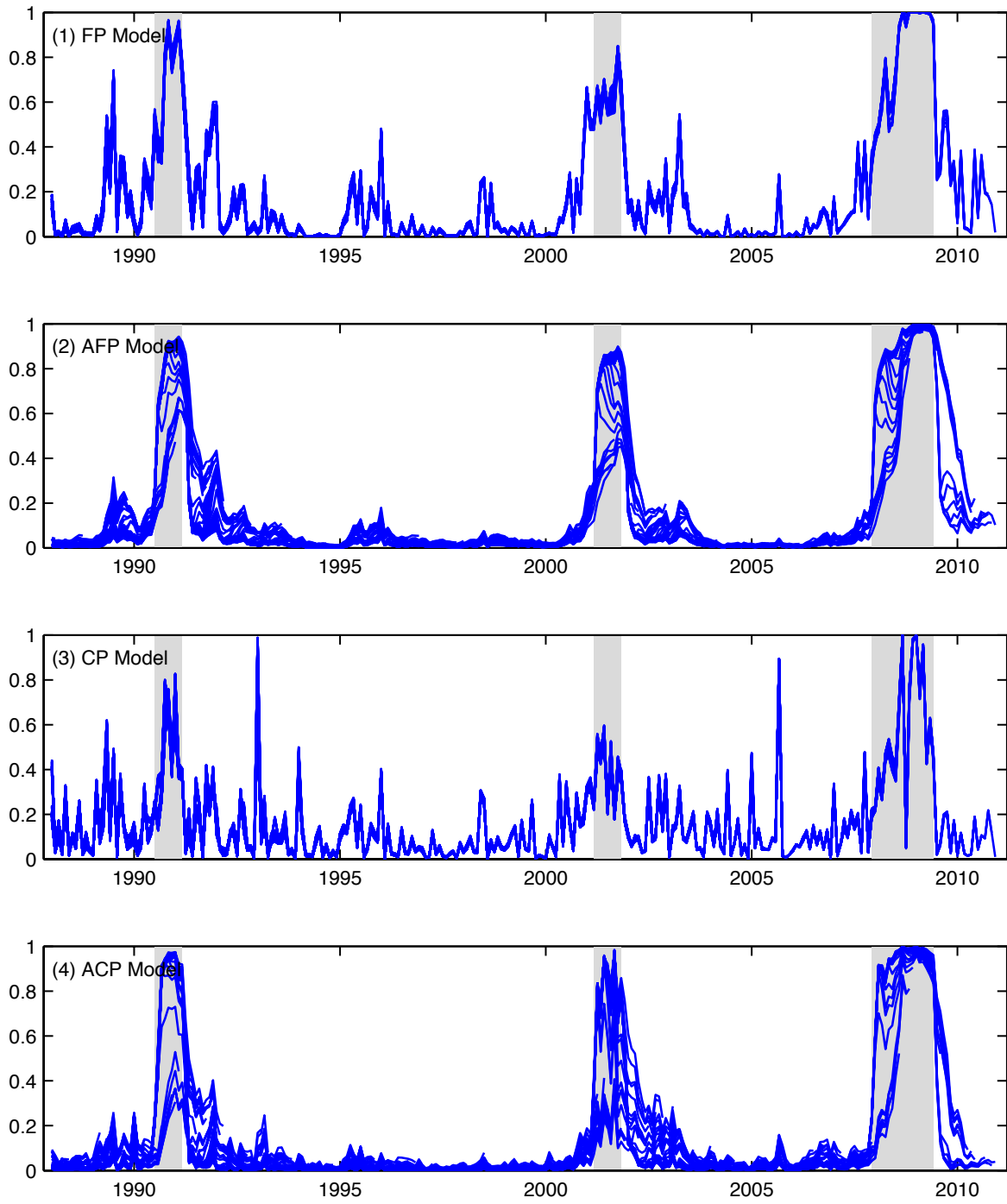


Figure 6: Posterior mean probabilities of recession (paths) from the four probit models for the hold-out sample period 1988:1 – 2010:12. Shaded areas denote NBER recession months.

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